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Robustness and Macroeconomic Policy

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Abstract

This paper considers the design of macroeconomic policies in the face of uncertainty. In recent years, several economists have advocated that when policymakers are uncertain about the environment they face and find it difficult to assign precise probabilities to the alternative scenarios that may characterize this environment, they should design policies to be robust in the sense that they minimize the worst-case loss these policies could ever impose. I review and evaluate the objections cited by critics of this approach. I further argue that, contrary to what some have inferred, concern about worst-case scenarios does not always lead to policies that respond more aggressively to incoming news than the optimal policy would respond absent any uncertainty.

Key Words: Robust Control, Uncertainty, Ambiguity, Attenuation Principle

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1 Introduction

A recurring theme in the literature on macroeconomic policy concerns the role of uncertainty. In practice, policymakers are often called to make decisions with only limited knowledge of the environment in which they operate, in part because they lack all of the relevant data when they are called to act and in part because they cannot be sure that the models they use to guide their policies are good descriptions of the environment they face. These limitations raise a question that economists have long grappled with: How should policymakers proceed when they are uncertain about the environment they operate in?

One seminal contribution in this area comes from Brainard (1967), who considered the case where a policymaker does not know some of the parameters that are relevant for choosing an appropriate policy but still knows the probability distribution according to which they are determined. Since the policymaker knows how the relevant parameters are distributed, he can compute the expected losses that result from different policies. Brainard solved for the policy that yields the lowest expected loss, and showed that when the policymaker is uncertain about the effect of his actions, the appropriate response to uncertainty is to moderate or attenuate the extent to which he should react to the news he does receive. This result has garnered considerable attention from both researchers and policymakers.[1]

However, over time economists have grown increasingly critical of the approach inherent in Brainard's model to dealing with uncertainty. In particular, they have criticized the notion that policymakers know the distribution of variables about which they are uncertain. For example, if policymakers do not understand the environment they face well enough to quite know how to model it, it seems implausible that they can assign exact probabilities as to what is the right model. Critics who have pushed this line of argument have instead suggested modelling policymakers as entertaining a class of models that can potentially describe the data, which is usually constructed as a set of perturbations around some benchmark model, and then letting policymakers choose policies that are "robust" in the sense that they perform well against all models in this class. That is, a robust policy is one whose worst performance across all models that policymakers contemplate exceeds the worst performance of all other policies.

The recommendation that macroeconomic policies be designed to be robust to a class of models remains controversial. In this paper, I review the debate over the appropriateness of robustness as a criterion for choosing policy, relying on simple illustrative examples that avoid some of the complicated technicalities that abound in this literature. I also address a question that preoccupied early work on robustness and macroeconomic policy, namely, whether robust policies contradict the attenuation result in the original Brainard model. Although early applications of robustness tended to find that policymakers should respond

[1] See, for example, the discussion of Brainard's result in Blinder (1998), pp. 9-13.

more aggressively to the news they do receive than they would otherwise, I argue that aggressiveness is not an inherent feature of robustness but is specific to the models these papers explored. In fact, robustness can in some environments lead to the same attenuation that Brainard obtained, and for quite similar reasons. This offers a useful reminder: Results concerning robustness that arise in particular environment will not always themselves be robust to changes in the underlying environment.

Given space limitations, I restrict my attention to these issues and ignore other applications of robustness in macroeconomics. For example, there is an extensive literature that studies what happens when private agents – as opposed to policymakers – choose strategies designed to be robust against a host of models, and examines whether such behavior can help resolve macroeconomic problems such as the equity premium puzzle and other puzzles that revolve around attitude toward risk. Readers interested in such questions should consult Hansen and Sargent (2008) and the references therein. Another line of research considers what policymakers should do when they know private agents employ robust decisionmaking, but policymakers are themselves confident about the underlying model of the economy. Examples of such analyses include Caballero and Krishnamurthy (2008) and Karantounias, Hansen, and Sargent (2009). I also abstract from work on robust policy in non-macroeconomic applications, for example treatment rules for a heterogeneous population where the effect of treatments is uncertain. The latter is surveyed in Manski (2010).

This article draws heavily on Barlevy (2009) and is similarly organized. As in that paper, I first review Brainard's (1967) original result. I then introduce the notion of robustness and discuss some of the critiques that have been raised against this approach. Next, I apply the robust control approach to a variant of Brainard's model and show that it too recommends that the policymaker attenuate his response to incoming news, just as in the original Brainard model. Finally, using simple models that contain some of the features from the early work on robust monetary policy, I offer some intuition for why concern for robustness can sometimes lead to aggressive policy.

2 The Brainard Model

Brainard (1967) studied the problem of a policymaker who wants to target some variable so that it will equal some prespecified level. For example, consider a monetary authority that wants to maintain inflation at some target rate or to steer short-run output growth toward its natural rate. These objectives may force the authority to intervene to offset shocks that would otherwise cause these variables to deviate from the desired targets. Brainard was concerned with how this intervention should be conducted when the monetary authority is uncertain about the economic environment it faces but can assign probabilities to all possible scenarios it could encounter. Note that this setup assumes the policymaker cares about meeting only a single target. By contrast, most papers on the design of macroeconomic policy assume policymakers try to meet multiple targets, potentially giving rise to tradeoffs between meeting conflicting targets that this setup necessarily ignores. I will discuss an example with multiple tradeoffs further below, but for the most part I focus on meeting a single target.

Formally, denote the variable the policymaker wants to target by y. Without loss of generality, we can assume the policymaker wants to target y to equal zero, and can affect it using a policy variable he can set and which I denote by r. In addition, y is determined by some variable x that the policymaker can observe prior to setting r. For example, y could reflect inflation, r could reflect the short-term nominal rate, and x could reflect shocks to productivity or the velocity of money. For simplicity, let y depend on these two variables linearly; that is,

$$y = x - kr, \tag{1}$$

where k measures the effect of changes in r on y and is assumed to be positive. Absent uncertainty, targeting y in the face of shocks to x is simple; the policymaker would simply have to set r to equal x/k to restore y to its target level of 0.

To incorporate uncertainty into the policymaker's problem, suppose y is also affected by random variables whose values the policymaker does not know, but whose distributions are known to him in advance. Thus, let us replace equation (1) with

$$y = x - (k + \varepsilon_k)r + \varepsilon_u, \tag{2}$$

where ε_k and ε_u are independent random variables with means 0 and variances and σ_k^2 and σ_u^2, respectively. This formulation lets the policymaker be uncertain both about the effect of his policy, as captured by the term ε_k that multiplies his choice of r, and about factors that influence y, as captured by the additive term ε_u. The optimal policy depends on how much loss the policymaker incurs from missing his target. Brainard assumed the loss is quadratic in the deviation between the actual value of y and its target – that is, the loss is equal to y^2 – and that the policymaker chooses r so as to minimize his expected loss, that is, to solve

$$\min_r E\left[y^2\right] = \min_r E\left[\left(x - (k + \varepsilon_k)\, r + \varepsilon_u\right)^2\right]. \tag{3}$$

Solving this problem is straightforward, and yields the solution

$$r = \frac{x}{k + \sigma_k^2/k}. \tag{4}$$

Uncertainty about the effect of the policy instrument r will thus lead the policymaker to attenuate his response to x relative to the case where he knows the effect of r on y with certainty. In particular, when $\sigma_k^2 = 0$, the policymaker will set r to undo the effect of x by setting $r = x/k$. But when $\sigma_k^2 > 0$, the policy will not fully offset x. This is Brainard's celebrated result: A policymaker who is unsure about how the policy instrument he controls influences the variable he wishes to target should react less to news about missing the target than he would if he were fully informed. By contrast, uncertainty about ε_u has no implications for policy, as evident from the fact that the optimal rule for r in (4) is independent of σ_u^2.

To understand this result, note that the expected loss in (3) is essentially the variance of y. Hence, a policy that leads y to be more volatile will be considered undesirable. From (2), the variance of y is equal to $r^2\sigma_k^2 + \sigma_u^2$, which is increasing in the absolute value of r. An aggressive policy that uses r to offset nonzero values of x thus implies a more volatile outcome for y, while an attenuated policy that sets r closer to 0 implies a less volatile outcome for y. This asymmetry introduces a bias toward less aggressive policies. Even though a less aggressive response to x would cause the policymaker to miss the target on average, he is willing to do so in order to make y less volatile. Absent this asymmetry, there would be no reason to attenuate policy. Indeed, this is precisely why uncertainty in ε_u has no effect on policy, since this source of uncertainty does not generate any asymmetry between aggressive and attenuated policy.

The fact that uncertainty about ε_u does not lead the policymaker to attenuate his response to news serves as a useful reminder that the attenuation principle is not a general result, but depends on what the underlying uncertainty is about. This point has been reiterated in subsequent work such as Chow (1973), Craine (1979), Soderstrom (2002), who show that uncertainty can sometimes imply that policymakers should respond to information about missing a target more aggressively than they would in a world of perfect certainty. The same caveat about drawing general conclusions from specific examples will turn out to apply equally to alternative approaches of dealing with uncertainty, such as robustness.

3 Robustness

As noted above, one often cited critique of the approach underlying Brainard's (1967) analysis is that it may be unreasonable to expect policymakers to know the distribution of the variables about which they are uncertain. For example, there may not be enough historical data to infer the likelihood of certain scenarios, especially those that have yet to be observed but remain theoretically possible. Likewise, if policymakers do not quite understand the environment they face, they will have a hard time assigning precise probabilities as to which economic model accurately captures the key features of this environment. Without knowing these probabilities, it will be impossible to compute an expected loss for different policy choices and thus to choose the policy that generates the smallest expected loss.

These concerns have led some economists to propose an alternative criterion for designing policy that does not require assigning probabilities to different models or scenarios. This alternative argues for picking the policy that minimizes the damage a policy could possibly inflict under any scenario the policymaker is willing to contemplate. That is, policymakers should choose the policy under which the largest possible loss across all potential scenarios is smaller than the maximal loss under any alternative policy. Such a policy ensures the policymaker will never have to incur a bigger loss than the bare minimum that cannot be avoided. This rule is often associated with Wald (1950, p. 18), who argued that this approach, known as the minimax or maximin rule, is "a reasonable solution of the decision problem when an *a priori* distribution in [the state space] Ω does not exist or is unknown." In macroeconomic applications, this principle has been applied to dynamic decision problems using techniques borrowed from the engineering literature on optimal control. Hence, applications of the minimax rule in macroeconomics are often referred to as robust control, and the policy that minimizes the worst possible loss is referred to as a robust policy.[2]

By way of introduction to the notion of robustness, it will help to begin with an example outside of economics where the minimax rule appears to have some intuitive appeal and which avoids some of the complications that arise in economic applications. The example is known as the "lost in a forest" problem, which was first posed by Bellman (1956) and which evolved into a larger literature that is surveyed in Finch and Wetzel (2004).[3] The lost in a forest problem can be described as follows. A hiker treks into a dense forest. He starts his trip from a road that cuts through the forest, and he travels one mile into the forest along a straight path that is perpendicular to the road. He then lies down to take a nap, but when he wakes up he realizes he forgot which direction he originally came from. He wishes to return back to the road – not necessarily the point where he started, but anywhere on the road where he can flag down a car and head back to town. Moreover, he would like to reach the road using the shortest possible route. If he knew which direction he came by originally, this task would be easy: He could reach the road in exactly one mile by simply retracing his original route in reverse. But the assumption is that he cannot recall from whence he came, and because the forest is dense with trees, he cannot see the road from afar. Thus, he must physically reach the road to realize he found it. The problem is more precisely described geometrically, as illustrated in panel (a) of figure 1. First, draw a circle of radius one mile around the hiker's initial location. The road the hiker is searching for is assumed to be an infinite straight line that lies tangent to this circle. The point of tangency lies somewhere along the circle, and the objective is to reach any point on the tangent line – not necessarily the point at which the line is tangent to the circle but any point on the line – using the shortest possible path starting at the center of the circle. Panel (a) in figure 1 illustrates three of the continuum of possible locations where the road might lie.

What strategy should the hiker follow in searching for the road? Solving this problem requires a criterion

[2] The term "robust control" is used by some interchangeably with the term "optimal decisionmaking under ambiguity." However, the latter terminology is more closely associated with the literature following Gilboa and Schmeidler (1989) that asks whether it is logically consistent for individuals to be averse to ambiguously posed choice problems in which the probability of various events remain unspecified. By contrast, robust control is motivated by the normative question of what a decisionmaker ought to do when faced with such ambiguity. I comment on the connection between these two literatures further below.

[3] The problem is sometimes referred to as the "lost at sea" problem. Richard Bellman, who originally posed the problem, is well known among economists for his work on dynamic programming for analyzing sequential decision problems.

to determine what constitutes the "best" possible strategy. In principle, if the hiker knew his propensity to wake up in a particular orientation relative to the direction he travelled from, he could assign a probability that his starting point lies in any given direction. In that case, he could pick the strategy that minimizes the expected distance he would need to reach the main road. But what if the hiker has no good notion of his propensity to lie down in a particular orientation or the odds that he didn't turn in his sleep? One possibility, to which I shall return below, is to assume all locations are equally likely and choose the path that minimizes the expected distance to reach any point along the road. While this restriction seems natural in that it does not favor any one direction over another, it still amounts to imposing beliefs about the likelihood of events the hiker cannot fathom. After all, it is not clear why assuming that the road is equally likely to lie in any direction should be a good model of the physical problem that describes how likely the hiker is to rotate a given amount in his sleep.

Bellman (1956) proposed an alternative criterion for choosing the strategy that does not require assigning any distribution to the location of the road. He suggested choosing the strategy that minimizes the amount of walking required to ensure reaching the road regardless of where it is located. That is, for any strategy, we can compute the longest distance one would have to walk to make sure he reaches the main road regardless of which direction the road lies. It will certainly be possible to search for the road in a way that ensures reaching the road after a finite amount of hiking: For example, the hiker could walk one mile in any particular direction, and then, if he didn't reach the road, turn to walk along the circle of radius one mile around his original location. This strategy ensures he will eventually find the road. Under this strategy, the most the hiker would ever have to walk to find the road is $1 + 2\pi \approx 7.28$ miles. This is the amount he would have to walk if the road was a small distance from where he ended up after walking one mile out from his original location, but the hiker unfortunately chose to turn in the opposite direction when he started to traverse the circle. Bellman suggested choosing the strategy for which the maximal distance required to guarantee reaching the road is shortest. The appeal of this rule is that it requires no more walking than is absolutely necessary to reach the road. While other criteria have been proposed for the lost in a forest problem, many have found the criterion of walking no more than is absolutely necessary to be intuitively appealing. But this is precisely the robust control approach. The worst-case scenario for any search strategy involves exhaustively searching through every wrong location before reaching the true location. Bellman's suggestion thus amounts to using the strategy whose worst-case scenario requires less walking than the worst-case scenario of any other strategy. In other words, the "best" strategy is the one that minimizes the amount of distance the hiker would need to run through the gamut of all possible locations for the road.

Although Bellman (1956) first proposed this rule as a solution to the lost in a forest problem, it was Isbell (1957) who derived the strategy that meets this criterion. Readers who are interested in the derivation of the optimal solution are referred to Isbell's original piece. It turns out that it is possible to search for the road in a way that requires walking no more than approximately 6.40 miles, along the path illustrated by the heavy line in panel (b) of figure 1. The idea is that by deviating from the circle and possibly reaching the road at a different spot from where the hiker originally left, one can improve upon the strategy of tracking the circle until reaching the road just at the point in which it lies tangent to the circle of radius one mile around the hiker's starting point.

An important feature of the lost in a forest problem is that the set of possibilities the hiker must consider in order to compute the worst-case scenario for each policy is an objective feature of the environment: The main road must lie somewhere along a circle of radius one mile around where the hiker fell asleep, we just don't know where. By contrast, in most macroeconomic applications, there is no objective set of models an agent should use in computing the worst-case scenario. Rather, the set of models agents can consider is typically derived by first positing some benchmark model and then considering all the models that are "close" in the sense that they imply a distribution for the endogenous variables of the model that is not too different from the distribution of these same variables under the benchmark model – often taken to be the entropy distance between the two distributions. Alternatively, agents can contemplate any conceivable model, but relying on a model that is further away from the benchmark model incurs a penalty in proportion to the entropy distance between that model and the benchmark one.[4] Thus, in contrast with the lost in a forest problem, economic applications often involve some arbitrariness in that the set of scenarios over which the worst-case scenario is calculated involves terms that are not tied down by theory – a benchmark model and a maximal distance or a penalty parameter that governs how far the models agents contemplate can be from this benchmark. One way of making these choices less arbitrary is to choose the maximal distance or penalty parameter in such a way that policymakers will not consider models that given the relevant historical time-series data can be easily rejected statistically in favor of the benchmark model.

The robust strategy is viewed by many mathematicians as a satisfactory solution for the lost in a forest problem. However, just as economists have critiqued the use of robustness in economics, one can find fault with this search strategy. The remainder of this section reviews the debate over whether robustness is an appropriate criterion for coping with uncertainty, using the lost in a forest problem for illustration.

One critique of robustness holds that the robust policy is narrowly tailored to do well in particular scenarios rather than in most scenarios. This critique is sometimes described as "perfection being the enemy of the good": The robust strategy is chosen because it does well in the worst-case scenario, even if that scenario is unlikely and even if the same strategy performs much worse than alternative strategies in most, if not all, remaining scenarios. Svennson (2007) conveys this critique as follows: "If a Bayesian prior probability measure were to be assigned to the feasible set of models, one might find that the probability

[4] Hansen and Sargent (2008) refer to these two approaches as constraint preferences and multiplier preferences, respectively.

assigned to the models on the boundary are exceedingly small. Thus, highly unlikely models can come to dominate the outcome of robust control."[5] This critique can be illustrated using the lost in a forest problem. In that problem, the worst-case scenario for any search strategy involves guessing each and every one of the wrong locations first before finding the road. Arguably, guessing wrong at each possible turn is rather unlikely, yet the robust policy is tailored to this scenario, and the hiker does not take advantage of shortcuts that allow him to search through many locations without having to walk a great distance.

The main problem with this critique is that the original motivation for using the robustness criterion to design policy is that the policymaker cannot assign a probability distribution to the scenarios he considers. Absent such a distribution, one cannot argue that the worst-case scenario is unlikely. That is, there is no way to tell whether the Bayesian prior probability measure Svennson alludes to is reasonable. One possible response is that even absent an exact probability distribution, we might be able to infer from common experience that the worst-case scenario is not very likely. For example, returning to the example of the lost in a forest problem, we do not often run through all possibilities before we find what we are searching for, so the worst-case scenario should be viewed as remote even without attaching an exact probability to this event. But such intuitive arguments are tricky. Consider the popularity of the adage known as Murphy's Law, which states that whatever can go wrong will go wrong. The fact that people view things going wrong at every turn as a sufficiently common experience to be humorously elevated to the level of a scientific law suggests they might not view looking through all of the wrong locations first as such a remote possibility. Even if similar events have proven to be rare in the past, there is no objective way to assure a person who is worried that this time might be different that he should continue to view such events as unlikely.

In addition, in both the lost in a forest problem and in many economic applications, it is not the case that the robust strategy performs poorly in all scenarios aside from the worst-case one. In general, the continuity present in many of these models implies that the strategy that is optimal in the worst-case scenario must be approximately optimal in nearby situations. Although robust policies may be overly perfectionist in some contexts, in practice they are often optimal in various scenarios rather than just the worst-case one. Insisting that a policy do well in many scenarios may not even be feasible, since there may be no single policy that does well in a large number of scenarios. More generally, absent a probability distribution, there is no clear reason to prefer a strategy that does well in certain states or in a large number of states, since it is always possible that those states are themselves unlikely.

Setting aside the issue of whether robust control policies are dominated by highly unlikely scenarios, economists have faulted other features of robustness and argued against using it as a criterion for designing

[5] A critique along the same spirit is offered in Section 13.4 in Savage (1954), using an example where the minimax policy is preferable to alternative rules only under a small set of states that collapses to a single state in the limit.

macroeconomic policy. For example, Sims (2001) argues that decisionmakers should avoid rules that violate the sure-thing principle, which holds that if one action is preferred to another action regardless of which event is known to occur, it should remain preferred if the event were unknown. As is well known, individuals whose preferences fail to satisfy the sure-thing principle can in principle be talked into entering gambles known as "Dutch books" they will almost surely lose.[6] But the robust control approach violates the sure-thing principle. Sims instead advocates that policymakers proceed as Bayesians; that is, they should assign subjective beliefs to the various scenarios they contemplate and then choose the strategy that minimizes the expected loss according to their subjective beliefs. Proceeding this way ensures that they will satisfy the sure-thing principle.[7] Al-Najjar and Weinstein (2009) describe other anomalies that arise from minimax behavior, which they find sufficiently compelling that they argue for rejecting preferences that naturally give rise to such behavior. The Bayesian prescription that Sims advocates as an alternative to robustness seems particularly natural for the lost in a forest problem, in which the hiker is always free to assume the road is equally likely to lie in any direction and then choose the search program that minimizes expected walking distance given this distribution. Indeed, when Bellman (1956) originally posed his problem, he suggested both minimizing the longest path (minmax) and minimizing the expected path assuming a uniform prior (min-mean) as possible solutions.

Do the anomalies that Sims and others point out provide sufficient grounds to reject robustness as a suitable criterion for choosing policy? As Siniscalchi (2009) notes in his discussion of Al-Najjar and Weinstein (2009), "there is no fundamental canon of rationality according to which every decisionmaker should feel similarly uncomfortable" with the implications of minimax strategies. Ultimately, whether one feels uncomfortable with these implications and prefers to act like a Bayesian is a matter of taste.[8] Indeed, there is some evidence that distaste for the Bayesian prescription advocated by Sims is quite common in

[6] See, for example, Yaari (1998).

[7] Sims (2001) goes on to argue that while robust control is an inappropriate policy recommendation, it might still aid policymakers who rely on simple procedural rules to guide their decisions rather than explicit optimization. The idea is to back out what beliefs would justify using the robust strategy, and then reflect on whether these beliefs seem reasonable. As Sims (2001, p. 52) notes, deriving the robust strategy "may alert decision-makers to forms of prior that, on reflection, do not seem far from what they actually might believe, yet imply decisions very different from that arrived at by other simple procedures." Interestingly, Sims' prescription cannot be applied to the lost in a forest problem: There is no distribution over the location of the road for which the minimax path minimizes expected distance. However, the lost in a forest problem offers an analogous interpretation consistent with Sims' view. Suppose that the cost of hiking can increase nonlinearly with effort. In that case, the hiker can ask what cost functions would support the minimax algorithm as a solution to the problem of minimizing expected effort, assuming the location of the road is distributed uniformly. This can alert him to paths that are different from mechanical search algorithms he already contemplated that were not derived from his true cost of effort, and to make him reflect on whether the cost functions that favor the minimax path seem reasonable.

[8] On this point, note that Yaari (1998) shows that expected utility preferences can also be subject to certain types of Dutch book manipulation. Rejecting minimax rules on the grounds that they allow for Dutch books while advocating alternative rules that allow for other types of Dutch books must therefore reflect a subjective distaste for certain types of Dutch books. But more generally, even if only minimax preferences allowed for Dutch books, the point is that if a policymaker who was made aware of this vulnerability preferred to stick to his original decision, we could not fault him for these preferences.

practice. Take the "Ellsberg paradox" laid out in Ellsberg (1961). This paradox is based on a thought experiment in which people are asked to choose between a lottery with a known probability of winning and another lottery featuring identical prizes but with an unknown probability of winning. Ellsberg argued that most people would not behave as if they assigned a fixed subjective probability to the lottery whose probability of winnings they did not know, and would instead be averse to participating in a lottery that was ambiguously specified. In support, he cites several prominent economists and decision theorists to whom he presented his thought experiment.[9] Subsequent researchers who conducted experiments offering these choices to real-life test subjects, starting with Becker and Brownson (1964), confirmed that people often fail to behave in accordance with Sims' prescription of adopting a Bayesian prior. Another example of the limited appeal of Bayesian decision rules is the aforementioned popularity of Murphy's Law. A believer in Murphy's Law would approach the lost in a forest problem expecting that they are cursed, so that they will always start their search in the worst possible location. But this implies the location of the road would depend on where they choose to search, which is inconsistent with assigning a subjective distribution to the location of the road. Thus, anyone who finds Murphy's Law appealing would be uncomfortable with imposing subjective beliefs over the location of the road. Since any recommendation on policy must respect the policymaker's tastes, we cannot objectively fault decisionmakers for not behaving like Bayesians.

Note that not all of those who have criticized the robustness criterion have advocated for Bayesian rules. For example, Savage (1951) argued that the minimax rule is unduly pessimistic, and offered an alternative approach that has come to be known as "minimax regret." This view argues for choosing not the policy that maximizes the lowest possible payoff that could ever be achieved under any conceivable scenario, but the policy that minimizes the largest possible regret, defined as the difference between the payoff under a given policy in a given state of the world and the payoff that would have been optimal to pursue in that state. In other words, a policy should seek to minimize losses relative to what could have been achieved.[10] This alternative approach has not been used in macroeconomic applications, but has been used in other economic applications, for example, in Manski (2004). Such applications are surveyed in Manski (2010), who considers decision problems where an uncertain decisionmaker must choose an appropriate treatment rule for a population that may respond differently to different treatments. Manski notes several features of the minmax-regret approach that differ from the recommendations of either the minmax or Bayesian approach. Unfortunately, little is known as to whether minimax regret exhibits similarly appealing features

[9] Ellsberg (1961, p. 655-6) cites these reactions as reflecting the spirit of Paul Samuelson's reply when confronted with the fact that his preferences were at odds with certain conventional axioms on preferences: "I'll satisfy my preferences. Let the axioms satisfy themselves." Interestingly, Ellsberg cites Samuelson as one who reported that he would not violate the Savage axioms when asked about the thought experiment concerning the two lotteries.

[10] For the lost in a forest problem, the optimal policy given a specific location for the road entails walking exactly one mile back to the hiker's starting location. In this case, the policy that minimizes regret turns out to be identical to the one that minimizes the loss from the worst-case outcome. But the two criterion often lead to different policy recommendations.

in macroeconomic applications that involve meeting targets as opposed to assigning treatments. Again, though, any preference for minimax regret is a matter of subjective tastes, just as with the preference for Bayesian decision rules.

The theme that runs through the discussion thus far is that if decisionmakers cannot assign probabilities to scenarios they are uncertain about, there is no inherently correct criterion on how to choose a policy. As Manski (2000, p. 421) put it, "there is no compelling reason why the decision maker should or should not use the maximin rule when [the probability distribution] is a fixed but unknown objective function. In this setting, the appeal of the maximin rule is a personal rather than normative matter. Some decision makers may deem it essential to protect against worst-case scenarios, while others may not." Thus, one can point to unappealing elements about robust control, but these do not definitively rule out this approach. What, then, is the appeal of robustness that defenders point to? Aside from the intuitive appeal some find in minimizing risk exposure, proponents of robustness often cite the work on ambiguity aversion, starting with the work of Gilboa and Schmeidler (1989), as motivation for robustness.[11] The latter literature is concerned not with the normative question of what a decisionmaker should do if he does not know the probability associated with various relevant scenarios, but with why people often seem averse to situations in which they do not know the probabilities of various outcomes. More precisely, this literature asks whether there exist coherent preferences over "acts" in the sense of Savage (1954) that are consistent with preferring clearly posed choice problems to ambiguously posed ones. Savage defined an "act" as a detailed description of the outcomes that would occur in every possible state of the world without any reference to the probability these states will be realized. Gliboa and Schmeidler asked if there are preferences that would tend to favor scenarios in which the probabilities of final outcomes are clearly stipulated over scenarios in which the probabilities of final outcomes are not clearly stipulated. For example, could individuals systematically prefer an act where in each state of the world the individual is offered the same lottery with known odds of winning each prize to an act where the prizes and odds of winning vary across states, so the odds of winning a particular prize depends on one's subjective perceptions of a given state occurring? Some of the preference specifications that can generate ambiguity aversion imply the agent behaves as if he contemplated a set of subjective probability distributions over the different states, and then chose the act that guaranteed the highest worst case expected utility over these distributions, that is, the strategy that was robust to the set of subjective beliefs he entertains. These preferences can rationalize why people would appear to choose strategies that are robust to the models they are subjectively willing to contemplate.

Of course, just as the appeal of Sims' directive to accept the sure-thing principle amounts to a matter of taste rather than rationality, there is no compelling justification for why a decisionmaker ought to

[11] Gilboa and Schmeidler (1989) consider only static decision problems. Various authors have since worked on extending their analysis to dynamic decision problems. See, for example, Epstein and Schneider (2003), Maccheroni, Marinacci, and Rustichini (2006), and Strzalecki (2009).

find compelling the particular axiomatic restrictions on preferences that give rise to minimax behavior. Ultimately, the aforementioned literature only shows that there exist coherent preferences that justify such decisions, implying that minimax behavior does not rest on logical contradictions. But this literature does not argue that these preferences are somehow natural, nor do they offer empirical evidence that these preferences are common. Instead, these papers largely appeal to the Ellsberg paradox, which can be reconciled by appealing to preferences that imply subjective multiple priors. But as Al-Najjar and Weinstein (2009) point out, other explanations for the Ellsberg paradox exist, and the paradox does not by itself confirm all of the restrictions on preferences needed to give rise to minimax behavior.

Equally important, the axiomatic approach provides a theory of robustness to *subjective* uncertainty. That is, it can tell under what conditions decision makers will behave as if they entertain a set of beliefs regarding the likelihood of various states, and then choose the policies that are robust against this set. But robust control is typically presented as a theory of what policymakers should do in the face of *objective* uncertainty, when they cannot fathom the environment they face. To appreciate this distinction, consider the lost in a forest problem. The axiomatic approach tells us that if individuals have certain preferences over proposals for different search algorithms, their decisions can be described as if they had a set of beliefs over the location of the road, and they chose their strategy to be robust to these beliefs. But there is no restriction that their preferences must carry over to other decision problems that involve a completely different set of Savage acts than the original lost in a forest problem. Thus, there is no guarantee that individuals who follow a minimax rule in the original lost in a forest problem will also follow a minimax rule if they had to search for a road located two miles away or if they also forgot the distance they originally hiked before falling asleep. By contrast, Bellman advocated the minimax search strategy as a general rule for any search problem in which the location of the object is unknown.

In sum, whether policymakers find robustness appealing is subjective. Given the sizable engineering literature on robust control, the notion of designing systems that minimize the worst-case scenario appears to have had some appeal among engineers. Interestingly, Murphy's Law is also an export from the field of engineering.[12] The two observations may be related: If worst-case outcomes are treated as everyday occurrences, robustness would seem like an appealing criterion. Policymakers who are equally nervous about worst-case outcomes would presumably like the notion of keeping their risk exposure to a bare minimum. But less nervous policymakers may find robustness unappealing upon reflection, either because of its implications in particular models or because it leads to the anomalous results pointed out by detractors.

[12] According to Spark (2006), the law is named after aerospace engineer Edward Murphy, who complained after a technician attached a pair of sensors in a precisely incorrect configuration during a crash test Murphy was observing. Engineers on the team Murphy was working with began referring to the notion that things will inevitably go wrong as Murphy's Law, and the expression gained public notoriety after one of the engineers used it in a press conference.

4 Robustness and the Brainard Model

Now that I have described what it means for a policy to be robust, I can return to the implications of robust control for macroeconomic policy. Various authors have solved for the robust policy in particular macro models, and have carried out comparative static exercises on how the robust policy changes with underlying features of the model, with the set of models the policymaker entertains, or with the penalty parameter that governs the disutility from using a model that is different from the benchmark. Not surprisingly, the results tend to be model-specific, and there are few general results. This is also true for the literature on optimal policy under uncertainty in the Brainard tradition, which assumes policymakers are uncertain about some parameters but still know the distribution. For example, Chow (1973) finds few general results when he considers a broader model that encompasses Brainard's framework.

However, early applications of robustness did have one result in common: In all of these applications, the robust policy tended to contradict the attenuation result in Brainard's model, and instead implied that uncertain policymakers should react more aggressively to news than they would in the absence of uncertainty. Examples of this result include Sargent (1999), Stock (1999), Tetlow and von zur Muehlen (2001), Giannoni (2002), and Onatski and Stock (2002). These findings were sometimes interpreted to mean that concern for robustness leads to more aggressive policy.[13] In this section, my aim is to dispel this interpretation. In particular, I argue that the lack of attenuation is not driven by concern for robustness *per se*, but by the environments these early papers considered. When we consider the implications of robustness in the same environment that Brainard considered, we obtain similar results to his.[14]

Consider the Brainard model above, but suppose now that the policymaker no longer knows the distribution of all of the parameters that he is uncertain about. More precisely, suppose the variable to be targeted, y, is still determined by (2):

$$y = x - (k + \varepsilon_k)r + \varepsilon_u. \tag{5}$$

Since the attenuation result only arises when the policymaker is uncertain about ε_k, I will continue to assume the policymaker knows the distribution of ε_u for simplicity. By contrast, I assume the policymaker only knows that the support of ε_k is restricted to some interval $[\underline{\varepsilon}, \overline{\varepsilon}]$ that includes 0, that is, $\underline{\varepsilon} < 0 < \overline{\varepsilon}$, and does not know its distribution. Thus, the effect of r on y can be less than, equal to, or higher than k, but the policymaker cannot ascribe probabilities to these events.[15]

[13] For example, Bernanke (2007) notes: "The concern about worst-case scenarios emphasized by the robust-control approach may likewise lead to amplification rather than attenuation in the response of the optimal policy to shocks."

[14] A similar point was made by Onatski (1999), who considers a closely related example to the one I discuss in this section.

[15] The case where a decisionmaker is uncertain about the value of one or more parameters in a model is known as structured

The robust control approach in this environment can be viewed as a two-step process. First, for each value of r, we compute its worst-case scenario over all values $\varepsilon_k \in [\underline{\varepsilon}, \overline{\varepsilon}]$, or the largest expected loss the policymaker could incur. Define this expected loss as $W(r)$; that is,

$$W(r) \equiv \max_{\varepsilon_k \in [\underline{\varepsilon}, \overline{\varepsilon}]} E\left[y^2\right] = \max_{\varepsilon_k \in [\underline{\varepsilon}, \overline{\varepsilon}]} \left\{(x - (k + \varepsilon_k)r)^2 + \sigma_u^2\right\}. \tag{6}$$

Second, we choose the policy r that implies the smallest value for $W(r)$. The robust strategy is defined as the value of r that solves $\min_r W(r)$; that is,

$$\min_r \max_{\varepsilon_k \in [\underline{\varepsilon}, \overline{\varepsilon}]} \left\{(x - (k + \varepsilon_k)r)^2 + \sigma_u^2\right\}. \tag{7}$$

Below, I describe the solution to this problem and provide some intuition for it. For a rigorous derivation, see Barlevy (2009). It turns out that the robust policy hinges on the lowest value that ε_k can assume. If $\underline{\varepsilon} \leq -k$, so the coefficient $k + \varepsilon_k$ can be either positive or negative, the solution to (7) is given by

$$r = 0.$$

If instead $\underline{\varepsilon} > -k$, so the policymaker is certain that the coefficient $k + \varepsilon_k$ is positive but is unsure of its exact value, the solution to (7) is given by

$$r = \frac{x}{k + (\underline{\varepsilon} + \overline{\varepsilon})/2}. \tag{8}$$

Thus, if the policymaker knows the sign of the effect of r on y, he should respond to changes in x in a way that depends on the extreme values ε_k can assume, that is, the endpoints of the interval $[\underline{\varepsilon}, \overline{\varepsilon}]$. But if the policymaker is unsure about the sign of the effect of policy on y, he should not respond to x at all.[16]

To better understand why concerns for robustness imply this rule, consider first the result that if the policymaker is uncertain about the sign of $k + \varepsilon_k$, he should altogether abstain from responding to x. This is related to Brainard's (1967) attenuation result: There is an inherent asymmetry in that a passive policy where $r = 0$ leaves the policymaker unexposed to risk from ε_k, while a policy that sets $r \neq 0$ leaves him exposed to such risk. When the policymaker is sufficiently concerned about this risk, which turns out to hinge on whether he knows the sign of the coefficient on r, he is better off resorting to a passive policy that

uncertainty. Unstructured uncertainty refers to the case where the decisionmaker is unsure about the probability distribution for the endogenous variables in the model, and considers any distribution that is some entropy distance from a benchmark model. The original finding that robust policy tends to be aggressive was demonstrated for both types of uncertainty.

[16] Interestingly, Kocherlakota and Phelan (2009) also provide an example where doing nothing is the minmax solution to a policy problem. They show that if policymakers rely on an interventionist mechanism, there is an environment in which intervention plays no useful role, since it does not allow the policymaker to achieve a superior outcome, but it admits an additional equilibrium that yields an inferior outcome. Thus, an interventionist mechanism is more vulnerable to bad outcomes than a mechanism that does nothing. The example here yields a similar outcome, but does not involve multiple equilibria.

protects him from this risk than trying to offset nonzero values of x.[17] However, the attenuation here is both more extreme and more abrupt than what Brainard found. In Brainard's formulation, the policymaker will always act to offset x, at least in part, but he will moderate his response to x continuously with σ_k^2. By contrast, robustness considerations imply a threshold level for the lower support of ε_k, which, if crossed, leads the policymaker to radically shift from actively offsetting x to not responding to it at all.

The abrupt shift in policy in response to small changes in ε_k demonstrates one of the criticisms of robust control cited earlier – namely, that this approach formulates policy based on how it performs in specific states of the world rather than how it performs in general. When $\underline{\varepsilon}$ is close to $-k$, it turns out that the policymaker is almost indifferent among a large set of policies that achieve roughly the same worst-case loss. When $\underline{\varepsilon}$ is just below $-k$, setting $r = 0$ performs slightly better under the worst-case scenario than setting r according to (8). When $\underline{\varepsilon}$ is just above $-k$, setting r according to (8) performs slightly better under the worst-case scenario than setting $r = 0$. When $\underline{\varepsilon}$ is exactly equal to $-k$, both strategies perform equally well in the worst-case scenario, as do all values of r that fall between these two extremes. However, the two strategies lead to quite distinct payoffs for values of ε_k that are between $\underline{\varepsilon}$ and $\overline{\varepsilon}$. Hence, concerns for robustness might encourage dramatic changes in policy to eke out small gains under the worst-case scenario, even if these changes result in substantially larger losses in most other scenarios. A dire pessimist would feel perfectly comfortable guarding against the worst-case scenario in this way. But in situations such as this, where the policymaker chooses his policy based on minor differences in how the policies perform in one particular case even though the policies result in large differences in other cases, the robust control approach has a certain tail-wagging-the-dog aspect to it that some may find unappealing.

Next, consider what concern for robustness dictates when the policymaker knows the sign of $k + \varepsilon_k$ but not its precise magnitude. To see why r depends on the endpoints of the interval $[\underline{\varepsilon}, \overline{\varepsilon}]$, consider figure 2. This figure depicts the expected loss $(x - (k + \varepsilon_k)\, r)^2 + \sigma_u^2$ for a fixed r against different values of ε_k. The loss function is quadratic and convex, which implies the largest loss will occur at one of the two extreme values for ε_k. Panel A of figure 2 illustrates a case in which the expected losses at $\varepsilon_k = \underline{\varepsilon}$ and $\varepsilon_k = \overline{\varepsilon}$ are unequal: The expected loss is larger for $\varepsilon_k = \overline{\varepsilon}$. But if the losses are unequal under some rule r, that value of r fails to minimize the worst-case scenario. This is because, as illustrated in panel B of figure 2, changing r will shift the loss function to the left or the right (it might also change the shape of the loss function, but for our purposes this can effectively be ignored). The policymaker should thus be able to reduce the largest possible loss over all values of ε_k in $[\underline{\varepsilon}, \overline{\varepsilon}]$. Although shifting r would lead to a greater loss if ε_k happened to

[17] An alternative intuition is as follows. Suppose the policymaker could achieve a lower loss function by using r. If the support of the coefficient includes k, an evil agent disposed to lower the policymaker's utility could always shut off the effect of r by setting the coefficient on r to zero. Thus, once the support of k includes 0, policymakers should not be able to use intervention to make things better off. What the results show is that in fact, the evil agent can make the policymaker worse off for choosing $r \neq 0$ when the coefficient on r can assume either positive or negative values.

equal $\underline{\varepsilon}$, since the goal of a robust policy is to minimize the largest *possible* loss, shifting r in this direction is desirable. Robustness concerns would therefore lead the policymaker to adjust r until the losses at the two extreme values were balanced, so that the loss associated with the policy being maximally effective is exactly equal to the loss associated with the policy being minimally effective.

When there is no uncertainty, that is, when $\underline{\varepsilon} = \overline{\varepsilon} = 0$, the policymaker would set $r = x/k$, since this would set y exactly equal to its target. When there is uncertainty, whether the robust policy will respond to x more or less aggressively than this benchmark depends on how the lower and upper bounds are located relative to 0. If the region of uncertainty is symmetric around 0 so that $\underline{\varepsilon} = -\overline{\varepsilon}$, uncertainty has no effect on policy. To see this, note that if we were to set $r = x/k$, the expected loss would reduce to $(x/k)^2\, \varepsilon_k^2 + \sigma_u^2$, which is symmetric in ε_k. Hence, setting r to offset x would naturally balance the loss at the two extremes. But if the region of uncertainty is asymmetric around 0, setting $r = x/k$ would fail to balance the expected losses at the two extremes, and r would have to be adjusted so that it either responds more or less to x than in the case of complete certainty. In particular, the response to x will be attenuated if $\overline{\varepsilon} > -\underline{\varepsilon}$, that is, if the potential for an overly powerful stimulus is greater than the potential for stimulus that is far too weak, and this response will be amplified in the opposite scenario.

This result begs the question of when the support for ε_k will be symmetric or asymmetric in a particular direction. If the region of uncertainty is constructed using past data on y, x, and r, any asymmetry would have to be driven by differences in detection probabilities across different scenarios – for example, if it is more difficult to detect k when its value is large than when it is small. This may occur if the distribution of ε_u were skewed in a particular direction. But if the distribution of ε_u were symmetric around 0, policymakers who rely on past data should find it equally difficult to detect deviations in either direction, and the robust policy would likely react to shocks in the same way as if k were known with certainty.

The above example shows that the robustness criterion does not inherently imply that policy should be more aggressive in the face of uncertainty. Quite to the contrary, the robust policy exhibits a more extreme form of the same attenuation principle that Brainard demonstrated, for essentially the same reason: The asymmetry between how passive and active policies leave the policymaker exposed to risk tends to favor passive policies. More generally, whether uncertainty about the economic environment leads to attenuation or more aggressive policy depends on asymmetries in the underlying environment. If the policymaker entertains the possibility that policy can be far too effective but not that it will be very ineffective, he will naturally tend to attenuate his policy. But if his beliefs are reversed, he will tend to magnify his response to news about potential deviations from the target level.[18]

[18]Rustem, Wieland, and Zakovic (2007) also consider robust control in asymmetric models, although they do not discuss the implications of these asymmetries for attenuation.

5 Robustness and Aggressive Rules

Given that robustness considerations do not necessarily imply more aggressive policies, which features of the macro models that were used in early applications of robust control account for the finding that robust policies tend to be more aggressive? The papers cited earlier consider different models, and their results are not driven by one common feature. To give some flavor of why some factors may lead robust policies to be aggressive, I now consider two simplified examples inspired by these papers. The first assumes the policymaker is uncertain about the persistence of shocks, following Sargent (1999). The second assumes the policymaker is uncertain about the tradeoff between competing objectives, following Giannoni (2002). I show that both features can tilt the robust policy toward reacting more aggressively to incoming news.

5.1 Uncertain persistence

One of the first to argue that concerns for robustness could dictate more aggressive policy was Sargent (1999). Sargent asked how optimal policy would be affected in a particular model when we account for the possibility that the model is misspecified – and especially the possibility that specification errors are serially correlated. To gain some insight on the implications of serially correlated shocks, consider the following extension of the Brainard model in which the policymaker attempts to target only one variable.[19]

Let y_t denote the value at date t of the variable that the policymaker wishes to target to 0. As in (1), I assume y_t is linear in the policy variable r_t and in an exogenous shock term x_t:

$$y_t = x_t - kr_t.$$

I now no longer assume the policymaker is uncertain about the effect of his policy on y_t. As such, it will be convenient to normalize k to 1. However, I now assume the policymaker is uncertain about the way x_t is correlated over time. That is, suppose x_t follows the AR(1) process

$$x_t = \rho x_{t-1} + \varepsilon_t,$$

where ε_t are independent and identically distributed over time with mean 0 and variance σ_ε^2, and ρ captures the autocorrelation of x_t. At each date t, the policymaker can observe x_{t-1} and condition his policy on its realization. However, he must set r_t before observing x_t. His uncertainty about x_t is due to two different factors. First, I assume he does not know ε_t at the time he chooses his policy. Second, to capture uncertainty

[19] Sargent assumed the policymaker cares about both inflation and unemployment, following the model he was discussing. Here, I continue to assume the policymaker cares about only one target. But my objective is not to replicate Sargent's results. Rather, it is to illustrate why a concern that shocks are persistent can lead to robust policies that tend to be aggressive.

about the persistence of variables over which the policymaker is uncertain, I assume the policymaker does not know the autocorrelation coefficient ρ that governs how x_{t-1} affects x_t.

Suppose the policymaker discounts future losses at rate $\beta < 1$ so that his expected loss is given by

$$E\left[\sum_{t=0}^{\infty}\beta^t y_t^2\right] = E\left[\sum_{t=0}^{\infty}\beta^t\left(x_t - r_t\right)^2\right].$$

If the policymaker knew ρ with certainty, his optimal strategy would be to set $r_t = E\left[x_t|x_{t-1}\right] = \rho x_{t-1}$. But in line with the notion that the policymaker remains uncertain about the degree of persistence, suppose he only knows that ρ falls in some interval $\left[\underline{\rho}, \overline{\rho}\right]$. Let ρ^* denote the midpoint of this interval:

$$\rho^* = \left(\underline{\rho} + \overline{\rho}\right)/2.$$

To emphasize asymmetries inherent to the loss function as opposed to the region of uncertainty, suppose the interval of uncertainty is symmetric around the certainty benchmark. An important and empirically plausible assumption in what follows is that $\rho^* > 0$; that is, the beliefs of the monetary authority are centered around the possibility that shocks are positively correlated.

Once again, we can derive the robust strategy in two steps. First, for each rule r_t, define $W(r_t)$ as the biggest loss possible among the different values of ρ; that is,

$$W\left(r_t\right) = \max_{\rho\in\left[\underline{\rho},\overline{\rho}\right]} E\left[\sum_{t=0}^{\infty}\beta^t y_t^2\right].$$

We then choose the policy rule r_t that minimizes $W(r_t)$; that is, we solve

$$\min_{r_t}\max_{\rho\in\left[\underline{\rho},\overline{\rho}\right]} E\left[\sum_{t=0}^{\infty}\beta^t y_t^2\right]. \tag{9}$$

Following Sargent (1999), I assume the policymaker is restricted in the type of policies r_t he can carry out: He must choose a rule of the form $r_t = ax_{t-1}$, where a is a constant that cannot vary over time. This restriction is meant to capture the notion that the policymaker cannot learn about the parameters over which he is uncertain and then change the way policy reacts to information as he observes x_t over time and potentially infer ρ. I further assume the expectation in (9) is the unconditional expectation of future losses; that is, the policymaker calculates his expected loss from the perspective of date 0. To simplify the calculations, I assume x_0 is drawn from the stationary distribution for x_t.

The solution to (9), subject to the constraint that $r_t = ax_{t-1}$, is derived in Barlevy (2009). The key result is that as long as $\rho^* > 0$, the robust policy would set a to a value in the interval that is strictly greater than the midpoint ρ^*. In other words, starting with the case in which the policymaker knows $\rho = \rho^*$, if we introduce a little bit of uncertainty in a symmetric fashion so that the degree of persistence can deviate equally in either direction, the robust policy would react more to a change in x_{t-1} in the face of uncertainty than it would react to such a change if the degree of persistence were known with certainty.

To understand this result, suppose the policymaker set $\rho = \rho^*$. As in the previous section, the loss function is convex in ρ, so the worst-case scenario will occur when ρ assumes one of its two extreme values, that is, when either $\rho = \underline{\rho}$ or $\rho = \overline{\rho}$. It turns out that when $a = \rho^*$, setting $\rho = \overline{\rho}$ imposes a bigger cost on the policymaker than setting $\rho = \underline{\rho}$. Intuitively, for any given ρ, setting $a = \rho^*$ will imply $y_t = (\rho - \rho^*)x_{t-1} + \varepsilon_t$. The expected deviation of y_t from its target given x_{t-1} will have the same expected magnitude in both cases; that is, $|(\rho - \rho^*)x_{t-1}|$ will be the same when $\rho = \overline{\rho}$ and $\rho = \underline{\rho}$, given $\left[\underline{\rho}, \overline{\rho}\right]$ is symmetric around ρ^*. However, the process x_t will be more persistent when ρ is higher, and so deviations from the target will be more persistent when $\rho = \overline{\rho}$ than when $\rho = \underline{\rho}$. More persistent deviations imply more volatile y_t and hence a larger expected loss. Since the robust policy tries to balance the losses at the two extreme values of ρ, the policymaker should choose a higher value for a to reduce the loss when $\rho = \overline{\rho}$.

The basic insight is that, while the loss function for the policymaker is symmetric in ρ around $\rho = 0$, if we focus on an interval that is centered around positive values of ρ, the loss function will be asymmetric. This asymmetry will tend to favor policies that react more to past shocks. In fact, this feature is not unique to policies guided by robustness. If we assumed the policymaker assigned a symmetric distribution to values of ρ in the interval $\left[\underline{\rho}, \overline{\rho}\right]$ and acted to minimize his expected losses, the asymmetry in the loss function would continue to imply that the value of a that solves $\min_a E\left[\sum_{t=0}^{\infty} \beta^t \left(\rho x_{t-1} - a x_{t-1} + \varepsilon_t\right)^2\right]$ will exceed ρ^*. Hence, the feature responsible for the aggressiveness of the robust policy is not disproportionate attention to worst-case scenarios, since even if the policymaker assigned small probabilities to extreme values of ρ, he might still respond aggressively to past shocks given such beliefs. Rather, because more persistent shocks create bigger losses than less persistent shocks, policymakers will want to minimize the damage in the event that shocks are persistent. As in the Brainard model, what tilts policy away from its certainty benchmark is an underlying asymmetry that favors certain types of policies over others.

5.2 Uncertain tradeoff parameters

Next, I turn to the Giannoni (2002) model. Giannoni focused on the case where policymakers are uncertain about the parameters in the underlying model the policymaker believes, specifically the parameters that dictate the effect of shocks on the endogenous variables in the model. If policymakers wish to target more than one of these variables, this can make them uncertain about the tradeoff between meeting their competing objectives, which in Giannoni's framework turns out to favor more aggressive policies. Up to now, I have conveniently assumed policymakers were only interested in targeting a single variable. However, tradeoffs in trying to target multiple variables can be important in designing robust policy, and so it is worth exploring a simplified example to illustrate some of these effects.

Consider the following simple static framework. Suppose the monetary authority cares about two variables, denoted y and π, where y and π represent the output gap and inflation, respectively. The monetary authority has a quadratic loss function in both terms:

$$\alpha y^2 + \pi^2. \tag{10}$$

The variables y and π are assumed to be related linearly, specifically

$$\pi = \lambda y + x, \tag{11}$$

where x is an observable shock.[20] This relationship implies a tradeoff between targeting π and y. In particular, if we set $\pi = 0$ as desired, then y would vary with x and deviate from 0. But if we set $y = 0$, then π would vary with x and deviate from 0. Substituting in (11) into the loss function allows us to express the policymaker's problem as choosing π to minimize the loss

$$\alpha \frac{(\pi - x)^2}{\lambda^2} + \pi^2.$$

Taking the first-order condition with respect to π gives us the optimal choices for y and π as

$$\pi = \frac{\alpha x}{\alpha + \lambda^2}, y = -\frac{\lambda x}{\alpha + \lambda^2}. \tag{12}$$

Suppose the policymaker were uncertain about λ, knowing only that it lies in some interval $\left[\underline{\lambda}, \overline{\lambda}\right]$. However, the policymaker still has to commit to a rule that determines π and y as functions of x. Given a value for π, the worst-case scenario over this range of λ is given by

$$\max_{\lambda \in \left[\underline{\lambda}, \overline{\lambda}\right]} \alpha \frac{(\pi - x)^2}{\lambda^2} + \pi^2.$$

For any choice of π except $\pi = x$, the worst case corresponds to $\lambda = \underline{\lambda}$. If $\pi = x$, the value of λ has no effect on the loss function. Hence, the robust strategy would be to choose a rule that sets π and y to their values in (12) as if $\lambda = \underline{\lambda}$. As long as the certainty benchmark λ lies in the interior of the uncertainty interval $\left[\underline{\lambda}, \overline{\lambda}\right]$, concerns for robustness will lead the policymaker to have y respond less to x and π respond more to x. Intuitively, when a shock x causes π to deviate from its target, a lower value of λ implies that pushing π back to its target rate of 0 would require that y deviate by a larger extent from its desired target. The worst-case scenario would thus amount to the policymaker fearing that meeting one target comes at a much higher cost in terms of missing the other target. Hence, the worst-case scenario would suggest that

[20] This relationship is meant to capture the Phillips curve relationship in Giannoni (2002), which implies inflation at date t depends on the output gap at date t, expected inflation at date $t+1$, and a shock term, so that $\pi_t = \lambda y_t + \beta E_t \pi_{t+1} + x_t$. Using a static model ignores an important contribution of Giannoni's work in explicitly incorporating a forward-looking term $E_t \pi_{t+1}$ that is a staple of New Keynesian models, but this simplification makes the analysis considerably more transparent.

the policymaker should not try to stabilize π too much for fear of missing his target for y. Put another way, the robust policy implies more aggressively stabilizing y and less aggressively stabilizing π.

Figure 3 illustrates this result graphically. The problem facing the policymaker is to choose a point from the line given by $\pi = \lambda y + x$. Ideally, it would like to move toward the origin, where $\pi = y = 0$. Changing λ will rotate the line from which the policymaker must choose, as depicted in the figure. A lower value of λ corresponds to a flatter curve. Given the policymaker prefers to be close to the origin, a flatter curve leaves the policymaker with distinctly worse options that are farther from the origin, since one can show that the policymaker would only choose points in the upper left quadrant of the figure. This explains why the worst-case scenario corresponds to the flattest curve possible. If we assume the policymaker must choose his relative position on the line before knowing the slope of the line (that is, before knowing λ), then the flatter the line could be, the greater his incentive will be to locate close to the π-axis rather than risk deviating from his target on both variables, as indicated by the path with the arrow. This explains why he will act more aggressively to isolate the effects of x on y.

Robustness concerns thus encourage the policymaker to proceed as if he knew λ was equal to its lowest possible value. Note the difference from the two earlier models, in which the robust policy recommended balancing losses associated with two polar opposite scenarios. Here, the policy equates *marginal* losses for a particular risk (the loss from letting y deviate a little more from its target and the loss from letting π deviate a little more), namely the risk that λ will be low so that stabilizing inflation comes at a large cost of destabilizing output.

6 Conclusion

My objective in this paper was to review the robustness criterion that some macroeconomists have advocated for designing policy when policymakers are at a loss to assign probabilities to the scenarios they contemplate. The key theme I have stressed is that in the absence of objective probabilities, there is no right criterion by which to choose policy. Critics of robust control are really just expressing their own subjective views about some of the implications of this approach, just as advocates of robust control are expressing their own subjective views that this approach has some appealing features. While there is no one right answer for whether robustness is an appropriate criterion for designing policy, even critics of robustness are willing to acknowledge that deriving the robust policy and understanding its properties can be useful, not necessarily because policymakers should adopt this policy but because solving for this rule can alert policymakers to where their policies might go amiss and whether acting to minimize the potential for losses is costly. Since the exact features of robust policies will depend on the underlying environment, typically one would have to solve for the minimax policy in each application rather than rely on general results on the nature of robust policy rules. Indeed, one of the points of this survey was that concern for robustness does not inherently contradict the attenuation principle derived by Brainard (1967), contrary to what some have inferred based on the results that emerged from the particular models used in early applications of robust control.

My survey focused exclusively on the normative question of how macroeconomic policymakers should behave when they are uncertain about the environment they face. However, equally important questions left out from this survey hinge on whether policymakers and agents do in fact exhibit a preference for robustness and choose policies this way. For example, Ellison and Sargent (2009) argue that concern for robustness can help to understand monetary policy and why the Federal Open Market Committee seems to ignore projections made by their staff even though they know these are good forecasts of future economic conditions. In addition, various authors have investigated the nature of optimal policy when private agents follow robust decision rules, both when policymakers themselves face no uncertainty and when policymakers are themselves concerned about robustness. Even if there are compelling arguments for why robustness may not always be desirable, evidence that agents have a taste for robustness may still force policymakers to take robustness concerns into account when designing macroeconomic policy.

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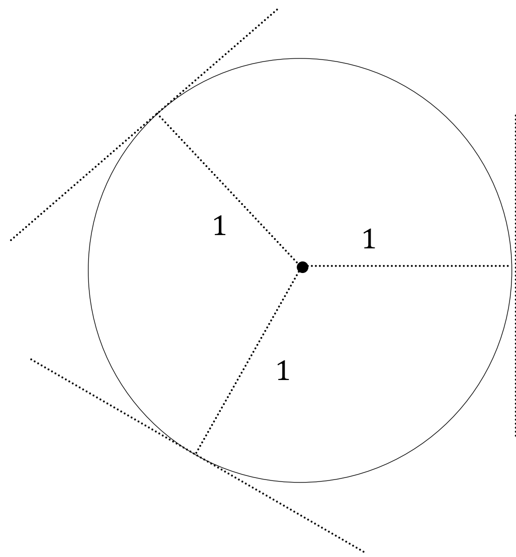
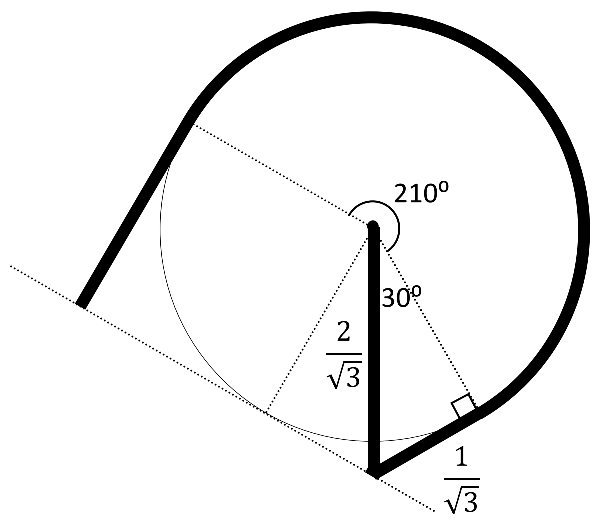
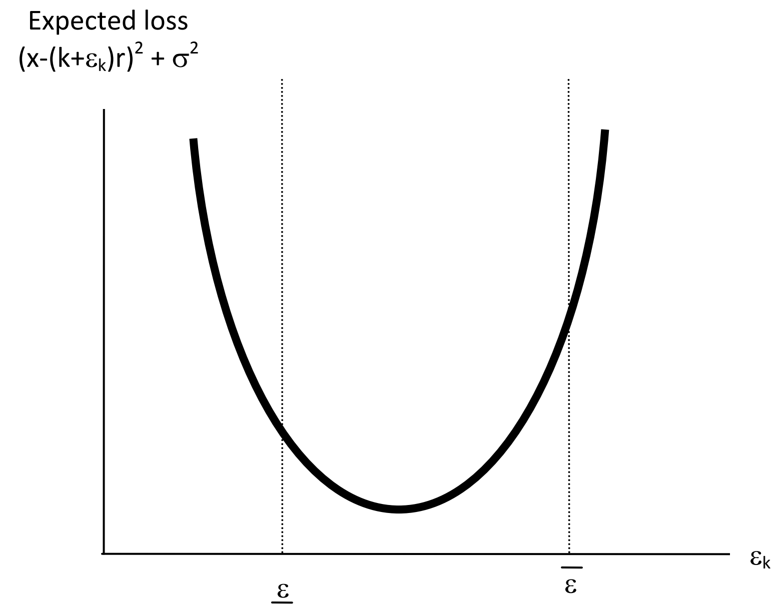


(a) Graphical representation of the problem

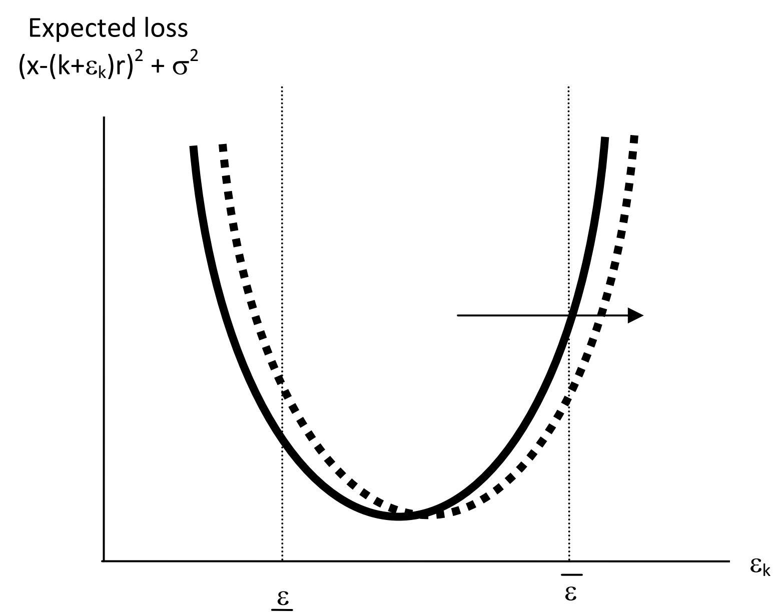


(b) Min-max path, derived by Isbell (1957)

Figure 1: Graphical Representation of the "Lost in a Forest" Problem



(a) Expected loss function with unequal expected losses at extremes



(b) Adjusting r to balance expected losses at the two extremes

Figure 2: Loss Functions for the Robust Brainard Model

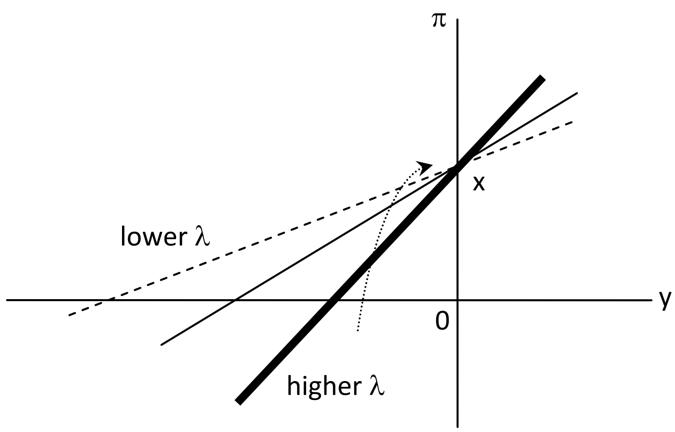


Figure 3: Graphical Representation of Uncertain Tradeoffs Model

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