



Federal Reserve Bank of Chicago

Bubbles and Leverage: A Simple and Unified Approach

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1 Introduction

It is generally, if not universally, believed that major asset price booms and busts are intimately caught up with variations in the terms and extent of borrowing to fund risky asset purchases. One important strain of thought attributes the genesis of so-called “bubble” episodes to relaxation of borrowing costs due to some combination of financial liberalization, central bank loosening, and increased capital flows from abroad. Likewise, credit tightening has been credited with ending such asset price booms as the pre-1929 stock market rally in the United States and the “Japanese Bubble”. Another strain, some of which was fueled by attempts to understand the 2007-2009 financial crisis, focuses on an endogenous “leverage cycle” which is regarded as an amplification mechanism for major asset price fluctuations (and perhaps their transmission to the real economy). However, systematic and (especially) simultaneous consideration of both the theoretical and empirical linkages between major asset price fluctuations and variations in both the terms and extent of leverage is scarcely to be found in the literature – even more so when real investment in the asset in question is included in the mix.

In work in progress, we document empirical patterns in a number of key time series associated with asset booms and busts (often informally referred to as “bubble” episodes), particularly series that inform our understanding of the role of credit, interest rates, and leverage in the genesis and demise of the “bubbles”. Our ultimate goal is to analyze these episodes in light of theory. In this paper, we lay out a simple framework that captures much of what the theoretical literature has to say about the role of credit in systemically important asset booms and busts, and in addition we suggest ways which to incorporate physical investment in the bubble asset as well as monetary policy.

There are basically two serious economic theories of the role of credit in major asset price fluctuations. The first, that we will refer to as Miller-Geanakoplos-Simsek, combines heterogeneous beliefs about fundamentals and at least partial limits on short sales to achieve a particular notion of “overvaluation” namely that the market price of the asset reflects the beliefs of the most optimist agents, who are the “natural “buyers”. The extent of that “overvaluation” depends on the ability of the natural buyers to borrow – which depends importantly on the uncertainty of the potential lenders about the downside of the assets, as well as on the net worth of the borrowers.

The second economic theory of leveraged asset price booms, which we will refer to as Allen-Gale-Barlevy, focuses on the shifting of risk from borrowers to lenders associated with the possibility of default. Those authors define a controversial but rigorous definition of a “bubble” situation in which the asset is priced at greater than its (social) fundamental value because of the risk-shifting.

One important accomplishment of our analysis is that it encompasses both the Miller-Geanakoplos-Simsek and Allen-Gale-Barlevy. Roughly speaking, in the presence of sufficient leverage, disagreement about upside uncertainty generates elevated asset prices by the first mechanism, and disagreement about dispersion elevates asset prices via the second mechanism. The latter occurs because agents who believe that the asset has a high potential payoff but particularly sharp downside uncertainty are natural buyers due to their perception that they are shifting risk. Our second major innovation is the incorporation of physical investment in the asset within the model of leveraged asset purchases, which allows us to endogenize the interest rate in an essential way. Our third innovation is the incorporation of a notion of monetary policy without the introduction of "money". Instead, the central bank subsidizes lending by buying loan contracts, possibly causing the interest rate to fall short of the natural rate, and raising asset prices and investment. With this unified model, we can discuss much of what has been said in the theoretical literature linking leverage and asset prices, and we can do it in a sufficiently simple way as to eventually admit comparison between theory and data.

2 A Simple Static Model

2.1 Overview

There are two types of assets: coconuts and trees. Agent i is endowed with T_i trees and n_i coconuts. At the end of the period, trees yield coconuts and agents consume. Agent i can store a non-negative amount of coconuts, s_i , at storage rate f . They borrow b from other agents at rate r and buy quantity q trees. There is no short selling. That is, $q_i \geq -T_i$. Agent i believes there are two states in the world, which we'll call the bad state and the good state. In the good state, trees yield U_i coconuts, in the bad state, trees yield L_i

coconuts with probabilities π and $(1 - \pi)$ respectively. This means agent i values the trees at

$$V_i = \pi U_i + (1 - \pi)L_i$$

In our model, we have two different types of agents, indexed by i and j . For now, they differ only in beliefs about the possible states of the world. More specifically, they differ in parameters U , L , and V . Later, we will adjust other parameters. As in Geanakoplos (date), agents face a borrowing constraint subject to a zero-value-at-risk haircut. That is, for agent i , borrowing from agent j ,

$$b_i \leq \frac{(q_i + T_i)L_j}{1 + r}$$

Where q_i is the number of trees purchased by agent i . That is, agent i has $q_i + T_i$ trees. Agent j believes that each unit of tree will pay off, at minimum, L_j . Then, discounting to the present, lending up to

$$\frac{(q_i + T_i)L_j}{1 + r}$$

is completely riskless, from agent j 's point of view. Each agent maximizes a utility function as follows

$$U_i = s_i(1 + f) - b_i(1 + r) + V_i(q_i + T_i)$$

Where as above

$$V_i = \pi U_i + (1 - \pi)L_i$$

In addition to the borrowing constraint above, there is a budget constraint, a nonnegativity constraint on storage, and a short-selling constraints. These constraints are as follows

$$\begin{aligned}
-pq_i + b_i - s_i + n_i &\geq 0 \\
\frac{(q_i + T_i)L_j}{1+r} - b_i &\geq 0 \\
s_i &\geq 0 \\
q_i + T_i &\geq 0
\end{aligned}$$

Often a constraint will bind for one agent but not the other. More specifically, the short sales constraint will often bind for the seller while the borrowing constraint will bind for the buyer. To clearly illustrate the main features of this model, we solve two trivial examples, then construct the more demand curve and supply curves for trees which can be used to understand the general behavior of the model.

2.2 Two Simple Examples

Suppose there are two agents and that $\pi = (1 - \pi) = 5$

Also suppose that $n_i = n_j = T_i = T_j = 1$ and that $f = 0$

Finally, suppose that $V_i = L_i = U_i = U_j = 1$ and that $L_j = 1$ and $U_j = 3$, so $V_j = 2$

Then the collateral constraint doesn't bind, a first order condition with respect to quantity implies

$$p = \frac{V_j}{(1+r)}$$

So the price is entirely determined by the optimist's valuation. The short sales constraint binds, and the pessimist sells his entire stock of trees while the optimist buys it. That is, the equilibrium is

$$\{p, q_i, q_j, b_i, b_j, s_i, s_j\} = \{2, -1, 1, -1, 1, 2, 0\}$$

This example is particularly easy because the collateral constraint doesn't bind for the optimist. A more interesting case is when the collateral constraint does bind.

Let's suppose instead that the optimist values the asset at 7 instead of 2. That is, $L_j = 1$ and $U_j = 13$, so $V_j = 7$. Two things happen. First, the optimist can no longer afford all of the asset at price

$$p = \frac{V_j}{(1+r)}$$

Since he only has $n = 1$ coconuts and can only borrow, at most, $b = 2$ more of them. But, since the optimists can still pay more than the pessimists think the asset is worth, equilibrium price is simply whatever the optimist can pay. That is

$$pq = b + c$$

In this case, $b = 2$ and $c = 1$, so $p = 3$. This example illustrates a key insight of the model. Since in equilibrium

$$b_i = \frac{(q_i + T_i)L_j}{1+r}$$

We see that b is completely unrelated to the optimist's valuation of the asset. Then it's clear that when the borrowing constraint binds, an increase in optimism has no effect on the price of the asset, a shock we call "Exciting Good News". It's also worth noting that if the seller had a lower initial stock of trees then the price would be higher. That is, suppose $T_i = .1$. Then the borrowing constraint would not bind.

In these examples we have seen the following: first, absent of borrowing constraints, we obtain the standard result that the most optimistic agent will completely determine the price of the asset. Second, with a binding borrowing constraint, the optimist will simply "pay what he can" to the pessimist. Finally, given a price, there exists some quantity of trees, q^* , where the collateral constraint will not bind for quantities purchased below q^* . With these observations, we can now construct the demand curve for this asset.

2.3 Constructing Supply and Demand

First, imagine the constraint is not binding. Then the demand curve is a horizontal line at V_i , where agent i buys the asset in equilibrium (ie, he is the

"optimist). Then it is only interesting to imagine the constraint is binding. Our first task is to solve for q^* . Given a price, p , then, when the constraint is binding, $pq = b + c$. Substituting in the borrowing constraint in place of b and solving for q yields the following expression.

$$q_i = \frac{N_i + \frac{T_i L_j}{(1+r)}}{p - \frac{L_j}{(1+r)}}$$

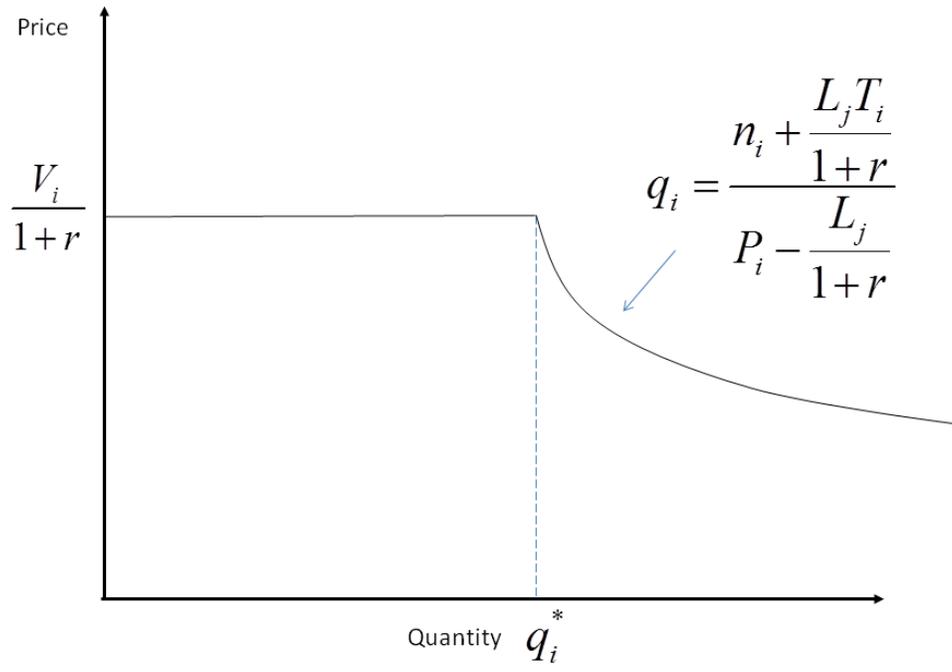
Then this describes the affordable quantity as a function of price, once we're in a cash in the market regime. For $q_i \leq q_i^*$, the demand curve is flat as before. For all $q_i > q_i^*$ which implies $p < V_i$ this will describe the slope of the demand curve after the borrowing constraint binds. In our simple, first example, this simplifies to

$$q = \frac{2}{(P - 1)}$$

Then the buyer's demand curve can be drawn as follows.

Interestingly, the buyer's demand curve depends in part on the seller's belief about the asset. The seller's supply curve is similarly simple to construct. The supply curve will be horizontal before the short sales constraint binds, reflecting the fact that the pessimist is still the marginal holder of the asset. At the short sales constraint, anything above his valuation is suitable.

Buyer's demand curve



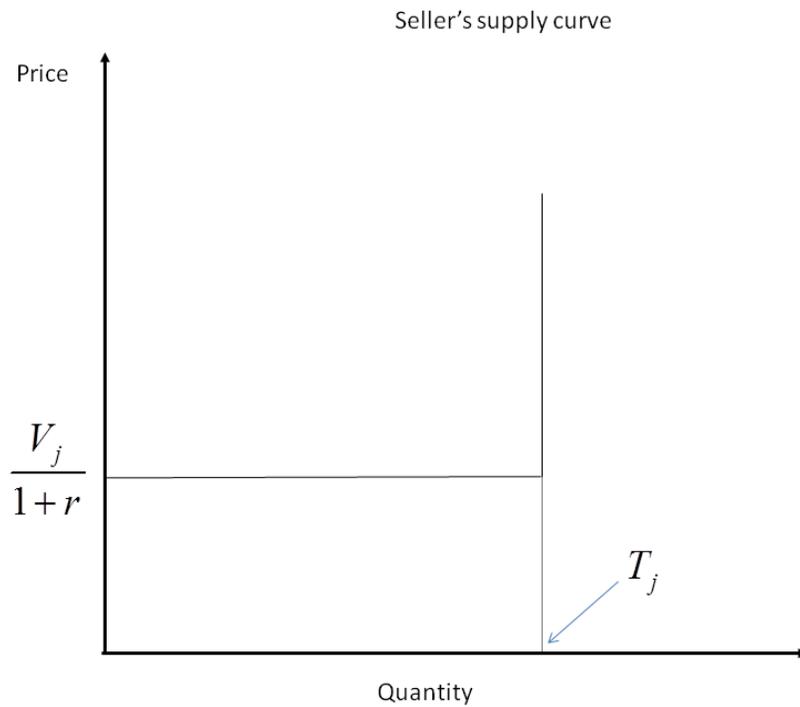
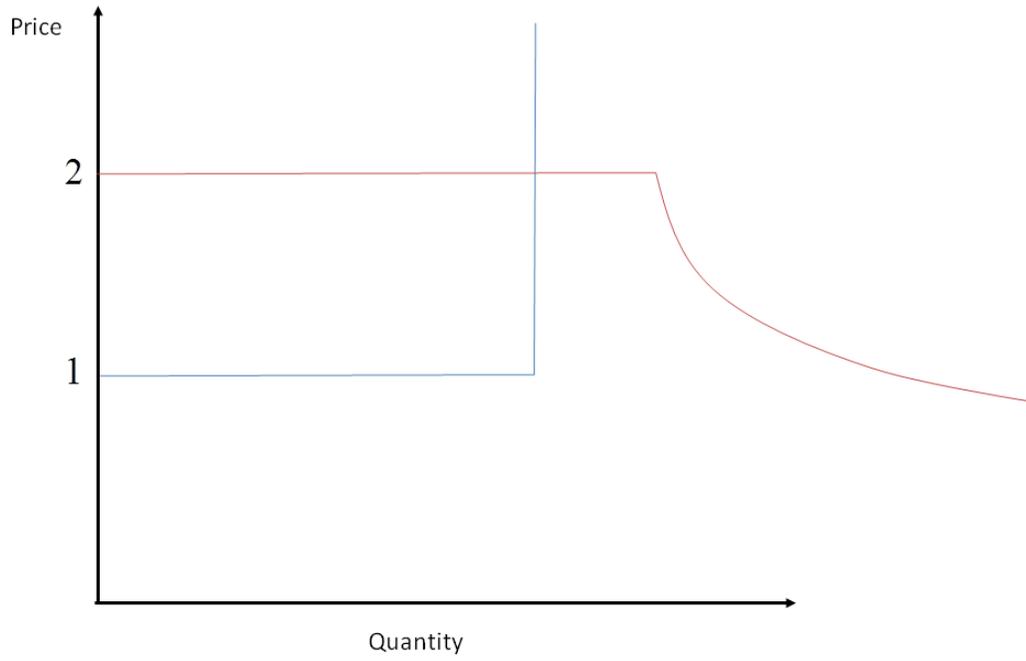
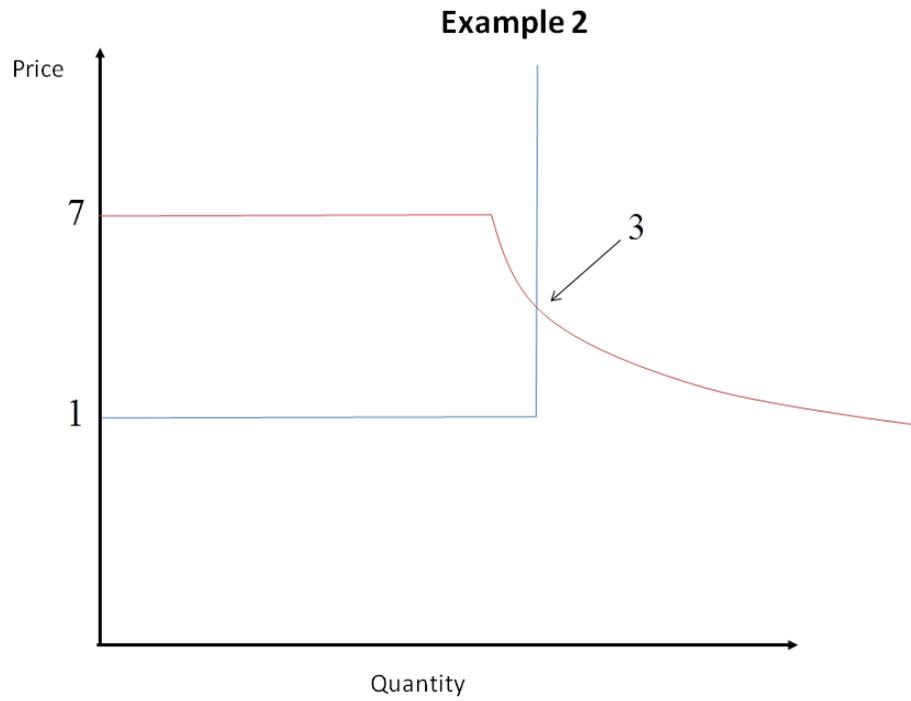


Figure 2

Next we examine the demand and supply curves together. Figure 3 shows the equilibrium in our first example, and figure 4 shows the equilibrium in our second.

Example 1





Generally, our analytical focus is on the collateral constrained case. We would like to think about the effects of various shocks on equilibrium price, and present comparative statics below.

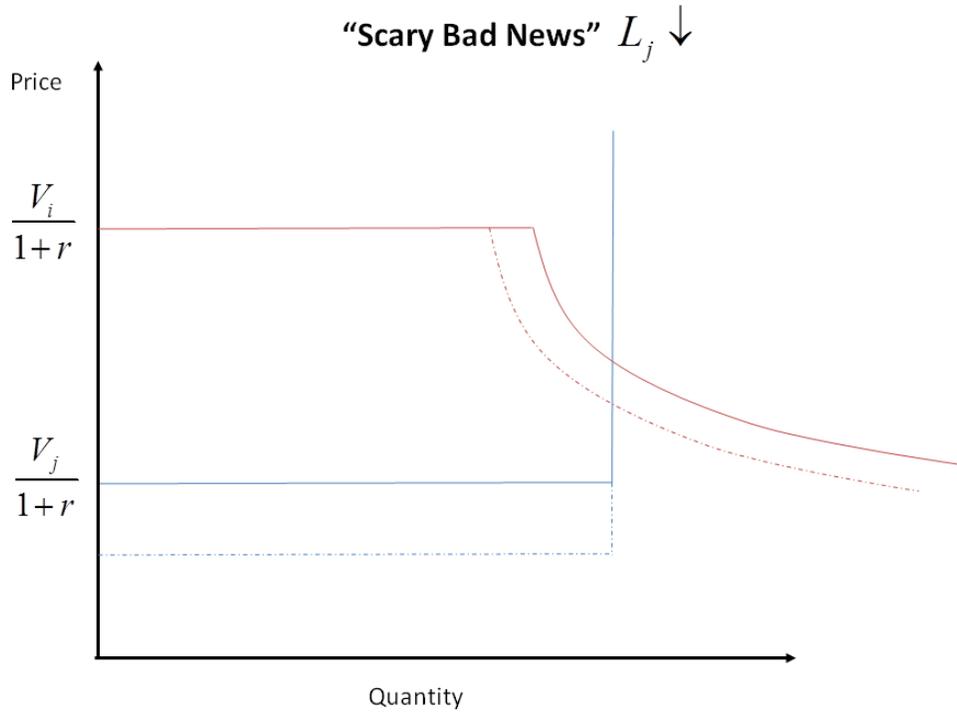


Figure 5

This, a “Scary Bad News” shock, is a shock to the beliefs of the seller about the asset’s worst-case-scenario payoff. If the worst case scenario becomes even worse, then they will be less willing to lend. When collateral constrained, pricing is cash in the market, so the equilibrium price will also decline. Since leverage is

$$\frac{p}{n_i} = \frac{b_i + n_i}{n_i}$$

Then this also implies a reduction in leverage. Since the effect of the shock is symmetrical so long as the binding constraints remain the same, any loosening of capital requirements, identical to an increase in L_j , will increase asset prices.

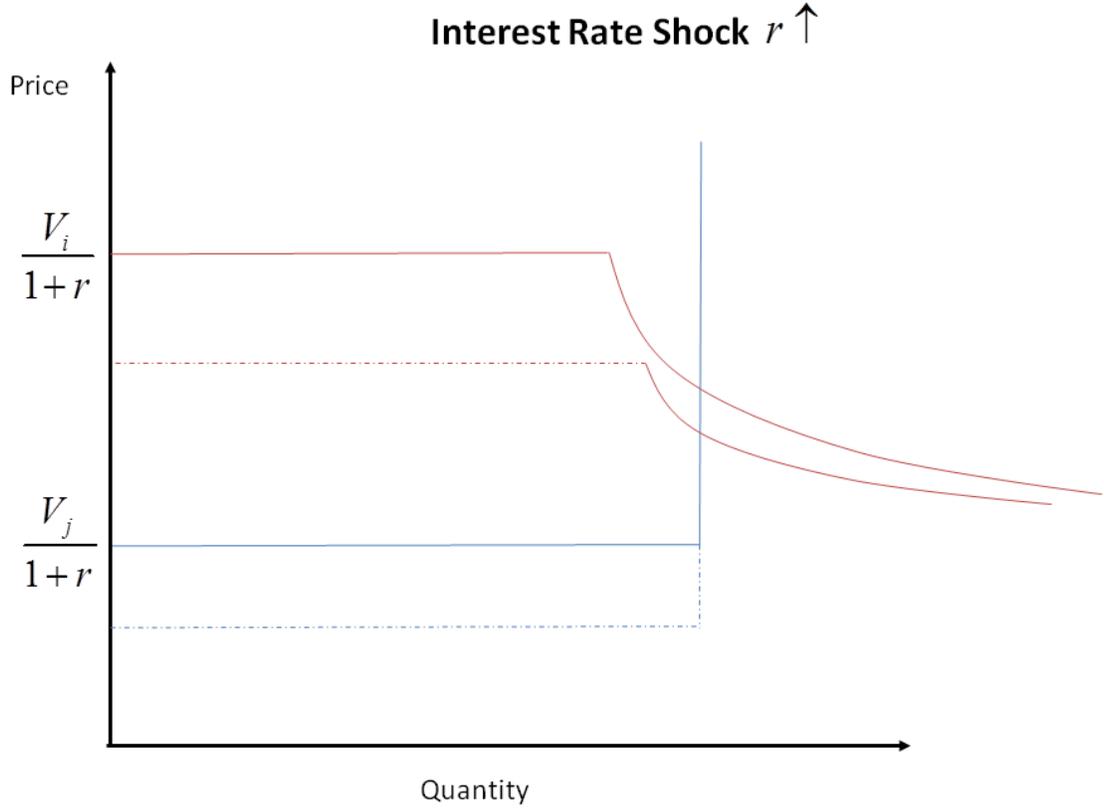


Figure 6

Here there is an exogenous shock to the return on storage. Effects are qualitatively, when collateral constrained, identical to a “Scary Bad News” shock. The equilibrium price is lower and leverage decreases. To see why the collateral constraint binds sooner requires more care. First remember that

$$q_i = \frac{N_i + \frac{T_i L_j}{(1+r)}}{p - \frac{L_j}{(1+r)}}$$

The point at which this will first start to bind then should be what the optimist would pay for the asset. That is, $\frac{V_i}{1+r}$, substituting this into the expression for q_i then yields q_i^* . Multiplying top and bottom by $(1+r)$ will then make it clear that an increase in r raises q_i^* . However, this is only because discounting by a higher interest rate makes the price optimists are willing to pay lower in the first place.

Finally, note that when pricing is cash in the market, changes in optimism from the point of view of the buyer are irrelevant. That is, changes in the natural buyer's beliefs are relevant if and only if the collateral constraint is not binding.

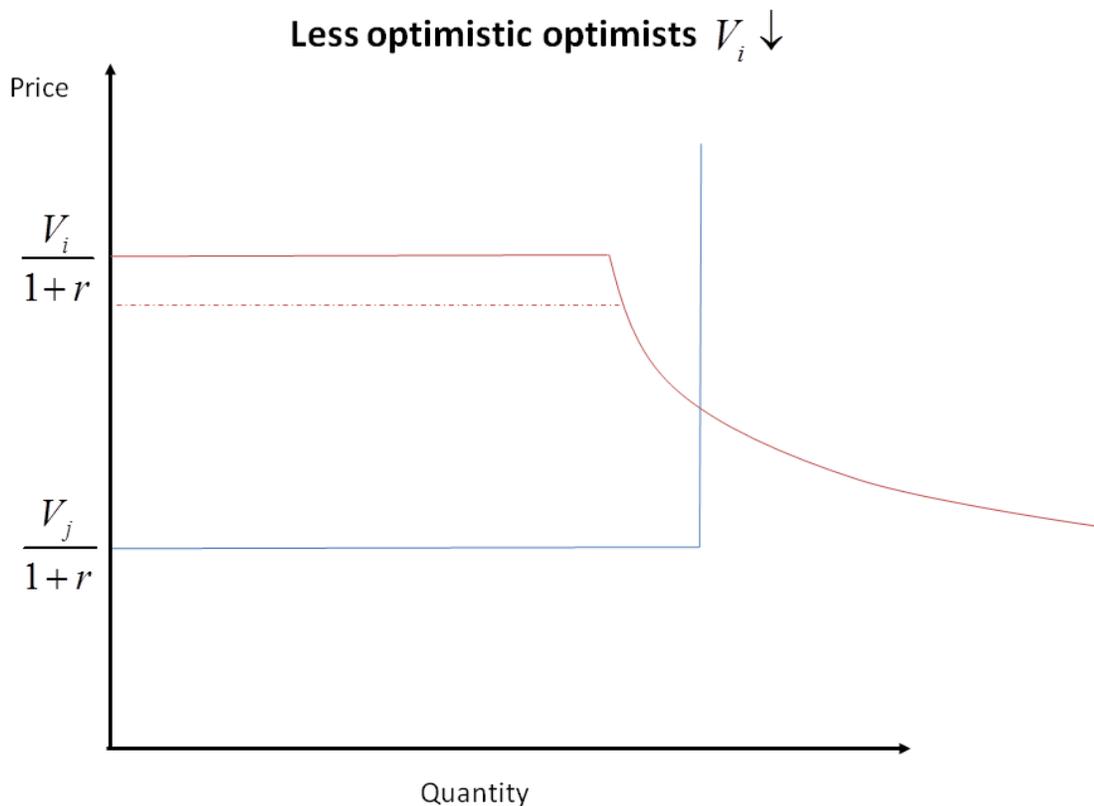


Figure 7

2.4 Two Features

2.4.1 A Special Case: Risk Shifting

We now slightly modify our model to accommodate risk shifting. Suppose again that $\pi = (1 - \pi) = .5$ for both agents. Then suppose that the first

agent has beliefs $U_i = L_i = V_i = 1$. Then suppose the second agent has beliefs $L_j = 0, U_j = 2, V_j = 1$. Risk shifting occurs because the first agent can only seize assets that are created by the second agent's tree. The problem is identical to the more general problem described at the beginning of the section, with one small change. Agent 2 instead maximizes the following utility function.

$$U_j = s_j(1 + f) - .5b_j(1 + r) + V_j(q_j + T_j)$$

From agent j 's perspective, since the tree yields nothing in the bad state, there is only a .5 probability that he will have to pay back any loans. However, agent i believes he is lending risklessly. We find the equilibrium is

$$\{p, q_i, q_j, b_i, b_j, s_i, s_j\} = \{1.5, -1, 1, -2, 2, .5, 1.5\}$$

This simple example showcases our key point. Namely, agent j has a fundamental, or "unlevered" valuation of the asset that differs from his "levered", or equilibrium valuation. This discrepancy is at the heart of risk-shifting models of bubbles such as Allen & Gale and Barlevy.

2.4.2 A Novel Symmetry: No Naked Shorts

So far we have had absolute restrictions on short-selling. For simplicity this is useful, but need not be the case. In fact, this analytical framework lends itself to a natural, less-restrictive constraint on short-selling: no naked shorts. This is best explained in terms of the borrowing constraints. From earlier, we saw the following constraint

$$b_i \leq \frac{(q_i + T_i)L_j}{1 + r}$$

Here, agent i is constrained in his borrowing by the worst possible outcome from the point of view of agent j . Agent i promises to pay back agent j what he borrows, b . This is constrained because agent j thinks there is a minimum amount that each unit of collateral might pay, L^j , so he wants to prevent the possibility of default. Now let's imagine the analogue. Here, the optimist (i) is getting a promise of a payment, in coconuts, of whatever the yield is on the asset after it pays dividends. Suppose then that the pessimist can

shortsell. It's possible that the price of the asset will increase enough that the pessimist (j) can't afford to deliver the coconuts he promised the optimist (i). By forbidding no naked shorts, we are ensuring that, even in the best state of the world, the optimist believes the pessimist can afford to pay him all he is owed. Instead of the short selling constraint originally shown above, it will change to

$$q_i \geq \frac{\frac{n_i(1+r)}{H_j}}{\frac{p(1+r)}{H_j} - 1}$$

Note that if the optimist believed the best and worst states were identical, that is, $H_j = U_j$, and the optimist was not credit-constrained, that is, $p = \frac{H_j}{1+r}$, the quantity purchased would become infinite. This short selling restriction acts through changing prices in the credit-constrained equilibrium. To see this, start from cash in the market pricing, that is

$$pq_i = b_i + n_i$$

Now note that neither endowments, n_i , nor borrowing, b_i , is affected by changing short-selling restrictions. However, q_i increases, leading to a lower (or possibly higher) equilibrium price.

3 A Richer Model

With the analytical framework shown above in the background, we extend the model in two key ways. First, instead of the interest rate being fixed by an exogenous parameter f , it is instead pinned by a concave technology for storage. Second, agents can now create trees of their own, an activity we refer to as "investment". The model still features two types of agents, agent i and agent j . As before, it will be helpful to refer to these agents as the "pessimist" and the "optimist" respectively. The model proceeds as follows.

1. Agents are endowed with trees and coconuts.
2. Agents take out loans at the interest rate and buy trees from other agents using coconuts. Agents can use coconuts in two different ways: first, they can build trees using coconuts, investment. They can also

choose to store their coconuts which will yield some amount of coconuts at the end of the period.

3. After all transactions have taken place, investment yields trees to agents, and storage yields coconuts. The returns on the trees are realized. Agents then settle debts and consume.

The following is a list of various variables we will refer to throughout this paper.

c_i - consumption

y - payoff to owning trees at the end of the model.

b_i - borrowing

q_i - quantity of trees purchased

s_i - riskless investment

I_i - quantity of coconuts invested in trees

T_i - trees agent i is endowed with

n_i - coconuts agent i is endowed with

r - interest rate

p - price of trees

Agents maximize consumption, all of which occurs at the end of the period. They also satisfy their budget constraint.

$$s_i + pq_i + I_i = n_i + b_i$$

This says that riskless investment (storage), plus what you pay for trees, plus investment in risky trees has to be equal to your endowment of coconuts plus what you borrow. Agents maximize consumption, which is as follows.

$$c_i = \alpha \frac{\ln(1 + s_i)}{1 + s_j} + y(T_i + q_i + \gamma \frac{\ln(1 + I_i)}{1 + I_j}) - b_i(1 + r)$$

Basically, this says that what you spend and what you eat has to be equal to your coconuts plus your borrowing. You consume, pay back debts, invest

in risky assets, buy risky assets, and invest in riskless assets. This is equal to your dividends from risky assets, plus borrowing, plus endowments, plus riskless dividends. Note the how the investment technology functions. At the beginning of the period, agent i puts in I_i coconuts. At the end of the period, agents get back $\gamma \frac{\ln(1+I_i)}{1+I_j}$ trees.

Note that y , the payoff in coconuts from holding trees, is a random variable. Agents differ only in their beliefs about y . All agents believe there are two possible outcomes: trees will yield a high payoff, H_i , or a low payoff, L_i , and these each occur with probably $\frac{1}{2}$. Note that agents' beliefs about states are not necessarily equal. That is, L_i is not necessarily equal to L_j . This means there can be up to four states in the model. However, much of our analysis can be mapped into a two-state setup, with varying probabilities.

There are numerous sometimes-binding constraints in the model. First, there is a short-selling constraint.

$$q_i + T_i \geq 0 \tag{1}$$

That is, agents cannot sell any trees that they were not endowed with. Note that this constraint will change if we allow trees created through investment to be sold. Allowing trees created through risky investment to be sold will not qualitatively alter our results, and for now we omit it.

For the optimist, the following constraints are relevant.

$$b_j \leq \frac{(q_j + T_j)L_i}{(1+r)} \tag{2}$$

For the optimist, riskless investment constraints will be binding.

$$s_j \geq 0 \tag{3}$$

For the pessimist, risky investment constraints will be binding.

$$I_i \geq 0$$

In the next section, we'll work on solving this analytically. When the problem is simplified sufficiently, we'll solve a couple of examples by hand. Then we'll examine the effects of various changes in beliefs on equilibrium outcomes.

4 Cash in the Market Pricing

4.1 The general problem

This section solves an example of the model with cash in the market pricing. We start from the assumption that parameters are such that the following constraints are binding for the optimist (agent j)

- The borrowing constraint
- Nonnegative riskless investment

For the pessimist, (agent i)

- The short selling constraint
- Nonnegative risky investment

Now let's begin to solve the problem. Before analyzing first order conditions, we'll do a large amount of legwork substituting in binding constraints.

There are three different possible scenarios. First, the optimist may have so few assets that he, even at the pessimist's valuation, cannot afford to buy all of the pessimist's trees. It is also possible that the optimist has so many assets that he can afford to buy all the pessimist's trees at his own valuation. Here we examine an in-between situation. The optimist has enough funds to buy all of the trees, but not at his own valuation. This is cash-in-the-market pricing. Since we're in the cash in the market regime, we immediately know that $q_i = -T_i$. Likewise, $b_j = \frac{(q_j + T_j)L_i}{(1+r)}$. Also note that $s_j = 0$ and $I_i = 0$. Combined with market clearing conditions, the following are immediately determined.

$$q_i = -T_i \tag{4}$$

$$q_j = T_i \tag{5}$$

$$b_j = \frac{(q_j + T_j)L_i}{(1+r)} \tag{6}$$

$$b_i = -\frac{(q_j + T_j)L_i}{(1+r)} \tag{7}$$

$$s_j = 0 \tag{8}$$

$$I_i = 0 \tag{9}$$

Let's substitute these equations into the budget constraints

$$-pT_i + s_i = -\frac{(T_j + T_j)L_i}{(1+r)} + n_i \quad (10)$$

$$I_j + pT_j = \frac{(T_j + T_j)L_i}{(1+r)} + n_j \quad (11)$$

This is quite a bit of immediately simplification, but here we have two equations and four unknowns, so we'll have to go to the first order conditions.

The first condition to look at is the easy one. For the pessimist's problem, we obtain the following

$$\frac{\partial U}{\partial s_i} = \frac{\alpha}{1 + s_{1,i}} - \lambda_i = 0 \quad (12)$$

$$\frac{\partial U}{\partial b_i} = -(1+r) + \lambda_i = 0 \quad (13)$$

Combing (12) and (13), we can relate storage done by the pessimist to the interest rate as follows

$$\frac{\alpha}{1 + s_{1,i}} = 1 + r \quad (14)$$

Substituting into (10) yields the following

$$-pT_i + \frac{\alpha}{1+r} - 1 = -\frac{(T_j + T_j)L_i}{(1+r)} + n_i \quad (15)$$

Now we're a little closer to success. There are 2 equations, but now only 3 unknowns. We now turn to the optimist's first order conditions. We will need most of them.

$$\frac{\partial U}{\partial b_j} = -(1+r) + \lambda_j - \Phi_j = 0 \quad (16)$$

$$\frac{\partial U}{\partial I_j} = \frac{\gamma E_i[y]}{1 + I_j} - \lambda_j = 0 \quad (17)$$

$$\frac{\partial U}{\partial q_j} = E_j[y] - p\lambda_j + \frac{\Phi_j L_i}{1+r} = 0 \quad (18)$$

Note we already substituted in the fact that $I_i = 0$ after we took derivatives. This system of equations introduces 2 new variables, λ_j , the lagrange multiplier on the optimists budget constraint, and Φ_j , the lagrange multiplier on the optimist's borrowing constraint.

We can immediately solve for λ_j as a function of risky investment using (16)

$$\frac{\gamma E_j[y]}{1 + I_j} = \lambda_j \quad (19)$$

Now we can do the same for Φ_j combining (16) and (19)

$$\Phi_j = \frac{\gamma E_j[y]}{1 + I_j} - (1 + r) \quad (20)$$

Making use of both (19) and (20), we can substitute into (18) to obtain our final equation. This yields

$$E_j[y] - p \frac{\gamma E_j[y]}{1 + I_j} + \frac{\frac{\gamma E_j[y] L_i}{1 + I_j} - (1 + r) L_i}{1 + r} = 0$$

We multiply both sides of the equation by $1 + I_j$ to further simplify.

$$E_j[y](1 + I_j) - p \gamma E_j[y] + \frac{\gamma E_j[y]}{1 + r} - (1 + I_j) L_i = 0 \quad (21)$$

Now let's combine with the budget constraints to get a system of 3 equations and 3 unknowns.

$$E_j[y](1 + I_j) - p \gamma E_j[y] + \frac{\gamma E_j[y] L_i}{1 + r} - (1 + I_j) L_i = 0 \quad (22)$$

$$-p T_i + \frac{\alpha}{1 + r} - 1 = -\frac{(T_j + T_j) L_i}{(1 + r)} + n_i \quad (23)$$

$$I_j + p T_j = \frac{(T_j + T_j) L_i}{(1 + r)} + n_j \quad (24)$$

Before experimenting with different parameter choices, we make one more simplification. The bottom two equations in (22) are the pessimist and optimist's constraints, respectively. We can combine them together to obtain the following expression for investment.

$$I_j = n_j + n_i + 1 - \frac{\alpha}{1+r} \quad (25)$$

This is perhaps the most important equation in our model. It shows that investment is an increasing function of the interest rate.

4.2 Basic CIM Example

We begin by picking a few basic, reasonable parameters. They are as follows

$$T_j = 1$$

$$T_i = 1$$

$$n_j = 2$$

$$n_i = 2$$

$$L_i = 1$$

$$E_j[y] = 4$$

$$\gamma = \frac{1}{2}$$

$$\alpha = 5$$

Substituting these parameters into the system of equations (22) above and simplifying leads to the following expressions.

$$4(1 + I_j) - 2p + \frac{2}{1+r} - (1 + I_j) = 0 \quad (26)$$

$$-p + \frac{5}{1+r} = -\frac{2}{(1+r)} + 3 \quad (27)$$

$$I_j + p = \frac{2}{(1+r)} + 2 \quad (28)$$

We first combine (43) and (44) to obtain an expression for I_j in terms of the interest rate.

$$I_j + \frac{5}{1+r} = \frac{2}{(1+r)} + 2 - \frac{2}{(1+r)} + 3$$

This simplifies down to

$$I_j = 5 - \frac{5}{1+r} \quad (29)$$

This is (25), as derived above. Next, let's solve for I_j in (42)

$$3 + 3I_j = 2p - \frac{2}{1+r}$$

Multiplying the optimists budget constraint (44) by 2 and combining with the above yields the following system

$$3 + 3I_j = 2p - \frac{2}{1+r} 2I_j + 2p = \frac{4}{(1+r)} + 4$$

which simplifies to

$$I_j = \frac{2}{5(1+r)} + \frac{1}{5} \quad (30)$$

Then, setting (30) and (29) equal yields

$$\frac{2}{5(1+r)} + \frac{1}{5} = 5 - \frac{5}{1+r}$$

Multiplying both sides by 5

$$\frac{2}{(1+r)} + 1 = 25 - \frac{25}{1+r}$$

Combining like terms

$$\frac{27}{(1+r)} = 24$$

so

$$1+r = \frac{27}{24}$$

And

$$r = \frac{1}{8} \quad (31)$$

From here, we can quickly obtain I_j and p . Using (29) we see

$$I_j = \frac{5}{9} \quad (32)$$

$$p = \frac{29}{9} \quad (33)$$

4.3 Reassuring Good News

Let's now examine the effect of a change in beliefs about the low end. Put differently, what would happen if the optimists remained identical but the pessimists perceived less tail risk? Suppose all exogenous processes remain constant except for L_i , which increases to 1.5. We will show that there is an increase in prices, investment, the interest rate, and leverage, relative to the previous example. This is because the optimist is able to borrow more, which makes the pessimist store less, driving up the interest rate. Since the price is constrained by the optimists' borrowing capacity, price and investment simultaneously increase. Since leverage is simply price divided by the endowed coconuts, leverage will also increase.

Our first expression for investment in (25) remains identical.

$$I_j = 5 - \frac{5}{1+r} \quad (34)$$

However, the other expression will change. Let's examine the system of equations that characterizes the solution.

$$4(1 + I_j) - 2p + \frac{3}{1+r} - \frac{3}{2}(1 + I_j) = 0 \quad (35)$$

$$-p + \frac{5}{1+r} = -\frac{3}{(1+r)} + 3 \quad (36)$$

$$I_j + p = \frac{3}{(1+r)} + 2 \quad (37)$$

We can simplify (35) to the following.

$$\frac{5(1 + I_j)}{2} = 2p - \frac{3}{1+r}$$

We combine with (37), as before.

$$\frac{5(1 + I_j)}{2} = 2p - \frac{3}{1 + r}$$

$$2I_j + 2p = \frac{6}{(1 + r)} + 4$$

which simplifies to

$$\frac{9I_j}{2} = \frac{3}{1 + r} + \frac{3}{2}$$

We then solve for I_j as follows

$$I_j = \frac{2}{3(1 + r)} + \frac{1}{3}$$

Equating our two expressions for I_j .

$$\frac{2}{3(1 + r)} + \frac{1}{3} = 5 - \frac{5}{1 + r}$$

Some simplification can yield

$$\frac{2}{(1 + r)} + 1 = 15 - \frac{15}{1 + r}$$

to

$$\frac{17}{(1 + r)} = 14$$

Which finally implies

$$1 + r = \frac{17}{14}$$

And that means

$$r = \frac{3}{14} \tag{38}$$

We can then substitute this back to get price and investment as before.

$$I_j = \frac{15}{17} \tag{39}$$

$$p = \frac{61}{17} \tag{40}$$

4.4 Monetary Policy

We consider the effect of monetary policy in our model to examine the effect of the fed deviating from the natural rate. While the natural rate in our model rises alongside prices, it is still possible that the fed could keep interest rates from rising *as fast* as the natural rate. In our model, we define the natural rate as the rate that prevails in the world with no monetary policy.

There are two ways we imagine monetary policy to function. The first way is through the monetary authority buying up (or selling) riskless assets, which acts as an exogenous increase in economy-wide storage. The second way is that the monetary authority can subsidize (or tax) borrowing. To see the effect on this, consider again the three equations which characterize the solution of our CIM examples (22).

$$\begin{aligned} E_j[y](1 + I_j) - p\gamma E_j[y] + \frac{\gamma E_j[y]}{1 + r} - (1 + I_j)L_i &= 0 \\ -pT_i + \frac{\alpha}{1 + r} - 1 - \bar{s}_t &= -\frac{(T_j + T_j)L_i}{(1 + r)} + n_i \\ I_j + pT_j &= \frac{(T_j + T_j)L_i}{(1 + r)} + n_j \end{aligned}$$

Note, the only difference is that the pessimist's borrowing constraint now includes a term for outside storage. Let's first examine the effect of an exogenous increase in riskless investment, \bar{s} .

Mechanically, with the same constraints binding, we suspect that the interest rate will go down, because of (14), which is now

$$\frac{\alpha}{1 + s_i + \bar{s}_t} = 1 + r \quad (41)$$

However, it is possible that the pessimist will just store less and lend more, since borrowing will increase because of (2). Thus, the effect on the interest rate, even with the same binding constraints, is ambiguous. Additionally, the impact on prices is ambiguous, since price is also determined by the interest rate. To understand this problem, we will solve a modified version of our first example. That is, pick for parameters the following

$$T_j = 1$$

$$T_i = 1$$

$$n_j = 2$$

$$n_i = 2$$

$$L_i = 1$$

$$E_j[y] = 4$$

$$\gamma = \frac{1}{2}$$

$$\alpha = 5$$

$$\bar{s}_t = 1$$

Substituting these parameters into the system of equations above and simplifying leads to the following expressions.

$$4(1 + I_j) - 2p + \frac{2}{1+r} - (1 + I_j) = 0 \quad (42)$$

$$-p + \frac{5}{1+r} = -\frac{2}{(1+r)} + 4 \quad (43)$$

$$I_j + p = \frac{2}{(1+r)} + 2 \quad (44)$$

Our first equation is (25). This now says

$$I_j = n_j + n_i + 1 + \bar{s}_t - \frac{\alpha}{1+r}$$

Plugging in numbers yields the following expression

$$I_j = 6 - \frac{5}{1+r}$$

Now recall that in section "Basic CIM Example" we derived our other expression for I_j using only the optimist's first order conditions and budget constraint. Thus, that equation is unaffected by monetary policy. It is as follows

$$I_j = \frac{2}{5(1+r)} + \frac{1}{5}$$

Setting these two expressions equal yields

$$\frac{2}{5(1+r)} + \frac{1}{5} = 6 - \frac{5}{1+r}$$

Then we can multiply both sides by 5

$$\frac{2}{(1+r)} + 1 = 30 - \frac{25}{1+r}$$

Combining like terms yields

$$\frac{27}{(1+r)} = 29$$

And finally,

$$\frac{27}{29} = 1 + r$$

Then, it looks like the fed has driven the interest rate negative, which is certainly less than what we had before. Price clearly increases, as does investment. We verify this is the case below.

$$I_j = 6 - \frac{5 * 29}{27}$$

which simplifies to

$$I_j = \frac{17}{27}$$

Note that this is an increase in risky investment, relative to before. Solving for price shows

Similarly, price increases as follows. The optimist's budget constraint says

$$I_j + p = \frac{2}{(1+r)} + 2$$

Substituting in yields

$$\frac{17}{27} + p = \frac{(2)(29)}{27} + 2$$

Which simplifies to

$$p = \frac{95}{27}$$

This is an increase in price, as shown before. It should be clear that a tightening of monetary policy works in the reverse, decreasing prices, increasing interest rates, and decreasing lending.

Alternatively, we can specify fed policy as a subsidy to borrowing. Suppose that the pessimist lends amount b_i . Then we suppose the fed buys a fraction, say, κ , of those loans at price σ . Then the pessimists' budget constraint will loosen and they will store more, driving down the interest rate. This will work identically to before. To see this, examine the conditions characterizing the solution once more.

$$\begin{aligned}
E_j[y](1 + I_j) - p\gamma E_j[y] + \frac{\gamma E_j[y]}{1 + r} - (1 + I_j)L_i &= 0 \\
-pT_i + \frac{\alpha}{1 + r} - 1 &= -\frac{(T_j + T_j)L_i}{(1 + r)} + n_i + \frac{\sigma (T_j + T_j)L_i}{\kappa (1 + r)} \\
I_j + pT_j &= \frac{(T_j + T_j)L_i}{(1 + r)} + n_j
\end{aligned}$$

Note that, again, only the pessimist's borrowing constraint changes. Also note that the relationship between interest rates and storage is

$$\frac{\alpha}{1 + s_i} = 1 + r$$

That is, while the outcome is identical, the mechanism is different. Previously, the fed directly influenced the interest rate by investing in riskless assets. Here, the fed subsidizes lenders, who then do the same.

We solve an example with similar parameters to before.

$$T_j = 1$$

$$T_i = 1$$

$$n_j = 2$$

$$n_i = 2$$

$$L_i = 1$$

$$E_j[y] = 4$$

$$\gamma = \frac{1}{2}$$

$$\alpha = 5$$

$$\frac{\sigma}{\kappa} = \frac{1}{4}$$

Substituting in to our equation yields

$$4(1 + I_j) - 2p_t + \frac{2}{1+r} - (1 + I_j) = 0 - p_t + \frac{5}{1+r} = -\frac{2}{(1+r)} + 3 + \frac{2}{4(1+r)}I_j + p_t = \frac{2}{(1+r)} +$$

Again, we can get our version of (25) so that

$$I_j = 5 + \frac{1}{2(1+r)} - \frac{5}{1+r}$$

Which simplifies to

$$I_j = 5 - \frac{9}{2(1+r)}$$

Our other expression for investment is, again, unchanged. Setting them equal says

$$5 - \frac{9}{2(1+r)} = \frac{2}{5(1+r)} + \frac{1}{5}$$

Multiplying both sides by 10 yields

$$50 - \frac{45}{(1+r)} = \frac{4}{(1+r)} + 1$$

Combining like terms

$$49 = \frac{49}{(1+r)}$$

And, surprisingly, the parameters we've chosen have driven the interest rate to exactly zero.

Similarly, investment and price can be shown to be as follows

$$I_j = \frac{1}{2}$$

and

$$p = 3.5$$

Again, this is an increase in price, investment, and a decrease in the interest rate relative to our benchmark example.

5 Next Steps

In this paper we have constructed a simple framework that incorporates - often by way of examples - much of what has been said about the linkages between interest rates, leverage, investment, and asset prices. This is intended as a first installment payment on a larger project linking theory with data. Our eventual goal is to analyze the data we have compiled on empirical episodes of booms and busts in asset prices in light of the model. Our preliminary analysis of the data shows a rather robust empirical pattern: asset prices, interest rates, leverage, and physical investment in the asset all rise steadily during the up phases of the asset price cycle, and fall steadily and continually during the unwinding.

We will ultimately seek to understand which disturbances are most likely to account for the empirical pattern noted above. Prima facie it does not appear that exogenous loosening of credit is the mechanism driving bubbles - at least once they are in progress - nor does it appear that rises in asset prices straightforwardly generate their own leverage. The model is most suggestive of a type of joint endogeneity in which some third cause accounts for both the rise in leverage and interest rates and the rise in asset prices.

As a "teaser", we note that one prima facie appealing mechanism, that we call "exciting good news", says that the boom is fueled by an increase in upside uncertainty on the part of the natural buyers. The model, however, tells us that as long as both the short sale constraint and the collateral constraint bind, this cannot be the answer. Although the natural buyer would like to increase his borrowing to further leverage his position in the risky asset, the lender will not allow it. A more promising channel, which we call "reassuring good news", focuses on a reduction in downside uncertainty on the part of lenders. Hence the model gives some prima facie support to the notion that the seeds of the financial crisis that preceded the Great Recession

lay in complacency on the part of lenders, perhaps part of the legacy of the Great Moderation.

6 References

Allen, Franklin and Douglas Gale, 2000. "Bubbles and Crises" *Economic Journal*, 110(460), January, p236-255.

Barlevy, Gadi, 2011. "A Leverage-based Model of Speculative Bubbles" Federal Reserve Bank of Chicago Working Paper.

Fostel, Ana and Geanakoplos, John, 2013. "Leverage and Default in Binomial Economies: A Complete Characterization" Working Paper.

Geanakoplos, John, 2010. "The Leverage Cycle" *NBER Macroeconomics Annual*, 24(1), p1-66.

Miller, Edward. 1977. "Risk, Uncertainty, and Divergence of Opinion" *Journal of Finance*, 32(4) p1151-1168.

Simsek, Alp. 2013. "Belief Disagreements and Collateral Constraints" *Econometrica*, 81(1) p1-53.

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