



Federal Reserve Bank of Chicago

## **Equilibrium Bank Runs Revisited**

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## Abstract

Peck and Shell (2003) show that it is possible to get a bank run in a Diamond-Dybvig environment. The mechanism they use, however, is not an optimal one. When an optimal mechanism is used, the bank run equilibrium disappears.

## 1 Introduction

Although Diamond and Dybvig's (1983) seminal article is associated with bank runs, it's actually difficult to generate them. For example, when there is no aggregate risk, they demonstrate that a bank run equilibrium cannot exist when the deposit contract is appropriately designed. The optimal contract is a "standard" deposit contract augmented by a suspension of convertibility if too many people want to withdraw early. In the second part of their article, they assert, but do not demonstrate, that deposit contracts will be subject to bank runs when there is aggregate risk. It was not until Green and Lin (2003), GL, that an optimal deposit contract under aggregate risk was fully characterized. GL take a mechanism design approach and demonstrate that the optimal deposit contract does not have a bank run equilibrium. Subsequently, Peck and Shell (2003), PS, modify the GL environment, and produce a bank run equilibrium.

In a departure from GL, PS assume that depositors do not know their positions in the service queue. This seems important. Among other things, it means that GL's powerful backward induction argument—that appears to eliminate bank run equilibria—does not apply. One can interpret PS's modifications as generalizing the GL environment. In particular, if depositors do not know their positions in the service

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queue, as in GL, then, in principle, the mechanism (or planner) can choose to either inform or not inform depositors regarding their positions. (GL can be interpreted as restricting the mechanism to always inform depositors about their positions in the queue.)

Independent of how one views the GL environment vis-à-vis the PS environment, the mechanism that GL adopt is optimal for their economic environment. Their mechanism is a direct revelation mechanism, where each depositor announces his private information or type to the planner. PS also use a direct revelation mechanism. But for their more general economic environment, the direct revelation mechanism may not be an optimal one. I pursue this idea by constructing an *indirect* mechanism and show that it *uniquely* implements the best allocation, or at least an allocation that is arbitrarily close to it. In other words, my indirect mechanism does not admit a bank run equilibrium. This result reinforces an earlier observation: When deposit contracts are appropriately designed, bank runs are hard to come by in the Diamond-Dybvig environment.

A bank run equilibrium can arise in a GL environment when depositors' types are correlated and allocations are implemented by a direct revelation mechanism, see Ennis and Keister (2009b).<sup>1</sup> Cavalcanti and Monteiro (2011) examine indirect mechanisms in this environment and demonstrate that the best allocation can be uniquely implemented in dominant strategies. Their backward induction argument, however, will not work in the more general PS environment, where depositors do not know their positions in the queue.<sup>2</sup> The indirect mechanism that I construct can uniquely implement the best allocation for either GL- and PS-type environments in Nash equilibrium strategies.

The paper is organized as follows. The next section describes the economic environment. Section 3 characterizes the best implementable allocation. Sections 4 and 5 construct mechanisms that uniquely implements it. Some concluding comments are offered in the final section.

## 2 Environment

There are three dates: 0, 1 and 2. The economy is endowed with  $Y > 0$  units of date-1 goods. A constant returns to scale technology transforms  $y$  units of date-1 goods into  $yR > y$  units of date-2 goods.

There are  $N$  *ex ante* identical agents. An agent is one of two types  $t \in T = \{1, 2\}$ : patient,  $t = 1$ , or impatient,  $t = 2$ . The utility function for an impatient agent is  $u(c^1)$  and the utility function for a patient agent is  $v(c^1 + c^2)$ , where  $c^1$  is date-1

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<sup>1</sup>GL assume that depositor types are identically and independently distributed.

<sup>2</sup>Cavalcanti and Monteiro (2011) propose an alternative indirect mechanism when they examine a PS environment. In one example, they show that their indirect mechanism uniquely implements the best implementable allocation. However, in another example, their indirect mechanism has a bank run equilibrium.

consumption and  $c^2$  is date-2 consumption.  $u$  and  $v$  are increasing, strictly concave, and twice continuously differentiable. Agents maximize expected utility.

The number of patient agents in economy is drawn from the probability distribution  $\pi = (\pi_0, \dots, \pi_N)$ , where  $\pi_n > 0$ ,  $n \in \{1, \dots, N\} \equiv \mathbb{N}$ , is the probability that there are  $n$  patient agents. A *queue* is the vector  $t^N = (t_1, \dots, t_N) \in T^N$ , where  $t_k \in T$  is the type of agent that occupies the  $k^{\text{th}}$  position/coordinate in the queue. Let  $P_n = \{t^N \in T^N \mid \#2 \in t^N = n\}$  and  $Q_n = \{j \mid t_j = 2 \text{ for } t^N \in P_n\}$ , where ‘ $\#2$ ’ is the number of patient agents.  $P_n$  is the set of queues with  $n$  patient agents and  $Q_n$  is the queue positions of the  $n$  patient agents in  $t^N \in P_n$ . The probability that  $t^N \in P_n$  is  $\pi_n / \#P_n = \pi_n / \binom{N}{n}$ , where  $\#P_n$  is the number of queues  $t^N \in P_n$ . This specification implies that all potential queues with  $n$  patient agents are equally likely. Agents are randomly assigned a position in the queue, where the (unconditional) probability that an agent is assigned to position  $k$  is  $1/N$ . For convenience, call the agent assigned to position  $k$  agent  $k$ .

The queue realization,  $t^N$ , is observed by no one: not by any of the agents nor the planner. Each agent, however, privately observes his type  $t \in T$ .

The timing of events and actions is as follows. At date 0, the planner constructs a mechanism that determines how date-1 and date-2 consumption are allocated among the  $N$  agents, and queue  $t^N$  is realized. A mechanism is a set of announcements,  $M$  and  $A$ , and an allocation rule,  $c = (c^1, c^2)$  where  $c^1 = (c_1^1, \dots, c_N^1)$  and  $c^2 = (c_1^2, \dots, c_N^2)$ . At date 1, agents sequentially meet the planner, starting with agent 1. In a meeting with agent  $k$ , the planner announces  $a_k \in A$  and agent  $k$  responds with  $m_k \in M$ . Only agent  $k$  and the planner can directly observe  $a_k$  and  $m_k$ . (But the planner can reveal  $(a_k, m_k)$  to agent  $j \geq k$  via announcement  $a_j$ , if he wishes.) There is a sequential service constraint at date 1, which means the planner allocates date-1 consumption to agent  $k \in \mathbb{N}$  based on the announcements of agents  $j \leq k$ , i.e.,  $c_k^1(m^{k-1}, m_k)$ , where  $m^{k-1} = (m_1, \dots, m_{k-1})$ .<sup>3</sup> Agents consume the date-1 good at their date-1 meetings with the planner. After all agents have met the planner, the planner simultaneously allocates the date-2 consumption good to each agent based on all of the date-1 announcements made by the agents, i.e., agent  $k$  receives  $c_k^2(m^N)$ , where  $m^N = (m_1, \dots, m_N) \in M^N$ .

### 3 Best Weakly Implementable Allocation

An allocation is weakly implementable if it is an outcome to some equilibrium of the mechanism; it is strongly (or uniquely) implementable if it is an outcome to every equilibrium of the mechanism. Among the set of weakly implementable allocations, the best weakly implementable allocation provides agents with the highest expected

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<sup>3</sup> $c_k^1$  is also a function of  $a_k$ . The notation in the text anticipates the result that the best implementable allocation is consistent with  $a_k = \emptyset$ , i.e., the planner does not make an announcement (or does not reveal any information) to agents. As a result, the best implementable allocation is only a function of agents’ announcements.

utility. To characterize the best weakly implementable allocation, it is without loss of generality to restrict the planner to use a direct revelation mechanism, where agents make truthful announcement,  $m_k = t_k \in M^D = \{1, 2\}$ . The economy-wide welfare—which is the expected utility of an agent before he learns his type—associated with allocation rule  $c$  when agents use strategies  $m_k \in M^D$  is

$$\sum_{n=0}^N \frac{\pi_n}{\binom{N}{n}} \sum_{t^N \in P_n} \sum_{k=1}^N U [c_k^1(m^{k-1}, m_k), c_k^2(m_1^N), t_k], \quad (1)$$

where

$$U [c_k^1(m^{k-1}, m_k), c_k^2(m_1^N), t_k] = u [c_k^1(m^{k-1}, m_k)] \text{ if } t_k = 1$$

and

$$U [c_k^1(m^{k-1}, m_k), c_k^2(m_1^N), t_k] = v [c_k^1(m^{k-1}, m_k) + c_k^2(m^N)] \text{ if } t_k = 2$$

The allocation rule  $c$  is feasible, i.e., there exists sufficient resources to pay for  $c$  for all  $m_k \in M^D$ ,  $k \in \mathbb{N}$ , if

$$R \left( Y - \sum_{k=1}^N c_k^1(m^{k-1}, m_k) \right) \geq \sum_{k=1}^N c_k^2(m^N). \quad (2)$$

Allocation rule  $c$  must be incentive compatible in the sense that agent  $k$  has no reason to announce  $m_k \neq t_k$ . Since impatient agent  $k$  only values date-1 consumption, he always announces  $m_k = 1$ .<sup>4</sup> When  $A = \emptyset$ ,<sup>5</sup> patient agent  $k$  has no incentive to depart from the strategy  $m_k = 2$ , assuming that all other agents  $j$  announce  $m_j = t_j$ , if

$$\begin{aligned} & \sum_{n=1}^N \hat{\pi}_n \sum_{t^N \in P_n} \frac{1}{n} \sum_{k \in Q_n} v [c_k^1(t^{k-1}, 2) + c_k^2(t^{k-1}, 2, t_{k+1}^N)] \geq \\ & \sum_{n=1}^N \hat{\pi}_n \sum_{t^N \in P_n} \frac{1}{n} \sum_{k \in Q_n} v [c_k^1(t^{k-1}, 1) + c_k^2(t^{k-1}, 1, t_{k+1}^N)] + \delta, \end{aligned} \quad (3)$$

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<sup>4</sup>This anticipates the result that the best weakly implementable allocation provides zero date-1 consumption to patient agents, which implies that the incentive compatibility constraint for impatient agents is always slack.

<sup>5</sup>To characterize the best weakly implementable allocation, one wants to choose from the largest possible set of incentive compatible allocations. This occurs when  $A = \emptyset$ , i.e., the planner makes no announcements. In particular, when  $A = \emptyset$ , there is only one incentive compatibility constraint for all patient agents, (3). When  $A \neq \emptyset$ , there will be distinct incentive constraints for agents  $k$  who receive information  $a_k$  from the mechanism. For example, if  $a_k = k$ , i.e., the planner announced to each agent his place in the queue, then there would be  $N$  incentive compatibility constraints for patient agents; one for each queue position. Since an appropriately weighted average of these distinct incentive constraints reduces to the single incentive constraint (3), the set of incentive compatible allocations when  $A \neq \emptyset$  is a subset of the set of incentive compatible allocations when  $A = \emptyset$ .

where  $x_i^j = (x_i, \dots, x_j)$ ,  $\delta \geq 0$  is a parameter, and

$$\hat{\pi}_n = \frac{\pi_n / \binom{N}{n}}{\sum_{n=1}^N \pi_n / \binom{N}{n}}$$

is the conditional probability that agent  $k$  is in a specific queue that has  $n$  patient agent. The  $1/n$  terms that appear in (3) reflect that a patient agent has a  $1/n$  chance of occupying each of the patient queue positions in  $Q_n$ .

Denote the solution to the problem

$$\max_c (1) \text{ subject to } (2) \text{ and } (3), \quad (4)$$

where  $m_k = t_k$  for all  $k \in \mathbb{N}$  in (1) and (2),

as  $c^*(\delta) = (c^{1*}(\delta), c^{2*}(\delta))$ . When agents use truth-telling strategies,  $c^*(\delta)$  has the feature that impatient agents consume only at date 1 and patient agents consume only at date 2. The best-weakly implementable allocation is  $c^*(0)$ ; the allocation rule  $c^*(0)$  corresponds to the analysis contained in PS's Appendix B.

Both PS and Ennis and Keister (2009b) demonstrate, by example, that mechanism  $(M^D, c^*(0))$  can have two equilibria: one where agents play truth-telling strategies,  $m_k = t_k$  for all  $k \in \mathbb{N}$ , and another where agents play bank run strategies,  $m_k = 1$  for all  $k \in \mathbb{N}$ .<sup>6</sup> The bank run equilibria arise in these examples because the direct revelation mechanism  $(M^D, c^*(0))$  is not an optimal mechanism. An optimal mechanism may be a direct mechanism with  $A \neq \emptyset$  or an indirect mechanism, (or both).

## 4 Direct Mechanisms with $A \neq \emptyset$

When  $c^*(0)$  cannot be uniquely implemented by the direct mechanism  $(M^D, c^*(0))$ , the optimal mechanism may be a direct mechanism with  $A \neq \emptyset$ , i.e.,  $(A, M^D, c^*(0))$ . Consider first the example provided by Ennis and Keister (2009b), where, as in PS, agents do not know their place in the queue. Ennis and Keister (2009b) assume the preference specification of GL, which implies that incentive constraint (3) does not bind for the allocation rule  $c^*(0)$ . In addition, we know from GL that when  $A = \mathbb{N}$  and  $a_k = k$ , i.e., the planner announces the agent's position in the queue, none of the  $N$  incentive compatibility constraints for patient agents bind for the allocation rule  $c^*(0)$ . This means that mechanism  $(A = \mathbb{N}, M^D, c^*(0))$  can weakly implement the best allocation in  $c^*(0)$ . And the main result of GL implies that mechanism  $(A = \mathbb{N}, M^D, c^*(0))$  can strongly implement  $c^*(0)$ . Therefore,  $(A = \mathbb{N}, M^D, c^*(0))$  is an optimal mechanism;  $(M^D, c^*(0))$  admits a bank run equilibrium only because it is a suboptimal mechanism.

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<sup>6</sup>The Ennis and Keister (2009b) example that I refer to is their bank run example in section 4.2 of their paper, where agents do not know their position in the queue, as in PS, but where the utility functions of patient and impatient agents are the same, as in GL.

Consider now the example provided by PS in their Appendix B. Nosal and Wallace (2009) show that the best weakly implementable allocation,  $c^*(0)$ , is not weakly implementable if the direct mechanism is characterized by  $A = \mathbb{N}$  and  $M = \{1, 2\}$ . This implies that the mechanism used by PS,  $(M^D, c^*(0))$ , is an optimal *direct* mechanism. But the optimal mechanism may not be a direct mechanism.

## 5 Indirect Mechanisms

Suppose that mechanism  $(M^D, c^*(0))$  weakly, but not strongly, implements the best allocation in  $c^*(0)$ , and that mechanism  $(A, M^D, c^*(0))$ , where  $A \neq \emptyset$ , cannot weakly implement the best allocation in  $c^*(0)$ . Since a direct mechanism cannot uniquely implement the best allocation in  $c^*(0)$ , I construct an indirect mechanism that can uniquely implement an allocation that is arbitrarily close to  $c^*(0)$ , i.e., allocation  $c^*(\delta)$ , where  $\delta$  is arbitrarily close to zero.

The indirect mechanism  $(M^I, c)$  has  $M^I \in \{1, 2, g\}$ . One can think of the payoff associated with the announcement  $g$  as providing the depositor with a (minimum) guaranteed payoff in date 2. This is in contrast to the date 2 payoff of agent  $k$  who announces  $m_k = 2$  when allocation rule  $c^*(0)$  is in place; his minimum guaranteed payoff is the lowest possible date-2 payoff associated with announcing  $m_k = 2$ . Before I describe the payoffs associated with announcements, the following notation is needed. The allocation rule for the indirect mechanism is  $c = (c^1, c^2)$ , and the date- $s$  payoff to agent  $j$  who announces  $m_j$  is denoted as  $c_j^s|_{m_j}$ . Define  $Z$  as the set of queue positions for agents who announce  $g$ , i.e.,  $Z = \{j|m_j = g\}$  and  $\#Z$  as the number of agents in  $Z$ . Define  $\hat{m}^{k-1}$  as the message vector of length  $k-1$  where for each  $j \leq k-1$ ,  $\hat{m}_j = 1$  if either  $m_j = 1$  or  $m_j = g$ , and  $\hat{m}_j = 2$  if  $m_j = 2$ .

I now specify the allocation rule  $c$  for the indirect mechanism  $(M^I, c)$ . The basic construction of  $c$  uses  $c^*(\delta)$ , where  $\delta > 0$  is arbitrarily small. To reduce notational clutter I will suppress the ‘ $\delta$ ’ when using allocations in  $c^*(\delta)$  to describe  $c$ . If agent  $j$  announces  $m_j = g$ , then

$$\begin{aligned} c_j^1|_g &= 0 \\ c_j^2|_g &= c_j^{1*}(\hat{m}^{j-1}, 1)(1 + \varepsilon(\#Z)) \text{ for all } j, \varepsilon > 0, \end{aligned} \tag{5}$$

where  $0 < \varepsilon(1) < \varepsilon(2) < \dots < \varepsilon(\#Z) < \dots < \varepsilon(N)$ , and  $R > 1 + \varepsilon(N)$ . The date-2 payoffs are feasible since  $R > 1 + \varepsilon(N)$ . Note that the date-2 payoff from announcing  $m_j = g$  is guaranteed to be at least  $c_j^{1*}(\hat{m}^{j-1}, 1)(1 + \varepsilon(1))$ . I will assume that  $\varepsilon(N)$  is arbitrarily small.

If agent  $j$  announces  $m_j = 1$ , then

$$\begin{aligned} c_j^1|_1 &= \begin{cases} c_j^{1*}(\hat{m}^{j-1}, 1) & \text{if } j < N \\ c_j^{1*}(\hat{m}^{j-1}, 1) + \Delta & \text{if } j = N \end{cases}, \\ c_j^2|_1 &= 0. \end{aligned} \tag{6}$$

Allocation rule (5) has the feature that if some agents announce  $m_k = g$ , then the planner accumulates “excess goods” since  $R > 1 + \varepsilon(j)$  for all  $j \in \mathbb{N}$ . The total amount of this excess after all agents make their date-1 announcements, denoted as  $\Delta$ , is

$$\Delta = \sum_{z \in Z} \frac{(R - 1 - \varepsilon(\#Z)) c_z^{1*}(\hat{m}^{z-1}, 1)}{R}.$$

According to (6), agents who announce  $m_k = 1$  and occupy the first  $N - 1$  positions in the queue receive the consumption payoff that they would get under the direct revelation mechanism  $(M^D, c^*(\delta))$ , assuming that  $\hat{m}^{k-1}$  is used as the announcement vector. Agent  $N$  who announces  $m_N = 1$  receives an additional consumption payment of  $\Delta$ .

Finally, if agent  $j$  announces  $m_j = 2$ , then

$$\begin{aligned} c_j^1|_2 &= 0, \\ c_j^2|_2 &= \begin{cases} c_j^{2*}(\hat{m}^N) & \text{if } j < N \\ c_j^{2*}(\hat{m}^N) + \Delta R & \text{if } j = N \end{cases}. \end{aligned} \quad (7)$$

The structure of the payments associated announcing  $m_j = 2$  resembles that of announcing  $m_j = 1$ , except that in the former positive payments are made at date 2 and in the latter at date 1. Note that the allocation rule (5)-(7) has the planner sometimes throwing away goods. This happens when agent  $N$  announces  $m_N = g$ .

**Proposition 1** *The indirect mechanism  $(M^I, c)$  uniquely implements in Nash equilibrium an allocation that is arbitrarily close to the best weakly implementable allocation in  $c^*(0)$ .*

**Proof.** First, there does not exist an equilibrium where all patient agents  $j$  randomize between announcing  $m_j = 1$  and  $m_j = 2$  or where all patient agents  $j$  announce  $m_j = 1$  with probability one. Suppose that such an equilibrium exists. Then, suppose that patient agent  $k$  defects from proposed play and announces  $m_k = g$  with probability one. The payoff associated with this announcement, given by (5), is  $c_k^{1*}(m^{k-1}, 1)(1 + \varepsilon(1))$ , which strictly exceeds the proposed equilibrium payment associated with announcing  $m_k = 1$ ,  $c_k^{1*}(m^{k-1}, 1)$ ; a contradiction.

Second, there does not exist an equilibrium where all patient agents  $j$  announce  $m_j = g$  with probability one. Suppose such an equilibrium exists. Then, the equilibrium expected utility to patient agent  $k$  is

$$\sum_{n=1}^N \hat{\pi}_n \sum_{t^N \in P_n} \frac{1}{n} \left\{ \sum_{k \in Q_n} v [c_k^{1*}(\hat{m}^{k-1}, 1)(1 + \varepsilon(n))] \right\}. \quad (8)$$



Suppose that patient agent  $k$  defects from the proposed equilibrium and announces  $m_k = 1$ . Using (6), his expected utility is

$$\sum_{n=1}^N \hat{\pi}_n \sum_{t^N \in P_n} \frac{1}{n} \left\{ \sum_{k \in Q_n} v[c_k^{1*}(\hat{m}^{k-1}, 1)] + \phi \sum_{\substack{j \in Q_n \\ j \neq k, N}} (R - 1 - \varepsilon(n - 1)) c_j^{1*}(\hat{m}^{j-1}, 1) \right\}, \quad (9)$$

where

$$\phi = \begin{cases} 1 & \text{if } k = N, \\ 0 & \text{otherwise} \end{cases}.$$

Note that as  $\varepsilon(N) \rightarrow 0$ , the difference between (9) and (8) is

$$\sum_{n=2}^N \hat{\pi}_n \sum_{t^N \in P_n} \frac{1}{n} \left\{ \sum_{k \in Q_n} v[c_k^{1*}(\hat{m}^{k-1}, 1)] + \phi \sum_{\substack{j \in Q_n \\ j \neq k, N}} (R - 1) c_j^{1*}(\hat{m}^{j-1}, 1) \right\} - v[c_k^{1*}(\hat{m}^{k-1}, 1)] > 0.$$

Hence, for any given  $N$ ,  $\pi$ , and  $R > 1$ , the mechanism can choose  $\varepsilon(N) > 0$  sufficiently small so that the value of (9) strictly exceeds that of (8), a contradiction.

Third, there does not exist an equilibrium where patient agents  $j$  randomize over announcing  $m_j = g$  and other announcements. Suppose that the proposed equilibrium has patient agents announcing  $m_j = g$  with probability  $\sigma_g$ , where  $0 < \sigma_g < 1$ . Since patient agent  $k$  randomizes he must be indifferent between announcing  $m_k = g$  and announcing  $m_k = 1$  and/or  $m_k = 2$ , (depending on the specification of the proposed equilibrium). However, given (5), if agent  $k$  announces  $m_k = g$  with probability one, he can increase his expected payoff, compared to the proposed equilibrium payoff, since the expected number of agents who announce  $m_j = g$  increases compared to the proposed equilibrium. Therefore, there cannot be an equilibrium where patient people announce  $m_j = g$  with probability  $\sigma_g$ , where  $0 < \sigma_g < 1$ .

Finally, consider an equilibrium where agents of type  $t_j$  announce  $m_j = t_j$  with probability one. Since  $\delta > 0$  in contract  $c^*(\delta)$ , incentive constraint (3) implies that all patient agents  $j$  strictly prefer to announce  $m_j = 2$  to  $m_j = 1$ . Note that patient agent  $k$  strictly prefers to announce  $m_k = g$  to  $m_k = 1$  when all other agents  $j \in \mathbb{N} \setminus \{k\}$  announce  $m_j = t_j$ . But for any  $\delta > 0$ , there exists an  $\varepsilon(N) > 0$  sufficiently small so that patient agent  $k$  strictly prefers announcing  $m_k = 2$  to  $m_k = g$ , (since  $\delta > 0$  implies that agent  $k$  *strictly* prefers announcing  $m_k = 2$  to  $m_k = 1$ ). Therefore, for  $\delta > 0$  arbitrarily small,  $c^*(\delta) \approx c^*(0)$ , and the unique equilibrium for mechanism  $(M^I, c)$  is characterized by  $m_j = t_j$  for all  $j \in \mathbb{N}$ . ■

Agents do not know their positions in the queue for the indirect mechanism  $(M^I, c)$ . Suppose that the economic environment is modified so agents not only learn their type, but they also (somehow) learn their position in the queue. Proposition 1 and the basic proof remains valid for the modified economic environment, where agents know their positions in the queue.<sup>7</sup>

<sup>7</sup>Of course, the allocation rule  $c^*(\delta)$  for the modified environment may be different than the

## 6 Final Comments

In a way, the message that underlies this paper is a rather negative one: A well designed deposit contract can prevent bank run equilibria in the classic Diamond-Dybvig environment. The message is negative because the Diamond-Dybvig model is supposed to be a model of banking instability. Green and Lin (2000, 2003) conjectured that the overlapping generations nature of depositors in the real world and/or moral hazard associated with the people who operate banks may prevent agents from using efficient mechanisms, which has implications for banking instability. These conjectures, unfortunately, do not appear to be supported by subsequent research.<sup>8</sup> An important assumption in the Diamond-Dybvig environment is that the planner can *ex ante* commit to implement contract allocations. Relaxing this assumption may result in bank run equilibria; see, for example, Ennis and Keister (2009a). Perhaps assuming that agents cannot not fully commit is a fruitful avenue for future work.

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solution to (4). The best weakly implementable allocation for the new environment is given by  $\max_c$  (1) subject to (2) and  $N$  incentive compatibility constraints, one for each patient agent in position  $j \in \mathbb{N}$  in the queue.

<sup>8</sup>The Peck and Shell (2003) can be interpreted as a way of modeling overlapping generations in the sense that there is no “last” depositor in an overlapping generations model, and in Peck and Shell (2003) depositors do not know if they are the last depositor. Andolfatto and Nosal (2008) assume that one of the agents operates the bank in a Diamond-Dybvig environment, and find that there do not exist bank run equilibria.

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