The optimal price of money

Pedro Teles

Introduction and summary

One of the basic monetary policy issues facing the monopolist supplier of currency is what price to charge for its use. The price paid for the use of currency, by households or firms, is the foregone interest on less liquid, but riskless, assets such as short-term government bonds. Thus, the question of what price to charge for the use of currency is identified with the question of what is the optimal nominal interest rate.

According to Friedman (1969), monetary policy ought to be conducted so that the resulting nominal interest on short-term, less liquid assets is zero. The argument for the Friedman rule is very simple: Since the cost of supplying money is negligible,[1] the price charged for its use should also be very close to zero.

The first best argument of Friedman (1969) was challenged by Phelps (1973) on the basis that a positive nominal interest rate generates tax revenues for the government. According to Phelps (1973), since the alternative sources of revenue also create distortions, liquidity should be taxed like any other good. This public finance argument motivated a literature on the optimal inflation tax in a second-best environment, where the government is constrained to finance exogenous government expenditures by recourse to distortionary taxes. Somewhat surprisingly, the recent literature on the optimal inflation tax has argued that, even in a second-best environment, it is optimal not to use the inflation tax, so that the Friedman rule is still optimal. Why is this the case? Why shouldn't liquidity be taxed like any other good, as argued by Phelps (1973)?

In this article, I review some of the results obtained in the literature on the optimality of the Friedman rule. I base the analysis on Correia and Teles (1996, 1999) and De Fiore and Teles (2003), which have built on work by Kimbrough (1986), Guidotti and Végh (1993), and Chari, Christiano, and Kehoe (1996), among others.

I start by analyzing a simple environment where liquidity services are modeled as a final good, so that agents gain utility from consumption, leisure, and real balances, measured by the stock of money deflated by the price level. This is the context in which the argument of Phelps (1973) was made. According to Phelps, an application of the Ramsey (1927) principles of taxation of final goods, would mean that tax distortions should be distributed across goods, including liquidity services. Since the public finance principles, such as Ramsey (1927), were applied to costly goods, I allow for the possibility that money is costly to supply. I assume that the utility function satisfies the conditions for uniform taxation of final goods, established by Atkinson and Stiglitz (1972). In that case it is optimal to tax money, at the same proportionate rate as the consumption good. Thus, the price charged for the use of money, the nominal interest rate, *int*, should be equal to the cost of producing real balances, *c*, marked up by the optimal common tax rate, τ^*, on real balances and consumption,

$$int^* = c(1 + \tau^*).$$

As the cost of producing money, *c*, is reduced, so is the optimal price charged for the use of money, int^*. When the cost is zero, $c = 0$, the optimal nominal interest rate is also zero,

$$int^* = 0.$$

Thus, even if the optimal proportionate tax on money is positive and relatively high, because the production costs of money are very small, the optimal price charged for money and therefore the implicit unit tax

Pedro Teles is a senior economist at the Federal Reserve Bank of Chicago.

may also be very small. In this case, it is clear that the reason for the optimality of the Friedman rule is the fact that money is costless.

Since real balances measure the purchasing power of money, it is more appropriate to use, as its measure, the stock of money deflated by the price level gross of consumption taxes, rather than net of these taxes. The reason for this is that the consumption taxes are typically paid using the same means of payment as that used to purchase the consumption goods. A small modification of the model described above considers this measure of real balances. The fact that money balances are deflated by the price level gross of taxes implies that the price paid for the use of real balances is now the nominal interest rate marked up by the consumption tax. Under the conditions for uniform taxation, in order to guarantee that real money is taxed at the same rate as consumption goods, the nominal interest rate ought to be equal to the production cost of real balances c,

$$int^* = c.$$

If the production cost c is negligible, then the nominal interest rate should be zero. Thus, also in this case, money is optimally taxed at a positive proportionate rate. However, the total price charged for money when the cost of producing money is zero is still zero,

$$int^*(1 + \tau^*) = 0.$$

Again in this case, the Friedman rule is optimal because of the assumption of a negligible production cost of money.

In the examples just described, liquidity was treated as a final good like any other consumption good. In reality, liquidity is valued because it reduces transaction costs. Modeling money as an input in the production of transactions, rather than as an argument in the utility function, has implications for the optimal inflation tax when money is costly to produce. Under the assumption that money is costly, if the transactions technology is constant returns to scale, real balances should not be taxed.[2] Thus, the optimal tax rate on real balances is

$$\tau^* = 0.$$

This is in the spirit of Diamond and Mirrlees' (1971) taxation rules, whereby it is not optimal to tax intermediate goods when the technology is homogenous of degree one. If instead, the degree of homogeneity of the transactions technology is different from one, as in the case of the transactions technology proposed by Baumol (1952) and Tobin (1956), then it is optimal to set a non-zero tax on the use of real balances,

$$\tau^* \lesseqgtr 0.$$

The optimal proportionate tax or subsidy does not approach zero as the cost of producing money becomes arbitrarily small. However, in the limit, when $c = 0$, the price of using money, that is, the nominal interest rate marked up by the consumption tax, $int^*(1 + \tau^c)$, is zero,

$$int^*(1 + \tau^c) = c\,(1 + \tau^*) = 0.$$

The Friedman rule is optimal. Thus, in this environment as well, it is the costless nature of money that justifies not taxing real balances. I review these results based on Correia and Teles (1996, 1999).

The analysis in this article compares, in welfare terms, consumption taxes to the inflation tax and leaves out income taxes. The reason for this is that, under reasonable assumptions on the transactions technology, consumption and income taxes are equivalent tax instruments, and so the result on the optimal inflation tax is unchanged whether one or the other alternative tax is considered. That is not the case when one uses the standard specification of the transactions technology, first proposed by Kimbrough (1986). Consequently, the issue of which alternative tax instrument one considers has received some attention in the literature. When the alternative tax is an income tax, the Friedman rule is optimal, while when the alternative is a consumption tax, the conditions for the optimality of the Friedman rule are more restrictive. Mulligan and Sala-i-Martin (1997) used this fact to argue for the fragility of the Friedman rule. I review their claim, which is assessed in De Fiore and Teles (2003).

The policy implications from the analysis in this article should be taken with some caution, since the analysis abstracts from the role of monetary policy as stabilization policy, justified by the presence of nominal rigidities that are assumed away in the analysis. In models with those frictions, although there are simple structures where the Friedman rule is still optimal (see Correia, Nicolini, and Teles, 2001), in more complex staggered price-setting environments, the optimality of the Friedman rule is lost. Nevertheless, the optimal inflation rate is still a very low number. Another aspect of monetary policy that this analysis abstracts from is the issue of commitment. The assumption here is that the policymaker can commit to future policy. If that is not the case, the policy suggestions in this article will not be of much use.

A simple model of liquidity as a final good

The first model I consider is a simple money-in-the-utility-function model. In such models, agents use real balances because they provide utility directly. This assumption is useful in the context of the analysis in this article to assess the public finance argument, originally made by Phelps (1973), that liquidity should be taxed like any other good.

The preferences of the households depend on consumption, leisure (defined here as time not devoted to the production of the consumption good), and real balances. In a first version of the model, I define real balances as the nominal balances deflated by the price level net of consumption taxes. The goods are produced with time and, for the sake of understanding the implications of money being a costly good, there is also a time cost of real balances.[3] The government must finance exogenous expenditures with either consumption taxes or the inflation tax. A positive inflation tax is levied whenever the price charged for the use of money is higher than the cost of producing it, that is, when the nominal interest rate is higher than the time cost of producing real balances. When that cost is zero and the interest rate is also zero, the Friedman rule is followed. When the cost is positive, a modified Friedman rule, which takes into account that money is costly, sets the nominal interest rate equal to the cost of real balances.

In this model the nominal interest rate creates a distortion between real balances and leisure when it differs from the cost of producing real balances. A non-zero consumption tax creates a distortion between consumption and leisure. In this model where real balances are a final good, a direct application of the Ramsey (1927) principles of taxation would suggest that real balances ought to be taxed like any other good. Indeed, under the conditions on preferences established by Atkinson and Stiglitz (1972), the two goods, consumption and money, should be taxed at the same proportionate rate. Therefore, under those conditions, the nominal interest rate should be equal to the production cost of money marked up by the proportionate tax levied on the consumption goods. This means that even for a very small cost of producing money, the modified Friedman rule is not exactly optimal. It is approximately optimal, though.

The Friedman rule is optimal in the limit case where the cost of supplying money is exactly zero. As the cost of producing money approaches zero, the consumption tax converges to a finite and strictly positive number, and thus the optimal price charged for the use of money converges to the production cost, that is, zero. In this case, it is clear that the optimality of the Friedman rule hinges on the assumption that money is costless. The formal analysis of this problem is described in box 1.

Money is deflated by the price level gross of consumption taxes

Above, I assumed that liquidity services were represented, as a final good, by the stock of nominal money deflated by the price level net of taxes. However, if consumption taxes are paid with money, the liquidity services of money are more appropriately described by the stock of money deflated by the price level gross of consumption taxes. What are the implications of considering this measure of real balances?

If liquidity services are measured by money deflated by the price level gross of consumption taxes, money is implicitly taxed at the same rate as consumption, and so the cost of using money is no longer the nominal interest rate, but rather the interest rate marked up by the consumption tax. The relative price of real balances in units of time is $i_t(1 + \tau_{ct})$, while the relative price of consumption in units of time is $(1 + \tau_{ct})$. Under the conditions for uniform taxation of Atkinson and Stiglitz (1972), the optimal nominal interest rate is equal to the cost of supplying real balances,[4]

$$i_t = \alpha.$$

Does this mean that the Friedman rule is optimal? Not really. In this context, a modified Friedman rule should take into account the implicit taxation of money, resulting from the need to use money to pay taxes. The modified Friedman rule is such that the total cost of using money equals the cost of supplying it,

$$i_t(1 + \tau_{ct}) = \alpha.$$

Thus, in order for the modified Friedman rule to hold, the nominal interest rate would have to include a subsidy to money at the same rate as the consumption tax that would compensate for the implicit taxation of real balances.

Under the conditions for uniform taxation, this policy is not optimal. However, as the cost of supplying money approaches zero, the two policies coincide. The optimal policy is the Friedman rule of a zero nominal interest rate. Again, in this case the Friedman rule is optimal because money has a zero cost of production. The Ramsey problem in this environment is formalized in box 2.

A monetary model with a transactions technology

The money-in-the-utility-function models analyzed in the previous sections can be interpreted as equivalent representations of models where money reduces the transactions costs that households have

BOX 1

The Ramsey problem in a money-in-the-utility-function model

In this model with money in the utility function, preferences depend on consumption c_t, real balances $\tilde{m}_t = \frac{M_t}{P_t}$, where P_t is the price level net of consumption taxes, and time not devoted to producing the consumption good that I call leisure, h_t^v,

$$1)\quad \sum_{t=0}^{\infty} \beta^t V(c_t, \frac{M_t}{P_t}, h_t^v).$$

The technology to produce consumption uses time only and is linear with a unitary coefficient.

The representative household chooses a sequence $\{c_t, h_t^v, M_t, B_t\}_{t=0}^{\infty}$, where B_t are nominal securities that pay $(1 + i_t)B_t$ units of money in period $t + 1$, that satisfies the budget constraint and maximizes utility in equation 1, given a sequence of prices, $\{P_t, i_t\}_{t=0}^{\infty}$ and initial nominal wealth $W_0 \equiv M_{-1} + (1 + i_{-1})B_{-1}$. For simplicity, I assume that the initial wealth is zero, $W_0 = 0$. The budget constraint is described by the following sequence:

$$2)\quad \begin{aligned} M_{t+1} + B_{t+1} &\le M_t - (1+\tau_{ct})P_t c_t + P_t(1-h_t^v) \\ &\quad + (1+i_t)B_t,\ t \ge 0 \\ M_0 + B_0 &\le W_0 \end{aligned}$$

together with a no-Ponzi games condition. The variable τ_{ct} is the consumption tax rate.

The government finances an exogenous sequence of government expenditures, $\{g_t\}$, by setting tax rates on the consumption good, $\{\tau_{ct}\}$, as well as the nominal interest rates, $\{i_t\}$. The resource constraints in this economy are given by

$$3)\quad c_t + g_t \le 1 - h_t^v - \alpha \tilde{m}_t,\ t \ge 0,$$

where α is the cost in units of time of supplying one unit of real money. It is a standard assumption in the literature that this cost is zero, $\alpha = 0$.

The intertemporal budget constraint for consumers can be written as

$$4)\quad \sum_{t=0}^{\infty} \frac{Q_t P_t}{Q_0 P_0}(1+\tau_{ct})c_t + \sum_{t=0}^{\infty} \frac{Q_t P_t}{Q_0 P_0} i_t \tilde{m}_t \le \sum_{t=0}^{\infty} \frac{Q_t P_t}{Q_0 P_0}(1-h_t^v),$$

where $Q_t = \frac{1}{(1+i_0)\dots(1+i_t)}$, $t \ge 0$. Maximizing equation 1 subject to equation 4, I obtain the following marginal conditions:

$$5)\quad \frac{V_c(t)}{V_{h^v}(t)} = 1 + \tau_{ct},\ t \ge 0$$

$$6)\quad \frac{V_{\tilde{m}}(t)}{V_c(t)} = \frac{i_t}{(1+\tau_{ct})},\ t \ge 0$$

$$7)\quad \frac{\beta^t V_{h^v}(t)}{V_{h^v}(0)} = \frac{Q_t P_t}{Q_0 P_0},\ t \ge 0.$$

The marginal conditions 5–7, the budget constraint, 4 satisfied with equality, and the resource constraints, 3, determine the set of feasible and implementable allocations, $\{c_t, h_t^v, \tilde{m}_t\}_{t=0}^{\infty}$, intertemporal prices $\left\{\frac{Q_t P_t}{Q_0 P_0}\right\}_{t=0}^{\infty}$, and taxes $\{\tau_{ct}, i_t\}_{t=0}^{\infty}$. This is the set of competitive equilibriums, such that the government finances exogenous government expenditures with consumption and inflation taxes. The government solves a Ramsey problem, by choosing in this set the path for the quantities, prices, and taxes that maximizes welfare, thus minimizing the excess burden of taxation.

The two intratemporal marginal conditions 5 and 6 and the resource constraint 3 determine the quantities of consumption, leisure, and real balances in each period $t \ge 0$ as functions of the taxes.[1] Once $\{c_t, h_t^v, \tilde{m}_t\}_{t=0}^{\infty}$ are determined as functions of the taxes $\{\tau_{ct}, i_t\}_{t=0}^{\infty}$, I can use condition 7 to determine the path of the intertemporal prices, $\left\{\frac{Q_t P_t}{Q_0 Q_0}\right\}_{t=0}^{\infty}$ as functions of the taxes $\{\tau_{ct}, i_t\}_{t=0}^{\infty}$. The paths of taxes must satisfy the government's budget constraint, which can be obtained from the households' budget constraint, 4 with equality, and the resource constraints. This strategy of solving the system of competitive equilibrium equations is the dual approach. Because the system is linear in the taxes and prices, a primal approach is more efficient, where the taxes and prices are expressed as functions of the quantities, and substituted in the households' budget constraint.

BOX 1 (CONTINUED)

Thus, I substitute the tax rates and prices, $\left\{\tau_{ct}, i_t, \frac{Q_t P_t}{Q_0 P_0}\right\}_{t=0}^{\infty}$ using conditions 5–7 into the budget constraint, 4 satisfied with equality, and obtain the implementability condition

8) $$\sum_{t=0}^{\infty}\beta^t[V_c(t)c_t+V_{\tilde{m}}(t)\tilde{m}_t-V_{h^v}(t)(1-h_t^v)]=0.$$

The Ramsey problem will then be simplified to consist of the choice of the path of quantities, $\{c_t, h_t^v, \tilde{m}_t\}_{t=0}^{\infty}$, that satisfies the implementability condition 8 and the resource constraint 3 and maximizes welfare. The taxes and prices that decentralize the optimal solution can then be obtained from equations 5–7. The following are first order conditions:

9) $$V_c(t)+\psi[V_c+V_{cc}(t)c_t-V_{h^v c}(t)(1-h_t) +V_{\tilde{m}c}(t)\tilde{m}_t]=\lambda_t,\ t\geq 0$$

10) $$V_{h^v}(t)+\psi[V_{h^v}(t)+V_{ch^v}(t)c_t-V_{h^v h^v}(t)(1-h_t) +V_{\tilde{m}h^v}(t)\tilde{m}_t]=\lambda_t,\ t\geq 0$$

11) $$V_{\tilde{m}}(t)+\psi[V_{\tilde{m}}(t)+V_{c\tilde{m}}(t)c_t-V_{h^v\tilde{m}}t)(1-h_t) +V_{\tilde{m}\tilde{m}}(t)\tilde{m}_t]=\alpha\lambda_t,\ t\geq 0,$$

where ψ and $\beta^t\lambda_t$ are the multipliers of the implementability constraint and the time t resource constraint, respectively.

Suppose the utility function is additively separable in leisure and homogeneous in consumption and real balances,[2] so that it can be written as

12) $$V(c,\tilde{m},h)=u(c,\tilde{m})+v(h^v),$$

where u is homogeneous of degree k. Then the first order conditions of the Ramsey problem in equations 9 and 10 become

13) $$V_c(t)[1+\psi(1+k)] = \lambda_t$$

14) $$V_{\tilde{m}}(t)[1+\psi(1+k)]=\alpha\lambda_t.$$

From these, I obtain $\frac{V_{\tilde{m}}(t)}{V_c(t)}=\alpha$. Thus, the optimal policy will not distort the marginal choice between consumption and real balances.[3] The way to decentralize this solution is to set the same proportionate tax on consumption and money. Since, from equation 6, the relative price of $\tilde{m}$ in units of consumption is $\frac{i_t}{(1+\tau_{ct})}$, the optimal interest rate is $i_t = (1+\tau_{ct})\alpha$, so that it imposes a tax on real balances, at the same rate as the consumption tax.

In this context, where there is a cost of producing real balances, it makes sense to consider a modified Friedman rule that takes into account the production cost of money and corresponds to a zero tax on money. According to that modified rule, the nominal interest rate should equal the cost of producing real balances, $i_t = \alpha$. The modified Friedman rule is not optimal. This is true for any $\alpha > 0$ since the tax rate on consumption is bounded away from zero.

Consider a sequence of problems where the cost of supplying real balances, α, approaches zero. Since the optimal tax rate on consumption is bounded above, as the cost of supplying real balances becomes arbitrarily low, the optimal interest rate approaches zero. Thus, in the limit, the Friedman rule is optimal. In this case, it is clear that the reason the Friedman rule is optimal is the standard assumption of a zero cost of producing real balances.

[1]If the taxes and the government expenditures are constant over time, $\tau_{ct} = \tau_c$, $i_t = i$, and $g_t = g$, then the allocation will be stationary. This steady state will be characterized by $\frac{1}{\beta}=(1+i)\frac{P_t}{P_{t+1}}, t\geq 0$, so that inflation will be constant as well. In this stationary economy, inflation will be equal to the growth rate of money supply. In order for the nominal interest rate to be equal to zero, so that the Friedman rule is followed, it must be the case that the growth rate of money supply is negative and equal to $\beta - 1$.

[2]The conditions for uniform taxation of Atkinson and Stiglitz (1972) are separability of leisure and homotheticity in the consumption goods.

[3]In one specification of $u(c,\tilde{m})=\frac{c^{1-\sigma}}{1-\sigma}+B\frac{\tilde{m}^{1-\sigma}}{1-\sigma}$, homogeneity corresponds to equal elasticity. In this case, where the goods have the same price elasticity, the tax rates ought to be the same.

BOX 2

The Ramsey problem with real balances measured by money deflated by the price level gross of taxes

The utility function is

$$15)\quad \sum_{t=0}^{\infty}\beta^t V\left(c_t,\frac{M_t}{(1+\tau_{ct})P_t},h_t^v\right)$$

and the resource constraints are given by

$$16)\quad c_t+g\leq 1-h_t^v-\alpha m_t,\ t\geq 0,$$

where α is the cost in units of time of supplying one unit of real money, $m_t\equiv\frac{M_t}{(1+\tau_{ct})P_t}$.

The representative household maximizes utility in equation 15, subject to the intertemporal budget constraint

$$17)\quad \sum_{t=0}^{\infty}\frac{Q_tP_t}{P_0}(1+\tau_{ct})c_t+\sum_{t=0}^{\infty}\frac{Q_tP_t}{P_0}i_t(1+\tau_{ct})m_t \leq \sum_{t=0}^{\infty}\frac{Q_tP_t}{P_0}(1-h_t^v).$$

The marginal conditions are

$$18)\quad \frac{V_c(t)}{V_{h^v}(t)}=1+\tau_{ct},\ t\geq 0$$

$$19)\quad \frac{V_m(t)}{V_c(t)}=i_t,\ t\geq 0$$

$$20)\quad \frac{\beta^t V_{h^v}(t)}{V_{h^v}(0)}=\frac{Q_tP_t}{Q_0P_0},\ t\geq 0.$$

Proceeding as before, I obtain the implementability condition

$$21)\quad \sum_{t=0}^{\infty}\beta^t[V_c(t)c_t+V_m(t)m_t-V_{h^v}(t)(1-h_t^v)]=0.$$

The two Ramsey problems are identical once $\tilde{m}_t$ is replaced for m_t. Thus, when the utility function is additively separable in leisure and homogeneous in consumption and real balances, we have

$$\frac{V_m(t)}{V_c(t)}=\alpha.$$

As before, the optimal fiscal policy will not distort the marginal choice between consumption and real balances. However, in this case, since the relative price of real balances in terms of consumption is i_t, the optimal solution will be decentralized with

$$i_t=\alpha,$$

so that the nominal interest rate does not include a tax or a subsidy on money. Money is being taxed implicitly at the same rate as consumption.

As the cost of supplying real balances becomes arbitrarily low, the optimal interest rate approaches zero. In the limit, the price charged for the use of money is zero. The Friedman rule is optimal in that limiting case.

to incur. That interpretation is the common justification for the assumption that real balances are a final good. In this section, I analyze a standard model with a transactions technology, derive the equivalent money-in-the-utility-function model, and show that the restrictions imposed by the transactions technology structure would have implications for the optimal inflation tax if money was a costly good. In the models above, the optimal policy imposed the same proportionate distortion between real balances and leisure as between consumption and leisure; however, when money is modeled as an input into a constant returns to scale transactions technology, it is no longer optimal to distort the marginal choice between real balances and leisure, so that a modified Friedman rule is optimal. The latter result is an application of the optimal taxation rules of Diamond and Mirrlees (1971), whereby intermediate goods should not be taxed when consumption taxes are available and the technology is constant returns to scale.

Since the monetary aggregate that facilitates transactions is, by assumption, the stock of money deflated by the price level gross of consumption taxes, real balances are implicitly taxed at the consumption tax rate, so that the relative price of money in terms of leisure is $i_t(1+\tau_{ct})$. The relative price of consumption is $(1+\tau_{ct})$. When the transactions technology is constant returns to scale, the optimal policy is to set the price of money equal to its cost,

$$i_t(1+\tau_{ct})=\alpha,$$

so that the optimal nominal interest rate includes a subsidy at the consumption tax rate. A modified Friedman rule, which takes into account both the cost of producing real balances and the implicit tax on real balances resulting from the need to use money to pay the consumption taxes, is optimal.

The principle that when there are consumption taxes it is not optimal to tax intermediate goods only holds if the technology is constant returns to scale, and, therefore, there are no implicit profits. Under the assumption that money is costly, if the transactions technology is not constant returns to scale, then the modified Friedman rule will no longer be optimal. In this case there are positive or negative implicit profits, introducing a trade-off between the lump-sum taxation of profits and the production distortions. In order to reduce profits, it will be optimal to either tax or subsidize money, depending on the degree of homogeneity.

The optimal policy is

$$22)\quad i_t = \frac{\alpha}{(1+\tau_{ct})}\left[\frac{1}{1+\frac{\psi U_h(t)[k-1]}{\lambda_t}}\right],$$

where ψ is the multiplier of the implementability condition, equation 33 in box 3, measuring the excess burden of taxes; λ_t is the multiplier of the resource constraint at time t; $U_h(t)$ is the marginal utility of leisure; and k is the degree of homogeneity of the transactions technology.[5] Clearly, if the transactions technology is constant returns to scale, so that $k = 1$, the modified Friedman rule is optimal. If $k > 1$, money should be subsidized, and if $k < 1$, money should be taxed. As α approaches zero, the Friedman rule is optimal for any value of the degree of homogeneity of the transactions technology. Thus, the modified Friedman rule is not generally optimal; it is optimal only for a zero cost of producing money.

Even if the two models, money in the utility function or transactions technology, give disparate results on the optimal inflation tax when money is costly, the limiting result, when the cost of producing money is zero, is the same. The Friedman rule is optimal, independent of the modeling assumption, because money is costless to produce.

The result that in models with transactions technologies it is optimal not to tax costless money was first obtained by Kimbrough (1986) and then extended by Chari, Christiano, and Kehoe (1996) and Correia and Teles (1996). The public finance exercise in these last two papers was a comparison between the income and inflation taxes. If instead the option is between the inflation tax and a consumption tax, other issues arise concerning the specification of the transactions technology. I discuss these issues, addressed by De Fiore and Teles (2003), in the next section.

The Ramsey problem in this section is formalized and solved in box 3.

How do consumption taxes affect the transactions technology?

In the previous section, I showed that, when the cost of producing money is zero, the Friedman rule is optimal for transactions technologies of any degree. This is the same result that Correia and Teles (1996) obtained in comparing the inflation tax with an income tax. In the set-up described above,[6] the consumption and income taxes are equivalent fiscal instruments. This means that the allocations that can be implemented are the same with any of the two taxes, so that the optimal inflation tax does not depend on which tax is considered. This result is in contrast with recent literature, in particular Mulligan and Sala-i-Martin (1997), who argue that the optimality of the Friedman rule is a fragile result because it depends on the alternative tax instrument. In this section, I clarify this point, based on De Fiore and Teles (2003).

Under the standard specification of the transactions technology, as originally proposed by Kimbrough (1986) and later used by Guidotti and Végh (1993) and Mulligan and Sala-i-Martin (1997), among others, the consumption and income taxes are, indeed, not equivalent fiscal instruments. Moreover, when the alternative tax instrument is the consumption tax, the conditions for the Friedman rule to be optimal are more restrictive. The reason for these contrasting results is that the standard specification of the transactions technology does not impose that money be unit elastic with respect to the price level gross of consumption taxes, as I assumed in equation 24 in box 3,

$$s_t = l\left(c_t, \frac{M_t}{(1+\tau_{ct})P_t}\right).$$

The transactions technology specified by Kimbrough (1986) is the following:

$$s_t = l\left(c_t(1+\tau_{ct}), \frac{M_t}{P_t}\right).$$

If the function l is homogeneous, then it can be written as

BOX 3

The Ramsey problem in a transactions technology model

In a monetary model with a transactions technology, the preferences of the representative household depend only on consumption and leisure, where leisure does not include the time used for transactions. They are given by

$$23)\quad \sum_{t=0}^{\infty}\beta^t U(c_t, h_t),$$

where U is an increasing concave function, c_t are consumption goods, and h_t is leisure at time t. The households supply labor $1 - h_t - s_t$, where s_t is time spent in transactions.

Transactions are costly since they require time that could otherwise be used for production. The amount of time devoted to transactions increases with consumption, c_t, and decreases with real money balances,

$m_t = \dfrac{M}{(1+\tau_{ct})P_t}$, where P_t is the price of the good before taxes, according to the following transactions technology:

$$24)\quad s_t \geq l\left(c_t, \frac{M_t}{(1+\tau_{ct})P_t}\right).$$

According to this transactions technology, money is unit elastic with respect to the price level gross of consumption taxes. In addition to standard assumptions to ensure that the problem is concave, it is assumed that the function l is homogeneous of degree $k \geq 0$.

The budget constraints of the households can be written as:

$$25)\quad \sum_{t=0}^{\infty}\frac{Q_t P_t}{Q_0 P_0}(1+\tau_{ct})c_t + \sum_{t=0}^{\infty}\frac{Q_t P_t}{Q_0 P_0}(1+\tau_{ct})i_t m_t \leq \sum_{t=0}^{\infty}\frac{Q_t P_t}{Q_0 P_0}(1-h_t - l(c_t, m_t)),$$

where $Q_t = \dfrac{1}{(1+i_0)\ldots(1+i_t)}$.

The resource constraints are

$$26)\quad c_t + g_t \leq 1 - h_t - l(c_t, m_t) - \alpha m_t.$$

This model can be written as an equivalent money-in-the-utility-function model by defining h_t^v as the total time used for leisure and transactions, $h_t^v \equiv h_t + s_t$.

The model can thus be written in the form presented in the last section. The preferences, the resource constraints, and the budget constraint are given by the following expressions:

$$27)\quad \sum_{t=0}^{\infty}\beta^t V(c_t h_t^v, m_t) = \sum_{t=0}^{\infty}\beta^t U(c_t h_t^v - l(c_t, m_t))$$

$$28)\quad c_t + g \leq 1 - h_t^v - \alpha m_t$$

$$29)\quad \sum_{t=0}^{\infty}\frac{Q_t P_t}{Q_0 P_0}(1+\tau_{ct})c_t + \sum_{t=0}^{\infty}\frac{Q_t P_t}{Q_0 P_0}(1+\tau_{ct})i_t m_t \leq \sum_{t=0}^{\infty}\frac{Q_t P_t}{Q_0 P_0}(1-h_t^v).$$

From equation 27, it becomes clear that the assumptions above that the utility function is separable in h_t^v and homogeneous in consumption and real balances are not easily justifiable.

The private problem is defined by the maximization of condition 27, subject to condition 29. The households' problem must satisfy the following marginal conditions:

$$30)\quad \frac{V_c(t)}{V_{h^v}(t)} = \frac{U_c(t) - U_h(t) l_c(t)}{U_h(t)} = 1 + \tau_{ct},\ t \geq 0$$

$$31)\quad \frac{V_m(t)}{V_c(t)} = -\frac{U_h(t)}{U_c(t) - U_h(t) l_c(t)} l_m(t) = i_t,\ t \geq 0$$

$$32)\quad \frac{\beta^t V_{h^v}(t)}{V_{h^v}(0)} = \frac{Q_t P_t}{Q_0 P_0},\ t \geq 0.$$

Conditions 30–32, 29 with equality, and 28, determine the set of feasible and implementable allocations, $\{c_t, h_t, m_t\}_{t=0}^{\infty}$, prices $\left\{\frac{Q_t P_t}{Q_0 P_0}\right\}_{t=0}^{\infty}$, and taxes $\{\tau_{ct}, i_t\}_{t=0}^{\infty}$.

Using the fact that $l(t)$ is homogeneous of degree k, so that $ks_t = l_c(c_t, m_t)c_t + l_m(c_t, m_t)m_t$, the implementability condition 21 in box 2 can be written as

$$33)\quad \sum_{t=0}^{\infty}\beta^t [U_c(t) - U_h(t)(1 - h_t^v + kl(t))] = 0.$$

BOX 3 (CONTINUED)

The Ramsey problem is, therefore, the choice of quantities, $\{c_t, h_t^v, m_t\}_{t=0}^{\infty}$, that maximize welfare, represented by the utility function (condition 27), and satisfy the implementability condition 33 and the resource constraints 28.

The first order conditions of this problem imply the following condition,

$$34)\quad \frac{V_m(t)}{V_{h^v}(t)} = -l_m(t) = \alpha \frac{1}{1+\frac{\psi U_h(t)[k-1]}{\lambda_t}},$$

where ψ and $\beta^t\lambda_t$ are the multipliers of the implementability condition and the resource constraints, respectively. Then, when $k = 1$, there is no distortion imposed between real money and leisure. If $k < 1$, money should be taxed, and if $k > 1$, money should be subsidized.

Since real money is implicitly taxed, because money is needed to pay taxes, the implementation of this solution requires that money be subsidized. Since the private problem has

$$35)\quad \frac{V_m(t)}{V_{h^v}(t)} = -l_m(t) = i_t(1+\tau_{ct}),\ t \geq 0,$$

when $k = 1$, the optimal policy is

$$36)\quad i_t = \frac{\alpha}{(1+\tau_{ct})}.$$

This is the modified Friedman rule, in this context where there is a cost of producing real balances and they are implicitly taxed.

Whether the modified Friedman rule is optimal or not, as α is made arbitrarily low the Friedman rule is optimal. If $k \neq 1$, there is a tax or a subsidy that is, in absolute value, bounded above and away from zero, as α becomes arbitrarily low.[1,2] So in this case too, it is the negligible cost of money that justifies the zero tax on money.

[1]As shown in Correia and Teles (1996), the term $\frac{\psi U_h(t)(k-1)}{\lambda_t}$ measures the marginal effect of real balances on the implicit profits in the production of transactions. By taxing or subsidizing real balances, the planner aims to reduce profits.

[2]If the transactions technology is Baumol–Tobin, so that $k = 0$, then it is optimal to set a positive tax on money balances.

$$37)\quad s_t = (1+\tau_{ct})^k\, l\left(c_t, \frac{M_t}{(1+\tau_{ct})P_t}\right),$$

where k is the degree of homogeneity. Notice that, under this specification and not under equation 24, for $k > 0$, it is possible to reduce time used for transactions without adjusting the real quantity of transactions measured in units of the consumption good, c_t, and without changing the real quantity of money required to buy those goods, $\frac{M_t}{(1+\tau_{ct})P_t}$. An extreme example of this is when c_t and $\frac{M_t}{(1+\tau_{ct})P_t}$ are kept constant, while setting $\tau_{ct} = -1$. As a result, transactions will be zero, $c_t(1 + \tau_{ct}) = 0$, and so time used for transactions will also be zero.

Under the standard specification of the transactions technology in equation 37, it may be optimal to use the consumption tax to reduce the volume of transactions and save on resources. In particular, when both consumption and income taxes are allowed, it is optimal, under certain conditions, to fully tax income and subsidize consumption in order to eliminate transaction costs. When this is so, the government performs the full volume of transactions on behalf of the private agents. When the taxes on income are excluded, then it may be optimal to set a positive inflation tax, so that the consumption tax may be lower, and it may be possible to save on the volume and cost of transactions.

Box 4 describes the formal solution of the Ramsey problem in this alternative environment.

Conclusion

From a Ramsey perspective on the optimum quantity of money, the optimality of the Friedman rule owes its robustness to the costless nature of money. In general, if money was a costly good, a positive price should be charged for its use, and this price in general should be distorted. It is not clear whether this distortion should involve subsidizing or taxing money, but still there should in general be one. As the cost of producing money becomes arbitrarily low, the proportionate distortion is in absolute value bounded above and away from zero. Thus, it is the costless nature of money that justifies the optimality of the Friedman rule.

BOX 4

The Ramsey problem with the standard specification of the transactions technology

The preferences in the equivalent money-in-the-utility-function model are

$$38)\quad \sum_{t=0}^{\infty}\beta^{t}V(c_t,(1+\tau_{ct}),m_t h_t^v)\equiv$$

$$\sum_{t=0}^{\infty}\beta_t U\left(c_t,h_t^v-(1+\tau_{ct})^k l(c_t,m_t)\right).$$

The conditions of the private problem are the same as described by conditions 17 and 18–20 in box 2. The implementability condition and the resource constraint are also the same, respectively, as conditions 21 and 16. It is useful to write the marginal conditions of the private problem, conditions 18 and 19, as

$$39)\quad \frac{U_c(t)-U_h(t)(1+\tau_{ct})^k l_c(t)}{U_h(t)}=1+\tau_{ct},\ t\geq 0$$

and

$$40)\quad \frac{-U_h(t)(1+\tau_{ct})^k l_m(t)}{U_c(t)-U_h(t)(1+\tau_{ct})^k l_c(t)}=i_t,\ t\geq 0.$$

From these we have,

$$41)\quad -(1+\tau_{ct})^k l_m(t)=i_t(1+\tau_{ct}),\ t\geq 0.$$

The implementability condition can be written as:

$$42)\quad \sum_{t=0}^{\infty}\beta^t\left[U_c(t)-U_h(t)\left(1-h_t^v+(1+\tau_{ct})^k kl(t)\right)\right]=0.$$

The Ramsey problem is to maximize utility in condition 38, subject to conditions 16 and 42 and to the constraint that $\tau_{ct}=\tau(c_t,h_t^v,m_t)$ is defined implicitly by condition 39. The first order conditions imply

$$43)\quad -\{\lambda_t+\psi U_h(t)[k-1]\}\left[1+\tau_{ct})^k l_m(t)+k(1+\tau_{ct})^{k-1}\frac{\partial\tau_{ct}}{\partial m_t}l(t)\right]=\alpha\lambda_t,$$

since $\frac{\partial\tau_{ct}}{\partial h_t^v}=0$. The marginal effect of real balances on the consumption tax is

$$44)\quad \frac{\partial\tau_{ct}}{\partial m_t}=\frac{-(1+\tau_{ct})^k l_{cm}(t)}{\left[1+k(1+\tau_{ct})^{k-1}l_c(t)\right]}.$$

The second term on the left-hand side of equation 43, $k(1+\tau_{ct})^{k-1}\frac{\partial\tau_{ct}}{\partial m_t}l(t)$, is the impact on transactions time of a marginal increase in real balances through the effect on expenditures. If, at the Friedman rule, those effects are not zero, then the Friedman rule will no longer be optimal. That term is zero when either $k=0$, $l(t)=0$, or $l_{cm}(t)=0$. When it is not zero, even when $k=1$, it is not optimal to set the private benefit of money equal to the production cost, so that the result of zero taxation of intermediate goods of Diamond and Mirrlees (1971) does not apply. The reason is that, in this case, the technology is directly affected by the tax instruments.

NOTES

[1]The nominal production costs of currency as a percentage of its nominal value are approximately 0.12 percent. They are relatively high for small denomination bills (2.18 percent for \$1 bills) but very low for higher denomination bills (less than 0.01 percent for \$100 bills). The cost of coins is 0.94 percent.

[2]The transactions technology uses real balances and time to produce transactions measured by consumption.

[3]I assume a constant cost per unit of real balances. One rationale for this is the assumption that the production of real balances uses bills of different real denominations in fixed proportions and with constant returns, and that there is a constant time cost of producing bills of each real denomination. The second assumption would be more easily justified if the nominal denominations were indexed to the price level at zero cost. In reality, this indexation is costly.

[4]Notice that even if the measure of real balances is different, I maintain the assumption of a constant time cost per unit of real balances.

[5]The assumption that the transactions technology is homogenous of degree k means that when real balances and consumption are multiplied by λ, time used for transactions is multiplied by λ^k.

[6]The specification for the transactions technology in that section is as in De Fiore and Teles (2003).

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