Loss Given Default and Economic Capital

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Introduction

Lenders know that any loan might default. They also know that the default rate of any portfolio can rise. When the default rate rises, the portfolio might experience losses. Lenders must have capital to absorb the possible loss.

A second effect can compound\ the loss. This is the variation in loss given default, or LGD. LGD is the fraction of exposure lost when a loan defaults. Lenders find that when the default rate rises, the LGD rate tends to rise as well. When thinking about possible loss, variation of the default rate is only half the picture. The variation of LGD can be just as important.

This chapter develops a framework for understanding the coordinated rise of the default rate and the LGD rate and their effect on economic capital. Both rates connect to an underlying systematic "risk factor." As will be seen, this brings into being two distributions of LGD. These distributions play distinct roles in the analysis that leads back to economic capital. In this analysis, two pitfalls stem from themes developed earlier, and each leads to an understatement of risk. These pitfalls are ignoring a source of systematic risk while measuring LGD and conflating the two distributions of LGD while estimating expected LGD.

Loss given default and ELGD

We begin with a simple model of a portfolio containing loans or other products that might default. The definition of default might differ between products, or it might allow discretion in determining whether a default has occurred. At any given time, though, we assume that either a particular loan has defaulted or it has not.

For a defaulted loan, loss given default (LGD) is the proportion of exposure that is lost. LGD is an economic concept; it does not necessarily correspond to the amounts reported under current financial reporting practices. Usually, LGDs are imagined as taking values between zero and one.

This seemingly straightforward definition of LGD contains subtleties that complicate measuring LGD in specific situations. Rather than focus on these, we stay with the broad properties of LGD. We assume that in the absence of default there is neither loss nor potential for gain. Thus, there is no loss due to downgrade and no gain due to upgrade. We also assume that the relevant economic loss can be measured.

Until LGD is measured, it is a random variable. Much of what follows involves an exploration of the distribution of random LGD, but an important observation can be made at the outset.

The observation stems from the definition: LGD is independent of default. Two random variables are independent if knowledge of the value of one of them tells nothing about the value of the other. In this case, the first random variable is the default indicator. The default indicator equals one in the event of default and equals zero otherwise. The second random variable is LGD. LGD is imagined, before default, to have some expected value, distribution, or set of likely values. If the loan in fact defaults, there is no effect on the expected value, distribution, or set of likely values—consistent with its name, loss *given* default imagines the default event to have occurred. If the imagined event becomes real, there is no new information; the occurrence of the default has no effect on the distribution of LGD. By definition, LGD is independent of the default event that brings it into being.[1]

The independence of LGD and default allows a simple bit of math that provides some insight. Credit loss requires a default; then, loss is equal to LGD. Stated symbolically,

(1) $$L = D \cdot LGD$$

where L is the loss on a loan, and D is the default indicator. Since the factors on the right hand side are independent, the expectation operator passes through as follows:

(2) $$E\,[L] = E\,[D] \cdot E\,[LGD], \text{ or } EL = PD \cdot ELGD$$

This says that the expected loss on a loan equals its expected default rate (usually denoted "PD" for "probability of default") times the loan's expected LGD rate. For example, if a loan has probability of default equal to 5% and expected LGD equal to 40%, its expected loss is 2%.

It is important to note the difference between the symbols: LGD is a random variable that has some distribution, and ELGD is the expectation of that random variable. Thus, ELGD is a moment or population parameter, and LGD itself is random. Similarly, PD is the population parameter of the random variable D. It is also important to note that (2) is specific to a given loan. Therefore, the expectation is of the distribution of the LGD of the loan. That distribution depends on the loan's distribution of default, as shown in a later section.

As we have seen, there is independence between an LGD and the default that brings it into being. Correlation does enter the picture, but only at the portfolio level. A given LGD might be correlated with other LGDs or with other defaults. As a practical matter, if in a given year there have been a number of defaults involving a certain kind of collateral, and if the LGD on these defaults has been greater than usual, then it is likely that subsequent defaults involving similar collateral will also produce LGDs that are greater than usual. Separately, an unusually large number of defaults, by itself, might also bring about LGDs that are greater than usual. Thus, "correlated LGD" stems from analysis at the portfolio level. This analysis uses portfolio risk models that provide an estimate of economic credit capital.

[1] Other variables, such as the value of collateral, might depend on the default event. In addition, LGD might depend on anything but the default event. For example, the reason for default (say, fraud by senior management) may tell something about LGD.

Economic credit capital

The concept of economic capital quantifies the risk faced by a financial institution over a defined period—usually one year. All types of loss, not just credit loss, are included in the economic capital concept. While keeping this in mind, this chapter analyzes economic credit capital but refers simply to economic capital.

Economic capital is analogous to the value-at-risk (VaR) analysis used to control risk on financial trading floors. Like market VaR, economic capital is a high percentile of the loss distribution.[2] This chapter mimics market VaR analysis in another way as well. We assume that the portfolio has its exposures for the entire analysis period. This means there are no deals maturing before the end of the year, and as a consequence there is no reinvestment risk. The risk to the institution is equal to the risk in its current portfolio.

This chapter illustrates economic capital using the 99.9th percentile loss. This means the loss occurring once in a millennium—***if*** the model reflects all the elements of reality. But a mathematical model simplifies and stylizes reality. Therefore, the nominal percentile is meaningful to the model, but it has a weaker relationship to the reality being modeled. The situation resembles the role of "implied volatility" in option pricing models. Different models, which stylize the market in different ways, produce different implied volatilities. A given implied volatility has meaning only in the context of the model that implies it. In a similar way, the percentile of a loss distribution has meaning in the context of the model that produces the distribution. The probability that the actual portfolio loses more than economic capital could be greater or less than 0.10%.

Still, the loss being estimated is far beyond the loss recorded at the average bank. Therefore, the economic capital model necessarily performs a kind of extrapolation based on a statistical distribution of losses.

This loss distribution depends on so-called "risk factors". In a market VaR model the factors are easily conceived; they are the prices of securities or derivatives that closely resemble portfolio holdings. In an economic capital model, the risk factors are more abstract. They are simply the driving forces behind variations in default and LGD—whatever makes the rates rise and fall. There is no need to reduce the risk factors to transformations of economic data such as GDP, stock prices, or interest rates.

Asymptotic single risk factor models

The simplest economic capital model employs only a single risk factor that affects other quantities in the model.[3] This risk factor is a random variable that subjects the portfolio to periods of greater or lesser credit loss. We refer to the risk factor as Z, and allow it to affect both the default rate and the LGD rate. One can imagine the Z's at different banks to be more or less the same, especially if the banks are large and well diversified.

[2] An alternative definition of economic capital removes EL from the loss distribution before finding the percentile.
[3] This section draws heavily from Gordy.

A single factor model is the simplest framework, and it provides the only way to think about the economic capital of a loan without also thinking about the make-up of the portfolio. To see this, suppose there were two risk factors, one that affects obligors based in the US and one that affects obligors based in the UK. Suppose that the obligor of a new loan is based in the UK. If the loan were added to a portfolio of US loans the new loan adds a bit of diversification, but if it were added to a portfolio of UK loans it adds no diversification. Thus, the risk of the new loan depends on the portfolio it joins. If there are two or more risk factors, the risk of a loan depends on the characteristics of the portfolio. To discuss economic capital without knowing the make-up of the portfolio one must have in mind a single factor model. Fortunately, single factor credit models are believed to capture much of the risk seen in multifactor models.

Another assumption is needed to discuss the economic capital of a loan in isolation: the portfolio is large enough to average away sampling variation. This assumption can be justified by the law of large numbers. If the portfolio has enough loans, and if the loans are similar enough in exposure amount, the expected rates of default and LGD can be assumed to occur. A given loan might or might not default, but there are enough loans that on average the expected default rate is observed. Similarly, a defaulted loan might have any LGD, but there are enough defaults that on average the expected LGD rate is observed.

Thus, default and LGD vary from year to year because of variation in a single risk factor, Z. The rates of default and LGD are said to be conditioned on Z. A greater value of Z brings about a greater conditionally expected default rate and a greater conditionally expected LGD rate. Since Z has an effect on every loan in the portfolio, it is said to be the "systematic" risk factor. This distinguishes Z from the "idiosyncratic" risk factors that contribute sampling noise in a portfolio that is not of asymptotic size.

If Z reflects "stress" conditions—if it is drawn at the 99.9^{th} percentile of its distribution—both the LGD rate and the default rate are at the 99.9^{th} percentiles of their respective distributions. Their product, the loss rate, is at 99.9^{th} percentile of its distribution as well. We can summarize this by saying that in a single factor model, economic capital for a loan equals its stress default rate times its stress LGD rate.

This context readily reveals the potential importance of LGD for economic capital. The stronger the effect of Z on the LGD rate, the greater will be stress LGD, and the greater will be economic capital.

Evidence of coordinated variation

This section examines evidence from Moody's Default Risk Service, which provides the rating history of every Moody's-rated loan or bond. For instruments that default, it provides both the date of the default event and the price of the instrument a few weeks later. The percentage difference between par and the post-default price measures LGD.

The data sample comprises nineteen years, 1983-2001. The years are separated into "good" (low default) years and "bad" (high default) years. The purpose of separating is to see what happens to LGD when the default rate is high. Therefore, the definition of a bad year is somewhat arbitrary. We take a bad year to mean any year where the default rate is greater than 4%, but some other separator could be used instead. Using 4% identifies four bad years (1990, 1991, 2000, and 2001) that have a total number of defaults approximately equal to the total number of defaults in the fifteen good years.

On the LGD side, Moody's observes a "debt type" for each defaulted loan or bond. A debt type designation is rather detailed, for example, one type of loan is "guaranteed senior secured term loan B". There are thousands of debt types in all, but less than one hundred of them have ever experienced a default. Among these, forty-nine debt types share a key trait: they have experienced at least one default in a good year and at least one default in a bad year.

For each of these forty-nine debt types, we calculate the average LGD during good years and, separately, the average LGD during bad years. The averages are plotted in Figure 1.[4] The position of a "bubble" reflects the two LGD averages. The size of a bubble reflects the total number of LGDs observed within the debt type. Smaller bubbles represent debt types that have not had many defaults.

Figure 1 provides evidence that default and LGD respond systematically to a common factor. The general pattern is that that most of the bubbles appear above the 45-degree line. This means that for most debt types, LGD has been greater in high default years. The exceptions are debt types represented by smaller sized bubbles. Representing fewer defaults, some of these bubbles may appear below the 45-degree line only by chance. Though the identity of the risk factor remains abstract, Figure 1 suggests that something causes the LGD rate to rise at the same time as the default rate.

Modeling LGD and default rates

Returning to the simple model introduced earlier, we tie the default rate and the LGD rate to Z. In other approaches to economic capital modeling, the default and LGD associated to any particular deal respond to the systematic risk factor.[5] By taking the average within a year, the conditionally expected rates of default and LGD can be derived as functions of Z. These functions have certain common-sense properties regardless of the specifics of the model that brings the functions into being:

- The functions are monotonic. Worse conditions produce a greater default rate and a greater LGD rate. We assume without loss of generality that greater Z represents worse conditions; the functions are therefore upward sloping.

[4] Figure 1 originally appeared in Frye, A False Sense of Security.
[5] See, for example, Pykthin.

- As Z rises without limit, both rates (default and LGD) approach 100%. In the worst imaginable state (collision with a large asteroid, perhaps), all obligors default and all LGDs equal 100%.

- As Z falls without limit, both rates approach zero. As long as there is some default and loss, things could always be better.

Rather than focus on the derivation of these functions, and on the complexities of the derivation, this chapter takes a direct approach. We assume that we already have the default rate and the LGD rate as functions of Z. Figure 2 displays the functions we will be using as examples. Both the LGD function and the default function exhibit the properties outlined above. They have not been calibrated to data, but they are useful to show the basics of working with LGD.

To complete the model, we need to specify the statistical distribution of the risk factor Z. The most common assumption, and what we will use for purposes of illustration, is that Z follows a standard normal distribution. Thus, each year we imagine a random realization of normal Z; then, Z implies the default rate and the LGD rate as shown in Figure 2.

The most common realizations of Z are near zero. These imply conditionally expected default rates near 4%. Figure 2 shows that much greater default rates can occur. Weighting all the conditionally expected default rates by their probabilities, the "unconditionally" expected default rate—in other words, PD—equals 5%. Both functions in Figure 2 are quite sensitive to Z. When standard normal Z takes its 99.9th percentile value of 3.09, it produces the stress default rate of 28% and the stress LGD rate of 75%.

LGD distributions

Using a standard statistical technique one can convert the functions of Figure 2 to probability distributions. That is because the annual rates are monotonic functions of random Z, which has a known probability distribution. These conditions allow application of the "change of variable" technique from elementary distribution theory.

The distribution of annual LGD appears in Figure 3. As in any probability distribution, the horizontal axis shows values of the random variable (LGD) and the vertical axis shows the relative frequency of the values.

Figure 3 also shows the distribution of LGD by *deal*, and it is quite different from the distribution of annual LGD. The distribution of LGD by deal depends on both the distribution of annual LGD and the default function.

To begin the derivation, consider an LGD of 32%. About 11% of years have conditionally expected LGD greater than 32%. However, loans with LGD greater than 32% are much more common because the conditions that produce high annual LGD also produce a great many defaulted loans.

When LGD is equal to 32%, Figure 2 shows that Z equals 1.25 and the associated default rate equals 10%. This is twice the "normal" rate of default, which is 5% (and equal, of course, to PD). Therefore, where LGD equals 32% in Figure 3, the distribution of LGD by deal is twice as high as the distribution of annual LGD. Reasoning this way up and down the range of LGD leads to the derivation of the full distribution of LGD by deal.

Sidebar: Deriving the distribution of LGD by deal at a single point

- The normal default rate, PD, equals 5%.
- In a year with Z = 1.25,
    - the default rate equals 10%, and
    - the annual LGD rate equals 32%.
- When LGD = 32%, LGDs are produced at twice the normal rate.

- Therefore, in Figure 3 where LGD = 32%, LGD by deal has twice the relative frequency of annual LGD.

To better see the difference between the two distributions plotted in Figure 3, imagine having LGDs from defaulted loans in a ten year period containing nine good (low default, low LGD) years and one bad (high default, high LGD) year. Though only one year in ten is bad, the bad year produces more than 10% of the defaults. Generalizing, bad years have extra weight in determining the distribution of LGD by deal. Since the bad years have large LGDs, the extra weight goes to the right of the distribution.

A different default function in Figure 2 would produce a different distribution of LGD by deal in Figure 3. For example, if default were not sensitive to Z, the default rate equals PD irrespective of Z. In that case, the distribution of LGD by deal is identical to the distribution of annual LGD, instead of being shifted as in Figure 3. In general, though, we assume that default rate is sensitive to Z.

It is not exaggerated to say that a bank lives by the distribution of LGD by deal, but it dies by the distribution of annual LGD. A bank lives by making profitable loans. The pricing for each should include expected loss. As we have seen, the expected loss (EL) for a loan equals PD for the loan times expected LGD of the loan. This expectation uses the distribution of LGD by deal. ELGD is shown in Figure 3 at 26%.

By contrast, a bank dies in an adverse year. In a year when Z is at its 99.9th percentile, loss equals economic capital, which equals the stress default rate times the stress LGD rate. Stress LGD can found from Figure 2, and it is noted in Figure 3. Stress LGD—the 99.9th percentile of annual LGD—equals 75%.[6]

[6] This observation may be obscured by the thickness of the line used to depict Figure 3.

Table: Features of the two LGD distributions

Distribution	Mode	Mean	99.9^{th} percentile
Annual LGD	6%	16%	75%
LGD by deal	13%	26%	Not useful

As we have seen, if annual LGD depends on Z, this gives rise to a probability distribution of annual LGD. If as well the default rate depends on Z, this gives rise to a distinct distribution of LGD by deal. These two distributions of LGD warrant some observations.

- ELGD and stress LGD come from different distributions. ELGD is expected LGD by deal. Stress LGD the 99.9^{th} percentile of annual LGD.
- Of the two distributions, the distribution of annual LGD is intrinsically simpler. Annual LGD can depend directly on Z. The distribution of LGD by deal, and therefore ELGD, depend on both the distribution of annual LGD and the annual default rate function.
- Expected annual LGD is less than ELGD. This is because years with greater LGDs contain more LGDs.
- Using data, the average of annual average LGDs ("time weighted average" LGD) tends to understate ELGD. The time weighted average produces an estimate of expected annual LGD, not of ELGD.

Before moving on it is worth noting that the distributions developed in this section are distributions of conditional expectations. They show systematic risk—the risk created by variation in the systematic risk factor Z. In addition to this, an institution with a finite-sized portfolio has idiosyncratic risk that is essentially the random sampling error around the conditional expectations. Idiosyncratic risk tends to diversify away as the size of the portfolio increases.

Capital models, capital functions, and elasticity

The above logic produces economic capital for a particular type of loan. In the example, the loan has ELGD = 26% and PD = 5%. These are the so-called unconditional expectations. Both LGD and the default rate vary by year depending on Z, leading to their conditional expectations. Economic capital is the product of the conditional expectations at the stress level of Z. For the above loan, this is 21% = 75% x 28%.

In principle, the same approach provides economic capital for any other type of loan. For a different type of loan, the functions portrayed in Figure 2 are different, leading to different expectations (i.e., PD and ELGD) and different stress values. Ideally, the functions result from a statistical estimation process that calibrates the model to default and LGD data, with special attention to the sensitivities to Z. That calibration lies outside the scope of this chapter.

In practice, sampling error has an effect on a bank portfolio. Sampling error, combined with the diversity of characteristics of loans in the portfolio, make the exact distributions like those in Figure 3 difficult or impossible to derive. Therefore, a practical economic capital model takes a different track. It specifies default and LGD functions similar to those in Figure 2, and then performs a Monte Carlo simulation to build up the picture of the distribution of loss. Economic capital for the entire portfolio is found at the desired percentile.

Rerunning the model adding a single loan produces the economic capital for that loan. Generalizing for a range of loans, the result can be stated as an economic capital function that depends on the characteristics of loans. The function resembles

(3) $K = f$ (PD, ELGD, other characteristics)

Using the example functions and assuming the asymptotic single risk factor model, we found earlier that $K = 21\% = f(5\%, 26\%)$.

An economic capital function usually states capital at the margin. It tells the effect on capital if a loan of the specified type is added to or removed from the portfolio. Because the portfolio is not "asymptotic" in size and because it depends in practice on more than one risk factor, economic capital in general depends on the other loans in the portfolio. Holding fixed the other loans, a capital function like (3) can always be determined, though it generally cannot be written in closed form because it effectively summarizes the output of an extensive Monte Carlo simulation.

The capital function tells the extra capital required when a deal is added the portfolio, or the capital released when a deal is removed. Often, a bank wants to judge both effects at once, for example, if it contemplates changing the mix of characteristics in its portfolio. The resulting change in capital depends on the sensitivities of the capital function to its inputs.

Sensitivities are most usefully stated as elasticities. Long the bane of introductory economics students, elasticity is a measure of sensitivity resembling slope, but stated in percentage change terms. It is usually the case that the elasticities of a capital function are more-or-less constant over some range of inputs. For example, a bank might contemplate exiting a credit having PD equal to 5% and making a similar loan to a credit having PD equal to 2%. Suppose this exchange decreases marginal capital by 50%. Then it probably is the case that an exchange from a loan having PD of 0.5% to one having PD of 0.2% also reduces capital by about 50%.

One elasticity is very readily identified for the asymptotic portfolio: if LGD does not depend on Z, the elasticity of economic capital with respect to ELGD equals 1.0. If expected LGD is no greater in stress conditions than otherwise, the capital function looks like $K = \text{ELGD} \times f(\text{PD})$. The elasticity of capital with respect to ELGD equals 1.0, because if ELGD doubles, capital doubles as well. In the more realistic case where LGD responds to Z, the elasticity of capital with respect to ELGD is generally less than 1.0. A change in ELGD causes a less-than-proportional change in capital.

First implementation pitfall: measuring LGD

The foregoing discussion provides a general approach for understanding the effect of LGD on economic capital. The final sections discuss two pitfalls that can be encountered when working with loss data. Either of these pitfalls can lead to a significant understatement of risk.

The first pitfall is to ignore a form of systematic risk while measuring LGD. LGD has been defined as economic concept, but it must be measured in some way using data that can be observed. Several methods have been put forth.

"Market value" measurements are generally crystallized soon after the default event. They require a financial market willing to bid for the loan soon after it defaults. By contrast, "workout value" measurements wait, possibly for years, for the defaulted contract ultimately to settle in some way. The ultimate cash flows are discounted back to the time of default; LGD is then measured as the proportional difference between the exposure amount and the discounted value of the ultimate cash flows. Intermediate approaches measure LGD at some other time, for example, as soon as the collateral is seized and appraised.

Any of these approaches might result in a workable assessment of economic loss, as long as all systematic effects are properly included. But when the market and workout approaches are compared in this regard, the workout approach seems more likely to err.

Market value measurements reflect uncertainty about the ultimate cash flows of the defaulted loan contract. Market participants make an effort to estimate them and to discount them back at risk-adjusted rates. Errors doubtless occur in the valuation process. If the errors are systematic, the evidence produced by market measures of LGD is flawed.

An example of a systematic market pricing error would be if defaulted loans tend to be under-priced in high default episodes. This would exaggerate LGD while the default rate is elevated; it would over-state risk and economic capital. However, if it is true that markets systematically under-price defaulted loans during high default periods, a market participant that discovers this fact can make extraordinary returns by buying loans rather than selling them. Therefore, significant deviations of market prices from economic values should be self-correcting to some degree. Without good evidence that defaulted loans have been systematically mis-priced, it seems reasonable to assume that market prices reflect an economic valuation process that properly takes into account the systematic influence of the underlying risk factor.

Workout value measurements of LGD discount the ultimate cash flows after they are known. But the cash flows are known only after they are received. Before the cash flows are received, they are unknown and risky. Risk is greater in a high default period. Greater risk brings with it greater spreads to the risk-free rate. Therefore, during a high default period the economic discount spread is greater than at other times. The first pitfall is to ignore the systematic variation in spread when discounting the cash flows obtained in the workout process.

A greater discount spread stems from greater systematic risk. In a high default period, similar obligors are apt to default, and loans with similar collateral are apt to be defaulted; systematic influences are generally more important in a high default period than otherwise. More broadly, it is more difficult to diversify away from defaulted assets in a high default period because at that time defaults are more likely to arise for systematic reasons. Still more broadly, during a high default period a defaulted loan correlates more strongly with the market portfolio, simply because the market portfolio contains more defaulted loans.

If constant spreads are used to discount workout cash flows, the first pitfall taints the LGD data. If that data are used to calibrate a capital model, the model understates risk. It might seem possible, though, to work around the first pitfall. The work-around would attempt to get the discount rate "right on average" without making it cyclically sensitive. Though individual LGDs would be mis-measured, perhaps they could be used in an economic capital model properly calibrated with full regard for systematic risk. The hope would be that, if the mismeasured LGDs produced accurate estimates of ELGD, the economic capital model could still be accurate, if the model were calibrated properly.

The work-around depends on getting the spread "right on average," and this is more difficult than it sounds. Some observations can be made at the outset. First, the right spread is not the original contractual spread on the defaulted loan; it is riskier to hold defaulted loans than to hold other loans. Second, the right spread is not derived from the bank's overall cost of capital; it is riskier to hold defaulted loans than to hold the bank's typical asset. Third, the right spread is not a simple average of the spreads observed through the credit cycle. A greater number of defaulted loans are exposed to the greater discount spreads during a high default period; the time to resolution may also be longer. Both magnify the effect of the greater spreads observed in high default periods.

In all, the discount rate used for measuring LGD by the workout method cannot ignore systematic variation in risk. Choosing the right discount rate is subtle and important.

Second implementation pitfall: estimating ELGD

Even if every measured LGD reflects full economic loss, there is a problem with estimating ELGD. The problem is that average historical LGD is a downward-biased estimator of ELGD. A related point was made earlier, namely, that time weighted average LGD is a downward-biased estimator. The current point cuts deeper. "Default weighted" LGD is also a downward-biased estimator of ELGD. That is, the average of LGD data is a biased estimator of expected LGD.

The above statement seems to conflict with elementary statistical theory, so an explanation is due. Statistical theory speaks about random sampling. For example, imagine the set of all LGDs experienced in 1000 years of data. If one LGD is drawn at random, there is a good chance that it comes from a year having a large number of defaults.

The difficulty, when it comes to the real world, is that LGDs are ***not*** drawn at random. Instead, a few years are drawn at random, and LGDs are sampled intensively from those years. This process is biased against sampling large LGDs. Large LGDs tend to occur in high default years. These years contain more than an average number of LGDs, but they have only an average probability of being sampled. If the years are sampled randomly, the large LGDs within high default years tend to be under-sampled.

The bias does not occur in the hypothetical case where default does not depend on Z and the expected default rate equals PD in all years. With no systematically high default years, there is no systematic under-sampling of large LGDs. But if default varies systematically, default-weighed average LGD is a downward-biased estimator of ELGD. Fortunately, the bias declines as the sample period extends, and the average is asymptotically unbiased.

The small-sample estimation bias is illustrated in two sets of simulations. Each set of simulations uses a Vasicek default function with PD equal to 0.5%, and each set has an LGD function that produces ELGD equal to 10% and stress LGD equal to 30%. The difference between the two sets is that they use different "asset correlation." Asset correlation controls the degree of systematic variation of default; in terms of Figure 2, greater asset correlation means the default function is steeper. Asset correlation equals 5% in the first set of simulations and 21% in the second.[7] In both sets of simulations, the portfolio is asymptotically fine-grained, so that on each simulation run the conditionally expected rates occur without sampling noise.

Figure 4 shows for each set of simulations the average default-weighted estimator among 50,000 simulation runs. If the average were unbiased, Figure 4 would show lines at 10% throughout. In fact, the upper line, representing the first set of simulations that have asset correlation equal to 5%, shows little departure from 10%. With as few as five years of simulated data, the average estimate equals 9.2%. Though this is slightly less than the true value of ELGD, an error of this size might readily be tolerated.

The lower line in Figure 4, where asset correlation equals 21%, tells a different story. The average estimate is well below the true value of 10% when the number of years is low to moderate. With five years of data, mean estimated ELGD is about 6.6%. A statistician would use over 150% of the estimated value to create an unbiased estimator of ELGD. With ten years of data, mean estimated ELGD is about 7.7%; even then, ELGD is 130% of the mean estimated level.

These simulations show that the degree of bias in the default-weighted estimator depends on the value of asset correlation (or on the steepness of the default function in Figure 2). As the number of years increases, the bias in the default-weighted average declines. In

[7] Asset correlation of 5% is estimated in Frye, Depressing Recoveries. The level of 21% is a not atypical, following the common practice of equating asset correlation to the correlation of equities or of deleveraged equities; see the CreditMetrics Technical Document.

any case, the percentage impact of the bias is apt to be greatest for loans having low ELGD.

Conclusion

Evidence has shown that when the default rate is elevated, the LGD rate also tends to be elevated. Tying both rates to a single systematic risk factor, we derive the distributions of the default rate and of annual LGD. The 99.9th percentiles of these distributions provide the stress rates; their product equals economic capital.

Expected LGD, or ELGD, is derived from the distribution of LGD by deal. The distribution of LGD by deal is derived from the distribution of annual LGD and the default rate function. Relating economic capital to the characteristics of loans (such as PD and ELGD) defines the economic capital function. The sensitivities of the economic capital function to its inputs are usefully summarized by the associated elasticities.

Two pitfalls of working with LGD data can lead to understatements of economic capital. First, if the effect of systematic risk is ignored when selecting the discount rate for ultimate cash flows, LGD is understated at times when risk has been high. Extrapolating to a high percentile, the economic capital model understates risk. Second, if the two distributions of LGD are not kept distinct, the average of historical LGD is imagined to be a good estimate of ELGD. In fact, average LGD is a downward-biased estimator of ELGD, though the bias declines as the sample period increases.

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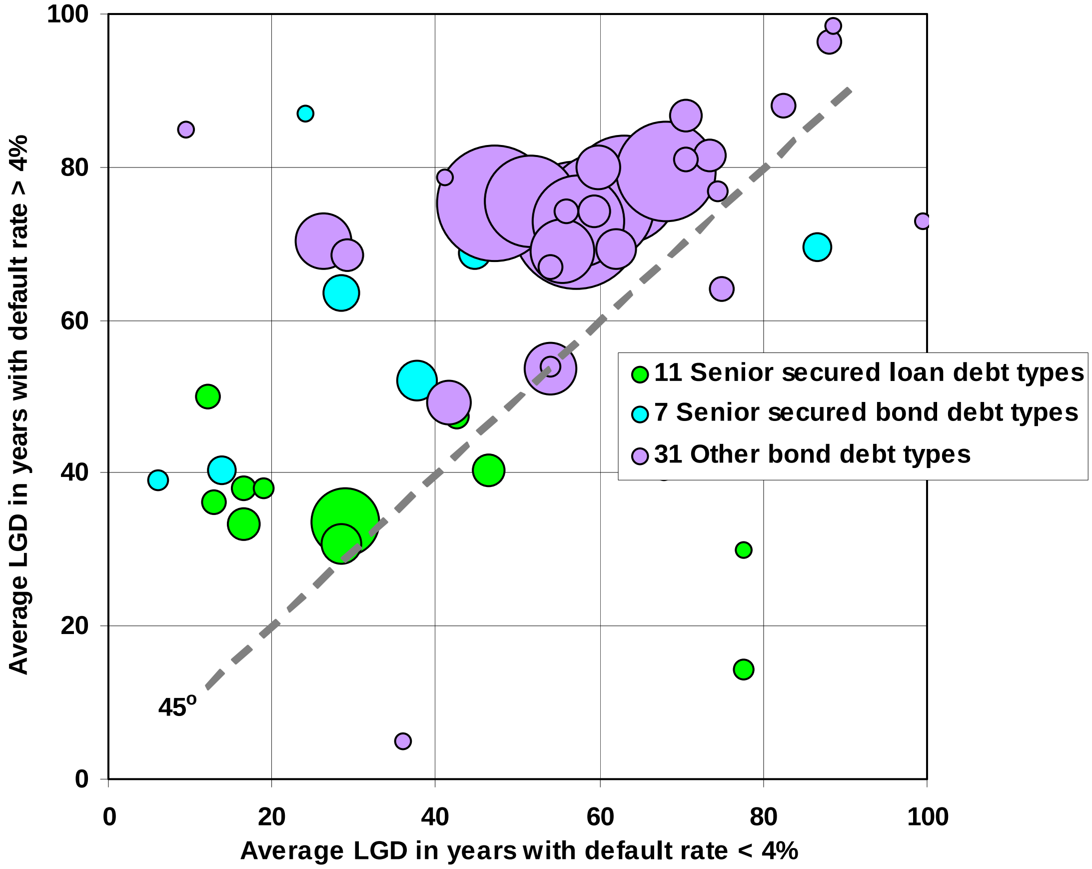
Figure 1. LGD in good years and bad years



Bubble size indicates number of LGDs in a debt type
Source: Moody's Default Risk Service

Figure 2. LGD and default as functions of risk factor z

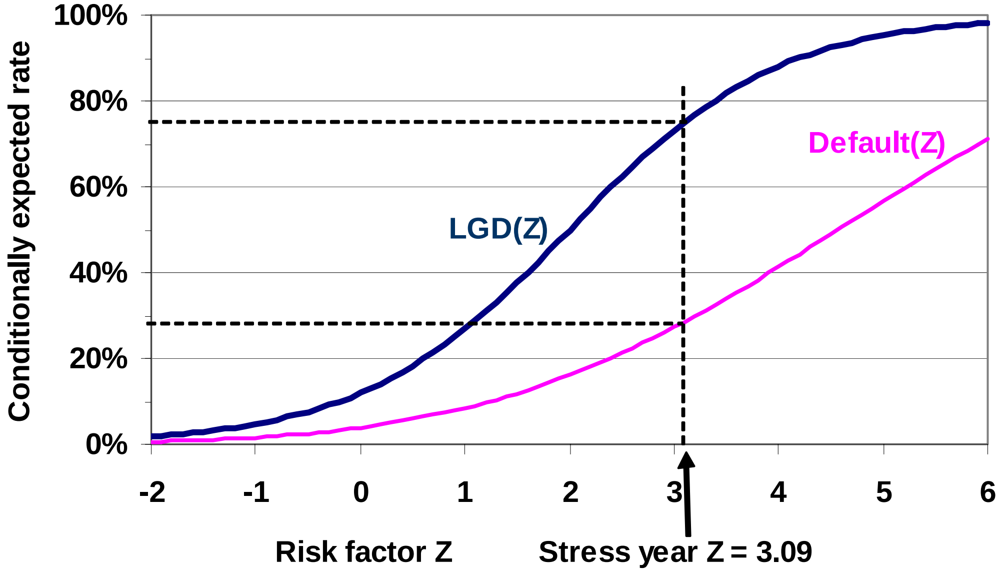


Figure 3. Two distributions of LGD

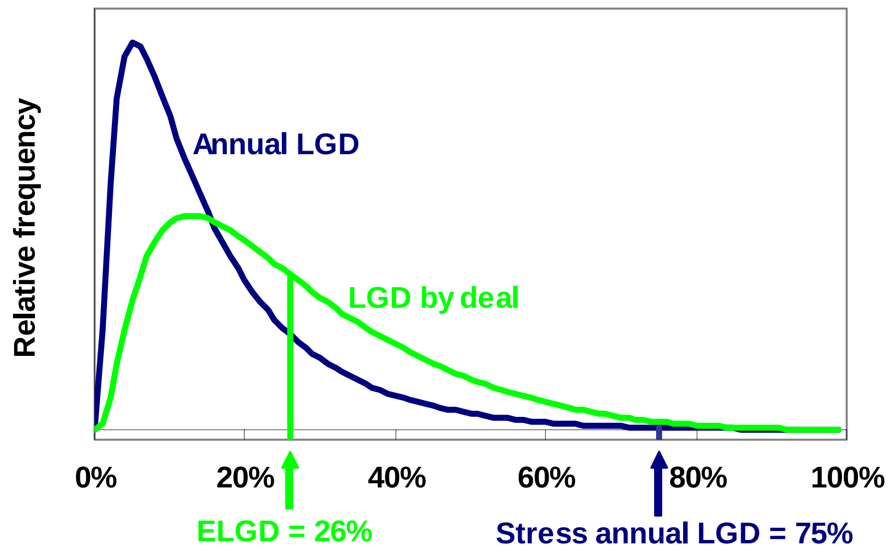


Figure 4. Simulated default-weighted LGD
ELGD = 10%, Stress LGD =30%