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when the Stock and Labor Markets
are Cointegrated**

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Abstract

We study portfolio choice when labor income and dividends are cointegrated. Economically plausible calibrations suggest young investors should take substantial short positions in the stock market. Because of cointegration the young agent's human capital effectively becomes "stock-like." However, for older agents with shorter times-to-retirement, cointegration does not have sufficient time to act, and thus their human capital becomes more "bond-like." Together, these effects create hump-shaped life-cycle portfolio holdings, consistent with empirical observation. These results hold even when asset return predictability is accounted for.

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THE OPTIMAL PORTFOLIO CHOICE PROBLEM OVER THE LIFE CYCLE has received considerable attention in political, financial, and academic circles. Yet, in spite of the vast work on this topic, there is still much disagreement across empirical observation, conventional wisdom, and the predictions of most of the academic literature.

While the level of stock market participation has increased significantly over the decades, several studies report that risky asset holdings have typically been low at young ages, and then either increasing or hump-shaped over the life cycle (see, for example, Ameriks and Zeldes (2001), Faig and Shum (2002), Heaton and Lucas (2000), and Poterba and Samwick (2001)). In contrast, conventional wisdom maintains that for reasonable levels of risk aversion, young agents should place a large proportion of their wealth into the market portfolio, and this proportion should decline as the agent nears retirement. Indeed, one often-quoted strategy suggested by financial advisors is that investors should place $(100 - \text{age})\%$ of their wealth in a well-diversified equity portfolio (see, for example, Malkiel (1996, p. 418)).

Both empirical observation and conventional wisdom seem at odds with the academic literature. First, early academic studies such as Merton (1969) and Samuelson (1969) conclude that a long-lived agent should hold a constant fraction of her wealth in the risky asset throughout her life. Second, when calibrated to historical values of the equity premium and stock market return volatility, as well as “reasonable” levels of risk-aversion, these models predict that the appropriate proportion of wealth placed in the risky asset is counterfactually large—sometimes higher than 100%. Third, these models generate little heterogeneity in stock market participation even if there is significant variation in risk aversion across agents. These results, however, are derived under many restrictive assumptions, including power utility, independent and identically distributed (i.i.d.) returns on the risky and risk-free investments, the absence of market frictions, and perhaps most importantly, the absence of labor income.

In an attempt to reconcile theory and observation, many of the restrictive assumptions underlying the Merton (1969) and Samuelson (1969) results have been progressively relaxed.¹ For instance, several studies examine the effect of labor income on portfolio choice over the life cycle. For many agents, the “wealth” (i.e., the certainty-equivalent present value) tied up in terms of future wages dwarfs their financial wealth. As such, one might suspect that optimal portfolio choice that takes labor income into account may generate significantly different predictions. Interestingly, however, most existing studies find that incorporating labor income into the optimal portfolio decision only serves to reinforce the puzzle. Indeed, most models attribute “bond-like” qualities to the future flow of labor income. That is, these models predict that through their labor income, agents implicitly hold a large position in the risk-free asset, which implies that they should take an even more aggressive position in the risky asset with their cash-on-hand

¹Some papers in this vein examine the implications of time variation in the riskless interest rate or in the equity premium, and/or different utility functions for the portfolio choice problem. See, for example, Balduzzi and Lynch (1999), Barberis (2000), Brandt (1999), Brandt et al. (2005), Brennan et al. (1997), Brennan and Xia (2000), Campbell et al. (2004), Campbell and Viceira (1999, 2001), Cvitanic et al. (2006), Dammon et al. (2003), Guidolin and Timmermann (2006), Kandel and Stambaugh (1996), Kim and Omberg (1996), Liu (2005), Michaelides (2003), Samuelson (1991), Schroder and Skiadas (1999), Wachter (2002), and Xia (2001).

compared to those models that ignore labor income. Early papers include Bodie, Merton and Samuelson (1992, BMS), who consider portfolio choice in the context of an endogenous leisure/labor trade-off. More recently, a number of researchers employ micro data to calibrate the individual labor income process. (See, for example, Campbell et al. (2001, CCGM), Cocco, Gomez, and Maenhout (2005, CGM), Davis and Willen (2000), Haliassos and Michaelides (2003), Jagannathan and Kocherlakota (1996), and Viceira (2001)).² With the particular distributional assumptions made in those papers (essentially, labor income and stock returns follow autonomous Markov i.i.d. or AR(1) processes), they find that only counterfactually high correlations between shocks to labor income and stock returns, or the possibility of disastrous labor income shocks (see, for example, CGM), can explain young investors' low holdings of the risky asset.

Note, however, that the labor income specification in these models may be unnecessarily restrictive. In particular, if the contemporaneous correlation $\rho_{R_M, L}$ between market returns and changes to aggregate labor income flow is specified to be low (consistent with the data), then these models force longer-term correlations to be low as well. Such specifications also force the correlation ρ_{R_M, R_L} between the return to the market portfolio and the return to human capital (which equals the sum of current labor income and the unobservable "capital gain" to human capital) to be low.

In contrast to these papers, we specify aggregate labor income to be cointegrated with dividends.³ Such a specification is consistent with empirically observed low contemporaneous correlations $\rho_{R_M, L}$ between market returns and changes to aggregate labor income flow. However, this specification permits correlations ρ_{R_M, R_L} between returns to human capital and market returns to be significantly higher.

The notion that returns to human capital and market returns should be highly correlated is not new. For example, Baxter and Jerman (1997, BJ) test for the existence of cointegration by using data on aggregate employee compensation and GDP growth (in contrast to using dividends on the market portfolio, as we do in this paper). Although they only find weak statistical evidence in support of cointegration, as is often the case in tests of cointegration, they proceed under the economically plausible assumption that such a relation exists and investigate the implications for international portfolio choice. Assuming a constant discount rate, they find that the present values of capital income and labor income exhibit a high correlation, in excess of 90%. Using a very different argument, Campbell (1996) also reports a high correlation between human capital and market returns. In particular, he assumes that labor income follows an AR(1) process and has low contemporaneous correlation with stock dividends. However, he assumes that the same (highly time-varying) discount factor should be used to discount both labor income

²Several other contributions investigate the implications of human capital for asset pricing and portfolio choice. For instance, Merton (1971), Svensson and Werner (1993), and Koo (1998) study portfolio choice in the presence of nontraded labor income. Bodie et al. (2004), Chan and Viceira (2000), and Dybvig and Liu (2004) examine the portfolio choice problem in economies with flexible labor supply or voluntary retirement. Telmer (1993) investigates the variability of the intertemporal marginal rate of substitution in an incomplete market economy with uninsurable labor income shocks. Heaton and Lucas (1996) and Lucas (1994) study the equity premium in economies with aggregate and idiosyncratic labor income shocks, transaction costs, as well as borrowing and short-sales constraints.

³Many other recent papers have assumed that labor income and dividend flows are cointegrated. See, for example, Santos and Veronesi (2006) and Menzly, Santos and Veronesi (2004).

and dividends. In his model, the high correlation between human capital and market returns is effectively due to the common and highly varying discount factor.⁴

In this paper, we investigate the implication of cointegration between aggregate labor income and dividends on the market portfolio for life-cycle portfolio choice. Although related to the work of BJ, our analysis differs significantly from theirs in many respects. First, they consider an infinitely-lived representative agent who has a claim to aggregate labor income. Thus, their analysis does not generate implications for the life-cycle behavior of finitely lived individual agents. Second, their analysis ignores the fact that individual agents face significant idiosyncratic labor income shocks (see, for example, Carroll and Samwick (1997, CS), CGM, and Gourinchas and Parker (2002, GP)) that are not captured by looking at aggregate averages alone. Third, they do not solve for the optimal portfolio choice. Rather, they focus on the one-period return of an investor who seeks a world value-weighted (i.e., diversified) portfolio. Finally, they estimate human capital by exogenously setting the discount rate that is used to discount labor income to a constant.

In contrast, we investigate the optimal portfolio and consumption choices over the life cycle for an agent with constant relative risk aversion who earns nontradable labor income. The agent's labor income has two components. The first component is aggregate labor income, which itself is specified to be cointegrated with the dividend process. The second component captures both life-cycle predictability (i.e., labor income tends to increase with age when the agent is young, and then tends to decline as the agent approaches retirement) and idiosyncratic labor income shocks. Combined, these two components are calibrated to be consistent with the results of CCGM and CGM. We use a dynamic programming approach to solve for the consumption and portfolio allocation rules. We also report the present value of labor income for the optimizing agent by discounting future labor income at her marginal utility.

Contrary to both conventional wisdom and much of the previous literature, and consistent with empirical evidence, our model predicts that the optimal portfolio strategy over the life cycle is hump-shaped. The intuition for this result is as follows. The inverse of the mean reversion coefficient controlling the cointegration, $\frac{1}{\kappa}$, provides a time scale for the agent. If the number of years of remaining employment is larger than this time scale (i.e., if the agent is young), then the return on the agent's human capital is highly exposed to market returns. Furthermore, most of the young agent's wealth is tied up in future labor income. Thus, she will find herself overexposed to market risk, in which case it will be optimal for her to take a *short* position in the market portfolio. As the agent ages, however, the cointegration between labor income and dividends has less time to act. As such, for older agents the present value of future labor income progressively acquires bond-like properties. Hence, as we move forward in time, the agent places a larger fraction of her financial wealth into the risky asset to offset the larger implicit bond position she has through her labor income wealth.

As the agent approaches retirement, however, there are two partially offsetting effects. First, for short

⁴Other papers that investigate a link between aggregate labor income and asset prices include Mayers (1974), Fama and Schwert (1977), Black (1995), Jagannathan and Wang (1996), Lettau and Ludvigson (2001a), and Santos and Veronesi (2006).

times-to-retirement, cointegration does not have sufficient time to act. Thus, human capital takes on bond-like features, as in, for example, BMS, CGM, CCGM, and GM. Second, the residual value of future labor income shrinks, since the agent has fewer years left to work, and therefore the value of the bond position implicit in her human capital decreases. Eventually, this second effect prevails, in which case the agent starts to reduce her stock holdings to buy more of the risk-free asset. This switch creates a hump in her investment strategy, and explains the downward-sloping part of her life-cycle profile. Just prior to retirement, the present value of future labor income for the agent is zero, and hence the optimal portfolio decision approaches the Merton (1969) solution (which ignores labor income).

As mentioned previously, the main determinant of the peak location in the hump-shaped portfolio holdings is the time scale of the cointegration. Based on this factor alone, we expect the peak to occur approximately $\frac{1}{\kappa}$ years before retirement. Hence, for our baseline case of $\kappa = 0.15$, portfolio holdings should peak approximately 6.6 years before retirement. The results we find below are remarkably close to this prediction, especially since other factors (such as the magnitude of idiosyncratic risk) are expected to play a role in the location of the peak.

Note that several previous studies also propose models in which young agents do not participate in the stock market. However, this result typically obtains by assuming a sizeable entry cost (see, for example, Abel (2001), CGM, and GM). We emphasize that our prediction follows without assuming any entry costs. Further, as we demonstrate below, the qualitative conclusions of our findings are very robust across a wide range of parameter inputs.

In support of our model, we find evidence that aggregate labor income and dividends on the market portfolio are cointegrated. Specifically, by using data from 1929 to 2004, we reject the unit root (i.e., $\kappa = 0$) hypothesis at reasonable significance levels. We acknowledge we cannot reject the unit root assumption with typical significance levels using only post World War II data. As is well known, however, it is econometrically very difficult to distinguish between these two hypotheses, as unit root tests are notorious for lacking power. Still, we consider an investigation of the implication of such a cointegrated relation for life-cycle portfolio choice a worthwhile endeavor for several reasons. First, cointegration is assumed by most macroeconomic models.⁵ Second, cointegration is economically plausible. Indeed, as BJ point out, if labor and capital income were to have independent trends, then the ratio of labor income to capital income would either grow without bound or approach zero asymptotically, and the labor share would approach either zero or one; these implications seem implausible. Third, our model specification reduces to traditional models (i.e., no cointegration) in the limit $\kappa \rightarrow 0$. Econometrically, it is difficult to distinguish between $\kappa = 0$ and, say, $\kappa = 0.05$ given only a few decades of data. Indeed, for $\kappa = 0.05$, we only expect to see the effects of cointegration over a time frame of $\frac{1}{0.05} \approx 20$ years, implying that with 60 years of data, we only have about $\frac{60}{20} \approx 3$ independent data points. Yet, as we show below, the models with $\kappa = 0$ or $\kappa = 0.05$ generate significantly different predictions for the optimal portfolio decision of a

⁵For instance, a model with a Cobb-Douglas production function predicts that returns to physical and human capital are perfectly correlated even in the short run.

young agent.⁶ Since these two models are difficult to distinguish econometrically, it seems important to investigate the implications of both.

Our results hold for reasonable levels of the agent's risk aversion coefficient. Following CCGM, CGM, and Gomes and Michaelides (2005, GM), we choose $\gamma = 5$ for our baseline case. Qualitatively similar results obtain if we set $\gamma = 4$. However, a less risk-averse agent (for example, $\gamma = 3$) finds it optimal to invest heavily in stocks in spite of the long-run cointegration effect. Hence, we find that even small differences in relative risk aversion can generate substantially different predictions. This result is consistent with empirical observation that asset holdings and stock market participation exhibit a high degree of heterogeneity. In contrast, most models that do not account for this long-run cointegration conclude that young agents over a wide range of risk aversion levels should hold a large proportion of their financial wealth in risky securities.

Finally, we extend our analysis to account for return predictability. Intuitively, the fact that low current returns generate higher future expected returns implies that stock ownership creates its own hedge, making stocks even more desirable than when it is assumed that returns are i.i.d. Even so, our main conclusions remain qualitatively unchanged. That is, we still find that, assuming reasonable parameter estimates for expected returns and risk-aversion coefficients, young agents should short the market even in the presence of stock return predictability.

The recent literature offers many alternative explanations for the limited stock market participation puzzle.⁷ The explanation we offer here, while different, can be viewed as complementary to these. In particular, our paper emphasizes that long-run cointegration between aggregate labor income and aggregate dividends has a first-order effect on the optimal portfolio decisions of an agent over the life cycle.⁸

The rest of the paper is organized as follows. In Section I, we present the life-cycle portfolio choice model. In Section II, we generalize the model to account for stock return predictability. We explain the details of the model calibration in Section III. In Section IV we determine optimal portfolio and consumption choice by numerically solving the Hamilton-Jacobi-Bellman equation. Sensitivity analysis suggests that the main qualitative result is robust to a wide range of parameter calibrations. We conclude

⁶In some respects, this is analogous to the approach of Bansal and Yaron (2004), who show that consumption dynamics with small but persistent drifts are econometrically difficult to distinguish from i.i.d. consumption dynamics, but generate significantly different risk premia.

⁷See, for example, Abel (2001), Curcuru et al. (2004), Davis, Kubler and Willen (2005), Faig and Shum (2002), GM, Guo (2004), Heaton and Lucas (1997, 2000), Hong et al. (2004), Hsu (2003), Lynch and Tan (2006), and Storesletten, Telmer, and Yaron (2001). Among these studies, those that are most closely related to ours are Lynch and Tan (2006) and Storesletten, Telmer, and Yaron (2001), who examine the implications of time variation in the moments of the labor income dynamics. They find that predictability of labor income growth at the business cycle frequency can generate negative hedging demand for stocks and therefore more realistic stock holdings implications.

⁸In a recent paper, Lustig and Van Nieuwerburgh (2005) attribute the residuals of consumption growth innovations that cannot be explained by their model to news about expected future returns on human wealth, and conclude that stock and labor income returns are negatively correlated. Interestingly, this finding would appear to deepen the limited stock market participation puzzle, since it implies that optimal holdings of the risky portfolio for the young agent would be even larger than the levels obtained when it is assumed that labor income is bond-like.

in Section V.

I. A Model with Cointegrated Dividends and Labor Income

Let the dividend process $D(t)$ of the risky asset follow a geometric Brownian motion, that is,

$$\frac{dD}{D} = g_D dt + \sigma dz_3. \quad (1)$$

Using Ito's lemma, the log-dividend process $\hat{d}(t) \equiv \log [D(t)]$ follows

$$d\hat{d}(t) = \left(g_D - \frac{\sigma^2}{2} \right) dt + \sigma dz_3. \quad (2)$$

Further, let the dynamics of the economy's pricing kernel $\Lambda(t)$ have a constant risk free rate r and constant price of risk λ ,⁹

$$\frac{d\Lambda}{\Lambda} = -r dt - \lambda dz_3. \quad (3)$$

The date- t stock price $P(t)$ of the risky asset can be determined from

$$\Lambda(t) P(t) = E_t \left[\int_t^\infty ds \Lambda(s) D(s) \right]. \quad (4)$$

As is well known, the solution to this expectation is

$$P(t) = \frac{D(t)}{r + \lambda\sigma - g_D}. \quad (5)$$

Hence, this economy supports a constant dividend yield $\delta \equiv \frac{D(t)}{P(t)}$:

$$\delta = r + \lambda\sigma - g_D. \quad (6)$$

Given that $D(t)$ is proportional to $P(t)$, it follows that the stock price follows the same dynamics as the dividend process

$$\frac{dP}{P} = g_D dt + \sigma dz_3. \quad (7)$$

It is useful to also introduce the gain process $S(t)$, which accounts for the total return (capital gain plus dividend):

$$\frac{dS(t)}{S(t)} \equiv \frac{dP(t) + D(t) dt}{P(t)} = \mu dt + \sigma dz_3, \quad (8)$$

where we define the expected return as $\mu \equiv (r + \lambda\sigma)$. Note from equation (6) that $\mu = \delta + g_D$. Defining the log-gain process $s(t) \equiv \log S(t)$, we have from Ito's lemma:

$$ds = \left(\mu - \frac{\sigma^2}{2} \right) dt + \sigma dz_3. \quad (9)$$

⁹Below, we will introduce other Brownian motions that affect aggregate quantities. As such, it is likely that such Brownian motions belong in our pricing kernel. We emphasize, however, that including these into the pricing kernel has no effect on our results.

We note that this simple model predicts the counterfactual result that the volatility of the dividend growth rate, σ , is identical to the stock return volatility. We emphasize, however, that only the stock return volatility is relevant for the agent's portfolio decision. As such, we fix σ to match historical stock return volatility in our calibration below.

For what follows, it will be useful to note that if we integrate equations (2) and (9), we find that for all dates t ,

$$\begin{aligned}\hat{d}(t) &= s(t) + \hat{d}(0) - s(0) - (\mu - g_D) t \\ &= s(t) + \hat{d}(0) - s(0) - \delta t.\end{aligned}\tag{10}$$

Next, we specify the dynamics for the labor income process. Define the current labor income flow for an individual as $L(t)$. It is convenient to introduce log-labor as $\ell(t) = \log L(t)$. Since CCGM, CGM, CS, and GP find idiosyncratic labor shocks to be proportional to the level of labor income, it follows that an individual's income is a product of two numbers: $L_1(t)$, the aggregate income associated with this agent's career choice, and $L_2(t)$, her idiosyncratic shocks. As such, her log-labor flow is a sum of these two factors, that is,

$$\ell(t) = \ell_1(t) + \ell_2(t),\tag{11}$$

where $\ell_1(t) = \log L_1(t)$ and $\ell_2(t) = \log L_2(t)$.

We now need to specify the dynamics for $\ell_1(t)$ and $\ell_2(t)$. We choose the process for the aggregate state variable $\ell_1(t)$ such that it captures two features. First, consistent with observation, contemporaneous correlations between market returns and aggregate shocks to labor income are low. Second, consistent with Santos and Veronesi (2006) and Menzly, Santos, and Veronesi (2004), aggregate labor income and aggregate dividends are cointegrated. In particular, define the difference between the logs of these two variables as $y(t)$:

$$y(t) \equiv \ell_1(t) - \hat{d}(t) - \bar{\ell}d.\tag{12}$$

The constant $\bar{\ell}d$ should be interpreted as the long-run log-ratio of aggregate labor income to dividends. Note that if we use equation (10) to replace $\hat{d}(t)$, and if we choose (without loss of generality) $s(0) \equiv \bar{\ell}d + \hat{d}(0)$, we can rewrite equation (12) as

$$y(t) = \ell_1(t) - s(t) + \delta t.\tag{13}$$

This equivalent definition will be useful when we account for stock return predictability in the next section.

To capture the notion of cointegration (i.e., long-run dependence) between labor income and dividends, we assume that $y(t)$ is a mean-reverting process,

$$dy(t) = -\kappa y(t) dt + \nu_1 dz_1(t) - \nu_3 dz_3(t),\tag{14}$$

where z_1 is a standard Brownian motion independent of z_3 .

Further, we specify that the dynamics of the logarithm of the idiosyncratic shocks be an arithmetic Brownian motion,

$$d\ell_2(t) = \left(\alpha(t) - \frac{\nu_2^2}{2} \right) dt + \nu_2 dz_{2,i}(t), \quad (15)$$

where $z_{2,i}$ is a standard Brownian motion independent of both z_1 and z_3 . The subscript i is used to emphasize that this shock is idiosyncratic, in contrast to the aggregate shocks z_1 and z_3 . That is, we follow CCGM, CGM, CS, GP, and many others and assume that the idiosyncratic labor income component is subject to permanent shocks. Further, we introduce time dependence in the drift in (15) to capture the finding in the literature that the drift of an individual's labor income is a function of her age. Specifically, we choose

$$\alpha(t) = \alpha_0 + \alpha_1 t, \quad (16)$$

where α_0 and α_1 are calibrated to capture the hump shape of earnings over the life cycle (see, for example, CGM and CS).

From equations (11) and (12), and using Ito's lemma, we can write

$$\ell(t) = y(t) + \hat{d}(t) + \bar{\ell}d + \ell_2(t) \quad (17)$$

$$d\ell(t) = \left(-\kappa y(t) + g_D - \frac{\sigma^2}{2} + \alpha(t) - \frac{\nu_2^2}{2} \right) dt + \nu_1 dz_1(t) + \nu_2 dz_{2,i}(t) + (\sigma - \nu_3) dz_3(t). \quad (18)$$

Since z_1 and $z_{2,i}$ are orthogonal to the stock return shock z_3 , equations (9) and (18) imply that the contemporaneous correlation between stock market and labor income shocks is $\text{corr}(ds, d\ell) = \frac{(\sigma - \nu_3)}{\sqrt{\nu_1^2 + \nu_2^2 + (\sigma - \nu_3)^2}}$. Thus, in the special case $(\sigma - \nu_3) = 0$, labor income is contemporaneously uncorrelated with market returns. We use this case as our benchmark case to emphasize that short-term correlations are unnecessary for generating labor income dynamics that are 'stock-like'. Instead, what is crucial is the long-term cointegration.

Finally, using Ito's lemma we find

$$\frac{dL}{L} = \left(-\kappa y(t) + g_D - \frac{\sigma^2}{2} + \alpha(t) + \frac{\nu_1^2}{2} + \frac{(\sigma - \nu_3)^2}{2} \right) dt + \nu_1 dz_1(t) + \nu_2 dz_{2,i}(t) + (\sigma - \nu_3) dz_3(t).$$

Note that in previous studies, most authors specify the labor process in levels rather than in changes. Furthermore, it is common to specify the model in discrete time rather than continuous time. It can be shown, however, that in the limit $\kappa \rightarrow 0$, our specification is nearly identical to these standard models.

A. Empirical Motivation for the Labor Income Model

As mentioned previously, the specification (15) for the idiosyncratic labor income component ℓ_2 is consistent with the evidence in earlier studies that examine the properties of labor income at the micro level. As such, here we focus on providing empirical motivation for the dynamics of the aggregate labor income component ℓ_1 , and in particular for the cointegration in equation (14).

To construct a proxy for ℓ_1 , we use Lettau and Ludvigson's (2001a, 2001b) definition for labor income, which is, in short, the sum of wages and salaries, transfer payments, and employer contributions for employee pension and insurance, net of employee contributions for social insurance and taxes.¹⁰ We use yearly data from 1929 to 2004 to form the total labor income series (the data are from the National Income and Product Accounts (NIPA) tables compiled by the Bureau of Economic Analysis). To compute a per capita measure of labor income, we divide the total labor income series by the population measure reported in the NIPA tables. Finally, we deflate the per capita labor income series by using the seasonally adjusted personal consumption expenditures (PCE) deflator (1992=100) released by the Bureau of Economic Analysis.

Following the approach of Fama and French (1988b), we construct a proxy for the aggregate log-dividend process \hat{d} . We obtain monthly dividend series from returns, with and without dividends, on the value-weighted market index released by the Center for Research in Security Prices (CRSP). We deflate the dividend series by the PCE. Finally, to avoid the seasonal differences in dividend payments, we construct yearly dividend series by summing the 12 monthly dividends paid out during each calendar year from 1929 to 2004.

We perform an augmented Dickey-Fuller (ADF) test to check whether the variable $y = \ell_1 - \hat{d} - \bar{\ell d}$ is stationary. Specifically, we estimate the ADF regression model

$$\Delta y(t) = \xi_1 + \xi_2 t + \xi_3 y(t-1) + \sum_{j=1}^L \Phi_j \Delta y(t-j) + \epsilon(t), \quad (19)$$

where $\Delta y(t) = y(t) - y(t-1)$, $y = \ell_1 - \hat{d} - \bar{\ell d}$, by ordinary least squares (OLS).

The results are in Table I below. The first two rows of the table report results for a unit root test under the assumption that the error terms $\epsilon(t)$ in equation (19) are serially uncorrelated (i.e., $L = 0$). In the first row, the ξ_2 coefficient is fixed at zero, while in the second row we allow for the presence of a time trend, as is customary to do in unit root tests. The 10% asymptotic critical values for these Dickey-Fuller tests are -2.57 and -3.13, respectively (see, for example, Davidson and MacKinnon (1993), p. 708). As such, we find some evidence against the unit root hypothesis for the 1929 to 2004 sample period (in the $L = \xi_2 = 0$ case, the τ statistic is -2.77, which compares favorably with the 10% asymptotic critical value, -2.57). However, we cannot reject the unit root hypothesis for the 1947 to 2004 sample period.

Because the assumption of zero correlation in the regression error terms is most likely untenable, in the third row of Table I we report ADF test results for the case in which the regression model contains the lagged term $\Delta y(t-1)$, which is equivalent to allowing for first-order autocorrelation in the error term of the Dickey-Fuller regression model. Interestingly, the R^2 for this regression improves considerably, which lends support to this model extension. Further, the ADF τ statistic is -4.16 for the 1929 to 2004 sample period. As such, we can reject the unit root hypothesis at a reasonable confidence level (the 2.5% asymptotic critical value is -4.08). This finding is robust to incorporating a second-order lagged term, $\Delta y(t-2)$,

¹⁰This definition reflects the 2003 revision of the NIPA data series by the Bureau of Economic Analysis. More details are available from Martin Lettau's web page at <http://pages.stern.nyu.edu/~mlettau/>.

which we find to be insignificant. We note, however, that these results are not robust to the choice of the sample period. Specifically, we still cannot reject the unit root hypothesis when we use post-World War II data.

INSERT TABLE I ABOUT HERE.

In sum, we find evidence suggesting that log-aggregate labor income, ℓ_1 , and log-dividend, \hat{d} , are cointegrated, consistent with our model equation (14). We acknowledge that this result is not robust to the choice of sample period. However, the cointegration effect can act at very low frequencies. For small values of κ , it might take many decades for an agent's wages to catch up with the performance of the economy. Thus, given the relatively short post-World War II sample period it is not surprising that this effect might go undetected by the ADF test, which is notorious for its lack of power.¹¹ Further, economic intuition provides strong support for the notion that labor and capital income are cointegrated. Therefore, we proceed under the assumption that the variable y is stationary.

Finally, comparing equation (14) and equation (19), we see that the estimates for the ξ_3 coefficient reported in the first two rows of Table I yield a measure for the speed of the mean-reversion coefficient, κ , in equation (14). Specifically, after we account for the transformation from discrete time to continuous time, we find that $\kappa = -\log(1 + \xi_3)$. If we rely on the post-World War II sample period, we find that this coefficient is as small as $\kappa = -\log(1 - 0.0464) = 0.0475$. If, instead, we rely on the (more informative) 1929 to 2004 full sample period and we allow for the presence of a time trend in the regression model, we obtain a value as large as $\kappa = -\log(1 - 0.1855) = 0.2052$. Below, we will use this evidence in choosing a range for the κ coefficient in our calibration exercise.

B. The Agent

The current financial wealth of the agent is placed in two securities, with a proportion π placed in the risky asset and $(1 - \pi(t))$ placed in the risk-free asset. As such, her wealth dynamics follow

$$\begin{aligned} dW(t) &= -C(t) dt + (1 - \pi(t)) W(t) \frac{dB(t)}{B(t)} + \pi(t) W(t) \left(\frac{dP(t) + D(t) dt}{P(t)} \right) + L(t) dt + \beta W(t) dz_{4,i}(t) \\ &= -C(t) dt + (1 - \pi(t)) W(t) \frac{dB(t)}{B(t)} + \pi(t) W(t) \frac{dS(t)}{S(t)} + L(t) dt + \beta W(t) dz_{4,i}(t). \end{aligned} \quad (20)$$

The last term captures the notion of transient shocks to the agent's wealth. In contrast to most discrete time models, it is simpler in our continuous time model to capture these transient shocks in the wealth process rather than in the labor income process. Consistent with intuition, and the numerical results of CGM, we report below that this term has a negligible effect on the agent's consumption and portfolio choices for a wide range of reasonable parameter estimates for β .

¹¹BJ use the same ADF approach to test whether the ratio of labor income to capital income is a stationary random variable. They use annual data on labor and capital income for Japan, Germany, the United Kingdom, and the United States from 1960 to 1993. For each of the four countries they consider, they cannot reject the unit root hypothesis (our results for the post-World War II period based on U.S. dividend and labor income data are similar to those that BJ report in their unpublished Appendix).

With this specification, the wealth dynamics of equation (20) can be written as

$$\frac{dW(t)}{W(t)} = \left(r + \pi(t)(\mu - r) + \frac{L(t)}{W(t)} - \frac{C(t)}{W(t)} \right) dt + \pi(t)\sigma dz_3(t) + \beta dz_{4,i}(t). \quad (21)$$

We assume that the agent has standard constant relative risk aversion (CRRA) utility function. As such, her objective function is

$$J(t, W(t), L(t), y(t)) \equiv \max_{\{C, \pi\}} \mathbb{E}_t \left[\int_t^T du e^{-\psi u} \frac{(C(u))^{1-\gamma}}{1-\gamma} + \epsilon^\gamma e^{-\psi T} \frac{(W(T))^{1-\gamma}}{1-\gamma} \right]. \quad (22)$$

Note that we do not explicitly model the post-retirement consumption and investment decision of the agent. Instead, in our application below we calibrate the bequest function to capture the retirement-years consumption that the agent wants to save for (an approach similar to that of GP). This is equivalent to modeling the post-retirement consumption and investment decisions under the assumption that the agent receives a fixed income flow like, for example, a retirement annuity. After retirement, the agent's problem simplifies to a version of the Merton model, which does not affect the pre-retirement solution.

The Hamilton-Jacobi-Bellman (HJB) equation is (dropping time arguments to simplify notation)

$$\begin{aligned} 0 = & e^{-\psi t} \frac{C^{1-\gamma}}{1-\gamma} + J_t + W J_W \left(r + \pi(\mu - r) + \frac{L}{W} - \frac{C}{W} \right) + \frac{1}{2} W^2 J_{WW} (\sigma^2 \pi^2 + \beta^2) \\ & + L J_L \left(-\kappa y + g_D - \frac{\sigma^2}{2} + \alpha(t) + \frac{1}{2} \nu_1^2 + \frac{1}{2} (\sigma - \nu_3)^2 \right) + \frac{1}{2} L^2 J_{LL} (\nu_1^2 + \nu_2^2 + (\sigma - \nu_3)^2) \\ & - \kappa y J_y + \frac{1}{2} J_{yy} (\nu_1^2 + \nu_3^2) + W L J_{WL} (\sigma - \nu_3) \pi \sigma - \nu_3 \sigma \pi W J_{Wy} + L J_{Ly} (\nu_1^2 - \nu_3 (\sigma - \nu_3)). \end{aligned} \quad (23)$$

The first-order conditions for the two controls are

$$0 = e^{-\psi t} C^{-\gamma} - J_W \quad (24)$$

$$0 = W J_W (\mu - r) + W^2 J_{WW} \sigma^2 \pi + W L J_{WL} \sigma (\sigma - \nu_3) - \nu_3 \sigma W J_{Wy}, \quad (25)$$

leading to the conditions

$$C = \left(e^{\psi t} J_W \right)^{-\frac{1}{\gamma}} \quad (26)$$

$$\pi = -\frac{W J_W (\mu - r) + W L J_{WL} \sigma (\sigma - \nu_3) - \nu_3 \sigma W J_{Wy}}{W^2 J_{WW} \sigma^2}. \quad (27)$$

Note that equation (26) provides a simple mapping between consumption C and J_W . Below, we take advantage of this relation by performing our numerical analysis using C (and its partial derivatives) rather than J . This improves the stability of the numerics as can be understood by noting that equation (27) implies that the proportion of wealth placed into the risky asset must be estimated from numerical estimates of the *second* derivative of the value function. Instead, using equation (26), we can rewrite equation (27) as

$$\pi = \frac{\frac{C}{W} (\mu - r) - \frac{L}{W} C_L \sigma (\sigma - \nu_3) \gamma + \frac{1}{W} \gamma \nu_3 \sigma C_y}{\gamma \sigma^2 C_W}. \quad (28)$$

That is, we can determine π by using only the first derivatives of C .

As is well known, the CRRA utility function possesses a scaling feature, which allows us to eliminate one of the state variables. In particular, for any value of λ we can write

$$C(\lambda W, \lambda L, y, t) = \lambda C(W, L, y, t). \quad (29)$$

Intuitively, this states that if an agent were twice as rich and had twice the labor income, then she would optimally choose to consume twice as much. If we choose $\lambda = \frac{1}{W}$, then we can write

$$C\left(1, \frac{L}{W}, y, t\right) = \frac{1}{W} C(W, L, y, t). \quad (30)$$

For what follows, it is convenient to define $X \equiv \frac{L}{W}$ and $c(X = \frac{L}{W}, y, t) \equiv C\left(1, \frac{L}{W}, y, t\right)$. Thus, we can interpret c as the consumption scaled by wealth:

$$c\left(X = \frac{L}{W}, y, t\right) = \frac{C(W, L, y, t)}{W}. \quad (31)$$

Using standard rules to change variables, we find that the optimal portfolio decision can be written in terms of c as

$$\pi = \frac{\mu - r}{\gamma \sigma^2} + \left(\frac{\mu - r}{\gamma \sigma^2} + \frac{\nu_3}{\sigma} - 1 \right) \frac{X c_X}{(c - X c_X)} + \frac{\nu_3}{\sigma} \frac{c_y}{(c - X c_X)}. \quad (32)$$

The first term is the well-known result from Merton (1969). The other terms capture the effects of stochastic labor income and cointegration. For the case in which there is no cointegration (i.e., $\kappa = 0$), it is straightforward to show that $c_y = 0$ and hence the last term drops out.¹²

Note that it might be difficult for an agent to sell securities short. Thus, following GM, Storesletten, Telmer, and Yaron (2001), and many others, we impose the constraint that π lies within the $\pi_{min} = 0$ and $\pi_{max} = 1$ bounds. In Section IV, we relax this constraint and allow the agent to take short positions up to 100% of her financial wealth, i.e., $\pi_{min} = -1$ and $\pi_{max} = 2$.

Just as we use equation (26) to rewrite the first-order condition on π , we use this equation to rewrite the Bellman equation. In particular, we first differentiate the HJB equation with respect to W , using the envelope condition to simplify the equation. A change of variables then gives

$$\begin{aligned} 0 = & -c_t + \left(\frac{1}{\gamma} \right) (r - \psi) c - (r + \pi(\mu - r) + X - c + \beta^2) (c - X c_X) \\ & - \frac{1}{2} (\pi^2 \sigma^2 + \beta^2) \left[X^2 c_{XX} - (\gamma + 1) c^{-1} (c - X c_X)^2 \right] + \pi \sigma (\sigma - \nu_3) (1 + \gamma) X c_X \end{aligned} \quad (33)$$

¹²Further, note that for the special case in which $\mu - r = \gamma \sigma^2$ and $\nu_3 = 0$, we find $\pi(t) = 1$. In that case, the agent invests 100% of her wealth in risky assets irrespective of the correlation between labor income and stock returns, which is driven by ν_1, ν_2 ! This is a very specific case, where absent any labor income investors would want to invest everything in the stock market (the Merton portfolio is $(\mu - r)/(\gamma \sigma^2) = 1$). With $\nu_3 = 0$ we can think of labor income as giving the agent a random number (determined by ν_1, ν_2) of shares of stock—the agent has no incentive to deviate from his position. Alternatively, we can think in terms of the two effects on the agent's risky asset holding decision that play a role when increasing exposure to labor income risk, namely background risk versus diversification motive. These effects offset each other exactly in this special case.

$$\begin{aligned}
& -Xc_x \left[-\kappa y + g_D - \frac{\sigma^2}{2} + \alpha(t) + \frac{1}{2}\nu_1^2 + \frac{1}{2}(\sigma - \nu_3)^2 \right] - \frac{1}{2}(\nu_1^2 + \nu_3^2) \left[c_{yy} - (\gamma + 1)c^{-1}c_y^2 \right] \\
& - \frac{1}{2} \left[\nu_1^2 + \nu_2^2 + (\sigma - \nu_3)^2 - 2\pi\sigma(\sigma - \nu_3) \right] \left[X^2c_{xx} - (\gamma + 1)c^{-1}X^2c_x^2 \right] + \kappa y c_y \\
& + \nu_3\sigma\pi \left[c_y - Xc_{xy} - (\gamma + 1)c^{-1}c_y(c - Xc_x) \right] - (\nu_1^2 - \nu_3(\sigma - \nu_3)) \left[Xc_{xy} - (\gamma + 1)c^{-1}Xc_xc_y \right].
\end{aligned}$$

The terminal condition is

$$c(X, y, T) = \epsilon^{-1} \quad \forall (X, y). \quad (34)$$

C. Present Value of Labor Income

The first-order condition with respect to consumption for the HJB equation yields¹³

$$J_W = U_C = e^{-\delta t} C^{-\gamma}. \quad (35)$$

Thus, the time- t present value of the agent's labor income is

$$V(t) = \mathbf{E}_t \left[\int_t^T ds e^{-\delta(s-t)} \left(\frac{C(s)}{C(t)} \right)^{-\gamma} L(s) \right]. \quad (36)$$

Below, we estimate the present value to labor income in equation (36) by using the Monte Carlo method.

Given that V_t is a function of only three state variables, namely, y , L , and W , we can write the stochastic component of dV as

$$\begin{aligned}
dV_{stochastic} &= V_y dy_{stochastic} + V_L dL_{stochastic} + V_W dW_{stochastic} \\
&= (\nu_1 V_y + \nu_1 L V_L) dz_1 + \nu_2 L V_L dz_{2,i} \\
&\quad + (-\nu_3 V_y + (\sigma - \nu_3) L V_L + \pi \sigma W V_W) dz_3 + \beta W V_W dz_{4,i}.
\end{aligned} \quad (37)$$

Although there are no traded securities that correlate with the z_1 , $z_{2,i}$, and $z_{4,i}$ sources of risk, as a thought experiment, consider three ‘‘pseudo-securities’’ X_j , $j = 1, 2$, and 4, such that

$$\begin{aligned}
\frac{dX_j(t)}{X_j(t)} &= r dt + \sigma dz_{j,i}^Q(t) \\
&= (r + \lambda_j(t)\sigma) dt + \sigma dz_{j,i}(t), \quad j = 1, 2, \text{ and } 4.
\end{aligned} \quad (38)$$

The coefficients $\lambda_j(t)$, $j = 1, 2$, and 4, are the risk premia on these pseudo-securities.¹⁴ We note that if all these claims were traded, then these risk premia would be pinned down by the observable price processes.

¹³In Section I.B, we impose short-selling constraints. Note, however, that such constraints do not affect the first-order condition with respect to consumption for the HJB equation. As such, discounting at J_W is still identical to discounting at U_C , which yields equation (36). Of course, the first-order condition with respect to the portfolio holding π and the optimal value of consumption will be affected by the presence of short-selling constraints. We impose these constraints numerically when solving for the optimal investment π and consumption C .

¹⁴For simplicity, we assume these securities pay no dividends and we normalize their diffusion coefficients to be constant (equal to σ). This ensures that the securities span all sources of risk.

In that case markets would be complete and the portfolio problem would have a simple solution (for example, Duffie (2001)). It is well known that when markets are incomplete, the portfolio problem can be characterized by a complete markets problem in a fictitiously completed market in which the risk premia of the added securities are such that, at the optimum, the agent does not want to hold them (He and Pearson (1991), Karatzas et al. (1991)). The corresponding risk premia, given the optimal value function, are determined by $\lambda_j(t) dt = - \left(\frac{dX_j(t)}{X_j(t)} \right) \cdot \left(\frac{dJ_W(t)}{J_W(t)} \right)$. Using equation (35), we obtain

$$\lambda_j(t) dt = - \left(\frac{1}{e^{-\delta t} C(t)^{-\gamma}} \right) d \left(e^{-\delta t} C(t)^{-\gamma} \right) \cdot dZ_j(t). \quad (39)$$

We then consider a replicating portfolio consisting of an investment θ_S in the stock S , θ_B in the risk-free asset B , and θ_{X_j} in X_j , $j = 1, 2$, and 4:

$$V^{Rep} = \theta_S S + \theta_B B + \theta_{X_1} X_1 + \theta_{X_2} X_2 + \theta_{X_4} X_4. \quad (40)$$

The stochastic component of dV^{Rep} is

$$dV_{stochastic}^{Rep} = \theta_S S \sigma dz_3 + \theta_{X_1} X_1 \sigma dz_1 + \theta_{X_2} X_2 \sigma dz_{2,i} + \theta_{X_4} X_4 \sigma dz_{4,i}. \quad (41)$$

Thus, by matching coefficients in (37) and (41) we conclude that the proportion of the agent's human capital implicitly tied up in the stock market is

$$\frac{\theta_S S}{V} = \frac{-\nu_3 V_y + (\sigma - \nu_3) LV_L + \pi \sigma W V_W}{\sigma V}. \quad (42)$$

Finally, we determine the correlation coefficient between returns to human capital and stock returns, which we denote by ρ . By combining (9) with (37), we obtain

$$\rho = \frac{-\nu_3 V_y + (\sigma - \nu_3) LV_L + \pi \sigma W V_W}{\sigma_V}, \quad (43)$$

where $\sigma_V^2 = (\nu_1 V_y + \nu_1 LV_L)^2 + (\nu_2 LV_L)^2 + (-\nu_3 V_y + (\sigma - \nu_3) LV_L + \pi \sigma W V_W)^2 + (\beta W V_W)^2$.

In Section IV, we evaluate (36), (42), and (43) for reasonable model coefficients, and we illustrate the effect of cointegration between the labor market and the stock market on the agent's human capital.

II. Accounting for Predictability in Returns

It is well documented that there is long-run predictability in stock returns. The fact that low returns generate higher future expected returns implies that stock ownership creates its own hedge, making stocks even more desirable than when it is assumed that returns are i.i.d.. An equivalent way to convey this intuition is to note that predictability lowers the return variance per unit time over longer horizons, even though the unconditional equity premium is unchanged. Below, we investigate whether our findings regarding optimal stock holdings are robust when long-run return predictability is taken into account.

First, we document that the variable y defined in equation (14) is successful at predicting future stock returns. In fact, we find that its predictive power is similar to that of the dividend yield—a variable that is often used in the literature (see, for example, Cochrane (2005)) to capture asset return predictability. We therefore extend the model to account for this predictability in stock returns.¹⁵

A. Empirical Motivation for the Model with Return Predictability

We consider the regression model

$$\frac{s(t+h) - s(t)}{h} = \beta_0 + \beta_1 \xi(t) + \epsilon(t+h), \quad (44)$$

where h denotes the predictability horizon ($h = 1, \dots, 5$ years). The dependent variable is the cumulative return (inclusive of distributions) on the CRSP value-weighted market index.¹⁶ As in Fama and French (1988b), the annual cumulative returns are non overlapping. The two- to five-year returns are overlapping annual (end-of-year) observations. The data are from 1929 to 2004.

In Table II, we report estimation results for several different predictive variables. In the first case, the predictive variable is the (logarithmic) dividend yield, that is, $\xi(t) = \hat{d}(t) - p(t-1)$, with $p(t) \equiv \log(P(t))$. In this case, the series of dividends D is constructed as discussed previously in Section A, while P is the level of the CRSP value-weighted market index. Next, we consider two different measures of the variable y and we explore their predictive power towards equity index returns. The first measure relies on the definition in equation (12), that is, $y(t) = \ell_1(t) - \hat{d}(t) - \bar{\ell}d$. Here, the per capita aggregate labor income $\ell_1(t)$ is computed according to the Lettau and Ludvigson (2001a, 2001b) definition, as explained in Section A. The second measure of y relies on the definition in equation (13), that is, $y(t) = \ell_1(t) - s(t) + \delta t$. In this latter case, s is the log-level of the CRSP value-weighted market index, inclusive of all distributions, and δ is the sample average of the yearly dividend yield from 1929 to 2004.

The main finding is that the variable y has considerable power in explaining future equity index returns. The best results are obtained when y is measured as in equation (13). Specifically, the R^2 coefficient for the regressions that use $y = \ell_1 - s + \delta t$ as a predictive variable is higher than the R^2 of the regressions that use the dividend yield as a predictive variable. The explanatory power of the y variable measured as $\ell_1 - \hat{d} - \bar{\ell}d$ is somewhat lower, but still non negligible.

In sum, this evidence motivates the choice of using y to capture return predictability in our analysis of the portfolio choice problem over the life cycle.

INSERT TABLE II ABOUT HERE.

¹⁵The fact that we can capture the predictability in stock returns with the same variable y used to model cointegration between dividend and labor income is related to recent empirical evidence in Santos and Veronesi (2006) and Julliard (2005). This approach allows us to incorporate predictability in stock returns in the model without expanding the number of state variables. This is a significant computational advantage given the difficulty in solving higher-order nonlinear differential equations.

¹⁶We deflate the nominal index value by using the by using the seasonally adjusted personal consumption expenditures (PCE) deflator (1992=100) released by the Bureau of Economic Analysis.

B. Modeling Predictability

Recall from equations (13) and (14) that we have

$$y(t) = \ell_1(t) - s(t) + \delta t. \quad (45)$$

$$dy(t) = -\kappa y(t) dt + \nu_1 dz_1(t) - \nu_3 dz_3(t). \quad (46)$$

Motivated by the evidence in Section A above, we capture predictability by replacing equation (9) with

$$ds(t) = \left(\mu - \frac{\sigma^2}{2} + \phi y(t) \right) dt + \sigma dz_3(t), \quad (47)$$

where the parameter ϕ captures the strength of the predictability. Integrating equation (47), we find

$$\mathbb{E}_0 [s(T)] = s(0) + \left(\mu - \frac{\sigma^2}{2} \right) T + \frac{\phi}{\kappa} (1 - e^{-\kappa T}) y(0) \quad (48)$$

$$\text{Var}_0 [s(T)] = \left[\frac{\phi^2 \nu_1^2}{\kappa^2} + \left(\frac{\phi \nu_3}{\kappa} - \sigma \right)^2 \right] T - \left(\frac{\phi^2}{2\kappa^3} \right) (\nu_1^2 + \nu_3^2) (3 - e^{-\kappa T}) (1 - e^{-\kappa T}) + \frac{2\phi \nu_3 \sigma}{\kappa^2} (1 - e^{-\kappa T}). \quad (49)$$

Note that over long periods the expected return per unit time $\left(\mu - \frac{\sigma^2}{2} \right)$ is the same as in the no-predictability model. In contrast, the variance per unit time, which equals σ^2 for all maturities in the no-predictability case, decreases with time in this model, as we demonstrate in Figure 1 below.

INSERT FIGURE 1 ABOUT HERE.

Analogous to equation (18) and the equations that follow, we have

$$d\ell(t) = \left(-(\kappa - \phi) y(t) + \mu - \delta - \frac{\sigma^2}{2} + \alpha(t) - \frac{\nu_2^2}{2} \right) dt + \nu_1 dz_1(t) + \nu_2 dz_{2,i}(t) + (\sigma - \nu_3) dz_3(t). \quad (50)$$

Equation (50) implies that $\ell(t)$ is normally distributed. Its mean and variance are, respectively,

$$\mathbb{E}_0 [\ell(T)] = \ell(0) - y(0) \left(1 - \frac{\phi}{\kappa} \right) (1 - e^{-\kappa T}) + \left(\mu - \delta - \frac{\sigma^2}{2} + \alpha_0 - \frac{\nu_2^2}{2} \right) T + \frac{\alpha_1}{2} T^2 \quad (51)$$

$$\begin{aligned} \text{Var}_0 [\ell(T)] &= \left(\frac{\nu_1^2 + \nu_3^2}{\kappa^2} \right) (\phi - \kappa)^2 \left[T - \frac{1}{2\kappa} (3 - e^{-\kappa T}) (1 - e^{-\kappa T}) \right] + (\nu_1^2 + \nu_2^2 + (\sigma - \nu_3)^2) T \\ &\quad + \frac{2(\kappa - \phi)}{\kappa} (\nu_3(\sigma - \nu_3) - \nu_1^2) \left[T - \frac{1}{\kappa} (1 - e^{-\kappa T}) \right]. \end{aligned} \quad (52)$$

When we set $\nu_2 = 0$ in equation (52), we obtain the variance of the *aggregate* labor income component, $\text{Var}_0 [\ell_1(T)]$. Figure 1 depicts $\text{Var}_0 [\ell_1(T)] / T$ for different values of the ϕ coefficient and as a function of the investment horizon T . The plots illustrate that due to the cointegration relation, the variance of the growth rates in the logarithmic gain process and aggregate labor income terms converge as the investment horizon T increases. Further, with predictability the variance of these growth rates are reduced significantly in the long run.

Normality implies that $E_0[L(T)] = e^{E_0[\ell(T)] + \frac{1}{2}\text{Var}_0[\ell(T)]}$. We use this formula to choose $\{\alpha_0, \alpha_1\}$ to best fit the empirical findings of CGM. Further, using Ito's lemma we find

$$\frac{dL}{L} = \left[(\phi - \kappa) y(t) + \mu - \delta - \frac{\sigma^2}{2} + \alpha(t) + \frac{\nu_1^2}{2} + \frac{(\sigma - \nu_3)^2}{2} \right] dt + \nu_1 dz_1(t) + \nu_2 dz_{2,i}(t) + (\sigma - \nu_3) dz_3(t).$$

Following the same argument as in the previous section, we find that the change-of-variable HJB equation is nearly the same as in equation (33):

$$\begin{aligned} 0 = & -c_t + \left(\frac{1}{\gamma}\right) (r - \psi) c - (r + \pi(\mu + \phi y - r) + X - c + \beta^2) (c - Xc_x) \\ & - \frac{1}{2} (\pi^2 \sigma^2 + \beta^2) \left[X^2 c_{xx} - (\gamma + 1) c^{-1} (c - Xc_x)^2 \right] + \pi \sigma (\sigma - \nu_3) (\gamma + 1) Xc_x \\ & - Xc_x \left[(\phi - \kappa) y + \mu - \delta - \frac{\sigma^2}{2} + \alpha(t) + \frac{1}{2} \nu_1^2 + \frac{1}{2} (\sigma - \nu_3)^2 \right] - \frac{1}{2} (\nu_1^2 + \nu_3^2) \left[c_{yy} - (\gamma + 1) c^{-1} c_y^2 \right] \\ & - \frac{1}{2} \left[\nu_1^2 + \nu_2^2 + (\sigma - \nu_3)^2 - 2\pi \sigma (\sigma - \nu_3) \right] \left[X^2 c_{xx} - (\gamma + 1) c^{-1} X^2 c_x^2 \right] + \kappa y c_y \\ & + \nu_3 \sigma \pi \left[c_y - Xc_{xy} - (\gamma + 1) c^{-1} c_y (c - Xc_x) \right] - (\nu_1^2 - \nu_3 (\sigma - \nu_3)) \left[Xc_{xy} - (\gamma + 1) c^{-1} Xc_x c_y \right]. \end{aligned} \quad (53)$$

From the first order condition, the optimal proportion invested in the risky asset is

$$\pi = \frac{\mu + \phi y - r}{\gamma \sigma^2} + \left(\frac{\mu + \phi y - r}{\gamma \sigma^2} + \frac{\nu_3}{\sigma} - 1 \right) \frac{Xc_x}{(c - Xc_x)} + \frac{\nu_3}{\sigma} \frac{c_y}{(c - Xc_x)}. \quad (54)$$

The terminal condition is

$$c(X, y, T) = \epsilon^{-1} \quad \forall (X, y). \quad (55)$$

III. Model Calibration

To illustrate the implications of our model, we consider a realistic calibration of its coefficients.

1. Labor Income Dynamics

The parameter κ is the key cointegration coefficient that links aggregate labor income ℓ_1 and dividends \hat{d} , in that it determines the speed of the mean reversion of the variable y towards its long-run mean. In Section I.A, we find evidence that κ is imprecisely measured and varies considerably depending on the sample period, with point estimates that range from approximately 0.05 to over 0.20. As such, below we use $\kappa = 0.15$ for our baseline case. We check the robustness of our simulation results when κ takes values at the lower ($\kappa = 0.05$) and upper bounds ($\kappa = 0.20$) of its empirical range.

The variance of the total labor income process is determined by three coefficients, ν_1 , ν_2 , and ν_3 . Of these, ν_3 captures the short-run correlation between labor income and returns innovations. As noted previously, this correlation is given by $\text{corr}(ds, d\ell) = \frac{(\sigma - \nu_3)}{\sqrt{\nu_1^2 + \nu_2^2 + (\sigma - \nu_3)^2}}$. In our baseline case, we fix $\nu_3 = \sigma = 0.16$, which yields a zero contemporaneous correlation between labor income growth and stock market returns. A low correlation is consistent with the empirical evidence reported in,

for example, CGM, Davis and Willen (2000), and Fama and Schwert (1977). In the next section we illustrate the robustness of our results to different values of this coefficient.

The coefficient ν_2 determines the variance of the idiosyncratic labor income component, ℓ_2 . We calibrate ν_2 to match the magnitude of the typical permanent income component's variance, as measured in previous studies that model the labor income process of individual households by using micro data from the Panel Study of Income Dynamics (PSID). For instance, CGM report values for the standard deviations of the permanent idiosyncratic shocks that range from 0.1 to 0.13, depending on the household's education level. CS and GP's estimates range from 0.11 to 0.21, depending on the household's occupation and education level. Storesletten, Telmer, and Yaron (2004) document that the conditional standard deviation of the permanent shocks increases from 0.12 to 0.21 as the economy moves from peak to trough. As such, in our baseline case we set $\nu_2 = 0.15$.

Finally, to illustrate the calibration of the ν_1 coefficient, it is worth noting that from equation (18) the total variance of the labor income process is $\nu_1^2 + (\sigma - \nu_3)^2 + \nu_2^2$. This total variance can be decomposed into aggregate, $\nu_1^2 + (\sigma - \nu_3)^2$, and idiosyncratic, ν_2^2 , components. In our baseline case, the idiosyncratic component is $\nu_2^2 = 0.0225$. As such, we choose ν_1 so that the ratio of aggregate to permanent idiosyncratic variance shocks is very small (on the order of 1-to-10), consistent with the evidence in CCGM. Specifically, we fix $\nu_1 = 0.05$, which implies that in the baseline case the aggregate variance component equals $\nu_1^2 + (\sigma - \nu_3)^2 = 0.0025$. In section IV, we document that a larger value of ν_2 , which implies an even smaller ratio of aggregate to permanent idiosyncratic variance shocks, also yields qualitatively similar risky asset holdings.

2. *Deterministic Life-Cycle Labor Income Profile*

We calibrate the coefficients in the drift term $\alpha(t)$ in (15) to reproduce the typical income pattern due to the predictable growth component described in CS. We consider a twenty-year-old college-educated agent, $t = 0$, who will work until her retirement date at age 65, $T = 45$. We assume that her $t = 0$ annual labor income is \$15,000 in 1992 USD and we set $\alpha_0 = 0.0581$ and $\alpha_1 = -0.0024$, which imply the deterministic labor income profile depicted in Figure 2.

INSERT FIGURE 2 ABOUT HERE.

3. *Transitory Income Shocks*

The transitory income component documented in CGM, CS, GP, and others is built into our model through the term proportional to $dz_{4,i}$ in the wealth dynamics (20). For our baseline case, we fix $\beta = 0.02$, which implies that most transient fluctuations are within $\pm 2\beta$, that is, $\pm 4\%$, of the current value of wealth. Thus, for an average wealth of, say, \$300,000, most transient shocks will be within $\pm \$12,000$ per year, with a typical yearly shock of $\pm \$6,000$, consistent with the results of, for example, CGM.

4. *Risky Asset and Riskfree Bond*

Consistent with Mehra and Prescott (1985), we fix the real risk-free interest rate at 1% and we assume a 6% risk premium for the risky asset investment, that is, $r = 1\%$ and $\mu = 7\%$ in real terms. As we will show later, lower estimates of the risk premium make our results stronger, in that optimal stock holdings are even lower. Further, we calibrate the σ coefficient to match the sample standard deviation of stock returns, $\sigma = 16\%$, and we set g_D to match the average growth rate in dividends, $g_D = 1.8\%$.

As for stock return predictability, below we compute life-cycle portfolio holdings for both the cases in which $\phi = 0$ (that is, i.i.d. returns) and $\phi > 0$. Fama and French (1988a) show that return predictability accounts for around 25% of the variance of the three to five-year return on the CRSP value-weighted market index. When we fix the model coefficients to their baseline value and $\phi = 0.08$, equation (49) yields that the three-year return variance is reduced by approximately 24% compared to the i.i.d. case, while the five-year return variance drops by nearly 40% (this result is illustrated in Figure 1 above). This analysis is also consistent with the empirical evidence discussed in Section A. Specifically, at the five-year horizon, the estimates of the regression coefficient β_1 , which links the cumulative per year return to the predictive variable y , ranges from about 0.07 to approximately 0.09, depending on how y is measured. The value $\phi = 0.08$ falls in the middle of this empirical range.

Below, we also compute life-cycle portfolio holdings for values of ϕ as high as 0.1. When $\phi = 0.1$, the three-year return variance is reduced by approximately 31% and the five-year return variance by nearly 52%, compared to the i.i.d. case. Such variance decay is approximately twice as high as that documented in Fama and French (1988a). Further, $\phi = 0.1$ is higher than the upper end of the empirical range found in our univariate predictability regressions at the five-year horizon. For all these reasons, this high value of ϕ should over estimate the degree of predictability in stock returns and therefore provide a useful benchmark to check the robustness of our results.

5. Preferences

The critical parameter in the CRRA utility function is the risk aversion coefficient γ . Mehra and Prescott (1985) argue that reasonable values of γ are smaller than 10. As in CCGM, CGM, and GM, we use $\gamma = 5$ for our baseline case. In the next section, we document the sensitivity of our results to different γ values.

The magnitude of the remaining coefficients in the value function (22) is less controversial. Following CCGM, CGM, and GM, we fix $\delta = 0.04$. Cagetti (2003), Dynan, Skinner, and Zeldes (2002), and Hurd (1989) examine the implications of a bequest motive on lifetime saving and consumption decisions. In our application, we follow an approach similar to that of GP and do not explicitly model the agent's behavior during her retirement years. In this case, ϵ determines the number of years of retirement consumption that the investor wants to save for. Accordingly, we calibrate ϵ to generate a wealth accumulation profile over the life cycle that is consistent with the evidence documented in, for example, Cagetti (2003), for college-educated households. This approach results

in $\epsilon = 8$.

6. *Initial Conditions*

We consider a twenty-year-old agent endowed with \$5,000 of cash-on-hand in 1992 USD, that is, $W(0) = 5$. As mentioned previously, the agent’s $t = 0$ annual labor income is \$15,000 in 1992 USD, that is, $L(0) = 15$. We fix $y(0)$ at its “steady state” most likely value, that is, $y(0) = 0$, and without loss of generality we initialize the logarithm of the stock market gain process at zero, that is, $s(0) = 0$.

IV. Simulation Results

With the exception of a few special cases,¹⁷ analytic solutions for the life-cycle portfolio choice problem are typically not available. As such, we solve our problem numerically, by using standard finite-difference methods; see, for example, Ames (1977) and Candler (1999).

Here, we only sketch the numerical solution approach. More details are available in Appendix A. We solve the consumption problem (33) backwards, starting from the time $T = 45$ terminal condition (34) and going all the way back to the initial date $t=0$. At each tenth of a year, we save the values of c and π on an X - and y -grid. To obtain representative wealth, consumption, investment, and X profiles, we simulate 200,000 W , L , y , and X paths from their dynamics at the frequency of one-tenth of a year. In the simulations, we fix the controls π and c at the values obtained by interpolating our π and c solutions on the points of the X - and y -grid. Then, we average the realizations of the W , C , π , and X paths. Finally, we use analytic solutions for $E_t[y_s]$ and $E_t[L_s]$, $t \leq s \leq T$ to determine the representative y and L life-cycle patterns.

A. *Baseline Case*

In Figure 3, Panel A, we report the representative life-cycle wealth, consumption, and labor income profiles that result from our baseline calibration of the model. As expected, accumulated wealth increases over the life of the agent, and her consumption grows proportionally. Further, the representative individual’s labor income profile exhibits the typical pattern identified by, for example, CGM for a college-educated household.

INSERT FIGURE 3 ABOUT HERE.

Most interestingly, Panel B of Figure 3 depicts the representative stock holdings, π , over the life cycle. Contrary to the findings of much of the previous literature, and consistent with empirical evidence, we find that a young agent should not invest in the risky asset. However, as the agent ages, the optimal proportion of wealth in risky stocks increases. Intuitively, the inverse of the mean reversion coefficient controlling

¹⁷Among recent studies, see, for example, Duffie et al. (1997), Liu and Loewenstein (2002), Liu, Longstaff and Pan (2003), Schroder and Skiadas (2003, 2005).

the cointegration provides a time scale for the agent. If the number of years of remaining employment is larger than this time scale (that is, if the agent is young), then the return on the agent's human capital is highly exposed to market returns. Furthermore, most of the young agent's wealth is tied up in future labor income. As such, she will find herself overexposed to market risk, in which case it will be optimal for her to short the market portfolio, analogous to the infinitely lived representative agent in BJ who faces no idiosyncratic labor shocks. Since we impose short-sale constraints, the agent chooses to invest her entire liquid wealth in the risk-free bond. If, instead, the number of years of remaining employment is smaller than this time scale (that is, if she is middle aged), then the return on her human capital is not highly exposed to market returns—that is, her future labor income is more bond-like than stock-like. As such, she will find it optimal to invest more in the risky asset than a young agent.

When the agent approaches retirement, however, we observe two partially offsetting effects. First, for short times-to-retirement the cointegration effect does not have sufficient time to act. Thus, as in BMS, CGM, CCGM, and GM, human capital becomes bond-like. Second, the residual value of future labor income shrinks, since the agent has fewer years left to work, and therefore the value of the bond position implicit in her human capital decreases. For sufficiently short times-to-retirement, this second effect prevails, and the agent starts to reduce her stock holdings in order to buy more of the risk-free asset. This switch creates a hump in her investment strategy, and explains the downward-sloping part of her life-cycle profile. Just before retirement, the present value of future labor income for the agent is zero, and therefore the optimal portfolio decision approaches the Merton (1969) solution (which ignores labor income).

As mentioned previously, the time scale of the cointegration is the main determinant of the peak location in portfolio holdings. Based on this factor alone, we expect the peak to occur approximately $\frac{1}{\kappa}$ years before retirement. Hence, for our baseline case of $\kappa = 0.15$, we expect portfolio holdings to peak approximately 6.6 years before retirement. The peak in portfolio holdings depicted in Panel B occurs around that age. This is remarkable, especially because other factors (such as the magnitude of idiosyncratic risk) are expected to play a role in the location of the peak. This result is also roughly consistent with previous empirical studies, that find life-cycle portfolio holdings peak around age 50 to 60.¹⁸

¹⁸A vast literature studies the link between age and risky asset holdings. The evidence is often mixed, partly because of the well-known identification problem that makes it impossible to disentangle age effects, time effects, and cohort (that is, birth year) effects on portfolio choice. Several studies set cohort effects to zero. For instance, Campbell (2006) follows this approach to study the 2001 Survey of Consumer Finances (SCF) data. He estimates coefficients on age and squared-age, which imply that portfolio holdings peak when the agent is in her late fifties. Similarly, when using SCF data from 1989 to 1998 and setting cohort effects to zero, Ameriks and Zeldes (2001) find that portfolio holdings peak when the agent is in her early fifties.

B. Human Capital

We use equation (36) to compute the value of a twenty-year-old agent's human capital. Following the same method discussed previously, we simulate 500,000 wealth and consumption paths and we average across these simulated paths to evaluate (36). For a twenty-year-old agent, in the baseline case this approach results in a present value of labor income, V , of approximately \$178,000. Further, we numerically differentiate V with respect to y , L , and W , and use our estimates of V_y , V_L , and V_W to compute the fraction of the agent's human capital tied up in the stock market, as illustrated in (42). We find this fraction to be approximately one-half.¹⁹ At first blush, this fraction might not seem high enough to generate our findings, since the optimal retired agent holds about that much in stock, in which case it would seem that the agent's implicit holdings match her desired holdings, and therefore with her remaining cash-on-hand she should also invest about half of it in the risky asset. However, this estimate does not account for her implicit holdings in the three pseudo-securities X_1 , X_2 , and X_4 introduced in Section I.C. Figure 4 below shows the decomposition of the replicating portfolio for human capital into its various holdings of stock, pseudo-securities, and the risk-free money market. We find that the positions in X_1 , X_2 , and X_4 implicit in the agent's human capital are 13.9%, 87.6%, and 0.4%, respectively. Clearly, human capital is mostly equivalent to a long position in the stock market portfolio and in permanent idiosyncratic risk, which is hedged with X_2 . The transient idiosyncratic shocks driven by z_4 and hedged with X_4 represent only a very small fraction of the replicating portfolio. Hence, they do not affect the shadow value of labor income very much. We emphasize that the pseudo-securities' risk-premia are determined endogenously such that, given their labor income, agents do not want to trade in these securities.²⁰ Interestingly, through her human capital the agent's implicit holding in the risk-free asset is approximately -51%. That is, the agent's present value of labor income is a very leveraged security. On the other hand, for an agent approaching retirement human capital becomes small. Thus, her position in these pseudo-securities approaches zero, which explains her long position in the stock market.

INSERT FIGURE 4 ABOUT HERE.

Related, we measure the correlation of stock returns and the returns to human capital. Using equation (43), for a twenty-year-old agent we find a correlation coefficient of $\rho \cong 50\%$. That is, due to the idiosyncratic labor income shocks, the correlation is much lower than what is found by BJ and Campbell (1996) at the aggregate level. Still, it is sufficiently high to have a first-order effect on the agent's portfolio choice decisions.

Finally, in Figure 5, Panel A, we illustrate how the agent's human capital evolves over the life cycle. For values of t from 0 to 45, we use (36) to compute the present value of the future stream of labor income,

¹⁹Lucas (2005) studies how firms should value and hedge defined-benefits pension plan obligations when earnings growth and stock returns are positively correlated over long horizons. Consistent with our findings, she concludes that a large share of a hedge portfolio for active workers would be invested in stocks, with the share in stocks declining as employees age.

²⁰An alternative interpretation is the following. Suppose the agent had no labor income, but instead could invest in these pseudo-securities (with risk premia as determined above). Then she would want to invest precisely in the portfolio represented in Figure 4.

V_t . We note that the fraction of the agent’s labor income tied up in the risky asset is roughly constant at 50% throughout the first half of her life, and it rapidly goes to zero as she approaches retirement. Further, we note that the present value of human capital has a hump-shaped profile. That is, although young agents face a larger stream of future labor income, they discount such cash flows more than older agents. This occurs for three reasons. First, the predictable labor income component has a hump-shaped profile. As such, when the agent is young, higher labor income cash flows occur at older ages, and therefore are subject to greater time discounting. Second, as the agent ages, she faces lower idiosyncratic labor income risk. To validate this intuition, we use equation (39) to compute the risk premium on the permanent idiosyncratic labor income shocks over the agent’s life cycle. We find that λ_2 has a downward-sloping profile. It is approximately 7% when the agent is young, and it approaches zero when the agent retires. This effect is common to other models with idiosyncratic labor income risk, for example, CCGM, CGM, CS, and GM, among others. Third, in our model human capital has pronounced stock-like features, and thus commands a higher discount rate, for young agents, whereas it acquires bond-like properties, and thus is discounted at a lower rate, for older agents. Due to this third effect, which is determined by the long-run cointegration of labor income and stock market performance, the value of human capital peaks at a later point in the agent’s life compared to standard models considered in previous studies. This intuition is confirmed by the evidence in Figure 5, Panel B, which shows that the correlation of stock returns and the returns to human capital remains high and basically constant over the first half of the agent’s life, and it rapidly drops as the agent approaches retirement.

INSERT FIGURE 5 ABOUT HERE.

C. Robustness of Results

Here we examine the sensitivity of our results to changes in parameter estimates. We find that qualitatively similar results hold for a substantial range of parameter estimates.

C.1. Speed of Mean Reversion and the Equity Premium

In Figure 6, we explore the robustness of our results to the magnitude of the κ coefficient. Consistent with the intuition discussed in Section A, we see that larger values of κ increase the agent’s exposure to stock market risk and thus reduce her stock holdings. However, even a small value of κ has first-order effects on the life-cycle π profile.

INSERT FIGURE 6 ABOUT HERE.

CCGM, CGM, and GM set the equity premium equal to 4%, a value that is motivated based on the observation that stock prices have tended to increase relative to corporate earnings over recent years. Thus, in Figure 7 we illustrate the life-cycle π profile when $r = 1\%$ and $\mu = 5\%$. Interestingly, a lower value of the equity premium makes our results even stronger. Specifically, it is worth noting that with this model calibration a young agent chooses not to invest in the stock market even if the κ coefficient is as low as 0.05, as compared to the $\kappa = 0.15$ of the baseline case.

INSERT FIGURE 7 ABOUT HERE.

C.2. Contemporaneous Correlation of Stock Returns and Aggregate Labor Income Shocks

We noted previously that our baseline calibration implies a zero contemporaneous correlation of stock returns and growth rates in labor income. In Figure 8, we illustrate the effect of non zero contemporaneous correlations. We consider two cases, $\nu_3 = 0.18$ and 0.14 , which imply correlations of approximately $\pm 13\%$, respectively. Consistent with previous studies, we note that even such high values of correlations have limited impact on the agent's stock holdings, compared to the long-run cointegration effect.

INSERT FIGURE 8 ABOUT HERE.

C.3. Persistent Idiosyncratic Labor Income Shocks

We note that an increase in the idiosyncratic labor income variance (through an increase in ν_2) has two possibly opposite effects on the investor's desired portfolio holdings. First, it increases background risk, which all else equal leads to a decrease in desired risky asset holdings. Second, it provides a diversification motive, which might induce the agent to increase her demand for the risky asset. The latter effect could potentially counter balance the effect due to the long-run cointegration-like behavior of the aggregate labor income with the market portfolio. In Figure 9, we show that a value of ν_2 as high as 0.20 (the upper end of the empirical range documented in the literature) attenuates but does not eliminate our main result. Interestingly, the picture shows that investors with an investment horizon of approximately 12 years are in fact indifferent to a change in ν_2 . This duration-like feature may be due to a near perfect offsetting of the two effects (diversification motive vs. background risk) noted above.

INSERT FIGURE 9 ABOUT HERE.

C.4. Transitory Idiosyncratic Labor Income Shocks

It is generally agreed that transient idiosyncratic labor income shocks have negligible implications for the optimal portfolio choice problem solution. We confirm this result by considering values of β as small as zero and as large as 0.04 (twice the value used in our baseline case). In either case, we find life-cycle portfolio holdings that are nearly identical to those obtained in the baseline case (not reported).

C.5. Relative Risk Aversion

In Figure 10, we document the sensitivity of our results to changes in the relative risk aversion coefficient. Note that even for a young agent with relative risk aversion of $\gamma = 4$, human capital has stock-like features. In this case, stock holdings retain the same hump-shaped profile over the life cycle. However, a less risk-averse agent (for example, $\gamma = 3$) perceives her human capital to be more bond-like, in spite of the long-run cointegration effect. Hence, even at a young age she invests heavily in the risky asset. As she gets older, the present value of her human capital declines relative to the value of her liquid wealth. Thus,

we see her π profile decline as she approaches retirement. As such, consistent with empirical evidence our model exhibits a high degree of heterogeneity in stock market participation even for small differences in the risk aversion coefficient.

INSERT FIGURE 10 ABOUT HERE.

C.6. Short-Sale Constraints

The recent development of derivatives markets as well as the proliferation of Exchange Traded Funds (ETFs) makes it easier for an agent to take short positions in the market portfolio. Thus, in Figure 11 we illustrate the typical life-cycle investment profile when the short-sale constraint is relaxed. Consistent with the intuition developed in Sections IV.A and IV.B, we find that a young agent chooses to short the market portfolio to hedge the long position in the stock market implicit in her human capital.

INSERT FIGURE 11 ABOUT HERE.

C.7. Predictability

Figure 12 illustrates the effect of return predictability on the agent's life-cycle portfolio holdings. We focus on the case in which the equity premium is 4%, a value that is also frequently used in the literature (see, for example, CCGM, CGM, and GM). In Panel A, we fix the predictability coefficient at the baseline value $\phi = 0.08$. As anticipated, predictability creates a hedging demand for the risky asset, which makes the agent more willing to invest in the stock market. Specifically, in the $\gamma = 5$ baseline case, the young investor chooses to allocate her entire financial wealth in the risky asset. As the investor ages, her risky asset holding converges to the Merton solution. However, if we increase the coefficient of risk aversion to $\gamma = 7.5$ (a value that is still considered acceptable in the literature),²¹ we find that the effect of cointegration between the stock and labor markets is sufficiently powerful to offset the hedging demand due to return predictability. That is, the young agent chooses to sell the market portfolio short to hedge the risk associated with her human capital position.

INSERT FIGURE 12 ABOUT HERE. In Panel B, we fix the coefficient of relative risk aversion at $\gamma = 7.5$ and we illustrate the sensitivity of life-cycle portfolio holdings to different values of the predictability coefficient ϕ . Clearly, as we decrease ϕ the agent's risky asset holdings converge to those found in the case of i.i.d. returns. Further, we note that for sufficiently high values of ϕ the hedging demand for the risky asset dominates and the young agent chooses to invest all her financial wealth in the stock market. We note, however, that an economically plausible model calibration that features higher values of the κ coefficient, possibly in combination with a lower variance of the idiosyncratic labor income shocks captured by the ν_2 coefficient and a higher risk aversion coefficient γ , would reverse this result (not reported). In conclusion, even in the presence of return predictability we can identify a variety of economically plausible calibrations that are consistent with the behavior of many young agents who choose to invest little or no

wealth in the stock market.
 Mehra and Prescott (1985) restrict the value of γ to be a maximum of ten. Bansal and Yaron (2004) use $\gamma = 7.5$ and $\gamma = 10$ in their analysis of the equity premium.

V. Conclusions

Conventional wisdom maintains that young investors should invest heavily in the stock market. Most theoretical investigations concur. Furthermore, most models suggest that labor income is more bond-like than stock-like, implying that even higher optimal proportions of wealth should be placed into holdings of the risky asset if labor income is taken into account. In this paper, we consider a model in which the aggregate component of an agent's labor income is cointegrated with the dividend process on the market portfolio, while the individual labor income component is subject to significant permanent idiosyncratic shocks. In contrast to much of the previous literature, we find that the optimal portfolio choice for the young investor is to take a substantial short position in the risky portfolio. This occurs because in our model the value of the claim to labor income is effectively a highly leveraged security with large implicit exposure to the market portfolio. Our main findings are robust to a wide array of economically plausible calibrations and are qualitatively unchanged when we allow for the presence of predictability in the risky asset's return.

One obvious extension of our paper is to include housing.²² Quan and Titman (1997) find that the real estate market is cointegrated with the stock market. This evidence suggests that if one were to incorporate housing into the portfolio choice problem and model this cointegration, the optimal investment in stocks would become even more negative.

Although this paper focuses on the individual's optimal portfolio and consumption choices taking the risk premium of the market as given, our findings might have important implications for general equilibrium models that attempt to explain the equity premium puzzle.²³ Indeed, as Basak and Cuoco (1998) point out, by taking as given that a large proportion of investors do not participate in the stock market, one need only attribute reasonable levels of risk aversion to those agents that do invest in stocks in order to explain the historical equity premium. Our results indicate that it is optimal for a large proportion of agents in the economy to short, or at least not participate, in the equity market. Thus, the exogenous specification of Basak and Cuoco (1998) might be justifiable in a general equilibrium setting that considers two classes of agents that endogenously choose to participate in the stock market depending on their risk aversion and long-run exposures to aggregate risk.²⁴

Further, since we find that in the presence of cointegration the investment horizon has a dramatic impact on portfolio holdings, it would be interesting to explore within an equilibrium model the interaction

²²Several recent studies investigate the implications of real estate holdings for asset pricing. See, for example, Cocco (2005), Flavin and Yamashita (2002), Fugazza et al. (2006), Hu (2005), Davidoff (2006), and Yao and Zhang (2005).

²³Related work includes Mankiw and Zeldes (1991), Polkovnichenko (2004), Vissing-Jørgensen (2002), Guvenen (2004), and Vissing-Jørgensen and Attanasio (2003). Although not directly related, Bansal and Yaron (2004) investigate how long-run AR(1) processes for earnings flow can explain historical equity premiums.

²⁴More specifically, the fraction of human capital implicitly tied up in the stock market might vary by occupation. This effect, which is captured by different values of the κ coefficient in our model, has a significant impact on portfolio holdings. Further, we demonstrate above that small differences in risk aversion can also yield heterogeneity in stock market participation in our model.

of various cohorts or overlapping generations of households whose labor income is cointegrated with long-term market performance.²⁵ Within this setting, it would be interesting to examine the effect of possible changes to the Social Security system, for example, the possibility of moving to a privatized retirement system in which retirement contributions earn market-based rates of return (see, for example, Abel (2001) and CCGM).

Finally, our model suggests that labor income artificially generates a negative net supply of risk-free securities. This prediction contrasts with the typical approach of assuming that the risk-free security is in zero net supply. We save these interesting questions for future research.

Appendix A: Numerical Solution Approach

We solve the optimal portfolio and consumption problem (32), (33), and (34) by using the alternate direction implicit (ADI) finite-difference method (see, for example, Ames (1977)). Following Candler (1999), we treat the nonlinear terms in (33) explicitly, thus reducing the problem to a sequence of tri-diagonal systems of linear equations that can be solved easily using standard numerical methods.²⁶

As noted previously, via some transformations we are able to reduce the state space from four state variables to two, namely, X and y . We evaluate the solution on a discrete state-space grid. For y , we set the lower bound of the domain at $y_{min} = y_0 - 5\sigma(y)$, and the upper bound at $y_{max} = y_0 + 5\sigma(y)$, where $y_0 = 0$ and $\sigma(y) = \sqrt{(\nu_1^2 + \nu_3^2)/2\kappa}$. We then construct the y -grid with a $\Delta y = 0.05$ mesh. For X , we use $X_{min} = 0$ and $X_{max} = 10$, and construct the corresponding X -grid using a $\Delta X = 0.05$ mesh.

We solve the problem backwards, starting from the time $T = 45$ terminal condition (34) and going all the way back to the initial date $t = 0$. We use a time step $\Delta t = 0.001$, which is further broken down into time increments of length $\Delta t/2$ in each of the two steps of the ADI algorithm.

We note that our numerical approach is robust to the choice of the time- and space-grid parameters. For instance, we have verified that setting $x_{max} = 20$, $y_{min} = y_0 - 7\sigma(y)$, $y_{max} = y_0 + 7\sigma(y)$, and $\Delta t = 0.0005$ results in the same numerical solution for c and π .

The boundary conditions are treated as follows. First, we note that at $X_{min} = 0$ labor income is zero. Thus, when returns are i.i.d. the Merton (1969) closed-form solution for optimal consumption holds and provides an exact boundary condition, which we impose in our finite-difference approach. When returns are predictable, as modeled in Section II, there is no closed-form solution for optimal consumption.²⁷ Thus, when returns are predictable we impose the condition $\frac{\partial^3 c(X,y)}{\partial X^3} = 0$, $X = X_{min}$. Further, we note that the second derivative of consumption with respect to the X state variable vanishes as X increases.

²⁵For related work, see, for example, Constantinides, Donaldson and Mehra (2002), Guvenen (2004), and Storesletten, Telmer, and Yaron (2001).

²⁶We test our numerical approach in the special case of the Merton (1969) model, for which a closed-form solution is known. In that case, the approximation error generated by the numerical solution method for the agent's consumption/investment policies is nearly zero.

²⁷A closed-form solution for optimal consumption is available in the special case in which intermediate consumption is zero (see, for example, Kim and Omberg (1996) and Liu (2005)) or under the condition of complete markets (see, for example, Liu (2005)). Unfortunately, neither of these conditions is satisfied in our application.

Thus, we impose the condition

$$\frac{\partial^2 c(X, y)}{\partial X^2} = 0, \quad X = X_{max}. \quad (56)$$

Economic intuition does not offer exact boundary conditions at y_{min} and y_{max} . After some experimentation, we find that the third derivative of consumption with respect to the variable y vanishes as y approaches the boundaries of its domain. Thus, we impose the conditions

$$\frac{\partial^3 c(X, y)}{\partial y^3} = 0, \quad y = y_{min} \text{ and } y = y_{max}. \quad (57)$$

We check the robustness of the solution to this approach by extending the range of the y -domain, finding identical results. Further, we note that using a discretization of (33) that relies only on internal points at y_{min} and y_{max} yields results identical to those obtained by imposing the boundary condition (57).

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Table I
ADF Test Results

We estimate the model

$$\Delta y(t) = \xi_1 + \xi_2 t + \xi_3 y(t-1) + \sum_{j=1}^L \Phi_j \Delta y(t-j) + \epsilon(t),$$

where $\Delta y(t) \equiv y(t) - y(t-1)$ and $y = \ell_1 - \hat{d} - \bar{\ell}d$. The proxy for ℓ_1 is computed using Lettau and Ludvigson's (2001a, 2001b) definition for labor income. We use yearly data from 1929 to 2004 to form the total labor income series (the data are from the National Income and Product Accounts (NIPA) tables compiled by the Bureau of Economic Analysis). We then divide the total labor income series by the population measure reported in the NIPA tables. Finally, we deflate the per capita labor income series by using the seasonally adjusted personal consumption expenditures (PCE) deflator (1992=100) released by the Bureau of Economic Analysis.

The proxy for the aggregate log-dividend process \hat{d} is constructed from return data on the value-weighted market index released by the Center for Research in Security Prices (CRSP). We deflate the dividend series by the PCE. Finally, to avoid the seasonal differences in dividend payments, we construct yearly dividend series by summing the 12 monthly dividends paid out during each calendar year from 1929 to 2004.

	1929–2004			1947–2004		
	ξ_3	τ	$R_{Adj.}^2$	ξ_3	τ	$R_{Adj.}^2$
$L = 0, \xi_2 = 0$	-0.1097	-2.77	8.29%	-0.0464	-1.15	0.58%
$L = 0$	-0.1855	-2.86	9.72%	-0.1401	-1.93	3.00%
$L = 1$	-0.2645	-4.16	24.04%	-0.1731	-2.44	17.77%
$L = 2$	-0.2397	-3.33	22.34%	-0.1697	-2.20	14.35%

Table II
Return Predictability Regressions

We estimate the model

$$\frac{s(t+h) - s(t)}{h} = \beta_0 + \beta_1 \xi(t) + \epsilon(t+h),$$

where h denotes the predictability horizon ($h = 1, \dots, 5$ years). The dependent variable is the real cumulative return (inclusive of distributions) on the CRSP value-weighted market index. The annual cumulative returns are nonoverlapping. The two- to five-year returns are overlapping annual (end-of-year) observations. The first predictive variable is $\xi(t) = \hat{d}(t) - p(t-1)$, that is, the logarithmic dividend yield. The other predictive variables are two different measures of the variable y , as given in equations (13) and (14). They are $\xi(t) = \ell_1(t) - \hat{d}(t) - \bar{\ell}d$ and $\xi(t) = y(t) = \ell_1(t) - s(t) + \delta t$, where δ is the average yearly dividend yield. The sample period is 1929 to 2004. Standard errors estimates are robust with respect to both autocorrelation and heteroskedasticity. Coefficient t -ratios are in square brackets.

Predictive variable:	β_0	$h = 1$ year		$R^2_{Adj.}$
		$[t\text{-ratio}]$	β_1	
$\xi = \hat{d} - p$	0.3026	[1.89]	0.0738	1.26%
$\xi = \ell_1 - \hat{d} - \bar{\ell}d$	0.0601	[2.96]	0.1460	3.51%
$\xi = \ell_1 - s + \delta t$	0.0740	[3.55]	0.1400	6.51%
$h = 2$ years				
$\xi = \hat{d} - p$	0.3274	[2.33]	0.0817	4.66%
$\xi = \ell_1 - \hat{d} - \bar{\ell}d$	0.0602	[3.10]	0.1322	6.19%
$\xi = \ell_1 - s + \delta t$	0.0733	[3.88]	0.1381	13.43%
$h = 3$ years				
$\xi = \hat{d} - p$	0.3225	[2.62]	0.0798	7.30%
$\xi = \ell_1 - \hat{d} - \bar{\ell}d$	0.0633	[3.60]	0.1106	7.06%
$\xi = \ell_1 - s + \delta t$	0.0736	[4.32]	0.1212	17.12%
$h = 4$ years				
$\xi = \hat{d} - p$	0.3281	[2.80]	0.0806	10.07%
$\xi = \ell_1 - \hat{d} - \bar{\ell}d$	0.0678	[4.34]	0.0871	6.19%
$\xi = \ell_1 - s + \delta t$	0.0756	[4.91]	0.1076	19.54%
$h = 5$ years				
$\xi = \hat{d} - p$	0.3164	[2.55]	0.0764	11.59%
$\xi = \ell_1 - \hat{d} - \bar{\ell}d$	0.0712	[4.95]	0.0709	6.06%
$\xi = \ell_1 - s + \delta t$	0.0766	[5.57]	0.0925	20.68%

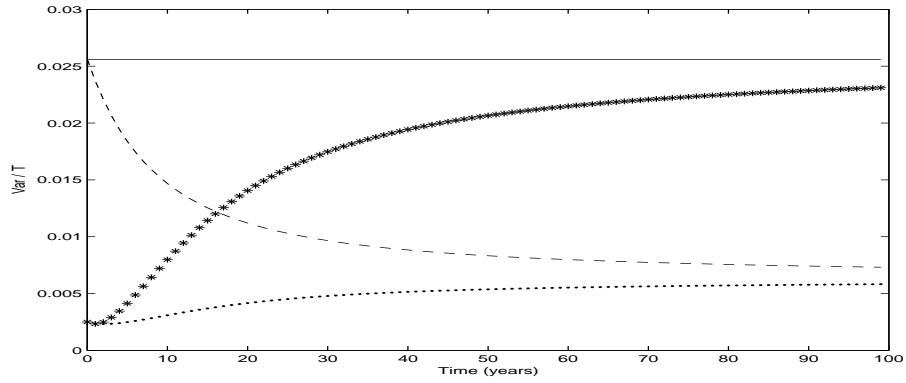


Figure 1. Return and aggregate labor income variance per unit time. The plots depict the variance of the risky asset’s return inclusive of distributions and the variance of the aggregate labor income component, both measured per unit time, for different values of the ϕ coefficient and as a function of the investment horizon T . The continuous (—) and dashed (- -) lines depict $\text{Var}_0 [s(T)] / T$ for $\phi = 0$ and $\phi = 0.08$, respectively. The plots marked with stars (***) and dots (...) depict $\text{Var}_0 [\ell_1(T)] / T$ for $\phi = 0$ and $\phi = 0.08$, respectively. When $\phi = 0$, returns are i.i.d., while a positive value of ϕ (for example, $\phi = 0.08$) captures the effect of return predictability. The other model coefficients are fixed at the baseline values discussed in Section III below.

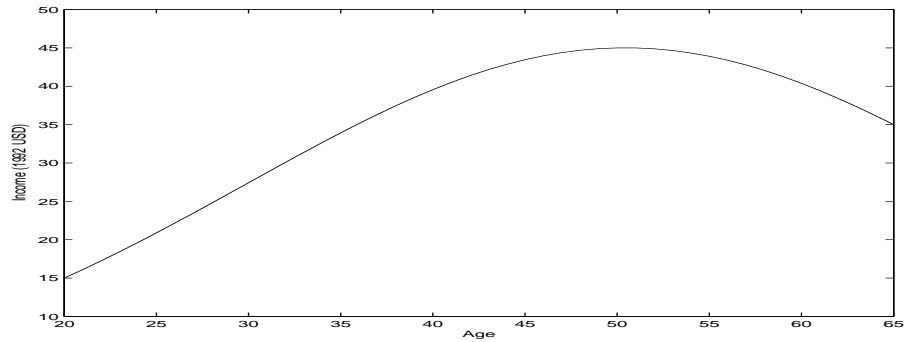
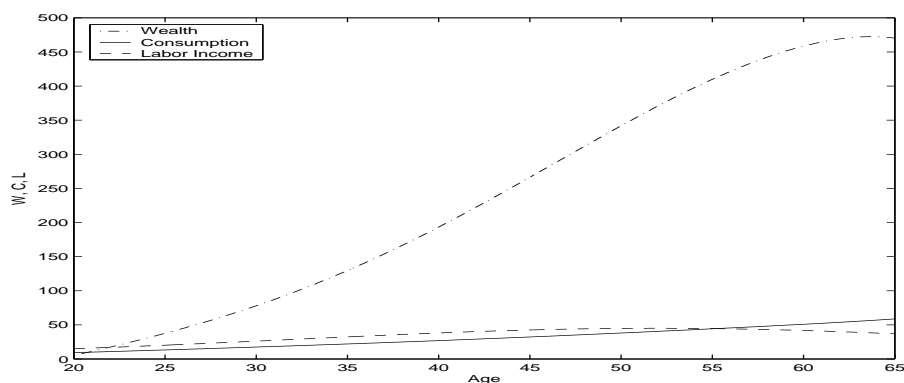
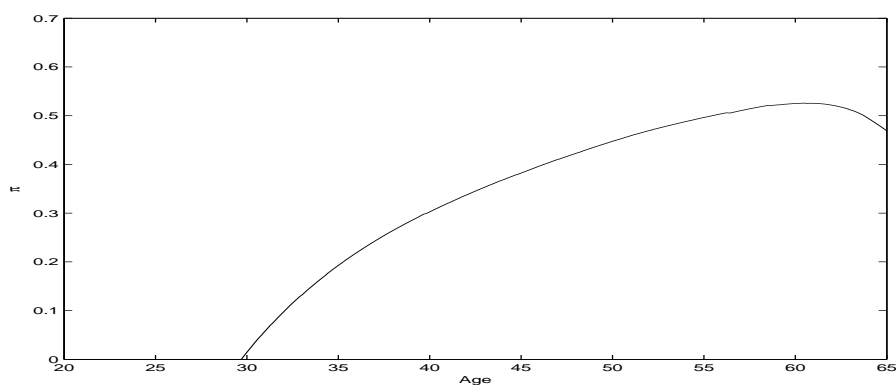


Figure 2. Deterministic labor income profile. The plot depicts the life-cycle deterministic labor income profile that results from our calibration of the $\alpha(t)$ term in (15). The agent enters the job market at age 20, earning an annual income of \$15,000 in 1992 USD, and retires at age 65.



Panel A: Wealth, consumption, and labor income in 1992 USD



Panel B: Stock holdings

Figure 3. Wealth, consumption, labor income, and stock holdings profiles. The plots depict the life-cycle profiles of wealth, consumption, labor income, and stock holdings for the baseline case parameters.

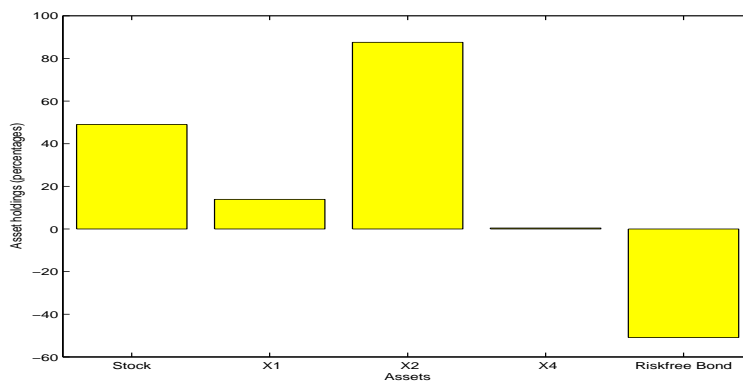
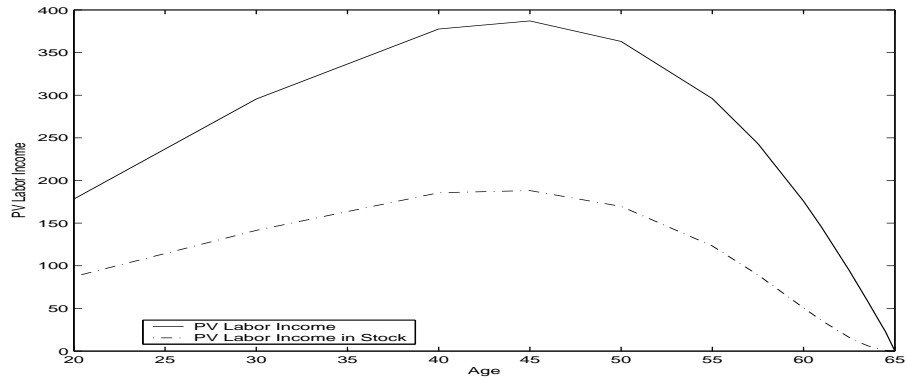
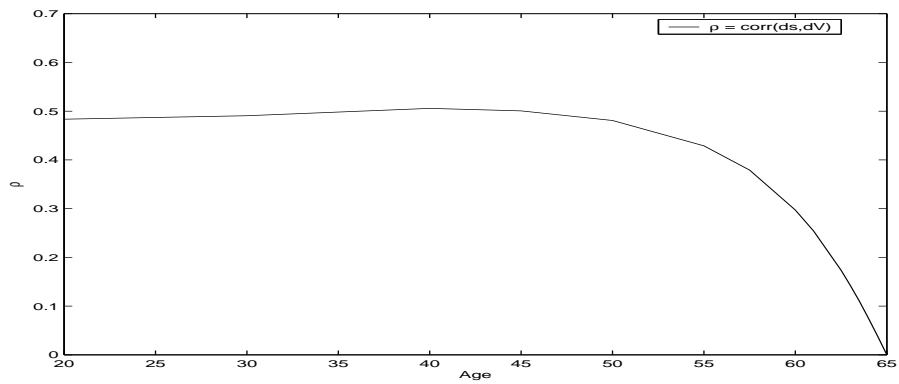


Figure 4. The components of human capital. The histogram depicts the investments in the various securities (risky asset S , pseudo-securities X_j , $j = 1, 2$, and 4, and risk-free money market B) that replicate the long position in human capital (that is, the present value of future labor income flows).



Panel A: Present value of labor income and present value of labor income tied up in stock



Panel B: Correlation of stock returns and returns to human capital

Figure 5. Human capital. The properties of human capital for the baseline case parameters.

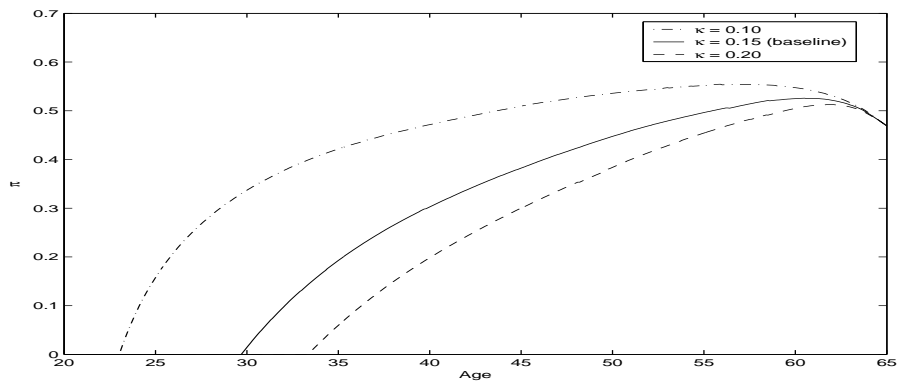


Figure 6. Life-cycle profiles of stock holdings. Sensitivity to the κ coefficient.

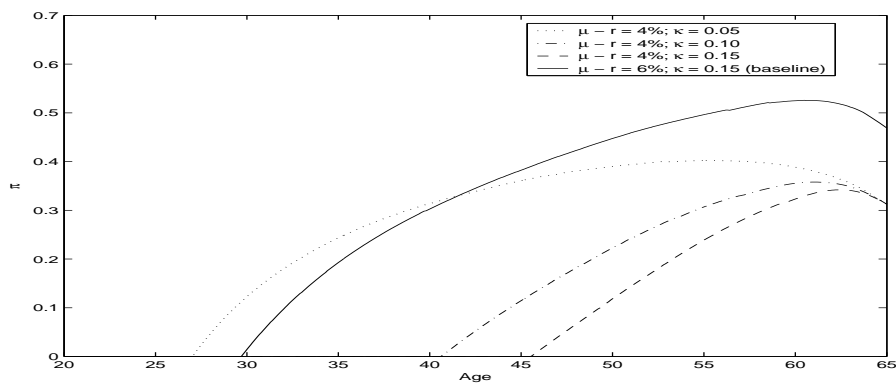


Figure 7. Life-cycle profiles of stock holdings. Sensitivity to the κ coefficient and the risk premium.

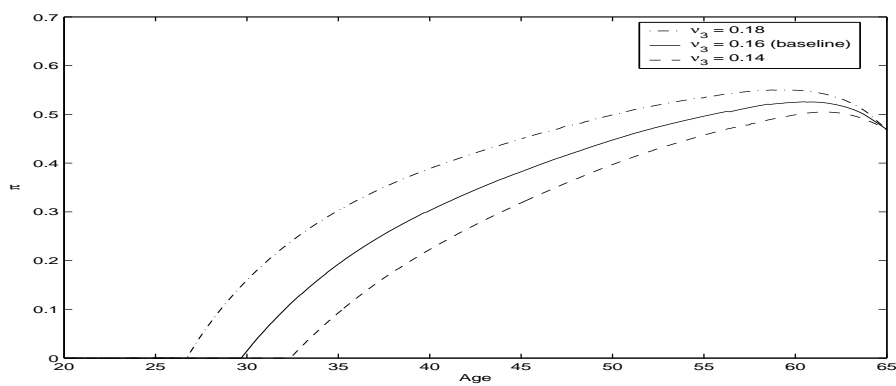


Figure 8. Life-cycle profiles of stock holdings. Sensitivity to the ν_3 coefficient.

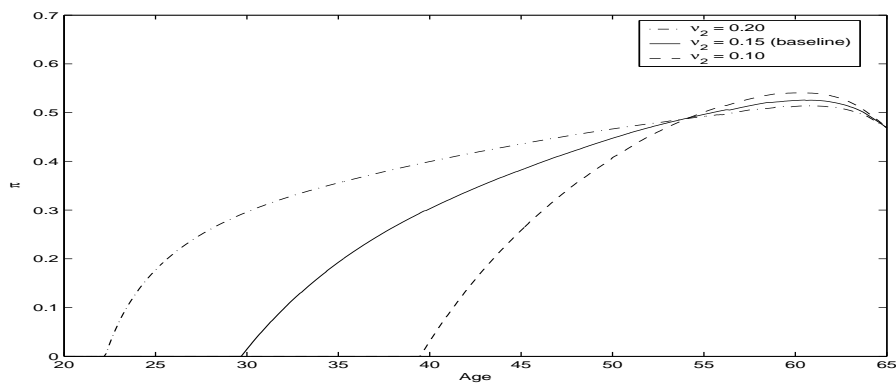


Figure 9. Life-cycle profiles of stock holdings. Sensitivity to the ν_2 coefficient.

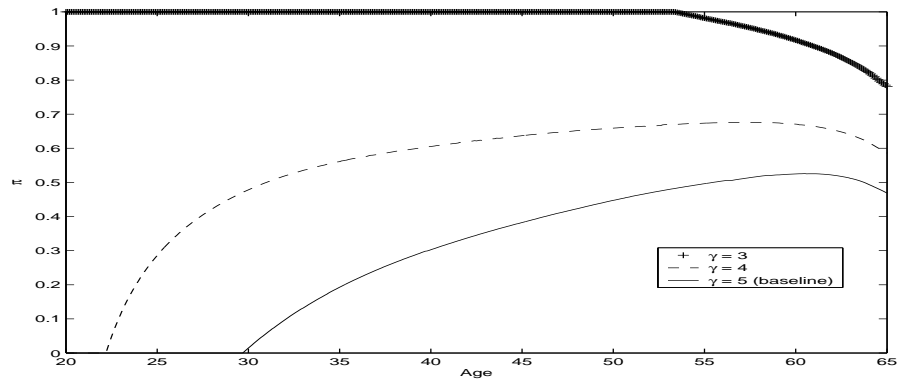


Figure 10. Life-cycle profiles of stock holdings. Sensitivity to the relative risk aversion coefficient γ .

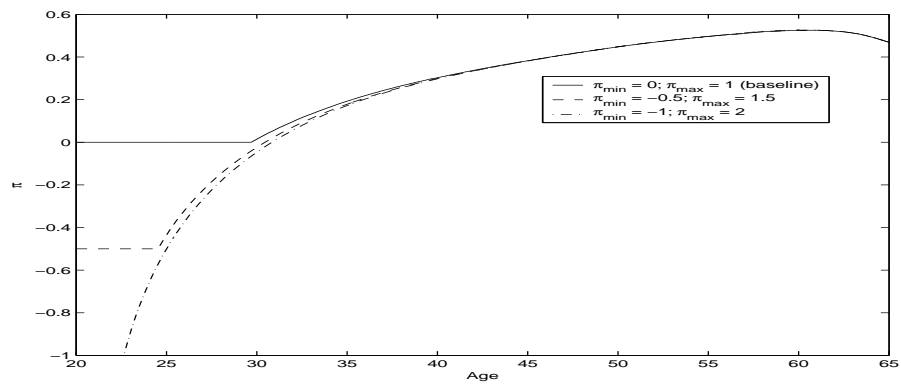
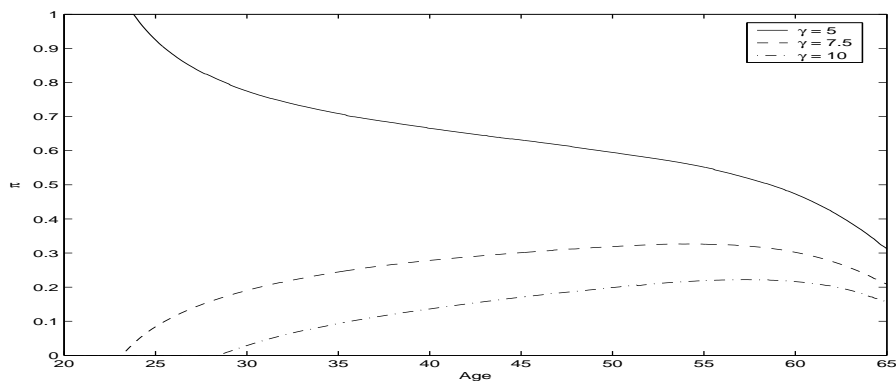
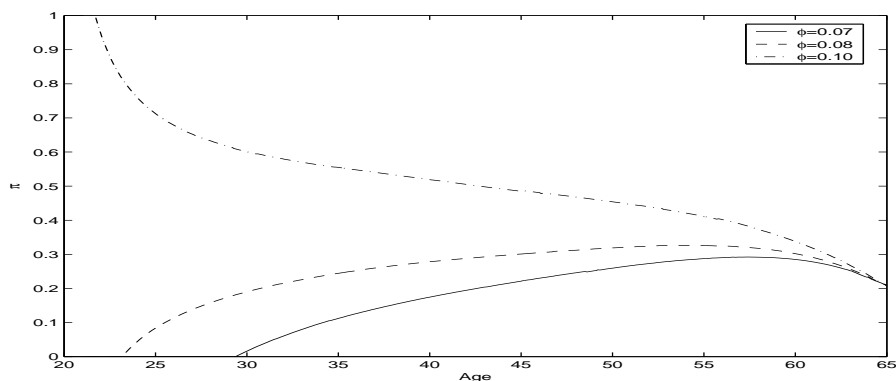


Figure 11. Life-cycle profiles of stock holdings. Sensitivity to short-sale constraints.



Panel A: Sensitivity to the risk aversion coefficient, γ . The return predictability coefficient is $\phi = 0.08$.



Panel B: Sensitivity to the risk aversion coefficient, ϕ . The risk aversion coefficient is $\gamma = 7.5$.

Figure 12. Life-cycle profiles of stock holdings. The plots depict life-cycle stock holdings when the stock market's return is predictable. In both panels, the risk premium is $\mu - r = 4\%$, while the other model coefficients are fixed at their baseline values.

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