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Capital Requirements and Competition in the Banking Industry

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Abstract

This paper focuses on the interaction between regulation and competition in an industrial organisation model. We analyse how capital requirements affect the profitability of two banks that compete as Cournot duopolists on a market for loans. Bank management of both banks choose optimal levels of loans provided, equity ratio and effort to reduce loan losses so as to maximise profits. From the regulator's point of view, the free market solution is not optimal as private banks do not take into account the consumer surplus and the social cost of bankruptcy (financial stability aspects). It is shown that capital requirements may improve welfare, even under conditions that both banks would never default. Moreover, we find that higher capital requirements impose a higher burden on the inefficient bank than on the efficient one, even though the requirement may only be binding for the efficient bank. If the inefficient bank chooses a strategy that might result in bankruptcy, capital requirements are particularly welfare improving.

Keywords: Cournot duopoly, Capital requirements, Profit paradox

J.E.L. Code: G28, E44, L16

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1 Introduction

The financial services industry is generally seen as unique in the sense that the importance of a sound financial system has probably led to more regulatory interference in this industry than in any other. Traditionally, this interference has primarily been of the "command and control" type, in which the regulator states what regulated banks can and cannot do. This direct approach has been called into question as the distortionary effects of this kind of regulation may be severe. The main reason for the distortions is the informational asymmetry. Regulators do not observe all the actions of the bank, and therefore can in general not impose a first best solution. As the regulations can not be tailor made, they might have unintended consequences in that the regulations might affect the behaviour of regulated banks thereby creating moral hazard problems.

Recently, the emphasis in regulation has shifted towards the "incentive-compatible" approach. This approach seeks to align the incentives of the firm's owners and operators with social objectives. By means of sticks and carrots the bank management is encouraged to fulfil the regulators objectives. The regulator spells out desired outcomes and then allows the firm to determine how best to achieve these goals. The 1995 amendment of the Basle Accord, known as the "internal models approach" can be seen as an example of this tendency. This amendment allows banks to use their internal risk models to determine the capital requirement for market risk. Regulators backtest the reported risk levels to see if they are reliable. If they are, future capital requirements may be reduced, whereas too low reported risk levels will result in an increase.

Although theoretically superior, the practical problems related to the incentive compatible approach are numerous. Especially if exceptionally large risks that occur with very low probability are to be taken into account, finding the optimal penalty scheme is far from trivial.[1] These very large risks are potentially the most dangerous for the solvability of the bank, but as there frequency is so low, the validity of the models predicting these risks can only be tested over very long time periods. As data are often not available, backtesting these models is hardly feasible. Consequently, at least part of the regulation will probably remain of the command and control type, and within this category capital controls are the most popular instrument. Moreover, also under the incentive compatible approach, capital requirements form an important instrument.

Moral hazard issues have been at the centre of much of the recent research on regulation

[1] See Marshall and Prescott (2000) for a feasible scheme if the risk profile of assets is known to the supervisor.

of financial institutions.[2] Most of this literature focuses on situations where there is just one bank. Not much research has been conducted so far in the interaction of banks in a regulated environment. Especially for the discussion whether it is important to treat all financial institutions equally (to create a "level playing field") the interaction is crucial. One paper that does address the interaction is Boot, Dezelan, and Milbourn (2000). They find that capital requirements affect the profitability of efficient banks more than inefficient banks. The efficient bank is hit hardest if there are competitors that are not regulated, less in monopoly setting and the least when his competitors are also regulated.

In this paper we challenge the results found by Boot, Dezelan, and Milbourn (2000), using a similar industrial organisation model in which two banks compete as Cournot duopolists for risky loans. The main differences with Boot et al. are that we assume that the banks face a private cost of bankruptcy, and that the deposit insurance fund will only pay if the bank indeed fails. Under these conditions we find that higher capital requirements impose a higher burden on the inefficient bank than on the efficient one. If the capital requirement is only binding for the efficient bank, this bank might even improve its profitability. Especially if the inefficient bank might choose a fail strategy in the absence of regulation, the introduction of capital requirements improves the profitability of the good bank.

The rest of this paper is organised as follows. Section 2 first describes the model. Then the effects of capital requirements on a monopoly bank are given, followed by the results for a duopoly under the assumption that both banks would never fail. The final subsection outlines the effects of capital requirements in the case that the inefficient bank would choose a fail strategy in the absence of regulation. Section 3 summarises the main conclusions, Appendix A gives the mathematical solutions of the optimal behaviour, and Appendix B provides proofs of the propositions.

2 The model

Our basic model is a static industrial organisation model in which banks compete for risky loans as Cournot duopolists. In period zero banks attract equity and deposits, provide loans, and decide on their monitoring effort. In period one the state of the world is revealed, loans are repaid subject to credit losses, bank employees and depositors are subsequently paid, whereas shareholders get what is left. Although dynamic aspects are not explicitly modeled,

[2] See for instance Dewatripont and Tirole (1994) and Freixas and Rochet (1997), Bhattacharya, Boot, and Thakor (1998) for literature overviews.

banks implicitly value continuation as they face a private cost of default.

2.1 Characterisation of the loan market

The main activity of banks in our model is the provision of risky loans. The riskiness of the loans is due to possible credit losses. The per-unit loan losses of bank τ, denoted L_τ, depend inversely on the per-unit effort (m_τ) bank τ has put in monitoring the loans, and on the state of the economy, for which we assume two possible outcomes:[3]

$$L_\tau = \begin{cases} \frac{1}{3(1+m_\tau)} & \text{with probability } 1/10 \\ \frac{1}{15(1+m_\tau)} & \text{otherwise} \end{cases} . \tag{1}$$

In the absence of monitoring one third of the principal of the loan is lost in the worst state of the world whereas in the good state one fifteenth is lost. The per-unit costs of monitoring the loans (V_τ) are assumed linear in the effort put into it:

$$V_\tau = \alpha_\tau m_\tau, \tag{2}$$

where α_τ is a bankspecific parameter. We assume there are potentially two banks in the market, one good bank (indexed G) and one bad bank (indexed B). The good bank is more efficient than the bad bank ($\alpha_G < \alpha_B$).[4]

The loans are financed by insured deposits, for which the interest rate r is assumed fixed, and equity. Funding by equity is relatively expensive for banks as equity holders require a risk premium ρ.[5] The shareholders get all residual proceeds, if positive, from the loans after all other expenses have been paid.

Each bank competes as a Cournot duopolist and chooses a quantity of loans to produce, given by Q_τ, where $\tau \in \{G, B\}$. The per-unit price of loans, denoted P, is determined by the inverse demand function:

$$P(\Omega, Q_G, Q_B) = \Omega - Q_G - Q_B \tag{3}$$

[3]The results are qualitatively similar if a uniform loss distribution is assumed instead. For this more difficult distribution it is no longer possible to find an analytical solution for the situation in which the bad bank might fail in a bad state of the world. Numerical solutions can still be found however.

[4]Throughout the paper, the names good bank and efficient bank are used interchangeably, as are bad bank and inefficient bank.

[5]The assumption of a fixed risk premium can be relaxed. Similar, but more complex, solutions result if the risk premium is a linear function of the standard deviation of returns or the probability of default.

where Ω is the intercept in the price function. This intercept can be interpreted as the maximum price the marginal consumer would be willing to pay for the loan in the absence of loan supply.

The bank management is assumed to maximise expected extraordinary profits. By this we mean profits in excess of the minimum required expected profit needed to be able to attract equity. As the required rate of return on equity is higher than that on deposits, without further assumptions the bank would never be inclined to attract equity. We provide a positive role to equity as a buffer against bad states of the world by assuming that the bank faces a private cost of bankruptcy (D_τ). The motivation for this cost is twofold. From the point of view of the individual banker, bankruptcy ruins his reputation and thereby his expected future income. In terms of shareholder value the bankruptcy cost represents the loss of the franchise value (i.e. the capitalised value of expected future profits) of the bank in case of bankruptcy (Hellmann, Murdock, and Stiglitz 2000).[6] We will assume that the bankruptcy cost for the good bank are sufficiently high, such that it will never be profitable for this bank to pursue strategies that might lead to bankruptcy. A higher cost of default for the most efficient bank is reasonable given the higher franchise value for this bank.

The regulator is entitled to impose minimum capital requirements (δ_{min}) that limit the fraction of loans financed by deposits. The resulting maximisation problem reads as follows:

$$\max_{Q_\tau, m_\tau, \delta_\tau} E(\Pi_\tau) = \begin{cases} Q_\tau(P - L_\tau - V_\tau - (1+r) - \delta_\tau \rho) & \text{if } P \geq L_\tau + V_\tau + (1-\delta_\tau)(1+r) \\ -Q_\tau \delta_\tau (1 + r + \rho) - D_\tau & \text{otherwise} \end{cases} \tag{4}$$

such that $\delta_\tau \geq \delta_{min}$.

where $E(\Pi_\tau)$ denotes the expected profit of bank τ and δ_τ represents the bank specific equity ratio.

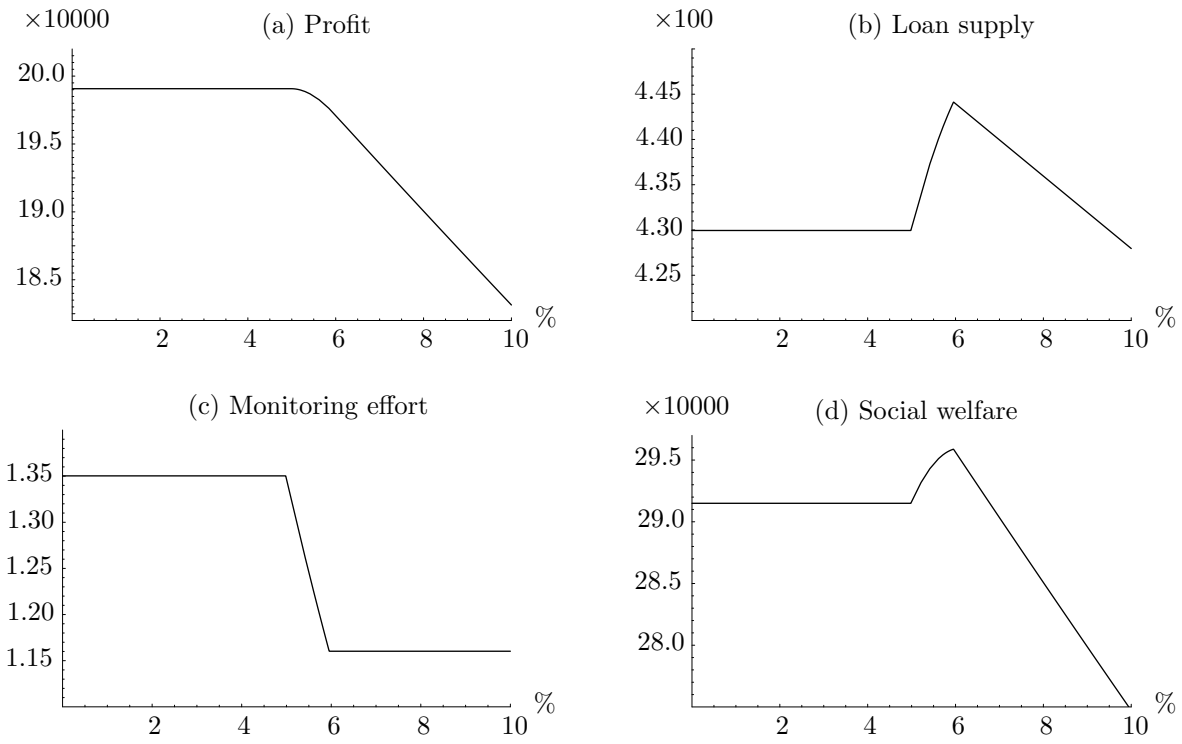
2.2 Solution for a monopoly

The first case considered is the one where the bad bank is not efficient enough to make a profit.[7] This benchmark case is illustrative for the effects of capital requirements on bank behaviour and profit, as the additional effects of the interaction between banks is not relevant then. All mathematical derivations are given in Appendix A. In the main text most results will be shown graphically. The calibrated numbers that were used to generate the figures

[6] The implicit assumption behind this argument is that the link between shareholders and the bank remains extant after period one. This is the case if not all equity is returned to the shareholders in period one.

[7] In the model, this is the case if both α_B and D_B are sufficiently high.

Figure 1: Optimal results under monopoly for various capital requirements



are shown in Table 1. The risk free deposit rate is assumed to be 4% annually and the risk premium is 8% annually. Reducing credit losses by one half will cost the bank 2%. Demand is such that the maximum interest rate to be paid on loans is 20%.

Table 1: Parameters used for monopoly

r	ρ	Ω	α_G	D_G
4/100	8/100	6/5	1/50	1

Figure 1 shows the results based on optimal bank behaviour for various capital requirements. The bank faces two constraints that must be met. On the one hand the minimum capital requirement and on the other the no default constraint. The figure shows three different phases, related to the importance of these constraints. During the first phase (here as long as $\delta_{min} < 5.0\%$), only the nofail constraint is binding, whereas the capital constraint is not. In the second phase both the capital requirement and the nofail constraint are binding. In the third phase (starting at 6%), only the capital requirement is binding. The influence of the capital requirements on bank behaviour is given in Proposition 1.

Proposition 1 *If the optimal equity ratio of the bank in absence of regulation is higher than*

zero, the introduction of a binding capital requirement always reduces the optimal monitoring effort level and increases the loan supply as long as:

$$\delta_{min} < \frac{2\sqrt{\alpha_G}\left(\frac{\sqrt{7+7r+25\rho}}{\sqrt{1+r+\rho}} - \sqrt{7}\right)}{5\sqrt{3}\rho} \tag{5}$$

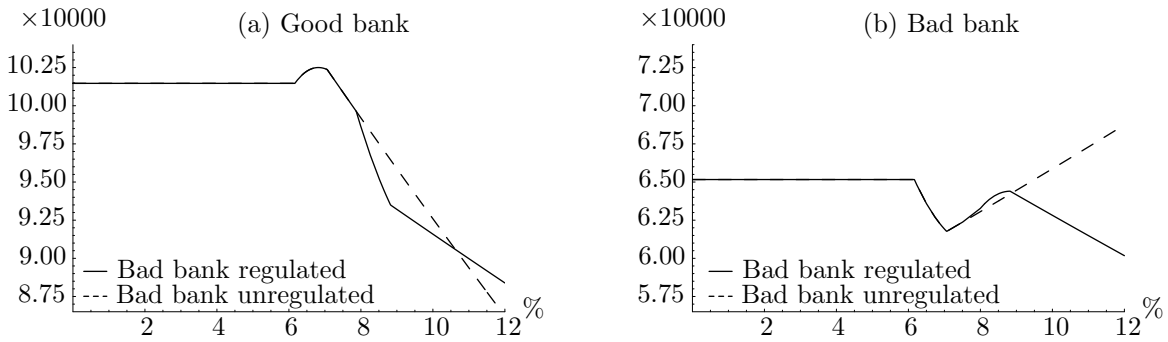
Proof: As the only advantage of equity over deposits is that equity reduces the probability of failure, the fact that the optimal equity ratio without regulation is higher than zero implies that the nofail constraint is binding. This fail probability can be reduced either by holding more equity, by putting more effort in monitoring or by reducing loan supply (increasing the price). In equilibrium the marginal cost of the three ways to fulfil the constraint will be the same. An increase in the equity ratio will alleviate the nofail constraint and, as the shadow prices for effort and supply are nonzero with respect to this constraint, will lead to a decrease in effort (Panel c) and an increase in loan supply (Panel b). Once the minimum capital requirement has risen above the point where the nofail constraint is no longer binding, further increase of the minimum equity ratio will reduce the loan supply as the increased financing costs is the only mechanism then. Equation 5 follows directly from equating the optimal output solutions for the regulated (Equation 16) and the unregulated (Equation 12) case. For a more extensive proof, including proof that the capital requirement in Equation 5 is indeed higher than the voluntarily held capital ratio, see Appendix B. □

Panel a of Figure 1 shows the equilibrium profit for the monopoly bank. Indeed profit is decreasing from the point on where the capital requirement becomes binding. Nevertheless, Social welfare, computed as the sum of consumer surplus and bank profit, increases initially due to the capital requirement (Panel d). This result is due to the fact that the bank increases output, and thereby increases the consumer surplus. This result indicates potential beneficial effects of capital requirements, even in the case where banks would never fail.

2.3 Solution for two never failing banks

The second case to be considered is the one where both banks are in the market and where the private cost of default are sufficiently high for both banks not to gamble. As banks risking bankruptcy are rather the exception than the rule, this exploration might tell us something about the impact of regulation on sound banks. Two cases will be considered. Under the standard scenario both banks are regulated. Under the alternative, only the good bank is regulated. The latter case is particularly interesting for analysing the level playing

Figure 2: Optimal profit nonfailing banks for various capital requirements



field. Boot, Dezelan, and Milbourn (2000) found that efficient banks bear a higher cost of regulation than inefficient banks, especially if the bad bank is not regulated. Surprisingly enough, for moderate minimum capital requirements we find exactly the opposite.

Figure 2 shows the equilibrium profit of the two banks, using the calibration from Table 2. The calibration for the good bank is the same as before. The monitoring costs for the inefficient bank are assumed to be 25% higher than those for the efficient bank. Reducing expected credit losses by one half costs the inefficient bank 2.5 percent of the principal of the loan, whereas for the efficient bank these costs are only 2 percent. The graphs for the two nonfailing regulated banks display five different phases, based on the relevance of the two constraints facing the banks. In the first phase, the nofail constraint is binding for both banks, whereas the capital constraint is not. In the second phase (capital requirements between 6.2% and 7.1%) the nofail constraint is still binding for both banks, whereas the capital requirement is only binding for the efficient bank. In the third phase the good bank only faces a binding capital constraint and the bad bank only a binding nofail constraint. If the bad bank is not regulated (dashed lines), this is the last phase. In the fourth phase (between 7.9% and 8.8%) the capital constraint is binding for both and the nofail constraint only for the inefficient bank. In the last phase only the capital constraint is binding for both banks.

Table 2: Parameters for two nonfailing banks

r	ρ	Ω	α_G	D_G	α_B	D_B
4/100	8/100	6/5	1/50	1	1/40	1

The most surprising result shown in Figure 2 is that although in first instance (phase two) the capital requirement is only binding for the efficient bank, this bank increases is profit,

whereas the profit of the inefficient bank drops. This *profit paradox* is our Proposition 2.

Proposition 2 *In the unregulated case, the optimal capital ratio for the efficient bank is lower than for the inefficient one. If the capital requirements are just binding for the efficient bank and not binding for the inefficient one, profits of the inefficient bank will always drop, whereas the profit of the restricted bank may rise relative to the unregulated case.*

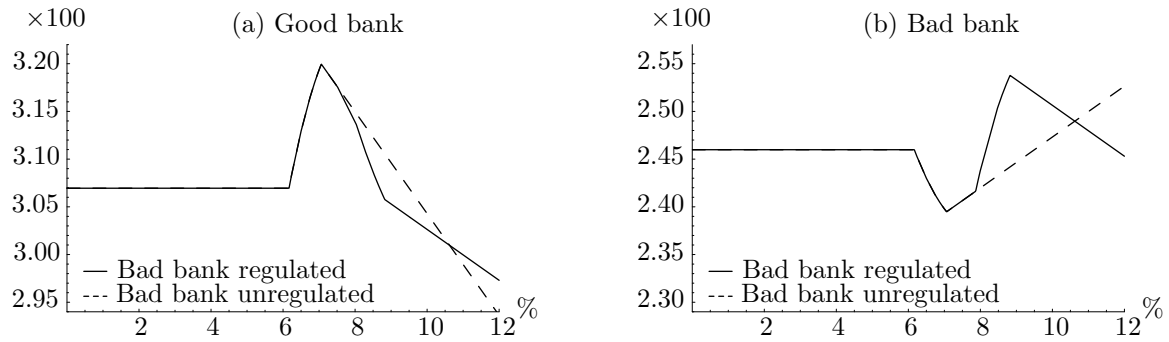
Proof: The capital requirement hits the efficient bank first as the alternative for holding equity, that is increasing monitoring effort, is less expensive for this bank. A formal proof is given in Appendix B. The drop in profits for the inefficient bank follows directly from Proposition 1. As the efficient bank will increase its output, the bad bank is confronted with a lower residual demand. That the good bank might increase its profit is shown in Figure 2. □

The temporary increase in profit for the restricted bank in phase two is due to a shift in market share from the inefficient bank to the efficient one, see Figure 3. The question then arises why the efficient bank needs a regulator in order to be able to get this market share. Why should he not declare to voluntarily held more capital and thereby achieve more output. The reason is that such a declaration is not credible. Taken the optimal response of the other bank as given, the efficient bank could improve further by deviating from the announced target. If the bank reduces both its loan supply, its effort and its capital ratio, its profits can increase. Consequently, the restricted equilibrium is not a Cournot Nash equilibrium of the unrestricted game.[8]

Whether the profits of the efficient bank will indeed increase depends on the reaction of the other bank. Two opposing forces are at work. On the one hand the capital costs of the bank increase due to the forced increase of its equity ratio. This will lead to lower profit, as shown for the monopoly case. The increase in the market share on the other hand increases the restricted bank's profit. For our parameterization, the market share argument dominates in first instance. However, if the minimum capital requirement is increased further (even within phase two when the nofail constraint is still binding) profits of the efficient bank decline as the financing cost argument starts dominating. Beyond the point where the nofail constraint is no longer binding, the good bank starts losing market share again as the increased financing costs decrease its optimal output. In the fourth phase, where the bad bank also

[8]If the efficient bank could credibly declare its capital ratio and loan supply, as the leader in a Stackleberg game, its optimal profit in the unregulated case would in general be much higher than the maximum profit in the restricted Cournot game. In that case imposing a binding capital requirement on the leader will always reduce its profit.

Figure 3: Optimal loan supply nonfailing banks for various capital requirements



faces a binding capital constraint if it is regulated, this process is accelerated as the nofail constraint on the bad bank is loosening. Consequently, the bad bank can afford a lower price and therefore increases output (Figure 3). Here again, the profit paradox appears. The bad bank is better off if it is regulated whereas the good bank is worse off (Figure 2). For higher capital requirements, these result are reversed. The developments of profits in the final phase are given in Proposition 3.

Proposition 3 *The marginal percentage loss in profits for the good bank in the final phase due to additional capital requirements is the highest if the good bank faces an unregulated competitor, lower under monopoly and the lowest in a fully regulated duopoly. Compared with the profit losses of the regulated bad bank in duopoly, the losses of the good bank are higher in absolute terms, but lower in relative terms. An unregulated bad bank will increase profits if the minimum capital requirements for the good bank are increased in the final phase.*

Proof: See Appendix B. □

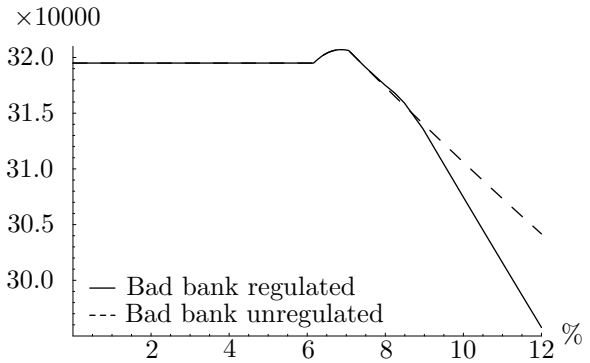
This last phase is particularly important as opponents of capital requirements usually claim that they are set too high. The proposition claims that even under these circumstances the regulated bad bank is not better off than the good bank. However, if there are unregulated competitors, too high capital requirements will have distortionary effects as market share is shifted from the efficient to the inefficient bank. Consequently, very high capital requirements will endanger the level playing field if not all financial institutions are regulated. The same conclusions hold for the model of Boot, Dezelan, and Milbourn (2000), but they do not mention the (most relevant) results for the relative profits.

In terms of optimal monitoring effort the results for the two banks are similar to those for the monopoly bank as optimal effort is not affected by the output of the other bank. Only

the question to what extent the nofail constraint is binding is relevant here.

Figure 4 shows the welfare implications of regulation by means of capital requirements. Not surprisingly, the duopoly solution results in higher welfare than monopoly (Figure 1d). Moreover, as in the monopoly case, capital requirements may be beneficial even in the case that no bank will ever fail. The optimal welfare level is reached at 6.87%, which is within phase two.

Figure 4: Social welfare for various capital requirements on never failing banks



2.4 Solution if the bad bank might fail

As the main role of bank regulation is to prevent banks from failing, the biggest influence of regulation might be expected for cases where banks might fail in the absence of regulation. Indeed, that turns out to be the case (Proposition 4). In our model a profitable fail strategy is achieved by adjusting the private cost of bankruptcy for the bad bank (D_B). Table 3 shows the parameterization for this case. Apart from the lower value for D_B it is identical to the nofail case. D_B is parameterized such that the bank makes a positive profit in the good state of the world but fails in the bad state. In this subsection, it will be assumed that both banks are regulated.[9]

Table 3: Parameters if bad bank might fail

r	ρ	Ω	α_G	D_G	α_B	D_B
4/100	8/100	6/5	1/50	1	1/40	1/200

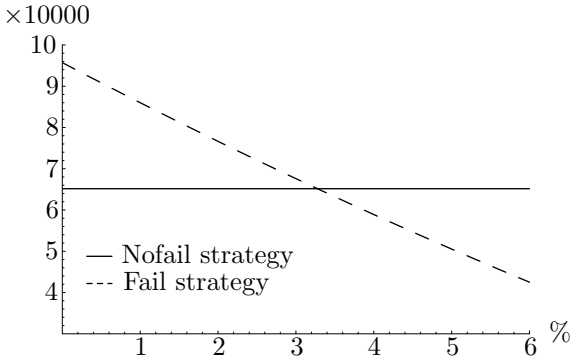
[9]This assumption seems reasonable as an institution with a nonzero failprobability would not be able to attract funds at the riskless deposit rate unless if it falls under a deposit insurance scheme. It that case it is most likely also regulated.

Proposition 4 *The existence of a bank that chooses a fail strategy strongly affects the profitability of the good bank. Capital requirements can be used to prevent the bad bank from choosing the fail strategy, thereby positively affecting the profitability of the good bank.*

Proof: See Figures 5 and 6a. □

In the unregulated case, the bad bank may either gamble for a good state of the world and finance its loans completely with deposits (fail strategy), or attract just enough capital to avoid bankruptcy (nofail strategy). The latter strategy is discussed in the previous subsection. Due to the limited liability of the bank, and the deposit insurance that ensures a fixed deposit rate, under the fail strategy only the outcome in the good state matters for the expected profit of the bank. Consequently, this bank can afford a lower price for loans and therefore can attract a large market share (Figure 6b). As monitoring effort is only paying out in the good state of the world, optimal monitoring effort is lower for the fail strategy (Figure 6c). The introduction of capital requirements introduces a financing cost for the bank. The higher the minimum equity ratio, the more money the bank loses in the bad state of the economy. After some point it becomes more profitable to prevent failure altogether. This is shown graphically in Figure 5.

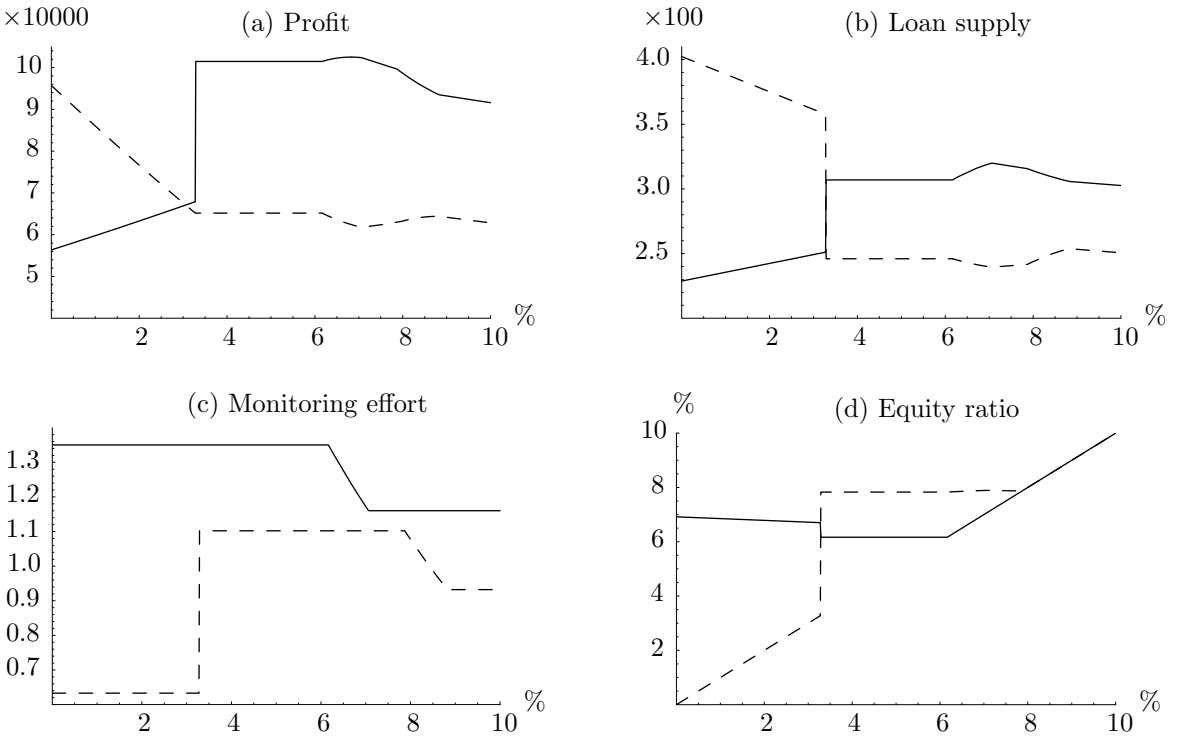
Figure 5: Optimal profit bad bank for various capital requirements



The strategy the bad bank chooses has a huge impact on the profitability of the good bank (Figure 6a). The lower marginal cost of the bad bank, which is due to the limited liability of the bank management, enables it to gain a large market share (Figure 6b). Once the capital requirements become too high for the fail strategy to be profitable (here from 3.3% on), we are back in the results of the previous subsection.

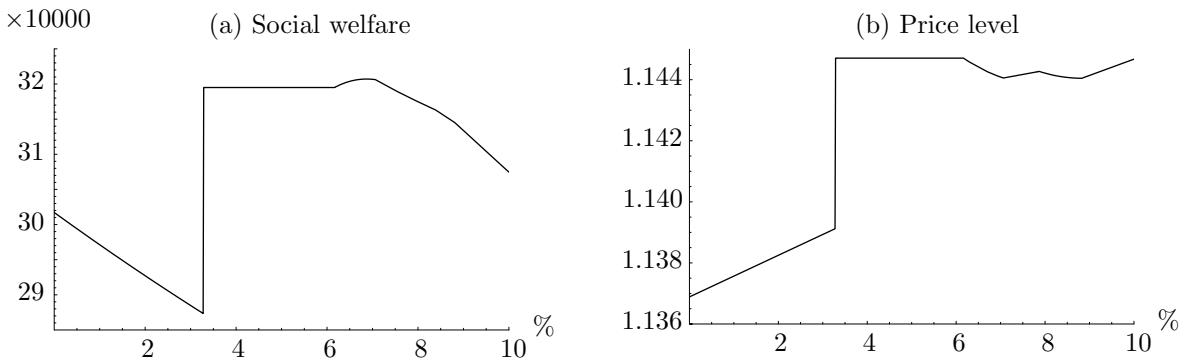
Figure 7a shows the social welfare implications of capital requirements, where welfare is computed as the sum of profits of the two banks, the consumer surplus and the cost of deposit insurance. The consumer surplus will be higher under the bankruptcy regime as the total

Figure 6: Optimal results good bank (solid lines) and possibly failing bank (dashed lines) for various capital requirements



amount of loans supplied is higher and consequently the price of loans is lower (Figure 7b). Nevertheless, even without taking the social cost of bankruptcy into account, the welfare levels under the bankruptcy regime are substantially lower than under the nofail regime. Introducing capital requirements will be strongly welfare improving if they are high enough to prevent the bad bank from following a fail strategy.

Figure 7: Social welfare and price level for various capital requirements



3 Conclusions

In the paper the competitive distortions of capital requirements are investigated. Contrary to Boot, Dezelan, and Milbourn (2000) we find that capital requirements impose a higher burden on inefficient banks than on efficient ones. Especially if the inefficient bank would pursue a fail strategy in absence of regulation, capital requirements strongly improve the profitability of the efficient bank.

Regarding the importance of a level playing field, our results indicate that as long as the competitors that are not regulated are not pursuing fail strategies, and if the capital requirements are not too high, binding capital requirements may even increase the profit of the regulated bank whereas the profit of the unregulated bank will drop. The situation that the unregulated competitor follows a fail strategy can not well de analysed in our model as one of the assumptions is that deposit rates are fixed. If the competitor is indeed protected by deposit insurance, he is not likely to be unregulated.

All in all, the distortionary effects of capital requirements seem to be mild. One should keep in mind however, that moral hazard problems hardly arise in this model as the asset side of the model is fixed. Banks can only invest in one kind of loans. An interesting extension to the model might be the introduction of a second loan market with different credit risk. If the regulator is not able to differentiate between the markets, higher capital requirements might result in more risk taking as it increases the funding costs of loans. Especially if the regulator is not able to regulate all banks equally, the non-regulated bank might be able to get a large proportion of the low-risk market, leaving the high-risk market for the regulated banks. This will be left for future research.

A Derivation of optima

The calculations made for this paper were all done in Mathematica 4.0 (Wolfram 1999). The complete notebook is available from the author upon request.

If the bank never defaults Equation 4 boils down to:

$$\max_{Q_\tau, m_\tau, \delta_\tau} E(\Pi_\tau) = Q_\tau \left(\Omega - Q_\tau - Q_\chi - \alpha_\tau m_\tau - (1+r) - \frac{7}{75(1+m_\tau)} - \delta_\tau \rho \right) \quad (6)$$

$$\text{such that} \quad \Omega - Q_\tau - Q_\chi \geq (1-\delta_\tau)(1+r) + \alpha_\tau m_\tau + \frac{1}{3(1+m_\tau)} \quad (7)$$

$$\text{and} \quad \delta_\tau \geq \delta_{min}. \quad (8)$$

The derivation of optima for bank τ (and bank χ) is always performed in a similar fashion. If either of the two constraints is binding it is solved for either δ_τ, Q_τ or m_τ and subsequently substituted in the profit function (Equation 6). The remaining decision variables are determined by solving the first order conditions together with the first order conditions for the rival bank. Finally, the second order conditions for both banks are checked to assure the solution is indeed a maximum.

A.1 The monopoly case

The solutions are presented in general form, including a second bank (indexed χ), in order to be able to use the results later on. For the monopoly bank solution, Q_χ is zero and the index τ should be replaced by G.

If bank τ is not regulated, it will choose an equity ratio which is just high enough to avoid bankruptcy in the worst state of nature. Solving the nofail constraint (7) for δ_τ one gets:

$$\delta_\tau = \frac{1}{3(1+m_\tau)(1+r)} + \frac{1 + r + \alpha_\tau m_\tau + Q_\tau + Q_\chi - \Omega}{1+r} \quad (9)$$

This result is substituted in the profit function (6), the first order conditions with respect to m_τ and Q_τ are solved, second order conditions are checked for the only solution with positive outcomes for both output and effort (its a maximum), and this solution is substituted back

in the profit and equity ratio functions. The results are:

$$\Pi_\tau = \frac{\left(2\sqrt{\alpha_\tau(7+7r+25\rho)}+5\sqrt{3(1+r+\rho)}(1+r-\alpha_\tau+Q_\chi-\Omega)\right)^2}{300(1+r)} \tag{10}$$

$$\delta_\tau = \frac{5\sqrt{\alpha_\tau(1+r+\rho)}}{(1+r)\sqrt{3(7+7r+25\rho)}}+\frac{1+r-\alpha_\tau+Q_\chi-\Omega}{2(1+r)} \tag{11}$$

$$Q_\tau = -\frac{\sqrt{\alpha_\tau(7+7r+25\rho)}}{5\sqrt{3(1+r+\rho)}}+\frac{\Omega-Q_\chi-1-r+\alpha_\tau}{2} \tag{12}$$

$$m_\tau = \frac{\sqrt{7+7r+25\rho}}{5\sqrt{3\alpha_\tau(1+r+\rho)}}-1 \tag{13}$$

If both the capital requirement and the nofail constraint are binding, first the nofail constraint can be solved for Q_τ:

$$Q_\tau = \Omega - Q_\chi - \frac{1}{3(1+m_\tau)} - (1+r)(1-\delta_\tau) - \alpha_\tau m_\tau \tag{14}$$

This expression and the capital requirement are substituted in the profit function, after which the first order condition with respect to m_τ is solved. This gives three solutions, one of which is negative, and one is not a maximum (positive second derivative). The allowable solution reads:

$$m_\tau = \frac{(-1+i\sqrt{3})(18(\Omega-Q_\chi-1-r+\alpha_\tau)+43\delta_{min}(1+r)+25\rho\delta_{min})}{30A} - \frac{(1+i\sqrt{3})A}{30\alpha_\tau\delta_{min}(1+r+\rho)} - 1$$

where $i=\sqrt{-1}$ and

$$A=\left(\sqrt{\alpha_\tau^3\delta_{min}^3(1+r+\rho)^3(72900\alpha_\tau\delta_{min}(1+r+\rho)-(18(\Omega-Q_\chi-1-r+\alpha_\tau)+43\delta_{min}(1+r)+25\rho\delta_{min})^3)}\right.$$
$$\left.-270\alpha_\tau^2\delta_{min}^2(1+r+\rho)^2\right)^{1/3}$$

The imaginary parts in this equation exactly cancel, so this solution is real. This solution can be substituted in (14) and (6) to get solutions for output and profit respectively.

If the nofail constraint is no longer binding, the first order conditions of Equation 6 with respect to m_τ and Q_τ can be solved. The solution with positive effort and output turns out to be a maximum. The solution is as follows:

$$\Pi_\tau = \frac{\left(-2\sqrt{21\alpha_\tau}-15(1+r-\alpha_\tau+Q_\chi-\Omega+\rho\delta_{min})\right)^2}{900} \tag{15}$$

$$Q_\tau = \frac{-2\sqrt{21\alpha_\tau}-15(1+r-\alpha_\tau+Q_\chi-\Omega+\rho\delta_{min})}{30} \tag{16}$$

$$m_\tau = \frac{\sqrt{7}}{5\sqrt{3\alpha_\tau}}-1 \tag{17}$$

A.2 Two nonfailing banks

The method for solving the two bank case is similar to the one bank case for the unregulated situation. First solve the nofail constraints for the optimal equity ratio (Equation 9) substitute this is profit functions (6) and derive the first order conditions with respect to effort and output. The solutions for effort are identical to the monopoly case as in the unregulated case these do not depend on the output of the other bank (Equation 13). The first order conditions with respect to output should be solved simultaneously. The solution with positive output levels is indeed a maximum. The solutions are:

$$\Pi_\tau = \frac{\left(2\left(\sqrt{\alpha_\chi}-2\sqrt{\alpha_\tau}\right)\sqrt{7+7r+25\rho}-5\sqrt{3(1+r+\rho)}(1+r+\alpha_\chi-2\alpha_\tau-\Omega)\right)^2}{675(1+r)} \quad (18)$$

$$\delta_\tau = \frac{-2\sqrt{\alpha_\chi}(7+7r+25\rho)+2\sqrt{\alpha_\tau}(41+41r+50\rho)}{15\sqrt{3}(1+r)\sqrt{(1+r+\rho)(7+7r+25\rho)}}+\frac{1+r+\alpha_\chi-2\alpha_\tau-\Omega}{3(1+r)} \quad (19)$$

$$Q_\tau = \frac{2\left(\sqrt{\alpha_\chi}-2\sqrt{\alpha_\tau}\right)\sqrt{7+7r+25\rho}}{15\sqrt{3(1+r+\rho)}}-\frac{1+r+\alpha_\chi-2\alpha_\tau-\Omega}{3} \quad (20)$$

The solution for the good bank with both a binding capital constraint and a binding nofail constraint differs somewhat from the monopoly case as the optimal output level of the other bank depends on the loan supply of the constraint bank. Consequently, loan supply can not be used to solve for the nofail constraint, as the first order conditions of the rival bank would bring it back into the optimisation problem. Instead the effort level is used to solve for the nofail constraint:

$$m_\tau = \frac{1}{6\alpha_\tau}\Big(3(\Omega-Q_\tau-Q_\chi-(1+r)(1-\delta_{min})-\alpha_\tau)- \sqrt{-12\alpha_\tau\left(1-3(\Omega-Q_\tau-Q_\chi-(1+r)(1-\delta_{min}))+9\left(\Omega-Q_\tau-Q_\chi-(1+r)(1-\delta_{min})-\alpha_\tau\right)^2\right.}\Big) \quad (21)$$

This solution, together with the capital requirement are substituted in the profit function. The first order condition with respect to output is derived. The optimal reaction of the rival bank (Equation 12 with $\tau = B$ and $\chi = G$) is subsequently substituted in this first order condition. This gives a complicated expression that has to be solved with respect to own output. The problem has three solutions, two of which generate positive profit levels. For one of these two the capital requirement turns out not to be binding, so it is not a valid solution. According to the second order condition, the other one results indeed in a maximum. The solution is too messy to report however (5 pages of Mathematica output).

With this solution at hand, the solution for the other decision variables follows directly by substitution.

In the third phase, the good bank only faces a binding capital constraint whereas for the bad bank only the nofail constraint is binding. The solutions for optimal effort follow directly from the monopoly bank results (Equations 17 respectively 13) as these results do not depend on the actions of the rival bank (other than via the influence on the nofail constraint). After the optimal capital ratios (using Equation 8 respectively 9) and monitoring efforts are substituted in the profit functions, the first order conditions with respect to loan supply are derived and solved simultaneously. The results are:

$$\Pi_G = \frac{\left(\sqrt{1+r+\rho}\left(5\sqrt{3}(1+r+\alpha_B-2\alpha_G+2\rho\delta_{min}-\Omega)+4\sqrt{7\alpha_G}\right)-2\sqrt{\alpha_B(7+7r+25\rho)}\right)^2}{675(1+r+\rho)} \quad (22)$$

$$\Pi_B = \frac{\left(\sqrt{1+r+\rho}\left(5\sqrt{3}(1+r+\alpha_G-2\alpha_B-\rho\delta_{min}-\Omega)-2\sqrt{7\alpha_G}\right)+4\sqrt{\alpha_B(7+7r+25\rho)}\right)^2}{675(1+r)} \quad (23)$$

$$\delta_B = \frac{2\sqrt{3\alpha_B}(41+41r+50\rho)}{45(1+r)\sqrt{(1+r+\rho)(7+7r+25\rho)}}+\frac{15(1+r+\alpha_G-2\alpha_B-\rho\delta_{min}-\Omega)-2\sqrt{21\alpha_G}}{45(1+r)} \quad (24)$$

$$Q_G = \frac{2\sqrt{\alpha_B(7+7r+25\rho)}-\sqrt{1+r+\rho}\left(5\sqrt{3}(1+r+\alpha_B-2\alpha_G+2\rho\delta_{min}-\Omega)+4\sqrt{7\alpha_G}\right)}{15\sqrt{3(1+r+\rho)}} \quad (25)$$

$$Q_B = -\frac{4\sqrt{3\alpha_B(7+7r+25\rho)}}{45\sqrt{1+r+\rho}}-\frac{15(1+r+\alpha_G-2\alpha_B-\rho\delta_{min}-\Omega)-2\sqrt{21\alpha_G}}{45} \quad (26)$$

In the fourth phase the bad bank faces two binding constraints, whereas the good bank only faces a binding capital constraint. The solution method is similar to phase two. The optimal effort of the bad bank is determined as the solution to the binding nofail constraint (Equation 21). This result and the binding capital constraint are substituted in its profit function. The first order condition with respect to output is derived. In this first order condition, the optimal output of the good bank is substituted (Equation 16). The equation is solved for Q_B and the right solution is selected. Again, two of the three solutions result in a positive profit, but for one of them the capital constraint is not binding. According to the second order conditions the valid solution is indeed a maximum. Results for the good bank can subsequently be obtained by substituting this solution in the optimal results of the capital constrained bank (Equations 15 to 17).

In the last phase both bank only face a binding capital constraint. After substituting the capital constraints and the optimal effort levels (Equation 17) the first order conditions with

respect to output are solved. This results in the following:

$$\Pi_\tau = \frac{\left(2\sqrt{21}\left(\sqrt{\alpha_\chi}-2\sqrt{\alpha_\tau}\right)-15(1+r+\alpha_\chi-2\alpha_\tau-\Omega+\rho\delta_{min})\right)^2}{2025} \quad (27)$$

$$Q_\tau = \frac{2\sqrt{21}\left(\sqrt{\alpha_\chi}-2\sqrt{\alpha_\tau}\right)-15(1+r+\alpha_\chi-2\alpha_\tau-\Omega+\rho\delta_{min})}{45} \quad (28)$$

A.3 Failprobability higher than zero

If the bad bank chooses a fail strategy, its optimisation problem is as follows:

$$\max_{Q_B,m_B} E(\Pi_B) = Q_\tau\left(\frac{9}{10}\left(\Omega-Q_B-Q_G-\alpha_B m_B-(1-\delta_{min})(1+r)-\frac{1}{15(1+m_B)}\right)-(1+r+\rho)\delta_{min}\right)-\frac{D_B}{10}. \quad (29)$$

In order to solve the model, first calculate the first order conditions with respect to m_B and Q_B. Then substitute the optimal output of the good bank (Equation 12), assuming that the capital requirement is not binding for this bank. Next, solve the first order conditions and check second order conditions. The results for the bad bank are:

$$\Pi_B = -\frac{D_B}{10}+\left(\left(\sqrt{1+r+\rho}\left(45(\Omega-1-r+2\alpha_B-\alpha_G)-12\sqrt{15\alpha_B}-10\delta_{min}(1+r+10\rho)\right)+6\sqrt{3\alpha_G(7+7r+25\rho)}\right)^2\right)/\left(20250(1+r+\rho)\right) \quad (30)$$

$$Q_B = \frac{2\sqrt{3\alpha_G(7+7r+25\rho)}}{45\sqrt{1+r+\rho}}+\frac{45(\Omega-1-r+2\alpha_B-\alpha_G)-12\sqrt{15\alpha_B}-10\delta_{min}(1+r+10\rho)}{135} \quad (31)$$

$$m_B = \frac{1}{\sqrt{15\alpha_B}}-1 \quad (32)$$

The results for the good bank are obtained by substituting Equation 31 in the unregulated monopoly bank solutions (Equations 10 to 12):

$$\Pi_G = \left(\left(\sqrt{1+r+\rho}\left(45(\Omega-1-r-\alpha_B+2\alpha_G)+6\sqrt{15\alpha_B}+5\delta_{min}(1+r+10\rho)\right)-12\sqrt{3\alpha_G(7+7r+25\rho)}\right)^2\right)/\left(18225(1+r)\right) \quad (33)$$

$$\delta_G = \frac{\sqrt{3\alpha_G(7+7r+25\rho)}}{45(1+r)\sqrt{1+r+\rho}}+\frac{5\sqrt{\alpha_G(1+r+\rho)}}{(1+r)\sqrt{3(7+7r+25\rho)}}-\frac{45(\Omega-1-r-\alpha_B+2\alpha_G)+6\sqrt{15\alpha_B}+5\delta_{min}(1+r+10\rho)}{135(1+r)} \quad (34)$$

$$Q_G = \frac{-4\sqrt{3\alpha_G(7+7r+25\rho)}}{45\sqrt{1+r+\rho}}+\frac{45(\Omega-1-r-\alpha_B+2\alpha_G)+6\sqrt{15\alpha_B}+5\delta_{min}(1+r+10\rho)}{135} \quad (35)$$

B Proof of Propositions

B.1 Proposition 1

The fact that monitoring effort reduces after the capital constraint becomes binding follows directly from a comparison between Equations 13 and 17:

$$\frac{\sqrt{7+7r+25\rho}}{5\sqrt{3\alpha_\tau(1+r+\rho)}}-1>\frac{\sqrt{7}}{5\sqrt{3\alpha_\tau}}-1 \tag{36}$$

for every positive ρ.

The length of the range of capital constraints for which loan supply is increased (denoted $\delta_{\Delta Q}$) is given by the difference between the Equations 5 and 11:

$$\delta_{\Delta Q}=\frac{2\sqrt{\alpha_G}\left(\frac{\sqrt{7+7r+25\rho}}{\sqrt{1+r+\rho}}-\sqrt{7}\right)}{5\sqrt{3}\rho}-\frac{5\sqrt{\alpha_G(1+r+\rho)}}{(1+r)\sqrt{3(7+7r+25\rho)}}-\frac{1+r-\alpha_G+Q_B-\Omega}{2(1+r)} \tag{37}$$

In order to check whether this length is positive Ω is substituted by the minimum amount necessary for the bank to make a positive profit (solving Equation 10 equals zero for Ω) plus a nonnegative extra (denoted ϵ):

$$\Omega=1+r+Q_B-\alpha_G+\frac{2\sqrt{\alpha_G(7+7r+25\rho)}}{5\sqrt{3(1+r+\rho)}}+\epsilon \tag{38}$$

After substituting 38 in 37 one gets:

$$\delta_{\Delta Q}=\frac{\frac{15\epsilon\rho}{1+r}+4\sqrt{3\alpha_G}\left(\frac{7+7r+16\rho}{\sqrt{(1+r+\rho)(7+7r+25\rho)}}-\sqrt{7}\right)}{30\rho} \tag{39}$$

Only the part within brackets might be negative, but in fact is not either as:

$$\left(\frac{7+7r+16\rho}{\sqrt{(1+r+\rho)(7+7r+25\rho)}}\right)^2-7=\frac{81\rho^2}{(1+r+\rho)(7+7r+25\rho)}>0 \tag{40}$$

Consequently, for every positive risk premium there is a positive range of capital requirements for which the restricted bank will increase its output.

This result will also hold if there are more banks involved that do not face a binding capital constraint. The result is obtained under the assumption that the other bank will not react, which is the case in a monopoly. In a duopoly, if the other bank is not restricted, the only thing that changes for him is the increased output of its rival. As this decreases the residual demand for loans, this will lead to a decrease in its own supply, leading to an even higher increase of output of the restricted bank.

B.2 Proposition 2

In the unregulated case, the difference between the optimal capital ratios for the two banks (Equation 19) is equal to:

$$\delta_B - \delta_G = \frac{\left(\sqrt{\alpha_B} - \sqrt{\alpha_G}\right)\left(2\sqrt{3}(16+16r+25\rho) - 15\left(\sqrt{\alpha_B}+\sqrt{\alpha_G}\right)\sqrt{(1+r+\rho)(7+7r+25\rho)}\right)}{15(1+r)\sqrt{(1+r+\rho)(7+7r+25\rho)}} \tag{41}$$

In order to check positivity of the second term in the numerator, take the difference between the squared positive and negative term:

$$\begin{aligned}\left(2\sqrt{3}(16+16r+25\rho)\right)^2 - \left(15\left(\sqrt{\alpha_B}+\sqrt{\alpha_G}\right)\sqrt{(1+r+\rho)(7+7r+25\rho)}\right)^2 = \\ 3(1+r)^2\left(1024 - 525\left(\sqrt{\alpha_B}+\sqrt{\alpha_G}\right)^2\right) + 75\rho\left(32(1+r)+25\rho\right)\left(4 - 3\left(\sqrt{\alpha_B}+\sqrt{\alpha_G}\right)^2\right)\end{aligned} \tag{42}$$

A sufficient condition to guarantee a positive outcome is $(\sqrt{\alpha_B}+\sqrt{\alpha_G}) < \sqrt{4/3}$. This condition is always fulfilled as the optimal effort level would not be positive if it was not.

B.3 Proposition 3

Both for monopoly and for duopoly, the equilibrium profit for a bank that faces a binding capital constraint, but no default constraint, is equal to the square of its output (see Equations 15 and 16, 22 and 25 respectively 27 and 28). The partial derivatives of profit of the good bank with respect to the capital requirements under these circumstances are $-\rho Q_G$ for a monopoly, $-\frac{4}{3}\rho Q_G$ when confronted with an unregulated competitor and $-\frac{2}{3}\rho Q_G$ when faced with an regulated competitor. Consequently, in absolute terms the loss must be bigger under monopoly than under a regulated duopoly as output under monopoly can not be lower than under duopoly. Dividing by total profits ($= Q_G^2$) leads to the conclusion that the relative profit loss is highest if the good bank faces an unregulated competitor, smaller under monopoly and smallest if the bad bank is also regulated. The latter inequality holds as long as the output of the good bank under a regulated duopoly is at least two third of the output under monopoly. As the good bank is supposed to be more efficient ($\alpha_G < \alpha_B$), this is always the case (its output will be exactly two third of the monopoly output if both banks are equally efficient).

The absolute losses of the unregulated bad bank are higher than those of the good bank as $\frac{2}{3}\rho Q_G > \frac{2}{3}\rho Q_B$. In relative terms the opposite is true as $\frac{2\rho}{3Q_G} < \frac{2\rho}{3Q_B}$.

The profit of the unregulated bank increases in the minimum capital requirement as $\frac{\partial \Pi_B}{\partial \delta_{min}} = \frac{2\rho Q_B(1+r+\rho)}{3(1+r)}$ (see Equations 23 and 26), which is always positive.

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