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A Leverage-based Model of Speculative Bubbles

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Abstract

This paper develops an equilibrium model of speculative bubbles that can be used to explore the role of various policies in either giving rise to or eliminating the possibility of asset bubbles, e.g. restricting the use of certain types of loan contracts, imposing down-payment restrictions, and changing inter-bank rates. As in previous work by Allen and Gorton (1993) and Allen and Gale (2000), a bubble arises in the model because traders are assumed to purchase assets with borrowed funds. My model adds to this literature by allowing creditors and traders to enter into a more general class of contracts, as well as by allowing speculators to trade strategically.

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Introduction

The spectacular rise and fall of stock prices in the late 1990s and housing prices in the mid 2000s have been interpreted by many pundits as examples of asset bubbles. Economists typically use the term "bubble" to mean that the price of an asset differs from its "fundamental" value, i.e. the present discounted value of dividends generated by the asset. Whether these particular episodes accord with this definition is difficult to ascertain. However, the mere notion that asset prices may have become unhinged from their fundamental values during this period has affected policy discussions. For example, there are some who have criticized the aggressive easing pursued by the Federal Reserve in response to the 2001 recession on the grounds that it may have led to the emergence of bubbles in asset markets. Others have faulted the Fed in its regulatory capacity for permitting the proliferation of various exotic loan contracts that allegedly lured in speculators, e.g. offers of low initial or "teaser" rates that are eventually reset to higher rates over the duration of the loan. Even setting aside the question of whether assets were truly overvalued during this period, it is hard to evaluate the merit of these critiques. This is because they are based on intuitive arguments rather than a clearly articulated channel relating these policies to asset bubbles.

The main difficulty with modelling the connection between policy and asset bubbles is that bubbles can be ruled out in many standard economic models. This was most clearly demonstrated in Tirole (1982), who derived a set of conditions under which bubbles could be ruled out. Although there are several models which violate these conditions and allow bubbles to emerge, many of these have been criticized as implausible or not conducive for policy analysis. One prominent example are overlapping generation models of money such as Samuelson (1958) and Diamond (1967), which Tirole (1985) emphasized could be viewed as models of bubbles. Bubbles can only emerge in these models if the economy grows at least as fast as the riskless rate of return on savings; yet Abel, Mankiw, Summers, and Zeckhauser (1989) show that a generalization of this prediction is rejected empirically. Santos and Woodford (1997) further argue that the bubbles that emerge in these models are theoretically fragile, since they would cease to exist as long as even some agents who own a non-vanishing share of the aggregate endowment had infinite horizons. Other theoretical examples assume agents have different prior beliefs over the fundamental value of the asset, e.g. Harrison and Kreps (1978), Allen, Morris, and Postlewaite (1993), and Scheinkman and Xiong (2003), or that some agents trade in a way that does not depend on fundamentals, e.g. DeLong, Shleifer, Summers, and Waldmann (1990). But without a model for why agents disagree about fundamentals or ignore them when trading, it is hard to predict how changes in policy will affect trading. Nor is it obvious how confident policymakers should be in their own beliefs when they know private agents disagree about fundamentals.

An alternative theory of bubbles, which inspires the present paper, was developed by Allen and Gorton (1993) and Allen and Gale (2000). These papers emphasize the role of agency problems as a source of bubbles. More specifically, they consider environments in which agents enter into contracts with financiers who cannot monitor what borrowers do with the funds they borrow. Allen and Gorton (1993) show that this feature can give rise to a speculative bubble in which the price of an intrinsically worthless asset is repeatedly bid up until some random point at which it collapses. Their model assumes traders enter into profit-sharing contracts that entitle them to a fraction of any positive profits they earn. Unfortunately, this makes it difficult to use their model to analyze the effect of changing interest rates or the structure of debt repayments. In fact, if creditors were to offer debt contracts in their model, the speculative bubble would unravel, since agents who purchase the asset close to its peak price would no longer find it profitable to speculate. Allen and Gale (2000) develop a model in which creditors and traders use simple debt contracts in which a bubble does emerge. But theirs is not a model of "speculative" bubbles in the sense of Harrison and Kreps (1978), who define speculative behavior as a willingness to pay more for an asset for the option to resell it in the future. This distinction is important for thinking about the role of policy in sustaining bubbles, since agents may behave differently if they plan to sell an asset than if they plan to hold on to it.

This paper builds on this earlier work by constructing a model of speculative bubbles without restricting the set of contracts agents can enter. The terms of equilibrium contracts emerge endogenously, and will resemble debt contracts. In addition, I allow speculators to trade strategically. By contrast, Allen and Gorton (1993) assume traders have a bliss point over consumption, and so are eventually willing to sell the asset even though they expect its price to keep rising. While this assumption greatly simplifies their analysis, it also ignores the fact that creditors may attempt to design their contracts to affect the trading strategies of the speculators they lend to. After solving the model, I then use it to analyze the role of various policies in either facilitating or curtailing speculation, extending work by Allen and Gale (2004) exploring the implications of their 2000 model for the conduct of monetary policy.

My model reveals several new insights. First, it shows that contracts offering low rates for early repayment emerge endogenously when creditors expect some of their borrowers to speculate. These credit arrangements are thus a response to, rather than a cause of, speculation. Precluding lenders from offering these financial products will not only fail to curb speculation, but may end up exposing creditors to greater risk. In addition, the model can be used to gauge whether particular policies allow speculative bubbles to emerge or can be used to rein them in. For example, the model shows that a reduction in the real Federal Funds rate need not generate bubbles if it is temporary. This contradicts the argument cited above that the historically low rates set by the Fed in 2003 was the main culprit for the bubble in housing markets. Regardless of the true culprit, the model suggests that certain policies can be used to eliminate one if it emerges. In particular, raising rates or imposing down payment (or margin) requirements can both be used to prevent bubbles, although the latter policy is only effective if it is applied systematically rather than temporarily as is sometimes advocated. While these policies may curtail speculation, the model suggests they might also discourage beneficial trades, so that curbing speculation may be socially costly.

The paper is organized as follows. Section 1 reviews the difficulties in modelling bubbles and outlines the features of the present model that allow us to overcome these difficulties. Section 2 lays out the formal analysis. Section 3 solves the contracting problems between borrowers and financiers. Section 4 characterizes the equilibrium. Section 5 uses the model to analyze policy. Section 6 concludes.

1 Overview

Before turning to the formal analysis, it will be useful to review some of the difficulties in modelling speculative bubbles and how my model overcomes them. I consider an economy that is finite on various dimensions: the time horizon ends at a finite terminal date; the number of individuals is finite with probability one; and agents have finite amounts of resources. There is an asset in this economy that pays a single dividend at the terminal date. Agents arrive at random times prior to the terminal date, and can trade the asset only when they arrive. The question is whether the asset would ever trade above its expected payout. One reason agents might agree to buy an overvalued asset is to speculate: if they expect prices to keep rising before the terminal date, they might buy the asset in hope of selling it again before the terminal date. But this intuition ultimately fails. Since agents have finite resources, the price of the asset must be bounded. One can show this implies that as the date the asset is purchased approaches the terminal date, the profits from reselling the asset tend to zero. By contrast, the expected loss from failing to sell the asset before the terminal date is bounded away from zero. With finitely many agents, the probability an agent will fail to sell the asset is also bounded away from zero. Hence, no trader would agree to buy a bubble sufficiently close to the terminal date. But then agents would refuse to buy the asset earlier, knowing they would have no one to sell it to later. The bubble unravels, and the asset never trades above its expected value.[1]

Allen and Gorton (1993) pointed out that this unravelling need not occur if agents speculate with borrowed funds. In this case, traders no longer face any risk from failing to sell the asset: it is the creditor who puts up the funds to buy the asset, but can at most collect back the dividend. As long as speculators can keep part of the positive profits speculation can generate, they would be willing to buy the asset close to the terminal date. For a bubble to be an equilibrium, then, we must ensure that creditors agree to fund speculators, and that the contract they offer leaves speculators with some profits if speculation is successful. Since creditors expect to incur losses on speculators who purchase the asset close to the terminal date, they will never fund such speculators if they can readily identify them. I therefore need to assume there are some non-speculators who wish to borrow at similar times as speculators but can repay their debts, and that creditors cannot distinguish the two types. Allen and Gorton (1993) rely on a similar assumption.

Second, I need to ensure that the equilibrium contract allows speculators to keep some of the profits their actions can generate. Allen and Gorton (1993) achieve this by assuming a contract that gives borrowers a share of the profits if they sell the asset. While this contract is an equilibrium in their model, it will not survive as an equilibrium more generally. This is because while the expected profits from reselling the asset are positive, they tend to zero as we approach the terminal date. Creditors thus have an incentive to charge borrowers who show up close to the terminal date positive interest, since this would deter speculators. But then the bubble would unravel. My model instead relies on a feature borrowed from Allen and Gale

[1] Relaxing the assumption that the economy is finite can be used to get around this unravelling. For example, overlapping generation models of bubbles such as Tirole (1985) allow for an infinite horizon, infinitely many traders, and traders with arbitrarily large endowments. In general, it is not necessary to relax finiteness in all three dimensions to sustain a bubble.

(2000), namely that the dividend is stochastic and with some probability will be large. Competition among creditors will drive the interest charged to non-speculators down until it just covers the expected losses from speculators. As long as the highest realization of the dividend exceeds this rate, traders who buy the asset can guarantee themselves positive expected profits even if they don't sell the asset.

To recap, the essential elements of my model that allow a bubble to emerge are as follows:

1. Agents can buy assets using borrowed funds, and face limited liability.
2. There are some agents who are willing to borrow not for the purpose of speculation.
3. Creditors cannot distinguish between speculators and non-speculators.
4. The dividend on the asset is stochastic and with positive probability is large.
5. Credit markets are competitive.

These features are a hybrid of Allen and Gorton (1993) and Allen and Gale (2000). Indeed, at different times agents behave in line with either one model or the other. Agents who purchase the asset late hold on to it to see if it pays out a large dividend, just as in Allen and Gale (2000). Early speculators purchase the asset in the hope of selling it later at a higher price, just as in Allen and Gorton (1993).

2 The Model

The model is characterized as follows. Trade takes place in continuous time, with a finite horizon that terminates at some date normalized to 1. There is a single, indivisible unit of an asset that cannot be sold short and is endowed to some agent, henceforth known as the original owner, at date 0. The asset pays a single dividend d at the terminal date 1, where

$$d = \begin{cases} R_1 & \text{with probability } \epsilon \\ 0 & \text{with probability } 1-\epsilon \end{cases} \tag{1}$$

Both R_1 and ϵ are positive. Agents do not discount, so the fundamental value of the asset is just the expected payoff ϵR_1 at date 1. I focus on the limiting case where $\epsilon \to 0$ and R_1 is large, in a sense that will be made precise below. For example, the asset could represent real estate in a booming area that might be hit by a large migration wave, making land scarce and driving up the value of even the least desirable properties. Alternatively, the asset could represent an equity stake in a firm that owns a patent which may or may not pan out, but will be enormously profitable if it does. Since a large payoff is essential for sustaining a bubble, the model predicts that bubbles only emerge in certain environments, e.g. a booming region where land might become scarce or an era of technical change that allows for large rents.

As noted above, sustaining a bubble requires that agents who buy the asset near the terminal date do so with borrowed funds. I therefore assume that none of the agents who can trade in the asset own resources, and they must all turn to creditors for financing. I assume creditors cannot purchase the asset themselves, although I relax this assumption later. Both the number of creditors and the wealth of each creditor are assumed to be large, so that their collective resources exceed the amount agents wish to borrow. Agents can shop around among creditors, although they must enter into a contract with only one. Since exclusivity imposes fewer constraints on what a contract can achieve, they would be willing to commit this way.

Let $p(t)$ denote the price of the asset at time t. Traders know this path in advance and treat it as given. As in Allen and Gorton (1993), I focus on the question of whether the asset could trade above its fundamental value. Since the original owner would refuse to sell the asset for less than ϵR_1, I restrict attention to paths where $p(t) \geq \epsilon R_1$ for all $t \in [0, 1)$ to ensure the original owner can always earn at least as much from selling the asset as from holding on to it. Such a path will be defined as an equilibrium if traders are willing to buy the asset at all dates $t \in [0, 1)$, and if the asset will be traded with positive probability sometime before the terminal date. As I show in the next section, $p(t) = \epsilon R_1$ for all t conforms to this definition, i.e. it will be an equilibrium for the asset to only trade at its fundamental value. The question is whether the asset might also trade above this fundamental value. Formally, I define an asset as a *bubble* if there exists some date t such that the price of the asset exceeds its fundamental, i.e. $p(t) > \epsilon R_1$, and if the asset will be traded at this date with positive probability. I further define an asset to be a *speculative bubble* if there exists some date $s > t$ such that $p(s) > p(t)$ and there is positive probability that some agent would be willing to buy it at date s. Under this assumption, the option to resell the asset beyond date t has positive value, in line with the definition of speculation in Harrison and Kreps (1978). Since my goal is to explore the possibility of speculative bubbles, I restrict attention to paths $p(t)$ that satisfy the following assumptions:

Assumption A1: $p(t)$ is continuous and increasing in t for $t \in [0, 1)$.

Assumption A2: $\epsilon R_1 \leq p(t) \leq 1$ for all $t \in [0, 1)$.

That is, I shall henceforth limit attention to paths in which the price keeps rising up to the terminal date, although there can be speculative bubbles that violate these assumptions. The purpose of imposing an upper bound on $p(t)$ will become apparent below.

Trade in the asset proceeds as follows. Agents are assumed to arrive at specific dates and can only trade at these times. The advantage of this assumption is that it allows me to avoid dealing with agents trying to strategically time their trades. To be concrete, suppose agents reside on an island, and must commute to the center of the island to trade. Agents must leave for the center at date 0. They cannot contact a creditor or purchase the asset before they arrive. An agent can only buy the asset the instant he arrives, and must leave if he fails to acquire or once he has sold it. Agents who leave the center cannot return.

To make the model more realistic, I assume the number of traders and the dates at which they arrive are random. Neither assumption is necessary for sustaining a bubble. Without uncertainty, the original owner of the asset would wait to sell the asset to the last trader to arrive, when the price of the asset is highest. In the fully specified model, that trader would agree to buy the asset. Adding uncertainty serves to capture the fact that traders cannot time the market perfectly. Let N denote the number of potential traders, and

$t_1, ..., t_N$ denote the times at which these traders arrive. I assume N is distributed as a Poisson(λ), i.e.

$$\Pr(N = n) = \frac{e^{-\lambda}\lambda^n}{n!} \tag{2}$$

and that the arrival times of individuals are independent and uniformly distributed over $[0, 1]$, i.e.

$$\Pr(t_n \leq u) = u \text{ for } u \in [0, 1] \tag{3}$$

Under these assumptions, the number of traders who arrive before any date is independent of the number who arrive after it.[2] Hence, traders who choose between selling the asset and waiting will not use the number of traders who already arrived to decide, only the amount of time until the terminal date.

Recall that traders own no resources, and so will need financing from a creditor to buy the asset. I assume agents incur a tiny utility cost to enter into a financial contract. This ensures they only purchase the asset if they expect to earn strictly positive profits. However, creditors might not wish to offer such contracts. In fact, creditors strictly prefer not to finance speculators close to the terminal date. To see this, note that, as shown in the Appendix, the probability at least one agent arrives after some date $s \in [0, 1)$ is

$$Q(s) = 1 - e^{-\lambda(1-s)}$$

Since $\lim_{s \to 1} Q(s) = 0$, a speculator who buys the asset close to the terminal date will face low odds of selling it. Under Assumptions A1 and A2, the price $p(s) > \epsilon R_1$ for all $s \in (0, 1)$. Hence, even if the contract required the trader to hand over any dividends he earns, on average the creditor would not recoup the amount he lent out to buy the asset. Creditors would therefore refuse to knowingly finance speculators close to the terminal date. To sustain the bubble requires allowing for additional agents whom it will be profitable for creditors to finance, and the assumption that creditors cannot distinguish these from speculators.

The notion that speculators can blend in with other borrowers whom creditors would like to finance is plausible in certain contexts. Consider the case of real estate. It will be impossible for a creditor to know whether an agent borrowing to buy a house is doing so to speculate or because he really likes the house but has yet to earn enough income to buy it. If creditors can charge the latter type enough to offset the expected losses on speculators, they would agree to finance all agents who ask to borrow to buy real estate. Similarly, it might be difficult to distinguish speculators from traders who buy equity to implement some arbitrage strategy or because they plan to take over and improve productivity in the firm they invest in.

Formally, suppose that there is an additional group of agents in the economy, whom I conveniently model as entrepreneurs. That is, I assume these agents have access to a production technology that converts a single unit of output invested prior to date 1 into $R > 1$ units of output at date 1.[3] None of the remaining

[2] To see this, note that these assumptions are equivalent to assuming independent arrivals. In particular, if individuals arrive at a constant rate λ, so arrivals are independent by assumption, the number of arrivals in any period $[t_0, t_1]$ will be Poisson($\lambda(t_1 - t_0)$), and the arrival times will be distributed uniformly over $[t_0, t_1]$.

[3] Alternatively, we could view enterpreneurs as agents who derive utility of at least R from the asset, earn no income before date 1, and earn income R at date 1. This interpretation is more appropriate for thinking about the housing market.

agents know how to operate this technology. Entrepreneurs can only invest when they arrive at the center of the island. I assume there is a known number of entrepreneurs M, and that they too arrive at uniformly drawn times in $[0, 1]$. The unconditional probability that an arriving agent is a non-entrepreneur is thus $\phi \equiv \dfrac{\lambda}{M+\lambda}$. Entrepreneurs can also buy the asset, but have limited attention and cannot both run the project and trade the asset. I refer to agents as entrepreneurs and non-entrepreneurs to emphasize that whether they speculate is determined in equilibrium. However, my parameter restrictions ensure that in equilibrium entrepreneurs run projects and non-entrepreneurs speculate. To ensure entrepreneurs run projects, I assume the net return to a project exceeds the maximum profit they could earn from buying and selling the asset:

$$R - 1 > \lim_{t \to 1} p(t) - p(0). \tag{4}$$

Ensuring non-entrepreneurs will speculate requires two assumptions. First, I assume R_1 is sufficiently large:

$$R_1 \geq R. \tag{5}$$

Second, I assume lending is profitable for creditors, i.e.

$$(1 - \phi)(R - 1) - \phi > 0. \tag{6}$$

Hence, the return to giving one unit resource to a random agent and collecting all of his output if he is an entrepreneur is positive. In equilibrium, competition prevents creditors from charging entrepreneurs R. Speculators can thus guarantee themselves positive expected profits by pretending to be entrepreneurs and holding the asset to see if it pays R_1. Note that since $p(t) < 1$, assumption (4) implies (6) when $\phi < \frac{1}{2}$.

The last element of the model that must be described is the information available to creditors. Clearly, creditors must not be able to observe what an agent does with the funds he borrows. My assumption that $p(t) < 1$ for all t prevents creditors from screening out non-entrepreneurs by restricting the size of the loan, since traders who wish to buy the asset need fewer resources than entrepreneurs. I further need to assume that creditors do not independently learn an agent's type or actions after they extend credit. Otherwise, they could condition the contract on this information and punish speculators. Hence, creditors must not be able to observe an agent's exact wealth, from which they could deduce his actions. Once I assume creditors cannot observe wealth, it follows that a creditor would not be able to detect if a fellow creditor approached him and pretended to be an agent. Creditors must therefore take this possibility into account when designing their contracts. This feature is important, since it prevents creditors from paying non-entrepreneurs not to speculate: such a scheme would encourage creditors to pose as non-entrepreneurs.

Since agents can always claim to have run down their wealth if it were private information, creditors must have some information about wealth to secure repayment. I therefore allow creditors to observe if an agent has zero or positive wealth at date 1, but not the exact level of his wealth. To motivate this assumption, note that in practice creditors can sue agents who claim to have exhausted their wealth (and thus unable to pay), but have no legal standing if they make no such claims. Creditors can thus threaten to seize the agent's wealth if he claims to have run it down, but not make repayments contingent on his wealth.

One remaining issue is that given my setup, creditors may be able to use information from earlier contracts to learn the current agent's type. This is because the times at which previous agents arrived and the contract they chose may reveal when the asset was last traded, and thus whether its current owner would agree to sell it. To avoid this complication, I could assume creditors cannot observe previous loans. An equivalent but more plausible assumption is that creditors can observe previous loans, but that speculators buy different assets and creditors cannot observe what specific assets they bought. That is, suppose there were many islands like the one above, each with its own specific asset. The number of agents and their arrival times are independent across islands. Agents can only trade on their own island, but when they borrow they turn to a common pool of creditors who supply all islands. Creditors can observe previous loans, but not which island they involve. This amounts to assuming they cannot tell what asset an agent bought. In the limit as the number of islands becomes large, creditors who observe previous loans are no better able to predict the identity of current agents than if they couldn't observe this information.

To summarize, the model assumes agents travel to the center of their island to trade. When they arrive, they must decide whether to engage in economic activity and if so which. If an agent chooses an activity, he must contact a creditor for financing. Creditors have limited information, and cannot discern the type of an agent who approaches them. If an agent receives financing and opts to initiate a project, he does nothing until date 1 when it pays off. If an agent is financed and opts to purchase an asset, he must decide whether to sell it or wait whenever another trader arrives and offers to buy it. These decisions will depend on the price path $p(t)$, the distribution of the number of agents, and the terms of the contract they enter. I now turn to the contracting problem between creditors and agents.

3 Contracting

To analyze the contracting problem between agents and creditors, I follow the customary route of modelling a contract as a direct revelation mechanism in which those who have private information (in this case, agents) disclose it to those who do not (in this case, creditors), and the parties take actions and transfer resources depending on what information is disclosed. Such a contract is said to be incentive compatible if those who have private information are willing to disclose it truthfully to other parties under the contract. Let X denote the set of all incentive-compatible contracts. An incentive compatible contract $x \in X$ is said to be an equilibrium contract if there exists no other contract $x' \in X$ that is strictly preferred to x by some agents and which yields strictly positive expected profits to the creditor who offers it.

The most general contract in this environment would require the agent to reveal his private information at each date between t and 1 and stipulate transfers of resources between the creditor and agent given the history of these announcements. This private information amounts to the following: (1) whether he is an entrepreneur; (2) his actions since date t; and (3) his cumulative income since date t. Let $\omega \in \{e, n\}$ denote whether an agent is an entrepreneur or not, respectively. The agent's actions between date t and any date

$\tau \in [t, 1]$ can be summarized using a single variable $a_t(\tau)$ as follows:

$$a_t(\tau) = \begin{cases} \varnothing & \text{if the agent did nothing at date } t \\ t & \text{if the agent invested in the project at date } t \\ s \in (t, \tau] & \text{if the agent bought the asset at date } t \text{ and sold it at date } s \\ 1 & \text{if the agent bought the asset at date } t \text{ and has yet to sell it} \end{cases}$$

For notational convenience, define $a = a_t(1)$. Finally, let $y_t(\tau)$ denote the cumulative income the agent earned between dates t and τ. For $\tau < 1$, $y_t(\tau)$ can be deduced from $a_t(\tau)$. At date 1, if $a = 1$, cumulative income $y = y_t(1)$ is equal to $-p(t)$ with probability $1 - \epsilon$ and $R_1 - p(t)$ with probability ϵ. Otherwise,

$$y = \begin{cases} 0 & \text{if } a = \varnothing \\ R - 1 & \text{if } a = t \\ p(s) - p(t) & \text{if } a = s \in (t, 1) \end{cases}$$

The most general type of contract would require the agent to announce $\widehat{\omega} \in \{e, n\}$ at date t, $\widehat{a}_t(\tau)$ at each date $\tau \in [t, 1]$, and $\widehat{y}_t(1)$ at date 1. Such a contract is rather cumbersome. To simplify the analysis, it will prove convenient to restrict attention to a reduced class of simple contracts in which the agent makes announcements and engages in transfers at only two dates, t and 1, as follows:

1. At date t, the agent announces a type $\widehat{\omega} \in \{e, n\}$ and is given a transfer $x_t^0(\widehat{\omega}) \geq 0$. At this point, the agent can choose $a_t(t)$ from the set of actions $A_t(\widehat{\omega}, \omega)$, where

$$A_t(\widehat{\omega}, \omega) = \begin{cases} \{\varnothing, t, 1\} & \text{if } \omega = e \text{ and } x_t^0(\widehat{\omega}) \geq 1 \\ \{\varnothing, 1\} & \text{if } \{\omega = n \text{ and } p(t) \leq x_t^0(\widehat{\omega})\} \text{ or} \\ & \quad \{\omega = e \text{ and } p(t) \leq x_t^0(\widehat{\omega}) < 1\} \\ \{\varnothing\} & \text{if } x_t^0(\widehat{\omega}) < p(t) \end{cases}$$

2. At dates $\tau \in (t, 1)$, the agent makes no announcements, and chooses $a_t(\tau)$ from the set

$$A_\tau(\widehat{\omega}, \omega) = \begin{cases} \{\tau, 1\} & \text{if } a_t(\tau') = 1 \ \forall \tau' < \tau \text{ and a buyer for the asset arrives at date } \tau \\ \{1\} & \text{if } a_t(\tau') = 1 \ \forall \tau' < \tau \text{ and no buyer for the asset arrives at date } \tau \\ \left\{ \inf\limits_{s \in (0, \tau)} a_t(s) \right\} & \text{if } \exists \tau' < \tau \text{ s.t. } a_t(\tau') \neq 1 \end{cases}$$

3. At date 1, the agent announces $(\widehat{a}, \widehat{y})$ from a set of reports $\Omega(\widehat{\omega}, \omega, a, y)$ that an agent of type (ω, a, y) can report given he previously reported $\widehat{\omega}$, and then transfers $x_t^1(\widehat{\omega}, \widehat{a}, \widehat{y})$ to the creditor.

There are several reasons to restrict the set of reports $(\widehat{a}, \widehat{y})$ an agent can make at date 1. First, an agent cannot be made to transfer resources he doesn't have, i.e. he cannot announce $(\widehat{a}, \widehat{y})$ such that

$$x_t^0(\widehat{\omega}) + y < x_t^1(\widehat{\omega}, \widehat{a}, \widehat{y})$$

Second, since creditors can verify if agents exhaust their wealth, an agent cannot report any $(\widehat{a}, \widehat{y})$ such that $x_t^1(\widehat{\omega}, \widehat{a}, \widehat{y}) = x_t^0(\widehat{\omega}) + y$ and $\widehat{y} \neq y$. Finally, as I argue below, any general contract can be replicated with a simple contract by restricting the set of permissible reports Ω for each type appropriately.

A simple contract of this type will be defined as incentive compatible if it meets the following conditions:

IC-1: Agents prefer to report their type ω truthfully at date t:

$$\omega = \arg\max_{\widehat{\omega}} E\left[\max_{a_t(\tau)\in A_\tau(\widehat{\omega},\omega)} \max_{(\widehat{a},\widehat{y})\in\Omega(\widehat{\omega},\omega,a,y)} \left\{x_t^0(\widehat{\omega}) + y - x_t^1(\widehat{\omega},\widehat{a},\widehat{y})\right\}\right] \tag{7}$$

IC-2: Given they reveal ω truthfully at date t, agents prefer to report their actions and income (a, y) truthfully at date 1:

$$(a,y) = \arg\max_{(\widehat{a},\widehat{y})\in\Omega(\omega,\omega,a,y)} E\left[\max_{a_t(\tau)\in A_\tau(\omega,\omega)} x_t^0(\omega) + y - x_t^1(\omega,\widehat{a},\widehat{y})\right] \tag{8}$$

IC-3: No creditor has incentive to pretend to be an agent and enter a contract with other creditors.

Since creditors are assumed to have vast resources, they can meet any payment obligations in a contract offered to entrepreneurs or non-entrepreneurs, whose incomes are finite at all dates. A creditor would benefit from pretending to be an agent if the contract specifies a positive cumulative transfer $x_t^1(\widehat{\omega},\widehat{a},\widehat{y}) - x_t^0(\widehat{\omega})$. Hence, to satisfy (IC-3) requires that the net cumulative transfer of resources to the agent by date 1 can be positive only in the verifiable event that the agent has zero terminal wealth. Formally, if

$$x_t^0(\widehat{\omega}) + y - x_t^1(\widehat{\omega},\widehat{a},\widehat{y}) > 0$$

for some $(\widehat{\omega},\widehat{a},\widehat{y},y)$, then the contract must stipulate

$$x_t^1(\widehat{\omega},\widehat{a},\widehat{y}) \geq x_t^0(\widehat{\omega}). \tag{9}$$

I now argue that focusing on these simple contracts will allow us to analyze the general contracting problem. Recall that a general contract would ask agents to announce their private information $a_t(\tau)$ at *all* dates $\tau \in (t, 1)$ and stipulate transfers from the agent to the creditor that depend on the *entire* history of announcements, i.e. the date-τ transfer $x_t^\tau = x_t^\tau(\widehat{\omega}, \{\widehat{a}_t(s)\}_{s\in[t,\tau]})$. At date 1 the agent would also announce $\widehat{y} = \widehat{y}(1)$, then make a transfer $x_t^1(\widehat{\omega}, \{\widehat{a}_t(s)\}_{s\in[t,1]}, \widehat{y})$ to the creditor. The set of reports an agent can make at date τ would depend not only on his type but also on all past announcements, i.e.

$$\widehat{a}(\tau) \in \Omega(\widehat{\omega}, \omega, \{\widehat{a}_t(s)\}_{s\in[t,\tau)}, a_t(\tau)).$$

However, given the specification of the model, any outcome that can be achieved with a general contract can be achieved with a modified simple contract. This is because the set of decisions $A_\tau(\widehat{\omega},\omega)$ an agent can make at any date $\tau > t$ is unaffected by the possibility of transfers at these dates: the agent has no opportunities to purchase anything beyond date t, so transfers have no effect on what he can do. Transfers prior to date 1 can only serve to prevent an agent from pretending to be certain types, e.g. asking for immediate payment can preclude an agent from pretending he sold the asset earlier than he truly did. But we can capture these effects by restricting the set of reports $\Omega(\widehat{\omega},\omega,a,y)$ in the simple contract.

Formally, given a general contract, we can construct a simple contract that achieves the same outcome as follows. First, for each $(\widehat{\omega},\widehat{a},\widehat{y})$, we set the terminal transfer $x_t^1(\widehat{\omega},\widehat{a},\widehat{y})$ to equal

$$x_t^1\left(\widehat{\omega}, \{\widehat{a}_t(s)\}_{s\in[t,1]}, \widehat{y}\right) + \sum_{\{\tau\in(t,1)\ |\ x_t^\tau\neq 0\}} x_t^\tau\left(\widehat{\omega}, \{\widehat{a}_t(s)\}_{s\in[t,\tau]}\right).$$

This restriction ensures that announcing $(\widehat{\omega}, \widehat{a}, \widehat{y})$ yields the same payoffs under the simple contract and the general contract. We then modify the set $\Omega(\widehat{\omega}, \omega, a, y)$ in the simple contract to exclude any $(\widehat{a}, \widehat{y})$ for which there exists some date $\tau \in (t, 1]$ such that the pair $(\widehat{a}(\tau), \widehat{y}(\tau))$ does not belong to the set

$$\Omega\left(\widehat{\omega}, \omega, \{\widehat{a}(s)\}_{s\in[t,\tau)}, a_t(\tau)\right)$$

As long as we take into account the way in which the general contract limits what an agent can report, the incentives for the agent will be the same under the original general contract and under the modified simple one. To find the best set of outcomes that could be achieved with general contracts, we simply pare down $\Omega(\widehat{\omega}, \omega, a, y)$ to the minimal set of reports an agent could be restricted to in state (ω, a, y). Since this contract would be subject to the fewest incentive constraints, it will weakly dominate all other contracts.

To construct this minimal set, note that when an agent arrives at the island's center, he will have at most three options: invest in the project, purchase the asset, or do nothing. Suppose first that an agent invested in the project, i.e. $a = t$. Would it be possible to use transfers between dates t and 1 to detect if this type misrepresented himself? Suppose we force agents who announce $a_t(t) \neq t$ to transfer $x_t^0(\widehat{\omega}) - p(t)$ to the creditor soon after date t if they announce $a_t(t) = 1$ and $x_t^0(\widehat{\omega})$ if they announce $a_t(t) = \varnothing$, but we then adjust $x_t^1\left(\widehat{\omega}, \{\widehat{a}_t(s)\}_{s\in[t,1]}, \widehat{y}\right)$ to keep cumulative transfers between agents and creditors at date 1 unchanged. Any $a_t(t) \neq t$ could make these transfers, but $a_t(t) = t$ could not. Hence, we can use transfers at intermediate dates to prevent an agent who invests from misreporting that he bought the asset or did nothing. This implies that the minimal set for an agent who invests in the project is given by

$$\Omega(\widehat{\omega}, \omega, t, R-1) = \begin{cases} \{(t, R-1)\} & \text{if } x_t^0(\widehat{\omega}) \geq 1 \text{ and } \omega = e \\ \varnothing & \text{else} \end{cases}$$

Next, suppose an agent purchases the asset at date t, i.e. $a \in (t, 1]$. The first question is whether we can use transfers between dates t and 1 to detect if this type misrepresents that he bought the asset. Again, suppose we forced agents who announce $a_t(t) = \varnothing$ to transfer $x_t^0(\widehat{\omega})$ to the creditor at date t, but we adjust $x_t^1\left(\widehat{\omega}, \{\widehat{a}_t(s)\}_{s\in[t,1]}, \widehat{y}\right)$ to keep the cumulative transfers at date 1 unchanged. The agent would be able to make this transfer if he did nothing, but not if he purchased the asset. Hence, we can prevent an agent who buys the asset from misrepresenting himself as having done nothing. Since there is no way to use transfers to prove an agent does *not* have resources, there is nothing we could do between dates t and 1 to detect if a trader who bought the asset falsely reported that he invested whenever $x_t^0(\widehat{\omega}) \geq 1$. Thus, an agent who buys the asset can either report truthfully or, if $x_t^0(\widehat{\omega}) \geq 1$, that he invested in the project.

The second question is whether we can use transfers between dates t and 1 to detect if an agent who bought the asset misreported the date s at which he sold it. Suppose we forced agents who announce they sold the asset at date $s \in (t, 1)$ to transfer $x_t^0(\widehat{\omega}) + p(s) - p(t)$ resources at date s, but adjust $x_t^1\left(\widehat{\omega}, \{\widehat{a}_t(s)\}_{s\in[t,1]}, \widehat{y}\right)$ to keep the cumulative transfers between the agent and creditor unchanged. This transfer would be possible only for an agent who truly sold at s. So an agent cannot falsely claim to sell the asset at a date he did not. However, there is no way to verify that an agent did not sell the asset by date 1.

In sum, the minimal set of reports for an agent who bought the asset at date t and sold it at s is given by

$$\Omega(\widehat{\omega},\omega,s,p(s)-p(t)) = \begin{cases} \left\{ \begin{array}{c} (s,p(s)-p(t)),(t,R-1), \\ (1,-p(t)),(1,R_1-p(t)) \end{array} \right\} & \text{if } x_t^0(\widehat{\omega}) \geq 1 \\ \left\{ \begin{array}{c} (s,p(s)-p(t)), \\ (1,-p(t)),(1,R_1-p(t)) \end{array} \right\} & \text{if } p(t) \leq x_t^0(\widehat{\omega}) < 1 \\ \varnothing & \text{if } x_t^0(\widehat{\omega}) < p(t) \end{cases}$$

while the minimal set of reports for an agent who bought the asset and did not sell it by date 1 is the same whether $y = R_1 - p(t)$ or $y = -p(t)$, and is given by

$$\Omega(\widehat{\omega},\omega,1,y) = \begin{cases} \{(t,R-1),(1,-p(t)),(1,R_1-p(t))\} & \text{if } x_t^0(\widehat{\omega}) \geq 1 \\ \{(1,-p(t)),(1,R_1-p(t))\} & \text{if } p(t) \leq x_t^0(\widehat{\omega}) < 1 \\ \varnothing & \text{if } x_t^0(\widehat{\omega}) < p(t) \end{cases}$$

Finally, suppose an agent does nothing, i.e. $a = \varnothing$. Would it be possible to use transfers between dates t and 1 to detect if this type misrepresents himself? By the same argument as above, we can use transfers to prevent the agent from pretending that he bought the asset at date t and sold it at some date s. However, there is nothing we could do between dates t and 1 to prevent the agent from reporting that he bought the asset but failed to sell it, or from reporting that he invested in the project. This implies that the minimal set of reports for an agent who does nothing is given by

$$\Omega(\widehat{\omega},\omega,\varnothing,0) = \begin{cases} \{(\varnothing,0),(t,R-1),(1,-p(t)),(1,R_1-p(t))\} & \text{if } x_t^0(\widehat{\omega}) \geq 1 \\ \{(\varnothing,0),(1,-p(t)),(1,R_1-p(t))\} & \text{if } p(t) \leq x_t^0(\widehat{\omega}) < 1 \\ (\varnothing,0) & \text{if } x_t^0(\widehat{\omega}) < p(t) \end{cases}$$

Having constructed the minimal set Ω, I now derive some results that characterize the equilibrium contract. The proofs of the claims are delegated to an Appendix.

Claim 1: In equilibrium, an agent who does nothing or who holds on to an asset which pays no dividends at date 1 will have have zero terminal wealth.

According to this claim, the equilibrium contract would require confiscating all of the resources of agents who show no positive income. This follows directly from (IC-3): otherwise, creditors could take out loans, claim they made no positive income, and then pocket resources left to them under the contract.

The next few claims show that the decisions of agents are uniquely determined in equilibrium.

Claim 2: Let $\epsilon \to 0$. Then agents will be able to buy the asset under the equilibrium contract, i.e. there exists a $\widehat{\omega}$ such that $x_t^0(\widehat{\omega}) \geq p(t)$.

Claim 3: Let $\epsilon \to 0$. Then non-entrepreneurs who have the chance to buy the asset will do so in equilibrium.

Claim 4: Let $\epsilon \to 0$. Then $x_t^0(e) \geq 1$ under the equilibrium contract and entrepreneurs will invest in the project in equilibrium.

Claim 5: Under the equilibrium contract, expected profits to the creditor must be zero.

These results can be understood as follows. Assumption (6) ensures that creditors will find it profitable to lend to an agent of unknown type if they could collect all of his output were he an entrepreneur, regardless of how much they collect from a non-entrepreneur. Since competition among creditors drives equilibrium profits to zero, it follows that agents who claim to be entrepreneurs will be asked to repay less than R at date 1. Since $R_1 \geq R$, this implies a non-entrepreneur can guarantee himself a positive expected profit by pretending to be an entrepreneur, buying the asset, then holding on to it until date 1 to see if it pays out R_1 and repay back the amount demanded from entrepreneurs. Since creditors cannot pay non-entrepreneurs not to speculate, speculation must occur in equilibrium. All creditors can hope to do is minimize the costs of funding speculators by tailoring the terms of the contracts they offer.

In deriving the terms offered to the different agents, I first normalize some features of the contract that are not uniquely determined but whose exact specification is irrelevant. First, I assume that the equilibrium contract stipulates any announcement $(\widehat{\omega}, \widehat{a}, \widehat{y})$ which does not correspond to an actual type leaves the agent with zero wealth. Punishing patently untruthful reports to the maximum extent possible only serves to discourage misrepresentation, even if it is not always necessary. Second, I assume $x_t^0(e) = 1$ and $x_t^0(n) = p(t)$. From Claims 3 and 4 we know $x_t^0(e) \geq 1$ and $x_t^0(n) \geq p(t)$. If these inequalities were strict, we could always replace the original contract with a new contract $\widetilde{x}$ where

$$\begin{aligned}
\widetilde{x}_t^0(e) &= 1 \\
\widetilde{x}_t^0(n) &= p(t) \\
\widetilde{x}_t^1(\widehat{\omega}, \widehat{a}, \widehat{y}) &= \begin{cases} x_t^1(\widehat{\omega}, \widehat{a}, \widehat{y}) + 1 - x_t^0(e) & \text{if } \widehat{\omega} = e \\ x_t^1(\widehat{\omega}, \widehat{a}, \widehat{y}) + p(t) - x_t^0(n) & \text{if } \widehat{\omega} = n \end{cases}.
\end{aligned}$$

This contract leaves all agents with the same expected utility as the original contract x. Under these normalizations, the terms of the contract for an agent who announces $\widehat{\omega} = e$ at date t reduce to the net transfer $r_t^e = x_t^1(e, t, R-1) - x_t^0(e)$. From Claim 5, we know $r_t^e < R - 1$. The next claim establishes $r_t^e > 0$:

Claim 6: Under the equilibrium contract, $r_t^e = x_t^1(e, t, R-1) - x_t^0(e) > 0$.

Next, I turn to the terms for those who announce $\widehat{\omega} = n$. Let $V(n, \widehat{\omega})$ denote the payoff to a non-entrepreneur under the equilibrium contract if he announces himself to be type $\widehat{\omega}$ at date t. (IC-2) implies

$$V(n, n) \geq V(n, e).$$

The next claim establishes that this constraint will hold with equality in equilibrium.

Claim 7: In equilibrium, the incentive constraint for type n will be binding, i.e. $V(n, n) = V(n, e)$

Hence, a non-entrepreneur expects to earn the same under the equilibrium contract as he could earn by pretending to be an entrepreneur, buying the asset with the funds he receives, and then trading optimally given he must either pay back what he borrowed plus r_t^e or hand over all of his wealth. Denote the payoff to this strategy by $V_0(r_t^e)$. The next lemma suggests how to achieve $V_0(r_t^e)$ at the lowest cost to the creditor.

Lemma: The contract that provides non-entrepreneurs with utility $V_0(r_t^e)$ at the lowest cost to creditors will induce speculators to trade as if they bought the asset with their own funds.

In general, this approach might suggest encouraging speculators to wait a little and let the asset appreciate in price before selling it. But once we take into account when non-entrepreneurs can buy the asset, i.e. when the original owner of the asset would first agree to sell it, it turns out that creditors would prefer to have speculators sell the asset as soon as possible. Since (IC-3) requires agents to repay at least $x_t^0(n)$ if they sell the asset, creditors will not be able to induce speculators to sell to the first buyer they meet. The best creditors can do is get the speculator to sell the asset after some waiting period. I now argue that the contract with the shortest waiting period is the one that backloads payments to the maximal extent possible. Such a contract is characterized by two parameters: a cutoff time $T_t \in (t,1]$ and an amount R_t^n the agent must pay if he fails to sell the asset and it pays R_1. If the agent sells the asset, he will be asked to repay only what he borrowed if he sells it before date T_t, but to hand over all of his wealth if he sells it after T_t. Formally, if the agents sells the asset at some date s, the contract would specify

$$x_t^1(n, s, p(s) - p(t)) = \begin{cases} x_t^0(n) & \text{if } s < T_t \\ x_t^0(n) + p(s) - p(t) & \text{if } s \geq T_t \end{cases} \tag{10}$$

and if he does not sell it and it yields R_1, he would pay the amount

$$x_t^1(n, 1, R_1 - p(t)) = R_t^n \tag{11}$$

where $R_t^n = R_1$ if $T_t < 1$ but can be any value between $p(t) + r_t^e$ and R_1 if $T_t = 1$. That is, if the contract ever seizes all of the agent's wealth if he sells the asset, it must also seize all of his wealth if he does not sell the asset. Since $r_t^e < R - 1$, one can show that there exists a unique (T_t, R_t^n) that leaves the agent with utility $V_0(r_t^e)$. The next claim proves that along the equilibrium path, a backloaded contract comes closest to meeting the creditor's objective:

Claim 8: In equilibrium, the contract in (10) and (11) comes closest to replicating the trading strategy of an agent who buys the asset with his own funds among all contracts that deliver utility $V_0(r_t^e)$.

Note that a backloaded contract might look identical to the one offered to entrepreneurs. This is because if an agent decides to hold on to the asset, the only way to leave them indifferent to pretending to be entrepreneurs is to ask them to repay the same interest r_t^e at date 1. Non-entrepreneurs who purchase the asset close to the terminal date will in fact refuse to sell it in equilibrium. This is because selling the asset yields a profit that at best equals the capital gains on the asset, and those who purchase the asset near the terminal date do not expect it to appreciate enough to yield as much profits as waiting to see if the asset pays off R_1. This intuition is formalized in the next claim:

Claim 9: If $\epsilon > 0$, then there exists some date t^* such that $(T_t, R_t^n) = \left(1, x_t^0(n) + r_t^e\right)$ for all $t \in [t^*, 1]$.

The last claim reveals that speculators and entrepreneurs receive identical terms close to the terminal date. Would the terms appear distinct for agents who arrive at earlier dates? That depends on the path $p(t)$. If the price $p(t)$ does not appreciate much over time, speculators would prefer to hold on to the asset, and all agents would be asked to pay back what they borrowed plus r_t^e at date 1. This includes the special case where $p(t) = \epsilon R_1$ for all t, i.e. where the price of the asset equals its fundamental. Since this will be the equilibrium contract for all t, non-entrepreneurs will want to buy the asset but not sell it. The original owner of the asset would be indifferent about selling it, so the asset can trade hands at most once.

By contrast, if the price of the asset does appreciate significantly, agents would be willing to sell the asset in equilibrium. In particular, creditors will offer non-entrepreneurs a distinct contract in which they pay no interest if they sell the asset early, as opposed to $r_t^e > 0$, but a high interest exceeding r_t^e if they sell it late or not at all. The fact that the equilibrium contract is separating distinguishes this model from Allen and Gorton (1993) and Allen and Gale (2000), where all agents receive identical terms. The difference arises because agents in my model trade strategically, and creditors structure their contracts to affect trading strategies. As a result, creditors in my model might know exactly which of the agents they fund engage in speculation, based on the terms they chose. However, they cannot use this information against speculators, or else speculators would blend in with entrepreneurs and hide their intent to speculate.

4 Equilibrium

So far, I have characterized the terms contracts offered to entrepreneurs and non-entrepreneurs, respectively. In this section I show how to solve for the equilibrium, and then briefly discuss some of its features.

4.1 Solving for Equilibrium

Recall that at any date t, the contract offered to non-entrepreneurs can be summarized with two variables, T_t and R_t^n. Since these variables are chosen to deliver a utility of $V_0(r_t^e)$ to speculators, we can express these variables as functions of the rate r_t^e charged to entrepreneurs. To do so, we must first solve for $V_0(r_t^e)$. As shown in the proof of Claim 4, the optimal trading strategy for a speculator facing a contract stipulating a constant repayment r_t^e is sell the asset from some time σ_t^* on. To use this observation to obtain an expression for $V_0(r_t^e)$, note that the same proof of Claim 4 shows that the probability another trader will arrive after some date s given n traders arrived before s depends on s but not n, and is given by

$$Q(s) = 1 - e^{-\lambda(1-s)}. \tag{12}$$

This ensures speculators will not need to keep track of how many traders already arrived when considering whether they should sell the asset. The distribution of the arrival of the time of the first trader beyond date s given at least one trader arrives turns out to be similarly independent of how many traders arrived by date s, and the likelihood $f(x|s)$ that this arrival occurs at date $x > s$ is given by

$$\frac{\lambda e^{-\lambda(x-s)}}{1-e^{-\lambda(1-s)}}. \tag{13}$$

The value $V_0(r_t^e)$ under the optimal trading strategy is just the expected value of waiting until date σ_t^* and then selling the asset to the next trader to arrive, i.e.

$$Q(\sigma_t^*)\int_{\sigma_t^*}^{1}(p(x)-p(t)-r_t^e)\, f(x|\sigma_t^*)\, dx + \epsilon\left[1-Q(\sigma_t^*)\right](R_1-p(t)-r_t^e) \tag{14}$$

To solve for σ_t^*, note that if $\sigma_t^* < 1$, the agent must be just indifferent at σ_t^* between selling the asset and waiting to sell it to the next trader to arrive after σ_t^*, i.e.

$$p(\sigma_t^*)-p(t)-r_t^e = Q(\sigma_t^*)\int_{\sigma_t^*}^{1}(p(x)-p(t)-r_t^e)\, f(x|\sigma_t^*)\, dx + \epsilon\left[1-Q(\sigma_t^*)\right](R_1-p(t)-r_t^e) \tag{15}$$

We can therefore solve σ_t^* from (15) and use it to compute $V(r_t^e)$ in (14). If the value of σ_t^* that solves (15) exceeds 1, the agent must strictly prefer to hold on to the asset, and $V_0(r_t^e) = \epsilon(R_1 - p(t) - r_t^e)$.

Next, consider an agent who faces a backloaded contract characterized by (T_t, R_t^n). The optimal trading strategy given this contract once again involves selling the asset from some cutoff date s_t^* on. By a similar argument as before, the expected value under this strategy is given by

$$Q(s_t^*)\int_{s_t^*}^{T_t}(p(x)-p(t))\, f(x|s_t^*)\, dx + \epsilon\left[1-Q(s_t^*)\right](R_1-R_t^n) \tag{16}$$

and if the cutoff $s_t^* < 1$, it must satisfy the indifference condition

$$p(s_t^*)-p(t) = Q(s_t^*)\int_{s_t^*}^{T_t}(p(x)-p(t))\, f(x|s_t^*)\, dx + \epsilon\left[1-Q(s_t^*)\right](R_1-R_t^n) \tag{17}$$

If the value of s_t^* that solves (17) is larger than 1, the agent must strictly prefer to hold on to the asset, in which case his utility is $\epsilon(R_1 - R_t^n)$. To express (T_t, R_t^n) as implicit functions of r_t^e, we must equate (14) and (16). More precisely, given a value for r_t^e we would follow the following two-step procedure. First, we check whether at $T_t = 1$ there exists a value of $R_t^n \in [p(t) + r_t^e, R_1]$ that equates (14) and (16). If not, we set $R_t^n = R_1$ and search for the value of $T_t \in (t, 1]$ that equates them. Once we obtain (T_t, R_t^n) as functions of r_t^e, we can express the cutoff s_t^* as a function of r_t^e as well.

To solve for the value of r_t^e in equilibrium, we use the fact that the expected profits of a creditor are equal to zero in equilibrium, as shown in Claim 5. Let ϕ_t denote the unconditional probability that an agent who arrives at date t is a non-entrepreneur. Creditors earn a profit of r_t^e per entrepreneur and all of the profits the speculator earns beyond date T_t. Expected profits from lending at date t are thus

$$\phi_t\left\{Q(s_t^*)\int_{T_t}^{1}\left[p(x)-p(t)\right] f(x|s_t^*)\, dx + \left[1-Q(s_t^*)\right](\epsilon R_t^n - p(t))\right\} + (1-\phi_t)\, r_t^e \tag{18}$$

Since we can express T_t, R_t^n, and s_t^* as functions of r_t^e, expected profits in (18) depends entirely on this variable. Solving for equilibrium thus amounts to finding the value of r_t^e that sets (18) to zero. However, we first need to know ϕ_t, the probability that an agent who arrives at date t is a non-entrepreneur.

In general, ϕ_t will differ from ϕ. This is because the agent who owns the asset at date t might be unwilling to sell it, so the non-entrepreneur who arrives will be unable to buy it. Let $\pi(t)$ denote the probability that the agent who owns the asset at date t will be willing to sell it. Then the probability that an agent who wishes to borrow at date t intends to speculate is just

$$\phi_t = \frac{\phi\pi(t)}{1-\phi+\phi\pi(t)}$$

When $\pi(t) = 1$, this expression collapses to ϕ. But the probability that an agent will be able to buy an asset will generally not equal 1. First, as shown in the proof of Claim 8, the original owner of the asset follows a cutoff rule, and will agree to sell it only from some date s_0 on. Hence, $\pi(t) = 0$ for all $t \in [0, s_0)$, so that $\phi_t = 0$ as well. At $t = s_0$, a non-entrepreneur will necessarily be the first to show up when the asset is up for sale, so $\pi(t) = 1$. Beyond date s_0, an agent who shows up will be able to buy the asset with probability less than 1. The exact probability depends on the distribution of arrival times and the trading strategies of those who buy the asset before date t. That is, ϕ_t depends on the values of s_τ^* for all $\tau \in [s_0, t)$.

To express ϕ_t in terms of variables up to date t, it will help to describe ϕ_t in terms of order statistics. Consider a set of i.i.d. uniform random variables $\{T_1, ..., T_N\}$, where N has a Poisson(λ) distribution and is independent of T_1 through T_N. If we name agents using the numbers 1 through N, then T_n corresponds to the arrival time of individual n. Let $\{T_{(1)}, ..., T_{(N)}\}$ denote the ordered values in $T_1, ...T_N$, i.e. $T_{(n)}$ is the n-th smallest value in the set $\{T_1, ..., T_N\}$. The variable $T_{(n)}$ corresponds the time of the n-th arrival.

Next, consider a continuous function $S(t)$ where (i) $t < S(t) \leq 1$ for all $t \in [0, 1]$; and (ii) there exists a $t^* < 1$ such that $S(t) = 1$ for $t \in [t^*, 1]$. Let us use $S(\cdot)$ to construct a new sequence of random variables as follows. First, set $Y_0 \equiv 0$ and $L_0 = 0$. Next, for $j \geq 1$, define L_j and Y_j recursively as follows:

$$\begin{aligned} L_j &= \min\left\{n \leq N : T_{(n)} \geq S(Y_{j-1})\right\} \\ Y_j &= T_{(L_j)} \end{aligned}$$

Let $J+1$ denote the (random) number of variables Y_j thus constructed. Constructing the sequence $\{Y_j\}_{j=0}^J$ is analogous to compiling record values from a sequence of observations, where an observation is counted as a new record only if it exceeds the previous record value by a threshold that depends on the value of the previous record, i.e. Y_j must exceed $S(Y_{j-1})$ rather than Y_{j-1}.[4] Define $S(0) = s_0$, the date at which the original owner would agree to sell the asset, and set $S(t) = s_t^*$ for all $t \in (0, 1)$, i.e. the date at which a speculator who bought the asset at date t would first be willing to sell it. With these assumptions,

[4] For a survey of the statistical literature that defines and studies the behavior of record values derived from random sequences, see Arnold, Balakrishnan, and Nagaraja (1998).

the sequence $\{Y_j\}_{j=1}^J$ corresponds to the list of dates at which the asset is traded, since a sale occurs only if a trader arrives after the cutoff set by the last trader who purchased the asset. The expression $\Pr\left(Y_j = t \text{ for some } j \mid T_{(n)} = t \text{ for some } n\right)$ would then correspond to the unconditional probability $\pi(t)$ that an agent would be able to buy the asset at date t. Appendix B derives an analytical expression for $\pi(t)$ that involves a finite sum of integrals involving the function s_τ^* for $t \in [s_0, t)$. That appendix also describes a numerical algorithm for solving ϕ_t at all $t \in [s_0, 1]$.

To illustrate an application of this algorithm, consider the following numerical example: Set $\lambda = 7$, so there are 7 traders on average. Let ϕ equal 0.1, let $R = R_1 = 2$, the lowest value that is consistent with conditions (4) and (6), and let ϵ equal 0.05. Lastly, suppose the price path $p(t)$ is given by

$$p(t) = \max(\epsilon R_1, t)$$

which satisfies assumptions (A1) and (A2). For these particular values, the path for s_t^* turns out to be monotonically increasing in t. As discussed in Appendix B, this monotonicity simplifies the numerical analysis. The exact details behind the calculations are provided in Appendix B.

Figure 1 plots the implied s_t^*, $\pi(t)$, r_t^e, and R_t^n against t, the date at which the contract is signed (T_t equals 1 for all t and is therefore not shown). Note that the difference $s_t^* - t$ decreases with t, implying that traders who buy the asset later will be willing to sell it more quickly. This is because the chance of meeting another agent falls as we move closer to the terminal date, so holding on to the asset is less profitable.

The probability $\pi(t)$ that an agent who arrives at date t can buy the asset is non-monotonic in t. Starting from date s_0, when the original owner is first willing to sell it, $\pi(t)$ is decreasing in t. This is because as more time passes, the odds increase that another trader will swoop in to buy the asset from its original owner and not yet agree to sell it himself. But at the first instant the asset could trade for a second time, i.e. when an agent who bought the asset at s_0^* would first be willing to sell it, $\pi(t)$ begins to increase with t. This is because at later dates it is more likely that *if* someone bought the asset from the original owner he would now agree to sell it. The non-monotonicity of ϕ_t gives rise to non-monotonicity in the remaining terms r_t^e and R_t^n. However, r_t^e tends to rise over time, so entrepreneurs who arrive closer to the terminal date will be asked to pay higher rates. This is because the expected losses per speculator rise with t, so entrepreneurs must be charged more to offset these losses. By contrast, the interest $R_t^n - p(t)$ charged to speculators who hold on to the asset tends to decline with t, and eventually converges to r_t^e. This is because speculators who buy the asset later are less likely to sell it, and so inducing them to reveal their type requires exacting a smaller repayment in the increasingly more likely event they don't sell it.

4.2 Discussion

Before turning to the role of policy in the model, it is worth pausing to reflect on what the model has to say about speculative bubbles. For example, under what conditions would bubbles emerge? What should we expect to happen as the bubble unfolds?

With respect to what conditions allow a speculative bubble to emerge, the model is similar to Allen and Gorton (1993) and Allen and Gale (2000) in laying the blame for bubbles on an agency problem in which traders gamble with other people's money. In relating this back to Tirole (1982), it is worth noting that while my model violates two of his conditions for ruling out bubbles, only one of these allows bubbles to emerge. More precisely, contrary to Tirole, I assume both that there are infinitely many potential traders and that the initial allocation of resources is inefficient. Although the number of traders is potentially unbounded, the number of times the asset will trade in equilibrium is bounded by a finite number, as demonstrated in Appendix B. Tirole's argument for ruling out bubbles continues to hold when the number of times the asset changes hands is bounded, regardless of the number of traders. Hence, the reason a bubble can arise is that an inefficient initial allocation creates gains from trade between entrepreneurs and creditors that speculators can cut into. In addition, the model suggests that an important element for sustaining a bubble is that there be some probability that the asset yields a large payoff. Thus, unlike the monetary models cited in the Introduction, the model here suggests speculative bubbles should be associated with assets that may have some intrinsic value rather than with intrinsically worthless assets.

Next, consider the predictions of the model for what should happen as a bubble unfolds. One implication is that if the price of the asset appreciates enough, speculators will shift from relying on distinct contracts that reward early repayment to the same flat rate contracts offered to entrepreneurs. Another implication, which will become apparent in the next section, is that only agents who are highly leveraged will be willing to purchase the asset as we move closer to the terminal date. As we shall see, agents who use some of their own funds might be willing to purchase a bubble, but only early on, when the odds of finding another trader they can sell the asset to are still high. Finally, as we move closer to the terminal date, the frequency of trades tends to rise as speculators hold on to the asset for a shorter time. Hence, if we observe that the agents trading in an asset are almost exclusively leveraged traders using flat rate (as opposed to more exotic) contracts, and that assets are turning over more rapidly, we can infer that the point at which the price of the asset converges to its true value (once that value is revealed) is near.

5 Policy and Bubbles

So far, I have abstracted from policy in constructing the model. But the whole motivation for developing the model was to explore whether certain policies cause or can rule out the possibility of speculative bubbles. Accordingly, this section explores three types of policies. First, I consider restrictions on the type of contracts creditors can offer, inspired by claims that the use of exotic financial contracts encourages speculation. Next, I consider policies that force agents to use some of their own funds to buy assets, e.g. down-payment or margin requirements. Lastly, I consider changes in the opportunity cost of funds for lenders that affect the equilibrium terms at which agents borrow, i.e. interest rate policy.

5.1 Restrictions on the Type of Contracts Creditors can Offer

The equilibrium terms offered to speculators in the model bear a striking resemblance to financial contracts that have gained popularity in recent years, specifically loan contracts that offer low initial or "teaser" rates that are eventually reset to higher levels. Although the simple contracts I analyze assume transfers from the agent are only made at date 1, one can reinterpret these contracts as requiring agents to repay their obligations when they sell the asset. Under this interpretation, speculators who repay their debt early will be asked to pay a low interest rate, while speculators who repay their debt late will be asked to pay a high interest rate, one that may leave them with zero wealth. These types of contracts have recently been the target of heated criticism. On the one hand, some have argued that these types of contracts encourage speculation by luring in speculators with low initial rates. Others have decried the fact that onerous payment requirements may bankrupt borrowers. This has led to the suggestion that regulators ought to restrict the use of such contracts or prevent lenders from resetting payments to higher rates.

Since these types of contracts emerge endogenously in the model, it is natural to inquire what the model implies about the desirability of such policies. A key insight that emerges from the model is that these sorts of contracts can evolve in response to speculative bubbles as opposed to causing them. The reason creditors backload payments in the model is to induce agents who are already speculating to unload their asset more quickly by rewarding them with lower rates for selling the asset early. Given assumption (6), the equilibrium rate r_t^e must fall below $R - 1$ to ensure zero profits to creditors. Since $R_1 - p(t) \geq R - 1$, non-enterpreneurs can guarantee themselves positive expected profit by pretending to be entrepreneurs, buying the asset, and holding it to date 1. Restricting creditors to flat rate contracts will therefore not deter speculation. However, it will expose creditors to greater risk by increasing the chance that the traders they lend to hold on to the asset. Creditors would have to charge a higher r_t^e to entrepreneurs to offset these losses. This hurts entrepreneurs, but it also hurts speculators, who were originally indifferent to accepting a flat rate contract with a lower r_t^e. Since creditors continue to earn zero profits, they will be unaffected. Hence, no agent is made better off, and intervention merely increases the cost of providing credit.

To be sure, the model abstracts from various issues raised by proponents of eliminating such contracts. For example, one of the arguments against the use of such contracts is that onerous payments may force agents to prematurely liquidate their assets, driving down the price of all assets and leading otherwise solvent agents to default on their loans. The model obviously does not capture such coordination problems. Nevertheless, the model does highlight that allowing creditors to backload interest payments can serve a useful role, and restricting the contracts that creditors can offer may be socially costly.

5.2 Preventing Traders from Speculating with Borrowed Funds

An alternative to restricting the type of contracts lenders can offer is to preclude agents from purchasing assets with borrowed funds, at least temporarily when policymakers are concerned about bubbles. Since borrowing plays a central role in allowing a speculative bubble to emerge in the model, this seems like a natural policy to explore within the model. One example of such a policy is the initial margin requirement required of investors who purchase stocks. The authority to set margin requirement was entrusted to the Federal Reserve under the Securities and Exchange Act of 1934, and the Fed varied this rate nearly two dozen times within the forty-year period following the passage of this act. Although the Fed opted not to change this rate from 1974 on, there have been occasional calls that it should return to using margin requirements as a policy tool.[5] Another example of such a policy is a down-payment requirement towards the purchase of a home, although in the U.S. this requirement is not directly set by any regulatory agency.

Since agents in my model are endowed with no resources, requiring them to self-finance even a tiny fraction of their investment would preclude them from purchasing the asset. Down payment requirements can therefore discourage speculation by making the asset unaffordable to those who would trade in it. But the notion that small down payment requirements would eliminate all trade in the asset (and, it should be noted, the initiation of all projects) is extreme. Even if margin requirements were set to 100%, there would presumably be some agents who could afford to purchase stocks. Whether margin requirements can curb speculation more realistically depends on whether forcing agents to put up some of their own wealth would make trading in speculative bubbles unprofitable rather than unaffordable.

To explore this question, suppose that an agent who purchased an asset were required to pay a fraction $\theta \in [0, 1]$ of its cost and borrow the remainder. If the trader failed to sell the asset and it paid no dividend, he would end up losing $\theta p(t)$. For small but positive values of θ, agents who could afford to buy the asset would find it strictly profitable to do so. This is because they could always pretend to be entrepreneurs, hold on to the asset until date 1, and earn an expected payoff of

$$\epsilon \left[R_1 - r_t^e - p(t) \right] - \theta (1 - \epsilon) p(t)$$

Since $r_t^e < R - 1$ from assumption (6), and since $R_1 - p(t) \geq R - 1$, this payoff will be positive for small θ. Hence, a small margin requirement will do nothing to deter agents from wanting to speculate. But large margin requirements will discourage speculation. For example, if $\theta = 1$ so agents must pay for the asset with their own wealth, it will be unprofitable to buy the asset close to the terminal date: the agent would be unlikely to sell the asset, and holding the asset to the terminal date is unprofitable if $p(t) > \epsilon R_1$.

The above analysis suggests that a large margin requirement can potentially curb speculation. However, it turns out that such requirements must be imposed permanently rather than temporarily, as is sometimes advocated. To see why, note that agents who can borrow would be willing to buy the asset whenever margin requirements are lifted: even if margin requirements at future dates discourage further trade in the asset, it will still be profitable to pretend to be an entrepreneur, buy the asset when margin requirements are low,

[5] For example, Cecchetti (2005) writes: "For equity bubbles, economists have suggested adjusting margin requirements. Margin trading accounts for approximately 20% of total trading in US equity markets. Increasing the cost of these transactions during periods when prices have been rising quickly has the potential to keep bubbles from growing large." Kwan (2000) also discusses (and criticizes) the use of margin requirements for discouraging speculation.

then hold on to it on the off chance it pays R_1. But, more interestingly, if margin requirements were lifted prematurely, unconstrained agents might be willing to buy the asset with their own funds *even when these requirements are in effect*. This contrasts with Allen and Gorton (1993) and Allen and Gale (2000), where agents would always refuse to trade if required to do so using their own funds.

To illustrate this last claim, let us modify the model to allow creditors to trade in the asset directly. Since creditors are assumed to have ample resources, they can afford to buy the asset. If I can show that creditors would buy the asset using only their own funds, it would follow that traders who finance only a fraction of their asset purchases would also be willing to buy it, since they can shift some of the risk from buying the asset to their lenders. Let K denote the number of creditors who can trade in the asset, and suppose K has a Poisson(μ) distribution that is independent of the number of non-entrepreneurs N. The sum $N + K$ is therefore distributed as a Poisson$(\lambda + \mu)$. The value of μ is assumed to be large, in line with my assumption that there is a large number of creditors. Hence, if creditors offered to pay agents not to speculate, they would assign a large probability that an agent who approaches them at any point is a rival creditor pretending to be an agent. Creditors who wish to trade in the asset must travel to the center of the island, and their arrival times are again assumed to be uniformly distributed over $[0, 1]$. Upon their arrival, their only option is to buy the asset, similarly to non-entrepreneurs. The only difference with non-entrepreneurs is that they do not need to secure credit to buy the asset. In fact, since the equilibrium contract requires borrowers to make positive net transfers, wealthy agents will strictly prefer to buy the asset with their own funds than pretend to be an agent and borrow from another creditor. I assume travelling does not impede the ability of creditors to be contacted by agents who wish to borrow, i.e. agents can enter into a contract with any potential creditor even when the latter are commuting.

Suppose margin requirements are imposed only temporarily. That is, there is a positive margin requirement between date 0 and some date $t^* \in [0, 1)$, but no requirement between dates t^* and 1. Thus, only creditors can purchase the asset up to t^*, while non-entrepreneurs could buy the asset between dates t^* and 1 if they wish. The next claim characterizes the equilibrium outcome under this regime:

Claim 10: Suppose positive margin requirements are applied only at dates $t \in [0, t^*]$, and the price path $p(t)$ is consistent with (A1) and (A2). Then in any equilibrium,

i. Creditors will be willing to purchase the asset using their own wealth up to some date $t^{**} \in [0, 1]$.

ii. Non-entrepreneurs are willing to purchase the asset at any date, and can do so from date t^* on.

iii. If $t^* < 1$, then for λ sufficiently large, $t^* < t^{**}$. Hence, for all dates between 0 and 1, an agent who arrives at the center will be both willing and able to buy the asset with positive probability.

Claim 10 reveals even if traders were required to put up their own wealth, they might be willing to buy asset bubbles if they assigned a high probability (consistent with a large λ) of encountering other traders to whom they can sell the asset at a higher price. Traders do so because they wish to "ride the bubble" in the sense of Abreu and Brunnermeier (2003) and Temin and Voth (2004), i.e. to invest their wealth in an asset whose price is rising over time with the goal of selling it before it will "pop" – i.e. before the expected price of the asset collapses.

It is worth noting that Claim 10 is only concerned with whether traders are *willing* to buy the asset, not whether they actually buy them. As can be shown using the proof of Claim 8, the last date t^{**} at which creditors would be willing to buy the asset directly is also the date at which the original owners are first willing to sell it. Intuitively, opting not to sell the asset is equivalent to paying $p(t)$ to hold on to it. Hence, if an agent finds it profitable to buy the asset using his own funds, the original owner would find it profitable to keep it. Even though creditors are willing to buy the asset, they will only be able to buy it at the single date t^{**} when both they and the original owners are indifferent about holding the asset. This does not undermine the main insight from Claim 10, namely that there exist conditions in which agents would be willing to stake some of their own funds to buy the asset. As long as λ is sufficiently large, so that there are many traders who are willing to speculate, and the margin requirement falls below 100%, there would be an interval of time (rather than a single point) during which the asset could be actively traded even while the requirements are in effect.

5.3 Changing the Opportunity Cost of Funds

While margin requirements are sometimes mentioned as a tool for curbing speculation, the more common recommendation for curbing speculation holds that the Fed should raise interest rates when it suspects a bubble has emerged. To explore the consequences of such an action, let us reinterpret creditors in the model as banks that intermediate financial transactions rather than individuals who lend out their own wealth. The Fed can intervene in the Federal Funds market, which governs the rate banks charge one another for short-term (overnight) loans. In the model, this rate would correspond to the opportunity cost of funds for creditors. In particular, if a creditor in the model represents a bank with excess reserves, the creditor would always have the option of lending its funds out at the going Federal Funds rate. Similarly, if a creditor represents a bank with a lending opportunity but not the reserves to provide them, it could borrow reserves at the Federal Funds rate and would then need to subtract the cost of borrowing these funds from its profits. The model as specified up to now is a special case in which this opportunity cost is set to zero. But we could use the model to explore what would happen in the model under alternative paths for overnight rates. Note that in the model, the overnight rate is a real rate, whereas in practice the Fed can control the nominal rate. Hence, underlying my analysis is an assumption that the Fed can affect real rates.

Formally, let $r_t^{FF} \geq 0$ denote the instantaneous rate of return agents could earn on funds at date t, i.e. r_t^{FF} represents the limit of the return per unit of time on a loan due at date $t + \Delta$ as $\Delta \to 0$. I assume this path of r_t^{FF} is set by the monetary authority. For example, the Fed could commit to borrowing and lending to any agent at rate r_t^{FF}, ensuring that agents could earn this return on funds they lend out. Agents therefore treat this path as given. Let $R_{t,s}^{FF}$ denote the compounded return between dates t and s, i.e.

$$R_{t,s}^{FF} = \exp\left(\int_t^s r_x^{FF} dx\right)$$

The rate r_t^{FF} is assumed to be available to any agent, not just creditors. Thus, an agent who borrows funds to purchase an asset and then sells it can earn the rate r_t^{FF} on the funds at his disposal after selling the asset. This is equivalent to assuming a competitive banking sector in which depositors earn the same rate of return that the bank could by lending out their in the Federal Funds market.

Before proceeding with the analysis, I first need to examine how this modification affects the contracting problem between creditors and agents. When I previously assumed the cost of funds was equal to zero, I was able to simplify the contracting problem by restricting attention to simple contracts in which agents were only required to make announcements and transfers at dates t and 1. Since agents can earn the same instantaneous return of r_t^{FF} on their funds as creditors, we can still focus on this reduced class of contracts. The reason is that under this assumption, it is immaterial who holds on to resources that are not committed to an asset or a project: either party would be able to earn the return r_t^{FF}. The only reason not to wait until date 1 to make transfers remains that earlier transfers can be used to detect whether an agent misrepresented himself, and we can account for this by restricting the set of reports Ω for an agent.

While it is possible to continue restricting attention to simple contracts in which transfers occur only at two dates, the contracting problem will change a little when the cost of funds is positive. In particular, we need to modify the definition of income y to include interest income. Thus, we have

$$y = \begin{cases} \left(R_{t,1}^{FF} - 1\right) x_t^0(\widehat{\omega}) & \text{if } a = \varnothing \\ \left(R_{t,1}^{FF} - 1\right) \left[x_t^0(\widehat{\omega}) - 1\right] + R - 1 & \text{if } a = t \\ \left(R_{t,1}^{FF} - 1\right) \left[x_t^0(\widehat{\omega}) - p(t)\right] + R_{s,1}^{FF} \left[p(s) - p(t)\right] & \text{if } a = s \in (t, 1) \\ \left(R_{t,1}^{FF} - 1\right) \left[x_t^0(\widehat{\omega}) - p(t)\right] + d - p(t) & \text{if } a = 1 \text{, where } d \in \{0, R_1\} \end{cases}$$

Given this measure of income, I again define a contract to be incentive compatible if it provides incentives for agents to reveal their information truthfully and discourages creditors from pretending to be agents. The first requirement implies the same conditions (IC-1) and (IC-2) as before, provided we define y as above. The second requirement implies a slightly different formulation from (IC-3). For a creditor not to benefit from pretending to be an agent and borrowing, it must be the case that if there exists a y for which

$$x_t^0(\widehat{\omega}) + y - x_t^1(\widehat{\omega}, \widehat{a}, \widehat{y}) > 0$$

so that it is impossible to verify the agent exhausted his wealth, a creditor should not be able to earn positive profits by pretending to be an agent and announcing $(\widehat{\omega}, \widehat{a}, \widehat{y})$. Since the creditor can earn a return of $R_{t,1}^{FF}$ on what he borrows by holding it until date 1, this requires

$$x_t^1(\widehat{\omega}, \widehat{a}, \widehat{y}) \geq R_{t,1}^{FF} x_t^0(\widehat{\omega}) \tag{19}$$

Finally, expected profits to creditors under the contract must be nonnegative, i.e.

$$E\left[x_t^1(\omega, a, y)\right] \geq R_{t,1}^{FF} E\left[x_t^0(\omega)\right].$$

Note that profits must be nonnegative in expectation rather than for each realization of ω, since the contract is designed without knowing the agent's type.

Constraint (19) is key for understanding why an increase in the opportunity cost of funds can discourage speculation. Increasing the cost of funds induces creditors to demand higher repayments from agents, lowering the return to speculation. As long as the cost of funds is raised to a sufficiently high level, speculation will turn unprofitable and agents would be unwilling to trade. To see this, suppose that for all $\tau \in [t, 1)$ we set the Federal Funds rate r_τ^{FF} above the growth rate of the price of the asset, $\dot{p}_\tau / p_\tau$. From (19), the lower bound on what an agent would have to pay his creditor at date 1 grows faster than the price of the asset. The capital gains from buying the asset and selling it later at a higher price will therefore fail to cover the required interest payment. Agents might still be willing to borrow in order to buy the asset on the off chance that it pays off R_1. But as long as r_t^{FF} are set sufficiently high, the lower bound in (19) would exceed $R_1/p(t)$ as well, and profits would fail to cover the required interest payment to the creditor. Hence, if the instantaneous interest rate sufficiently exceeds the growth rate in the price of the asset, borrowers would never be willing to purchase the asset, and a bubble will not emerge in equilibrium.[6]

One difference between raising the cost of funds in this way and imposing high margin requirements is that the former renders speculation unprofitable not just for borrowers but also for those who can afford to buy the asset with their own funds. This is because if r_τ^{FF} exceeds the growth rate of the price of the asset $\dot{p}_t / p_t$, creditors would be better off lending their funds out at the Federal Funds rate than buying the asset at date t and selling it later. Essentially, raising the cost of funds above $\dot{p}_t / p_t$ deters speculation by creating an alternative activity more profitable than speculation: agents can earn a higher return on their wealth lending it out at the Federal Funds rate than they ever could by either speculating themselves or extracting the gains others earn while speculating, and no speculative trade ever takes place. Margin requirements, by contrast, preclude poor agents from speculating but do not make speculation inherently unprofitable. This is why speculation may continue to take place even when margin requirements are temporarily in effect: any agent who buys an asset can still expect to profitably sell it later on when margin requirements are relaxed so long as λ is large, and will therefore be willing to buy the asset. But as the next claim shows, for any value of λ, it will be possible to deter all trade in the asset while high rates are in effect, even if the increase in rates is only temporary:

Claim 11: Suppose $r_t^{FF} > 0$ for $t \in [0, t^*]$ and $r_t^{FF} = 0$ for $t \in (t^*, 1]$. Then in equilibrium, we have

[6] This result should not be confused with the result in Tirole (1985) that a bubble is unsustainable if the riskless interest rate exceeds the growth rate of the economy. In that paper, the price of the bubble necessarily grows at the same rate as the riskless interest rate. A bubble will not emerge if the interest rate exceeds the rate at which income grows, since eventually agents will not have enough income to keep buying the asset. Here, the argument is that a bubble will not exist if the riskless interest rate exceeds the rate at which the price of the bubble grows, and the growth rate of income plays no role.

i. Non-entrepreneurs would be willing to purchase the asset at any date beyond t^*

ii. For any date $\tau < t^*$, there exists a path for r_t^{FF} such that no agent would be willing to buy the asset on or before date τ, regardless of the value of λ.

The interest rate policy underlying Claim 11 involves offering a high return in the Federal Funds market that eclipses what agents could earn from riding the bubble, i.e. which exceeds the growth in the price of the asset. But the model suggests the Fed could discourage speculation even without setting such high rates. Recall that agents in the model purchase the asset either on the off chance that it pays a high dividend at date 1 or to sell it later for a higher price. Suppose we only raised interest rates for a short period of time around date 1, enough to ensure by (19) that an agent who holds the asset to date 1 would have to pay all of his earnings as interest payments if the asset paid off a positive dividend. Prior to this, we would keep the rates at an arbitrarily small level. In this case, no agent would wish to buy the asset as we come close enough to date 1, since the chance of meeting another agent before date 1 falls towards zero and holding the asset until date 1 is unprofitable. But if that case, agents would also refuse to buy the asset earlier: holding the asset to date 1 would not be profitable for them either, and they would not be able to sell the asset given agents in the future refuse to buy it. The bubble would therefore unravel – not because the returns to other investments are more attractive, but because eliminating the incentive for agents to purchase the bubble at its later stages makes it unprofitable to buy it earlier. By contrast, imposing a high initial margin requirement around date 1 will not cause the bubble to unravel. Non-entrepreneurs would remain willing to buy the asset before margin requirements are put into effect even though they expect not to sell the asset, since it would still be profitable to buy the asset on the off chance it pays a high dividend which exceeds the required interest payment under the equilibrium contract. A speculative bubble will still emerge, but will cease trading as soon as margin requirements are introduced.

Formalizing the above argument requires a slight modification to the model. Suppose that although all agents arrive on or before date 1, payoffs on investments are only realized at some date $1+\Delta$ where Δ can be taken to be arbitrarily small. Entrepreneurs who invest one unit of resources before date 1 thus receive R units of output at date $1+\Delta$, and the dividend on the asset accrues at date $1+\Delta$. However, the dividend is revealed at date 1. From this point on, one can show that the unique price at which agents would be willing to buy the asset is the dividend on the asset. In that case, we can establish the following result:

Claim 12: Suppose that $r_t^{FF} = \delta$ where $\delta > 0$ for all $t \in [0,1]$, while for $t \in [1, 1+\Delta]$, the path of r_t^{FF} is such that

$$R_{1,1+\Delta}^{FF} \equiv \exp\left(\int_1^{1+\Delta} r_t^{FF} dt\right) \geq \frac{R_1}{p(0)} \tag{20}$$

Under (A1) and (A2), the probability of trade along the equilibrium path will be zero, i.e. an asset whose price implies a speculative bubble will not be traded.

The reason for the delay between when dividends are revealed and when they are paid can be seen in (20). If we let $\Delta \to 0$, we would only be able to satisfy (20) by letting r_t^{FF} grow arbitrarily large as we approached date 1. A delay ensures that we can impose a large opportunity cost for holding the asset until the date it pays off a dividend regardless of when the asset is bought without requiring an infinite instantaneous rate.

The fact that it is possible to rule out a bubble by increasing rates for a relatively short period is interesting for two reasons. First, if increasing rates is costly, the fact that it is possible to eliminate bubbles by increasing rates for a short time may allow policymakers to deter speculation at a relatively small social cost. For example, one cost of raising rates is that entrepreneurs will be precluded from initiating projects that yield positive social value (since $R > 1$). More generally, we would need a model of the Federal Funds market to determine all of the effects of raising the real Federal Funds rate. However, it is not hard to imagine scenarios in which increasing the equilibrium Federal Funds rate would be costly. For example, suppose that aside from entrepreneurs and speculators, banks lend out their reserves for capital investment. When the Fed raises the real Federal Funds rate, it essentially makes it unprofitable for banks to lend to projects with a low marginal product of capital, and only projects with a high marginal product of capital would be funded, until in equilibrium banks were indifferent between lending to firms and lending at the going Federal Funds rate. Thus, raising rates would preclude not only investment projects by entrepreneurs but projects with a low but positive marginal product of capital. The fact that policymakers can limit their intervention to a relatively short period is therefore encouraging. However, since this intervention is postponed to some date in the future, it requires policymakers to commit to raising rates in the future. Since increasing rates is costly, policymakers may be reluctant to raise rates when the time comes, and promises to do so in the future may not be credible.

Another implication of Claim 12 is that a temporary rate cut need not automatically give rise to bubbles. As long as agents are forward-looking, whether a bubble can be sustained depends not just on current rates but also the path for rates in the future. In this sense, the claim that a temporary rate reduction must fuel a bubble may be misguided. If agents anticipate that currently low rates will be reversed in due time and that rates will eventually return to a level such that it will be unprofitable for agents to speculate on the off chance that the asset yields high dividends, a bubble should never emerge. This caveat is certainly important to keep in mind when evaluating the conduct of Fed policy in the recent past. Various critics have argued that the Fed created or exacerbated a housing bubble by cutting the Federal Funds rate from 2000 to 2003 and raising it back to only gradually over the course of the next two years. However, if agents viewed the rate cut as temporary and expected it would be reversed before housing prices might collapse, and if the eventual target for this rate would have been enough to make speculation unprofitable, neither the low rates in 2003 nor the gradual pace with which rates were raised should have mattered. Although the Fed did announce that its accommodative stance would remain for the "foreseeable future" when it lowered rates, the cut was always presented as a temporary measure that would eventually be reversed. Moreover, housing prices continued to rise at an above-average rate even after the Fed began raising rates, including when it ultimately reached its new target. The model suggests that neither lowering rates less aggressively nor raising rates more quickly would have necessarily eliminated the bubble, although raising the target to an even higher level might have done so.

6 Conclusion

In light of the dramatic increase and decrease in asset prices in the past decade, various pundits have criticized the Fed for fueling asset bubbles. One critique holds that the Fed's response to the 2001 recession by easing credit conditions fed into a housing bubble during the subsequent years, and the slow pace with which the Fed subsequently raised rates sustained this bubble. Another critique faults the Fed in its regulatory capacity for not preventing financing arrangements that lured in speculators and drove up asset prices. To investigate these claims, this paper constructed a model in which credit plays an essential role in allowing speculative bubbles to emerge. Yet rather than pointing to the Fed as the culprit, the model raises questions as to whether these policies were necessarily responsible for creating bubbles.

One implication of the model is that in a period of significant technological change that carries with it a hope of both great profits and great uncertainty as to which firms will be the ones to capture these profits, speculation may be unavoidable. This is because profits to successful firms are so high that without equally high interest rates, agents would always be willing to borrow and invest in such firms on the off chance that a particular firm is successful. By contrast, if the firm is unsuccessful, it is creditors who will incur the loss under the equilibrium contract. To the extent that the rise in stock prices for internet firms in the late 1990s corresponded to a true speculative bubble, such a bubble would have emerged without any preceding reduction in rates or easing of monetary policy. In response to this bubble, it might have been perfectly reasonable for creditors to loans with lucrative short-term teaser rates. The reason lenders offer such contracts is to induce speculators to turn over their investments more quickly than they would otherwise, thereby minimizing the risk exposure of creditors. So long as traders believed that the profits of successful internet firms would be fantastically high, there is little the Fed could have done to abate speculative trades without choking off credit altogether, and nothing it should have done to preclude the types of credit arrangements that emerged in the wake of this speculation.

Indeed, much of the focus of the criticism at the Fed concerns the actions of the Fed following the decline of stock prices in the mid 2000s rather than its actions during the run-up in stock prices. In response to deteriorating economic conditions, the Fed lowered rates to unprecedentedly low levels. Concomitant with this reduction was an acceleration in housing prices. Since the notion that investing in the right housing tract could yield fantastically high dividends seems less plausible than the notion that investing in the right startup firm could, the case against monetary policy for this particular episode seems stronger *a priori*. Yet the model raises some doubts on this view as well. First, to the extent that the target rate the Fed eventually settled on would have been enough to discourage speculation, a temporary rate reduction need not have given rise to a bubble that would not have occurred otherwise. This is because if agents anticipated that the Fed would eventually reach this target, whether rates were cut initially or not would not have made it profitable for agents to commence speculation when they know that eventually no buyer would be able to find a buyer for the assets they purchase. The model further suggests that the slow pace at which the Fed raised rates should have been immaterial for the sustainability of speculation. Again, once the Fed announced its intention to increase rates, so long as investors expected it to arrive at its new target before the bubble would become unsustainable, they would not have wished to trade speculatively in the asset. Although the Fed might be culpable for not having set a higher target rate that would have made a speculative bubble unsustainable, proceeding more quickly to the target it did set might not have mattered.

While the model offers some new insights on whether various policies facilitate or can be used to avoid bubbles, it suffers from some limitations. First, the model has the odd feature that agents know precisely when the bubble will (most likely) collapse. A more realistic model would allow the date at which dividends are revealed to be random. In addition, while the model shows that there are policies the Fed could pursue that can put an end to speculative trade in assets, it does not explore whether such intervention would be desirable. Exploring welfare turns out to be somewhat complicated. First, even if the policy needed to eliminate the bubble carried no cost, the welfare effects of bursting a bubble are subtle. Indeed, as the model is specified, a bubble merely redistributes income from some agents to others, so bursting a bubble does not generate a Pareto improvement. More strikingly, some of the models of bubbles mentioned in the Introduction suggest that bursting bubbles may be Pareto worsening. Hence, it is not obvious that bursting a bubble would be desirable even if it could be achieved at no cost. Second, even if we enriched the model so that bursting a bubble could be associated with a Pareto improvement, the question remains as to whether these gains are enough to offset the costs of the policy intervention that would be required in order to rein in the bubble. I tackle some of these welfare questions in a companion paper.

Appendix A: Proofs

Claim 1: In equilibrium, an agent who does nothing or who holds on to an asset which pays no dividends at date 1 will have have zero terminal wealth, i.e. $x_t^1(\widehat{\omega}, \widehat{a}, \widehat{y}) = x_t^0(\widehat{\omega}) + y$ if $y \leq 0$.

Proof of Claim 1: Suppose the agent does nothing. Since agents reveal their types truthfully in equilibrium, he would reveal himself to be type $(\omega, \varnothing, 0)$ at date 1. Suppose an agent who made this announcement were left with positive terminal wealth. Since $y = 0$, this would imply $x_t^0(\omega) > x_t^1(\omega, \varnothing, 0)$. But this contradicts (IC-3), which holds that $x_t^0(\widehat{\omega}) \leq x_t^1(\widehat{\omega}, \widehat{a}, \widehat{y})$ if terminal wealth is positive. Next, suppose an agent holds on to an asset which pays no dividend. In equilibrium, the agent would truthfully reveal his type $(\omega, 1, -p(t))$. Since his wealth is nonnegative, $x_t^0(\omega) - p(t) \geq x_t^1(\omega, 1, -p(t))$. But since $p(t) > 0$, then $x_t^1(\omega, 1, -p(t)) < x_t^0(\omega)$. (IC-3) then implies $x_t^1(\widehat{\omega}, \widehat{a}, \widehat{y}) = x_t^0(\omega) + y$, as claimed. ■

Claim 2: Let $\epsilon \to 0$. Then agents will be able to buy the asset under the equilibrium contract, i.e. there exists a $\widehat{\omega}$ such that $x_t^0(\widehat{\omega}) \geq p(t)$.

Proof: Suppose not, i.e. $x_t^0(\widehat{\omega}) < p(t)$ for all $\widehat{\omega}$. Agents would then be unable to purchase the asset or invest, and it follows from Claim 1 that agents have zero terminal wealth. Suppose one of the creditors were to offer the following contract $\widetilde{x}$:

$$\begin{aligned} \widetilde{x}_t^0(e) &= \widetilde{x}_t^0(n) \;=\; 1 \\ \widetilde{x}_t^1(\widehat{\omega}, \widehat{a}, \widehat{y}) &= \min\left(\frac{1+\varepsilon}{1-\phi}, 1+y\right) \text{ for all } (\widehat{\omega}, \widehat{a}, \widehat{y}) \end{aligned} \tag{A.1}$$

where $\varepsilon > 0$ is arbitrarily small. Since $\widetilde{x}_t^0(\widehat{\omega}) \geq 1$, an entrepreneur who accepts this contract will be able to invest in the project or purchase the asset. If he invests, his profits will equal

$$\widetilde{x}_t^0(e) + y - \widetilde{x}_t^1(e, t, R-1) = R - \frac{1+\varepsilon}{1-\phi}$$

Given our maintained hypothesis that $(1-\phi)(R-1) - \phi > 0$, profits will be positive for ε sufficiently small. Hence, an entrepreneur would strictly prefer this contract to the original equilibrium contract. In addition, for sufficiently small ϵ, the entrepreneur will strictly prefer investing in the project to buying the asset. To see this, suppose he bought the asset. If he held it until date 1, his expected profit would equal

$$\epsilon\left(R_1 + (1 - p(t)) - \frac{1+\varepsilon}{1-\phi}\right)$$

which goes to zero as $\epsilon \to 0$. If he instead sold the asset prior to date 1, the most he could earn is

$$1 + \lim_{s\to 1} p(s) - p(t) - \frac{1+\varepsilon}{1-\phi}$$

Given assumption (4), it follows that $R - 1 \geq \lim_{s\to 1} p(s) - p(t)$, so investing in the project is more profitable than purchasing the asset and selling it could ever be. Next, I argue that non-entrepreneurs will also opt for this contract. In particular, they could always guarantee themselves positive expected profits by buying the asset and holding it to maturity, which would net them

$$\begin{aligned}\epsilon\left(R_1+(1-p(t))-\frac{1+\varepsilon}{1-\phi}\right) &\geq \epsilon\left(R+(1-p(t))-\frac{1+\varepsilon}{1-\phi}\right)\\ &\geq \epsilon\left(R-\frac{1+\varepsilon}{1-\phi}\right)\end{aligned}$$

where recall that the last expression will be positive for small ε. Since the creditor loses at most 1 unit to non-entrepreneurs who engage in speculation, and since the fraction of agents who are non-entrepreneurs is at most ϕ, expected profits to the creditor are bounded below by

$$(1-\phi)\frac{\phi+\varepsilon}{1-\phi}-\phi=\varepsilon>0$$

Hence, there exists a contract that agents strictly prefer and which delivers positive expected profits to creditors. The original contract must therefore not have been an equilibrium. ■

Claim 3: Let $\epsilon \to 0$. Then non-entrepreneurs will buy the asset under the equilibrium contract.

Proof: Suppose not. Then it follows that non-entrepreneurs do nothing, in which case by Claim 1 we know they will have zero wealth. Since by Claim 2 they must be able to buy the asset, incentive compatibility requires that buying the asset should yield them zero profits under the equilibrium contract. Consider any $\widehat{\omega}$ for which $x_t^0(\widehat{\omega}) \geq p(t)$. If the agent announces $\widehat{\omega}$, then, he could afford to buy the asset. We already established that an agent who buys the asset will be found out if he announced $a = \varnothing$, but he can announce other actions without being caught. We therefore need to make sure that he would earn zero profits regardless of what he announces in period 1, i.e.

$$\begin{aligned}x_t^1(\widehat{\omega},1,R_1-p(t))-x_t^0(\widehat{\omega}) &\geq R_1-p(t)\\ x_t^1(\widehat{\omega},1,-p(t))-x_t^0(\widehat{\omega}) &\geq R_1-p(t)\\ x_t^1(\widehat{\omega},t,R-1)-x_t^0(\widehat{\omega}) &\geq R_1-p(t)\\ x_t^1(\widehat{\omega},s,p(s)-p(t))-x_t^0(\widehat{\omega}) &\geq p(s)-p(t)\end{aligned}$$

These conditions are sufficient, since $R_1 \geq R \geq 1 \geq \lim_{s\to 1} p(s)$ implies that if an agent earns zero profits from holding on to the asset he will also earn zero profits from selling it at some date s. Since an agent who announces that he sold the asset can be identified, for that announcement the only requirement is that his required transfer must exceed $p(s)-p(t)$. Given these restrictions on transfers, an entrepreneur who invests in the project must also earn zero profits, whether he invests in the project, buys the asset, or does nothing. But if all agents earn zero profits, both types would strictly prefer the alternative contract (A.1) introduced in the proof of Claim 2, and it would yield positive profits to the creditor who offers it. So the original contract could not have been an equilibrium. ■

Claim 4: Let $\epsilon \to 0$. Then $x_t^0(e) \geq 1$ under the equilibrium contract and entrepreneurs will invest in the project in equilibrium.

Proof: Suppose not. Then an entrepreneur will either do nothing or purchase the asset. From Claim 3, we know that in equilibrium non-entrepreneurs earn positive expected profits from buying the asset. So, if there is an equilibrium in which type e agents do not invest in the project, they will buy the asset when given the opportunity. It then follows from Claim 3 that all agents will purchase the asset.

The proof relies on showing that there exists a contract $\widehat{x}$ that only entrepreneurs would prefer to the original contract and which yields positive expected profits to creditors. The idea is to offer a contract that allows entrepreneurs to invest in the project but demand a higher transfer payment in return. Since entrepreneurs earn more from investing in a project than from buying the asset, they would prefer this new contract. Non-entrepreneurs, by contrast, avoid this contract because it demands a higher transfer. Since creditors earn positive profits from lending exclusively to entrepreneurs, the original contract could not have been an equilibrium. Formally, I first argue that there exists a contract $\widetilde{x}$ of the form

$$\begin{aligned} \widetilde{x}_t^0(e) &= \widetilde{x}_t^0(n) = p(t) \\ \widetilde{x}_t^1(\widehat{\omega},\widehat{a},\widehat{y}) &= \min\left(p(t)+r^*, p(t)+y\right) \text{ for all } (\widehat{\omega},\widehat{a},\widehat{y}) \in \Omega(\widehat{\omega},\omega,a,y) \end{aligned} \tag{A.2}$$

that yields the same expected profits to agents as the original contract, denoted V, where

$$V = E\left[\max_{a_t(\tau)\in A_\tau(\omega,\omega)} \left\{x_t^0(\omega) + y - x_t^1(\omega,a,y)\right\} \,\middle|\, a_t(t) = 1\right]$$

I then argue that we can find a sufficiently small ε such that a new contract $\widehat{x}$ of the form

$$\begin{aligned} \widehat{x}_t^0(e) &= x_t^0(n) = 1 \\ \widehat{x}_t^1(\widehat{\omega},\widehat{a},\widehat{y}) &= \min\left(1+r^*+\varepsilon, 1+y\right) \text{ for all } (\widehat{\omega},\widehat{a},\widehat{y}) \in \Omega(\widehat{\omega},\omega,a,y) \end{aligned} \tag{A.3}$$

will be strictly preferred by entrepreneurs to $\widetilde{x}$, and thus to the original contract x, but non-entrepreneurs prefer the original contract. Lastly, I show the creditor offering contract $\widehat{x}$ will earn positive expected profits.

To prove there exists an r^* such that (A.2) yields an expected profit of V to a trader, I first derive bounds for V, and then show we can choose r^* to achieve any value within the derived bounds. Consider the expression $x_t^0(\omega) + y - x_t^1(\omega,a,y)$. If $y \le 0$, we know from Claim 1 that $x_t^0(\omega) + y - x_t^1(\omega,a,y) = 0$. If $y > 0$, (IC-3) implies that $x_t^1(\omega,a,y) \ge x_t^0(\omega)$, so $x_t^0(\omega) + y - x_t^1(\omega,a,y) \le y$. In the opposite direction, agents cannot be left with negative wealth, i.e. $x_t^0(\omega) + y - x_t^1(\omega,a,y) \ge 0$. Hence, for $y \ge 0$, the contract must stipulate

$$0 \le x_t^0(\omega) + y - x_t^1(\omega,a,y) \le y$$

The first inequality implies $V \ge 0$. To derive an upper bound on V, I use the fact that a contract that maximizes $x_t^0(\omega) + y - x_t^1(\omega,a,y)$ at each $y \ge 0$ must also maximize the expected payoff to the agent who buys the asset. Hence, the maximum expected payoff occurs if $x_t^0(\omega) + y - x_t^1(\omega,a,y) = y$ for all $y \ge 0$. To compute a value for expected utility in this case, we need the optimal action of the agent. From Claim 3, we know that in equilibrium, agents will prefer to purchase the asset in equilibrium over doing nothing. Since entrepreneurs and non-entrepreneurs alike invest in the asset, the number of potential buyers on an island will be distributed over $\{M, M+1, ...\}$, and the probability that the total number is equal to some m in this set will be given by $\dfrac{\lambda^{m-M}e^{-\lambda}}{(m-M)!}$. So long as fewer than M traders have arrived, the original owner of the asset will be better off waiting to sell it at a higher price. Define $m(s)$ as the number of traders who arrived by date s, and let $Q(s,n)$ denote the probability that at least one trader arrives if n traders already showed up by date s, i.e. $Q(s,n) = \Pr(m(1) \geq n+1 \mid m(s) = n)$. Then we have

$$\begin{aligned} Q(s,n) &= \Pr(m(1) \geq n+1 \mid m(s) = n) \\ &= \frac{\Pr(m(1) \geq n+1 \cap m(s) = n)}{\Pr(m(s) = n)} \\ &= \frac{\sum_{k=n+1}^{\infty} \frac{\lambda^{k-M}e^{-\lambda}}{(k-M)!} \frac{(k-M)!}{n!(k-M-n)!} s^n (1-s)^{k-n}}{\sum_{k=n}^{\infty} \frac{\lambda^{k-M}e^{-\lambda}}{(k-M)!} \frac{(k-M)!}{n!(k-M-n)!} s^n (1-s)^{k-n}} \\ &= \frac{\lambda^n e^{-\lambda} s^n / n!}{\lambda^n e^{-\lambda} t^n / n!} \frac{e^{\lambda(1-s)} - 1}{e^{\lambda(1-s)}} \\ &= 1 - e^{-\lambda(1-s)} \end{aligned}$$

Hence, $Q(s,n)$ is independent of n, i.e. $Q(s,n) = Q(s)$. Next, let $F(x|n,s)$ denote the probability that the first arrival time after date s given n traders arrived by date s and there are at least $n+1$ traders occurs before date x. Then we have

$$\begin{aligned} 1 - F(x|s,n) &= \Pr(t_{n+1} \geq x \mid m(s) = n \cap m(1) \geq n+1) \\ &= \frac{\Pr(t_{n+1} \geq x \cap m(s) = n)}{\Pr(m(s) = n \cap m(1) \geq n+1)} \\ &= \frac{\sum_{k=n+1}^{\infty} \frac{e^{-\lambda}\lambda^{k-M}}{(k-M)!} \frac{(k-M)!}{n!(k-M-n)!} s^n (1-x)^{m-n}}{\sum_{k=n+1}^{\infty} \frac{e^{-\lambda}\lambda^{k-M}}{(k-M)!} \frac{(k-M)!}{n!(k-M-n)!} s^n (1-s)^{m-n}} \\ &= \frac{e^{\lambda(1-x)} - 1}{e^{\lambda(1-s)} - 1} \end{aligned}$$

which again is independent of n. Differentiating with respect to x yields the conditional probability density function of the first arrival time beyond s:

$$f(x|s) = \frac{\lambda e^{-\lambda(x-s)}}{1 - e^{-\lambda(1-s)}}$$

Now, suppose $x_t^0(\omega) + y - x_t^1(\omega, a, y) = y$ for all $y \geq 0$. Define $W(t,s)$ as the expected profits under this contract for an agent who bought the asset at date t, did not sell the asset up to date s, and chooses what to do with the asset optimally thereafter. Then $W(t,s)$ solves the following integral equation:

$$W(t,s) = Q(s) \int_s^1 \max(W(t,\tau), p(\tau) - p(t)) f(\tau|s) \, d\tau + (1 - Q(s)) \epsilon (R_1 - p(t))$$

I next argue that the optimal strategy for the agent will be a cutoff rule, i.e. the agent will sell the asset from some date s_t^* on, where $s_t^* \in (t, 1]$. Showing that the trader will choose to follow a cutoff rule is equivalent to showing that if $W(t,s) \geq p(s) - p(t)$, then $W(t,s') \geq p(s') - p(t)$ for all $s' < s$. Suppose instead that $W(t,s') < p(s') - p(t)$ for some $s' < s$. At date s', the agent always has the option of holding on to the asset until date s and proceeding optimally thereafter. This implies $W(t,s') \geq W(t,s)$. But then

$$p(s') - p(t) > W(t,s') \geq W(t,s) \geq p(s) - p(t).$$

This contradicts the fact that $p(s)$ is non-decreasing. Hence, agents who buy the asset at date t will hold on to it until some cutoff date s_t^*, and will sell it to the first trader who arrives on or after s_t^*. Hence, the expected value from buying the asset if $x_t^0(\omega) = x_t^1(\omega, a, y)$ for all $y \geq 0$ is just

$$\overline{V} = Q(s_t^*) \int_{s_t^*}^1 [p(\tau) - p(t)] f(\tau | s_t^*) d\tau + [1 - Q(s_t^*)] \epsilon (R_1 - p(t))$$

Since this is the most favorable contract the agent can receive, $\overline{V}$ forms on upper bound for V.

I now argue that for any $V \in [0, \overline{V}]$, there exists an r^* for which the contract $\widetilde{x}$ defined by (A.2) yields an expected utility equal to V. First, note that if we set $r^* = R_1 - p(t)$, the contract will always leave agents with zero terminal wealth. Next, if we set $r^* = 0$, the contract will be identical to the one I just argued yields a value of $\overline{V}$ to the trader. If I can show that expected profits to the trader are continuous in r^*, it would follow by the intermediate value theorem that for any $V \in \left[0, \overline{V}\right]$ there exists an $r^* \in (0, R_1 - p(t))$ such that the expected profits to the agent equal V. Let $W(t, s; r^*)$ denote the value of waiting at date s for an agent who bought the asset at date t and who faces the contract (A.2). Once again, $W(t, s; r^*)$ must satisfy the integral equation

$$W(t,s;r^*) = Q(s) \int_s^1 \max(W(t,\tau;r^*), p(\tau) - p(t) - r^*, 0) f(\tau|s) d\tau + (1 - Q(s)) \epsilon (R_1 - p(t) - r^*)$$

It follows that $W(t, s; r^*)$ is continuous (and even differentiable) in r^*. Hence, we can always find a contract of the form in (A.2) that yields the same value to an agent as the equilibrium contract.

Finally, suppose a creditor were to offer the contract $\widehat{x}$ defined by (A.3). In the limit as $\epsilon \to 0$, the amount an entrepreneur could earn under contract $\widetilde{x}$ defined by (A.2) is bounded above by $\lim_{s \to 1} p(s) - p(t) - r^*$. By contrast, under contract $\widehat{x}$ defined by (A.3) he could earn $R - 1 - r^* - \varepsilon$. Under the maintained hypothesis that $R - 1 > 1 \geq \lim_{s \to 1} p(s) - p(t)$, there exists an ε small enough such that the entrepreneur will strictly prefer (A.3) to (A.2) and hence to the original equilibrium contract. Non-entrepreneurs, however, will strictly prefer the original contract, since the expected profits under a contract of type (A.2) are decreasing in r^*. Hence, the expected profits to a creditor who offers contract (A.3) are $r^* + \varepsilon > 0$, suggesting the original contract could not have been an equilibrium. ■

Claim 5: Under the equilibrium contract, expected profits to the creditor must be zero.

Proof: Suppose not, i.e. a creditor expects to earn strictly positive profits in equilibrium. From Claims 3 and 4, we know that in equilibrium entrepreneurs will invest in the project and non-entrepreneurs will purchase the asset if it is up for sale. Let $\Sigma(t) \equiv \left\{ s \geq t \mid W(t,s) \leq p(s) - p(t) + x_t^0(n) - x_t^1(n, s, p(s) - p(t)) \right\}$ denote the set of dates at which a non-entrepreneur would (weakly) prefer to sell the asset under the equilibrium contract. Suppose first that the net expected non-negative transfers from those who announce themselves to be non-entrepreneurs is strictly positive, i.e. either

$$x_t^1(n, 1, R_1 - p(t)) > x_t^0(n)$$

or else

$$E\left[x_t^1(n, s, p(s) - p(t)) \mid s \in \Sigma(t)\right] > x_t^0(n).$$

Consider an alternative contract $\widetilde{x}$ where $\widetilde{x}_t^0(n) = x_t^0(n)$ but which offered slightly more favorable terms to non-entrepreneurs, i.e. either

$$\widetilde{x}_t^1(n, 1, R_1 - p(t)) = x_t^1(n, 1, R_1 - p(t)) - \varepsilon/\epsilon$$

or, if $x_t^1(n, 1, R_1 - p(t)) = x_t^0(n)$, then

$$E\left[\widetilde{x}_t^1(n, s, p(s) - p(t)) \mid s \in \Sigma(t)\right] = E\left[x_t^1(n, s, p(s) - p(t)) \mid s \in \Sigma(t)\right] - \varepsilon$$

for some small $\varepsilon > 0$. Since the original contract must have been incentive compatible, it follows that the net transfer of entrepreneurs is strictly positive, i.e.

$$x_t^1(e, t, R - 1) - x_t^0(e) > 0$$

If it were not positive, non-entrepreneurs would prefer to pass themselves off as entrepreneurs and take on a zero interest contract. As shown in Claim 4, this yields a payoff of $\overline{V}$, which is the upper bound on how much an agent can earn, whereas if expected non-negative transfers are positive, the utility to the agent from acting optimally will be below $\overline{V}$. Hence, for ε small enough, we could reduce $x_t^1(e, t, R - 1)$ by ε and still exceed $x_t^0(e)$. Thus, we set

$$\begin{aligned} \widetilde{x}_t^0(e) &= x_t^0(e) \\ \widetilde{x}_t^1(e, t, R - 1) &= x_t^1(e, t, R - 1) - \varepsilon \end{aligned}$$

Both parties will strictly prefer accepting contract $\widetilde{x}$ and telling the truth to accepting contract x and telling the truth. Moreover, since the original equilibrium contract must be incentive compatible and I subtract the same amount from the expected payoff of both types, non-entrepreneurs would continue to prefer telling the truth under contract $\widetilde{x}$ than passing themselves off as entrepreneurs. Finally, since entrepreneurs prefer to invest under the original contract, but expected payoffs to both investing and purchasing the asset are ε higher under contract $\widetilde{x}$, they would continue to prefer investing in the project under contract $\widetilde{x}$. Since profits to the creditor under the original contract were strictly positive, they will remain positive under the new contract for ε small enough. But then original contract could not have been an equilibrium.

Next, suppose $x_t^1(n, 1, R_1 - p(t)) = E\left[x_t^1(n, s, p(s) - p(t)) \mid s \in \Sigma(t)\right] = x_t^0(n)$. This is equivalent to giving them a contract of type (A.2) with $r^* = 0$. Since there will be some non-entrepreneurs who fail to sell an asset which proves to be worthless, the only way for the creditor to earn nonzero expected profits is if $x_t^1(e, t, R-1) > x_t^0(e)$, i.e. if entrepreneurs transferred positive resources to the creditor. Suppose a creditor offers the same contract but sets $\widetilde{x}_t^1(e, t, R-1) = x_t^1(e, t, R-1) - \varepsilon$ for some arbitrarily small ε that ensures $\widetilde{x}_t^1(e, t, R-1)$ is positive. Non-entrepreneurs will prefer their original contract, since pretending to be an entrepreneur under this contract is equivalent to a contract of type (A.2) with $r^* = x_t^1(e, t, R-1) - \varepsilon > 0$, while their original contract was equivalent to a contract of type (A.2) with $r^* = 0$. Entrepreneurs will prefer this alternative contract to the original equilibrium contract, and the expected profits to the creditor are $x_t^1(e, t, R-1) - \varepsilon > 0$, implying the original contract could not have been an equilibrium. ■

Claim 6: Under the equilibrium contract, $r_t^e = x_t^1(e, t, R-1) - x_t^0(e) > 0$.

Proof: Suppose not. Recall that an agent who reports an outcome that is not possible is assumed to be left with zero wealth. Given this, if a non-entrepreneur pretended to be an entrepreneur, the fact that $r_t^e = 0$ implies their expected utility would be the same as under a contract of type (A.2) with $r^* = 0$. Incentive compatibility requires that non-entrepreneurs prefer to disclose their information truthfully. But if the equilibrium contract set $x_t^1(n, \widehat{a}, \widehat{y}) > x_t^0(n)$ for any (a, y), the agent would be better off pretending to be an entrepreneur. Thus, if $r_t^e = 0$, then $x_t^1(n, \widehat{a}, \widehat{y}) = x_t^0(n)$ for any $\widehat{y} > 0$. Expected profits for the creditor are then equal to

$$-(1-\epsilon)\left[1 - Q(s_t^*)\right] p(t)$$

where s_t^* denotes the date at which an agent who bought the asset at date t would first sell the asset if facing a contract of type (A.2) with $r^* = 0$. Since $s_t^* > t \geq 0$ and $Q(x) < 1$ for $x > 0$, this expression is negative. But creditors must earn non-negative profits in equilibrium, so the original contract could not have been an equilibrium. ■

Claim 7: In equilibrium, the incentive constraint for type n will be binding, i.e. $V(n, n) = V(n, e)$

Proof: Suppose not, i.e. an agent strictly prefers to announce n than to announce e. From Claim 6, we know $x_t^1(e, t, R-1) - x_t^0(e) > 0$. Consider a contract $\widetilde{x}$ which offers offers slightly better terms to those who announce themselves to be entrepreneurs, i.e.

$$\widetilde{x}_t^1(e, t, R-1) - \widetilde{x}_t^0(e) = x_t^1(e, t, R-1) - x_t^0(e) - \varepsilon.$$

Since $V(n, n) > V(e, e)$, we can choose ε small enough so that non-entrepreneurs still prefer to reveal themselves truthfully under the original contract than to take the new contract and misrepresent themselves as entrepreneurs. In addition, suppose $\widetilde{x}$ offers identical terms to non-entrepreneurs or treats them worse by demanding a higher net transfer. Thus, we can assume that only entrepreneurs will be attracted to the new contract $\widetilde{x}$, and for ε small enough the expected profits to the creditor who offers it will be $x_t^1(e, t, R-1) - x_t^0(e) - \varepsilon$ which is strictly positive. Hence, the original contract could not have been an equilibrium. ■

Lemma: The contract that provides non-entrepreneurs with utility $V_0\left(r_t^e\right)$ at the lowest cost to creditors will induce speculators to trade as if they bought the asset with their own funds.

Proof: Given a value for $r_t^e \equiv x_t^1(e,t,R-1) - x_t^0(e)$, we can use the equation for $W\left(t,s;r_t^e\right)$ in the proof of Claim 4 evaluated at $s=t$ to compute $V(n,e)$, the expected utility to a non-entrepreneur who pretends to be an entrepreneur. Denote this value by $V_0\left(r_t^e\right)$. Consider the problem of choosing transfers $x_t^1(n,a,y)$ for $y \geq 0$ in order to maximize the expected profits of the creditor while keeping the expected utility of the non-entrepreneur equal to $V_0\left(r_t^e\right)$. This problem can be stated as follows. Define $\Sigma(t)$ as the set of dates under the contract x_t at which the agent would prefer to sell the asset. Let $Z(t)$ denote the probability that $\exists t_n \in \Sigma(t)$, and let $h(s)$ denote the distribution of the first arrival time in the set $\Sigma(t)$ conditional on at least one arrival. Then the creditor would choose $x_t^1(n,a,y)$ for $y \geq 0$ to maximize his expected payoff,

$$\max_{x_t^1(n,a,y)} \begin{array}{r} (1-\phi_t)\, r_t^e + \phi_t Z(t) \int_{\Sigma(t)} \left[x_t^1(n,s,p(s)-p(t)) - p(t)\right] h(s)\, ds + \\ \phi_t (1-Z(t)) \left(\epsilon x_t^1(n,1,R_1-p(t)) - p(t)\right) \end{array} \tag{A.4}$$

subject to the constraints

1. $Z(t) \int_{\Sigma(t)} \left[p(s) - x_t^1(n,s,p(s)-p(t))\right] h(s)\, ds + (1-Z(t))\, \epsilon \left(R_1 - x_t^1(n,1,R_1-p(t))\right) = V_0\left(r_t^e\right)$
2. $x_t^0(n) \leq x_t^1(n,a,y) \leq x_t^0(n) + y$

If we substitute the first constraint into the objective function, we can rewrite this problem as

$$\max_{x_t^1(n,a,y)\in[x_t^0(n),x_t^0(n)+y]} (1-\phi_t)\, r_t^e + \phi_t \left\{ Z(t) \int_{\Sigma(t)} [p(s)-p(t)]\, h(s)\, ds + (1-Z(t))(\epsilon R_1 - p(t)) - V_0\left(r_t^e\right) \right\}$$

Since $V_0\left(r_t^e\right)$ and r_t^e are constants, this problem identical to solving

$$\max_{x_t^0(n) \leq x_t^1(n,a,y) \leq x_t^0(n)+y} Z(t) \int_{\Sigma(t)} [p(s)-p(t)]\, h(s)\, ds + (1-Z(t))(\epsilon R_1 - p(t)) \tag{A.5}$$

The choices for $x_t^1(n,a,y)$ thus do not enter the objective function in (A.5) directly, but through their effect on the set $\Sigma(t)$, which in turn determines $Z(t) = \Pr(\exists t_n \in \Sigma(t))$ and $h(s)$. Choosing $\Sigma(t)$ is equivalent to the problem of an agent who buys the asset with his own funds and must decide when to sell it. ■

Claim 8: In equilibrium, the contract in (10) and (11) comes closest to replicating the trading strategy of an agent who buys the asset with his own funds among all contracts that deliver utility $V_0\left(r_t^e\right)$.

Proof: I begin by solving the creditor's problem from the proof of Claim 8. Define $\Pi(t,s)$ as the value for an agent who bought the asset at date t with his own funds, opts to wait at date s, and acts optimally thereafter. $\Pi(t,s)$ is thus analogous to the value $W(t,s)$ for an agent who purchases the asset with borrowed funds. The function $\Pi(t,s)$ satisfies a related integral equation to $W(t,s)$:

$$\Pi(t,s) = Q(s) \int_s^1 \max\left(\Pi(t,\tau), p(\tau) - p(t)\right) f(\tau|s)\, d\tau + (1-Q(s))(\epsilon R_1 - p(t))$$

Note the similarity to the problem of an agent who borrows funds but faces a zero-interest contract; the equation differs only in the last term, which involves $\epsilon R_1 - p(t)$ rather than $\epsilon R_1 - \epsilon p(t)$. Note also that by setting $p(t) = 0$, we capture the problem of the original owner who must decide when to sell the asset he was endowed with.

Once again, the optimal trading strategy involves a cutoff rule: an agent who bought the asset at date t would agree to sell the asset from some date σ_t on. To show this, it will again suffice to show that if $\Pi(t,s) \geq p(s) - p(t)$, then $\Pi(t,s') \geq p(s') - p(t)$ for all $s' < s$. Suppose instead that $\Pi(t,s') < p(s') - p(t)$ for some $s' < s$. At date s', the agent always has the option of holding on to the asset until date s and proceeding optimally thereafter. This implies $\Pi(t,s') \geq \Pi(t,s)$. But then

$$p(s') - p(t) > \Pi(t,s') \geq \Pi(t,s) \geq p(s) - p(t)$$

which contradicts the fact that $p(s)$ is assumed to be increasing.

We can thus rewrite (A.5) as choosing σ_t to maximize the expected payoff

$$Q(\sigma_t) \int_{\sigma_t}^{1} p(x) f(x|\sigma_t)\, dx + (1 - Q(\sigma_t))\, \epsilon R_1 - p(t) \tag{21}$$

Since $p(t)$ is a constant, it has no effect on the choice of σ_t. Thus, the optimal strategy would involve selling the asset from the same date σ^* on regardless of $p(t)$.

Since (A.5) coincides with the problem the original owner faces as to when to sell the asset, it follows that the original owner would sell the asset from date σ^* on. Hence, non-entrepreneurs can only buy the asset from date σ^* on. This implies that the unconstrained optimal contract a creditor would offer would induce the agent to sell the asset to the first arriving buyer. But the constraints on $x_t^1(n,a,y)$ turn out to binding. I now argue that because of these constraints, the equilibrium contract that emerges will be the maximally backloaded contract that yields a utility of $V_0(r_t^e)$ to the agent. Formally, the equilibrium contract will be the one that solves

$$\max_{x_t^0(n) \leq x_t(n,a,y) \leq x_t^0(n)+y} \sup\left\{s : x_t^1(n,s,p(s)-p(t)) = x_t^0(n)\right\}$$

subject to providing the agent with a utility of $V_0(r_t^e)$. Define $T_t \equiv \sup\left\{s : x_t^1(n,s,p(s)-p(t)) = x_t^0(n)\right\}$ under this contract. If $T_t < 1$, the contract will specify

$$x_t^1(n,a,y) = \begin{cases} x_t^0(n) + y & \text{if } a \geq T_t \\ x_t^0(n) & \text{else} \end{cases}$$

whereas if $T_t = 1$, the contract will specify

$$x_t^1(n,a,y) = \begin{cases} \in \left[x_t^0(n) + r_t^e, R_1\right] & \text{if } a = 1 \\ x_t^0(n) & \text{else} \end{cases}$$

For expositional convenience, let us refer to the maximally backloaded contract as x^*, and the set of dates during which an agent facing this contract would be willing to sell the asset by $\Sigma^*(t)$. It is easy to show that the optimal strategy given x^* is a cutoff rule, and so $\Sigma^*(t) = (s_t^*, 1)$ for some $s_t^* \leq T_t$. I now argue the contract x^* satisfies the following property. Pick any contract $x \neq x^*$ that yields the agent an expected utility of $V_0(r_t^e)$, and let $\Sigma(t;x)$ denote the set of dates at which an agent would choose to sell the asset. Then $\Sigma(t;x) \subset \Sigma^*(t)$. Thus, any date s that an agent would agree to sell the asset under some contract that gives the agent utility $V_0(r_t^e)$ is a date that he would agree to sell it if he faced the contract x^*.

To see this, pick any $s \in \Sigma(t;x)$ where x is a contract that yields the relevant utility. If $s \geq T_t$, the backloaded contract will guarantee zero terminal wealth no matter what the agent does from time s on, and so an agent facing x^* will be willing to sell the asset. Suppose then that $s < T_t$. By definition, at this date it must be the case that

$$p(s) - p(t) + x_t^0(n) - x_t^1(n, s, p(s) - p(t)) \geq W(t, s; x)$$

where $W(t,s;x)$ denotes the value of waiting given the contract x. We wish to compare $W(t,s;x)$ and the value to continuing under the weakly backloaded contract, $W^*(t,s)$. Using the expressions for $Q(s)$ and $f(x|s)$, we have

$$\begin{aligned}
W(t,s;x) &= Q(s)\int_s^1 \max\left[W(t,\tau;x), p(\tau) - p(t) + x_t^0(n) - x_t^1(n,\tau,p(\tau)-p(t))\right] f(\tau|s)\,d\tau \\
&\quad + (1-Q(s))\,\epsilon(R_1 + x_1) \\
&= \int_s^1 \max\left[W(t,\tau;x), p(\tau) - p(t) + x_t^0(n) - x_t^1(n,\tau,p(\tau)-p(t))\right] \lambda e^{-\lambda(\tau-s)}d\tau \\
&\quad + e^{-\lambda(1-s)}\epsilon(R_1 - p(t)) \\
&= e^{\lambda s}\left[\int_s^1 \max\left[W(t,\tau;x), p(\tau) - p(t) + x_t^0(n) - x_t^1(n,\tau,p(\tau)-p(t))\right] \lambda e^{-\lambda\tau}d\tau + e^{-\lambda}\epsilon(R_1 - p(t))\right]
\end{aligned}$$

If we differentiate this expression with respect to s, we get

$$\frac{\partial W(t,s;x)}{\partial s} = \lambda W(t,s;x) - \lambda \max\left[W(t,s;x), p(s) - p(t) + x_t^0(n) - x_t^1(n,s,p(s)-p(t))\right]$$

Now, consider any contract $x \neq x^*$. Suppose $W(t,s;x) = W^*(t,s)$ for some $s < T_t$. Since any contract must satisfy $x_t^1(n,s,p(s)-p(t)) \geq x_t^0(n)$ by IC-3, it follows that

$$\begin{aligned}
\max\left[W^*(t,s;x), p(s)-p(t)\right] &\geq \max\left[W^*(t,s;x), p(s) - p(t) + x_t^0(n) - x_t^1(n,s,p(s)-p(t))\right] \\
&= \max\left[W(t,s;x), p(s) - p(t) + x_t^0(n) - x_t^1(n,s,p(s)-p(t))\right]
\end{aligned}$$

Under contract x^*, the payoff from selling the asset at date $s < T_t$ is exactly $p(s) - p(t)$. Hence, $\max\left[W^*(t,s), p(s)-p(t)\right]$ corresponds to the second term in $\dfrac{\partial W(t,s;x)}{\partial s}$ when $x = x^*$. This implies that whenever $W(t,s;x) = W^*(t,s)$, then

$$\frac{\partial W^*(t,s)}{\partial s} \leq \frac{\partial W(t,s;x)}{\partial s}$$

Since $W^*(t,t) = W(t,t;x) = V_0(r_t^e)$, this is enough to ensure that

$$W^*(t,s) \leq W(t,s;x)$$

for all $s < T_t$. Hence, we have

$$\begin{aligned} p(s) - p(t) &\geq p(s) - p(t) + x_t^0(n) - x_t^1(n, s, p(s) - p(t)) \\ &\geq W(t, s; x) \\ &\geq W^*(t, s) \end{aligned}$$

These inequalities imply the agent would be willing to sell the asset and contract x^*.

Finally, since the creditor would like to have the agent to sell the asset at all dates, it follows that given two contracts x and x' that give the agent the same expected utility $V_0(r_t^e)$ such that $\Sigma(t; x) \subset \Sigma'(t; x')$, the creditor will prefer contract x' over contract x. The maximally backloaded contract thus solves the planner's problem. ■

Claim 9: If $\epsilon > 0$, then there exists some date t^* such that $(T_t, R_t^n) = (1, x_t^0(n) + r_t^e)$ for all $t \in [t^*, 1]$.

Proof: Consider any date t such that

$$p(t) > \lim_{s \to 1} p(s) - \epsilon(R_1 - R)$$

Under Assumption A2, this will be true for all t that are sufficiently close to 1. Since the rate $r_t^e < R - 1$ in equilibrium, the capital gain from selling the asset will not exceed the expected profits from holding on to the asset. Since the most the agent can expect to get from selling the asset is the capital gain, it follows that for these dates non-entrepreneurs will hold on to the asset until date 1. In that case, the only way to achieve a utility of $V_0(r_t^e)$ is to require the agent to repay r_t^e if the asset pays a dividend of R_1. ■

Claim 10: Suppose positive margin requirements are applied only at dates $t \in [0, t^*]$, and the price path $p(t)$ is consistent with (A1) and (A2). In equilibrium, we have

i. Creditors will be willing to purchase the asset using their own wealth up to some date $t^{**} \in [0, 1]$.

ii. Non-entrepreneurs are willing to purchase the asset at any date, and can do so from date t^* on.

iii. If $t^* < 1$, then for λ sufficiently large, $t^* < t^{**}$. Hence, for all dates between 0 and 1, an agent who arrives at the center will be both willing and able to buy the asset with positive probability.

Proof: I first derive the value to a creditor from purchasing an asset. First, note that if a creditor owns an asset, then by a similar argument to Claim 4, his optimal strategy will be a cutoff rule. That is, if we define $W_0(t, s)$ as the expected utility of a creditor who purchased the asset at date t, has held on to it until and including date s, and acts optimally thereafter, then once again $p(s) - p(t) \geq W_0(t, s)$ implies $p(s') - p(t) \geq W_0(t, s')$ for all $s' \leq s$. A creditor who purchased the asset at date t will therefore sell it from some date σ_t^* on. The expected profits to a creditor who purchased an asset at date t are given by

$$V_0(t) = \max_{\sigma_t^*} Q(\sigma_t^*) \int_{\sigma_t^*}^1 [p(x) - p(t)] f(x|\sigma_t^*) dx + (1 - Q(\sigma_t^*))(\epsilon R_1 - p(t))$$

where $Q(s)$ denotes the probability that at least one more agent who is willing and able to buy the asset will arrive after date s, and $f(x|s)$ denotes the density of the first such arrival conditional on there being such an arrival. A creditor will be willing to purchase the asset if maximizing the above expression over all σ_t^* yields a positive expression. Applying the envelope theorem, we have that

$$\frac{dV_0(t)}{dt} = -p'(t).$$

Hence, if there exists a t^{**} such that $V_0(t^{**}) = 0$, creditors will not find it profitable to purchase the asset beyond date t^{**}.

Beyond date t^{**}, the equilibrium will be identical to the case where creditors are unable to purchase the asset. Now, suppose $t^* < t^{**}$. We need to show that traders will be willing to buy the asset at any date $t \in [t^*, t^{**}]$. By the same logic as in Claims 1 through 4, we can show that in equilibrium entrepreneurs will invest in the project. Since a trader could always pretend to be an entrepreneur and hold the asset until maturity, his profits are bounded below by

$$\epsilon(R_1 - p(t) - r_t^e)$$

Creditors will earn r_t^e per entrepreneur. Since the fraction of entrepreneurs is at most ϕ, expected profits to a creditor are at least

$$(1-\phi)r_t^e - \phi$$

Using the same logic as in Claim 5, we know that expected profits will equal zero. But if $r_t^e \geq R_1 - p(t) \geq R - 1$, then

$$(1-\phi)r_t^e - \phi \geq (1-\phi)(R-1) - \phi > 0$$

i.e. profits would be strictly positive, so this could not be an equilibrium. Hence, $r_t^e < R_1 - p(t)$, and non-entrepreneurs will wish to purchase the asset from date t^* on.

Lastly, we wish to determine whether it is possible for $t^{**} > t^*$. A sufficient condition for this is that the expression maximized to generate $V_0(t)$ is positive when we evaluate it at $t = t^*$, i.e.

$$Q(\sigma_t^*) \int_{\sigma_t^*}^1 p(x) f(x|\sigma_t^*) dx + (1 - Q(\sigma_t^*)) \epsilon R_1 - p(t^*) > 0 \tag{A.6}$$

This is because $V_0(t)$ will be at least as large as the LHS of (A.6). Condition (A.6) will hold provided $Q(t^*)$ is sufficiently close to 1, since under assumption A2 we know that $\int_{t^*}^1 p(x) f(x|t^*) dx \geq p(t^*)$. Hence, a sufficient condition that ensures $t^{**} > t^*$ is if $Q(t^*) \to 1$. To exploit this condition requires an analytical expression for $Q(s)$. Using a similar argument to the one in Appendix B, one can show that the number of individuals who arrive at any interval is independent of the number of arrivals at other intervals.

The value of $Q(s)$ will depend on how t^{**} compares with t^*. Consider first the case where $t^{**} > t^*$. Then we can partition time into three intervals: in the interval $[0, t^*]$ only creditors would be willing and able to buy the asset; in the interval (t^*, t^{**}), both creditors and non-entrepreneurs would be willing and able to buy the asset; and in the interval $[t^{**}, 1]$, only non-entrepreneurs would be willing and able to buy the asset. The probability that at least one trader arrives beyond date s is equal to one minus the probability that there are zero arrivals in the interval $(s, 1]$. Hence, if $t^{**} > t^*$, we have

$$\begin{aligned} Q(s) &= 1 - e^{-\lambda(1-\max(t^{**},s))} e^{-(\lambda+\mu)(\max(t^{**},s)-\max(t^*,s))} e^{-\mu(\max(t^*,s)-s)} \\ &= 1 - e^{-\lambda(1-\max(t^*,s))-\mu(\max(t^{**},s)-s)} \end{aligned} \tag{A.7}$$

Next, consider the case where $t^* \geq t^{**}$. In this case, we can partition time into three intervals: in the interval $[0, t^{**}]$ only creditors would be willing and able to buy the asset; in the interval (t^{**}, t^*), neither creditors and non-entrepreneurs would be both willing and able to buy the asset; and in the interval $[t^*, 1]$, only non-entrepreneurs would be willing and able to buy the asset. The probability that at least one trader arrives beyond date s is equal to one minus the probability that there are zero arrivals in the interval $(s, 1]$. Hence, if $t^* \geq t^{**}$, we have

$$Q(s) = 1 - e^{-\lambda(1-\max(t^*,s))} e^{-\mu(\max(t^{**},s)-s)} \tag{A.8}$$

To show that for λ large enough it must be the case that $t^{**} > t^*$, suppose to the contrary $t^* \geq t^{**}$. From (A.8), we know that in this case, $Q(t^*) = 1 - e^{-\lambda(1-t^*)}$. But as $\lambda \to \infty$, this expression converges to 1, implying (A.6) will be satisfied. But then $V_0(t^*) > 0$, which implies $t^{**} > t^*$, a contradiction. ■

Claim 11: Suppose $r_t^{FF} > 0$ for $t \in [0, t^*]$ and $r_t^{FF} = 0$ for $t \in (t^*, 1]$. Then in equilibrium, we have

i. Non-entrepreneurs would be willing to purchase the asset at any date beyond t^*

ii. For any date $\tau < t^*$, there exists a path for r_t^{FF} such that no agent would be willing to buy the asset on or before date τ, regardless of the value of λ.

Proof: Part (i) follows directly from the fact that beyond date t^* we are back to the benchmark version of the model in which by Claims 1-4 we know that non-entrepreneurs are both willing and able to purchase the asset. For part (ii), suppose we set the path of r_t^{FF} for $t \in [0, t^*]$ so that $r_t^{FF} > \dot{p}_t / p_t$, and for $t \in (\tau, t^*]$ we further require that

$$R_{\tau,t^*}^{FF} \equiv \exp\left(\int_\tau^{t^*} r_t^{FF} dt\right) > \frac{R_1}{p(0)} \geq \frac{1}{p(0)}$$

Note that these conditions do not depend on the value of λ. We need to show that no agent would wish to buy the asset before date τ. Suppose an agent purchases the asset and holds it until maturity. By (19) if the agent keeps positive wealth he would have to pay at least $R_{\tau,t^*}^{FF} p(t)$. But then

$$R_{\tau,t^*}^{FF} p(t) \geq R_{\tau,t^*}^{FF} p(0) > R_1$$

which contradicts the assumption that the agent has positive wealth. So holding the asset until maturity will yield zero wealth. Next, suppose an agent sells the asset. The maximal profits he could earn from selling the asset are given by

$$\max_s \left\{ p(s) - R_{t,s}^{FF} p(t) \right\}$$

For all $s < t^*$, using the fact that $\dot{p}_t / p_t = \dfrac{d}{dt} \ln p_t$, we have

$$\begin{aligned} \exp\left(\int_t^s r_x^{FF} dx\right) &> \exp\left[\int_t^s \left(\frac{d}{dx} \ln p_x\right) dx\right] \\ &= \exp\left[\ln p(s) - \ln p(t)\right] \\ &= p(s)/p(t) \end{aligned}$$

and so

$$R_{t,s}^{FF} p(t) > p(s)$$

so that selling the asset before date τ yields zero profits. For $s \geq t^*$,

$$p(s) - R_{t,s}^{FF} p(t) = p(s) - R_{t,t^*}^{FF} p(t)$$

which is increasing in s. But then

$$R_{t,t^*}^{FF} p(t) \geq R_{\tau,t^*}^{FF} p(t) \geq R_{\tau,t^*}^{FF} p(0) \geq 1 \geq \lim_{s \to 1} p(s)$$

Thus, a non-entrepreneur could never earn positive profits from buying the asset, and given there is a tiny cost of entering into a financial contract, they would never wish to buy it. For an agent who does not have to borrow, the opportunity cost of buying the asset at date t and holding it until date s is $R_{t,s}^{FF} p(t)$. The same calculations then imply buying the asset yields less value than this opportunity cost. ■

Claim 12: Suppose that for all $t \in [0,1]$, $r_t^{FF} = \varepsilon$ for some $\varepsilon > 0$, and for $t \in [1, 1+\Delta]$, the path of r_t^{FF} is such that

$$R_{1,1+\Delta}^{FF} \equiv \exp\left(\int_1^{1+\Delta} r_t^{FF} dt\right) \geq \frac{R_1}{p(0)} \tag{A.10}$$

where $p(0) \geq \epsilon R_1$ is the price of the asset at date 0. Then under (A1) and (A2), the probability of trade along the equilibrium path will be zero, i.e. an asset whose price implies a speculative bubble will not be traded.

Proof: Define $T = \sup\{t \mid \text{Prob}(\text{asset sells at date } t) > 0\}$ as the supremum over all dates at which the asset might trade. We need to show that $T = 0$. Suppose that $T > 0$. Consider the expected profit from purchasing the asset at date $T - \varepsilon$. From Lemma B1 in Appendix B, the probability that at least one trader arrives in this interval is given by $Q = 1 - e^{-\lambda\varepsilon}$, which tends to 0 as $\varepsilon \to 0$. The expected payoff to an agent from buying the asset at date t is at most

$$Q \max\{p(T) - p(T - \varepsilon) - \delta, 0\} + \epsilon(1 - Q) \max\left\{R_1 - R_{1,1+\Delta}^{FF} p(t), 0\right\}$$

Since $p(t) > p(0)$, the second term is equal to zero. Since $p(t)$ is assumed the be continuous, there exists an $\varepsilon^* > 0$ such that for all $t \in (T - \varepsilon^*, T)$, we have $p(T) - p(t) < \delta$. But then we have an open interval in which no non-entrepreneur would purchase the asset given the tiny cost of a financial transaction. But then T would not be the supremum for the set of dates at which the asset trades. ■

Appendix B: Solving for ϕ_t

I begin with the following two lemmas:

Lemma B1: In any interval $[t, t+\Delta]$, the number of arrivals is Poisson with parameter $\lambda\Delta$ and is independent of the number of arrivals in any non-overlapping interval.

Proof: Pick any two intervals $[t_1, t_1+\Delta_1]$ and $[t_2, t_2+\Delta_2]$ such that $[t_1, t_1+\Delta_1] \cap [t_2, t_2+\Delta_2] = \varnothing$. Define $N([a,b])$ as the cardinality of the set $\left\{n : a \leq T_{(n)} \leq b\right\}$. Proving the lemma requires showing that

$$\mathbb{P} = \Pr\left(N\left([t_1, t_1+\Delta_1]\right) = n_1, N\left([t_2, t_2+\Delta_2]\right) = n_2\right) = \frac{e^{-\lambda\Delta_1}\left(\lambda\Delta_1\right)^{n_1}}{n_1!}\frac{e^{-\lambda\Delta_2}\left(\lambda\Delta_2\right)^{n_2}}{n_2!}$$

We can compute $\mathbb{P}$ by conditioning on the total number of arrivals N, as follows:

$$\begin{aligned}
\mathbb{P} &= \sum_{n=n_1+n_2}^{\infty} \Pr\left(N\left([t_1, t_1+\Delta_1]\right) = n_1, N\left([t_2, t_2+\Delta_2]\right) = n_2 | N = n\right)\Pr\left(N = n\right) \\
&= \sum_{n=n_1+n_2}^{\infty} \frac{n!}{n_1! n_2! \left(n - n_1 - n_2\right)!}\Delta_1^{n_1}\Delta_2^{n_2}\left(1 - \Delta_1 - \Delta_2\right)^{n-n_1-n_2}\frac{e^{-\lambda}\lambda^n}{n!} \\
&= e^{-\lambda}\frac{\left(\lambda\Delta_1\right)^{n_1}}{n_1!}\frac{\left(\lambda\Delta_2\right)^{n_2}}{n_2!}\sum_{n=n_1+n_2}^{\infty}\frac{\left(\lambda\left(1-\Delta_1-\Delta_2\right)\right)^{n-n_1-n_2}}{\left(n-n_1-n_2\right)!} \\
&= e^{-\lambda}\frac{\left(\lambda\Delta_1\right)^{n_1}}{n_1!}\frac{\left(\lambda\Delta_2\right)^{n_2}}{n_2!}e^{\lambda\left(1-\Delta_1-\Delta_2\right)} \\
&= \frac{e^{-\lambda\Delta_1}\left(\lambda\Delta_1\right)^{n_1}}{n_1!}\frac{e^{-\lambda\Delta_2}\left(\lambda\Delta_2\right)^{n_2}}{n_2!}
\end{aligned}$$

This establishes the claim. ■

Lemma B2: The probability density for the event that an agent arrives at date t is given by

$$f\left(T_{(n)} = t \text{ for some } n\right) = \lim_{\Delta\to 0}\frac{\Pr\left(N[t, t+\Delta] > 0\right)}{\Delta} = \lambda$$

and is independent of what happens at any other interval.

Proof: From Lemma B1, we know that the number of arrivals in $[t, t+\Delta]$ is independent of what happens at any other interval. The probability that there are zero arrivals in this interval are given by $\Pr\left(N\left([t, t+\Delta]\right) = 0\right) = e^{-\lambda\Delta}$. Hence, the probability that there is at least one arrival is given by $1 - e^{-\lambda\Delta}$. Dividing by Δ, taking the limit as Δ goes to zero, and applying L'Hopital's rule shows this expression is equal to λ. ■

Next, consider the particular event $\{Y_1 = y_1 \cap Y_2 = y_2 \cap \cdots \cap Y_k = t\}$, i.e. there are at least k values in $\{Y_j\}_{j=1}^{J}$, and the first k realizations are given by $y_1, y_2, ...,$ and t. This event implies there exists some integers $n_1 < n_2 < \cdots < n_k$ such that

$$T_{(n_1)} = y_1, T_{(n_2)} = y_2, ...T_{(n_k)} = t$$

and, in addition, that

$$N\left(\left(S\left(0\right),y_1\right)\right)=N\left(\left(S\left(y_1\right),y_2\right)\right)=N\left(\left(S\left(y_2\right),y_3\right)\right)=...=N\left(\left(S\left(y_{k-1}\right),t\right)\right)=0$$

Define $y_0 = 0$ and $y_k = t$. Using the two lemmas above, the probability density associated with the event $\{Y_1 = y_1 \cap Y_2 = y_2 \cap \cdots \cap Y_k = t\}$ is given by

$$\lambda^k \exp\left(-\lambda \sum_{m=1}^{k} \left(y_m - S\left(y_{m-1}\right)\right)\right)$$

Hence, the probability that $Y_1 = t$, implying that t is the first arrival after $S(y_0)$, is equal to $\lambda e^{-\lambda(t-S(y_0))}$, while the probability that $Y_k = t$ for $k > 1$ is given by the integral

$$\int_{(y_1,\ldots,y_{k-1})\in\mathbb{Y}_m(t)} \cdots \int \Pr\left(Y_1 = y_1 \cap Y_2 = y_2 \cap \cdots \cap Y_k = t\right)\ dy_{k-1}\cdots dy_1$$

where $\mathbb{Y}_k(t) = \{(y_1, ..., y_{k-1}) \mid y_1 \geq S(0), y_2 \geq S(y_1), ..., y_{k-1} \geq S(y_{k-2}), t \geq S(y_{k-1})\}$.

The probability density for the event that $\exists Y_j = t$ can be obtained by adding up over all possible realizations in which $Y_k = t$ for some t, i.e.

$$\begin{aligned}
\Pr\left(\exists Y_j = t\right) &= \sum_{k=1}^{\infty} \Pr\left(Y_1 = y_1 \cap Y_2 = y_2 \cap \cdots \cap Y_k = t\right) \\
&= \Pr\left(Y_1 = t\right) + \sum_{k=2}^{\infty} \int_{(y_1,\ldots,y_{k-1})\in\mathbb{Y}_m(t)} \cdots \int \Pr\left(Y_1 = y_1 \cap Y_2 = y_2 \cap \cdots \cap Y_k = t\right)\ dy_{k-1}\cdots dy_1 \\
&= \lambda e^{-\lambda(t-S(y_0))} + \sum_{k=2}^{\infty} \int_{(y_1,\ldots,y_{k-1})\in\mathbb{Y}_k(t)} \cdots \int \lambda^k \exp\left(-\lambda \sum_{m=1}^{k} \left[y_m - S\left(y_{m-1}\right)\right]\right) dy_{k-1}\cdots dy_1
\end{aligned}$$

To further simplify this expression, I now establish the following result:

Lemma B3: If $Y_k = t$ for some $t \in [0,1]$, then $k < K_t$ where K_t is finite.

Proof: Recall that $S(t) > t$ and $S(t) = 1$ for $t \in [t^*, 1]$ for some $t^* < 1$. Define

$$\Delta = \inf_{t\in[0,t^*]} \{S(t) - t\}$$

Since $S(t)$ is continuous, $S(t) > t$ for all $t \in [0, t^*]$ and the interval $[0, t^*]$ is compact, the infimum is achieved at some $t \in [0, t^*]$. It then follows that $\Delta > 0$. Define $K = \frac{t^*}{\Delta}$. Suppose we apply $S(\cdot)$ recursively at some point $t \in [0, 1]$. At each point, the lowest value we could achieve is $t + K\Delta$. By construction, then, after K iterations we would eventually exceed t^*, and one final application of S would reach 1. For any value of $k > K + 1$, then, it would not be possible for $S(y_{k-1}) < t$ when $t \leq 1$. ■

We can therefore express $\pi(t) = \Pr\left(\exists Y_k = t \mid \exists T_{(n)} = t\right)$ as a finite sum of integrals:

$$\begin{aligned}
\Pr\left(\exists Y_k = t \mid \exists T_{(n)} = t\right) &= \frac{\Pr\left(\exists Y_k = t \cap \exists T_{(n)} = t\right)}{\Pr\left(\exists T_{(n)} = t\right)} \\
&= \frac{\Pr\left(\exists Y_k = t\right)}{\Pr\left(\exists T_{(n)} = t\right)} \\
&= e^{-\lambda(t-S(y_0))} + \sum_{k=2}^{K_t} \int \cdots \int_{(y_1,\ldots,y_{k-1})\in\mathbb{Y}_k(t)} \lambda^{k-1} \exp\left(-\lambda \sum_{j=1}^{k} \left[y_j - S\left(y_{j-1}\right)\right]\right) dy_{k-1}\cdots dy_1
\end{aligned}$$

where we use the fact that $\Pr\left(\exists T_{(n)} = t\right) = \lambda$ from Lemma B2.

When $S(t)$ is monotonically increasing, this expression can be further simplified to

$$\pi(t) = e^{-\lambda(t-S(y_0))} + \sum_{k=2}^{K_t} \int_{S(0)}^{\left[S^{-1}\right]^{k-1}(t)} \int_{S(y_1)}^{\left[S^{-1}\right]^{k-2}(t)} \cdots \int_{S(y_{k-2})}^{S^{-1}(t)} \lambda^{k-1} \exp\left(-\lambda \sum_{j=1}^{k} \left[y_j - S\left(y_{j-1}\right)\right]\right) dy_{k-1}\cdots dy_1 \tag{22}$$

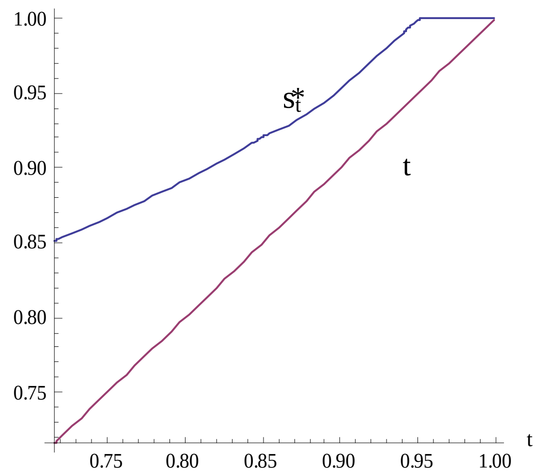
In addition, when $S(t)$ is monotonically increasing, the set $[0,1]$ can be partitioned into a finite set of intervals $[0,t_0]$, $(t_0,t_1]$, ..., $(t_K,1]$ where $t_k = S^{k+1}(0)$, so that if $t \in (t_k, t_{k+1}]$, then $\pi(t)$ is the sum of k terms, and depends on the values of $S(t)$ for $t \leq t_k$. This greatly simplifies the task of solving for $\pi(t)$ numerically: for $t \in (t_0, t_1]$, we have

$$\pi(t) = e^{-\lambda(t-S(y_0))} = e^{-\lambda(t-t_0)}$$

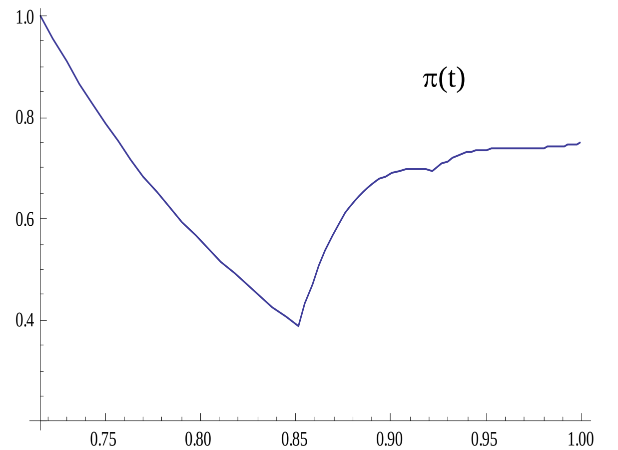
From $\pi(t)$, we can recover ϕ_t, r_t^e, and s_t^*. Hence, we can use a grid to interpolate the function $S(t)$ for all $t \in (t_0, t_1]$. We can then use this interpolated function to solve for $\pi(t)$ for any $t \in (t_1, t_2]$, and so on, until we recover the value of $\pi(t)$ at each interval. Note that $t_k = S^{k+1}(0)$ corresponds to the first date at which the asset could be traded for the $k+1$-th time, which occurs if the original owners sell at date $s_0 = S(0)$, those who buy from them sell at the s_t^* associated with $t = s_0$, i.e. $S^2(0)$, and so on. In calculating the numerical example reported in the text, I computed $\pi(t)$ as defined in (22) for a grid of 20 equally spaced points within each interval, starting first with $(t_0, t_1]$. Given $\pi(t)$, I could then solve for s_t^* at each of these points, and interpolate the function s_t^* over the entire interval. This interpolation was done with Mathematica using a linear spline. Once I have an interpolated function for all intervals $(t_0, t_1]$ through $(t_k, t_{k+1}]$, I can proceed to use it to calculate $\pi(t)$ over the interval $(t_{k+1}, t_{k+2}]$ using (22).

Figure 1: Equilibrium contract for a particular numerical example

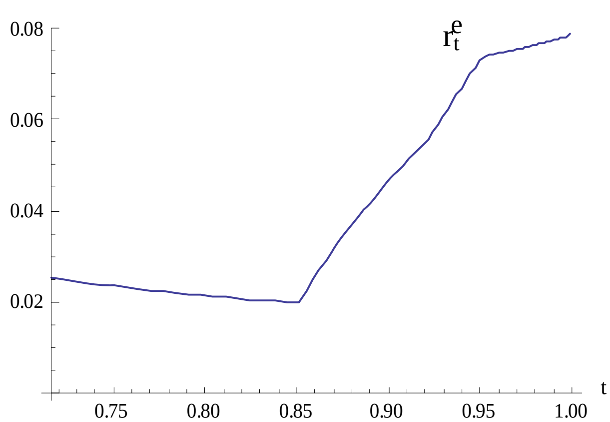
Numerical values used to generate the example: $\lambda = 7$; $\phi = .1$; $R = R_1 = 2$; $\in = .05$; and $p(t) = \min(t, \in R_1)$.



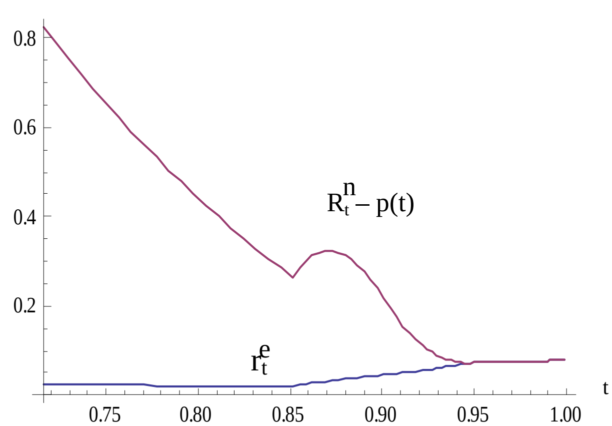
a. Optimal trading strategy: if buy asset at t, sell it from date s_t^* on



b. Probability that non-entrepreneurs arriving at date t will be able to buy the asset



c. Equilibrium rate r_t^e charged to entrepreneur as a function of the date t in which he arrived



d. Amount R_t^e - p(t) a speculator arriving at date t would pay if he failed to sell the asset

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