Gaps and Triangles*

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Abstract

In this paper, we derive principles of optimal cyclical monetary policy in an economy without capital, with a cash-in-advance restriction on household transactions, and with monopolistic firms that set prices one period in advance. The only distortionary policy instruments are the nominal interest rate and the money supply. In this environment, it is feasible to undo both the cash in advance and the price setting restrictions, but not the monopolistic competition distortion. We show that it is optimal to follow the Friedman rule, and thus offset the cash-in-advance restriction. We also

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find that, in general, it is not optimal to undo the price setting restriction. Sticky prices provide the planner with tools to improve upon a distorted flexible prices allocation.

Key words: Optimal cyclical monetary policy; Friedman rule; prices set in advance

JEL classification: E31; E41; E58; E62

1. Introduction

In this paper, we derive general principles on how to conduct short-run monetary policy, by analyzing a real business cycles model without capital, to which three main restrictions are added: monopolistic firms, a cash-in-advance restriction on household transactions, and a restriction on firms that prices must be set one period in advance. In this sticky price environment, monetary policy can affect allocations both through the path of the nominal interest rate and the path of the money supply. These are the only distortionary policy instruments that the government can use. In addition the government can raise lump sum taxes to finance exogenous public expenditures.

In this environment, it is possible to conduct money supply policy in order to undo the restriction of price stickiness, so that the allocations under sticky and flexible prices coincide. By setting the nominal interest rate to zero, the effect of the cash-in-advance restriction can also be eliminated. However, with the available policy instruments, because of the zero bound on the nominal interest rate, the mark-up distortion cannot be removed.

We first show that the optimal monetary policy includes setting the nominal interest rate to zero, i.e. following the Friedman rule. That is a robust result and thus extends the result in Ireland (1996). The major finding of the paper, though, is that in general it is not optimal to completely undo the effects of price stickiness. Optimal rules in this environment require the sticky price allocation to deviate from that of the flexible price. Thus, the policy that maximizes welfare, and similarly would minimize the Harberger triangles, does not close gaps.[1] Sticky prices provide the planner with policy tools to improve upon a distorted flexible price allocation.

[1] As is common in this literature, gaps are defined as the deviations between the flexible price and the sticky price allocations.

Under flexible prices, only the interest rate matters. The optimal monetary policy is to set the nominal interest rate to zero. That way the wedge between the marginal rate of substitution and the marginal rate of transformation is minimized in each date and state, and expected utility is maximized. The optimal allocation will be distorted by a constant mark-up.

Under sticky prices, it is also optimal to set the nominal interest rate to zero, and it is still the case that under particular conditions on preferences, technology, and the nature of shocks, constant markups are optimal. However, in general, the optimal allocation under sticky prices will be characterized by variable markups. These variable proportionate wedges cannot be attained under flexible prices because the nominal interest rate cannot be negative. Under sticky prices, the planner is able to use money supply policy to side-step the zero bound restriction on the nominal interest rate and achieve higher utility.

The literature on optimal monetary policy in an imperfectly competitive and sticky price world is relatively recent. Papers related to the current analysis include Ireland (1996), Carlstrom and Fuerst (1998a and1998b), King and Wolman (1998), Goodfriend and King (1997, 2000), Rotemberg and Woodford (1997, 1999), Kahn, King and Wolman (2000), Gali and Monacelli (1999), Erceg et al. (2000).

Our analysis builds on the work by Ireland (1996), Goodfriend and King (1997), Carlstrom and Fuerst (1998a and1998b), and Adão, Correia and Teles (1999). Our approach is in many ways orthogonal to the one in Rotemberg and Woodford (1997, 1999), Gali and Monacelli (1999) and Erceg et al (2000). These papers analyze an environment with a basic structure similar to ours, but with three main differences. They allow for fiscal instruments that undo the monopolistic competition distortion; the economies are cashless; and prices are set in a staggered fashion. The nominal rigidities are the only distortions so that the flexible price allocation, if feasible, is optimal.

King and Wolman (1998) study a model with staggered price setting, keep the monopolistic competition distortion but remove the nominal interest rate distortion by allowing for interest to be paid on currency. They show that the flexible price solution is optimal in the deterministic case. Kahn, King and Wolman (2000) allow for the money demand distortion and solve the optimal policy problem numerically. They show that the optimal allocation is quantitatively close to the flexible price allocation. Goodfriend and King (2000) analyze the case of a small open economy and discuss the analogy between the optimal monetary policy under sticky prices and the optimal taxation problem under flexible prices.

The remainder of the paper is organized as follows. In Section 2 we describe the model, define the equilibria under flexible and sticky prices, and describe how monetary policy affects the equilibria in the two environments. In Section 3, we define the planner's problem in the economy with sticky prices. We then determine the optimal allocation and policy in two steps. We first determine the optimal interest rate policy. For this policy there are multiple allocations, corresponding to different money supply policies. One of those implementable allocations is the flexible price allocation. We identify the conditions on preferences, technology and shocks under which the flexible price allocation solves the planner's problem. Since it is unlikely that those conditions are satisfied, in general the optimal allocations in the two environments will differ. The optimal policy under sticky prices achieves higher utility than under flexible prices. Section 4 contains concluding remarks.

2. The economy

The economy consists of a representative household, a continuum of firms indexed by $i \in [0,1]$, and a government or central bank. We consider a vector of shocks, $s_t = [\varkappa_t, z_t, v_t, G_t] \in S_t$, to preferences, $\varkappa_t$, technology, z_t, velocity, v_t, and government expenditures, G_t. The history of these shocks up to period t, $(s_0, s_1, ..., s_t)$, is denoted by $s^t \in S^t$. In order to simplify the exposition, we assume that the history of shocks has a discrete distribution.

Each firm produces a distinct, perishable consumption good, indexed by i. The production uses labor, according to a concave technology.

We impose a cash-in-advance constraint to the households' transactions with the timing as in Lucas and Stokey (1983). Each period is divided into two subperiods, with the assets market in the first subperiod and the goods market in the second.

2.1. The households

The households have preferences over composite consumption C_t, and leisure L_t, described by the expected utility function:

$$U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(C_t, L_t, \varkappa_t) \right\}, \tag{2.1}$$

where β is a discount factor, and $\varkappa_t$ is the preference shock. C_t is defined as

$$C_t = \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \theta > 1, \tag{2.2}$$

where $c_t(i)$ is the consumption level of good i, and θ is the elasticity of substitution between any two goods.

Households start period t with nominal wealth $\mathbb{W}_t$. They receive monetary transfers X_t, decide to hold money, M_t, and to buy B_t nominal bonds that pay $R_t B_t$ one period later. R_t is the gross nominal interest rate at date t. They also buy A_{t+1} units of state contingent nominal securities. Each security pays one unit of money at the beginning of period $t+1$ in a particular state. Let $Q_{t,t+1}$ be the beginning of period t price of these securities normalized by the probability of the occurrence of the state. Therefore, households spend $E_t Q_{t,t+1} A_{t+1}$ in state contingent nominal securities. Thus, in the assets market at the beginning of period t they face the constraint

$$M_t + B_t + E_t Q_{t,t+1} A_{t+1} - A_t - X_t \leq \mathbb{W}_t. \tag{2.3}$$

A fraction, $\frac{1}{v_t}$, of the purchases of consumption, $\int_0^1 P_t(i) c_t(i) di$, where $P_t(i)$ is the price of good i in units of money, must be made with money so that the following cash-in-advance constraint is satisfied,

$$\frac{1}{v_t} \int_0^1 P_t(i) c_t(i) di \leq M_t. \tag{2.4}$$

At the end of the period, the households receive the labor income $W_t N_t$, where $N_t = 1 - L_t$ is labor and W_t is the nominal wage rate. They also receive the dividends from the firms $\int_0^1 \Pi_t(i) di$, and pay lump sum taxes, T_t. Thus, the nominal wealth households bring to period $t+1$ is

$$\mathbb{W}_{t+1} \leq M_t + R_t B_t - \int_0^1 P_t(i) c_t(i) di + W_t N_t + \int_0^1 \Pi_t(i) di - T_t. \tag{2.5}$$

The households' problem is to maximize expected utility (2.1) subject to the restrictions (2.2), (2.3), (2.4) and (2.5), together with no-Ponzi games conditions on the holdings of assets.

Let $P_t = \left[\int P_{it}^{1-\theta} di\right]^{\frac{1}{1-\theta}}$. The following are first order conditions of this problem,[2] for all t and s^t,

$$\frac{c_t(i)}{C_t} = \left(\frac{P_t(i)}{P_t}\right)^{-\theta}, \tag{2.6}$$

$$\frac{u_L(t)}{u_C(t)} = \frac{W_t}{P_t}\frac{1}{R_t'}, \text{ where } R_t' \equiv 1 + \frac{1}{v_t}(R_t - 1), \tag{2.7}$$

$$\frac{u_C(t)}{P_t} = R_t' E_t\left[\frac{\beta u_C(t+1)}{P_{t+1}}\frac{R_{t+1}}{R_{t+1}'}\right], \tag{2.8}$$

and, for all t, $t+1$, s^t, s^{t+1},

$$Q_{t,t+1} = \frac{\beta\frac{u_C(t+1)}{P_{t+1}}\frac{R_{t+1}}{R_{t+1}'}}{\frac{u_C(t)}{P_t}\frac{R_t}{R_t'}}. \tag{2.9}$$

From (2.8) and (2.9), we have

$$E_t\left[Q_{t,t+1}\right] = \frac{1}{R_t}, \text{ for all } t \text{ and } s^t. \tag{2.10}$$

Condition (2.6) defines the demand for each of the goods i and condition (2.7) sets the intratemporal marginal rate of substitution between leisure and composite consumption equal to the real wage adjusted for the cost of using money. The cost of using money is $R_t' = 1 + \frac{1}{v_t}(R_t - 1)$, since only a fraction of goods, $\frac{1}{v_t}$, must be purchased with money. Condition (2.8) is a requirement for the optimal choice of risk-free nominal bonds. Condition (2.9) determines the price of one unit of money at time $t+1$, for each state of nature s^{t+1}, normalized by the conditional probability of occurrence of state s^{t+1}, in units of money at time t.

When $v_t = 1$ then $R_t' = R_t$. In this case, our model coincides with the standard, unitary velocity, cash-in-advance model. As $v_t \to \infty$, the cash constraint becomes irrelevant and the model reduces to a real model. Note that in that case $R_t' = 1$. It is also worth noting that when $R_t = 1$, the velocity shock does not affect the households' decision, since the cost of holding money will be zero.

[2] We denote by $u_j(t)$, the partial derivative of the function $u(C_t, L_t, \varkappa_t)$ with respect to the argument $j = C_t, L_t$.

2.2. Government

The government has to finance an exogenous stream of government purchases, G_t. These purchases result from the aggregation of the expenditures on each good produced in the economy,

$$G_t = \left[\int_0^1 g_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}, \theta > 0.$$

Given G_t and the prices on each good, $P_t(i)$, the government chooses the quantities, $g_t(i)$, to minimize total spending. Thus, the purchases of each good i must satisfy

$$\frac{g_t(i)}{G_t} = \left(\frac{P_t(i)}{P_t}\right)^{-\theta}. \tag{2.11}$$

The government finances G_t with lump sum taxes $T_t = P_t G_t$. These taxes are collected at the goods market. In addition, the government makes a lump-sum monetary transfer, X_t, to the households at the assets market.

As government debt is irrelevant in this environment, we choose to write the government budget constraint as a balanced budget constraint. Therefore

$$M_t^s = M_{t-1}^s + P_t G_t + X_t - T_t.$$

The money supply evolves according to $M_t^s = M_{t-1}^s + X_t$.

2.3. Firms

Each firm i has the production technology

$$y_t(i) \leq z_t F(n_t(i)), \tag{2.12}$$

where $y_t(i)$ is the production of good i, $n_t(i)$ is the labor used in the production of good i, and z_t is an aggregate technology shock. Good i can be used for private and public consumption, $y_t(i) = c_t(i) + g_t(i)$. The profits of firm i are $\Pi_t(i) = P_t(i) y_t(i) - W_t n_t(i)$.

We will consider now the maximization problem of the firms in the two environments, when firms can set prices contemporaneously and when prices have to be set one period in advance.

2.3.1. Under flexible prices

Under flexible prices, firm i maximizes the value of period t profits in units of money, $E_t Q_{t,t+1} \Pi_t(i)$, subject to the production function (2.12) and to the demand function,

$$\frac{y_t(i)}{Y_t} = \left(\frac{P_t(i)}{P_t}\right)^{-\theta}, \tag{2.13}$$

obtained from (2.6) and from (2.11), where $Y_t = C_t + G_t$.

The first order conditions of this problem are such that

$$P_t(i) = P_t = \frac{\theta}{\theta - 1} \frac{W_t}{z_t F_N(t)}. \tag{2.14}$$

The firms set a common price, equal to a constant mark-up over marginal cost.

2.3.2. Under sticky prices

We consider now the environment where firms set prices one period in advance and sell output on demand in period t at the previously chosen price. In this environment firm i solves the problem of choosing at $t-1$, the price $P_t(i)$ that maximizes the value of profits, $E_{t-1}[Q_{t-1,t} Q_{t,t+1} \Pi_t(i)]$, subject to (2.12) and (2.13). Using (2.8), (2.9) and the law of iterated expectations, the objective function of this problem can be rewritten as

$$E_{t-1}\left[\left(\frac{u_C(t+1)}{P_{t+1}} \frac{R_{t+1}}{R'_{t+1}}\right) \Pi_t(i)\right].$$

The price chosen by the firm is

$$P_t(i) = P_t = \frac{\theta}{(\theta - 1)} E_{t-1}\left[\eta_t \frac{W_t}{z_t F_N(t)}\right], \tag{2.15}$$

where

$$\eta_t = \frac{\left(\frac{u_C(t+1)}{P_{t+1}} \frac{R_{t+1}}{R'_{t+1}}\right) Y_t}{E_{t-1}\left[\left(\frac{u_C(t+1)}{P_{t+1}} \frac{R_{t+1}}{R'_{t+1}}\right) Y_t\right]}. \tag{2.16}$$

All firms set the price equal to a mark-up over the expected value of a weighted marginal cost.

2.4. Market clearing:

In each period there are markets for goods, labor, money, risk-free nominal bonds and state contingent nominal securities. Market clearing conditions are given by the following equations, for each date and state

$$c_t(i) + g_t(i) = y_t(i),$$

so that the demand for each of the goods is equal to the supply,

$$\int_0^1 n_t(i)\ di = N_t,$$

so that the total demand of labor is equal to the supply,

$$B_t = 0,$$

and

$$A_{t+1} = 0,$$

since nominal bonds and the contingent securities are in zero net supply, and

$$M_t^s = M_t.$$

From the cash in advance constraints (2.4) of the households and the market clearing conditions we have,

$$\frac{P_t C_t}{v_t} = M_{t-1}^s + X_t \equiv M_t^s. \tag{2.17}$$

2.5. Imperfectly competitive equilibria

Equilibria in the two environments, flexible prices and sticky prices, are defined as follows:

Definition 2.1. An equilibrium is a set of prices $\{(P_t(s^t), P_t(i)(s^t), W_t(s^t), Q_{t,t+1}(s^{t+1}), R_t(s^t))_{i\in[0,1],\ s^t\in S^t, s^{t+1}\in S^{t+1}}\}_{t=0}^{\infty}$, allocations $\{(Y_t(s^t), y_t(i)(s^t), C_t(s^t), c_t(i)(s^t), N_t(s^t), n_t(i)(s^t), M_t(s^t), B_t(s^t), A_{t+1}(s^{t+1}))_{i\in[0,1],\ s^t\in S^t,\ s^{t+1}\in S^{t+1}}\}_{t=0}^{\infty}$, initial nominal wealth $\mathbb{W}_0$, and policy variables $\left\{(X_t(s^t), T_t(s^t), G_t(s^t), g_t(i)(s^t))_{i\in[0,1],\ s^t\in S^t}\right\}_{t=0}^{\infty}$ such that: (i) Given the prices $\left\{(P_t(s^t), P_t(i)(s^t), W_t(s^t), Q_{t,t+1}(s^{t+1}), R_t(s^t))_{i\in[0,1],\ s^t\in S^t, s^{t+1}\in S^{t+1}}\right\}_{t=0}^{\infty}$,

initial nominal wealth $\mathbb{W}_0$, and policy variables $\left\{(X_t(s^t), T_t(s^t))_{s^t \in S^t}\right\}_{t=0}^{\infty}$ the sequences $\{(C_t(s^t), c_t(i)(s^t), N_t(s^t), M_t(s^t), B_t(s^t), A_{t+1}(s^{t+1}))_{i \in [0,1],\ s^t \in S^t, s^{t+1} \in S^{t+1}}\}_{t=0}^{\infty}$ solve the problem of the representative household.
(ii) (a) In the environment with flexible prices, given prices $\left\{(P_t(s^t), W_t(s^t))_{s^t \in S^t}\right\}_{t=0}^{\infty}$, and total output $\left\{(Y_t(s^t))_{s^t \in S^t}\right\}_{t=0}^{\infty}$, the sequence $\left\{P_t(i)(s^t)_{i \in [0,1],\ s^t \in S^t}\right\}_{t=0}^{\infty}$ solves the problem of the firms, as in Section 2.3.1.
(b) In the environment with sticky prices, given prices $\{(P_t(s^t), W_t(s^t), Q_{t-1,t}(s^t), Q_{t,t+1}(s^{t+1}), R_t(s^t))_{s^t \in S^t, s^{t+1} \in S^{t+1}}\}_{t=0}^{\infty}$, and total output $\left\{(Y_t(s^t))_{s^t \in S^t}\right\}_{t=0}^{\infty}$, the sequence $\left\{(P_t(i)(s^t))_{i \in [0,1],\ s^t \in S^t}\right\}_{t=0}^{\infty}$, solves the problem of the firms, as in Section 2.3.2. In this environment the prices are such that $P_t(s^{t-1}, s') = P_t(s^{t-1}, s'')$ for all $s^{t-1} \in S^{t-1}$, and $s', s'' \in S_t$, and $P_t(i)(s^{t-1}, s') = P_t(i)(s^{t-1}, s'')$ for all $i \in [0,1]$, $s^{t-1} \in S^{t-1}$, and $s', s'' \in S_t$.
(iii) All markets clear.

In the following section we characterize the set of equilibria defined above. In particular, we discuss how the sets of equilibrium allocations can be implemented using monetary policy in the two environments.

2.6. Implementable allocations under flexible and under sticky prices

Under flexible prices, for each path of the nominal interest rate there is a single equilibrium allocation. The set of implementable allocations is the set of equilibrium allocations associated with the different interest rate policy paths, $\{R_t\}$, with $R_t \geq 1$. This set can be described, for each date and state, by an intratemporal marginal condition and the feasibility condition.

Under flexible prices, the marginal condition of households, (2.7), and the price setting condition, (2.14), can be combined as

$$\frac{u_L(C_t, 1-N_t, \varkappa_t)}{u_C(C_t, 1-N_t, \varkappa_t)} = \frac{(\theta-1)}{\theta R_t'} z_t F_N(N_t), \text{ all } t \geq 0,\ s^t, \tag{2.18}$$

where $R_t' = \frac{R_t - 1 + v_t}{v_t}$. The level of the interest rate and the mark-up of the monopolistic competition affect the allocation on the same margin. Both introduce a wedge between the marginal rate of transformation and the marginal rate of substitution. Given a path for the gross nominal interest rate, $\{R_t\}$, condition (2.18) together with the feasibility constraint,

$$C_t + G_t = z_t F(N_t), \text{ all } t \geq 0,\ s^t, \tag{2.19}$$

determines the flexible price allocation $\{C_t, N_t\}$, for each date and state.

It is straightforward to see that the optimal allocation is implemented by setting the net interest rate equal to zero. With $R_t = 1$, the wedge is minimized for each date and state, and expected utility is maximized.

Each allocation can be implemented by multiple paths for the money supply. Thus the price level is not uniquely determined. This is the nature of the nominal indeterminacy under flexible prices. The price level and the money supply for each date and state must satisfy the cash-in-advance constraint, (2.4), that is repeated here

$$\frac{P_t C_t}{v_t} = M_t^s, \text{ all } t \geq 0,\, s^t,$$

and the intertemporal condition, (2.8),

$$\frac{u_C\left(C_{t-1}, 1 - N_{t-1}, \varkappa_{t-1}\right)}{P_{t-1} R'_{t-1}} = \beta E_{t-1} \frac{u_C(C_t, 1 - N_t, \varkappa_t) R_t}{P_t R'_t}, \text{ all } t \geq 1,\, s^{t-1}.$$

which, as stated, holds from period one on. At $t = 0$, there is one cash-in-advance constraint to determine both the price level and the money supply.

If the number of states at $t \geq 1$ is Φ_t, there are $\Phi_t + 1$ equations to determine $2\Phi_t$ variables, the time t price levels and money supplies. Thus, there is a continuum of sequences for the money supply and for the price level that satisfy these equations. For a given path of the price level, the nominal wage can be determined using condition (2.7). In summary, under flexible prices there is a single allocation implemented by a particular sequence of the nominal interest rate and there are multiple sequences for the money supply, the price level and the nominal wage associated with that allocation.

It is important to note that there is one equilibrium price level sequence where the price does not depend on the contemporaneous information. Given P_0, in period 0 there is one equation to determine M_0, and in each period $t \geq 1$, there are $\Phi_t + 1$ equations to determine $\Phi_t + 1$ variables, i.e., Φ_t money supplies and one price level. When the money supply is set in this way, and if firms were restricted to set prices in advance, that restriction would not be binding. This means that the set of flexible prices equilibrium allocations can be implemented under sticky prices.

In contrast to what happens under flexible prices, in the sticky price environment, for a given process of the nominal interest rate, there are multiple equilibrium allocations, each one associated with a different process for the money supply.

In the sticky price environment, the marginal condition of the households, (2.7), and the price setting restriction, (2.15), can be used to obtain

$$E_{t-1}\left[\eta_t \frac{\frac{u_L(C_t,1-N_t,\varkappa_t)}{u_C(C_t,1-N_t,\varkappa_t)}}{\frac{(\theta-1)}{\theta R_t'} z_t F_N(t)}\right] = 1, \text{ all } t \geq 1, \ s^{t-1}, \tag{2.20}$$

where η_t is given by (2.16). This condition restricts the average proportionate wedge between the marginal rate of transformation and the marginal rate of substitution. It can be manipulated, together with the intertemporal condition, (2.8), using the law of iterated expectations, to obtain the implementability condition

$$E_{t-1}\left[\frac{u_C(t)}{\frac{\theta R_t'}{(\theta-1)}} z_t F(t) - \frac{F(t)}{F_N(t)} u_L(t)\right] = 0, \text{ all } t \geq 1, \ s^{t-1}. \tag{2.21}$$

Given the nominal interest rate process, condition (2.21) together with (2.19) determine the set of implementable allocations $\{C_t, N_t\}$. Notice that for each date there are more unknowns ($2\Phi_t$) than equations ($1 + \Phi_t$). While there is a single allocation under flexible prices for a given path of the nominal interest rate, under sticky prices there are multiple allocations that can be implemented. Each of those allocations is implemented with a path for the money supply, $\{M_t\}$, $t \geq 0$, a path for the price level, $\{P_t\}$, $t \geq 1$, and a path for the nominal wage rate, which are obtained from

$$\frac{u_C\left(C_{t-1}, N_{t-1}, \varkappa_{t-1}\right)}{P_{t-1} R_{t-1}'} = \frac{\beta}{P_t} E_{t-1} \frac{u_C(C_t, N_t, \varkappa_t) R_t}{R_t'}, \text{ all } t \geq 1, \ s^{t-1},$$

the cash-in-advance constraint, (2.4), and the households' intratemporal condition, (2.7). The time 0 price level, P_0, is exogenous.[3]

In summary, in both environments, for a given path of the nominal interest rate, there is a continuum of money supply policies. Under sticky prices, each of these policies is associated with a different real allocation. This is the sense in which Carlstrom and Fuerst (1998a) call attention to the real indeterminacy associated with an interest rate policy under sticky prices. Our approach is very different, since we allow the planner to decide on the money supply and, therefore, to use the degrees of freedom implied by (2.21) and (2.19) to achieve the optimal allocation.

[3]Notice that the degrees of freedom are the same under sticky and flexible prices. Under flexible prices there are Φ_t price setting conditions each period that are replaced by one under sticky prices. However under sticky prices there is only one price level to determine in each period.

3. Optimal allocations under sticky prices

In this section, we characterize the optimal allocations under commitment to policy in the sticky price environment. These allocations maximize welfare in the set of implementable allocations described above.

As mentioned above, under sticky prices it is possible to use the money supply policy to replicate the set of allocations under flexible prices. This is stated in the following proposition.

Proposition 1: *The set of implementable allocations under sticky prices contains the corresponding set under flexible prices.*[4] *Thus, the optimal allocation under sticky prices makes the households at least as well off as under flexible prices.*

Proof. This proposition is straightforward since for each interest rate path $\{R_t\}$, the feasibility constraints are the same in two environments, and the implementability condition under sticky prices, (2.21), is the expected value of the constraints (2.18) of the flexible prices environment. ■

The optimal flexible price allocation will be the benchmark to which the optimal allocation under sticky prices will be compared. As we will show, in general the two optimal allocations do not coincide, so that it is not optimal to follow a policy that eliminates gaps. Sticky prices allow the planner to choose an allocation that is strictly better than the optimal allocation under flexible prices.

The planner's problem is to choose the sequences of consumption, C_t, labor, N_t, and interest rate R_t, that maximize (2.1) subject to the feasibility constraint (2.19), the implementability condition (2.21) and the condition that the gross nominal interest rate is greater than or equal to one, $R_t \geq 1$. The Lagrangian for the Ramsey problem under sticky prices can be written as

$$\begin{aligned} \mathsf{L} &= \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \Pr(s^t) u\left(C_t, 1 - N_t, \varkappa_t\right) \\ &\quad + \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \lambda_t(s^t) \Pr(s^t)(z_t F(N_t) - G_t - C_t) \end{aligned}$$

[4] This has been stated in Carlstrom and Fuerst (1998a), even though the emphasis is on policies where this possibility of targeting both the nominal interest rates and the price level is excluded. See also Adão, Correia and Teles (1999).

$$+\sum_{t=0}^{\infty}\sum_{s^{t-1}\in S^{t-1}}\beta^t\varphi_t(s^{t-1})\sum_{s^t\in S^t}\Pr(s^t)\left(\frac{u_C(t)}{\frac{\theta R_t'}{(\theta-1)}}(C_t+G_t)-\frac{F(t)}{F_N(t)}u_L(t)\right) \tag{3.1}$$

where $\beta^t\lambda_t(s^t)\Pr(s^t)$, $\beta^t\varphi_t(s^{t-1})\Pr(s^{t-1})$ are the multipliers of the resources constraint and the implementability condition, respectively, and $\Pr(s^t)$ is the probability of the history s^t.

In the next two subsections we analyze the planner's problem. We do it in two stages. We start by determining the optimal interest rate policy. We show that it is necessary to follow the Friedman rule, of setting the nominal interest rate to zero, in order to achieve the optimal allocation. The Friedman rule is also optimal under flexible prices. However, while under flexible prices there is a unique allocation for consumption and labor, under sticky prices there are multiple implementable allocations. In Section 3.2, we characterize the optimal allocation under sticky prices, and compare it to the optimal allocation under flexible prices.

3.1. The optimal interest rate policy

The following proposition states the optimal interest rate policy,

Proposition 2: *The optimal interest rate policy is the Friedman rule, i.e.* $R_t = 1$.

Proof. A marginal increase in the nominal interest rate has a negative impact on utility, which is given by

$$-\beta^t\varphi_t(s^{t-1})\Pr(s^t)\frac{u_C(t)}{\frac{\theta R_t'^2}{(\theta-1)}}\frac{1}{v_t}(C_t+G_t). \tag{3.2}$$

Given the non-negativity constraint on the net nominal interest rate and the fact that in this second best environment φ_t is always strictly positive, it is optimal to set $R_t' = R_t = 1$. ■

The optimal interest rate path under flexible prices coincides with the optimal interest rate path in the sticky price environment. As we restricted the fiscal instruments to lump sum taxes, the planner does not need to rely on distortionary taxes to finance public expenditures. Under flexible prices, if the nominal interest rate is set equal to its lower bound, the economy will be distorted by the monopolistic mark-up only. If the net nominal interest rate in any state is set higher than zero, then this worsens the distortion in that state and reduces expected utility. Under sticky prices, the wedge is not necessarily higher in that particular state.

However, it is still optimal to set the interest rate equal to its lower bound since otherwise the relevant average wedge is larger.

When the net nominal interest rate is zero, the level of real balances is indeterminate as the cash-in-advance constraint does not bind. Under flexible prices there is a unique equilibrium for consumption and labor. However, in a sticky price environment, at the Friedman rule, for one money supply path there are multiple equilibrium allocations. This is shown in a comment to Ireland (1996) by Carlstrom and Fuerst (1998b). The indeterminacy is no longer present for a positive but arbitrarily small interest rate. It seems to us that all that is economically relevant is the behavior of these economies as the interest rate approaches zero.[5]

3.2. Optimal policy for a given interest rate path

We pursue the analysis of the optimal money supply policy, taking as given a constant nominal interest rate R_t, that is greater than, but arbitrarily close to, one. In the limit R_t' is also equal to one for any velocity shock, v_t. We characterize the optimal wedges, i.e., the optimal deviation of the ratio of the marginal rate of substitution to the marginal rate of transformation from the first best level, $\frac{u_C(t)}{u_L(t)} z_t F_N(t) - 1$. Under flexible prices, when the nominal interest rate is one, these wedges are constant and equal to the markup, $\frac{\theta}{(\theta-1)} - 1$. Under sticky prices, the wedges are on average equal to that number, but can vary across states. We investigate whether it is optimal to conduct policy so that the wedges are constant. In other words, we want to establish whether it is optimal to use non-neutral monetary policy under sticky prices to replicate the flexible price allocation, i.e. to close gaps.

The optimal allocation is the allocation that maximizes the utility in the set defined by the feasibility constraint, (2.19), and the implementability condition, (2.21), for the optimal interest rate path. First order conditions of the planner's problem include

$$u_C(t)\left\{1+\varphi_t\left[\frac{u_{CC}(t)}{u_C(t)\frac{\theta}{(\theta-1)}}Y_t+\frac{1}{\frac{\theta}{(\theta-1)}}-\frac{u_{LC}(t)}{u_C(t)}\varpi(t)\right]\right\}=\lambda_t(s^t), \tag{3.3}$$

[5] We thank Robert Lucas and V.V. Chari for discussions on this issue. The approach we follow in this paper is normative. We do not model the game played by the government and the private agents. If we were to do so, the Friedman rule would be the policy in the unique subgame perfect equilibrium.

and

$$u_L(t)\left\{1+\varphi_t\left[\frac{u_{LC}(t)}{u_L(t)\frac{\theta}{(\theta-1)}}Y_t+\varpi_N(t)-\frac{u_{LL}(t)}{u_L(t)}\varpi(t)\right]\right\}=\lambda_t(s^t)z_tF_N(t), \quad (3.4)$$

where $\varpi(t)=\frac{F(t)}{F_N(t)}$ and $\varpi_N(t)$ is the derivative of $\varpi(t)$ with respect to N_t.

The optimality conditions can be written as

$$\frac{u_C(t)}{u_L(t)}z_tF_N(t)=\frac{1+\varphi_t\left[\frac{u_{LC}(t)C_t}{u_L(t)\frac{\theta}{(\theta-1)}}\frac{Y_t}{C_t}+\varpi_N(t)-\frac{u_{LL}(t)}{u_L(t)}\varpi(t)\right]}{1+\varphi_t\left[\frac{u_{CC}(t)C_t}{u_C(t)\frac{\theta}{(\theta-1)}}\frac{Y_t}{C_t}+\frac{1}{\frac{\theta}{(\theta-1)}}-\frac{u_{LC}(t)}{u_C(t)}\varpi(t)\right]}, \quad (3.5)$$

together with the implementability and the feasibility conditions, (2.19) and (2.21). Notice that the left hand side of condition (3.5) is one plus the optimal markup. We want to determine under which conditions on preferences, technology, and shocks, the allocation with constant markups satisfies the optimality conditions. We have seen that the constant markup allocation satisfies the feasibility and implementability condition (Proposition 1). It remains to be checked whether that allocation satisfies the marginal condition (3.5). Condition (3.5) can be written as

$$\begin{aligned}\frac{u_C(t)}{u_L(t)}z_tF_N(t)-1 \;=\; & \varphi_t\frac{\frac{u_C(t)}{u_L(t)}z_tF_N(t)}{\frac{\theta}{(\theta-1)}}\left(-\frac{u_{CC}(t)C_t}{u_C(t)}\frac{Y_t}{C_t}-1+\frac{u_{LC}(t)}{u_C(t)}\varpi(t)\right)+\\ & +\varphi_t\left(\varpi_N(t)-\frac{u_{LL}(t)}{u_L(t)}\varpi(t)+\frac{u_{LC}(t)C_t}{u_L(t)}\frac{Y_t}{C_t}\right). \quad (3.6)\end{aligned}$$

Since the allocation with constant markups is such that $\frac{u_C(t)}{u_L(t)}z_tF_N(t)=\frac{\theta}{(\theta-1)}$, it must be that that allocation satisfies the optimality conditions if and if the expression

$$D_t\equiv-\frac{u_{CC}(t)C_t}{u_C(t)}\frac{Y_t}{C_t}+\frac{u_{LC}(t)}{u_C(t)}\varpi(t)+\varpi_N(t)-\frac{u_{LL}(t)}{u_L(t)}\varpi(t)+\frac{u_{LC}(t)C_t}{u_L(t)}\frac{Y_t}{C_t} \quad (3.7)$$

is constant across states.[6] The following lemma states this result.

[6](3.6) can be written for the allocation under flexible prices as $\varphi_t=\frac{\frac{\theta}{(\theta-1)}-1}{D_t}$. At the optimal allocation, D_t must be constant so that φ_t is constant across states. Notice that we can write the optimal problem under flexible prices as we wrote the problem under sticky prices, (3.1) except that the multipliers are state dependent, $\varphi_t(s^t)$. When the optimal multipliers in that problem are constant across states, $\varphi_t(s^t)=\varphi_t(s^{t-1})$, i.e., when D_t is constant, then the two optimal solutions coincide.

Lemma 1: *The flexible price allocation satisfies the optimality conditions of the planner's problem under sticky prices iff the expression (3.7) is constant across states.*

In the lemma and propositions that follow we establish that in general the expression for D_t is not constant for the flexible price allocation. Thus, in general it is not optimal to set the wedges constant across states, or to replicate the flexible price allocation. We proceed in the following way: In Lemma 2 we assert the irrelevance of velocity shocks. In Proposition 3 we state that if there are no shocks to the share of government expenditures in the output, and if there are no preference shocks or they are multiplicative, then for most preferences and technologies used in the real business cycles literature, the flexible price allocation solves the planner's problem marginal conditions. However, once government expenditure shocks are considered the result is reversed. Thus in general the flexible price allocation is not optimal. This is stated in Proposition 4.

Lemma 2: *Velocity shocks are irrelevant for the determination of the optimal allocations.*

Proof: Since the optimal interest rate is $R_t = R'_t = 1$, then velocity shocks do not affect the set of implementable allocations. They also do not affect the preferences directly.¥

Proposition 3: *Let $G_t = GY_t$, $G \geq 0$, and $\varkappa_t = \varkappa$, or $\varkappa_t \neq \varkappa$, and $u(C_t, L_t, \varkappa_t) \equiv \varkappa_t u(C_t, L_t)$. The expression for D_t is constant for the flexible price allocation when*

(i) Preferences are described by monotonic transformations of

$$u = \frac{C_t^{1-\sigma}}{1-\sigma}\mathcal{F}(L_t),\ \mathcal{F}_L > 0,\ \sigma \geq 0,\ \sigma \neq 1, \tag{3.8}$$

which are preferences such that labor is constant across states in the flexible price allocation; or when

(ii) Preferences are described by monotonic transformations of additively separable and constant elasticity preferences

$$u = \frac{C_t^{1-\sigma}}{1-\sigma} - \alpha N_t^{\psi},\ \sigma \geq 0, \sigma \neq 1, \psi \geq 1,\ \alpha > 0, \tag{3.9}$$

and the technology is

$$F = \gamma N_t^{\varrho},\ \gamma > 0,\ \varrho \leq 1.$$

Proof: Let $\varkappa_t = \varkappa$. In case (i) the flexible price allocation satisfies $\frac{u_C(t)}{u_L(t)} = \frac{\frac{\theta}{(\theta-1)}}{z_t F_N(t)}$, which can be rewritten, using the resources constraint, as

$$\frac{(1-\sigma)\,\mathcal{F}(1-N_t)}{z_t F(N_t)\left(1-\frac{G_t}{Y_t}\right)\mathcal{F}_L(1-N_t)} = \frac{\frac{\theta}{(\theta-1)}}{z_t F_N(N_t)}.$$

Since $\frac{G_t}{Y_t} = G$, the labor allocations under flexible prices are constant across states. As D_t is invariant to monotonic transformations of the momentary utility (see Appendix), it is enough to check whether D_t is constant for $u = \log C_t + \mathcal{H}(L_t)$. In this case

$$D_t = \frac{1}{1-\frac{G_t}{Y_t}} + \varpi_N(t) - \frac{u_{LL}(t)N_t}{u_L(t)}\frac{\varpi(t)}{N_t}.$$

Since the labor allocation is constant, the elasticity of the marginal utility of labor, $\frac{u_{LL}(t)N_t}{u_L(t)}$, is constant, and so are $\varpi_N(t)$ and $\frac{\varpi(t)}{N_t}$. Thus, D_t is constant across states when the share of expenditures in output is constant, $\frac{G_t}{Y_t} = G$.

In case (ii), the labor allocation varies across states. However since the additional assumptions of Cobb-Douglas production and constant elasticity of the marginal utility of labor are made, the expression for D_t is still going to be constant across states. For this class of preferences the expression for D_t is

$$D_t = \sigma\frac{1}{1-\frac{G_t}{Y_t}} + \varpi_N(t) + (\psi-1)\frac{\varpi(t)}{N_t}.$$

Since $\frac{G_t}{Y_t} = G$ and since $\frac{\varpi(t)}{N_t}$ and $\varpi_N(t)$ are constants for the Cobb-Douglas production function, then D_t is constant across states. Notice that if $\sigma = 0$, then D_t is constant even if there are shocks to the share of government expenditures.

Now we consider $\varkappa_t \neq \varkappa$, and $u(C_t, L_t, \varkappa_t) \equiv \varkappa_t u(C_t, L_t)$. Since the shock is multiplicative and there are no intertemporal production linkages, the flexible price allocation does not depend on $\varkappa_t$. It is also immediate to see that D_t does not depend directly on $\varkappa_t$. Therefore D_t is constant for the preferences and technology described in (i) and (ii).¥

The classes of preferences (i) and (ii) in Proposition 3 include preferences commonly used in macroeconomics. Class (i) includes preferences that are aggregable and consistent with balanced growth such as the ones described by the instantaneous utility functions

$$u = \frac{(C_t {L_t}^{\psi})^{1-\sigma} - 1}{1-\sigma},\ \sigma > 0, \psi > 0,$$

or

$$u = \log C_t + \alpha L_t, \ \alpha > 0. \tag{3.10}$$

This second utility function, (3.10), is the one assumed in Ireland (1996). He does not consider government expenditure shocks, so that in his environment the flexible price allocation is optimal. The class (ii) of preferences includes the Greenwood, Hercowitz and Huffman (1988) utility function

$$u_t = \frac{\left(C_t - \alpha N_t^{\psi}\right)^{1-\xi} - 1}{1-\xi}, \ \xi \geq 0, \ \psi > 1, \ \alpha > 0. \tag{3.11}$$

These preferences correspond to the case where $\sigma = 0$, so that D_t is constant even if there are shocks to the share of government expenditures.

We now provide a simple example of aggregable preferences, outside of classes (i) and (ii) in Proposition 3, where, under the conditions in that proposition, the flexible price allocation is not optimal. Consider preferences described by $u = \alpha C_t + (L_t)^{\frac{1}{2}}$ and technology such that $F_N = 1$. Then, $D_t = \frac{1}{2L_t}$. Since in the flexible price allocation L_t depends on the state, D_t is state dependent. The sticky price optimal allocation achieves higher utility than the flexible price allocation does. For the parametrization of two equally probable i.i.d. shocks, $S = \{1, 2\}$, with $\alpha = 2$, and $\theta = 2$, the flexible price allocation is given by $C_t = 1.875$ in the good state and $C_t = .75$ in the bad state. The pair of consumptions, 1.845 in the good state and .834 in the bad state, satisfies the conditions of the planner's problem and gives a higher expected utility to the representative agent. In this particular case the social planner prefers a smoother output allocation to the flexible prices one.

So far we have assumed that government expenditures are colinear with the output. As was clear from the proof in Proposition 3, that is a necessary condition for the flexible price allocation to be optimal for the preferences and technologies identified in that proposition, except when $\sigma = 0$ in case (ii). This is stated in the following corollary.

Corollary: *For the classes of preferences and technologies described in Proposition 3, except when $\sigma = 0$ in case (ii), if $\frac{G_t}{z_t F(N_t)}$ is state dependent for the flexible price allocation, then D_t is not constant across states. Thus, the optimal allocation under sticky prices is different from the optimal allocation under flexible prices, and provides higher welfare.*

Proof: It is easy to verify that D_t is not constant when $\frac{G_t}{z_t F(N_t)}$ varies across states for the flexible price allocation. For preferences i), N_t in the flexible price allocation is now state dependent and D_t depends both on N_t and on $\frac{G_t}{z_t F(N_t)}$. For preferences and technology ii) with $\sigma \neq 0$, D_t depends only on $\frac{G_t}{z_t F(N_t)}$ and since this ratio is state dependent, D_t is also state dependent.¥

As stated in this corollary, if the share of government expenditures in output is not constant, the optimal solution under sticky prices will in general differ from the flexible price one. Since there is no reason to restrict government expenditures to be a constant share of output, we claim that in general the two optimal solutions are different. Since the flexible price allocation is implementable under sticky prices, it must be the case that the optimal policy under sticky prices dominates, in welfare terms, the optimal policy under flexible prices. Thus, sticky prices allow the planner to improve upon the optimal allocation under flexible prices.

We have analyzed an economy without capital. Allowing for capital accumulation should reinforce the result, that the optimal allocations in the two environments differ. We conjecture that this result holds whenever the share of any component of aggregate demand varies across states. Since in a business cycle model with capital, the investment share in the output reacts to technological shocks, we have additional reasons to infer that the optimal allocation under sticky prices provides higher utility than the optimal allocation under flexible prices.

In this section we have been assuming that the cash-in-advance constraint is binding. A positive nominal interest rate guarantees that. However when the nominal interest rate is zero, as is the case at the optimum, it is not possible to determine the allocations using money supply. For this reason, at the Friedman rule we can only claim that there is one allocation under sticky prices that dominates in welfare terms the optimal allocation under flexible prices. By a continuity argument, however it is still the case that, under sticky prices, there is a monetary policy that dominates the policy that eliminates gaps. There is a sufficiently small interest rate such that the optimal allocation under sticky prices can be determined using the money supply, and that optimal allocation provides higher utility than the optimal flexible price allocation at the Friedman rule.

We have chosen to identify the issue of the optimal monetary policy under sticky prices with the issue of whether the flexible price allocation is optimal. Since the flexible price allocation is characterized by constant markups, the analysis was centered on the conditions under which it is optimal to set the markups constant across states. For this reason there is a resemblance between the problem we analyzed and the one of determining whether taxes should be uniform in the Ramsey problem under flexible prices, where the government must finance exogenous government expenditures with distortionary taxes.[7] In one problem the government can use monetary policy to affect the markups across states while keeping some average of these markups constant, which is imposed by the consistency of the price setting decisions of firms. In the other problem the government can choose the taxes across states, and time, for a given average of these tax rates that satisfies the budget constraint. The results in both problems are also similar. In particular it is the case that for the utility function (3.8), and for (3.9) with Cobb-Douglas technology, it is optimal to set uniform taxes. However because the two problems are in fact different, the results are also different. On one hand the uniform taxation result holds for preferences (3.9) and Cobb-Douglas technology whatever is the process of government expenditures. On the other hand the result does not hold for monotonic transformations of the utility functions, and thus for the (3.11) preferences when $\xi \neq 0$.[8]

4. Concluding Remarks

In this paper, we focus on optimal cyclical monetary policy. We analyze a simple environment with short run non-neutrality of money and determine principles for the conduct of the monetary policy as stabilization policy.

The main result of the paper is that the optimal allocation under sticky prices is in general different from the optimal one under flexible prices. It also provides higher welfare, since the flexible price allocation is achievable under sticky prices. Thus, a simple welfare criterion of eliminating gaps is not optimal. We also show that, as under flexible prices, the Friedman rule must be followed in order to achieve the optimal allocation.

In the economies without capital that we analyze, if there were only technological shocks, velocity shocks, and multiplicative shocks to preferences, then, for most preferences and technology structures used in the real business cycles literature, it would be optimal to replicate the flexible price allocation, eliminating the distortions arising from sticky prices. Under the optimal zero nominal interest rate policy, the only remaining distortion would be the constant mark-up resulting from monopolistic competition. This is not a general result, though. When shocks to government expenditures are considered, the result is reversed. In general, it is no longer the case that the optimal allocations under flexible and sticky prices

[7] See Goodfriend and King (2000)

[8] See Correia, Nicolini and Teles (2000)

coincide. This means that it is optimal to set varying ex-post mark-ups across states.

The optimal solution under sticky prices, characterized both by the Friedman rule and by variable ex-post markups, cannot be decentralized in the environment with flexible prices, because under flexible prices monetary policy can only affect the wedges through the nominal interest rates and these are bounded away from zero. Under sticky prices it is possible to overcome that restriction by using the short run non neutrality of monetary policy.

Our results on the optimality of constant proportionate wedges resemble results in the optimal taxation literature on the optimality of uniform taxation. As in the taxation problem the optimal allocations are in general not characterized by constant proportionate wedges. However, again as in that problem, it may be the case that quantitatively the optimal allocation and the one with constant wedges do not differ significantly.

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5. Appendix

We want to show that

$$D_t = -\frac{u_{CC}(t)C_t}{u_C\,(t)}\frac{Y_t}{C_t} + \frac{u_{LC}(t)}{u_C\,(t)}\varpi(t) + \varpi_N(t) - \frac{u_{LL}(t)}{u_L(t)}\varpi(t) + \frac{u_{LC}(t)}{u_L(t)}C_t\frac{Y_t}{C_t}$$

is invariant to monotonic transformations F of the utility function u.

Let us define $V = F(u)$.

Then

$$\begin{aligned} V_C &= F_u u_C \\ V_{CC} &= F_{uu} u_C^2 + F_u u_{CC} \\ V_{CL} &= F_{uu} u_C u_L + F_u u_{LC} \\ V_L &= F_u u_L \\ V_{LL} &= F_{uu} u_L^2 + F_u u_{LL} \end{aligned}$$

and

$$\begin{aligned} D^V &= -\frac{(F_{uu} u_C^2 + F_u u_{CC}) C \frac{Y}{C}}{F_u u_C} + \frac{(F_{uu} u_C u_L + F_u u_{LC}) \varpi}{F_u u_C} + \varpi_N \\ &\quad - \frac{(F_{uu} u_L^2 + F_u u_{LL}) \varpi}{F_u u_L} + \frac{(F_{uu} u_C u_L + F_u u_{LC}) C \frac{Y}{C}}{F_u u_L} \end{aligned}$$

or

$$D^V = D - \frac{F_{uu} u_C C \frac{Y}{C}}{F_u} + \frac{F_{uu} u_L \varpi}{F_u} - \frac{F_{uu} u_L \varpi}{F_u} + \frac{F_{uu} u_C C \frac{Y}{C}}{F_u} = D.$$