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# A Theory of Demand Shocks

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**Abstract**

This paper presents a model of business cycles driven by shocks to consumer expectations regarding aggregate productivity. Agents are hit by heterogeneous productivity shocks, they observe their own productivity and a noisy public signal regarding aggregate productivity. The public signal gives rise to "noise shocks," which have the features of aggregate demand shocks: they increase output, employment and inflation in the short run and have no effects in the long run. The dynamics of the economy following an aggregate productivity shock are also affected by the presence of imperfect information: after a positive productivity shock output adjusts gradually to its higher long-run level, and there is a temporary negative effect on inflation and employment. A quantitative analysis suggests that noise shocks can produce sizeable amounts of short-run volatility. Moreover, a test based on survey data lends support to a central prediction of the model, regarding the overreaction of average expectations following a noise shock.

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# 1 Introduction

The idea that changes in consumer expectations have a causal effect on cyclical fluctuations is widespread both in business discussions and in policy debates. Recent dynamic stochastic general equilibrium models of the business cycle include a large number of shocks (to technology, monetary policy, preferences, etc.), but typically do not include expectational shocks as independent drivers of short-run fluctuations.[1] This paper explores the idea of expectation-driven cycles, looking at a model where technology determines equilibrium output in the long run, but consumers only observe noisy signals about technology in the short run. The presence of noisy signals produces expectational errors. This paper studies the role of these expectational errors in generating volatility at business cycle frequencies.

The model is based on two basic ingredients. First, consumers take time to recognize permanent changes in aggregate fundamentals. Although they may have good information on the current state of the individual firm or sector where they operate, they only have limited information regarding the long-run determinants of aggregate activity. Second, consumers have access to public information which is relevant to estimate long-run productivity. This includes news about technological innovations, publicly released macroeconomic and sectoral statistics, financial market prices, and public statements by policy-makers. However, these signals only provide a noisy forecast of the long-run effects of technological innovations. This opens the door to "noise shocks," which induce consumers to temporarily overestimate or underestimate the productive capacity of the economy.

In this paper, I derive the model's implications for the aggregate effects of actual technology shocks and noise shocks. In particular, the theory imposes restrictions on the relative responses of output and employment following the two types of shocks and it places an upper bound on the amount of short-run volatility that noise shocks can generate, for a given level of fundamental volatility.

The analysis is based on a standard new Keynesian model where I introduce both aggregate and idiosyncratic productivity shocks. The average level of productivity in the economy follows a random walk. However, agents cannot observe average productivity directly. They can only observe the productivity level in their own sector, which has a temporary idiosyncratic component, and a noisy public signal regarding average productivity. They also observe prices

[1] Recent exceptions include Danthine, Donaldson, and Johnsen (1998), Beaudry and Portier (2004), Jaimovich and Rebelo (2006), Christiano, Motto, and Rostagno (2006). The relation with these papers is discussed below.

and quantities which provide endogenous sources of information.

In this environment, a positive technology shock leads to a gradual adjustment in output to its new long run level, and to a temporary fall in employment and inflation. On the other hand, a positive noise shock leads to a temporary increase in output, employment and inflation.

The mechanism behind these effects is essentially based on the consumers' Euler equation. Current consumption depends positively on expected future consumption and negatively on the expected real interest rate. In equilibrium, agents expect future consumption to converge to a level determined by permanent changes in technology. Due to nominal rigidities, the real interest rate responds sluggishly to shocks. Therefore, consumption is mainly driven, in the short run, by changes in expectations about permanent productivity. After a technology shock, expectations respond less than one-for-one to the change in average productivity, given that consumers only observe noisy signals about it. Demand lags behind actual productivity, leading to a temporary fall in employment and to a deflationary pressure. A noise shock, on the other hand, has the features of a pure "aggregate demand shock." As consumers temporarily overstate the economy's productive capacity, demand increases while productivity is unchanged. This generates a temporary increase in employment and inflation.

To present my argument, I consider first a simple representative agent version of the model without dispersed information, where the idiosyncratic temporary productivity shocks are replaced by an *aggregate* temporary productivity shock. This basic model can be solved analytically and provides the essential intuition for the richer model with dispersed information. However, this version of the model requires large temporary productivity shocks to prevent agents from learning long-run productivity too quickly. Dispersed information provides a more realistic way to slow down aggregate learning and generate persistence in the model. When I turn to the model with dispersed information, I resort to numerical simulations. The computation of the model poses some technical challenges, reflecting the infinite regress problem that arises when agents "forecast the forecasts of others," as in Townsend (1983). To address this problem, I develop a method of indeterminate coefficients with a truncated state space.

I then turn to a basic quantitative exercise. Namely, I compare the model predictions with those of a simple bivariate VAR on US data, similar to Gali (1999). I choose the model's parameters to match the empirical output response to an identified long-run technology shock, and look at the size of the simulated output responses to noise shocks. I then compare the size of these responses with the "non-technology" shocks identified in the data. In this way, I show that noise shocks can account for a sizeable fraction of the "non-technology" or "demand-side" volatility observed in the data.

A novel element of a business cycle model based on learning and noise is that the choice of variance parameters can have rich, non-monotonic effects on the model's dynamics. This is because variance parameters affect not only the volatility of the shocks, but also the inference problem of the agents in the model. In particular, a crucial parameter for my quantitative exercise is the variance of the noise in the public signal. When this variance is either too small or too large, noise shocks generate small amounts of short-run volatility. In the first case, public signals are very precise and the economy converges immediately to the full information equilibrium. In the second case, public signals are very imprecise and private agents tend to disregard them in their inference. Therefore, intermediate levels of noise variance are required to generate sizeable amounts of short-run volatility.

Finally, I present a simple test which lends support to a central prediction of the model. According to the model, average expectations tend to underreact following an actual technology shock and to overreact following a noise shock. The reason is that, in the first case, consumers are optimistic, but actual productivity is even better than their expectations. In this case, producers tend to lower prices, leading to a stronger output response. In the second case, consumers are also optimistic, but actual productivity has not changed. Then producers tend to increase prices, leading to a weaker output response. To test this hypothesis I look at two measures of short-run expectations regarding aggregate activity, derived from the Survey of Professional Forecasters and from the Michigan Consumer Sentiment Survey, and I look at their responses to identified technology and non-technology shocks from a bivariate VAR. In both cases, non-technology shocks tend to have a relatively larger effect on expectations than technology shocks, although the difference is significant only when using the Survey of Professional Forecasters data.

The idea that expectations and expectational errors play a relevant role in explaining business cycles goes back, at least, to Pigou (1929) and Keynes (1936). This idea has received renewed attention in a number of papers, including Danthine, Donaldson, and Johnsen (1998), Beaudry and Portier (2004, 2006a and 2006b), Jaimovich and Rebelo (2006), Christiano, Motto, and Rostagno (2006).[2] These papers tend to emphasize the distinction between shocks to current and future productivity. In this paper, instead, I tend to emphasize the

[2] Blanchard (1993) and Cochrane (1994) are two early papers that point attention to endogenous movements in consumption as a driving force behind cyclical fluctuation.

difference between fundamental and noise shocks. The only fundamental shock in my model is a permanent shock which affects both current and future productivity. In this environment, I concentrate on telling apart the effects of an actual change in productivity from those of an expectational mistake.

Recent work on estimated dynamic stochastic general equilibrium models has identified intertemporal disturbances affecting the consumers' Euler equation as important drivers of the business cycle (Primiceri, Schaumburg, and Tambalotti, 2006). These intertemporal disturbances are somehow treated as a residual, as they are attributed to shocks to intertemporal preferences. In this paper, I provide an alternative foundation for shocks to the consumers' Euler equations, as shocks coming from changes in average expectations about long-run fundamentals.

The modelling approach in this paper is related to various strands of literature. The idea that imperfect information can cause sluggish adjustment in economic variables and generate fluctuations driven by expectational errors, goes back to Phelps (1969) and Lucas (1971, 1975). More recently, Woodford (2002), Mankiw and Reis (2002), and Sims (2003), have renewed attention to imperfect information and limited information processing as sources of inertial behavior.[3] Finally, a rich literature, starting with Morris and Shin (2002), has emphasized that, in environments with imperfect information, public sources of information can cause persistent deviations of economic variables from their fundamental values.[4] This paper puts together ideas from these literatures to build a model of the cycle based on noisy learning.[5]

Finally, the paper is related to the literature on optimal monetary policy with uncertain fundamentals.[6] That literature focuses on the central bank's uncertainty regarding these fundamentals, while here I focus on the private sector's uncertainty.

The paper is organized as follows. Section 2 presents the representative agent model with common information which is used to illustrate the basic mechanism of the paper. In Section 3, I introduce the model with dispersed information. In Section 4, I present numerical simulations of the model. In Section 5, I present the test based on survey data. Section 6 concludes.

[3] See also Collard and Dellas (2004), Moscarini (2004), Hellwig (2005), Adam (2006), Bacchetta and Van Wincoop (2006), Luo (2006), Maćkowiak and Wiederholt (2006), Reis (2006), Milani (2007), Nimark (2008).

[4] See Hellwig (2002), Angeletos and Pavan (2004), Amato, Morris and Shin (2005), Bacchetta and Van Wincoop (2005), Allen, Morris and Shin (2006).

[5] Kawamoto (2004) looks at the effect of technology shocks in an environment with imperfect information. His analysis does not feature noise shocks and focuses on the gradual adjustment of output after a technology shock. He independently derives the result that, under imperfect information, technology shocks lead to a fall in employment.

[6] See Aoki (2003), Orphanides (2003), Reis (2003), Svensson and Woodford (2003, 2005), Tambalotti (2003).

## 2 A basic model

Let me begin by considering a simple representative agent model with common information, which illustrates the basic mechanism of the paper. The model is a standard new Keynesian model with monopolistic competition and price setting *à la* Calvo (1983). In this environment, I introduce temporary and permanent aggregate technology shocks and assume that agents cannot distinguish the two shocks and receive a noisy public signal regarding the permanent shock. I then analyze the economy's dynamic behavior, focusing on the effect of the "noise shock" which corresponds to the noise component in the public signal.

**Preferences and technology.** The preferences of the representative consumer are given by

$$E\sum_{t=0}^{\infty}\beta^{t}U\left(C_{t},N_{t}\right),$$

with

$$U\left(C_{t},N_{t}\right)=\log C_{t}-\frac{1}{1+\zeta}N_{t}^{1+\zeta},$$

where $N_t$ are hours worked and $C_t$ is a composite consumption good given by

$$C_{t}=\left(\int_{0}^{1}C_{j,t}^{\frac{\gamma-1}{\gamma}}dj\right)^{\frac{\gamma}{\gamma-1}},$$

$C_{j,t}$ is the consumption of good $j$ in period $t$, and $\gamma>1$ is the elasticity of substitution among goods. Each good $j\in[0,1]$ is produced by a single monopolistic firm, which has access to the linear production function

$$Y_{j,t}=A_{t}N_{j,t}. \tag{1}$$

**Uncertainty.** The only source of exogenous uncertainty is the productivity parameter $A_t$. Let $a_t=\log A_t$. From now on, a lowercase variable will denote the log of the corresponding uppercase variable. Productivity has a permanent component, $x_t$, and a temporary component, $\eta_t$,

$$a_{t}=x_{t}+\eta_{t}, \tag{2}$$

where $\eta_t$ is an i.i.d. shock, normal, with zero mean and variance $\sigma_{\eta}^{2}$, and $x_t$ is a random walk process given by

$$x_{t}=x_{t-1}+\epsilon_{t}, \tag{3}$$

where $\epsilon_t$ is i.i.d., normal, with zero mean and variance $\sigma_{\epsilon}^{2}$. Each period all agents in the economy observe current productivity $a_t$ and the noisy signal $s_t$ regarding the permanent component of the productivity process, given by

$$s_t = x_t + e_t, \tag{4}$$

where $e_t$ is i.i.d., normal, with zero mean and variance $\sigma_e^2$. The three shocks $\eta_t$, $\epsilon_t$ and $e_t$ are mutually independent.

The noise term $e_t$ in the signal $s_t$ plays two roles: it prevents the agents from perfectly identifying permanent innovations to technology *and* it generates an independent source of variation in the agents' beliefs regarding $x_t$. As I will show below, both roles are relevant in determining the economy's cyclical behavior.

**Consumers.** I consider a simple "cashless" environment where consumers have access to a nominal one-period bond which trades at the price $Q_t$. The consumer's budget constraint is

$$Q_t B_{t+1} + \int_0^1 P_{j,t} C_{j,t} dj = B_t + W_t N_t + \int_0^1 \Pi_{j,t} dj, \tag{5}$$

where $B_t$ are nominal bonds' holdings, $P_{j,t}$ is the price of good $j$, $W_t$ is the nominal wage rate, and $\Pi_{j,t}$ are the profits of firm $j$. In equilibrium consumers choose consumption, hours worked, and bond holdings, so as to maximize their expected utility subject to (5) and a standard no-Ponzi-game condition. Nominal bonds are in zero net supply, so market clearing in the bonds market requires that $B_t = 0$.

From consumers' optimization it follows that the demand for good $j$ is

$$Y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\gamma} C_t, \tag{6}$$

where $P_t$ is the price index

$$P_t = \left( \int_0^1 P_{j,t}^{1-\gamma} dj \right)^{\frac{1}{1-\gamma}}. \tag{7}$$

**Firms.** Firms are allowed to reset prices only at random time intervals. Each period, a firm is allowed to reset its price with probability $1 - \theta$ and must keep the price unchanged with probability $\theta$. Firms hire labor on a competitive labor market at the wage $W_t$, which is fully flexible.

The firm's objective is to maximize the expected present value of its profits. Since the firms are owned by the consumers, this present value is computed using the stochastic discount factor $Q_{t+\tau|t} = \beta^\tau (C_{t+\tau}/C_t)^{-1}$. Let $P_t^*$ denote the optimal price for a firm who can adjust its price at time $t$. This firm maximizes

$$E_t \sum_{\tau=0}^{\infty} \theta^\tau Q_{t+\tau|t} \left[ P_{j,t+\tau} Y_{j,t+\tau} - W_{t+\tau} N_{j,t+\tau} \right],$$

subject to $P_{j,t+\tau} = P_t^*$, the technological constraint (1), and the demand relation (6). The firm takes as given the stochastic processes for $P_t$, $C_t$, and $W_t$, and the stochastic discount factor $Q_{t+\tau|t}$.

Aggregate real output is defined as nominal output divided by the price index $P_t$,

$$Y_t \equiv \frac{\int_0^1 P_{j,t} Y_{j,t} dj}{P_t}.$$

Substituting (6) and (7) on the right-hand side, it follows that $Y_t = C_t$, so aggregate output is equal to aggregate consumption. Inflation is defined as the change in the log of the price index $P_t$, that is,

$$\pi_t \equiv p_t - p_{t-1}.$$

**Monetary policy.** To complete the description of the environment, I need to specify a monetary policy rule. The central bank sets the short-term nominal interest rate, i.e., it sets the price of the one-period nominal bond, $Q_t$. Letting $i_t = -\log Q_t$, I can describe monetary policy in terms of choosing $i_t$ each period. For simplicity, I focus on a simple rule which responds only to current inflation

$$i_t = i^* + \phi \pi_t, \tag{8}$$

where $i^* = -\log \beta$ and $\phi$ is a constant coefficient chosen by the monetary authority.

## 2.1 Equilibrium

Following standard steps, the consumers' and the firms' optimality conditions and the market clearing conditions can be log-linearized and transformed so as to obtain two stochastic difference equations which characterize the joint behavior of output and inflation in equilibrium.[7]

In particular, the consumer's Euler equation and goods market clearing give the relation[8]

$$y_t = E_t[y_{t+1}] - i_t + E_t[\pi_{t+1}]. \tag{9}$$

The firm's optimal pricing condition can be manipulated so as to obtain

$$\pi_t = \lambda (w_t - p_t - a_t) + \beta E_t[\pi_{t+1}], \tag{10}$$

where $\lambda \equiv (1-\theta)(1-\beta\theta)/\theta$ is a constant parameter. The first term on the right-hand side reflects the effect of real marginal costs, captured by $w_t - p_t - a_t$, on the desired price-target

[7] See Gali (2007), Chapter 3.

[8] From now on, throughout the paper, I will omit constant terms in linear equations, whenever confusion is not possible.

of the firms who can adjust prices. Substituting in (10) the consumer's optimality condition for labor supply, $w_t - p_t - y_t = \zeta n_t$, and the labor market clearing condition, $n_t = y_t - a_t$, one obtains

$$\pi_t = \kappa (y_t - a_t) + \beta E_t [\pi_{t+1}], \tag{11}$$

where $\kappa \equiv \lambda (1 + \zeta)$.

Equations (9) and (11), together with the monetary rule (8), can be used to derive the equilibrium dynamics of $y_t$ and $\pi_t$. Let $x_{t|t}$ denote the agents' expectation regarding $x_t$ based on their information at date $t$, that is

$$x_{t|t} \equiv E_t [x_t].$$

To characterize the equilibrium, let me begin with the following conjectures regarding the one-step-ahead forecasts of output and inflation:

$$E_t [y_{t+1}] = E_t [x_t], \tag{12}$$

$$E_t [\pi_{t+1}] = 0. \tag{13}$$

Substituting these conjectures and the monetary policy rule (8) in (9) and (11) gives

$$\begin{aligned} y_t &= x_{t|t} - \phi \pi_t, \\ \pi_t &= \kappa (y_t - a_t). \end{aligned}$$

The first equation reflects the fact that current consumption, and hence current output, depend positively on the agents' expectations regarding the permanent component of technology and negatively on current inflation, which tends to raise the nominal interest rate and, given (13), the real interest rate. The second equation shows that current inflation depends positively on the difference between current output and "natural output," which is equal to $a_t$.[9] Rearranging, I obtain

$$y_t = \frac{1}{1+\phi\kappa} x_{t|t} + \frac{\phi\kappa}{1+\phi\kappa} a_t, \tag{14}$$

$$\pi_t = \frac{\kappa}{1+\phi\kappa} \left( x_{t|t} - a_t \right). \tag{15}$$

Equation (14) shows that realized output is a weighted average of productivity and the agents' expectation of the permanent component of productivity, $x_{t|t}$. The relative weights depend

[9] Natural output is defined as the output level that arises under flexible prices. Since prices are flexible in the limit case where $\theta \to 0$ and $\kappa \to \infty$, expression (14), below, shows that indeed natural output is equal to $a_t$.

on the monetary policy rule, captured by $\phi$. I will return to this relation below. Taking expectations on both sides of (14) and (15) at time $t-1$ and using the fact that $x_t$ is a random walk, confirms the initial conjectures (12) and (13).

## 2.2 Productivity shocks and noise shocks

While expressions (14) and (15) provide a compact characterization of the equilibrium behavior of output and inflation, to fully characterize the economy's response to the underlying shocks $(\epsilon_t, e_t, \eta_t)$, I need to derive an explicit expression for $x_{t|t}$. Each period, the agents observe two noisy signals regarding the aggregate state $x_t$: current productivity $a_t$ and the current signal $s_t$. Applying standard Kalman filtering techniques the dynamics of $x_{t|t}$ are given by

$$x_{t|t} = \rho x_{t-1|t-1} + (1-\rho)\left(\delta s_t + (1-\delta) a_t\right), \tag{16}$$

where $\rho$ and $\delta$ are scalars in $(0,1)$, which depend on the variance parameters $\sigma_\epsilon^2$, $\sigma_e^2$ and $\sigma_\eta^2$.[10] In particular, the parameter $\rho$ is increasing in $\sigma_e^2$ and $\sigma_\eta^2$, given that, when these variances are larger, $s_t$ and $a_t$ are less precise signals of $x_t$ and agents take longer to adjust their expectation $x_{t|t}$ to the true value of $x_t$. The parameter $\delta$, instead, depends on the ratio $\sigma_e^2/\sigma_\eta^2$, that is, on the relative precision of the two signals. The more precise is $s_t$, relative to $a_t$, the larger the value of $\delta$.

Now it is possible to study the effect of the three underlying shocks $\epsilon_t, e_t$, and $\eta_t$, by deriving the impulse-response functions of $y_t$, $n_t$, and $\pi_t$ to these shocks. Let me begin by considering a permanent productivity shock $\epsilon_t = 1$. The response of realized productivity, $a_{t+\tau}$, $\tau$ periods after the shock, is 1 for all $\tau \geq 0$. The response of the agents' expectation $x_{t+\tau|t+\tau}$ is equal to $\sum_{k=0}^{\tau} \rho^k (1-\rho)$. To derive this expression, iterate (16) forward and notice that, after the shock, both $s_{t+\tau}$ and $a_{t+\tau}$ increase permanently. Therefore, using (14), it follows that the

[10] The expressions for $\rho$ and $\delta$ are

$$\begin{aligned} \rho &= \frac{1/\sigma_x^2}{1/\sigma_x^2 + 1/\sigma_\eta^2 + 1/\sigma_e^2} \\ \delta &= \frac{1/\sigma_e^2}{1/\sigma_e^2 + 1/\sigma_\eta^2}, \end{aligned}$$

where $\sigma_x^2 \equiv Var_{t-1}[x_t]$ is the solution to the Riccati equation

$$\sigma_x^2 = \left(\frac{1}{\sigma_x^2} + \frac{1}{\sigma_\eta^2} + \frac{1}{\sigma_e^2}\right)^{-1} + \sigma_\epsilon^2.$$

I assume that the prior of the agents at time $t=0$ is $N\left(0, \sigma_x^2\right)$ so that the agents' learning problem has already reached its steady state and the coefficients $\rho$ and $\delta$ are time invariant.

impulse-response function for output is given by

$$\frac{dy_{t+\tau}}{d\epsilon_t} = \frac{1}{1+\phi\kappa}\sum_{k=0}^{\tau}\rho^k(1-\rho) + \frac{\phi\kappa}{1+\phi\kappa} \in (0,1).$$

The output response to a permanent productivity shock is positive, grows with $\tau$, and converges to 1 as $\tau \to \infty$. Since $n_t = y_t - a_t$, it is easy to derive the response of employment and see that it is equal to

$$\frac{dn_{t+\tau}}{d\epsilon_t} = \frac{1}{1+\phi\kappa}\left(\sum_{k=0}^{\tau}\rho^k(1-\rho) - 1\right) < 0,$$

giving a temporary negative response of employment which dies out as $\tau \to \infty$. Using (15), also inflation displays a temporary negative response, with an impulse-response function given by

$$\frac{d\pi_{t+\tau}}{d\epsilon_t} = \frac{\kappa}{1+\phi\kappa}\left(\sum_{k=0}^{\tau}\rho^k(1-\rho) - 1\right) < 0.$$

The reason for these responses is that agents are not able to immediately identify the permanent technology shock. Therefore, the expectation $x_{t|t}$ initially underreacts relative to the actual change in underlying productivity. This implies that consumers' demand, and thus output, catch up only gradually with the increased productivity of the economy. Along the transition path, firms tend to lower prices as they face lower marginal costs, and employment falls temporarily.

Let me turn to the noise shock $e_t$, which is a pure shock to expectations and does not affect productivity. Again, it is useful to first derive the responses of $a_{t+\tau}$ and $x_{t+\tau|t+\tau}$ to $e_t = 1$. The response of $a_{t+\tau}$ is clearly zero for all $\tau \geq 0$. The filtering equation (16) shows that the response of $x_{t+\tau|t+\tau}$ is now given by $\rho^\tau(1-\rho)\delta$ for $\tau \geq 0$. Using (14) it then follows that the response of output is

$$\frac{dy_{t+\tau}}{de_t} = \rho^\tau\frac{(1-\rho)\delta}{1+\phi\kappa} > 0,$$

the response of employment is

$$\frac{dn_{t+\tau}}{de_t} = \rho^\tau\frac{(1-\rho)\delta}{1+\phi\kappa} > 0,$$

and, using (15), the response of inflation is

$$\frac{d\pi_{t+\tau}}{de_t} = \rho^\tau\frac{\kappa(1-\rho)\delta}{1+\phi\kappa} > 0.$$

Therefore, output, employment, and inflation all increase in the short run and then revert to their initial values as $\tau \to \infty$.

The response to a temporary shock $\eta_t$ is richer, because in the first period the shock affects both the agents' beliefs and realized productivity, while in the following periods it only affects the agents' beliefs. Proceeding as in the previous cases, it is easy to show that the first period responses of output, employment, and inflation are, respectively,

$$\begin{aligned}
\frac{dy_t}{d\eta_t} &= \frac{(1-\rho)(1-\delta)+\phi\kappa}{1+\phi\kappa} > 0, \\
\frac{dn_t}{d\eta_t} &= \frac{(1-\rho)(1-\delta)-1}{1+\phi\kappa} < 0, \\
\frac{d\pi_t}{d\eta_t} &= \frac{\kappa\left((1-\rho)(1-\delta)-1\right)}{1+\phi\kappa} < 0.
\end{aligned}$$

In the following periods, the responses are all positive and equal, respectively, to

$$\begin{aligned}
\frac{dy_{t+\tau}}{d\eta_t} &= \rho^\tau \frac{(1-\rho)(1-\delta)}{1+\phi\kappa}, \\
\frac{dn_{t+\tau}}{d\eta_t} &= \rho^\tau \frac{(1-\rho)(1-\delta)}{1+\phi\kappa}, \\
\frac{d\pi_{t+\tau}}{d\eta_t} &= \rho^\tau \frac{\kappa(1-\rho)(1-\delta)}{1+\phi\kappa}.
\end{aligned}$$

After the effect on productivity has vanished, the effect of the shock $\eta_t$ is analogous to that of a noise shock, since it only affects agents' expectations.

This simple model suggests that a model where agents learn about long-run changes in productivity delivers rich implications about the conditional correlations of output, inflation, and employment, following different shocks. In particular, the impulse-responses derived above suggest that the noise shock has the flavor of an aggregate demand shock in traditional Keynesian models.

## 2.3 Remarks

Inspecting (14) immediately reveals that the two crucial parameters for the model's dynamics are $\kappa$, reflecting the importance of nominal rigidities in the model,[11] and $\phi$, reflecting the monetary policy response to inflation. When either $\kappa$ or $\phi$ are larger, equilibrium output tends to be closer to current productivity. In the flexible price limit (with $\theta \to 0$ and $\kappa \to \infty$), the long-run expectations of consumers only determine the real interest rate but have no impact on equilibrium output. This emphasizes that the role of consumer expectations on equilibrium output is very different depending on the degree of price stickiness. Nominal rigidities mute

[11] Notice that $\kappa$ is decreasing in $\theta$, for given $\beta$ and $\zeta$.

the response of the real interest rate and imply that shifts in consumers' expectations are translated into changes in current output. With flexible prices, instead, changes in expectations are completely absorbed by the real rate.

On the monetary policy side, as $\phi$ goes to infinity the equilibrium converges to the equilibrium of a flexible price economy irrespective of the value of $\kappa$. In that case, the central bank adjusts the nominal interest rate so as to mimic the movements in the real rate in the flexible price benchmark.[12] Notice that $\phi \to \infty$ corresponds to the optimal monetary policy in this environment, as it delivers both zero inflation and a zero output gap. In this sense, the demand shocks identified above are the result of a suboptimal policy rule. Extending the model, there are a number of reasons why optimal monetary policy may not be able to mimic the flexible price benchmark in this type of environment. For example, one could introduce mark-up shocks, affecting the pricing equation, and assume that the monetary authority can only observe $y_t$ and $\pi_t$. In this case, the monetary authority would not be able to identify the values of $a_t$ and $x_{t|t}$ (which are needed to compute the "natural rate") and would have to base its actions on its best estimates of these variables. The analysis of optimal monetary policy in such an environment is outside the scope of this paper.[13]

Notice that in the model there is a non-trivial relation between the variances $\sigma_\epsilon^2, \sigma_e^2$, and $\sigma_\eta^2$ and the output volatility generated, respectively, by the three shocks. In particular, consider the short-run (one period) output volatility due to noise shocks, which is equal to

$$\left(\frac{(1-\rho)\,\delta}{1+\phi\kappa}\right)^2 \sigma_e^2. \tag{17}$$

Notice that, as $\sigma_e^2$ approaches 0 the value of $(1-\rho)\,\delta$ converges to 1, since in the limit the signal $s_t$ conveys perfect information about $x_t$. When instead $\sigma_e^2$ goes to $\infty$, the expression $(1-\rho)\,\delta$ goes to 0, as the signal becomes completely uninformative.[14] In both cases, the expression in

[12] Substituting (15) into (8) shows that as $\phi \to \infty$ the nominal interest rate, and thus the real interest rate, converge to $x_{t|t} - a_t$.

[13] See Aoki (2003) and Svensson and Woodford (2003) for related exercises in environments where the private sector has full information. In Lorenzoni (2007), I analyze optimal monetary policy in an environment with dispersed information analogous to that in Section 3.

[14] Using the expressions in footnote 10, it is easy to show that

$$\lim_{\sigma_e^2 \to 0} \rho = 0, \; \lim_{\sigma_e^2 \to 0} \delta = 1,$$

and

$$\lim_{\sigma_e^2 \to \infty} \rho = \frac{2\sigma_\eta^2}{2\sigma_\eta^2 + \sigma_\epsilon^2 + \sqrt{\left(\sigma_\epsilon^2\right)^2 + 4\sigma_\eta^2\sigma_\epsilon^2}}, \; \lim_{\sigma_e^2 \to \infty} \delta = 0.$$

(17) goes to 0.[15] That is, when the signal is too precise or too imprecise, noise shocks tend to generate small levels of output volatility. In order for noise shocks to have a relevant cyclical effect, one needs to consider intermediate values for $\sigma_e^2$, so that agents put some weight on the signal $s_t$, while, at the same time, the noise $e_t$ is sufficiently volatile. This non-monotonic relation between the variance of the noise shocks and the output volatility they generate is a peculiar feature of a learning model of the business cycle. I will return to this point when looking at the quantitative implications of the model with dispersed information, in Section 4.2.

One undesirable feature of the simple model considered in this section is that, in order for noise shocks to have sizeable and persistent effects on output, the information given by current productivity, $a_t$, needs to be sufficiently imprecise. This means that I need relatively large temporary shocks, that is, a relatively large value of $\sigma_\eta^2$. From an analytical point of view, notice that, as $\sigma_\eta^2 \to 0$, $\delta$ goes to zero, agents only use current productivity to forecast $x_t$, and the expression in (17) goes to zero.[16] From a quantitative point of view, I performed numerical simulations, using parameters in the range of those used in Section 4 below. I found that, to obtain realistic responses to permanent technology shocks and noise shocks, both in terms of size and persistence, I need values of $\sigma_\eta$ about ten times larger than $\sigma_\epsilon$. This values appear highly unrealistic and, somehow, go in the opposite direction of the idea laid out in the introduction, which is to interpret long-run volatility as the outcome of technical change and short-run volatility as the outcome of expectational errors. Moreover, such large temporary productivity shocks would lead to a counterfactual negative correlation between output and employment at business cycle frequencies.[17]

The role of the temporary shock $\eta_t$ in the model of this section was essentially to add noise to the observation of $x_t$ by the representative agent. A realistic alternative is to assume that agents in the economy cannot observe aggregate productivity directly but only productivity in their specific sector. This provides them with a noisy signal about economy-level average innovations. To capture this idea, I modify the model above by introducing idiosyncratic productivity shocks and dispersed information. In particular, in the next section, the temporary shock $\eta_t$ will be

[15] To prove this statement, in the second case, notice that

$$\lim_{\sigma_e^2 \to \infty} ((1-\rho)\,\delta)^2\,\sigma_e^2 = \lim_{\sigma_e^2 \to \infty} \frac{1/\sigma_e^2}{\left(1/\sigma_x^2 + 1/\sigma_\eta^2 + 1/\sigma_e^2\right)^2} = 0.$$

[16] This follows from the expressions in footnote 10.

[17] This is due to the fact that productivity shocks are associated to falls in employment.

replaced by idiosyncratic temporary productivity shocks which vary across sectors but wash out in the aggregate. I will then assume that agents can only observe productivity in their own sector and noisy price and quantity signals about the aggregate economy.

# 3 The model with dispersed information

I now turn to the full model with heterogeneity and dispersed information. Consumers are located in a continuum of islands, indexed by $l \in [0,1]$. Each island is analogous to the economy described in the previous section, with a representative consumer who owns a continuum of price-setting firms producing differentiated goods indexed by $j \in [0,1]$. However, now islands are characterized by different productivity levels $A_{l,t}$. The consumer from island $l$ consumes the goods produced in a subset of other islands. This subset is denoted by $\mathcal{L}_{l,t} \subset [0,1]$ and is randomly selected by nature each period. Symmetrically, the firms in island $l$ are visited by a subset $\hat{\mathcal{L}}_{l,t} \subset [0,1]$ of consumers coming from other islands. Labor is immobile across islands, so the consumer located in island $l$ only works for the firms in island $l$.

Given this geography, I will make some crucial informational assumptions: agents in island $l$ only observe productivity, output, prices and wages in their own island, the prices of the goods in the consumption basket of the local consumer, and a public noisy inflation signal. With this information structure, agents only receive noisy price and quantity signals about the aggregate economy. The signal $s_t$, regarding the permanent component of the technology process is still present, and is publicly observed by all the agents in the economy.

**Preferences and technology.** Preferences are the same as in the previous section, except that consumption and labor supply now have an island index ($C_{l,t}$ and $N_{l,t}$) and the composite consumption good for island $l$ only includes the goods produced in the islands $\tilde{l} \in \mathcal{L}_{l,t}$, that is,

$$C_{l,t} = \left( \int_{\mathcal{L}_{l,t}} \int_0^1 C_{j,\tilde{l},l,t}^{\frac{\gamma-1}{\gamma}} dj d\tilde{l} \right)^{\frac{\gamma}{\gamma-1}},$$

where $C_{j,\tilde{l},l,t}$ is the consumption of variety $j$ produced in island $\tilde{l}$, by the representative consumer of island $l$, at time $t$.

The production function is

$$Y_{j,l,t} = A_{l,t} N_{j,l,t}, \tag{18}$$

where $N_{j,l,t}$ is the labor input and $A_{l,t}$ is the island-specific productivity.

**Uncertainty.** As in the basic model of Section 2, productivity $a_{l,t}$ has a permanent component and a temporary component, but the temporary component is now idiosyncratic to island $l$ and is denoted by $\eta_{l,t}$. Therefore, $a_{l,t}$ is given by

$$a_{l,t} = x_t + \eta_{l,t}.$$

For each island $l$, the idiosyncratic shock, $\eta_{l,t}$, is normal, with zero mean and variance $\sigma_\eta^2$, serially uncorrelated, and independent of the aggregate shocks $\epsilon_t$ and $e_t$. The cross sectional distribution of $\eta_{l,t}$ satisfies $\int_0^1 \eta_{l,t} dl = 0$. The process for $x_t$ and for the public signal $s_t$ are given by (3) and (4), as in Section 2.

Finally, there are two idiosyncratic shocks $\xi_{l,t}^1$ and $\xi_{l,t}^2$, which introduce noise in the endogenous price and quantity signals observed by the agents, and a shock $\omega_t$ to the public signal about aggregate inflation. For ease of exposition, I will discuss them in detail below.

Each period, consumers and firms located in island $l$ choose quantities and prices optimally on the basis of the information available to them which includes: the local productivity $a_{l,t}$, the public signal $s_t$, the price of the one-period nominal bond $Q_t$, the local wage rate $W_{l,t}$, the prices of all the goods in the consumption basket of the local consumer $\{P_{j,\tilde{l},t}\}_{j\in[0,1],\tilde{l}\in\mathcal{L}_{l,t}}$, the total sales of the local firms $\{Y_{j,l,t}\}_{j\in[0,1]}$, and the inflation index $\tilde{\pi}_t$, introduced below.

**Consumers.** The consumer in island $l$ owns the firms in the island and, thus, receives the profits $\int_0^1 \Pi_{j,l,t} dj$, where $\Pi_{j,l,t}$ are the profits of firm $j$ in island $l$. Nominal one-period bonds are the only financial assets traded *across* islands. Due to the presence of island-specific shocks, the consumer in island $l$ is now subject to uninsurable idiosyncratic income shocks. His budget constraint is

$$Q_t B_{l,t+1} + \int_{\mathcal{L}_{l,t}} \int_0^1 P_{j,\tilde{l},t} C_{j,\tilde{l},l,t} dj d\tilde{l} = B_{l,t} + W_{l,t} N_{l,t} + \int_0^1 \Pi_{j,l,t} dj, \tag{19}$$

where $B_{l,t}$ denotes holdings of nominal bonds. In equilibrium, consumers choose consumption, hours worked, and bond holdings, so as to maximize their expected utility subject to (19) and a no-Ponzi-game condition.

For each island, there are now two relevant price indexes. The first, is the local price index $P_{l,t}$, which includes all the goods produced in island $l$ and is equal to

$$P_{l,t} = \left( \int_0^1 P_{j,l,t}^{1-\gamma} dj \right)^{\frac{1}{1-\gamma}}.$$

The second, is the consumer price index $\overline{P}_{l,t}$, which includes all the goods consumed in island $l$, and is given by

$$\overline{P}_{l,t} = \left( \int_{\mathcal{L}_{l,t}} P_{\tilde{l},t}^{1-\gamma} d\tilde{l} \right)^{\frac{1}{1-\gamma}}.$$

The demand for good $j$ in island $\tilde{l} \in \mathcal{L}_{l,t}$ by consumer $l$ is then

$$C_{j,\tilde{l},l,t} = \left( \frac{P_{j,\tilde{l},t}}{\overline{P}_{l,t}} \right)^{-\gamma} C_{l,t}.$$

Aggregating the demand of all consumers in $\hat{\mathcal{L}}_{l,t}$, gives the demand for the good produced by firm $j, l$, which is equal to

$$Y_{j,l,t} = \int_{\hat{\mathcal{L}}_{l,t}} \left( \frac{P_{j,l,t}}{\overline{P}_{\tilde{l},t}} \right)^{-\gamma} C_{\tilde{l},t} d\tilde{l}. \tag{20}$$

The economy-wide price index is defined, conventionally, as

$$P_t = \left( \int_0^1 P_{l,t}^{1-\gamma} dl \right)^{\frac{1}{1-\gamma}}.$$

**Firms.** Firms are price-setters *à la* Calvo (1983), as in the baseline model of Section 2. Each period, on each island, a fraction $1-\theta$ of firms are allowed to reset their price. Let $E_{l,t}[.]$ denote the expectation of the agents located in island $l$. Let $P_{l,t}^*$ denotes the optimal price for a firm who can adjust its price in island $l$ at time $t$. The problem of this firm is to maximize

$$E_{l,t} \sum_{\tau=t}^{\infty} \theta^{\tau} Q_{t+\tau|t}^{l} \left( P_{j,l,t+\tau} Y_{j,l,t+\tau} - W_{l,t+\tau} N_{j,l,t+\tau} \right),$$

subject to $P_{j,l,t+\tau} = P_{l,t}^*$, the technological constraint (18) and the demand relation (20). The firm takes as given the stochastic processes for $W_{l,t}$ and for $\overline{P}_{\tilde{l},t}$ and $C_{\tilde{l},t}$ for all $\tilde{l} \in [0,1]$, and the stochastic discount factor of consumer $l$, given by $Q_{t+\tau|t}^{l} = \beta^{\tau} \left( C_{l,t+\tau}/C_{l,t} \right)^{-1}$.

**Endogenous signals.** Now I can discuss the endogenous price and quantity signals observed by the agents and introduce the sampling shocks $\xi_{l,t}^1$ and $\xi_{l,t}^2$. I assume that the random selection of islands in $\mathcal{L}_{l,t}$ is such that the consumer price index for island $l$ is, in log-linear approximation,

$$\overline{p}_{l,t} = p_t + \xi_{l,t}^1,$$

where $\xi_{l,t}^1$ is i.i.d., normal, with zero mean and variance $\sigma_{\xi,1}^2 > 0$, and satisfies $\int_0^1 \xi_{l,t}^1 dl = 0$. This assumption basically says that, each period, nature selects a biased sample of islands for each consumer $l$, so that the price index $\overline{p}_{l,t}$ is not identical to the aggregate price index $p_t$.

The role of this assumption is to limit the ability of agents to infer the aggregate shocks from their observation of the prices of the goods they buy.[18]

The demand faced by firm $j$ in island $l$, (20), can be rewritten, in log-linear approximation, as

$$y_{j,l,t} = \int_{\tilde{l} \in \hat{\mathcal{L}}_{l,t}} \left( c_{\tilde{l},t} + \gamma \overline{p}_{\tilde{l},t} \right) d\tilde{l} - \gamma p_{j,l,t}.$$

I assume that the random selection of the islands in $\hat{\mathcal{L}}_{l,t}$ is such that this expression is equal to

$$y_{j,l,t} = y_t + \gamma p_t - \gamma p_{j,l,t} + \xi_{l,t}^2, \tag{21}$$

where $\xi_{l,t}^2$ is i.i.d., normal, with zero mean and variance $\sigma_{\xi,2}^2 > 0$, and with $\int_0^1 \xi_{l,t}^2 dl = 0$. The underlying assumption is that the sample of consumers who buy goods in island $l$ is a biased sample, so that firms only receive a noisy signal regarding $y_t + \gamma p_t$. Again, the role of this assumption is to limit the agents' ability to infer aggregate shocks by observing the quantities produced in their island.[19]

**Monetary policy.** To define a monetary policy rule, I need to allow the central bank to observe some measure of realized inflation. Moreover, given that agents observe the nominal interest rate and there are no monetary policy shocks, if the central bank had access to a perfect measure of inflation the agents would be able to infer $\pi_t$ from $i_t$. Here, I address this issue by assuming that both the central bank and the private agents have access to a noisy measure of inflation

$$\tilde{\pi}_t = \pi_t + \omega_t,$$

where $\omega_t$ is an i.i.d. normal shock, with zero mean and variance $\sigma_\omega^2$. Once more, the role of the shock $\omega_t$ is to limit the agents' ability to infer aggregate shocks from aggregate inflation.

The nominal interest rate is set according to the rule

$$i_t = (1 - \rho_i)\, i^* + \rho_i i_{t-1} + \phi \tilde{\pi}_t, \tag{22}$$

where $\rho_i \in [0, 1)$ and $\phi$ are coefficients chosen by the monetary authority. This generalizes the rule (8), taking a step in the direction of realism, by allowing for inertia in the monetary response to inflation.

[18] Given that nature selects the island-prices $p_{\tilde{l},t}$ from the distribution $\{p_{\tilde{l},t}\}_{\tilde{l} \in [0,1]}$, consistency requires that the variance $\sigma_{\xi_1}^2$ is bounded above by the cross-sectional variance of prices across islands.

[19] As in the case of prices, consistency requires that the variance $\sigma_{\xi_2}^2$ is bounded above by the cross-sectional variance of $c_{l,t} + \gamma \overline{p}_{l,t}$ across islands.

### 3.1 Equilibrium

Unlike in the basic environment of Section 2, the equilibrium dynamics of inflation and output can no longer be derived analytically and I need to solve the model numerically. As before, I will study a log-linear approximation to a rational expectations equilibrium. In a setup with dispersed information a log-linear approximation helps in three dimensions: it simplifies the inference problem of the individual agents, it simplifies the state space for individual decision rules, and it simplifies aggregation.

**Individual optimality conditions.** Let me first derive the individual optimality conditions which will be used to characterize an equilibrium. The consumers' Euler equation takes the form[20]

$$c_{l,t} = E_{l,t}\left[c_{l,t+1}\right] - i_t + E_{l,t}\left[\overline{p}_{l,t+1}\right] - \overline{p}_{l,t}. \tag{23}$$

The two differences with equation (9) are that both expected consumption and expected inflation are island-specific. On the other hand, as I will argue in the next section, consumption in each island $l$ still tends to converge towards the common level dictated by the permanent productivity $x_t$. Therefore, through the term $E_{l,t}\left[c_{l,t+1}\right]$, $c_{l,t}$ is still driven by the agents' expectations of $x_t$, as in Section 2.

To complete the characterization of the consumption side, it is useful to write down the individual budget constraint in log-linearized form, which is

$$\beta h_{l,t+1} = h_{l,t} + p_{l,t} + y_{l,t} - \overline{p}_{l,t} - c_{l,t}, \tag{24}$$

where $h_{l,t} \equiv B_{l,t}/E_{l,t}\left[P_t Y_t\right]$ is the ratio of nominal bond holdings to expected aggregate nominal output. The variable $B_{l,t}$ is kept in levels rather than in logs, since it can take both positive and negative values.

Optimality for a firm who can update its price at date $t$ gives

$$p^*_{l,t} = (1-\beta\theta)\sum_{\tau=0}^{\infty}(\beta\theta)^{\tau} E_{l,t}\left[w_{l,t+\tau} - a_{l,t+\tau}\right], \tag{25}$$

where $w_{l,t+\tau} - a_{l,t+\tau}$ represents the marginal cost in nominal terms in island $l$. This condition can be rewritten in recursive form as

$$p^*_{l,t} = (1-\beta\theta)\left(w_{l,t} - a_{l,t}\right) + \beta\theta E_{l,t}\left[p^*_{l,t+1}\right].$$

[20] To obtain log-linear approximations of the optimality conditions, I take as a reference point the stochastic equilibrium of an economy with no heterogeneity and full information (i.e., where all variances except $\sigma^2_\epsilon$ are set to zero). The full derivations are in the supplementary material (Section 8).

The law of motion for the local price index is

$$p_{l,t} = \theta p_{l,t-1} + (1-\theta)\, p_{l,t}^{*}.$$

Rearranging the last two equations and using the consumer's optimality condition for labor supply, $w_{l,t} - \overline{p}_{l,t} - c_{l,t} = \zeta n_{l,t}$, the labor market clearing condition, $n_{l,t} = y_{l,t} - a_{l,t}$, and the demand relation (21), I then obtain

$$p_{l,t} - p_{l,t-1} = \lambda \left(\overline{p}_{l,t} + c_{l,t} - p_{l,t} - a_{l,t}\right) + \lambda\zeta \left(d_{l,t} - \gamma p_{l,t} - a_{l,t}\right) + \beta E_{l,t} \left[p_{l,t+1} - p_{l,t}\right], \tag{26}$$

where $\lambda \equiv (1-\theta)(1-\beta\theta)/\theta$ and

$$d_{l,t} \equiv y_t + \gamma p_t + \xi_{l,t}^{2}. \tag{27}$$

The quantity $d_{l,t}$ corresponds to the intercept of the demand function faced by the producers in island $l$ in period $t$ (in log-linear terms).

Expression (26) shows that prices in island $l$ tend to increase when either the consumption of the local consumer or the demand for the goods produced in island $l$ are high relative to the local productivity $a_{l,t}$. The consumption of the local consumer matters since it determines the location of the labor supply curve in island $l$. The demand of external consumers matters because it determines the amount of labor input required. Both variables jointly determine equilibrium wages and thus equilibrium marginal costs in island $l$.

The presence of imperfect information makes it impossible to aggregate (26) across islands and obtain a simple equation linking aggregate inflation to the aggregate output gap, as in (11). However, the underlying logic survives as I will show in Section 4.

**Learning and aggregation.** The economy's aggregate dynamics will be described in terms of the variables $z_t = (x_t, e_t, p_t, i_t)$. The state of the economy is captured by the infinite dimensional vector $\mathbf{z}_t = (z_t, z_{t-1}, ...)$. I am looking for a linear equilibrium where the law of motion for $\mathbf{z}_t$ takes the form

$$\mathbf{z}_t = A\mathbf{z}_{t-1} + B\mathbf{u}_t^1, \tag{28}$$

with

$$\mathbf{u}_t^1 \equiv \begin{pmatrix} \epsilon_t & e_t & \omega_t \end{pmatrix}',$$

and the appropriate rows of $A$ and $B$ conform with the law of motion of $x_t$, (3), expression (4) for the signal $s_t$, and the monetary policy rule (22).[21]

[21] See the appendix for explicit expressions for $A$ and $B$.

To solve for a rational expectations equilibrium, I conjecture that $p_{l,t}$ and $c_{l,t}$ follow the rules

$$\begin{aligned} p_{l,t} &= q_h h_{l,t} + q_p p_{l,t-1} + q_a a_{l,t} + q_d d_{l,t} + q_z E_{l,t}\left[\mathbf{z}_t\right], && (29) \\ c_{l,t} &= -\overline{p}_{l,t} + b_h h_{l,t} + b_p p_{l,t-1} + b_a a_{l,t} + b_d d_{l,t} + b_z E_{l,t}\left[\mathbf{z}_t\right]. && (30) \end{aligned}$$

The expressions (29) and (30) represent, respectively, the optimal pricing policy of the firms in island $l$ (aggregated across firms) and the optimal consumption policy of the representative consumer in island $l$. Notice that $h_{l,t}$, $p_{l,t-1}$, and $E_{l,t}\left[\mathbf{z}_t\right]$ are the relevant individual state variables to describe the average behavior of consumers and firms in island $l$. I need to keep track of $h_{l,t}$ because of the consumer's budget constraint, I need $p_{l,t-1}$ because of Calvo pricing, and I need $E_{l,t}\left[\mathbf{z}_t\right]$ to form agents' expectations about current and future values of the aggregate state $\mathbf{z}_t$. The dynamics of $E_{l,t}\left[\mathbf{z}_t\right]$ can be characterized recursively using the Kalman filter,

$$E_{l,t}\left[\mathbf{z}_t\right] = A E_{l,t-1}\left[\mathbf{z}_{t-1}\right] + C\left(\mathbf{s}_{l,t} - E_{l,t-1}\left[\mathbf{s}_{l,t}\right]\right),$$

where $\mathbf{s}_{l,t}$ is the vector of signals observed by the agents in island $l$,[22]

$$\mathbf{s}_{l,t} = \begin{pmatrix} a_{l,t} & s_t & \overline{p}_{l,t} & d_{l,t} & i_t \end{pmatrix}',$$

and $C$ is a matrix of Kalman gains, which is derived explicitly in the appendix. Let $\mathbf{z}_{t|t}$ denote the average expectation regarding the aggregate state $\mathbf{z}_t$, defined as

$$\mathbf{z}_{t|t} \equiv \int_0^1 E_{l,t}\left[\mathbf{z}_t\right] dl.$$

The individual updating rules can then be aggregated to find a matrix $\Xi$ such that

$$\mathbf{z}_{t|t} = \Xi \mathbf{z}_t. \tag{31}$$

In equilibrium aggregate output $y_t$ is given by

$$y_t = \boldsymbol{\psi} \mathbf{z}_t, \tag{32}$$

where $\boldsymbol{\psi}$ is a vectors of constant coefficients. The solution of the model requires finding matrices $A, B, C, \Xi$, and vectors $\boldsymbol{\psi}$, $\{q_h, q_p, q_a, q_d, q_z\}$, and $\{b_h, b_p, b_a, b_d, b_z\}$ that are consistent

[22] For computational reasons, it is convenient to include $i_t$ instead of $\tilde{\pi}_t$ in the set variables observed by the agents. This allows me to simplify the Kalman filter, since, in this way, aggregate shocks do not appear in the observation equation. Since the agents know the monetary policy rule, (22), they can recover $\tilde{\pi}_t$ from their observation of $i_t$ and $i_{t-1}$.

with agents' optimality, with Bayesian updating, and with market clearing in the goods, labor, and bonds markets. The details of the algorithm used for computations are in the appendix.

Computing an equilibrium requires dealing with the infinite histories $\mathbf{z}_t$. Here, I replace $\mathbf{z}_t$ with a truncated vector of states $\mathbf{z}_t^{[T]} = \{z_t, ..., z_{t-T}\}$. Numerical results show that when $T$ is sufficiently large the choice of $T$ does not affect the equilibrium dynamics. For the simulations presented below, I use $T = 50$. Kasa (2000) uses frequency domain methods to deal explicitly with infinite histories and explores in what cases infinite histories lead to a fully revealing equilibrium. In particular, he shows that in the model of Townsend (1983) with a continuum of industries, imperfect information does not go away when looking at infinite histories.[23] My numerical results suggest that my model belongs to the same class of models, given that increasing $T$ in my simulations does not lead to smaller expectational errors.[24]

# 4 Noise shocks and aggregate volatility

In this section, I use numerical simulations to explore some basic qualitative and quantitative implications of the model. The model is very stylized, so the main objectives of this section are: (i) to evaluate the model's ability to generate sizeable cyclical movements from noise in public information, and (ii) to point out a non-monotone relation between noise variance and noise-driven volatility in business cycle models based on learning. To this end, I will explore the joint short run dynamics of output and employment and compare the model's implications with those of a simple bivariate VAR.

The parameter $\beta$ is set equal to 0.99, so the time period can be interpreted as a quarter. The value of $\zeta$ is set to 0.5, corresponding to a Frisch labor elasticity of 2, and the value of the elasticity of substitution $\gamma$ is set to 7.5, which implies a mark-up of around 15%. The parameter $\theta$ is set equal to 2/3, corresponding to an average price duration of three quarters. These values are in the range of those used in existing DSGE studies with monopolistic competition and sticky prices. The parameters for the monetary policy rule are set at $\rho_i = 0.9$ and $\phi = 1.5$, which corresponds to a relatively inertial Taylor rule with a response to inflation broadly consistent with existing empirical estimates.[25]

It remains to choose values for the variances of aggregate and idiosyncratic shocks. Unlike

[23] See Section 2 of Kasa (2000).

[24] See also Rondina (2007) for an application of frequency domain methods to a related monetary environment and Nimark (2007), who uses a truncation method in the space of higher order expectations.

[25] See, for example, Clarida, Gali, and Gertler (2000), Christiano, Eichenbaum, and Evans (2005).

standard linearized models, models with imperfect information do not display a "certainty equivalence" property, that is, variance parameters do not simply determine the size of the shocks, leaving the behavioral responses unchanged. Variance parameters also affect the filtering problem faced by the agents in the model and thus contribute to determine the shape of the responses of the endogenous variables. Here, variance parameters are chosen so that the model can roughly replicate the impulse-responses derived from a simple bivariate VAR in output and employment, as I will explain below. The baseline values of the parameters are reported in Table 1.

| | | | |
|---|---|---|---|
| $\sigma_\epsilon$ | 0.0077 | $\sigma_\eta$ | 0.15 |
| $\sigma_e$ | 0.03 | $\sigma_{\xi,1}$ | 0.02 |
| $\sigma_\omega$ | 0.0006 | $\sigma_{\xi,2}$ | 0.11 |

Table 1 – Baseline parameters

A feature that immediately stands out is that idiosyncratic shocks are assumed to be large relative to aggregate shocks. This choice of parameters is made to prevent agents from learning the underlying aggregate shocks from their local observation of productivity, prices and quantities.[26] The specific values of $\sigma_\eta, \sigma_{\xi,1}$, and $\sigma_{\xi,2}$ have little effect on the model's aggregate implications as long as none of them is too small.[27]

### 4.1 Responses to the three shocks

Figure 1 depicts the responses of output, employment, inflation and the interest rate to the three shocks $\epsilon_t$, $e_t$ and $\omega_t$. In the first row of graphs, I plot both the response of $y_t$ and that of $y_{t+1|t} = \int E_{l,t}\left[y_{t+1}\right] dl$ representing the average expectation of next period's output $y_{t+1}$ (solid line for $y_t$, dashed line for $y_{t+1|t}$). In the second row, I plot the response of hours. In the third row, I plot both the response of actual inflation $\pi_t$ and of the noisy inflation measure $\tilde{\pi}_t$ (solid line for $\pi_t$, dashed line for $\tilde{\pi}_t$). In the fourth row, I plot the nominal interest rate $i_t$ and the average expected real rate, defined as $r_t \equiv i_t - \left(p_{t+1|t} - p_t\right)$ (solid line for $i_t$, dashed line for $r_t$). For all three shocks, I plot the responses to a 1-standard-error shock.[28]

[26] The assumption of large idiosyncratic shocks is not unrealistic, given recent empirical findings on firm-level volatility. For example, Comin and Philippon (2005) show that firm sales volatility is an order of magnitude larger than the aggregate volatility of GDP.

[27] For a given value of $\sigma_\eta$, the parameters $\sigma_{\xi,1}$ and $\sigma_{\xi,2}$ are chosen so as to ensure that they satisfy the bounds described in footnotes 18 and 19. In particular, I use (29) and (30) to evaluate the cross-sectional volatilities in prices and demand which are solely due to the idiosyncratic shocks $\eta_{l,t}$ and use these as conservative upper bounds for $\sigma_{\xi,1}$ and $\sigma_{\xi,2}$.

[28] In all figures, the scale of the responses is multiplied by 100 to make the graphs more readable.

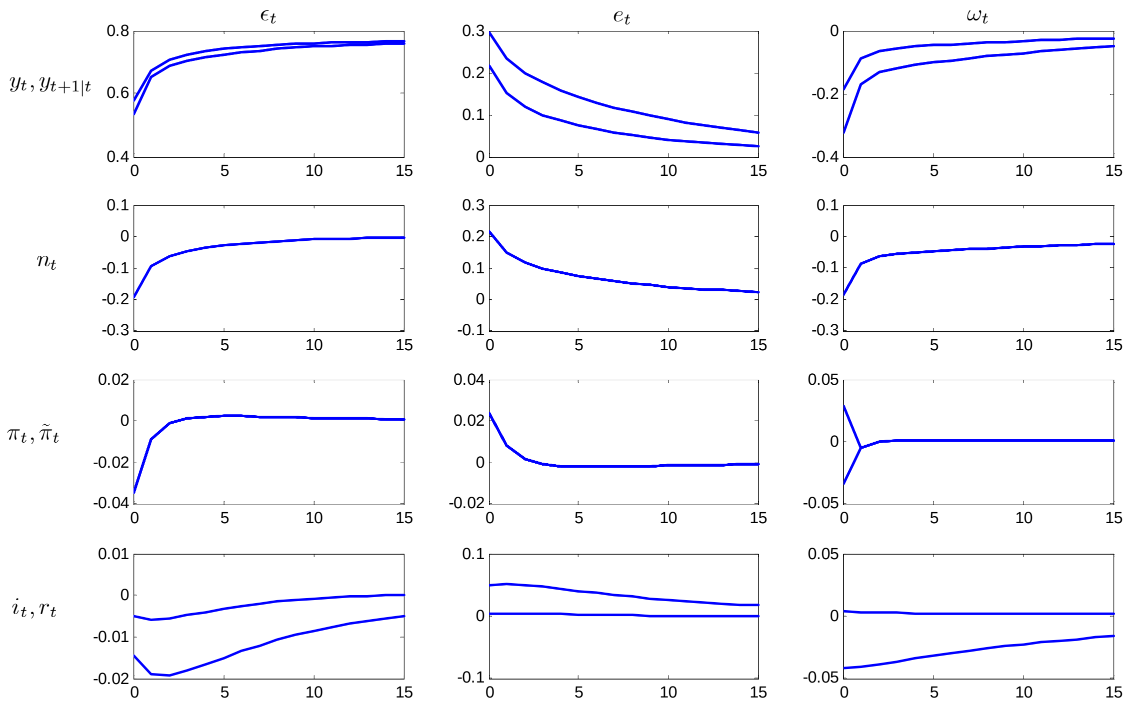


Figure 1: Impulse responses of output, employment, inflation and the interest rate.

From a qualitative point of view, the shocks $\epsilon_t$ and $e_t$ have similar effects as in the model with common information analyzed in Section 2. The technology shock leads to a gradual adjustment of output to its new long run level and to a temporary fall in employment and inflation. The noise shock $e_t$ leads to a joint, temporary increase in output, employment and inflation.

The intuition behind the output response to the noise shock is that forward looking consumers expect their future incomes and consumption levels to be driven by the permanent, common component of technology, $x_t$. A noise shock temporarily increases their expectation of $x_t$. This increases expected future consumption on the right-hand side of the Euler equation (23).[29] At the same time, the real interest rate, also on the right-hand side of (23), responds sluggishly, due to the combination of nominal rigidities and of a partially responsive mone-

[29] Aggregating across islands, this gives $\int E_{l,t}[c_{l,t+1}]\,dl$, which tends to move together with $y_{t+1|t}$. The two are not identical given that $y_{t+1|t} = \int E_{l,t}\left[\int c_{l,t+1}dl\right]dl$ and, under dispersed information, cross-sectional integration and the expectation operator are not interchangeable.

tary rule. This implies that the pressure from increased consumers' demand translates into a temporary increase in output and employment. The last row of graphs confirms this intuition, showing that the response of the real interest rate is relatively small after all three shocks, so that movements in consumption are dominated by movements in income expectations.

To understand the effects of the shock $\omega_t$ notice that this shock operates through two channels: a monetary policy channel and an information channel. First, a positive shock $\omega_t$ leads to a temporary increase in measured inflation and thus to a nominal interest rate increase. Second, since agents know that positive inflation signals that they are overestimating natural output, the shock $\omega_t$ leads agents to revise downwards their expectations about $x_t$. Both channels lead to a reduction in spending and aggregate output. However, in the parametrization described above, the information channel explains virtually all of the output decline following the shock. This can be seen both by observing the strong commovement of $y_t$ and $y_{t+1|t}$ after an $\omega_t$ shock (in the top right graph of Figure 1), and by observing the small response of the nominal interest rate (in the bottom right graph). Therefore, $\omega_t$ is essentially an additional noise shock, leading to qualitative responses analogous to those following an $e_t$ shock (with the opposite sign).

This leads to an interesting observation. The presence of a noisy public statistic (here about inflation) can have an ambiguous effect on noise-driven volatility. On the one hand, it allows agents to better estimate the economy's fundamentals. On the other hand, it introduces an additional source of correlated expectational errors. I will return to this point below and discuss further the interpretation of the shock $\omega_t$.

## 4.2 How much noise-driven volatility can the model generate?

In Figure 2 I report the impulse-responses obtained from a simple bivariate VAR of GDP and hours, using US quarterly data. To identify the technology shock I use a long-run identification restriction *à la* Blanchard and Quah (1989), following Gali (1999).[30] In Figure 3, I report the impulse-responses obtained from performing the same exercise on a 10,000 period sample generated using the simulated theoretical model, with the parameters specified above.[31]

[30] The data are from the Haver USECON database, the sample period is 1948:1-2006:3, the measure of output is the GDP quantity index (GDPQ), and the measure of hours worked is hours in the business sector (LXBH). Hours are detrended using a quadratic trend and the VAR is estimated using 4 lags. The dashed lines represent 10% confidence bands, computed following Sims and Zha (1999).

[31] See Canova (2007, Chapter 4.7) for a systematic discussion of this type of "first pass" evaluation of DSGE models using VARs.

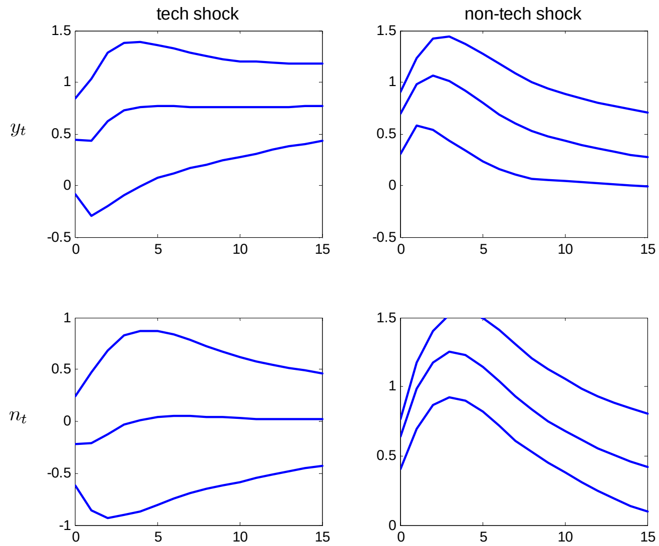


Figure 2: Impulse responses: bivariate VAR on US data, with a long-run restriction.

Comparing the top left panels of Figures 1 and 3 shows that the long-run identification strategy allows me to identify the productivity shock $\epsilon_t$ in the model, despite the presence of three shocks in the model. At the same time, the “non-technology” shock obtained from the VAR on the simulated series reflects the combined effect of the shocks $e_t$ and $\omega_t$. As argued above, both $e_t$ and $\omega_t$ are essentially two noise shocks and have similar implications for output and employment. Therefore, here I will not attempt to separate them with a richer identification strategy and I will concentrate on evaluating their joint role in generating short-run volatility.

Notice also that, in comparing the model with the data, I am implicitly attributing all non-technology disturbances in the empirical VAR to noise shocks. This is clearly an extreme assumption, as it leaves out a number of additional shocks that one would like to include on the demand side (e.g., monetary shocks and shocks to government spending). However, this assumption is in line with the spirit of the exercise, which is to generate as much demand-side volatility as possible, using *only* noise shocks.

The idea of my quantitative exercise is to choose $\sigma_\epsilon$ so that the model can replicate the empirical size of the long-run output response to an identified technology shock (around 0.78), and then look at the output response to the non-technology shock in the data and in the model,

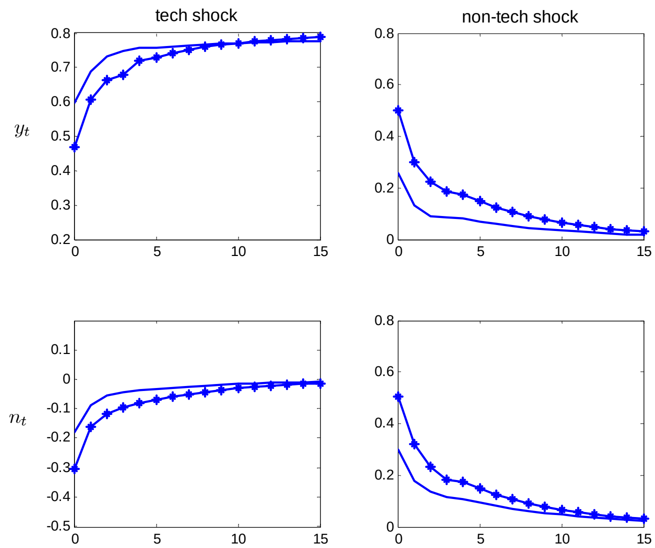


Figure 3: Impulse responses: bivariate VAR on simulated data, with a long-run restriction. Solid lines: baseline parametrization. Starred lines: high-variance parametrization.

for different values of $\sigma_e$ and $\sigma_\omega$. In Figure 3, I report two sets of results: those obtained under my baseline parametrization, $(\sigma_e, \sigma_\omega) = (0.03, 0.0006)$, represented by the solid lines, and those obtained under an alternative parametrization, with higher volatility for both shocks, $(\sigma_e, \sigma_\omega) = (0.04, 0.0015)$, represented by the starred lines. In the baseline parametrization, I choose the value of $\sigma_\omega$ so that the volatility of inflation due to $\omega_t$ is of comparable size as the volatility of inflation due to the other two shocks $\epsilon_t$ and $e_t$.[32]

Comparing figures 2 and 3 shows that the model is able to capture well some features of the empirical impulse response functions. In particular, two patterns are present both in the data and in the model (under either parametrization). First, following a non-technology shock, hours tend to increase roughly one-for-one with output. Second, following a technology shock, hours tend to fall and the drop in hours is of the same size as the difference between the impact response of output and its long-run response.[33]

[32] Interpreting $\tilde{\pi}_t$ as the initial release of inflation data and $\pi_t$ as the final revised value, this calibration is roughly consistent with the finding in Runkle (1998) that the variance of innovations in inflation are of the same order of magnitude as the revision error.

[33] The finding that hours respond negatively to an identified technology shock is the subject of a heated debate (Christiano, Eichenbaum, and Viguffson, 2003, Francis and Ramey, 2003, Chari, Kehoe, and McGrattan, 2004, Gali and Rabanal, 2004). This debate has highlighted the need of a theory-based rationale for identification assumptions. The present model has at least the virtue of being consistent with the identification assumptions

There are other features of the empirical responses which are not captured by the model, in particular, the hump-shaped responses of output and hours following the non-technology shock. However, at this stage of the analysis, the major quantitative challenge for the model is whether it is able to generate relatively large non-technology disturbances. The answer to this question depends on the parametrization. In both versions of the model the effects of non-technology shocks are smaller than in the data, but the model with larger volatilities for $e_t$ and $\omega_t$ generates a larger amount of short-run volatility (this is not as obvious as it sounds, as I will explain in a moment) and, on impact, generates an output response which is comparable to the empirical one (0.50 in the model, 0.69 in the data). Moreover, it also displays an output response to the technology shock more in line with the data, with a smaller effect on impact and a more gradual convergence afterwards. This is due to the fact that greater volatility in the public signals $s_t$ and $\tilde{\pi}_t$ implies that the agents take more time to learn the value of $x_t$.

Therefore, in terms of matching the empirical impulse-responses, the second parametrization seems preferable. However, this parametrization requires a relatively large value of $\sigma_\omega$.[34] The shock $\omega_t$ can be interpreted in several ways: as pure measurement error, as a "reduced-form" way of introducing mark-up shocks, or, more generally, as a summary for all the shocks and specification errors which make inflation a noisy measure of the distance between current output and natural output. Depending on the interpretation, one may be more or less happy about assuming large values for $\sigma_\omega$. The results in Figure 3 show that larger values of $\sigma_\omega$ help deliver larger amounts of noise-driven volatility.

Now suppose I keep $\sigma_\omega$ constant and I simply increase the variance $\sigma_e$. Would that increase demand-side volatility? Not necessarily. The reason for this is the non-monotonicity between variance parameters and noise-driven volatility, which was pointed out already in Section 2.3. As I increase the variance of the noise associated to a given signal, the quality of that signal deteriorates and agents put less and less weight on it. To illustrate this, in Figure 4 I plot the output response to the $\epsilon_t$ and $e_t$ shock for different values of $\sigma_e$, keeping all other parameters at their baseline levels.[35]

This figure illustrates well the rich effects that the choice of variance parameters has on the model dynamics. The maximum output response to the $e_t$ shock, arises for intermediate

in the VAR exercise.

[34] Recall that in the baseline parametrization $\omega_t$ accounts for about 50% of inflation volatility. Under the alternative parametrization it accounts for about 70% of it.

[35] To interpret the figure, recall that I always plot responses to a 1-standard-deviation shock.

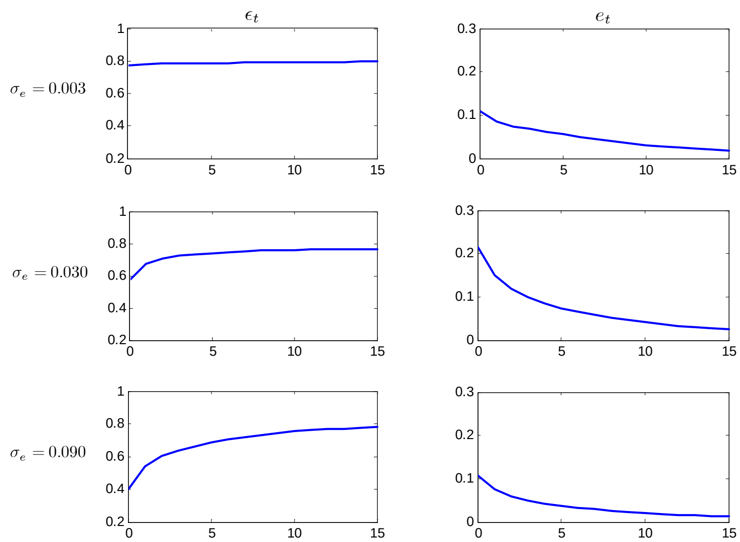


Figure 4: Output responses for different values of $\sigma_e$

values of $\sigma_e$ (around the baseline value of 0.030). Moreover, changing $\sigma_e$ also modifies the shape of the output response to a productivity shock, by changing the speed at which agents learn the underlying value of $x_t$. When $\sigma_e$ is larger, learning is slower and the output response to a technology shock is more gradual.

Summing up, the model ability to generate sizeable noise-driven shocks rests crucially on the assumptions made about the volatility of the various shocks. Clearly, the model is highly stylized so it is relatively easy for the agents in the model to figure out the underlying value of $x_t$ from observing public statistics of productivity and inflation.[1] A richer model which allows, for example, for monetary policy shocks and shocks to government expenditure, would make this inference problem more complicated. Monetary policy shocks would confound the inference problem of the agents, by breaking the tight connection between inflation and the estimation error of $x_t$. Therefore, introducing additional disturbances might be useful not only to increase directly demand-side volatility, but also to magnify the effect of noise shocks.

So far I have concentrated on the model's ability to replicate *conditional* correlations of output and employment, which are estimated using VAR methods. An alternative is to evaluate the model looking at its implication in terms of simple *unconditional* correlations. It is interesting to recast the conclusions of this section in terms of unconditional implications.

Notice that the model generates a negative correlation of output and employment following a technology shock and a positive correlation following a noise shock. Therefore, to obtain a positive unconditional correlation (as observed in the data), I need noise shocks to explain a large fraction of total volatility. In particular, under the baseline parametrization described above I get a correlation between output and employment which is basically zero (looking at HP-filtered simulated series), while under the high-variance parametrization used for Figure 3, the correlation is 0.25. Both fall short of the empirical correlation which is around 0.8. The model does better in terms of total employment volatility. The standard deviation of (HP-filtered) hours relative to that of output is 0.37 in the baseline parametrization and 0.62 in the high-variance parametrization. The corresponding empirical value is close to 1. Therefore, both conditional and unconditional moments show that, although noise shocks can generate a sizeable fraction of demand-side volatility, they are not enough to explain all of it in the present model.

## 4.3 An extension with decreasing returns and variable capacity

To push a step further my quantitative exploration, I consider a variant of the model with decreasing returns and variable capacity utilization. The purpose of this extension is twofold. First, it allows me to explore a version of the model with stronger strategic complementarity in pricing decisions. Second, it allows me to show that the model can deliver procyclical labor productivity following noise shocks.

The model is identical to the model presented in Section 3 except for the firm's technology. The production function is

$$Y_{j,l,t} = A_{l,t} U_{j,l,t}^{\alpha} N_{j,l,t}^{1-\alpha},$$

where $U_{j,l,t}$ is a measure of capacity utilization which is chosen by firm $j$ each period. To reach the level $U_{j,l,t}$ of capacity utilization, the firm needs $\chi_0 A_{l,t} U_{j,l,t}^{1+\chi}$ units of the local consumption good as an input, with $\chi > 0$.[36] This is a simple way of introducing a form of variable capacity without explicitly introducing capital in the model. It allows the firm to vary $U_{j,l,t}$ in response to increases in demand for their good.

The analysis of this case is presented in the supplementary material (Section 8). Here, I

[36] This formulation is analogous to that in Christiano, Eichenbaum, and Evans (2005), except for the fact that the capital stock is fixed (i.e. adjustment costs are infinite) and for the presence of $A_{l,t}$ in the cost of capital utilization function. I assume that the cost of capacity utilization moves one-for-one with local productivity to ensure that $U_{j,l,t}$ is stationary despite the non-stationarity of $A_{l,t}$.

directly present the simulation results. The value of $\alpha$ is set to a standard value of 0.33 and the value of $\chi$ to 0.1. The parameter $\chi$ determines how much the marginal cost of capacity utilization rises with the level of utilization. When $\chi$ is larger than a certain threshold $\hat{\chi}$, the response of $U_{j,l,t}$ is relatively inelastic. In this case, labor productivity tends to fall following a positive noise shock, because decreasing returns tend to dominate. When instead $\chi$ is below $\hat{\chi}$, the response of $U_{j,l,t}$ is sufficiently elastic, and labor productivity tends to increase following a positive noise shock. The value chosen implies that labor productivity is procyclical following a noise shock.[37]

Figure 5 reports the responses of the same variables reported in Figure 1 for the model with decreasing returns and variable capacity, except that in the second row I report the responses of labor productivity, $y_t - n_t$. The remaining model parameters are the same as in the baseline model except for $\sigma_\omega$ which is re-calibrated at 0.0004. The value of $\sigma_\omega$ has been reduced to keep the volatility of inflation noise in line with the inflation volatility generated by the other shocks, following my approach in the baseline parametrization.[38]

The most immediate difference between Figures 1 and 5 is that the inflation responses to the shocks $\epsilon_t$ and $e_t$ are smaller. This is due to the fact that, with decreasing returns in labor, the degree of strategic complementarity in pricing is stronger. This implies that, a given deviation of output from potential generates a milder response of inflation, in line with what happens in standard sticky price models.[39] Therefore, the real interest rate moves less and current consumption is closer to expected future consumption, narrowing the gap between the dotted and the solid lines in the top three panels. Comparing Figures 1 and 5 one can check that this implies larger real responses of output to noise shocks. However, there is an additional difference between the two cases. After re-calibrating the variance of $\omega_t$, the total effect of the noise shocks $\omega_t$ is larger in the economy with variable capacity. The reason for this is that, with smaller responses of inflation to $\epsilon_t$ and $e_t$, it gets harder for consumers to infer the size of the average expectational error from observing aggregate inflation. Even after adjusting downward the volatility of $\omega_t$, the joint combination of real and nominal signals observed by the agents remains less informative. This implies that the shock $\omega_t$ ends up having a bigger

[37] I experimented with various values of $\chi$ and obtained that, all else equal, the choice of $\chi$ does not affect much the total response of output to a noise shock, but it affects how this response is decomposed into changes in labor supply and changes in labor productivity.

[38] See footnote 32.

[39] See Woodford (2003, Chapter 3) for a discussion of the various determinants of strategic complementarity in pricing.

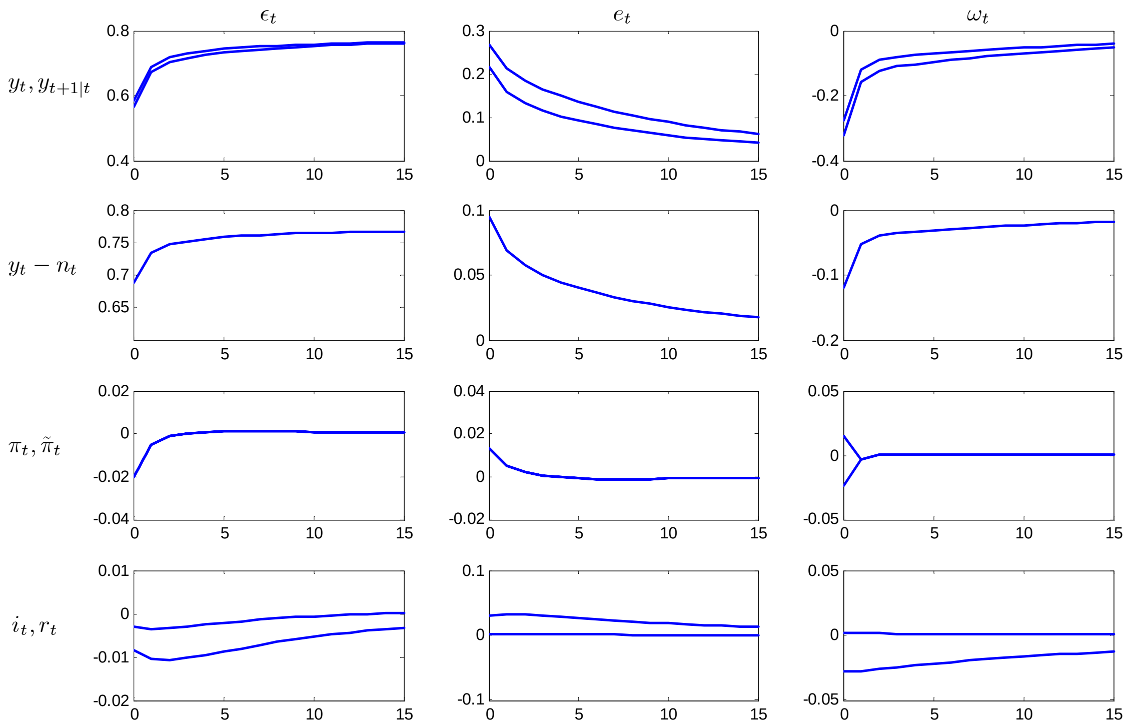


Figure 5: Impulse responses: model with decreasing returns and variable capacity.

effect on output.

This discussion emphasizes once more that, in an economy with imperfect information, changes in parameter values have rich effects, since these parameters also affect the inference problems of the individual agents. This is true for variance parameters, as shown in 4.2. But it also applies to other parameters, with relatively well-known effects in standard models (e.g., parameters affecting the degree of strategic complementarity in pricing), as they may have additional effects through informational channels.

## 5 A test on survey data

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A crucial distinction between a noise shock and an actual technology shock is that, when a noise shock hits, agents' expectations tend to overreact, while, when a technology shock hits, they tend to underreact. In both cases agents receive a positive signal regarding $x_t$. However, given imperfect information, in the first case the actual change in $x_t$ exceeds the expected change. In this case, firms lower prices, the expected real interest rate falls, and realized output ends up responding more than expected output. In the second case, firms tend to increase prices, the expected real interest rate increases, and realized output responds less. The difference $y_t - y_{t|t}$ is then positive in the first case and negative in the second case. This is illustrated in the top row of Figure 1 for my baseline parametrization.[40] This is a robust prediction of the model, which holds across a wide range of parameter values and captures the central role played by expectational errors in the model.

In this section, I attempt to test this prediction, using survey data to obtain a measure of the agents' average expectations, corresponding to $y_{t|t}$ in the model. The idea of the test is to use the simple bivariate semi-structural VAR introduced in the previous section to identify technology and non-technology shocks, and then to test the hypothesis that output expectations underreact following a technology shock and overreact following a non-technology shock. As measures of expectations, I use data from the Survey of Professional Forecasters (SPF) and from the Michigan Consumer Sentiment Survey (CSS). From the SPF, I take the median forecasts of nominal GDP and of the GDP deflator in the coming quarter, to form a forecast of real GDP.[41] From the CSS, I take the third component of the Index of Consumer Sentiment which reflects the consumers' expectations regarding the state of the economy in the coming 12 months.[42]

To make the coefficients easier to interpret, I normalize the technology and the non-technology shock from the VAR, so that each has a 1% impact effect on output. Then, letting $Y_t^e$ denote the expectation variable, I regress $Y_t^e$ on contemporaneous values of the technology and non-technology shocks, on lagged values of $Y_t^e$, and on lagged values of GDP and hours.[43] In Table 3, I report the regression coefficients of the two shocks, for each data set. In the same table, I also report the F-statistic for a test of the difference between the coefficients of the two shocks.

[40] To help the Euler equation interpretation of consumption movements, the figure displays the path of the one-period-ahead average forecast $y_{t+1|t}$. However, the path for the contemporary average forecast $y_{t|t}$ is virtually identical to that of $y_{t+1|t}$.

[41] The mnemonics for the two variables used are NGDP2 and PGDP2. The sample is 1968:4-2006:3.

[42] This is the component denominated "Business Condition, 12 months." The sample is 1960:1-2006:3.

[43] In the reported regressions I use 4 lags for $Y_t^e$, GDP and hours.

| Survey of Professional Forecasters Data | | |
|---|---|---|
| | coefficient | std. error |
| tech. shock | 0.94145 | 0.23805 |
| non-tech. shock | 1.57494 | 0.16126 |

Test of the difference between coefficients
F(1,134) = 4.73078 (significance level: 0.031)

| Consumer Sentiment Survey Data | | |
|---|---|---|
| | coefficient | std. error |
| tech. shock | 417.02179 | 208.88858 |
| non-tech. shock | 649.84454 | 151.91774 |

Test of the difference between coefficients
F(1,168) = 0.86605 (significance level: 0.353)

Table 2 – Estimate of the effect of identified shocks on expectations

These results lend support to the model's prediction, as both the coefficients for the technology and the non-technology shock are positive and the coefficient for the non-technology shock is larger using both data sets. However, the difference between the two coefficients is only significant when I use the SPF data. This may be due to the fact that the SPF variable is defined in terms of an explicit estimate of aggregate GDP, while the CSS index is an aggregate of qualitative responses, which is more loosely connected to the respondents' quantitative expectations about aggregate output.[44] This also implies that the values of the coefficients have a more meaningful interpretation in the case of the SPF data. In this case, the coefficients can be used for a stronger test of the model's predictions. That is, I can look at the absolute value of the two coefficients and not just at their difference. Also this version of the test provides support to the model's mechanism. In particular, the coefficient on the technology shock is smaller than 1 (although not significantly different from 1), and the coefficient on the non-technology shock is larger than 1 (and here the difference is statistically significant).

To check the robustness of the result, I have repeated the exercise using different VAR specifications to estimate the technology and non-technology shocks, obtaining very similar

[44] The index used here is based on the respondents answers to the question: "Now turning to business conditions in the country as a whole–do you think that during the next twelve months we'll have good times financially, or bad times, or what?" The index is equal to the percentage of positive answers minus the percentage of negative answers (plus 100).

results.[45] I have also tried different measures of expectations, obtaining similar qualitative results, although the difference in the coefficients is only significant when using one and two-quarter-ahead SPF forecasts.[46]

Notice that the VAR identification approach used here bunches together all non-technology shocks as if they were all driven by noise. Richer identification strategies, which are able to tease out monetary shocks and shocks to government spending, may yield sharper conclusions regarding the effect of noise shocks on expectations.

## 6 Concluding remarks

In this paper, I interpret the business cycle as a process of noisy learning by the consumers, who can temporarily overstate or understate the economy's productive capacity. This idea is incorporated into a standard dynamic general equilibrium model and gives rise to noise shocks which have the features of aggregate demand shocks.

For the sake of simplicity, the model makes sharp assumptions on the processes for aggregate and individual technology: the aggregate technology shock is a permanent level shock, the idiosyncratic shock is purely a temporary shock. Both assumptions could be relaxed. Introducing persistent shocks to the growth rate of aggregate TFP, rather than to the level, may help to better capture the uncertainty about medium run swings in productivity growth. This modification may lead to potentially larger consumption responses, given that the same short run increase in TFP would be associated to larger increases in the expected present value of income. Introducing persistence in idiosyncratic shocks would have two effects. On the one hand, it would induce agents to rely more on their private productivity signal, since it is a better predictor of future individual income. This would tend to reduce the effect of noise shocks. On the other hand, it would introduce serial correlation in the private productivity signal, inducing slower learning and possibly increasing the effect of noise shocks.

Also, the model features no capital and has a very limited role for financial markets. This choice was made to concentrate on the consumers' learning dynamics and to introduce dispersed information in the simplest setup. However, uncertainty about long-run technology

[45] I have tried a VAR using total employment (LE) and hours in the non-farm sector (LXNFH), and I have used output in the business and non-farm sectors instead of GDP (LXBO, LXNFO) paired with the respective hours' measure. I have also tried a bivariate VAR of output and inflation, still using a long-run restriction to identify the technology shock.

[46] I have tried the SPF median forecasts of output in the coming 2 to 3 quarters and I have tried the index of "Business Conditions, 5 years," from the Consumer Sentiment Survey.

innovations is clearly crucial for investment decisions. Adding investment may help to generate larger demand responses following a noise shocks, improving the model's ability to fit the data.

Finally, the model requires high levels of idiosyncratic uncertainty in order to generate relatively slow aggregate learning. One reason for this is that the agents in the model know exactly the model's structure and have unlimited capacity to acquire and process information. It would be interesting to extend the model to relax these assumptions. In particular, the model is well suited to the introduction of limited attention *à la* Sims (2003).[47]

The quantitative analysis in Section 4 suggests that noise shocks may generate sizeable levels of short run volatility. However, a number of issues remain open. In particular, how should one calibrate the idiosyncratic noise in the private signals observed by individual agents and the aggregate noise in the public signals? Is it possible to obtain direct measures of noise shocks to test their effect directly?

To calibrate idiosyncratic noise, one possibility is to look at measures of cross-sectional dispersion in quantities and prices.[48] However, this is subject to the caveat that agents' private information is probably richer than just individual price and quantity observations. Moreover, limited attention may imply that not all the private information is efficiently processed to forecast aggregate changes in fundamentals. Another potentially fruitful approach may be to calibrate the idiosyncratic informational parameters by looking at measures of cross-sectional dispersion in expectations, obtained from survey-based data.

To calibrate the noise in aggregate signals, an approach is to use data on revision errors in publicly released aggregate statistics.[49] These can also be used for direct tests of the transmission of noise shocks. Rodriguez Mora and Schulstald (2006) go in this direction and show that aggregate consumption responds more to public announcements regarding aggregate GDP than to actual movements in GDP, a result consistent with the approach in this paper.[50] However, aggregate statistics are only a subset of the public signals available to the private sector. Again, survey-based data on average expectations may help calibrate the total volatility in common noise. They can also be used for direct testing of the model implications, which is the strategy I followed in Section 5.

[47] See Maćkowiak and Wiederholt (2007).

[48] See footnote 26.

[49] See footnote 32.

[50] They look at the effect on aggregate consumption of changes in public statistics regarding GDP, including both the series representing the initial data release and the series for the revised data, which are published later but which are more precisely measured. In this way, they can identify a positive effect due to the noise included in the initial data release.

Finally, an alternative approach is to estimate the model solely looking at its implications for aggregate macroeconomic variables, as it is commonly done in estimated DSGE exercises. In this paper, it was possible to use a simple long-run identification assumption to compare the model with time series evidence on output and employment. In richer models, this is less likely to be feasible and interesting questions open up about estimation and identification in a framework where agents are uncertain about the model parameters.[51]

[51] See Collard, Dellas, and Smets (2007) and Blanchard, L'Hullier, and Lorenzoni (2008).

# 7 Appendix

In this appendix, I provide a more detailed characterization of the equilibrium of the model with dispersed information and describe the algorithm used for computations. The matrices $A$ and $B$ are given by

$$A = \begin{bmatrix} 1 \quad \mathbf{0} \\ 0 \quad \mathbf{0} \\ A_p \\ \begin{pmatrix} 0 & 0 & -\phi & \rho_i & \mathbf{0} \end{pmatrix} + \phi A_p \\ I \quad \mathbf{0} \end{bmatrix}, \quad B = \begin{bmatrix} 1 \quad 0 \quad 0 \\ 0 \quad 1 \quad 0 \\ B_p \\ \phi \left( B_p + \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \right) \\ \mathbf{0} \end{bmatrix},$$

where $A_p$ and $B_p$ are vectors to be determined.

## 7.1 Optimal decision rules

First, let me substitute the demand relation (21) (aggregated across the firms in island $l$) to rewrite the budget constraint (24) as

$$\beta h_{l,t+1} = h_{l,t} + (1-\gamma) p_{l,t} + d_{l,t} - \overline{p}_{l,t} - c_{l,t}, \tag{33}$$

where $d_{l,t}$ is defined by (27).

**Prices.** Using (29) to substitute for $p_{l,t+1}$ on the right-hand side of the optimal pricing condition (26) gives

$$\begin{aligned} \Lambda p_{l,t} \quad = \quad & p_{l,t-1} - \lambda (1+\zeta) a_{l,t} + \lambda \left( \overline{p}_{l,t} + c_{l,t} \right) + \lambda \zeta d_{l,t} + \\ & + \beta \left( q_h h_{l,t+1} + q_p p_{l,t} + q_a E_{l,t} [a_{l,t+1}] + q_d E_{l,t} [d_{l,t+1}] + q_z E_{l,t} [\mathbf{z}_{t+1}] \right) \end{aligned}$$

where $\Lambda \equiv 1 + \beta + \lambda (1 + \gamma\zeta)$. Use the budget constraint (33) to substitute for $h_{l,t+1}$ and (27) to substitute for $d_{l,t+1}$. The expected values of all aggregate variables dated $t+1$ on the right-hand side can be expressed in terms of $\mathbf{z}_t$, using (28), (32), and the fact that $E_{l,t} \left[ \mathbf{u}^1_{t+1} \right] = 0$. Moreover, the expected values of all idiosyncratic shocks dated $t+1$ are zero. Rearranging, I then obtain

$$\begin{aligned} \left( \Lambda - (1-\gamma) q_h - \beta q_p \right) p_{l,t} \quad = \quad & p_{l,t-1} + q_h h_{l,t} - \lambda (1+\zeta) a_{l,t} + \\ & + (\lambda\zeta + q_h) d_{l,t} + (\lambda - q_h) \left( \overline{p}_{l,t} + c_{l,t} \right) + \\ & + \beta \left( q_a \mathbf{e}_x + q_d (\boldsymbol{\psi} + \gamma \mathbf{e}_p) + q_z \right) A E_{l,t} [\mathbf{z}_t] . \end{aligned}$$

where $\mathbf{e}_x$ and $\mathbf{e}_p$ are unitary vectors which select, respectively, $x_t$ and $p_t$ from the state vector $\mathbf{z}_t$. Use (29) to substitute for $p_{l,t}$ on the left-hand side and (30) to substitute for $c_{l,t}$ on the right-hand side. Matching coefficients, this gives

$$q_p = \frac{1}{\Lambda - (1-\gamma) q_h - \beta q_p}, \tag{34a}$$

$$q_h = q_p \left( q_h + (\lambda - q_h) b_h \right), \tag{34b}$$

$$q_a = q_p \left( -\lambda (1+\zeta) + (\lambda - q_h) b_a \right), \tag{34c}$$

$$q_d = q_p \left( \lambda\zeta + q_h + (\lambda - q_h) b_d \right), \tag{34d}$$

$$q_z = q_p \left[ \beta \left( q_a \mathbf{e}_x + q_d (\boldsymbol{\psi} + \gamma \mathbf{e}_p) + q_z \right) A + (\lambda - q_h) b_z \right]. \tag{34e}$$

**Consumption.** Using (30) to substitute for $c_{l,t+1}$ on the right-hand side of the Euler equation (23) gives

$$c_{l,t} = -\overline{p}_{l,t} - i_t + b_h h_{l,t+1} + b_p p_{l,t} + b_a E_{l,t} [a_{l,t+1}] + b_d E_{l,t} [d_{l,t+1}] + b_z E_{l,t} [\mathbf{z}_{t+1}] .$$

As in the case of prices, use the budget constraint (33) to substitute for $h_{l,t+1}$ and (27) to substitute for $d_{l,t+1}$. The expected values of future aggregate variables can be expressed in terms of $\mathbf{z}_t$, using (28), (32), and the fact that $E_{l,t}\left[\mathbf{u}^1_{t+1}\right] = 0$. The expected values of future idiosyncratic shocks are zero. The resulting expression is

$$\begin{aligned} c_{l,t} &= -\overline{p}_{l,t} - i_t + \frac{b_h}{\beta}\left(h_{l,t} + d_{l,t} + (1-\gamma)\,p_{l,t} - \overline{p}_{l,t} - c_{l,t}\right) + \\ &\quad + b_p p_{l,t} + \left(b_a \mathbf{e}_x + b_d\left(\boldsymbol{\psi} + \gamma \mathbf{e}_p\right) + b_z\right) A E_{l,t}\left[\mathbf{z}_t\right], \end{aligned}$$

where $\mathbf{e}_x$ and $\mathbf{e}_p$ are the unitary vectors defined above. Rearranging, gives

$$\begin{aligned} c_{l,t} &= -\overline{p}_{l,t} + \frac{b_h}{\beta + b_h} h_{l,t} + \frac{b_h(1-\gamma) + \beta b_p}{\beta + b_h} p_{l,t} + \frac{b_h}{\beta + b_h} d_{l,t} + \\ &\quad + \frac{\beta}{\beta + b_h}\left[-\mathbf{e}_i + \left(b_a \mathbf{e}_x + b_d\left(\boldsymbol{\psi} + \gamma \mathbf{e}_p\right) + b_z\right) A\right] E_{l,t}\left[\mathbf{z}_t\right], \end{aligned}$$

where $\mathbf{e}_i$ is the unitary vectors which selects $i_t$ from $\mathbf{z}_t$. Use (30) to substitute for $c_{l,t}$ on the left-hand side and (29) to substitute for $p_{l,t}$ on the right-hand side. Matching coefficients, this gives

$$b_h = \frac{b_h}{\beta + b_h} + \varkappa q_h, \tag{35a}$$

$$b_p = \varkappa q_p, \tag{35b}$$

$$b_a = \varkappa q_a, \tag{35c}$$

$$b_d = \frac{b_h}{\beta + b_h} + \varkappa q_d, \tag{35d}$$

$$b_z = \frac{\beta}{\beta + b_h}\left[-\mathbf{e}_i + \left(b_a \mathbf{e}_x + b_d\left(\boldsymbol{\psi} + \gamma \mathbf{e}_p\right) + b_z\right) A\right] + \varkappa q_z, \tag{35e}$$

where

$$\varkappa \equiv \frac{b_h(1-\gamma) + \beta b_p}{\beta + b_h}.$$

## 7.2 Individual inference

To find the Kalman gains for the individual learning problem notice that the vector of signals $\mathbf{s}_{l,t} = \begin{pmatrix} a_{l,t} & s_t & \overline{p}_{l,t} & d_{l,t} & i_t \end{pmatrix}'$ can be written as

$$\mathbf{s}_{l,t} = F\mathbf{z}_t + G\mathbf{u}^2_{l,t}$$

where

$$\mathbf{u}^2_{l,t} \equiv \begin{pmatrix} \eta_{l,t} & \xi^1_{l,t} & \xi^2_{l,t} \end{pmatrix}',$$

the matrices $F$ and $G$ are

$$F \equiv \begin{bmatrix} \mathbf{e}_x \\ \mathbf{e}_x + \mathbf{e}_e \\ \mathbf{e}_p \\ \boldsymbol{\psi} + \gamma \mathbf{e}_p \\ \mathbf{e}_i \end{bmatrix}, G \equiv \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix},$$

$\mathbf{e}_x, \mathbf{e}_p$, and $\mathbf{e}_i$ where defined above, and $\mathbf{e}_e$ is the unitary vector which selects $e_t$ from $\mathbf{z}_t$. Bayesian updating for island $l$'s agents implies that

$$E_{l,t}\left[\mathbf{z}_t\right] = E_{l,t-1}\left[\mathbf{z}_t\right] + C\left(\mathbf{s}_{l,t} - E_{l,t-1}\left[\mathbf{s}_{l,t}\right]\right).$$

Define the variance-covariance matrices

$$\Sigma = \begin{bmatrix} \sigma_\epsilon^2 & 0 & 0 \\ 0 & \sigma_\nu^2 & 0 \\ 0 & 0 & \sigma_\omega^2 \end{bmatrix}, V = \begin{bmatrix} \sigma_\eta^2 & 0 & 0 \\ 0 & \sigma_{\xi,1}^2 & 0 \\ 0 & 0 & \sigma_{\xi,2}^2 \end{bmatrix},$$

and let $\Omega$ be defined as

$$\Omega = Var_{l,t-1}\left[\mathbf{z}_t\right].$$

Then the Kalman gains $C$ are given by

$$C = \Omega F' \left(F\Omega F' + GVG'\right)^{-1}, \tag{36}$$

and $\Omega$ must satisfy the Riccati equation

$$\Omega = A\left(\Omega - CF\Omega\right)A' + B\Sigma B'. \tag{37}$$

## 7.3 Fixed point

The average first order expectations regarding the state $\mathbf{z}_t$ can be expressed as a function of $\mathbf{z}_t$ itself as

$$\mathbf{z}_{t|t} = \Xi \mathbf{z}_t.$$

Using the updating equations and aggregating across consumers gives:

$$\mathbf{z}_{t|t} = \left(I - CF\right) A\mathbf{z}_{t-1|t-1} + CF\mathbf{z}_t.$$

Therefore, the matrix $\Xi$ must satisfy the condition

$$\Xi\mathbf{z}_t = \left(I - CF\right) A\Xi\mathbf{z}_{t-1} + CF\mathbf{z}_t, \tag{38}$$

for all $\mathbf{z}_t$. Aggregating the individual decision rules (29) and (30), I then obtain

$$\begin{aligned} p_t &= q_p p_{t-1} + q_a x_t + q_d\left(c_t + \gamma p_t\right) + q_z\Xi\mathbf{z}_t, \\ c_t &= -p_t + b_p p_{t-1} + b_a x_t + b_d\left(c_t + \gamma p_t\right) + b_z\Xi\mathbf{z}_t. \end{aligned}$$

Expressing everything in terms of the state $\mathbf{z}_t$, the equilibrium coefficients must satisfy

$$\left[\mathbf{e}_p - q_p\mathbf{e}_{p_{-1}} - q_a\mathbf{e}_x - q_d\left(\boldsymbol{\psi} + \gamma\mathbf{e}_p\right) - q_z\Xi\right]\mathbf{z}_t = 0, \tag{39}$$

$$\left[\mathbf{e}_p + \boldsymbol{\psi} - b_p\mathbf{e}_{p_{-1}} - b_a\mathbf{e}_x - b_d\left(\boldsymbol{\psi} + \gamma\mathbf{e}_p\right) - b_z\Xi\right]\mathbf{z}_t = 0, \tag{40}$$

for all $\mathbf{z}_t$.

An equilibrium is characterized by the vectors $A_p, B_p, \boldsymbol{\psi}$ describing the aggregate dynamics, the vectors $\{q_h, q_p, q_a, q_d, q_z\}$, and $\{b_h, b_p, b_a, b_d, b_z\}$ describing individual behavior, the matrices $C$ and $\Omega$ describing the individual learning problem, and the matrix $\Xi$ capturing the aggregate behavior of first order expectations. These objects characterize an equilibrium if they satisfy conditions (34)-(40).

## 7.4 Computation

To compute an equilibrium I apply the following algorithm. I start with some initial value for $\{A_p, B_p, \boldsymbol{\psi}, \}$. I derive the values of $\{q_h, q_p, q_a, q_d, q_z\}$ and $\{b_h, b_p, b_a, b_d, b_z\}$ which satisfy individual optimality, by substituting the prices and quantities (29) and (30) into (26), (23), and (24). Next, I solve for $C$ and $\Omega$ in the individual inference problem. Since the vector $\mathbf{z}_t^{[T]}$ is truncated, I set to zero the value of $z_{t-T-1}$ in $\mathbf{z}_{t-1}^{[T]}$ and replace (38) with

$$\Xi\mathbf{z}_t^{[T]} = \left(I - CF\right) A\Xi M\mathbf{z}_t^{[T]} + CF\mathbf{z}_t^{[T]},$$

where

$$M \equiv \begin{bmatrix} \mathbf{0} & I \\ \mathbf{0} & \mathbf{0} \end{bmatrix}.$$

This gives the following relation, which is used iteratively to compute $\Xi$,

$$\Xi = (I - CF)\, A\Xi M + CF.$$

I then apply the updating rule

$$\begin{aligned} A_p &= \left(q_p \mathbf{e}_{p_{-1}} + q_a \mathbf{e}_x + q_d \left(\boldsymbol{\psi} + \gamma \mathbf{e}_p\right) + q_z \Xi\right) A, \\ B_p &= \left(q_p \mathbf{e}_{p_{-1}} + q_a \mathbf{e}_x + q_d \left(\boldsymbol{\psi} + \gamma \mathbf{e}_p\right) + q_z \Xi\right) B, \\ \boldsymbol{\psi} &= b_p \mathbf{e}_{p_{-1}} + b_a \mathbf{e}_x + b_d \left(\boldsymbol{\psi} + \gamma \mathbf{e}_p\right) + b_z \Xi - \mathbf{e}_p, \end{aligned}$$

and repeat until convergence is achieved. The convergence criterion is given by the quadratic distance of the impulse-response functions of $y_t$ and $p_t$ to the shocks in $\mathbf{u}_t^1$ (with weights given by the variances of the shocks), under the old and updated values of $\{A_p, B_p, \boldsymbol{\psi}\}$.

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# 8 Supplementary material

## 8.1 Log-linearization

First, let me derive the optimality conditions in their original form. The Euler equation is

$$Q_t \frac{1}{\overline{P}_{l,t} C_{l,t}} = \beta E_{l,t} \left[ \frac{1}{\overline{P}_{l,t+1} C_{l,t+1}} \right], \tag{41}$$

the consumer budget constraint is, after some substitutions,

$$Q_t B_{l,t+1} + \overline{P}_{l,t} C_{l,t} = B_{l,t} + P_{l,t} Y_{l,t}, \tag{42}$$

the firm's optimality condition is

$$E_{l,t} \left[ \sum_{\tau=t}^{\infty} \theta^{\tau} Q^{l}_{t+\tau|t} \left( (\gamma - 1) P^*_{l,t} Y_{j,l,t+\tau} - \gamma \frac{W_{l,t+\tau}}{A_{l,t+\tau}} \frac{Y_{j,l,t+\tau}}{A_{l,t+\tau}} \right) \frac{Y_{j,l,t+\tau}}{P^*_{l,t}} \right] = 0. \tag{43}$$

As a reference point, I will consider the stochastic equilibrium of an economy where $\sigma^2_{\epsilon}$ is the same as in the economy I want to study, but all other variances are set to zero, implying that (i) there is no heterogeneity and (ii) there is complete information. Using stars to denote variables in the reference economy, I have $A^*_{l,t} = A_t \equiv e^{x_t}$ by definition. It is easy to show that in equilibrium

$$C^*_{l,t} = \bar{C} A_t, \quad Y^*_{j,l,t} = \bar{Y} A_t, \quad \frac{W^*_{l,t}}{P^*_{l,t}} = \bar{W} A_t,$$

the prices $P^*_{j,l,t}$ are all constant, the nominal bond price $Q^*_t$ is constant and equal to $\beta e^{-\frac{1}{2}\sigma^2_{\epsilon}}$, and bond holdings are constant and equal to zero. To check that these values form an equilibrium and to derive the values of the scalars $\bar{C}$, $\bar{Y}$, and $\bar{W}$, it is sufficient to substitute these expressions in (41)-(43).

Now let me go back to the economy with heterogeneity and dispersed information. I will focus on equilibria that have the following property: conditional on a given sequence of realized shocks $\{\epsilon_t\}$, all equilibrium quantities, relative prices, and rates of inflation move in a neighborhood of the corresponding values in the reference economy. Denote with hats log deviations of quantities and relative prices from their values in the reference economy, e.g., $\hat{c}_{l,t} \equiv \ln C_{l,t} - \ln C^*_{l,t}$. For the rates of inflation, let $\pi_{l,t}, \overline{\pi}_{l,t}$ and $\pi_t$ be defined, respectively, as $\pi_{l,t} \equiv \ln P_{l,t} - \ln P_{l,t-1}$, $\overline{\pi}_{l,t} \equiv \ln \overline{P}_{l,t} - \ln \overline{P}_{l,t-1}$, and $\pi_t = \ln P_t - \ln P_{t-1}$. Finally, recall that $h_{l,t} \equiv B_{l,t}/E_{l,t}[P_t Y_t]$. The linearized versions of (41)-(43) are obtained by taking Taylor expansions with respect to these variables, which are all zero in the reference economy.

To derive the log-linearized version of (41) multiply both sides by $\overline{P}_{l,t} \bar{C} A_{l,t}$ and take expectations, to obtain

$$Q_t E_{l,t} \left[ \frac{\bar{C} A_{l,t}}{C_{l,t}} \right] = \beta E_{l,t} \left[ \frac{\overline{P}_{l,t}}{\overline{P}_{l,t+1}} \frac{\bar{C} A_{l,t}}{C_{l,t+1}} \right],$$

which can be rewritten as

$$Q_t E_{l,t} \left[ \frac{\bar{C} A_t}{C_{l,t}} \frac{A_{l,t}}{A_t} \right] = \beta E_{l,t} \left[ \frac{\overline{P}_{l,t}}{\overline{P}_{l,t+1}} \frac{\bar{C} A_{t+1}}{C_{l,t+1}} \frac{A_{l,t}}{A_t} \frac{A_t}{A_{t+1}} \right],$$

and then as

$$Q_t E_{l,t} \left[ e^{-\hat{c}_{l,t} + \eta_{l,t}} \right] = E_{l,t} \left[ e^{-\overline{\pi}_{l,t+1} - \hat{c}_{l,t+1} + \eta_{l,t}} \right] \beta e^{-\frac{1}{2}\sigma^2_{\epsilon}}.$$

This gives the approximate relation

$$E_{l,t} \left[ \hat{c}_{l,t} + \eta_{l,t} \right] = -i_t + E_{l,t} \left[ \overline{\pi}_{l,t+1} + \hat{c}_{l,t+1} + \eta_{l,t} \right]. \tag{44}$$

The terms $E_{l,t}\left[\eta_{l,t}\right]$ on both sides cancel out. Moreover, the random walk assumption for $x_t$ implies that $E_{l,t}\left[a_t\right] = E_{l,t}\left[a_{t+1}\right] = E_{l,t}\left[x_t\right]$. Adding $\ln \bar{C} + E_{l,t}\left[x_t\right]$ on both sides of (44) then gives

$$E_{l,t}\left[c_{l,t}\right] = E_{l,t}\left[c_{l,t+1}\right] - i_t + E_{l,t}\left[\overline{p}_{l,t+1} - \overline{p}_{l,t}\right].$$

Given that $c_{l,t}$ and $\overline{p}_{l,t}$ are in the information set of consumer $l$ at time $t$, this gives (23) in the main text.

Proceeding in a similar way, (42) and (43) can be transformed to obtain (24) and (25).

## 8.2 Extension with decreasing returns and variable capacity

### 8.2.1 Profits and marginal costs

It is useful to introduce the following change of variables

$$V_{j,l,t} \equiv U_{j,l,t}^{\frac{1}{1+\chi}}.$$

Then, the technology takes the form

$$Y_{j,l,t} = A_{l,t} V_{j,l,t}^{\xi} N_{j,l,t}^{1-\alpha},$$

where $\xi \equiv \alpha / (1+\chi) \leq \alpha$. The profits of firm $j, l$ are

$$\Pi_{j,l,t} = P_{j,l,t} A_{l,t} V_{j,l,t}^{\xi} N_{j,l,t}^{1-\alpha} - \chi_0 P_{l,t} A_{l,t} V_{j,l,t} - W_{l,t} N_{j,l,t}.$$

Total costs minimization shows that

$$\chi_0 P_{l,t} A_{l,t} V_{j,l,t} + W_{l,t} N_{j,l,t} = \chi_0^{\tilde{\alpha}} \tilde{\alpha}^{-\tilde{\alpha}} (1-\tilde{\alpha})^{-(1-\tilde{\alpha})} (P_{l,t} A_{l,t})^{\tilde{\alpha}} W_{l,t}^{1-\tilde{\alpha}} \left(\frac{Y_{j,l,t}}{A_{l,t}}\right)^{1+\mu}$$

where

$$\begin{aligned} \tilde{\alpha} &\equiv \frac{\xi}{\xi + 1 - \alpha}, \\ \mu &\equiv \frac{\alpha - \xi}{\xi + 1 - \alpha}. \end{aligned}$$

The marginal cost of firm $j, l$ is then equal to

$$MC_{j,l,t} = \mu_0 \frac{1}{A_{l,t}} (P_{l,t} A_{l,t})^{\tilde{\alpha}} W_{l,t}^{1-\tilde{\alpha}} \left(\frac{Y_{j,l,t}}{A_{l,t}}\right)^{\mu}$$

where $\mu_0$ is a constant term. Taking logs and using the demand function for good $j, l$, this gives

$$mc_{j,l,t} = mc_{l,t} + \gamma\mu \left(p_{l,t} - p_{j,l,t}\right),$$

where $mc_{l,t}$ is given by

$$mc_{l,t} = p_{l,t} + (1-\tilde{\alpha})\left(w_{l,t} - p_{l,t} - a_{l,t}\right) + \mu\left(y_{l,t} - a_{l,t}\right). \tag{45}$$

### 8.2.2 Labor market equilibrium

The cost minimization problem analyzed above, gives the relative input demand

$$\frac{V_{j,l,t}}{N_{j,l,t}} = \frac{\xi}{1-\alpha}\frac{W_{l,t}}{P_{l,t}A_{l,t}}.$$

Substituting in the production function gives

$$Y_j = A_{l,t}\left(\frac{\xi}{1-\alpha}\frac{W_{l,t}}{P_{l,t}A_{l,t}}\right)^{\xi} N_{j,l,t}^{1-\alpha+\xi},$$

which, in logs, gives

$$n_{j,l,t} = (1+\mu)(y_{j,l,t} - a_{l,t}) - \tilde{\alpha}(w_{l,t} - p_{l,t} - a_{l,t}),$$

and, aggregating across firms in island $l$,

$$n_{l,t} = (1+\mu)(y_{l,t} - a_{l,t}) - \tilde{\alpha}(w_{l,t} - p_{l,t} - a_{l,t}).$$

The optimal labor supply condition is

$$w_{l,t} = c_{l,t} + \overline{p}_{l,t} + \zeta n_{l,t}.$$

Substituting and rearranging gives the following expression for real wages in island $l$:

$$w_{l,t} - p_{l,t} = a_{l,t} + \frac{1}{1+\tilde{\alpha}\zeta}\left(\overline{p}_{l,t} + c_{l,t} - p_{l,t} - a_{l,t} + \zeta(1+\mu)(y_{l,t} - a_{l,t})\right), \tag{46}$$

and the following expression for the capacity-input $v_{l,t}$:

$$\begin{aligned} v_{l,t} &= n_{l,t} + w_{l,t} - p_{l,t} - a_{l,t} = \\ &= \left(1+\mu+\frac{1-\tilde{\alpha}}{1+\tilde{\alpha}\zeta}\zeta(1+\mu)\right)(y_{l,t} - a_{l,t}) + \frac{1-\tilde{\alpha}}{1+\tilde{\alpha}\zeta}\left(\overline{p}_{l,t} + c_{l,t} - p_{l,t} - a_{l,t}\right). \end{aligned} \tag{47}$$

I can also derive an expression for aggregate employment which is

$$n_t = (1+\mu)(y_t - a_t) - \frac{\tilde{\alpha}}{1+\tilde{\alpha}\zeta}(c_t - a_t + \zeta(1+\mu)(y_t - a_t))$$

using (49), derived below, this gives

$$n_t = \left(\frac{1+\mu}{1+\tilde{\alpha}\zeta}\frac{\tau(1-\tilde{\alpha}) + (1-\tau)(1+\tilde{\alpha}\zeta)}{1+\tilde{\alpha}\zeta + \tau(1+\mu)(1+\zeta)} - \tilde{\alpha}\right)(c_t - a_t).$$

### 8.2.3 Firm-level demand

The demand for the output of firm $j, l$ now comes from two sources: other producers located in island $l$ and final consumers:

$$Y_{j,l,t} = \int_0^1 \left(\frac{P_{j,l,t}}{P_{l,t}}\right)^{-\gamma} \chi_0 A_{l,t} V_{j',l,t} dj' + \int_{\tilde{\mathcal{L}}_{l,t}} \left(\frac{P_{j,l,t}}{\overline{P}_{\tilde{l},t}}\right)^{-\gamma} C_{\tilde{l},t} d\tilde{l}.$$

In log approximation this gives

$$y_{j,l,t} = \tau(a_{l,t} + v_{l,t} - \gamma(p_{j,l,t} - p_{l,t})) + (1-\tau)\left(c_t - \gamma(p_{j,l,t} - p_t) + \xi_{l,t}^2\right),$$

where the consumers' demand factor is

$$d_{l,t} \equiv c_t + \gamma p_t + \xi^2_{l,t},$$

and the ratio $\tau$ is equal to the value of $\chi_0 A^*_t V^*_t / Y^*_t$ in the reference equilibrium with no heterogeneity. Aggregating across producers in island $l$ gives

$$y_{l,t} = \tau \left(a_{l,t} + v_{l,t}\right) + (1-\tau)\left(d_{l,t} - \gamma p_{l,t}\right).$$

Substituting (47) and rearranging gives

$$y_{l,t} - a_{l,t} = \nu_1 \left(\overline{p}_{l,t} + c_{l,t} - p_{l,t} - a_{l,t}\right) + \nu_2 \left(d_{l,t} - \gamma p_{l,t} - a_{l,t}\right), \tag{48}$$

where

$$\begin{aligned}
\nu_1 &\equiv \frac{1}{1+\tau(1+\mu)\frac{1+\zeta}{1+\tilde{\alpha}\zeta}} \tau \frac{1-\tilde{\alpha}}{1+\tilde{\alpha}\zeta}, \\
\nu_2 &\equiv \frac{1}{1+\tau(1+\mu)\frac{1+\zeta}{1+\tilde{\alpha}\zeta}} (1-\tau).
\end{aligned}$$

Aggregating gives a relation between $y_t - a_t$ and $c_t - a_t$:

$$y_t - a_t = \frac{\tau \frac{1-\tilde{\alpha}}{1+\tilde{\alpha}\zeta} + 1 - \tau}{1+\tau(1+\mu)\frac{1+\zeta}{1+\tilde{\alpha}\zeta}} \left(c_t - a_t\right). \tag{49}$$

#### 8.2.4 Optimal prices

Optimality for firm $j$ who can update at date $t$ gives

$$\begin{aligned}
p^*_{l,t} &= (1-\beta\theta) \sum (\beta\theta)^\tau E_{l,t}\left[mc_{j,l,t+\tau}\right] = \\
&= (1-\beta\theta) \sum (\beta\theta)^\tau E_{l,t}\left[mc_{l,t+\tau} + \gamma\mu p_{l,t}\right] - \gamma\mu p^*_{l,t},
\end{aligned}$$

which can be rewritten in recursive form as

$$p^*_{l,t} = \frac{1-\beta\theta}{1+\gamma\mu}\left(mc_{l,t} + \mu\gamma p_{l,t}\right) + \beta\theta E_{l,t}\left[p^*_{l,t+1}\right].$$

Combining this with

$$p_{l,t} = \theta p_{l,t-1} + (1-\theta)\, p^*_{l,t},$$

and rearranging, I obtain

$$p_{l,t} - p_{l,t-1} = \frac{1-\theta}{\theta}\frac{1-\beta\theta}{1+\mu\gamma}\left(mc_{l,t} - p_{l,t}\right) + \beta E_{l,t}\left[p_{l,t+1} - p_{l,t}\right].$$

Using (45) and (46), I get

$$mc_{l,t} - p_{l,t} = \frac{1-\tilde{\alpha}}{1+\tilde{\alpha}\zeta}\left(\overline{p}_{l,t} + c_{l,t} - p_{l,t} - a_{l,t}\right) + \left(\frac{1-\tilde{\alpha}}{1+\tilde{\alpha}\zeta}\zeta(1+\mu) + \mu\right)\left(y_{l,t} - a_{l,t}\right).$$

Finally, using (48) and rearranging, I obtain

$$p_{l,t} - p_{l,t-1} = \kappa_1 \left(\overline{p}_{l,t} + c_{l,t} - p_{l,t} - a_{l,t}\right) + \kappa_2 \left(d_{l,t} - \gamma p_{l,t} - a_{l,t}\right) + \beta E_{l,t}\left[p_{l,t+1} - p_{l,t}\right], \tag{50}$$

where

$$\begin{aligned}
\kappa_1 &\equiv \frac{1-\theta}{\theta}\frac{1-\beta\theta}{1+\mu\gamma}\left(\frac{1-\tilde{\alpha}}{1+\tilde{\alpha}\zeta} + \nu_1\left(\frac{1-\tilde{\alpha}}{1+\tilde{\alpha}\zeta}\zeta(1+\mu) + \mu\right)\right), \\
\kappa_2 &\equiv \frac{1-\theta}{\theta}\frac{1-\beta\theta}{1+\mu\gamma}\left(\frac{1-\tilde{\alpha}}{1+\tilde{\alpha}\zeta}\zeta(1+\mu) + \mu\right)\nu_2.
\end{aligned}$$

### 8.2.5 Net nominal income

The budget constraint now takes the form

$$Q_t B_{l,t+1} + \overline{P}_{l,t} C_{l,t} = B_{l,t} + NI_{l,t},$$

or, in log-linearized form

$$\beta h_{l,t+1} = h_{l,t} + ni_{l,t} - \overline{p}_{l,t} - c_{l,t}. \tag{51}$$

$NI_{l,t}$ represents net nominal income in island $l$ (the sum of nominal wages and profits), which can be written as

$$NI_{l,t} = W_{l,t} N_{l,t} + \int_0^1 \Pi_{j,l,t} dj,$$

or, equivalently, as

$$NI_{l,t} = \int_0^1 \left[ P_{j,l,t} Y_{j,l,t} - \chi_0 P_{l,t} A_{l,t} V_{j,l,t} \right] dj.$$

After substituting the demand for $V_{j,l,t}$ in this expression and taking a log-linear approximation, I obtain

$$ni_{j,l,t} = \frac{1}{1-\tau} \left( p_{j,l,t} + y_{j,l,t} \right) - \frac{\tau}{1-\tau} \left( p_{l,t} + a_{l,t} + \left(1-\tilde{\alpha}\right) \left( w_{l,t} - p_{l,t} - a_{l,t} \right) + \mu \left( y_{j,l,t} - a_{l,t} \right) \right).$$

Aggregating across $j$ and substituting (46)

$$\begin{aligned} ni_{l,t} \quad = \quad & p_{l,t} + a_{l,t} + \frac{1}{1-\tau} \left( y_{l,t} - a_{l,t} \right) + \\ & - \frac{\tau}{1-\tau} \frac{1-\tilde{\alpha}}{1+\tilde{\alpha}\zeta} \left( \overline{p}_{l,t} + c_{l,t} - p_{l,t} - a_{l,t} + \zeta \left(1+\mu\right) \left( y_{l,t} - a_{l,t} \right) \right) - \frac{\tau}{1-\tau} \mu \left( y_{l,t} - a_{l,t} \right). \end{aligned}$$

Substituting (48) and rearranging gives

$$ni_{l,t} = p_{l,t} + a_{l,t} + \upsilon_1 \left( \overline{p}_{l,t} + c_{l,t} - p_{l,t} - a_{l,t} \right) + \upsilon_2 \left( d_{l,t} - \gamma p_{l,t} - a_{l,t} \right), \tag{52}$$

with

$$\begin{aligned} \upsilon_1 \quad & \equiv \quad \left( \frac{1}{1-\tau} - \frac{\tau}{1-\tau} \left( \mu + \zeta \left(1+\mu\right) \frac{1-\tilde{\alpha}}{1+\tilde{\alpha}\zeta} \right) \right) \nu_1 - \frac{\tau}{1-\tau} \frac{1-\tilde{\alpha}}{1+\tilde{\alpha}\zeta}, \\ \upsilon_2 \quad & \equiv \quad \left( \frac{1}{1-\tau} - \frac{\tau}{1-\tau} \left( \mu + \zeta \left(1+\mu\right) \frac{1-\tilde{\alpha}}{1+\tilde{\alpha}\zeta} \right) \right) \nu_2. \end{aligned}$$

### 8.2.6 Optimal decision rules

The individual decision rules take the form (29) and (30) as in the baseline model. The consumer Euler equation takes the form (23) as in the baseline model. The optimal pricing condition and the budget constraint, instead, take the form (50) and (51).

**Prices** Substitute (29) on the right-hand side of (50) and rearrange to get

$$\begin{aligned} \Lambda p_{l,t} \quad = \quad & p_{l,t-1} - \left( \kappa_1 + \kappa_2 \right) a_{l,t} + \kappa_1 \left( \overline{p}_{l,t} + c_{l,t} \right) + \kappa_2 d_{l,t} + \\ & + \beta \left( q_h h_{l,t+1} + q_p p_{l,t} + \left( q_a \mathbf{e}_x + q_d \left( \boldsymbol{\psi} + \gamma \mathbf{e}_p \right) + q_z \right) A E_{l,t} \left[ \mathbf{z}_t \right] \right) \end{aligned} \tag{53}$$

where

$$\Lambda \equiv 1 + \beta + \kappa_1 + \gamma \kappa_2.$$

Using and (51) and (52), gives me

$$\beta h_{l,t+1} = h_{l,t} + p_{l,t} + a_{l,t} + \upsilon_1 \left(\overline{p}_{l,t} + c_{l,t} - p_{l,t} - a_{l,t}\right) + \upsilon_2 \left(d_{l,t} - \gamma p_{l,t} - a_{l,t}\right) - \overline{p}_{l,t} - c_{l,t}. \tag{54}$$

Substitute (54) in (53), (29) on the right-hand side of (53), and match coefficients to get

$$q_p = \frac{1}{\Lambda - q_h + q_h\left(\upsilon_1 + \gamma\upsilon_2\right) - \beta q_p}, \tag{55a}$$

$$q_h = q_p\left[q_h + \varpi b_h\right], \tag{55b}$$

$$q_a = q_p\left[q_h - (\kappa_1 + \kappa_2) - q_h\left(\upsilon_1 + \upsilon_2\right) + \varpi b_a\right], \tag{55c}$$

$$q_d = q_p\left[\kappa_2 + q_h\upsilon_2 + \varpi b_d\right], \tag{55d}$$

$$q_z = q_p\left[\beta\left(q_a\mathbf{e}_x + q_d\left(\boldsymbol{\psi} + \gamma\mathbf{e}_p\right) + q_z\right)A + \varpi b_z\right], \tag{55e}$$

where

$$\varpi \equiv \kappa_1 - q_h\left(1 - \upsilon_1\right).$$

**Quantities** Similar steps on the consumption side lead to

$$b_h = \frac{b_h}{\beta + b_h\left(1 - \upsilon_1\right)} + \varkappa q_h, \tag{56a}$$

$$b_p = \varkappa q_p, \tag{56b}$$

$$b_a = \frac{b_h}{\beta + b_h\left(1 - \upsilon_1\right)}\left(1 - \upsilon_1 - \upsilon_2\right) + \varkappa q_a, \tag{56c}$$

$$b_d = \frac{b_h}{\beta + b_h\left(1 - \upsilon_1\right)}\upsilon_2 + \varkappa q_d, \tag{56d}$$

$$b_z = \frac{\beta}{\beta + b_h\left(1 - \upsilon_1\right)}\left[-\mathbf{e}_i + \left(b_a\mathbf{e}_x + b_d\left(\boldsymbol{\psi} + \gamma\mathbf{e}_p\right) + b_z\right)A\right] + \varkappa q_z, \tag{56e}$$

where

$$\varkappa \equiv \frac{b_h\left(1 - \upsilon_1 - \gamma\upsilon_2\right) + \beta b_p}{\beta + b_h\left(1 - \upsilon_1\right)}.$$

The rest of the computation proceeds as in the main model (treated in the main Appendix), except that (34) and (10) are replaced by (55) and (56).