# Habit Persistence and Asset Returns in an Exchange Economy

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## Abstract

We examine asset prices and returns in the context of a version of the pure exchange economy studied in Lucas (1978) and Mehra and Prescott (1985). Our purpose is to identify the key channels by which changes in preferences affect the equity premium and the risk free rate and to develop intuition that is useful for understanding asset pricing in more complicated economies. Our analysis suggests that capital gains play a crucial role in generating empirically plausible mean equity premia.

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# 1. Introduction

We examine asset prices and returns in a pure exchange economy as in Lucas (1978) and Mehra and Prescott (1985). According to Mehra and Prescott (1985), it is difficult to account for the observed mean return on equity and the risk free rate in this type of environment if it is assumed that agents have power utility. Later, Constantinides (1990) showed that the free parameters in a habit persistence specification of utility with low risk aversion in wealth can be tailored to match both mean returns. Our purpose is to delineate and quantify the channels by which switching from power utility to habit persistence preferences has this effect on mean returns. We do this in order to develop intuition useful for understanding asset pricing in more complicated models. Our finding is that the rise in the equity premium arising from the switch in preferences operates primarily via the impact of the change on the dynamics of capital gains. Capital gains are identified with changes in the price of the underlying productive asset ('Lucas tree') in the exchange economy.

To understand the role played by capital gains, consider the classic covariance formula: the equity premium is negatively related to the conditional covariance between the one-period-ahead marginal utility of consumption and the rate of return on equity. In the models we study, variations in the rate of return on equity are primarily determined by variations in capital gains. A change in the specification of the model changes both arguments in the covariance. Thus, when we switch from power utility to habit persistence so as to raise the equity premium, there are two effects. On the one hand, the spread across states of nature of the one-period-ahead marginal utility of consumption increases. Holding fixed the stochastic process on the rate of return on equity, this raises the equity premium. We refer to this mechanism as the curvature channel, because it is determined by the degree of curvature in the utility function, measured as the elasticity of the marginal utility of consumption with respect to consumption. On the other hand, switching from power utility to habit persistence preferences changes the stochastic process on equity returns by altering the pattern of demand for assets across states of nature. Under habit persistence, households' demand for assets jumps by more in response to a positive

innovation in consumption opportunities than it does under power utility. Because the quantity of the productive asset is fixed in the exchange economy, this translates into increased capital gains in states when consumption is high, and smaller capital gains when consumption is low. Holding fixed the stochastic properties of the marginal utility of consumption, this also raises the equity premium. We refer to this as the *capital gains channel*.

Our computational experiments suggest that the capital gains channel warrants considerable attention. We find that over 90% of the increase in the mean equity premium resulting from a switch from power utility to habit persistence is due to the operation of the capital gains channel. Capital gains reflect the outlook for events extending into the distant future and so are influenced by many other features of the environment in addition to the curvature properties of the utility function. These features include such things as households' preferences over the intertemporal pattern of consumption and the persistence properties of households' consumption opportunities. Thus, although our computational experiments suggest that high curvature is a necessary ingredient for getting the equity premium, it is by no means sufficient. We dramatize this point by analyzing examples in which increased curvature in the utility function changes the stochastic process on equity returns in such a way that the equity premium is actually reduced.

On some dimensions, our work is closely related to that of Heaton (1995). We estimate our model's two habit persistence parameters based on the first moments of the asset returns of interest and we evaluate the model's performance based on its implications for second moments. As can be anticipated from Heaton and others, our ability to account for first moments of asset returns at the estimation stage comes at a cost of poor performance on second moments. What differentiates our work from Heaton's is its central emphasis on the curvature and capital gains channels as devices for understanding the impact on mean asset returns of a change in model specification.

The rest of the paper proceeds as follows. In the next section we describe the model economy.

This is followed by a qualitative analysis of the implications of habit persistence for mean asset returns. We also review the implications for risk aversion since, in practice, these play an

important role in assessing the plausibility of alternative preference specifications. After this we present our quantitative results and discuss their implications for risk aversion. In the final section we offer some concluding remarks.

# 2. The Exchange Economy

In this section we describe our version of the exchange economy studied in Lucas (1978) and Mehra and Prescott (1985). We suppose the economy is populated with households and firms who trade in competitive goods markets. Asset markets are of course complete. In what follows we first describe the problems of the households and firms and then we describe equilibrium.

## 2.1. Households

There are a continuum of infinitely lived, identical households who maximize expected discounted utility. Let  $\mathcal{E}_t$  denote the expectation operator, conditional on the information available at time t. At every date t, the representative household values consumption from that point forward according to:<sup>1</sup>

$$\mathcal{E}_t \sum_{j=t}^{\infty} \beta^{j-t} \frac{(C_j - X_j)^{1-\phi} - 1}{1 - \phi}, \tag{1}$$

where  $X_j$  represents the *habit stock*, which evolves as follows:

$$X_j = hX_{j-1} + bC_{j-1}. (2)$$

In (1),  $0 < \beta < 1$  is the household's discount factor. For the purposes of our analysis, we define power utility preferences as the case  $\phi > 0$ , h = b = 0, and habit persistence preferences as the case  $\phi \equiv 1$ , and either h or  $b \neq 0$ .

<sup>&</sup>lt;sup>1</sup>This (standard) specification of the habit persistence utility function has the distinctive feature that the present discounted value of the utility of a consumption sequence is nonmonotone in the consumption of any particular period. This reflects the fact that, although the period utility function is increasing in current consumption, period utility at later dates is decreasing in current consumption. This latter effect dominates at high values of consumption. In the simulations computed for this paper, consumption is always in the region of increasing marginal utility.

At every date t, the household must satisfy the following budget constraint:

$$B_t + S_t + C_t \le (1 + r_t^e)S_{t-1} + (1 + r_{t-1}^f)B_{t-1}, \tag{3}$$

where  $B_t$  and  $S_t$  denote period t acquisition of two types of one-period assets, denominated in consumption units. Their rates of return are  $1 + r_t^f$  and  $1 + r_{t+1}^e$ , respectively. The rate of return on  $S_t$  is conditional on the realization of the date t + 1 state of nature, and the rate of return on  $B_t$  is not. The problem of the household is as follows: at every date t, it takes  $S_{t-1}$ ,  $B_{t-1}$ ,  $X_t$  and  $\{r_j^e, r_{j-1}^f; j \geq t\}$  as given and maximizes (1) subject to (2) and (3), by choice of  $\{B_j, S_j, C_j; j \geq t\}$ .

# **2.2.** Firms

Firms possess the following technology for converting capital,  $K_t$ , into output,  $Y_t$ :

$$Y_t = Z_t K_t, (4)$$

where

$$Z_t = Z_{t-1} \exp(\theta_t). \tag{5}$$

The random variable  $\theta_t$  follows the autoregressive process

$$\theta_t = (1 - \rho)\overline{\theta} + \rho\theta_{t-1} + \epsilon_t, \tag{6}$$

and  $\epsilon_t \sim N(0, \sigma^2)$ , for all  $t \geq 0$ . Capital does not depreciate, and there exists no technology for increasing or decreasing its magnitude. The aggregate, per capita stock of capital is a constant, equal to K > 0.

We assume that firms have a one-period planning horizon. In order to operate capital in period t + 1, a firm must purchase it in period t. To do so, it issues equity  $S_t$ , subject to the

following financing constraint:

$$P_{k,t}K_{t+1} \le S_t,\tag{7}$$

where  $P_{k,t}$  is the date t price of capital, denominated in consumption units, and  $K_{t+1}$  represents the quantity of capital the firm plans to use. Let  $\pi_{t+1}$  denote the firm's period t+1 revenues, net of expenses, denominated in period t+1 consumption units. Revenues include the sale of output,  $Y_{t+1}$ , plus the sale of the capital stock,  $P_{k,t+1}K_{t+1}$ . The firm's expenses are limited to its obligations on equity,  $(1+r_{t+1}^e)S_t$ . Its choice variables are  $S_t$  and  $K_{t+1}$ , and it takes  $P_{k,t}$  and the state contingent objects  $r_{t+1}^e$  and  $P_{k,t+1}$  as given. The firm's outlays in each state of the world must not exceed its revenues:

$$\pi_{t+1} = Y_{t+1} + P_{kt+1}K_{t+1} - (1 + r_{t+1}^e)S_t \ge 0.$$
(8)

The firm's problem at date t is to maximize, by choice of  $S_t$  and  $K_{t+1}$ , the value of  $\pi_{t+1}$  across states of the world, subject to (4)-(8). This implies that the financing constraint, (7), is satisfied as a strict equality in equilibrium. Linear homogeneity of the firm's objective, together with the weak inequality in (8), imply the equilibrium condition,  $\pi_{t+1} = 0$  for all t + 1, and for all states of nature, so that:

$$1 + r_{t+1}^e = \frac{Z_{t+1} + P_{k,t+1}}{P_{k,t}}. (9)$$

## 2.3. Equilibrium

We adopt the normalization that the number of households and firms is one. Then the goods market clearing conditions for this economy are

$$C_t = Y_t, \ K_t = K. \tag{10}$$

Since households are identical, market clearing in risk free bonds implies  $B_t = 0$  in equilibrium. Market clearing in the equity market is reflected in (7) holding with equality. A sequence-ofmarkets competitive equilibrium is defined in the usual way.

The objects in equilibrium are obtained as follows. First,  $C_t = Z_t K$ . We find prices by combining the household's first-order condition for  $S_t$  with (5) and (9) to get:

$$p_{k,t} = \mathcal{E}_t p_{c,t+1} \exp(\theta_{t+1}) [1 + p_{k,t+1}], \tag{11}$$

where  $p_{k,t} \equiv P_{k,t}/Z_t$ . In addition,

$$p_{c,t+1} = \beta \frac{\Lambda_{c,t+1}}{\Lambda_{c,t}},\tag{12}$$

where  $\Lambda_{c,t}$  denotes the derivative of (1) with respect to  $C_t$ . This is computable given the solution for  $C_t$  described above. We then find  $p_{k,t}$  by specifying it to be a function of  $\theta_t$  and solving for the fixed point of the functional equation, (11). To approximate the solution to this and other functional equations, we use the nonlinear methods described in Judd (1992) and Christiano and Fisher (1994). Given  $P_{k,t} = p_{k,t} Z_t$ , we solve for  $r_{t+1}^e$  using (9). Finally,

$$1 + r_t^f = \frac{1}{\mathcal{E}_t p_{c,t+1}}. (13)$$

# 3. Preferences and Asset Returns

Here we review the risk aversion properties of habit persistence preferences and compare them to other preference specifications. We also discuss the implications of habit persistence for the risk-free rate, and present our formal decomposition of the equity premium into curvature and capital gains channels.

## 3.1. Risk Aversion

Since at least Mehra and Prescott (1985), asset pricing models have been evaluated based in part on their implications for risk aversion. In this section, we discuss the implications of habit persistence for risk aversion, and we contrast these with the risk aversion implications of alternative specifications of utility. The alternatives we consider include power utility and a

version of the habit persistence specification discussed by Abel (1990). In this specification the stock of habit,  $X_t$ , is exogenous to the individual household's consumption. We refer to this as external habit persistence.

It is important to distinguish at least three concepts of risk aversion. Two of these include risk aversion in wealth and risk aversion in consumption. Each measures a household's aversion to a particular kind of gamble. The gambles differ in the extent to which the household has access to credit markets for dealing with the outcome of the gamble. Risk aversion in consumption (wealth) measures a household's aversion to a gamble that occurs after (before) the current period credit markets have closed. We contrast these measures with another one in common use, which is based on the curvature of the utility function (i.e., the elasticity of the marginal utility of consumption with respect to consumption).

The above three measures of risk aversion coincide in the case of external habit persistence and power utility. Since accounting for the equity premium requires high curvature in utility, using these utility functions results in counterfactually high risk aversion in wealth. Constantinides (1990) pointed out that with habit persistence preferences, the three concepts of risk aversion are disentangled. Thus, for example, with habit persistence and  $\beta$  close to unity, risk aversion in wealth is close to unity, independent of b and b. At the same time, by increasing b and b curvature is increased, raising the equity premium without implying counterfactually high risk aversion in wealth.

To gain intuition into why curvature in the utility function can be high while risk aversion in wealth is low under habit persistence, recall that the amount a household is willing pay to avoid a fair bet on its wealth is directly related to the utility loss that the household suffers in the adverse state of the world. If the household were forced to accept an immediate drop in consumption, the loss of such a bet would be very painful, given the short-term exogeneity of the habit stock and high curvature. However, the habit persistence household can avoid this. Though the present value of its total lifetime consumption must fall with the loss of a bet, recourse to credit markets enables the household to slow the fall in actual consumption so that

the habit stock can fall. This is why the disutility occasioned by the loss of a bet on wealth may be relatively small for a household with habit persistence preferences.

A household with external habit, but high curvature, does not benefit from the habit persistence household's strategy for dealing with the loss of a gamble. For this type of household, the habit stock is exogenous for all time, and so recourse to credit markets represents a much less effective cushion against the loss of a bet. As a result, for asset pricing analyses based on external habit, risk aversion in wealth is typically high. For example, in the formulation studied by Campbell and Cochrane (1997), risk aversion in wealth is 80 (see also Weil 1992). This contrasts with a value of roughly unity for the habit persistence preferences studied in this paper.

As this discussion suggests, in general risk aversion depends on the nature of credit markets. In the analysis of this subsection, we assume households can borrow or lend as much as they like, subject to an intertemporal budget constraint. To keep the analysis simple, we rule out aggregate uncertainty, and we assume that the rate of interest faced by the household is constant. This last assumption is compatible with the experiments we perform because they represent gambles offered to an individual household. Since a household is small in our economy, the outcome of such a gamble has no impact on aggregates. If, by contrast, the outcome of a gamble affected all the households in the same way, then of course the fixed interest rate assumption would be untenable in the environment we consider. This is because there exists no technology for shifting resources intertemporally in our environment. We first consider the case of habit persistence preferences. We then turn to the case of external habit and power utility.

#### 3.1.1. Habit Persistence

At date 0, the household has a given stock of wealth,  $W_0$  (=  $S_{-1} + B_{-1}$ ), and habit,  $X_0$ , and seeks to maximize (1) subject to (2) and the following intertemporal budget constraint:

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t C_t = (1+r)W_0. \tag{14}$$

Let  $\gamma \equiv [\beta (1+r)]^{\frac{1}{\phi}}$ , and we assume:

$$h, b > 0, h + b < \gamma < 1 + r, \beta \gamma^{1-\phi} < 1.$$
 (15)

For later purposes, it is useful to note that (15) implies  $h < \gamma$ ,  $\beta h \gamma^{-\phi} < 1$  and h < 1 + r. All the parameterizations considered in this paper satisfy (15). After equation (2) we define habit persistence preferences as having  $\phi = 1$ . In this subsection, we find it convenient to deviate from this and to consider habit persistence preference in which, potentially,  $\phi \neq 1$ .

Let  $\Lambda_{c,t}$  denote the derivative of (1) with respect to  $C_t$ :

$$\Lambda_{c,t} = (C_t - X_t)^{-\phi} - \frac{b}{h} \sum_{j=1}^{\infty} (\beta h)^j (C_{t+j} - X_{t+j})^{-\phi}.$$
(16)

The Euler equation for the household's problem is  $\Lambda_{c,t} = \beta(1+r^e)\Lambda_{c,t+1}$ , for t=0,1,... This is satisfied by the following class of policies:

$$(C_t - X_t) = Q\gamma^t. (17)$$

The parameter Q is determined by the requirement that (17) also satisfy the intertemporal budget equation. To find Q, first evaluate the present value of both sides of (17), and make use of the fact that the present value of consumption must equal  $(1 + r)W_0$ . Then rearrange the resulting expression to get:

$$Q = (1 + r - \gamma) W_0 - \frac{1 + r - \gamma}{1 + r} \bar{X}_0, \tag{18}$$

where  $\bar{X}_0$  is the present value of  $X_0, X_1, \dots$ . This equation does not yet determine Q, since  $\bar{X}_0$  involves current and future household consumption. Taking into account (2) and imposing the

budget constraint again, we get<sup>2</sup>:

$$\bar{X}_0 = \frac{1+r}{1+r-h} X_0 + b \frac{1+r}{1+r-h} W_0. \tag{19}$$

Substituting (19) into (18), we get that, for given  $W_0$  and  $X_0$ ,

$$Q(W_0, X_0) = \frac{(1+r-\gamma)\left\{ [1+r-(h+b)]W_0 - X_0 \right\}}{(1+r-h)}.$$
 (20)

Discounted utility associated with the policy, (17), is (apart from an additive constant):

$$v(W_0, X_0) = \frac{Q(W_0, X_0)^{(1-\phi)}/(1-\phi)}{1 - \beta \gamma^{1-\phi}}.$$
(21)

It is illuminating to derive the optimal response of consumption to a change in wealth. In particular, note from (17) and (20) that

$$\frac{dC_0}{dW_0} = (1+r-\gamma)\left(\frac{1+r-(h+b)}{1+r-h}\right).$$

There are two interesting features of this expression. First, when b=0 and  $\gamma=1$ , then  $dC_0/dW_0=r$ , which is exactly the prediction of standard permanent income theory. Second,  $dC_0/dW_0$  is decreasing in b and b. That is, with habit persistence, the optimal response to a

$$X_t = h^t X_0 + b \sum_{i=1}^t h^{i-1} C_{t-i},$$

for t = 1, .... The key to obtaining (19) lies in evaluating

$$Z_0 = \sum_{t=1}^{\infty} \left(\frac{1}{1+r}\right)^t \sum_{i=1}^t h^{i-1} C_{t-i}.$$

A simple rearrangement of the terms in this sum yields:

$$Z_0 = \frac{1}{1+r} \sum_{t=0}^{\infty} \left[ \sum_{i=0}^{\infty} \left( \frac{h}{1+r} \right)^i \right] \left( \frac{1}{1+r} \right)^t C_t.$$

Expression (19) follows trivially.

<sup>&</sup>lt;sup>2</sup>To get (19), note first that (2) implies

decrease in wealth is to use financial markets to bring down consumption slowly so that the stock of habit has a chance to fall.

# Risk Aversion in Wealth

We now consider risk aversion in wealth. In period 0, prior to the closing of credit markets, the household is unexpectedly confronted with the following gamble: it wins or loses  $W_0\mu$  with probability 1/2 assigned to each outcome. Let  $\nu$  denote the largest fraction of the household's wealth that it would be willing to sacrifice to avoid this gamble:

$$v(W_0(1-\nu), X_0) = \frac{1}{2} \left[ v(W_0(1+\mu), X_0) + v(W_0(1-\mu), X_0) \right].$$
 (22)

Take a first-order Taylor series expansion of the expression on the left of the equality about  $\nu = 0$ , and a second-order Taylor series expansion of the expression on the right about  $\mu = 0$ , and then solve for  $\nu$ :

$$\nu = RRA_W \frac{1}{2}\mu^2. \tag{23}$$

Here,

$$RRA_W = -\frac{W_0 v_{WW}}{v_{WW}},\tag{24}$$

where  $v_W$  and  $v_{WW}$  denote the first and second derivatives of v in (21) with respect to its first argument, evaluated on a steady state growth path. Now if  $\mu = 0.1414$ , then  $\frac{1}{2}\mu^2 = .01$ , so that a habit persistence household faced with a 50-50 chance of losing or increasing its wealth by 14 percent would be willing to pay  $RRA_W$  (=  $\nu \times 100$ ) percent of its wealth to avoid the gamble.

Evaluating the derivatives in the definition of the relative risk aversion in wealth,  $RRA_W$ , we get:

$$RRA_W = \frac{\phi}{1 - \frac{b(1+r-\gamma)}{(1+r-(h+b))(\gamma-h)}}.$$
 (25)

This has been evaluated along a steady state growth path, in which the habit stock to wealth ratio is  $b(1 + r - \gamma)/(\gamma - h)$ . Equation (25) is the discrete-time analog of the steady-state

<sup>&</sup>lt;sup>3</sup>See Ferson and Constantinides (1991) for a discussion of the case where h = 0.

formula for risk aversion provided in provided in Constantinides (1990).

In the case,  $\phi = \beta = 1$ ,

$$RRA_W = 1, (26)$$

independent of the values of the habit persistence parameters, h, b.4

# Risk Aversion in Consumption

Let  $C_0, C_1, ...$ , denote the solution to the household's problem. After solving this problem and after credit markets have closed, the household is unexpectedly confronted with the following gamble: it wins or loses  $\mu C_0$  units of consumption goods with probability 1/2 assigned to each outcome. The timing assumption about credit markets implies that  $W_1 = (1+r)W_0 - C_0$  whether the household wins or loses. Risk aversion in consumption is measured by the fraction,  $\nu$ , of  $C_0$  the household is willing to sacrifice with probability one in order to avoid this gamble. That is,  $\nu$  solves<sup>5</sup>:

$$U\left(C_{0}(1-\nu)-X_{0}\right)+\beta v\left(W_{1},hX_{0}+bC_{0}(1-\nu)\right)$$

$$=\frac{1}{2}\left\{U\left(C_{0}(1-\mu)-X_{0}\right)+\beta v\left(W_{1},hX_{0}+bC_{0}(1-\mu)\right)\right.$$

$$\left.+U\left(C_{0}(1+\mu)-X_{0}\right)+\beta v\left(W_{1},hX_{0}+bC_{0}(1+\mu)\right)\right\},$$
(27)

where  $U(x) = \left[x^{(1-\phi)} - 1\right]/(1-\phi)$ . Take a first-order Taylor series expansion of the expression on the left of the equality about  $\nu = 0$ , and a second-order Taylor series expansion of the expression on the right about  $\mu = 0$ , and then solve for  $\nu$ :

$$\nu = RRA_C \frac{1}{2}\mu^2,$$

<sup>&</sup>lt;sup>4</sup>The case,  $\phi = \beta = 1$  violates (15), since it implies  $\gamma = 1 + r$ . The results attributed to this case in the text obviously also hold approximately for values of  $\phi$  and  $\beta$  which are close to unity, but consistent with (15).

<sup>&</sup>lt;sup>5</sup>Implicitly, we assume here that the household may reoptimize its consumption plan starting in period 1 to reflect the impact of the gamble on next period's habit stock. One can imagine another concept of risk aversion in which this reoptimization is not permitted. In our economy, this concept would be appropriate for a fictitious social planner who faces a gamble on aggregate consumption and who cannot adjust consumption in later periods because capital is not intertemporally transferrable.

where

$$RRA_C = -\left[\frac{U_{cc} + \beta b^2 v_{XX}}{U_c + \beta b v_X}\right] C. \tag{28}$$

Here,  $U_{cc} = -\phi Q^{-(1+\phi)}$  and  $U_c = Q^{-\phi}$ , where Q is given in (20). Evaluating this expression:

$$RRA_{C} = \frac{\phi C}{Q} \left[ \frac{1 + \frac{\beta b^{2}}{1 - \beta \gamma^{1 - \phi}} \left(\frac{1 + r - \gamma}{1 + r - h}\right)^{2}}{1 - \frac{\beta b}{1 - \beta \gamma^{1 - \phi}} \frac{1 + r - \gamma}{1 + r - h}} \right], \quad \frac{C}{Q} = \frac{1 + r - h}{1 + r - (h + b) - \frac{b}{\gamma - h} (1 + r - \gamma)}. \tag{29}$$

In the case,  $\phi = \beta = 1$ ,

$$RRA_C = \frac{1+r-h}{1+r-(h+b)}. (30)$$

Note that risk aversion in consumption is unbounded above as  $h+b \to 1+r$ , while risk aversion in wealth always remains at unity (see (26)).

#### Curvature

Finally, we relate the above two concepts of risk aversion to the standard one, RRA, which corresponds to curvature, or, the elasticity of the marginal utility of consumption with respect to consumption:

$$RRA = -\frac{C_t \Lambda_{cc,t}}{\Lambda_{c,t}},$$

evaluated along a steady state growth path. Here,  $\Lambda_{cc,t}$  is the derivative of  $\Lambda_{c,t}$  with respect to  $C_t$ . Note from (16) that, along a nonstochastic steady-state growth path where  $C_t = \gamma C_{t-1}$ ,

$$\Lambda_{c,t} = C_t^{-\phi} s^{-\phi} q_c, \ \Lambda_{cc,t} = -\phi C_t^{-(\phi+1)} s^{-(1+\phi)} q_{cc}.$$

Here

$$q_c = 1 - \frac{b\beta/\gamma^{\phi}}{1 - \beta h/\gamma^{\phi}}, \ q_{cc} = 1 + \frac{b^2\beta/\gamma^{\phi+1}}{1 - h^2\beta/\gamma^{\phi+1}}, \ s = \frac{\gamma - (h+b)}{\gamma - h},$$

where s is the steady-state value of  $(C_t - X_t)/C_t$ . Note that  $q_{cc}$  is increasing, and  $q_c$  and s are

decreasing, in b and h. Consequently,

$$RRA = \frac{\phi q_{cc}}{sq_c}$$

is increasing in b and h. It is easily verified that, in the case,  $\phi = \beta = 1$ ,

$$RRA = RRA_C \frac{1 + \frac{b^2}{(1+r)^2 - h^2}}{1 - \frac{b}{1+r-h}} > RRA_C.$$

Evidently, RRA and  $RRA_C$  are different. However, each has the property of being unbounded above as  $h + b \to 1 + r$ . This establishes the properties of habit persistence perferences stated in the introduction of this section.

# 3.1.2. External Habit, Power Utility

Under external habit persistence, households also have preference specification (1) and (2), except that  $C_{t-1}$  in (2) is economy-wide average consumption, rather than the household's own consumption. Thus, the habit stock is viewed by the household as evolving exogenously. Under these circumstances, the derivative of (1) with respect to  $C_t$  is (16) without the future terms, i.e., it is  $\tilde{\Lambda}_{c,t} = (C_t - X_t)^{-\phi}$ . The intertemporal Euler equation is  $\tilde{\Lambda}_{c,t} = \beta(1 + r^e)\tilde{\Lambda}_{c,t+1}$ , for  $t = 0, 1, \ldots$ , which is satisfied by a consumption policy of the form, (17). Since  $\bar{X}_0$  is viewed as exogenous to the household, (18) gives the value of Q which determines its consumption plan. We denote this by  $\tilde{Q}(W_0, \bar{X}_0)$ . Along a steady state growth path,  $X_{t+1} = \gamma X_t$ , for  $t = 0, 1, \ldots$ , in which case  $\bar{X}_0 = [(1+r)/(1+r-\gamma)]X_0$ . Then, combining (17) and (18) yields the following policy function on a steady state growth path:

$$C_0 = (1 + r - \gamma)W_0. (31)$$

The value of the household's optimal plan, given  $W_0$  and  $\bar{X}_0$ , is (apart from an additive constant):

$$\tilde{v}(W_0, \bar{X}_0) = \frac{\tilde{Q}(W_0, \bar{X}_0)^{(1-\phi)}/(1-\phi)}{1-\beta\gamma^{1-\phi}}.$$

It is easily verified that  $\tilde{v}(W_0, \bar{X}_0) = v(W_0, X_0)$  and  $\tilde{Q}(W_0, \bar{X}_0) = Q(W_0, X_0)$  when all house-holds in the economy adopt identical consumption plans. As a result, in an economy with a single type of agent, the optimal consumption sequences are invarient to whether habit persistence is internal or external.

Now consider the risk aversion of a household with external habit:

$$RRA_W \equiv -\frac{W_0 \tilde{v}_{WW}}{\tilde{v}_W} = -\frac{W_0 \tilde{\Lambda}_{cc} (1 + r - \gamma)}{\tilde{\Lambda}_c} = -\frac{C_0 \tilde{\Lambda}_{cc}}{\tilde{\Lambda}_c} \equiv RRA.$$
 (32)

Here,  $\tilde{\Lambda}_{cc}$  denotes the derivative of  $\tilde{\Lambda}_c$  with respect to  $C_t$ , evaluated along a steady state growth path. Also, the third equality in (32) makes use of (31). It is obvious from (28) that  $RRA_C = RRA$ , since the analog of  $v_X$  and  $v_{XX}$  is zero here. Thus, with external habit and (since the results are also true for  $X_t \equiv 0$ ) with power utility,

$$RRA_W = RRA_C = RRA$$
.

This establishes the properties of external habit and power utility perferences stated in the introduction of this section.

#### 3.2. The Risk-Free Rate

Consider (13) along a nonstochastic steady-state growth path in which  $C_t = C_{t-1} \exp(\bar{\theta})$ :

$$1 + r_t^f = \begin{cases} \frac{\exp(\phi\bar{\theta})}{\beta}, & \text{for power utility} \\ \frac{\exp(\bar{\theta})}{\beta}, & \text{for habit persistence.} \end{cases}$$
 (33)

As we argued above, with habit persistence preferences, curvature can be increased by not as b or h. To raise curvature with power utility requires increasing  $\phi$ . We infer from (33) that raising curvature has a very different impact on the risk-free rate, depending on whether one adopts habit persistence or power preferences.

The intuition for this difference between the two utility functions is simple. With power utility and positive consumption growth, the future marginal utility of consumption is low compared with the marginal utility of present consumption. Increasing  $\phi$  intensifies this, so that a higher interest rate is required to discourage households from attempting to reallocate consumption from the future to the present. The impact of increasing b or h is quite different. This has the effect of increasing the future habit stock and, other things the same, this raises the marginal utility of future consumption, reducing the incentive to reallocate consumption toward the present.

In sum, accounting for the equity premium by increasing curvature in utility is more likely to avoid counterfactual implications for the risk-free rate if it is done by increasing b or h in habit persistence preferences, than if it is done by increasing  $\phi$  in a power utility function. For a further discussion of related issues, see Weil (1989, 1992) and Campbell and Cochrane (1995).

### 3.3. The Curvature and Capital Gains Channels

The curvature and capital gains channels are associated with the two arguments in the conditional covariance expression for the equity premium:

$$r_t^{ep} = \frac{\mathcal{E}_t(1 + r_{t+1}^e)}{1 + r_t^f} = 1 - Cov_t \left( \beta \frac{\Lambda_{c,t+1}}{\Lambda_{c,t}}, \frac{Z_{t+1} + P_{k,t+1}}{P_{k,t}} \right), \tag{34}$$

where  $Cov_t(x, y)$  denotes the date t conditional covariance between x and y. Let  $\Delta Er_t^{ep}$  denote the change in mean of the equity premium due to a change in preferences. Our decomposition is:

$$\Delta E r_t^{ep} = \delta_{\Lambda} E r_t^{ep} + \delta_{P_k} E r_t^{ep}, \tag{35}$$

where  $\delta_{\Lambda}Er_{t}^{ep}$  and  $\delta_{P_{k}}Er_{t}^{ep}$  measure the curvature channel and the price of capital channel, respectively. We define  $\delta_{\Lambda}Er_{t}^{ep}$  as the change in the mean equity premium due to a change in the utility function, holding fixed the distribution of  $(Z_{t+1} + P_{k,t+1})/P_{k,t}$  and  $C_{t+1}$  across dates and states of nature. The capital gains channel,  $\delta_{P_{k}}Er_{t}^{ep}$ , is simply defined as the residual:  $\delta_{P_{k}}Er_{t}^{ep} = \Delta Er_{t}^{ep} - \delta_{\Lambda}Er_{t}^{ep}$ .

# 4. Quantitative Results

In this section we present our quantitative results. First we discuss our method for assigning values to the model parameters. Second, we document the importance of the capital gains channel. We do this by exhibiting the sensitivity of the equity premium to the persistence of consumption growth in the power utility model. Also, we use the decomposition in (35) to quantify the magnitude of the curvature and capital gains channels under habit persistence. Third, we document the ability of habit persistence preferences to account for key features of asset prices in our exchange economy.

## 4.1. Parameter Values

We adopt the normalization, K = 1. The equilibrium consumption process (that is, the technology shock  $Z_t$ ) was chosen to be consistent with the observed mean, standard deviation, and the autocorrelation of quarterly U.S. per capita consumption growth.<sup>6</sup> This requires setting

$$\overline{\theta} = 0.0045, \sigma = 0.0053, \rho = 0.34.$$
 (36)

We set  $\beta = 0.99999$  to maximize the model's ability to account for the observed risk-free rate. Conditional on these parameter values, the two habit persistence parameters were set to

<sup>&</sup>lt;sup>6</sup>Our measure of consumption is private consumption of nondurables and services, plus a measure of the service flow from the stock of durables. The data cover the period 1959.1 to 1989.4, and are discussed in Christiano (1988) and Fisher (1994). Consumption growth at different levels of time aggregation have different autocorrelation patterns (see Heaton 1993, 1995). Accounting for this phenomenon is beyond the scope of this paper.

optimize the model's implications for the mean equity premium and risk-free rate. Our metric for this is  $\mathcal{L}(\psi)$ , where:

$$\mathcal{L}(\psi) = [\hat{\nu}_T - f(\psi)] \, \hat{V}_T^{-1} \, [\hat{\nu}_T - f(\psi)]' \,, \tag{37}$$

and  $\psi = (b, h)$ . Also,  $\hat{\nu}_T$  is the 2 × 1 vector of point estimates for the risk-free rate and the equity premium reported in Cecchetti, Lam, and Mark (1993) (CLM), and the 2 × 2 matrix  $\hat{V}_T$  is their estimate of the underlying sampling variance. Finally, f is the model's implied risk-free rate and equity premium, given  $\psi$ . We executed this mapping by computing the average of these variables across 500 artificial data sets, each of length 120. We considered  $\psi \in \Psi$ , a grid of points, b, h, in the unit box, having the property that  $C_t \leq X_t$  and  $\Lambda_{c,t} \leq 0$  are never observed in the Monte Carlo simulations used to evaluate f. Let

$$J = \mathcal{L}(\hat{\psi}_T),\tag{38}$$

where  $\hat{\psi}_T$  minimizes  $\mathcal{L}(\psi)$  over  $\psi \in \Psi$ . In practice, we could not find values of  $\psi$  that set J = 0. We find that  $\hat{\psi}_T = (0.58, 0.3)$ , with J = 0.37.

# 4.2. Power Utility

The results of analyzing the economy with power utility are summarized in Figure 1. That figure indicates (see "U.S. data") the sample averages for the risk-free rate and the equity premium taken from CLM. In addition, we report 1 percent and 5 percent confidence ellipses, based on CLM's reported  $\hat{V}_T$  matrix. Results for several versions of the exchange economy with power utility are presented.

The curve marked "power utility, consumption growth autocorrelation = 0.34" adopts the parameter values in (36). Moving from left to right, each letter "o" reports  $f(\psi)$  for a different

<sup>&</sup>lt;sup>7</sup>Our estimation procedure is similar to Heaton (1995)'s, who also takes into account sampling uncertainty in estimates of asset market moments, but ignores sampling uncertainty in the underlying endowment (consumption) process.

value of  $\phi$ , with  $\phi = 1, 2, 3, 4, 5, 10, 15, 20, 25, 30, 35$ . There are two basic findings here. First, consistent with the nonstochastic analysis reported above, increasing curvature with power utility preferences produces a rise in the average risk-free rate. Second, increasing curvature results in a *fall* in the equity premium. For  $\phi$  exceeding 5, the equity premium is negative, with equity actually being a good hedge against risk.

To understand this result, we studied three other versions of our model. First, we repeated the calculations with  $\rho = -0.34$  (see "power utility, consumption growth autocorrelation = -0.34"). This change in the autocorrelation of consumption growth has essentially no effect on the monotone relationship between the risk-free rate and curvature, but the effect on the equity premium is substantial. Now the equity premium rises monotonically with curvature. Second, we simulated a version of the model in which the parameters of the equilibrium consumption growth process are taken from Mehra and Prescott (1985). They based their parameter values on annual U.S. data covering the period 1889–1978, in which the first-order autocorrelation of consumption growth is -0.14 (see "Mehra-Prescott, consumption growth autocorrelation = -0.14"). Note that now the risk-free rate initially rises, then falls, as curvature rises. In addition, consistent with the version of our model with  $\rho = -0.34$ , the equity premium rises monotonically with curvature. Third, we altered the Mehra-Prescott parameterization by switching the sign on  $\rho$  (see "Mehra-Prescott, consumption growth autocorrelation = 0.14"). This has a very large impact on the equity premium. It now falls sharply with increased curvature. Significantly, none of the perturbations considered places the power utility model anywhere close to the U.S. data.

We infer from these computational experiments that the autocorrelation of consumption growth is critical for determining whether higher curvature produces a positive or negative equity premium. This finding impressively illustrates the importance of the capital gains channel in determining the equity premium, since, by construction, the curvature channel plays no role with variations in  $\rho$ .

Insight into the role of  $\rho$  can be obtained by making use of a simple permanent income-

type argument, and the covariance formula in (34). The sign of  $r_t^{ep}$  depends on the sign of the conditional covariance between  $\Lambda_{c,t+1}$  and  $Z_{t+1} + P_{k,t+1}$ . When technology growth is positively autocorrelated, a date t+1 state of nature in which  $Z_{t+1}$  is high signals an even greater rise in technology at later dates, and thus a rise in households' long-run consumption opportunities. Under power utility, households have an incentive to adjust consumption immediately to its long-run potential, and this implies a large jump in desired  $C_{t+1}$ . However, to increase consumption by more than output, households must reduce their accumulation of equity; this in turn translates into a reduced demand for capital. The latter, in view of the fixed supply of capital, translates into a fall in its price,  $P_{k,t+1}$ . If this price effect is strong enough to overcome the jump in  $Z_{t+1}$  itself—a result that is more likely, the greater  $\phi$  is—then the conditional covariance in (34) would be positive, implying the negative equity premiums that we see in Figure 1.

If technology growth is negatively autocorrelated instead, then a high  $Z_{t+1}$  signals a smaller increase—perhaps even a reduction—in long-run consumption prospects. Under these circumstances, adjusting consumption to its long-run potential dictates shifting  $C_{t+1}$  up by less than the rise in date t+1 output, thus giving rise to an increased demand for capital. This drives up  $P_{k,t+1}$ , guaranteeing that the covariance in (34) is negative and that the equity premium is positive.<sup>8</sup>

We think these results make it clear that, to understand the sensitivity of the equity premium to  $\rho$ , one must understand the impact of changes in  $\rho$  on the dynamics of the price of capital, that is, the capital gains channel.

## 4.3. Habit Persistence

Results for analyzing the economy with habit persistence utility are summarized in Figure 2. The figure reproduces the empirical observations and confidence ellipsoids from Figure 1. The mean equity premium and risk-free rate, corresponding to a subset of  $(b, h) \in \Psi$ , are also reported. To

These observations can be illustrated with a simple example. Let  $C_{t+1}/C_t = \exp(\theta_{t+1})$ , with  $\theta_t = \bar{\theta} + \varepsilon_t + \rho \varepsilon_{t-1}$ , and  $\varepsilon_t \sim IIN(0, \sigma_{\varepsilon})$ . Then  $P_{k,t+1} = \mathcal{E}_{t+1} \sum_{j=1}^{\infty} \beta^j \Lambda_{c,t+1+j} C_{t+j+1}/\Lambda_{c,t+1}$ . In the power utility case, this implies  $d \log(P_{k,t+1})/d\varepsilon_{t+1} = 1 + (1-\phi)\rho$ , which may be negative if  $\rho > 0$  and  $\phi > 1$ .

gain insight into the relation of b and h with asset returns, we find it useful to arrange the results in a particular way. That is, we consider instances of (b, h) that imply nonstochastic steady-state values of  $X_t/C_t$ , denoted by x, equal to 0.85, 0.83, 0.81, and 0.30. For the last two values of x, we consider h = 0, 0.10, 0.20, ..., 0.90, 0.95. For x = 0.85 and x = 0.83, only the first 9 and 8 values of h are reported, respectively. Note that for each value of x, the equity premium/risk-free rate combinations form a half-ellipse. For small values of h, the equity premium and risk-free rate are both decreasing in h, for given x. For larger values of h, the equity premium continues to be decreasing in h, but the risk-free rate now begins to increase. Note also that the half-ellipses shift down and get smaller with decreasing x. As x gets even smaller, the half-ellipses converge to an equity premium of 0.03 percent and a risk-free rate of 1.8 percent, after rounding.

The point in Figure 2 (and among all  $b, h \in \Psi$ ) closest to the U.S. data is b = 0.58, h = 0.3, with x = 0.83. At this point, the risk-free rate is 1.68 percent and the equity premium 6.86 percent. This is close to the U.S. numbers, once sampling uncertainty is taken into account. Statistical results for this model economy are provided in Table 1. The column labeled "No Habit" corresponds to the parameterization  $\phi = 1$ , b = h = 0, while "Habit" corresponds to the model with habit persistence, evaluated at the estimated parameters. The U.S. numbers are also reported in Table 1. Note that, although the model does well in accounting for the mean risk-free rate and the equity premium, it does less well in accounting for the variance of these objects.

Table 1 reports various statistics of interest and compares them with the analogue quantities for the US data. A result that is anticipated by, among others, Cochrane and Hansen (1992), Heaton (1995), is that the volatility of asset returns is too high in the economy with habit persistence preferences, relative to the data. Note, too, that the correlation between consumption growth and equity returns is predicted to be very large in the model, compared with the roughly zero number in the data. The low correlation might well stem from the fact that the stock return used in our empirical investigation is poorly correlated with the 'true' return on the

<sup>&</sup>lt;sup>9</sup>Given values of x and h, the value of b is implied by the condition  $x = b/(\exp(\bar{\theta}) - h)$ .

wealth portfolio. This same argument will not resolve all the variance anomolies depicted in Table 1, however, For instance, Hansen and Jagannathan (1991) and Cochrane and Hansen (1992) formulate the equity return puzzle without requiring data on the return on the wealth portfolio. Instead their characterization focuses on Sharpe ratios: ratios of mean excess returns to their standard deviations. The Sharpe ratio implied by our habit persistence calibration is about one third that of the historical data. Also, the variation in the risk free rate implied by the model is about three time that of the data.

Finally, to quantify the curvature channel, we considered the impact of the change from power utility to our estimated habit persistence specification, *i.e.* from (b = h = 0) to (b = 0.58, h = 0.3). For this,  $\delta_{\Lambda} E r_t^{ep} = 1.0004340 - 1.0000281$ , and  $\Delta E r_t^{ep} = 1.0067107 - 1.0000281$ , so that  $\delta_{\Lambda} E r_t^{ep} / \Delta E r_t^{ep} = 0.06$ . Thus, of the full increase in the equity premium (here expressed at a quarterly rate), 6 percent is accounted for by the curvature channel, and 94 percent is accounted for by the capital gains channel.

# 5. Implications for Risk Aversion in Consumption

As has been emphasised previously, it is common to evaluate preference based explanations of asset return behavior by studying the risk aversion implications of the candidate preference specification. We have already documented that for our specification of habit persistence  $RRA_W$  is about unity. Presumably this falls within a range usually considered acceptable by economists. According to our definition of  $RRA_C$  given above, with our estimated habit parameters, b = 0.58 and h = 0.30, a household is willing to give up 7.3 percent of consumption to avoid a fair, 10 percent gamble. Some perhaps will view this as a high degree of consumption risk aversion. Does this mean that, necessarily, to account for the observed equity premium, high  $RRA_C$  is required? The answer may be yes. But, there are at least three reasons to think that the answer might actually be no. All of these reasons build, in different ways, on the notion that the information observed by the economic analyst and that observed by households differ in some way.

First, from the analysis above, it is clear that the details of the consumption process matter

a lot for determining how much  $RRA_c$  is required to account for the equity premium. Yet, there is little confidence in the quality of this data (see Wilcox (1992).) Gibbons (1989) cites this low quality as a reason for ignoring consumption altogether in evaluating asset pricing models. The range of uncertainty about the consumption data when these quality considerations are integrated with the usual sampling uncertainty may include parameterizations of consumption which permit accounting for the observed equity premium with low  $RRA_c$ .

Second, suppose all the features of the univariate stochastic process underlying the consumption data were known accurately. Quah (1990) has shown that, even a process in which the univariate representation is a first order autoregression in growth rates with positive AR(1) parameter is consistent with an unobserved components representation in which transitory shocks play a very large role. In the statistical environment like the one studied by Quah, as long as agents observe the two underlying components driving consumption, their demand for equity may be driven in an important way by the transitory component, possibly leading to a large premium on equity.

Third, as is well known, various transformations are applied to the data, which are likely to have the implication that measured consumption displays more persistence than the actual consumption choices made by agents. The fact that the data are aggregated over time is perhaps the prime example of this possibility. Thus, agents could be living in an environment with relatively little persistence in consumption, which could be reflected in a high equity premium, even though published data exhibit substantial persistence due to time aggregation. For a quantitative investigation of this idea in a closely related context, a discussion of the "Deaton paradox" for consumption, see Christiano (1989). Also, see Heaton (1995) for an analysis of habit persistence preferences in an exchange economy in which temporal aggregation is explicitly taken into account.

# 6. Conclusion

We have studied a version of the exchange economies investigated by Lucas (1978) and Mehra and Prescott (1985). We found that the key to generating a large mean equity premium in the general equilibrium models considered is to produce the 'right' dynamic behavior in the price of capital. In particular, innovations in the price of capital must be large, and negatively correlated with the marginal utility of consumption. Under these circumstances, equity is a bad hedge against risk, and thus requires a large premium to induce households to hold it. To get the appropriate movements in the price of capital, we require that (i) households have a strong incentive to buy assets when the marginal utility of consumption is low, and to sell when the marginal utility of consumption is high, and (ii) a technology which frustrates these desires. In the endowment economy, habit persistence preferences deliver (i) and the fixed capital and labor assumptions deliver (ii).

We believe this intuition is valuable for model builders interested in integrating research on asset pricing with research on business cycles. In fact, we have already found this intution to be useful. For example, Boldrin, Christiano and Fisher (1995) build on the results discussed here to show that it is possible to construct a real business cycle model which is consistent with mean asset returns. In addition, Christiano and Fisher (1995) take advantage of these insights to investigate the implications of Tobin's q and mean asset returns for business cycle analysis. In both of these papers, assumptions on the production structure are introduced to mimic the inelastic supply of capital that drives the equity premium findings in the exchange economy. These assumptions lead to improvements in the business cycle implications of the models considered.

We now briefly discuss some of the limitations of the analysis. First, consistent with the intuition described above, we find that our model implies consumption growth and the rate of return on equity are highly correlated - more so than in the data. This is a long-standing puzzle for the type of equilibrium model used here. One possible resolution, which deserves formal investigation, is that the discrepancy reflects measurement error in consumption data,

or in the price data used to convert nominal returns into real returns.<sup>10</sup> Alternatively, the resolution to the puzzle may lie in a discrepancy between the marginal utility of consumption and consumption itself. There is such a discrepancy in the model of this paper, however, it is not sufficiently large quantitatively to resolve the puzzle. A second shortcoming of the model is that it overpredicts the volatility of returns. This is an important shortcoming which deserves further attention.

 $<sup>^{10}</sup>$ A quantitative analysis of the impact of measurement error in prices appears in Christiano (1989). For a formal, maximum likelihood approach to estimation and testing when there is measurement error in the data, see Sargent (1989).

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Table 1. Financial Statistics

Statistic	U.S. Data	No Habit	Habit
$\overline{-{\cal E}r^e}$	7.82	1.84	8.54
	(1.80)	(0.01)	(0.05)
$\mathcal{E}r^f$	1.19	1.81	1.68
	(0.81)	(4e-3)	(0.12)
$\mathcal{E}\left(r^{e}-r^{f} ight)$	6.63	0.02	6.86
,	(1.78)	(0.01)	(0.14)
$\sigma_{r^e}$	19.53	2.32	40.0
	(1.32)	(0.01)	(0.2)
$\sigma_{\it r^f}$	5.27	0.72	15.0
	(0.74)	(2e-3)	(0.1)
$\sigma_{r^e-r^f}$	19.02	2.23	36.5
	(1.73)	(0.01)	(0.1)
$ ho(\Delta \log C, r^e)$	0.02	1.00	0.75
	(0.11)	(1.e-6)	(0.002)

Notes: (i) The "U.S. Data" column contains estimates of the indicated statistics with standard errors in parentheses, over the period 1892–1987 based on data provided by Nelson Mark (see Cecchetti, Lam, and Mark (1993)). (ii) All rate of return statistics are annualized and in percent terms. (iii) Results for the models are based on 500 replications of sample size 120; Monte Carlo standard errors are reported in parentheses. The latter are the standard deviation, across replications, of the associated statistics, divided by  $\sqrt{500}$ . (iv) "No Habit" corresponds to power utility, with  $\phi=1$ ; "Habit" correspond to habit persistence utility, with  $\phi=1$  and 'estimated' b,h.



