Federal Reserve Bank of Chicago

# Adverse Selection, Risk Sharing and Business Cycles

*Marcelo Veracierto*

REVISED
October 2017

WP 2014-10

# Adverse Selection, Risk Sharing and Business Cycles

Marcelo Veracierto*

*Federal Reserve Bank of Chicago, U.S.A*

October, 2017

**Abstract:** I consider a real business cycle model in which agents have private information about their stochastic value of leisure. For the case of logarithmic preferences I provide an analytical characterization of the solution to the associated mechanism design problem. Moreover, I show a striking irrelevance result: That the stationary behavior of all aggregate variables are exactly the same in the private information economy as in the full information case. I then introduce a new computational method to show that the irrelevance result holds numerically for more general CRRA preferences.

**Keywords:** Adverse selection, risk sharing, business cycles, private information, social insurance, optimal contracts, computational methods, heterogeneous agents.

## 1 Introduction

At least since the seminal paper by Krusell et al. (1998) there has been a long literature analyzing the effects of exogenous forms of market incompleteness on aggregate fluctuations. The purpose of this paper is to take a more primitive approach by exploring the effects of restrictions to perfect risk sharing but when these restrictions arise optimally in response to information frictions. In order to do this the paper merges two basic benchmarks in the macroeconomics and private information literatures: A standard real business cycle (RBC) model and a Mirleesian economy. The mechanism design problem for the resulting economy is then solved for and its business cycle fluctuations compared to those of the full information case. The paper is not only interested in evaluating the

*I thank V.V. Chari, Chris Phelan, Venky Venkateswaran and participants at various seminars and conferences for useful comments. The views express here do not necessarily reflect the position of the Federal Reserve Bank of Chicago or the Federal Reserve System. Address: Federal Reserve Bank of Chicago, Research Department, 230 South LaSalle Street, Chicago, IL 60604. E-mail: mveracierto@frbchi.org. Phone: (312) 322-5695.

effects of private information on aggregate fluctuations, but in characterizing the behavior of the resulting optimal contracts and in exploring the implications for the optimal amount of consumption and employment inequality over the business cycle.

The model used is a simple RBC model with private information. Agents value consumption and leisure and receive idiosyncratic shocks to their value of leisure. These shocks, which are i.i.d. over time and across individuals, are assumed to be private information. The production technology is standard. Output, which can be consumed or invested, is produced with capital and labor using a Cobb-Douglas production function subject to an aggregate productivity shock. The aggregate shock follows an AR(1) process.

Following the literature, a dynamic contract is given a recursive formulation in which its state is given by a promised value to the agent. Given the current state, the contract specifies current consumption, current hours worked and next-period promised values as a function of the value of leisure reported by the agent. Since the model has a large number of agents and the shocks to the value of leisure are idiosyncratic, the social planner needs to keep track as a state variable the whole distribution of promised values across individuals. Given this distribution, the aggregate stock of capital and the aggregate productivity level, the social planner seeks to maximize the present discounted utility of agents subject to incentive compatibility, promise keeping and aggregate resource feasibility constraints.

For the case in which the utilities of consumption and leisure are both logarithmic (a benchmark case in the RBC literature), the paper provides a sharp analytical characterization of the solution to the mechanism design problem. Consumption, hours worked and next-period promised values are decreasing functions of the reported value of leisure. Moreover, the utility of consumption, utility of leisure and next-period promised values are all linear, strictly increasing functions of the current promised value. The slopes of these functions are all independent of the reported value of leisure, and while the utilities of consumption and leisure have a common slope less than one, the slope of next-period promised values is equal to one (as a consequence, promised values follow a random walk). Over the business cycle all of these functions shift vertically while maintaining constant the differences between the high and low values of leisure. In turn, the distributions of promised values and log-consumption levels shift horizontally over the business cycle while maintaining their shapes. While consumption inequality is constant over the business cycle, the dispersion of the distribution of log-hours worked is countercyclical. In terms of aggregate dynamics the paper finds a striking irrelevance result: The business cycle fluctuations of all macroeconomic variables (i.e. aggregate output, consumption, investment, hours worked and capital) are exactly the same under private information as under full information. That is, once the information frictions are dealt with in an optimal way they have no implications for the aggregate dynamics of the economy.

For preferences other than the log-log case, analytical results are no longer available and the model must be solved for numerically. The high dimensionality of the state space, which includes the distribution of promised values across individuals, makes computations difficult. A second contribution of the paper is to develop a strategy that makes this problem tractable. In fact, the computational method described here is not only applicable to the model in this paper but to a wide class of economies with heterogeneous agents and aggregate uncertainty.[2] The basic strategy is to parametrize individual decision rules as spline approximations and to keep long histories of the spline coefficients as state variables. Starting from the deterministic steady state distribution, the history of decision rules implied by the spline coefficients is then used to obtain the current distribution of individuals across individual states. This is done performing a large number of Monte Carlo simulations. I then linearize the first order conditions with respect to the coefficients of the spline approximations and solve the resulting linear rational expectations model using standard methods.

Applying this computational method to the economy with logarithmic preferences recovers all of the analytical results proved earlier on. Since nothing in the computational method takes advantage of the particular functional form of the utility function, this provides significant evidence about the accuracy of the method. Having established its accuracy the method is then used to analyze more general preferences. However, for all the CRRA preferences considered the same basic result is obtained: The stationary behavior of all macroeconomic variables in an economy with private information is numerically indistinguishable from the same economy with full information.

Dynamic optimal contracts under private information have been used to study a variety of issues in macroeconomics. For example, they have been used to study optimal consumption inequality (e.g. Atkeson and Lucas 1992, Green 1987, etc.), optimal unemployment insurance (e.g. Hopenhayn and Nicolini 1997, Kocherlakota 2004, etc.), and taxation (e.g. Golosov et al. 2007, Farhi and Werning 2012, etc.). However, any interactions with aggregate fluctuations have been mostly neglected. A notable exception is Phelan (1994) who considered a model in which agents take hidden actions that, together with the realization of a public i.i.d. aggregate shock and an unobservable i.i.d. idiosyncratic shock, determine their observed output levels. Assuming that actions are taken prior

[2]The computational method should be applicable to any model in which agents have smooth decision rules, are subject to idiosyncratic uncertainty, and in which the aggregate shocks are small and follow autorregressive processes.

to the realization of the aggregate shock, that agents have CARA preferences and that agents have a constant probability of dying, he was able to characterize the model analytically. He found two important results: that the cross-sectional distribution of consumption levels depends on the entire history of aggregate shocks and that there is a well defined long-run distribution over cross-sectional consumption distributions.

My model differs from Phelan (1994), not only because it has hidden types (adverse selection) instead of hidden actions (moral hazard), but because it has a neoclassical production function with persistent aggregate shocks. Besides these differences, an apparent similarity is that even in my model with logarithmic preferences the cross-sectional distributions of consumption and leisure depends on the entire history of aggregate shocks. However, this is only due to the presence of capital. Without it I would get that these cross-sectional distributions only depend on the current realization of aggregate productivity.

In fact the lack of memory in the case of no capital and logarithmic preferences has already been shown by DaCosta and Luz (2013) in a related setting. In that paper DaCosta and Luz consider a finite horizon version of Phelan's economy in which actions are taken after the realization of aggregate productivity, agents have CRRA preferences, and agents live as long as the economy. Contrary to Phelan (1994), their cross-sectional distribution of consumption becomes degenerate as the time horizon of the economy becomes large. Interestingly, DaCosta and Luz find that when log preferences are used that the cross-sectional distribution of consumption does not depend on the entire history of aggregate shocks but on the current realization. However, when the elasticity of intertemporal substitution is different than one, the cross-sectional distribution of consumption has memory of the past history. A major contribution of this paper over DaCosta and Luz (2013) for the case of logarithmic preferences is that, in addition to analyzing an economy with capital and persistent aggregate shocks, I provide a tight analytical characterization of the optimal contracts and an equivalence result with the full information economy. For preferences different from the logarithmic case, I am able to compute solutions for infinite horizon economies instead of two-periods cases.

The equivalence with the full information economy in terms of aggregate variables is related to a result in Farhi and Werning (2012). In that paper Fahri and Werning also consider a Mirleesian economy similar to the one in this paper except that it has no aggregate productivity shocks, idiosyncratic shocks are persistent and the social planner is only allowed to optimize with respect to the consumption allocations (labor allocations are taken to be beyond his control). Starting from the steady state of a Bewley economy they perform the dynamic public finance experiment of evaluating the welfare gains of moving to an optimal consumption plan. They show that when preferences are logarithmic, along the transitionary dynamics of the model all aggregate variables behave exactly the same as in a representative agent economy. Interestingly, I obtain a similar equivalence result when optimizing with respect to labor as well as consumption and when the economy is subject to aggregate productivity shocks. However, contrary to Farhi and Werning (2012), the equivalence result with a representative agent economy only holds for the long-run stationary equilibrium of the model. The transitionary dynamics from an arbitrary initial state will generally differ from the representative agent case.[3]

The paper is organized as follows. Section 2 builds intuition for the main results in the paper by analyzing a simple static economy. Section 3 describes the dynamic economy. Section 4 describes the mechanism design problem for this economy in recursive form. Section 5 establishes the irrelevance result for the log-log case. Section 6 characterizes the cyclical behavior of the cross-sectional amount of consumption and employment inequality in the log-log case. Section 7 describes the computational method. Section 8 presents the numerical results. Finally, Section 9 concludes the paper.

## 2 A static economy

This section analyzes the optimal provision of social insurance and incentives in a static economy. The purpose is to build intuition towards one of the main results in the paper: The irrelevance of private information for aggregate allocations in the case of logarithmic preferences.

The economy is populated by a unit measure of agents with preferences given by

$$E\left\{u\left(c\right)+sn\left(1-h\right)\right\}$$

where $c$ is consumption, $h$ is hours worked, $s$ is the idiosyncratic value of leisure and $u$ and $n$ are continuously differentiable, strictly increasing and strictly concave utility functions. The idiosyncratic value of leisure $s$ takes two possible values: $s_L$ and $s_H$, with $s_L < s_H$. Realizations of $s$ are i.i.d. across individuals and are distributed according to a distribution function $\psi = (\psi_L, \psi_H)$. A key assumption is that $s$ is private information of the individual.

[3] Strictly speaking, the transitionary dynamics differ from those of a representative agent economy with stationary preferences. Section 5 shows that the transitionary dynamics generally coincide with those of a representative agent economy with preferences that shift over time.

Output is produced according to the following production function:

$$Y = e^z F(H),$$

where $Y$ is aggregate output, $H$ is aggregate hours worked and $F$ is continuously differentiable, strictly increasing, concave and satisfies the Inada conditions.

The mechanism design problem is the following:

$$\max \sum_s \left[u_s + s n_s\right] \psi_s \tag{2.1}$$

subject to

$$\sum_s u^{-1}(u_s)\, \psi_s \leq e^z F(H), \tag{2.2}$$

$$H \leq \sum_s \left[1 - n^{-1}(n_s)\right] \psi_s, \tag{2.3}$$

$$u_L + s_L n_L \geq u_H + s_L n_H, \tag{2.4}$$

where equation (2.2) is the aggregate feasibility constraint for the consumption good, equation (2.3) is the aggregate feasibility constraint for hours worked and equation (2.4) is the binding incentive compatibility constraint.[4] I formulate the planning problem in terms of the utilities of consumption and leisure (instead of consumption and leisure levels) in order to obtain a convex feasible set, which is crucial for characterizing the solution using first order conditions.

The unique solution to this problem satisfies equations (2.2)-(2.4) and the following first order conditions:

$$0 = \psi_L - \lambda \frac{1}{u'(c_L)} \psi_L + \lambda \xi, \tag{2.5}$$

$$0 = \psi_H - \lambda \frac{1}{u'(c_H)} \psi_H - \lambda \xi, \tag{2.6}$$

$$0 = s_L \psi_L - \lambda q \frac{1}{n'(1 - h_L)} \psi_L + s_L \lambda \xi, \tag{2.7}$$

$$0 = s_H \psi_H - \lambda q \frac{1}{n'(1 - h_H)} \psi_H - s_L \lambda \xi, \tag{2.8}$$

$$q = e^z F'(H), \tag{2.9}$$

where $\lambda$, $\lambda q$, and $\lambda \xi$ are the Lagrange multipliers of equations (2.2), (2.3) and (2.4), respectively, and where $c_s = u^{-1}(u_s)$ and $1 - h_s = n^{-1}(n_s)$. Observe that in these equations (and in the rest of the paper) a variable $x_s$ is denoted $x_L$ when $s = s_L$ and $x_H$ when $s = s_H$.

[4] It can be shown that the truth-telling constraint for an agent with the high value of leisure will not be binding under the optimal allocation. See Section 2 in the Technical Appendix for the details.

From equations (2.6) and (2.8) we have that

$$q\frac{1}{n'(1-h_H)} = s_H\frac{1}{u'(c_H)} + \frac{(s_H - s_L)\lambda\xi}{\lambda\psi_H}.$$

Hence,

$$q > s_H\frac{n'(1-h_H)}{u'(c_H)}. \tag{2.10}$$

Since the marginal rate of substitution of leisure for consumption is less than the shadow wage rate $q$, it follows that under an optimal plan agents with the high value of leisure are "taxed" their labor supply. On the contrary, from equations (2.5) and (2.7) we have that

$$q = s_L\frac{n'(1-h_L)}{u'(c_L)}. \tag{2.11}$$

That is, the labor supply decision of agents with the low value of leisure is undistorted.

Consider now the social planner problem of this same economy but under full information. This problem is to maximize equation (2.1) subject to equations (2.2) and (2.3). Setting $\xi = 0$ in equations (2.5)-(2.9) we get that the optimal allocation under full information satisfies:

$$C^* = e^z F(H^*), \tag{2.12}$$

$$H^* = h_L^*\psi_L + h_H^*\psi_H, \tag{2.13}$$

$$\lambda^* = u'(C^*), \tag{2.14}$$

$$0 = s_L - \lambda^* q^* \frac{1}{n'\left(1-h_L^*\right)}, \tag{2.15}$$

$$0 = s_H - \lambda^* q^* \frac{1}{n'\left(1-h_H^*\right)}, \tag{2.16}$$

$$q^* = e^z F'(H^*). \tag{2.17}$$

That is, under full information agents' consumption is fully insured and, while $h_H^* < h_L^*$, equations (2.14)-(2.16) imply that both types of agents face zero labor supply taxes.

A crucial question is under what conditions the aggregate allocation of the full information economy $(C^*, H^*)$ is identical to that of the private information economy. To see this, lets try to seek a solution to equations (2.2)-(2.9) that satisfy that $H = H^*$. In fact, in doing so it will be convenient to rewrite those equations as follows:

$$c_L\psi_L + c_H\psi_H = C^* \tag{2.18}$$

$$h_L\psi_L + h_H\psi_H = H^* \tag{2.19}$$

$$u(c_L) + s_L n(1 - h_L) = u(c_H) + s_L n(1 - h_H) \tag{2.20}$$

$$0 = \psi_L - \lambda \frac{1}{u'(c_L)} \psi_L + \lambda \xi, \tag{2.21}$$

$$1 = \lambda \left[ \frac{1}{u'(c_L)} \psi_L + \frac{1}{u'(c_H)} \psi_H \right] \tag{2.22}$$

$$0 = s_L \psi_L - \lambda q^* \frac{1}{n'(1 - h_L)} \psi_L + s_L \lambda \xi, \tag{2.23}$$

$$\bar{s} \left[ \frac{1}{u'(c_L)} \psi_L + \frac{1}{u'(c_H)} \psi_H \right] = q^* \left[ \frac{1}{n'(1 - h_L)} \psi_L + \frac{1}{n'(1 - h_H)} \psi_H \right] \tag{2.24}$$

where, using equations (2.9) and (2.17), $q$ has already been substituted by $q^*$ and $\bar{s} = s_L \psi_L + s_H \psi_H$. Observe that equation (2.22) is obtained by adding equations (2.5) and (2.6) and that equation (2.24) is obtained by adding equations (2.7) and (2.8) and using (2.22).

Equations (2.18)-(2.24) form a system of 7 equations in 6 unknowns: $c_L$, $c_H$, $h_L$, $h_H$, $\lambda$ and $\xi$. As a consequence, a solution will generally not exist. In particular, suppose that we have a solution $(c_L, c_H, h_L, h_H, \lambda, \xi)$ to equations (2.18)-(2.23). Then only by chance equation (2.24) would also be satisfied. However, there is an exception: when $1/u'$ is a linear function of $c$ and $1/n'$ is a linear function of $1 - h$. Observe that in this case equation (2.24) reduces to

$$\bar{s} \frac{1}{u'(c_L \psi_L + c_H \psi_H)} = q^* \frac{1}{n'(1 - h_L \psi_L - h_H \psi_H)} \tag{2.25}$$

and, using equations (2.18) and (2.19), to the following:

$$\bar{s} \frac{1}{u'(C^*)} = q^* \frac{1}{n'(1 - H^*)}. \tag{2.26}$$

But this equation is guaranteed to hold since $C^*$ and $H^*$ correspond to a solution of the full information planning problem. To see this, multiply equation (2.15) by $\psi_L$ and equation (2.16) by $\psi_H$, add them and use equation (2.14) to get:

$$\bar{s} \frac{1}{u'(C^*)} = q^* \left[ \frac{1}{n'(1 - h_L^*)} \psi_L + \frac{1}{n'(1 - h_H^*)} \psi_H \right]. \tag{2.27}$$

Equation (2.26) now follows from equation (2.27) and the linearity of $1/n'$.

This argument has established that logarithmic functional forms for both $u$ and $n$ are generally needed to get identical aggregate allocations under private and full information. Moreover, equation (2.26) indicates that under logarithmic preferences the aggregate allocation of the private information economy coincides with the aggregate allocation of a representative agent model with preferences given by

$$\ln(C) + \bar{s} \ln(1 - H).$$

Furthermore, from equations (2.12), (2.17) and (2.26) it can be verified that aggregate hours worked $H^*$ are independent of aggregate productivity $z$ (a standard result under separable and log of consumption preferences). In addition, equations (2.2)-(2.9) imply that $h_L$ and $h_H$ are independent of $z$ while $c_L$ and $c_H$ vary proportionately with it. It follows that the cross sectional variances of log-hours worked and of log-consumption levels are independent of $z$.

It is also useful to observe from equations (2.14), (2.18) and (2.22) that under logarithmic preferences $\lambda = \lambda^*$. From equations (2.5), (2.6), (2.14) and the concavity of $u$ we then see that $c_H < C^* < c_L$. From equations (2.7), (2.8), (2.15), (2.16), the concavity of $n$ and the fact that $q = q^*$ we also see that $1 - h_L < 1 - h_L^*$ and that $1 - h_H^* < 1 - h_H$. Since, $1 - h_L^* < 1 - h_H^*$ if follows that under private information agents not only receive less insurance in terms of consumption levels but also in terms of leisure levels. Thus, while aggregate allocations are identical in the private and full information cases, there are important differences in the individual allocations.

# 3 The dynamic economy

The previous section showed that when preferences are logarithmic (both in consumption and in leisure), that the presence of private information becomes irrelevant for the optimal aggregate allocation of a static economy. In what follows I explore if this irrelevance result can be extended to a dynamic setting. There are three reasons for doing this. First, a static economy with logarithmic preferences is quite uninteresting from a macroeconomic point of view since, as was previously mentioned, aggregate hours are not affected by the realization of aggregate productivity. Second, in a dynamic setting the social planner uses intertemporal rewards and punishments to induce truthful revelation in addition to the intratemporal elements already present in a static environment. It is unclear whether logarithmic preferences will be able to jointly aggregate these intertemporal and intratemporal margins into those of a representative agent economy with full information. Third, even if private information under logarithmic preferences plays no role for aggregate allocations it seems important to characterize the cyclical behavior of the optimal amount of cross-sectional inequality in consumption and hours worked within the realm of a realistic business cycle model. The reason is that the presence of private information may be able to explain cross-sectional cyclical observations that a representative agent model is not able to address. For these reasons, this section incorporates the private information structure of the previous section into a standard real business cycle model and characterizes its optimal allocation.

The economy is populated by a unit measure of agents subject to stochastic lifetimes. Whenever an agent dies he is immediately replaced by a newborn, leaving the aggregate population level constant.[5] The preferences of an individual born at date $T$ are given by

$$E_T \left\{ \sum_{t=T}^{\infty} \beta^{t-T} \sigma^{t-T} \left[ u\left(c_t\right) + s_t n\left(1-h_t\right) \right] \right\}, \tag{3.1}$$

where $\sigma$ is the survival probability, $0 < \beta < 1$ is the discount factor, and $u$ and $n$ have the same properties as in the static economy. Realizations of the idiosyncratic value of leisure $s_t \in \{s_L, s_H\}$ are assumed to be i.i.d. not only across individuals but also across time.[6]

Output, which can be consumed or invested, is produced with the following production function:

$$Y_t = e^{z_t} K_{t-1}^{\gamma} H_t^{1-\gamma}$$

where $Y_t$ is output, $z_t$ is aggregate productivity, $K_{t-1}$ is capital and $H_t$ is hours worked. The aggregate productivity level $z_t$ follows a standard AR(1) process given by:

$$z_{t+1} = \rho z_t + \varepsilon_{t+1},$$

where $0 < \rho < 1$ and $\varepsilon_{t+1}$ is normally distributed with mean zero and standard deviation $\sigma_\varepsilon$.

Capital is accumulated using a standard linear technology given by

$$K_t = (1-\delta) K_{t-1} + I_t,$$

where $I_t$ is gross investment and $0 < \delta < 1$.

# 4 Recursive mechanism design problem

This section provides a recursive formulation to the problem of a social planner that seeks to maximize utility subject to incentive compatibility, promise keeping and resource feasibility constraints. In order to do this it will be important to distinguish between two types of agents: young and old. A young agent is one that has been born at the beginning of the current period. An old agent is one that has been born in some previous period.

[5] As in Phelan (1994), the stochastic lifetimes guarantee that there will be a stationary distribution of agents across individual states. While there are other ways to generate this outcome, the advantage of having stochastic lifetimes will become apparent in Section 7 once the computational method is described.

[6] In principle it would be desirable to introduce persistence to the idiosyncratic shocks. However, this would require specifying a mechanism design problem with not only promise keeping constraints, but with threat keeping constraints as well (see Fernandes and Phelan 2000). As a consequence, all allocation rules would be two dimensional, making the analysis and computations quite more complicated.

The social planner decides recursive plans for both types of agents. The state of a recursive plan is the value (i.e. discounted expected utility) that the agent is entitled to at the beginning of the period. Given this promised value, the recursive plan specifies the current utility of consumption, the current utility of leisure and next period promised values as functions of the value of leisure currently reported by the agent. The social planner is fully committed to the recursive plans that he chooses and agents have no outside opportunities available.

A key difference between the young and the old is in terms of promised values. Since during the previous period the social planner has already decided on some recursive plan for a currently old agent, he is restricted to deliver the corresponding promised value during the current period. On the contrary, the social planner is free to deliver any value to a currently young agent since this is the first period that he is alive. Reflecting this difference, I will specify the individual state of an old agent to be his promised value $v$ and his current value of leisure $s$. His current utility of consumption, utility of leisure and next-period promised values are denoted by $u_{os}(v)$, $n_{os}(v)$ and $w_{os}(v, z')$, respectively. In turn, the individual state of a young agent is solely given by his current value of leisure $s$. His current utility of consumption, utility of leisure and next-period promised values are denoted by $u_{ys}$, $n_{ys}$ and $w_{ys}(z')$, respectively. Observe that next-period promised values of young and old agents are allowed to be contingent on the realization of next-period aggregate productivity $z'$.

The aggregate state of the economy is given by the triplet $(z_t, K_{t-1}, \mu_t)$, where $z_t$ is the aggregate productivity level, $K_{t-1}$ is the stock of capital, and $\mu_t$ is a measure describing the number of old agents across individual promised values $v$.[7] The social planner seeks to maximize the weighted sum of welfare levels of current and future generations of young agents (the welfare levels of old agents are predetermined by their promised values at the beginning of the period). In recursive form, the social planner problem is described by the following Bellman equation:

$$V(z_t, K_{t-1}, \mu_t) = \max \left\{ (1-\sigma) \sum_s \left[ u_{yst} + s n_{yst} + \beta\sigma E_t (w_{ys,t+1}) \right] \psi_s + \theta E_t V(z_{t+1}, K_t, \mu_{t+1}) \right\} \tag{4.1}$$

subject to:

$$(1-\sigma) \sum_s u^{-1}(u_{yst}) \psi_s + \int \sum_s u^{-1} \left[ u_{ost}(v) \right] \psi_s d\mu_t + K_t - (1-\delta) K_{t-1} \leq e^{z_t} K_{t-1}^{\gamma} H_t^{1-\gamma}, \tag{4.2}$$

$$H_t \leq (1-\sigma) \sum_s \left\{ 1 - n^{-1}(n_{yst}) \right\} \psi_s + \int \sum_s \left\{ 1 - n^{-1} \left[ n_{ost}(v) \right] \right\} \psi_s d\mu_t, \tag{4.3}$$

[7] Throughout the paper I follow the convention that a variable is dated $t$ if it becomes known at date $t$.

$$u_{yLt} + s_L n_{yLt} + \beta\sigma E_t \left[w_{yL,t+1}\right] \geq u_{yHt} + s_L n_{yHt} + \beta\sigma E_t \left[w_{yH,t+1}\right] \tag{4.4}$$

$$u_{oLt}(v) + s_L n_{oLt}(v) + \beta\sigma E_t \left[w_{oL,t+1}(v)\right] \geq u_{oHt}(v) + s_L n_{oHt}(v) + \beta\sigma E_t \left[w_{oH,t+1}(v)\right], \tag{4.5}$$

$$v = \sum_s \left\{u_{ost}(v) + s n_{ost}(v) + \beta\sigma E_t \left[w_{os,t+1}(v)\right]\right\} \psi_s, \tag{4.6}$$

$$\mu_{t+1}(B) = \sigma \sum_s \int_{\{(v,s):\ w_{os,t+1}(v) \in B\}} \psi_s d\mu_t + (1-\sigma)\sigma \sum_{s:\ w_{ys,t+1} \in B} \psi_s, \tag{4.7}$$

where $E_t$ denotes expectation conditional on $z_t$ and $\beta\sigma < \theta < 1$ is the welfare weight of the next-period generation relative to the current-period generation. Equation (4.2) describes the aggregate feasibility constraint for the consumption good. It states that the total consumption of young and old agents, plus aggregate investment cannot exceed aggregate output.[8] Equation (4.3) is the aggregate labor feasibility constraint. It states that the input of hours into the production function cannot exceed the total hours worked by young and old agents. Equations (4.4) and (4.5) are the binding incentive compatibility constraints of young and old agents, respectively. Equation (4.6) is the promise keeping constraint. It states that the recursive plan for an old agent with promised value $v$ must provide him an expected utility equal to that promised value. Equation (4.7) is the law of motion for the measure of old agents across promised values. It states that the number of old agents that at the beginning of the following period will have a promised value in the Borel set $B$ is given by the sum of two terms. The first term sums all currently old agents that receive a next-period promised value in the set $B$ and do not die. The second term does the same for all currently young agents. Observe that since next-period promised values $w_{os,t+1}(v)$ and $w_{ys,t+1}$ are contingent on the realization of next-period aggregate productivity $z_{t+1}$, that the same is true for the measure $\mu_{t+1}$.

Since the objective function in equation (4.1) is linear and increasing and equations (4.2)-(4.7) define a convex feasible set, the solution to the social planning problem is unique.[9] This solution satisfies equations (4.2)-(4.7) and the following first order conditions:

$$0 = \psi_L - \lambda_t \frac{1}{u'(c_{yLt})} \psi_L + \lambda_t \xi_{yt}, \tag{4.8}$$

$$0 = \psi_H - \lambda_t \frac{1}{u'(c_{yHt})} \psi_H - \lambda_t \xi_{yt}, \tag{4.9}$$

$$0 = s_L \psi_L - \lambda_t q_t \frac{1}{n'\left[1 - h_{yLt}\right]} \psi_L + s_L \lambda_t \xi_{yt}, \tag{4.10}$$

[8] Observe that, given the constant probability of dying $1-\sigma$ and the immediate replacement with newborns, the number of young agents in the economy is always equal to $1-\sigma$.

[9] For a proof, see Section 1 in the Technical Appendix.

$$0 = s_H \psi_H - \lambda_t q_t \frac{1}{n' [1 - h_{yHt}]} \psi_H - s_L \lambda_t \xi_{yt}, \tag{4.11}$$

$$0 = \beta \sigma \psi_L + \lambda_t \beta \sigma \xi_{yt} - \theta \lambda_{t+1} \sigma \psi_L \eta_{t+1} (w_{yL,t+1}), \tag{4.12}$$

$$0 = \beta \sigma \psi_H - \lambda_t \beta \sigma \xi_{yt} - \theta \lambda_{t+1} \sigma \psi_H \eta_{t+1} (w_{yH,t+1}), \tag{4.13}$$

$$0 = -\frac{1}{u' [c_{oLt} (v)]} \psi_L + \xi_{ot} (v) + \eta_t (v) \psi_L, \tag{4.14}$$

$$0 = -\frac{1}{u' [c_{oHt} (v)]} \psi_H - \xi_{ot} (v) + \eta_t (v) \psi_H, \tag{4.15}$$

$$0 = -q_t \frac{1}{n' [1 - h_{oLt} (v)]} \psi_L + s_L \xi_{ot} (v) + \eta_t (v) s_L \psi_L, \tag{4.16}$$

$$0 = -q_t \frac{1}{n' [1 - h_{oHt} (v)]} \psi_H - s_L \xi_{ot} (v) + \eta_t (v) s_H \psi_H, \tag{4.17}$$

$$0 = \lambda_t \beta \sigma \xi_{ot} (v) + \lambda_t \eta_t (v) \beta \sigma \psi_L - \theta \lambda_{t+1} \sigma \psi_L \eta_{t+1} [w_{oL,t+1} (v)], \tag{4.18}$$

$$0 = -\lambda_t \beta \sigma \xi_{ot} (v) + \lambda_t \eta_t (v) \beta \sigma \psi_H - \theta \lambda_{t+1} \sigma \psi_H \eta_{t+1} [w_{oH,t+1} (v)], \tag{4.19}$$

$$0 = q_t - e^{z_t} K_{t-1}^{\gamma} (1 - \gamma) H_t^{-\gamma}, \tag{4.20}$$

$$0 = -\lambda_t + \theta E_t \left\{ \lambda_{t+1} \left[ e^{z_{t+1}} \gamma K_t^{\gamma - 1} H_{t+1}^{1-\gamma} + 1 - \delta \right] \right\}, \tag{4.21}$$

where $\lambda_t$, $\lambda_t q_t$, $\lambda_t \xi_{yt}$, $\lambda_t \xi_{ot} (v)$ and $\lambda_t \eta_t (v)$ are the Lagrange multipliers of equations (4.2)-(4.6), respectively.

Since $\eta_{t+1}$ is strictly increasing, $u$ and $n$ are strictly concave, and equations (4.4) and (4.5) hold with equality, equations (4.14)-(4.19) imply that

$$c_{oHt} (v) < c_{oLt} (v), \tag{4.22}$$

$$h_{oHt} (v) < h_{oLt} (v), \tag{4.23}$$

$$w_{oH,t+1} (v) < w_{oL,t+1} (v), \text{ almost surely.} \tag{4.24}$$

and equations (4.8)-(4.13) imply a similar relation for young agents.[10] These relations are quite intuitive. They state that when an agent (young or old) reports a high value of leisure, the planner allows him to enjoy more leisure but, in compensation, he receives less consumption and is promised a worse treatment in the future.

Since equations (4.8)-(4.11) are the same as equations (2.5)-(2.8) it follows that equations (2.10) and (2.11) hold for young agents. Actually, it is straightforward to verify that equations (4.14)-(4.17) imply that equations (2.10) and (2.11) also hold for old agents. That is, similarly to the

[10] See Section 2 in the Technical Appendix for a proof that $\eta_{t+1}$ is strictly increasing and other details.

static economy, agents with the high value of leisure have their labor supply decision distorted while agents with the low value of leisure do not (irrespective of the agents being young or old).

From equations (4.14), (4.15), (4.18), (4.19) and (4.21) we get that for every $s$,

$$u'\left[c_{ost}\left(v\right)\right]=\beta E_t\left\{\frac{e^{z_{t+1}}\gamma K_t^{\gamma-1}{H_{t+1}}^{1-\gamma}+1-\delta}{\frac{1}{u'\left[c_{oL,t+1}\left(w_{os,t+1}\left(v\right)\right)\right]}\psi_L+\frac{1}{u'\left[c_{oH,t+1}\left(w_{os,t+1}\left(v\right)\right)\right]}\psi_H}\right\}, \tag{4.25}$$

a relation known in the Dynamic Public Finance literature as the Inverse Euler equation. Applying Jensen's inequality to equation (4.25) we get that

$$u'\left[c_{ost}\left(v\right)\right]<\beta E_t\left\{\left(r_{t+1}+1-\delta\right)\left[u'\left[c_{oL,t+1}\left(w_{os,t+1}\left(v\right)\right)\right]\psi_L+u'\left[c_{oH,t+1}\left(w_{os,t+1}\left(v\right)\right)\right]\psi_H\right]\right\}, \tag{4.26}$$

where $r_{t+1}=e^{z_{t+1}}\gamma K_t^{\gamma-1}{H_{t+1}}^{1-\gamma}$. That is, there is a wedge in the intertemporal Euler equations of old agents. Using equations (4.8), (4.9), (4.12), (4.13) and (4.21) we derive similar relations to equations (4.25) and (4.26) but for young agents. We conclude that, irrespective of being young or old, under an optimal allocation agents have their intertemporal decisions distorted.

# 5 An irrelevance result under logarithmic preferences

The previous section described the mechanism design problem in recursive form. However, date 0 is special because it has no ongoing recursive plans in place on which promised values must be delivered. As a consequence, all agents at date 0 must be treated as young. The date-0 mechanism design problem is thus given by

$$\max\left\{\sum_s\left[u_{ys0}+sn_{ys0}+\beta\sigma E_t\left(w_{ys1}\right)\right]\psi_s+\theta E_0V\left(z_1,K_0,\mu_1\right)\right\} \tag{5.1}$$

subject to

$$\sum_s u^{-1}\left(u_{ys0}\right)\psi_s+K_0-\left(1-\delta\right)K_{-1}\leq e^{z_0}K_{-1}^{\gamma}H_0^{1-\gamma}, \tag{5.2}$$

$$H_0\leq\sum_s\left\{1-n^{-1}\left(n_{ys0}\right)\right\}\psi_s \tag{5.3}$$

$$u_{yL0}+s_Ln_{yL0}+\beta\sigma E_0\left[w_{yL,1}\right]\geq u_{yH0}+s_Ln_{yH0}+\beta\sigma E_0\left[w_{yH,1}\right] \tag{5.4}$$

$$\mu_1\left(B\right)=\sigma\sum_{s:\ w_{ys1}\in B}\psi_s, \tag{5.5}$$

where $V$ is the value function in equation (4.1) and $(z_0,K_{-1})$ is taken as given. The solution to this problem satisfies equations (4.2)-(4.21) for $t\geq1$, equations (5.2)-(5.5), and equations (4.8)-(4.13) and (4.20)-(4.21) for $t=0$.

By contrast, consider the following non-stationary representative agent planning problem:

$$\max E_0 \left\{ \sum_{t=0}^{\infty} \alpha_t \theta^t \left[ u(C_t) + \bar{s} n(1 - H_t) \right] \right\} \tag{5.6}$$

subject to:

$$C_t + K_t - (1-\delta) K_{t-1} \leq e^{z_t} K_{t-1}^{\gamma} H_t^{1-\gamma}. \tag{5.7}$$

where $\alpha_t > 0$ is a deterministic preference shifter with positive limit and $(z_0, K_{-1})$ is taken as given. Its solution is characterized by equation (5.7) and the following first order conditions:

$$\bar{s} n'(1 - H_t) = u'(C_t) e^{z_t} K_{t-1}^{\gamma} (1-\gamma) H_t^{-\gamma}, \tag{5.8}$$

$$1 = \theta \frac{\alpha_{t+1}}{\alpha_t} E_t \left\{ \frac{u'(C_{t+1})}{u'(C_t)} \left[ e^{z_{t+1}} \gamma K_t^{\gamma-1} H_{t+1}^{1-\gamma} + 1 - \delta \right] \right\}. \tag{5.9}$$

In what follows I show that when $u$ and $n$ are both logarithmic, the optimal aggregate allocation of the economy with private information coincides with the solution to the representative agent planning problem (5.6) under a particular sequence $\{\alpha_t\}_{t=0}^{\infty}$.

Under logarithmic $u$ equations (4.8)-(4.9) imply that the average consumption of young agents is given by

$$C_t^y = \sum_s c_{yst} \psi_s = \frac{1}{\lambda_t}, \text{ for } t \geq 0, \tag{5.10}$$

and equations (4.14)-(4.15) imply that the average consumption of old agents is given by

$$C_t^o = \frac{1}{\sigma} \int \sum_s c_{ost}(v) \psi_s d\mu_t = \frac{1}{\sigma} \int \eta_t(v) d\mu_t, \text{ for } t \geq 1. \tag{5.11}$$

From equations (4.10)-(4.11) and the integral of equations (4.16)-(4.17) we have under logarithmic $n$ that

$$\begin{aligned} (1-\sigma) \bar{s} \frac{1}{\lambda_t} + \bar{s} \int \eta_t(v) d\mu_t &= (1-\sigma) q_t \left[ (1 - h_{yLt}) \psi_L + (1 - h_{yHt}) \psi_H \right] \\ &+ q_t \int \left[ (1 - h_{oLt}(v)) \psi_L + (1 - h_{oHt}(v)) \psi_H \right] d\mu_t, \text{ for } t \geq 1 \end{aligned} \tag{5.12}$$

and

$$\bar{s} \frac{1}{\lambda_0} = q_0 \left[ (1 - h_{yL0}) \psi_L + (1 - h_{yH0}) \psi_H \right]. \tag{5.13}$$

Since aggregate consumption is given by

$$C_t = \begin{cases} (1-\sigma) C_t^y + \sigma C_t^o, \text{ for } t \geq 1, \\ C_0^y, \text{ for } t = 0, \end{cases} \tag{5.14}$$

it follows from equations (4.3),(5.3) and (5.10)-(5.14) that

$$\bar{s} C_t = q_t (1 - H_t), \text{ for } t \geq 0, \tag{5.15}$$

which is the representative agent intratemporal condition (5.8). Observe that we have obtained this aggregate optimality condition for the same reason as in the static economy: The intratemporal optimality conditions in the private information economy are linear under logarithmic preferences.

Deriving the representative agent intertemporal Euler equation (5.9) for a suitable sequence $\{\alpha_t\}_{t=0}^{\infty}$ is somewhat more involved. From equations (4.7), (4.12)-(4.13), (4.18)-(4.19) and (5.5) we have that

$$\begin{aligned}\int \eta_{t+1}(v)\,d\mu_{t+1} &= \sigma \int \sum_s \eta_{t+1}\left[w_{os,t+1}(v)\right]\psi_s d\mu_t + (1-\sigma)\,\sigma \sum_s \eta_{t+1}\left(w_{ys,t+1}\right)\psi_s \\ &= \frac{\lambda_t \beta\sigma}{\theta\lambda_{t+1}} \int \eta_t(v)\,d\mu_t + (1-\sigma)\frac{\beta\sigma}{\theta\lambda_{t+1}}, \text{ for } t \geq 1 \end{aligned} \tag{5.16}$$

and

$$\int \eta_1(v)\,d\mu_1 = \sigma \sum_s \eta_1\left(w_{ys,1}\right)\psi_s = \frac{\beta\sigma}{\theta\lambda_1}. \tag{5.17}$$

Let $\{\rho_t\}_{t=1}^{\infty}$ be defined as follows:

$$\begin{aligned}\rho_1 &= \frac{\beta\sigma}{\theta}, \\ \rho_{t+1} &= \frac{\beta\sigma}{\theta}\rho_t + (1-\sigma)\frac{\beta\sigma}{\theta}, \text{ for } t \geq 1.\end{aligned} \tag{5.18}$$

From equations (5.16) and (5.17) it follows that

$$\rho_t = \lambda_t \int \eta_t(v)\,d\mu_t, \text{ for } t \geq 1.$$

Then, the ratio of the average consumption of old agents to the average consumption of young agents is given by

$$\frac{C_t^o}{C_t^y} = \frac{\frac{1}{\sigma}\int \eta_t(v)\,d\mu_t}{\frac{1}{\lambda_t}} = \frac{1}{\sigma}\rho_t, \text{ for } t \geq 1,$$

and consequently,

$$\frac{C_t}{C_{t+1}} = \frac{(1-\sigma)\,C_t^y + \sigma C_t^o}{(1-\sigma)\,C_{t+1}^y + \sigma C_{t+1}^o} = \frac{(1-\sigma)+\rho_t}{(1-\sigma)+\rho_{t+1}}\frac{C_t^y}{C_{t+1}^y} = \frac{(1-\sigma)+\rho_t}{(1-\sigma)+\rho_{t+1}}\frac{\lambda_{t+1}}{\lambda_t}, \text{ for } t \geq 1. \tag{5.19}$$

Also, observe that

$$\frac{C_0}{C_1} = \frac{C_0^y}{(1-\sigma)\,C_1^y + \sigma C_1^o} = \frac{1}{(1-\sigma)\,C_1^y + \rho_1 C_1^y}\frac{1}{\lambda_0} = \frac{1}{(1-\sigma)+\rho_1}\frac{\lambda_1}{\lambda_0}. \tag{5.20}$$

Defining $\{\alpha_t\}_{t=0}^{\infty}$ as

$$\alpha_t = \begin{cases} 1, \text{ for } t = 0 \\ (1-\sigma)+\rho_t, \text{ for } t \geq 1, \end{cases} \tag{5.21}$$

equations (5.19) and (5.20) can then be written as

$$\frac{C_t}{C_{t+1}} \frac{\alpha_{t+1}}{\alpha_t} = \frac{\lambda_{t+1}}{\lambda_t}, \text{ for } t \geq 0. \tag{5.22}$$

From equations (4.21) and (5.22) it follows that the intertemporal Euler equation of the representative agent (5.9) holds for $t \geq 0$. Since equations (4.2) and (5.2) imply equation (5.7), we have thus established the following Lemma.

**Lemma 1** *Suppose that $u$ and $n$ are logarithmic. Define $\alpha = \{\alpha_t\}_{t=0}^{\infty}$ as in equation (5.21). Then, the optimal aggregate allocation of the economy with* ***private information*** *is identical to the optimal allocation of the representative agent economy with preference shifters $\alpha$.*

Observe that the optimal allocation of the full information economy can be obtained by dropping the incentive compatibility constraints (4.4), (4.5) and (5.4) and setting $\xi_{yt}$ and $\xi_{ot}(v)$ to zero in all first order conditions. Also observe that none of those incentive compatibility constraints or positive values for $\xi_{yt}$ or $\xi_{ot}(v)$ were used in the derivations of equations (5.15) and (5.22). We thus have a second important Lemma.

**Lemma 2** *Suppose that $u$ and $n$ are logarithmic. Define $\alpha = \{\alpha_t\}_{t=0}^{\infty}$ as in equation (5.21). Then, the optimal aggregate allocation of the economy with* ***full information*** *is identical to the optimal allocation of the representative agent economy with preference shifters $\alpha$.*

Since the optimal aggregate allocations of the economy with private information and the economy with full information are equal to the same object we have the following Corollary:

**Corollary 3** *Suppose that $u$ and $n$ are logarithmic. Then, the optimal aggregate allocation of the economy with private information is identical to the optimal aggregate allocation of the economy with full information.*

The reason why the allocations of the private information and full information economy do not aggregate to a representative agent economy with stationary preferences (and preference shifters are generally needed) is because the social planner is allowed to discount the welfare of future generations at a different rate than private agents discount future utility. In fact, if we set the relative Pareto weight $\theta$ to the private discount factor $\beta$ we see from equation (5.18) that $\rho_t = \sigma$ for all $t \geq 1$ and from equation (5.21) that $\alpha_t = 1$ for all $t \geq 0$. That is, in this case the representative agent economy has standard stationary preferences.

Independently of the value of $\theta$, however, from equation (5.18) we verify that $\rho_t$ converges to a positive value and, therefore, that $\alpha_t$ converges to a positive value as well.[11] Since it is well known that the solution to the representative agent economy with stationary preferences (constant $\alpha_t$) converges to a stationary stochastic process, from Lemmas 1 and 2 we can say the same about the aggregate optimal allocations of the economies with private and full information. Thus, we have the following Corollary.

**Corollary 4** *Suppose that u and n are logarithmic. Then, the aggregate optimal allocations of the economies with private and full information converge to a stationary stochastic process. Moreover, this stationary process is the one associated to a representative agent economy with stationary preferences (zero preference shifters).*

## 6 Cross-sectional heterogeneity under logarithmic preferences

The previous section showed that private information is irrelevant for aggregate business cycle fluctuations when preferences are logarithmic. However, even in this case the lack of perfect insurance generates endogenous heterogeneity across individual agents that may help understand cross-sectional features of the business cycle that economies with full information cannot address.

In order to study cross-sectional properties of the business cycle we need a sharper characterization of the private information optimal stationary allocation. The next Lemma provides such characterization. In what follows, for any variable $x_t$, $\Delta x_t$ is defined to be the differences between $x_t$ and its deterministic steady state value $\bar{x}$.

**Lemma 5** *Suppose that u and n are logarithmic. Then, the stationary solution to the private information planning problem satisfies that*

$$u_{yst} = \bar{u}_{ys} - \Delta \ln \lambda_t \tag{6.1}$$

$$n_{yst} = \bar{n}_{ys} - \Delta \ln \lambda_t - \Delta \ln q_t \tag{6.2}$$

$$w_{ys,t+1} = \bar{w}_{ys} - \frac{\Delta \ln \lambda_{t+1} + \Delta \pi_{t+1}}{b} \tag{6.3}$$

$$u_{ost}(v) = \bar{u}_{os} + bv + \Delta \pi_t \tag{6.4}$$

$$n_{ost}(v) = \bar{n}_{os} + bv + \Delta \pi_t - \Delta \ln q_t \tag{6.5}$$

$$w_{os,t+1}(v) = \bar{w}_{os} + v + \frac{\Delta \ln \lambda_t + \Delta \pi_t}{b} - \frac{\Delta \ln \lambda_{t+1} + \Delta \pi_{t+1}}{b} \tag{6.6}$$

$$\ln \eta_t(v) = \bar{\pi} + bv + \Delta \pi_t \tag{6.7}$$

[11] Recall that $\theta$ was assumed to be greater than $\beta\sigma$.

*where* $0 < b = \frac{1-\beta\sigma}{1+\bar{s}} < 1$.

***Proof:*** *Guess that*

$$\begin{aligned} 0 &= \Delta\pi_t + \bar{s}\Delta n_{ot} + \beta\sigma E_t\left[\Delta w_{o,t+1}\right] \\ \Delta \ln \xi_{yt} &= -\Delta \ln \lambda_t \\ \ln \xi_{ost}(v) &= \bar{\xi}_{os} + bv + \Delta\pi_t \\ \Delta \ln V_t &= -\Delta \ln \lambda_t - \Delta\pi_t, \end{aligned}$$

*where* $V_t = \int e^{bv} d\mu_t$, *and verify that all constraints and first order conditions are satisfied both on and off steady state.*[12]■

Equations (6.1)-(6.3) indicate that for young agents the utility of consumption, the utility of leisure and next-period promised values shift over the business cycle by amounts that are independent of the reported type. Equation (6.4) states that $u_{oLt}(v)$ and $u_{oHt}(v)$ are linear parallel functions that shift vertically over the business cycle by amounts that are independent of the reported type. While, equations (6.5) and (6.6) show that the same is true for the utility of leisure and next-period promised values, the slopes of $w_{oL,t+1}(v)$ and $w_{oH,t+1}(v)$ are equal to one. Thus, promised values follow a random walk process with innovations that depend on the realization of the idiosyncratic and aggregate shocks.[13]

I now turn to characterize the behavior of the distributions of promised values, consumption levels and hours worked implied by the optimal allocation rules described in Lemma 5. Observe, from equations (4.7) and (6.6) that for every interval $(a_1, a_2)$ the steady state distribution $\bar{\mu}$ satisfies that:

$$\bar{\mu}\left[(a_1, a_2)\right] = \sigma \sum_s \psi_s \bar{\mu}\left[(a_1 - \bar{w}_{os}, a_2 - \bar{w}_{os})\right] + (1-\sigma)\sigma \sum_{s:\ \bar{w}_{ys} \in (a_1, a_2)} \psi_s. \tag{6.8}$$

Define

$$\Delta_t = \frac{\Delta \ln \lambda_t + \Delta\pi_t}{b}. \tag{6.9}$$

[12] See Section 3 in the Technical Appendix for the details.

[13] Even with no aggregate fluctuations promised values follow a random walk. However, contrary to Atkeson and Lucas (1992) an immizerizing result is not obtained because of the stochastic lifetimes. As people die and are replaced by young agents, there is enough "reversion to the mean" in promised values that an invariant distribution is obtained (see Phelan 1994). The immizerizing result actually applies within each cohort of agents: Within each cohort the distribution of promised values keeps spreading out more and more over time.

From equations (4.7), (6.3) and (6.6) we then have that for every interval $(a_1 - \Delta_{t+1}, a_2 - \Delta_{t+1})$:

$$\begin{aligned} \mu_{t+1}\left[(a_1 - \Delta_{t+1}, a_2 - \Delta_{t+1})\right] &= \sigma \sum_s \psi_s \mu_t \left[(a_1 - \Delta_t - \bar{w}_{os}, a_2 - \Delta_t - \bar{w}_{os})\right] \\ &\quad + (1-\sigma)\,\sigma \sum_{s:\ \bar{w}_{ys} \in (a_1, a_2)} \psi_s. \end{aligned} \tag{6.10}$$

From equations (6.8) and (6.10) it then follows that for every interval $(a_1, a_2)$:

$$\mu_t\left[(a_1 - \Delta_t, a_2 - \Delta_t)\right] = \bar{\mu}\left[(a_1, a_2)\right]. \tag{6.11}$$

That is, $\mu_t$ is merely a $\Delta_t$ horizontal translation of the steady state distribution $\bar{\mu}$. In particular, since promised values increase during a boom, $\mu_t$ shifts to the right during such an episode. We thus have the following Lemma.

**Lemma 6** *The dispersion of the cross-sectional distribution of promised values is constant over the business cycle.*

Now let's consider the associated behavior of the cross-sectional distribution $\phi_t$ of utilities of consumption $u_t$. From equations (6.1) and (6.4) we have that $\phi_t$ satisfies that for every Borel set $B$,

$$\phi_t(B) = \sum_s \int_{\{v:\ \bar{u}_{os} + bv + \Delta\pi_t\ \in\ B\}} \psi_s d\mu_t + \sum_{s:\ \bar{u}_{ys} - \Delta \ln \lambda_t\ \in\ B} \psi_s.$$

It follows that for every interval $(a_1, a_2)$,

$$\bar{\phi}\left[(a_1, a_2)\right] = \sum_s \psi_s \bar{\mu}\left[\left(\frac{a_1 - \bar{u}_{os}}{b}, \frac{a_2 - \bar{u}_{os}}{b}\right)\right] + \sum_{s:\ \bar{u}_{ys}\ \in\ (a_1, a_2)} \psi_s$$

and

$$\begin{aligned} &\phi_t\left[(a_1 - \Delta \ln \lambda_t, a_2 - \Delta \ln \lambda_t)\right] \\ = &\sum_s \psi_s \mu_t \left[\left(\frac{a_1 - \Delta \ln \lambda_t - \bar{u}_{os} - \Delta\pi_t}{b}, \frac{a_2 - \Delta \ln \lambda_t - \bar{u}_{os} - \Delta\pi_t}{b}\right)\right] + \sum_{s:\ \bar{u}_{ys}\ \in\ (a_1, a_2)} \psi_s. \end{aligned}$$

From equations (6.9) and (6.11) we then have that

$$\phi_t\left[(a_1 - \Delta \ln \lambda_t, a_2 - \Delta \ln \lambda_t)\right] = \bar{\phi}\left[(a_1, a_2)\right]. \tag{6.12}$$

Thus, $\phi_t$ is also a $\Delta \ln \lambda_t$ horizontal translation of the steady state distribution $\bar{\phi}$. Since $u$ is logarithmic, we then have the following Lemma.

**Lemma 7** *The dispersion of the cross-sectional distribution of log-consumption levels is constant over the business cycle.*

Finally, let's turn to characterizing the behavior of the cross-sectional distribution $\zeta_t$ of utilities of leisure $n_t$. From equations (6.1) and (6.2) we have that cyclical shifts in $n_{yst}$ differ from the cyclical shifts in $u_{yst}$ by the amount $-\Delta \ln q_t$. From equations (6.4) and (6.5) we also see that $n_{ost}(v)$ is parallel to $u_{ost}(v)$ and that its vertical shifts differ from those in $u_{ost}(v)$ by the amount $-\Delta \ln q_t$. Following the same steps as those used to derive equation (6.12) we thus have that,

$$\zeta_t \left[ (a_1 - \Delta \ln \lambda_t - \Delta \ln q_t, a_2 - \Delta \ln \lambda_t - \Delta \ln q_t) \right] = \bar{\zeta} \left[ (a_1, a_2) \right].$$

That is, $\zeta_t$ is a $\Delta \ln \lambda_t + \Delta \ln q_t$ horizontal translation of the steady state distribution $\bar{\zeta}$. Since the utilities of leisure decrease during a boom, it follows that $\zeta_t$ shifts to the left during such an episode.

Observe that the log of hours worked are related to utilities of leisure according to $\ln(h) = \ln(1 - e^n)$. Since this is a strictly decreasing and strictly concave function it follows that when the distribution of utilities of leisure shifts to the left, that the dispersion of the distribution of log hours decreases. Thus, we have our last Lemma.

**Lemma 8** *The dispersion of the cross-sectional distribution of log-hours worked is countercyclical.*

There is a considerable empirical literature analyzing the behavior of consumption and labor income inequality over time. While most of the literature has focused on trends a few studies have considered business cycle frequencies as well. Heathcote et al. (2010) is a recent example. A key finding in that paper is that U.S. labor earnings inequality widens sharply in recessions and that this is driven by an increase in labor supply inequality (since the cross-sectional distribution of wages is not much affected). Krueger et al. (2010) reported similar findings for eight other countries considered in their study. This empirical evidence is broadly in line with the theoretical results obtained in this section. In particular, Lemma 8 indicates that the model's labor supply inequality increases during recessions and, since all agents earn the same wage rate $q$, that this translates into an increase in labor income inequality.[14] The empirical evidence on the cyclical behavior of consumption inequality is less clear. Summarizing the international evidence, Krueger et al. (2010) reported that most recessions are accompanied by an uptick in consumption inequality that is much smaller than the associated increase in earnings inequality. Focusing on the U.S. Great

[14] Discussing labor income and wages actually requires specifying a decentralization. To fix concepts it may be useful to consider a simple decentralization in which households have a continuum of members and all the dynamic contracting is done within the family. Output is produced by competitive firms and the market structure consists of spot markets for labor and capital, and a complete set of Arrow securities.

Recession of 2007-2009, Krueger et al. (2016) found that consumption inequality increased during that recession as well. However, using a structural factor model Giorgi and Gambetti (2017) found that TFP shocks generate pro-cyclical movement in U.S. consumption inequality. Given these opposing results it seems that the acyclical consumption inequality described in Lemma 7 represents a rough compromise between the different empirical studies.

# 7 Computations

The previous sections were able to provide a full characterization of the solution to the mechanism design problem because of the particular preferences considered. However, when preferences differ from the logarithmic case such characterization is no longer possible and the model must be solved for numerically. This is a nontrivial task because of the high dimensionality of the state space. In this section I introduce a new method for computing equilibria of models with heterogeneous agents and aggregate shocks and apply it to the model considered in this paper. An important advantage of this computational method over existing alternatives in the literature is not only that it keeps track of an arbitrarily good approximation to the distribution of agents over individual states, but that the law of motion for this distribution is exact (no approximation errors are introduced there).[15] Thus, the method promises to be extremely useful for computing equilibria in cases where the distribution of individual states matters.[16]

Before proceeding to describe its details it will be useful to sketch the main ingredients of the computational method. Instead of keeping track of the distribution of promised values $\mu$ as a state variable, what the computational method keeps track of is a long history of individual decision rules $w_{os}$ and $w_{ys}$. Since the individual decision rules $w_{os}$ are parametrized as spline approximations, the computational method only needs to keep track of a long but finite history of spline coefficients. The current distribution of promised values is then recovered by simulating the evolution of a large number of agents (and their descendants) over time using the history of individual decision

[15]See Algan et al. (2014) for a survey of alternatives. A recent method that parametrizes the distribution of agents but still linearizes its law of motion is described in Winberry (2016).

[16]While incorporating the exact law of motion for the distribution of agents is a big gain in accuracy, the method is extremely slow compared to the alternatives. While this makes estimation unfeasible, calibration works perfectly fine. The reason is that by calibrating the deterministic steady state of the model the method needs to be applied only once, after all parameters have been determined.

rules kept as state variables.[17] The next period distribution of promised values is then obtained by simply updating by one period the history of individual decision rules using the decision rules chosen during the current period. All first order conditions and aggregate feasibility constraints are then linearized with respect to the spline coefficients describing current and past individual decision rules.[18] This delivers a linear rational expectations model which, despite of its high dimensionality, can be solved for using standard methods.

The method is actually a generalization of the approach used in Veracierto (2002) for computing business cycles of (S,s) economies. In that paper, histories of past decision rules were also used as state variables. However, under (S,s) adjustments lower and upper adjustment thresholds could be used to parametrize the complete individual decision rules. In this paper, decision rules are smooth functions and therefore parametrized as spline approximations. Also, in Veracierto (2002) the (S,s) adjustments together with a finite number of idiosyncratic shocks led to a finite support for the distribution of agents and, therefore, to a finite dimensional aggregate state. Here, the support of the distribution of agents is a continuum.

## 7.1 Computing the deterministic steady state

While computing the deterministic steady state of the model is completely standard, this section describes the algorithm in detail since this will introduce objects and notation that will be needed later on.

Observe that the shadow value of labor $q$ is known from the steady state versions of equations (4.20) and (4.21). In particular it is given by

$$q = (1-\gamma)\left\{\frac{1}{\gamma}\left[\frac{1}{\theta} - 1 + \delta\right]\right\}^{\frac{\gamma}{1-\gamma}}.$$

Given this value of $q$, the steady state decision rules for old agents can then be solved for. To this end, I find it convenient to use cubic spline approximations and iterate with the steady state

[17] Because of the stochastic lifetimes, the truncation introduced by the finite history of decision rules generates arbitrarily small approximation errors as the length of the history becomes large. In fact, when this length becomes large the distribution used for drawing initial promised values for the simulations becomes irrelevant (although, in practice, I use the invariant distribution of the deterministic steady state).

[18] This is the computationally most intensive part of the method. The reason is that we need to take numerical derivatives with respect to each spline coefficient in the history, and each of these calculations requires simulating the evolution of a large panel of agents over the entire history of individual decision rules kept as state variables.

versions of equations (4.14)-(4.19).[19] In order to do this, I first restrict the promised values to lie on a closed interval $[v_{\min}, v_{\max}]$ and define an equidistant vector of grid points $(v_j)_{j=1}^{J}$, with $v_1 = v_{\min}$ and $v_J = v_{\max}$.[20] Given the function $\eta$ from the previous iteration, which is used to value next period promised values in the steady state versions of equations (4.18) and (4.19), the values of $[u_{os}(v_j), n_{os}(v_j), w_{os}(v_j), \xi_o(v_j), \eta(v_j)]_{j=1}^{J}$ that satisfy the steady state versions of equations (4.14)-(4.19) are then solved for at the grid points $(v_j)_{j=1}^{J}$. Once these values are found, the functions are extended to the full domain $[v_{\min}, v_{\max}]$ using cubic splines.[21] The iterations continue until the values for $[u_{os}(v_j), n_{os}(v_j), w_{os}(v_j), \xi_o(v_j), \eta(v_j)]_{j=1}^{J}$ converge. Observe, that this solution does not depend on any other endogenous values, so it forms part of the steady state.

Given the steady state solution for $\eta$ the steady state decisions for young agents can be solved for next. This is straightforward: conditional on a value for $\lambda$, the steady state versions of equations (4.8)-(4.13) can be solved for the finite numbers of unknowns $(u_{ys}, n_{ys}, w_{ys}, \xi_y)$ in one step (no iterations are needed here). Later on I will have to provide the side condition that $\lambda$ must satisfy for this to form part of the steady state.

The steady state version of equation (4.7) describes the recursion that the invariant $\mu$ has to satisfy. This equation corresponds to the case of a continuum of agents. However, I find it convenient to work with a large, but finite number of agents, and perform the recursion for this case. In particular, consider a large but finite number of agents $I$ and endow them with promised values in the interval $[v_{\min}, v_{\max}]$. Using the functions $w_{os}$ and the values $w_{ys}$ already obtained, simulate the evolution of the promised values of these $I$ agents and their descendants for a large number of periods $T$. To be precise, if agent $i$ was promised a value $v$ at the beginning of the current period (conditional on being alive), then his promised value (or his descendant's, in case the agent dies) at the beginning of the following period will be given by:

$$v' = \begin{cases} w_{os}(v), \text{ with probability } \sigma\psi_s, \\ w_{ys}, \text{ with probability } (1-\sigma)\psi_s, \end{cases} \tag{7.1}$$

Simulating the $I$ agents for $T$ periods using equation (7.1) we obtain a realized distribution $(\bar{v}_i)_{i=1}^{I}$ of promised values (conditional on being alive) across the $I$ agents. Observe that the last iteration of equation (7.1) also gives the corresponding realized values of leisure $(\bar{s}_i)_{i=1}^{I}$ across the $I$

[19] Observe that the shadow value of consumption $\lambda$ does not appear in the steady state version of these equations,

[20] When restricting promised values to lie in the interval $[v_{\min}, v_{\max}]$, the first order conditions (4.12)-(4.13) and (4.18)-(4.19) change by incorporating inequalities that check for corner solutions.

[21] In practice, I use the monotonicity preserving cubic splines described by Steffen (1990).

agents. The joint realized distribution of promised values and values of leisure $(\bar{v}_i, \bar{s}_i)_{i=1}^{I}$ can then be used to compute statistics under the invariant distribution. In particular, aggregate consumption can be obtained as

$$C = \sigma \frac{1}{I} \sum_{i=1}^{I} u^{-1} \left[ u_{o,\bar{s}_i} \left( \bar{v}_i \right) \right] + (1-\sigma) \sum_{s} u^{-1} \left( u_{ys} \right) \psi_s. \tag{7.2}$$

To understand this expression, suppose that we are at the beginning of period $T+1$. The joint realized distribution $(\bar{v}_i, \bar{s}_i)_{i=1}^{I}$ now corresponds to agents that were alive in the previous period, and thus a fraction $\sigma$ of them will have survived and a fraction $(1-\sigma)$ of them will have died. The first term in equation (7.2) corresponds to those who have survived. It averages the consumption of these agents and multiplies the result by the probability of surviving $\sigma$. The second term corresponds to those who have died and thus have been replaced by young agents. It averages the consumption of young agents and multiplies the result by the probability of dying $(1-\sigma)$.

Aggregate hours worked can be similarly computed as

$$H = \sigma \frac{\sum_{i=1}^{I} \left[ 1 - n^{-1} \left[ n_{o,\bar{s}_i} \left( \bar{v}_i \right) \right] \right]}{I} + (1-\sigma) \sum_{s} \left[ 1 - n^{-1} \left( n_{ys} \right) \right] \psi_s. \tag{7.3}$$

Observe that by a law of large numbers equations (7.2) and (7.3) will become arbitrarily good approximations to the steady state versions of equations (4.2) and (4.3) as $I$ and $T$ tend to infinity.

Given aggregate hours worked, aggregate capital can be then obtained from the fact that the social planner equates the marginal productivity of capital to its shadow price. In particular, from the steady state version of equation (4.21) we have that aggregate capital is given by

$$K = \left( \frac{\gamma}{\frac{1}{\theta} - 1 + \delta} \right)^{\frac{1}{1-\gamma}} H. \tag{7.4}$$

The last equation that needs to be satisfied is the feasibility condition for consumption,

$$C + \delta K = K^{\gamma} H^{1-\gamma}. \tag{7.5}$$

This is the side condition mentioned above for the shadow value of consumption $\lambda$. The shadow value of consumption determines the consumption, hours worked and promised values of young agents, and therefore each of the variables in equation (7.5). Therefore, $\lambda$ must be changed until equation (7.5) holds.

## 7.2 Computing business cycle fluctuations

As has already been mentioned, computing business cycle fluctuations requires linearizing the first order conditions and aggregate feasibility constraints with respect to a convenient set of variables.

Linearizing equations (4.2)-(4.21) present different types of issues. As a consequence, I classify them into different categories.

The first category is constituted by equations that only involve scalar variables. Equations (4.4), (4.8)-(4.11) and (4.20)-(4.21) fall into this category. For example, consider equation (4.9). This equation is a function of $\{\lambda_t, u_{yHt}, \xi_{yt}\}$, which are all scalars. Linearizing this equation around the deterministic steady state values $\{\bar{\lambda}, \bar{u}_{yHt}, \bar{\xi}_{yt}\}$ poses no difficulty.[22]

The second category is constituted by a continuum of equations that only involve scalar variables. Equations (4.5)-(4.6) and (4.14)-(4.17) fall into this category. Consider, for example, equation (4.15). This equation depends on $\{u_{oHt}(v), \xi_{ot}(v), \eta_t(v)\}$ which are all scalars. The problem is that there is one of these equations for every value of $v$ in the interval $[v_{\min}, v_{\max}]$. In this case the "curse of dimensionality" is solved by considering this equation only at the grid points $(v_j)_{j=1}^J$ that were used in the computation of the deterministic steady state. It is now straightforward to linearize each of these $J$ equations with respect to $\{u_{oHt}(v_j), \xi_{ot}(v_j), \eta_t(v_j)\}$ at their deterministic steady state values $\{\bar{u}_{oH}(v_j), \bar{\xi}_o(v_j), \bar{\eta}(v_j)\}$. Extending $\{u_{oHt}(v), \xi_{ot}(v), \eta_t(v)\}$ to the full domain $[v_{\min}, v_{\max}]$ using cubic splines will make equation (4.15) hold only approximately outside of the grid points $(v_j)_{j=1}^J$. The quality of this approximation will depend on how many grid points $J$ we work with.

The third category is constituted by equations that involve both scalars and functions. Equations (4.12) and (4.13) fall in this category. For example, consider equation (4.13). This equation depends on $\lambda_t$, $\xi_{yt}$, $\lambda_{t+1}$, $w_{yH,t+1}$ and on the function $\eta_{t+1}$, which is a high dimensional object. In this case the "curse of dimensionality" is broken by considering that $\eta_{t+1}$ is a spline approximation and, therefore, is completely determined by the finite set of values $\{\eta_{t+1}(v_j)\}_{j=1}^J$, i.e. the value of the function at the grid points. The equation can then be linearized with respect to $\left[\lambda_t, \xi_{yt}, \lambda_{t+1}, w_{yH,t+1}, \{\eta_{t+1}(v_j)\}_{j=1}^J\right]$ at the steady state values $\left[\bar{\lambda}, \bar{\xi}_y, \bar{\lambda}, \bar{w}_{yH}, \{\bar{\eta}(v_j)\}_{j=1}^J\right]$.

The fourth category is a combination of the previous two: it is constituted by a continuum of equations that involve both scalars and functions. Equations (4.18) and (4.19) fall in this category. For example, consider equation (4.19). Similarly to the third category, this equation depends on the scalars $\lambda_t$, $\xi_{ot}(v)$, $\lambda_{t+1}$, $w_{oH,t+1}(v)$ and on the function $\eta_{t+1}$. Similarly to the second category there is one of these equations for every value of $v$ in the interval $[v_{\min}, v_{\max}]$. Given these similarities we can use the same strategy. In particular, we can consider this equa-

[22] Although in this case derivatives can be taken analytically, throughout the section derivatives are assumed to be numerically obtained.

tion only at the grid points $(v_j)_{j=1}^{J}$ and linearize each of these $J$ equations with respect to $\left[\lambda_t, \xi_{ot}(v_j), \lambda_{t+1}, w_{oH,t+1}(v_j), \{\eta_{t+1}(v_k)\}_{k=1}^{J}\right]$ at the deterministic steady state values $\left[\bar{\lambda}, \bar{\xi}_o(v_j), \bar{\lambda},\right.$ $\left.\bar{w}_{oH}(v_j), \{\bar{\eta}(v_k)\}_{k=1}^{J}\right]$.

The fifth category is much more complicated. It is constituted by equations that involve scalars and integrals of variables with respect to the distribution $\mu_t$. Equations (4.2) and (4.3) fall in this category. For example, consider equation (4.2). This equation depends on the real numbers $u_{yL,t}$, $u_{yH,t}$, $z_t$, $K_t$, $K_{t-1}$, and $H_t$, and on the integrals $\int u^{-1}[u_{ost}(v)]\, d\mu_t$. To make progress it will be important to represent these integrals with a convenient finite set of variables. In order to do this, I will follow a strategy that is closely related to the one that was used in Section 7.1 for computing statistics under the invariant distribution. In particular, consider the same large but finite number of agents $I$ that was used in that section and endow them with the same realized distribution of promised values $(\bar{v}_i)_{i=1}^{I}$ that was obtained when computing the steady state. Now, assume that these agents populated the economy $M$ time periods ago and consider the history

$$\{w_{oL,t-m}, w_{oH,t-m}, w_{yL,t-m}, w_{yH,t-m}\}_{m=0}^{M},$$

which describes the allocation rules for next-period promised values that were chosen during the last $M$ periods (where $t$ is considered to be the current period). Observe that since $w_{oL,t-m}$ and $w_{oH,t-m}$ are spline approximations, this history can be represented by the following finite list of values:

$$\left\{[w_{oL,t-m}(v_j)]_{j=1}^{J}, [w_{oH,t-m}(v_j)]_{j=1}^{J}, w_{yL,t-m}, w_{yH,t-m}\right\}_{m=0}^{M}. \tag{7.6}$$

Using the history of allocation rules for next-period promised values, we can simulate the evolution of promised values for the $I$ agents and their descendants during the last $M$ time periods to update the distribution of promised values from the initial $(\bar{v}_i)_{i=1}^{I}$ to a current distribution $(v_{i,t})_{i=1}^{I}$.

In particular, we can initialize the distribution of promised values at the beginning of period $t-M-1$ as follows:

$$v_{i,t-M-1} = \bar{v}_i,$$

for $i = 1, ..., I$. Given a distribution of promised values at the beginning of period $t-m-1$, the distribution of promised values at period $t-m$ is then obtained from the following equation:

$$v_{i,t-m} = \begin{cases} w_{os,t-m}(v_{i,t-m-1}), \text{ with probability } \sigma\psi_s, \\ w_{ys,t-m}, \text{ with probability } (1-\sigma)\psi_s, \end{cases} \tag{7.7}$$

for $i = 1, ..., I$. Proceeding recursively for $m = M, M-1, ..., 0$, we obtain a realized distribution of promised values $(v_{i,t})_{i=1}^{I}$ at the beginning of period $t$.

Observe that the last iteration of equation (7.7) also gives the corresponding realized values of leisure $(s_{it})_{i=1}^{I}$ across the $I$ agents. The joint realized distribution of promised values and values of leisure $(v_{it}, s_{it})_{i=1}^{I}$ can then be used to compute statistics under the distribution $\mu_t$. In particular, equation (4.2) can be re-written as:

$$0 = (1-\sigma)\left[e^{u_{yL,t}}\psi_L + e^{u_{yH,t}}\psi_H\right] + \sigma\frac{1}{I}\sum_{i=1}^{I} e^{u_{os_{it}}(v_{it})} + K_t - (1-\delta)K_{t-1} - e^{z_t}K_{t-1}^{\gamma}H_t^{1-\gamma}. \quad (7.8)$$

Since $u_{oL,t}$ and $u_{oH,t}$ are splines approximations, they can be summarized by their values at the grid points $(v_j)_{j=1}^{J}$. Therefore, equation (7.8) can be linearized with respect to

$$\begin{aligned} & z_t, K_t, K_{t-1}, H_t, u_{yL,t}, u_{yH,t}, \left[u_{oL,t}(v_j)\right]_{j=1}^{J}, \left[u_{oH,t}(v_j)\right]_{j=1}^{J}, \\ & \left\{\left[w_{oL,t-m}(v_j)\right]_{j=1}^{J}, \left[w_{oH,t-m}(v_j)\right]_{j=1}^{J}, w_{yL,t-m}, w_{yH,t-m}\right\}_{m=0}^{M} \end{aligned} \quad (7.9)$$

at their deterministic steady state values.

Observe that equation (7.9) provides a large but finite list of variables. In particular, there are $(M+1)(2J+2)$ variables in the second line of equation (7.9). Taking numerical derivatives with respect to each of these variables requires simulating $I$ agents over $M$ periods. As a consequence, linearizing equation (7.8) requires performing a massive number of Monte Carlo simulations. While this seems a daunting task it is easily parallelizable. Thus, using massively parallel computer systems can play an important role in reducing computing times and keeping the task manageable.[23]

The last category of equations has only one element: equation (4.7), which describes the law of motion for the distribution $\mu_t$. While daunting at first sight, this equation is greatly simplified by our approach of representing the distribution $\mu_t$ using the history of values given by equation (7.6). In fact, updating the distribution $\mu_t$ is merely reduced to updating this history. In particular, the date-$(t+1)$ history can be obtained from the date-$t$ history and the current values of $\left[w_{oL,t+1}(v_j)\right]_{j=1}^{J}$, $\left[w_{oH,t+1}(v_j)\right]_{j=1}^{J}$, $w_{yL,t+1}$ and $w_{yH,t+1}$ using the following equations:

$$\left[w_{os,(t+1)-m}(v_j)\right]_{j=1}^{J} = \left[w_{os,t-(m-1)}(v_j)\right]_{j=1}^{J} \quad (7.10)$$

$$w_{ys,(t+1)-m} = w_{ys,t-(m-1)} \quad (7.11)$$

for $s = L, H$ and $m = 1, ..., M$. Observe that the law of motion described by equations (7.10) and (7.11) is already linear, so no further linearization is needed. Also observe that the variables that are $M$ periods old in the date-$t$ history are dropped from the date-$(t+1)$ history. Thus, the law of motion described by equations (7.10)-(7.11) introduces a truncation. However, the consequences

[23] In practice, I heavily rely on GPU computing for performing the Monte Carlo simulations.

of this truncation are expected to be negligible. The reason is that the truncation only affects the agents that had survived for $M$ consecutive periods, and given a sufficiently small survival probability $\sigma$ and/or a sufficiently large $M$ there will be very few of these agents. Aside from this negligible truncation there are no further approximations errors in the representation of the law of motion given by equation (4.7). As has been already stated, this is the crucial benefit of using the computational method described here.

Once equations (4.2)-(4.21) have been linearized with respect to the variables described above, we are left with a linear rational expectations model that is large but that can be solved using standard methods.

# 8 Numerical results

This section uses the computational method just described to explore the quantitative properties of different private information economies and compare them to those of their full information counterparts. In order to do this I first select parameter values for the benchmark economy with logarithmic preferences. Economies with more general preferences will be considered later on.

## 8.1 Parametrization

Except for the private information, the basic structure of the model corresponds to a standard real business cycle model. In fact, under logarithmic preferences the basic structure of the model is identical to the one in Cooley and Prescott (1995). For this reason, I calibrate all parameters associated with the neoclassical growth model to the same observations as theirs. In order to simplify computations, the model time period is selected to be one year.

Following Cooley and Prescott (1995) the labor share parameter $1-\gamma$ is set to 0.60, the depreciation rate $\delta$ is chosen to reproduce an investment-capital ratio $I/K$ equal to 0.076, and the social discount factor $\theta$ is chosen to reproduce a capital-output ratio $K/Y$ equal to 3.32. The values of leisure $s_L$ and $s_H$ are chosen to satisfy two criteria: that aggregate hours worked $H$ equal 0.31 (another observation from Cooley and Prescott 1995) and that the hours worked by old agents with the high value of leisure and the highest possible promised value $n_{oH}(v_{\max})$ be a small but positive number. The rationale for this second criterion is that I want to maximize the relevance of the information frictions while keeping an internal solution for hours worked. The probability of drawing a high value of leisure $\psi_H$ is chosen to maximize the standard deviation of the invariant distribution of promised values. It turns out that a value of $\psi_L = 0.50$ achieves this. The survival probability $\sigma$ is chosen to generate an expected lifespan of 40 years. In turn, the individual discount factor $\beta$ is chosen to be the same as the social discount factor $\theta$. In terms of the parameters for the aggregate productivity stochastic process, $\rho$ is chosen to be 0.95 and the variance of the innovations to aggregate productivity $\sigma_\varepsilon^2$ is chosen to be $4 \times 0.007^2$ (as in Cooley and Prescott 1995).

Table 1

Parameter values

| Structural | Computational |
|---|---|
| $s_L = 1.513$ | $v_{\min} = -28.5$ |
| $s_H = 2.047$ | $v_{\max} = -11$ |
| $\psi_L = 0.50$ | $T = 1,000$ |
| $\theta = 0.9574$ | $M = 273$ |
| $\beta = 0.9574$ | $I = 8,388,608$ |
| $\sigma = 0.975$ | $J = 20$ |
| $\gamma = 0.40$ | |
| $\delta = 0.076$ | |
| $\rho = 0.95$ | |
| $\sigma_\varepsilon = 0.014$ | |

While the above parameters are structural, there are a number of computational parameters to be determined. The number of grid points in the spline approximations $J$, the total number of agents simulated $I$, the length of the simulations for computing the invariant distribution $T$, and the length of the histories kept as state variables when computing the business cycles $M$ are all chosen to be as large as possible, while keeping the computational task manageable and results being robust to non-trivial changes in their values.[24] The lower and upper bounds for the range of possible promised values $v_{\min}$ and $v_{\max}$ in turn were chosen so that the fraction of agents in the intervals $[v_1, v_2]$ and $[v_{J-1}, v_J]$ are each less than 0.1%. Thus, truncating the range of possible values at $v_{\min}$ and $v_{\max}$ should not play an important role in the results.

Table 1 describes all parameter values. It turns out that under the computational parameters specified in this table the linearized system described in Section 7.2 has about $12,000$ variables, a

[24]Given the value selected for the survival probability $\sigma$, less than 0.1% of individuals survive more than $M$ periods. Thus, the truncation imposed by keeping track of a finite history of decision rules introduces a very small approximation error.

large system indeed.

### 8.2 Results under log-log preferences

Before turning to business cycle dynamics I illustrate different features of the model at its deterministic steady state. Figure 1.A shows the invariant distribution of promised values across the $J-1$ intervals $[v_j, v_{j+1}]_{j=1}^{J-1}$ defined by the grid points of the spline approximations. While it is hard to see at this coarseness level, the distribution is approximately symmetrical. More importantly, we see that the invariant distribution puts very little mass at extreme values. As a consequence, in what follows I will report allocation rules only between the 7th and 15th ranges of the histogram. The reason is not only that there are very few agents at the tails of the distribution for them to matter, but being close to the artificial bounds $v_{\min}$ and $v_{\max}$ greatly distorts the shape of the allocation rules.

While not apparent in Figure 1.A, the invariant distribution of promised values generates too little heterogeneity. The standard deviation of the cross-sectional distribution of log-consumption levels and log-hours worked are 0.04 and 0.35, respectively. This compares with values of 0.50 and 0.82 reported by Heathcote et al. (2010) for 1981 (the year of lowest consumption heterogeneity in their sample).[25] The reason for the small amount of heterogeneity is that there is no persistence in the idiosyncratic shocks: The only way that the model can generate large deviations from the mean is through long streams of repeated bad shocks or good shocks, and these are unlikely to happen.[26]

Figure 1.B reports the utility of consumption for old agents $u_{oL}(v)$ and $u_{oH}(v)$ across promised values $v$, as well as those of young agents $u_{yL}$ and $u_{yH}$ (which are independent of $v$). We see that, in all cases the utility of consumption is higher when the value of leisure is low. Both $u_{oL}$ and $u_{oH}$ are strictly increasing in the promised value $v$, are linear (with slope less than one) and parallel to each other. Moreover, the vertical difference between $u_{oL}$ and $u_{oH}$ is the same as between $u_{yL}$ and $u_{yH}$.

Figure 1.C reports the utility of leisure for old agents $n_{oL}(v)$ and $n_{oH}(v)$ across promised values $v$, as well as those of young agent $n_{yL}$ and $n_{yH}$. We see that in all cases leisure is lower when the value of leisure is low. Both $n_{oL}$ and $n_{oH}$ are strictly increasing in the promised value $v$, are linear (with slope less than one) and parallel to each other. Moreover, the vertical difference between $n_{oL}$

[25] See their Figures 10 and 13.

[26] Given the unrealistic amount of cross-sectional heterogeneity that the model generates there is no point in reporting other features of the cross section, such as optimal labor and capital wedges.

and $n_{oH}$ is the same as between $n_{yL}$ and $n_{yH}$. In turn, Figure 1.D reports the next-period promised values for old agents $w_{oL}(v)$ and $w_{oH}(v)$ across promised values $v$, as well as those of young agent $w_{yL}$ and $w_{yH}$. We see that in all cases next-period promised values are higher when the value of leisure is low. Both $w_{oL}$ and $w_{oL}$ are strictly increasing in the promised value $v$, are linear (with slope equal to one) and parallel to each other. We also see that the vertical difference between $w_{oL}$ and $w_{oH}$ is the same as between $w_{yL}$ and $w_{yH}$. Thus, Figure 1 verifies the analytical results given by the steady state versions of equations (4.22)-(4.24) and Lemma 5.

The discussion of business cycle dynamics that follows will be centered around the analysis of the impulse responses of different variables to a one standard deviation increase in aggregate productivity. Figure 2.A shows the impulse responses of the utility of consumption of young agents $u_{yL}$ and $u_{yH}$. We see that both impulse responses are identical and that their shape qualitatively resembles one for aggregate consumption in a standard RBC model. Figure 2.B shows the impulse response of the utility of consumption of old agents with a low value of leisure $u_{oL}(v)$, at each of the eleven grid points $(v_j)_{j=6}^{16}$. While the figure shows eleven impulse responses, only one of them is actually seen because they happen to overlap perfectly. This means that, in response to the aggregate productivity shock, the function $u_{oL}$ depicted in Figure 1.B shifts vertically over time. Figure 2.C, which does the same for $u_{oH}$, is identical to Figure 2.B. Thus, $u_{oH}$ also shifts vertically over time and its increments are the same as those of $u_{oL}$.

Figure 3 is analogous to Figure 2, except that they depict the behavior of the utility of leisure. Figure 3.A shows that the impulse responses of $n_{yL}$ and $n_{yH}$ are identical and that they resemble the response of leisure in a standard RBC model, while Figures 3.B and 3.C indicate identical vertical shifts of the functions $n_{oL}$ and $n_{oH}$ in response to the aggregate productivity shock.

Turning to promised values, Figure 4.A shows that the impulse responses of $w_{yL}$ and $w_{yH}$ coincide. In turn, Figures 4.B and 4.C show that $w_{oL}$ and $w_{oH}$ shift vertically by identical amounts in response to an aggregate productivity shock. Thus, taken together, we see that Figures 2 -4 reproduce the analytical results of Lemma 5.

Figure 5.A shows the impulse responses of the cross sectional standard deviations of promised values, log-consumption and log-hours worked. We see that in response to a positive aggregate productivity shock the standard deviations of promised values and log-consumption remain flat while the standard deviation of log-hours worked decreases. Thus, Figure 5.A reproduces the analytical results of Lemmas 6, 7 and 8.

Finally, Figure 5.B shows the impulse responses of aggregate output $Y$, aggregate consumption $C$, aggregate investment $I$, aggregate hours worked $H$ and aggregate capital $K$ in the benchmark economy with private information. Figure 5.C reports the impulse responses for the same variables but for the representative agent economy. We see that both sets of impulse responses are identical. Thus, Figures 5.B and 5.C reproduce the analytical result of Corollary 4.

We have verified that while the computational method was not designed to exploit any of the properties of the logarithmic case, it is able to exactly reproduce the analytical results derived for this case. This suggests that the computational method introduced in this paper could be quite useful not only for analyzing other functional forms, but as a general method for computing aggregate fluctuations of economies with heterogeneous agents.

### 8.3 Extension to other preferences

This section generalizes the preferences of agents to the following form:

$$E_T \left\{ \sum_{t=T}^{\infty} \beta^{t-T} \sigma^{t-T} \left[ \frac{c_t^{1-\varphi} - 1}{1-\varphi} + s_t \frac{(1-h_t)^{1-\alpha} - 1}{1-\alpha} \right] \right\},$$

where $\varphi \neq 1$ and $\alpha \neq 1$. Since under this general functional form analytical results are no longer available the computational method becomes essential to evaluate these preferences.

Table 2

Steady state macroeconomic variables

| $(\alpha, \varphi)$ | Information | $Y$ | $C$ | $I$ | $H$ | $K$ |
|---|---|---|---|---|---|---|
| $(1, 1)$ | Private | 0.69155 | 0.51706 | 0.17449 | 0.31074 | 2.2959 |
| | Full | 0.69155 | 0.51706 | 0.17449 | 0.31074 | 2.2959 |
| $(1, 2)$ | Private | 0.56302 | 0.42096 | 0.14206 | 0.25299 | 1.8692 |
| | Full | 0.56305 | 0.42098 | 0.14207 | 0.25300 | 1.8693 |
| $(2, 1)$ | Private | 0.89539 | 0.66947 | 0.22592 | 0.40234 | 2.9727 |
| | Full | 0.89551 | 0.66956 | 0.22595 | 0.40239 | 2.9731 |
| $(2, 2)$ | Private | 0.76319 | 0.57062 | 0.19257 | 0.34293 | 2.5338 |
| | Full | 0.76327 | 0.57068 | 0.19259 | 0.34297 | 2.5341 |

Without recalibrating other parameters different values for $\varphi$ and $\alpha$ have been considered. However, in all cases similar results were obtained. For concreteness I here report results for unit deviations from the $\varphi = 1$ and $\alpha = 1$ case. For each of these cases Table 2 reports the deterministic steady state values of all macroeconomic variables for the economies with private information and full information. We see that in each parametrization all variables are nearly identical in both information scenarios.

In order to streamline the analysis of business cycle dynamics I consider the $\varphi = 2$ and $\alpha = 2$ as a representative case. Figure 6.A reports that, contrary to the log-log case, the cross sectional distribution of promised values now follows a non-trivial dynamics: Instead of being constant, the standard deviation of promised values decreases significantly in response to a positive aggregate productivity shock. Despite of this the information frictions still turn out to be irrelevant for aggregate dynamics. Figure 6.B reports the impulse responses of all macroeconomic variables in the economy with private information while Figure 6.C does the same for the economy with full information. We see that both sets of impulse responses are identical. Thus, similarly to the log-log case, the stationary behavior of the aggregate variables of the economy is not affected by the presence of information frictions.

## 9 Conclusions

The paper analyzed the effects of restrictions to risk sharing on macroeconomic dynamics when these restrictions are not exogenously imposed but arise endogenously as the optimal response to the presence of private information. For this purpose, the paper brought together two benchmark models in the microeconomics and information economics literatures, respectively: A real business cycle model and a Mirleesian economy. In particular, the paper considered a RBC model in which agents are subject to i.i.d. idiosyncratic shocks to their value of leisure and these shocks are private information. In this framework the paper analyzed the mechanism design problem of maximizing utility subject to incentive compatibility, promise keeping and aggregate feasibility constraints.

For the case of log-log preferences, a standard case in the RBC literature, the paper obtained sharp analytical characterizations. In particular, the utility of consumption, the utility of leisure and next-period promised values are all linear functions of current promised values. Over the business cycle these functions shift vertically in such a way that the distributions of promised values and log-consumption shift horizontally while maintaining their shapes. However, consistent with empirical evidence, the cross-sectional dispersion of log-hours worked is countercyclical. A striking result of the paper is that under logarithmic preferences the business cycle fluctuations of all macroeconomic variables are exactly the same under private information as under full information.

For preferences other than the log-log case analytical results are no longer available. To analyze these other cases the paper developed a novel method for computing equilibria of economies with heterogeneous agents. Its basic strategy is to parametrize individual decision rules as spline approximations and to keep long histories of the spline coefficients as state variable. The model is then linearized with respect to these variables and solved. Two advantages of the computational method over alternatives is that it approximates the current distribution of promised values arbitrarily well and that the law of motion for this distribution is exact. While the method does not take advantage of the log-log structure of preferences it is shown to reproduce all the analytical results for this case, providing significant evidence of its accuracy. Applying this method to other preference specifications still produces an irrelevance result for aggregate dynamics: While the distribution of promised values may now change its shape over the business cycle, the business cycle fluctuations of all macroeconomic variables are still unaffected by the presence of private information.

The paper opens wide possibilities for future research. The irrelevance result for macroeconomic dynamics was obtained under i.i.d shocks. It would be extremely interesting to explore if this result extends to the case of persistent shocks, specially given the small amount of cross-sectional heterogeneity that the i.i.d. shocks were found to generate. Also, the irrelevance result was obtained under a very particular framework (although a very interesting one, since the Mirleesian structure considered constitutes a benchmark case in the public finance literature). It is an open question if information frictions could play an important role in aggregate dynamics in alternative settings, such as economies with moral hazard and unemployment insurance. The computational method developed in this paper should prove extremely useful not only to evaluate these alternatives but to compute more general models with heterogeneous agents and aggregate fluctuations.

## References


Algan, Y., O. Allais, W. J. D. Haan and P. Rendhal, "Solving and Simulating Models with Heterogeneous Agents and Aggregate Uncertainty," in K. Schmedders and K. L. Judd, eds., *Handbook of Computational Economics*volume 3 (North-Holland, 2014), 277–324.

Atkeson, A. and R. E. Lucas, "On Efficient Distribution With Private Information," *Review of Economic Studies* 59 (1992), 427–453.

Cooley, T. F. and E. C. Prescott, "Economic growth and business cycles," in T. F. Cooley, ed., *Frontiers of business cycle research* (Princeton: Princeton University Press, 1995), 1–38.

DaCosta, C. and V. Luz, "The Private Memory of Aggregate Shocks," mimeo, Fundacao Getulio Vargas, 2013.

Farhi, E. and I. Werning, “Capital Taxation: Quantitative Explorations of the Inverse Euler Equation,” *Journal of Political Economy* 120 (2012), 398 – 445.

Fernandes, A. and C. Phelan, “A Recursive Formulation for Repeated Agency with History Dependence,” *Journal of Economic Theory* 91 (April 2000), 223–247.

Giorgi, G. D. and L. Gambetti, “Business Cycle Fluctuations and the Distribution of Consumption,” *Review of Economic Dynamics* 23 (January 2017), 19–41.

Golosov, M., A. Tsyvinski and I. Werning, “New Dynamic Public Finance: A User’s Guide,” in *NBER Macroeconomics Annual 2006, Volume 21* NBER Chapters (National Bureau of Economic Research, Inc, 2007), 317–388.

Green, E. J., “Lending and the Smoothing of Uninsurable Income,” in E. C. Prescott and N. Wallace, eds., *Contractual arrangements for intertemporal trade* (Minneapolis: University of Minnesota Press, 1987), 3–25.

Heathcote, J., F. Perri and G. L. Violante, “Unequal We Stand: An Empirical Analysis of Economic Inequality in the United States: 1967-2006,” *Review of Economic Dynamics* 13 (January 2010), 15–51.

Hopenhayn, H. A. and J. P. Nicolini, “Optimal Unemployment Insurance,” *Journal of Political Economy* 105 (April 1997), 412–38.

Kocherlakota, N., “Figuring out the Impact of Hidden Savings on Optimal Unemployment Insurance,” *Review of Economic Dynamics* 7 (July 2004), 541–554.

Krueger, D., K. Mitman and F. Perri, “On the Distribution of the Welfare Losses of Large Recessions,” Staff Report 532, Federal Reserve Bank of Minneapolis, July 2016.

Krueger, D., F. Perri, L. Pistaferri and G. L. Violante, “Cross Sectional Facts for Macroeconomists,” *Review of Economic Dynamics* 13 (January 2010), 1–14.

Krusell, P., A. A. Smith and Jr., “Income and Wealth Heterogeneity in the Macroeconomy,” *Journal of Political Economy* 106 (October 1998), 867–896.

Phelan, C., “Incentives and Aggregate Shocks,” *Review of Economic Studies* 61 (1994), 681–700.

Steffen, M., “A Simple Method for Monotonic Interpolation in One Dimension,” *Astronomy and Astrophysics* 239 (1990), 443 – 450.

Veracierto, M. L., "Plant-Level Irreversible Investment and Equilibrium Business Cycles," *American Economic Review* 92 (2002), 181–197.

Winberry, T., "A Toolbox for Solving and Estimating Heterogeneous Agent Macro Models," mimeo, University of Chicago, 2016.

Figure 1: Deterministic steady state

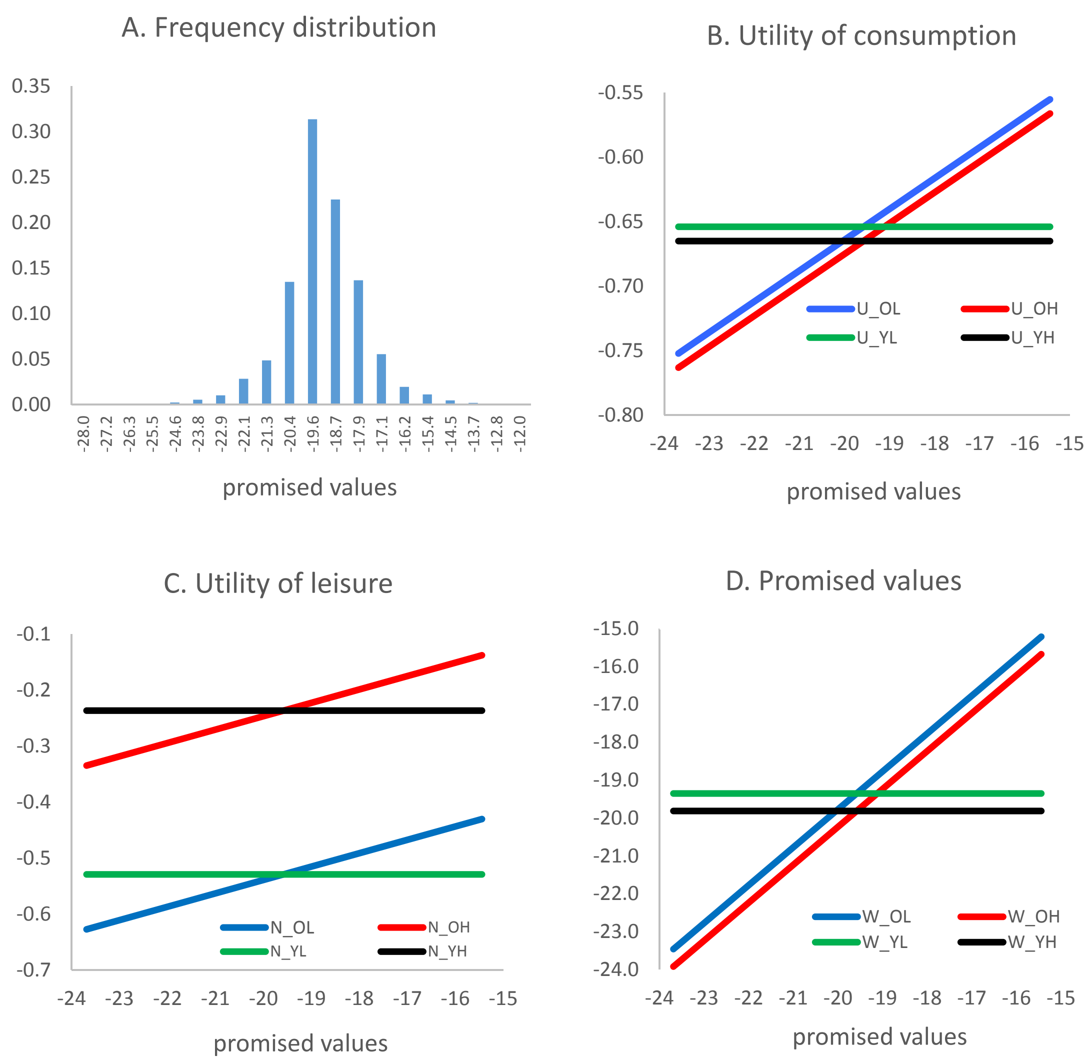


Figure 2: Impulse responses of utility of consumption

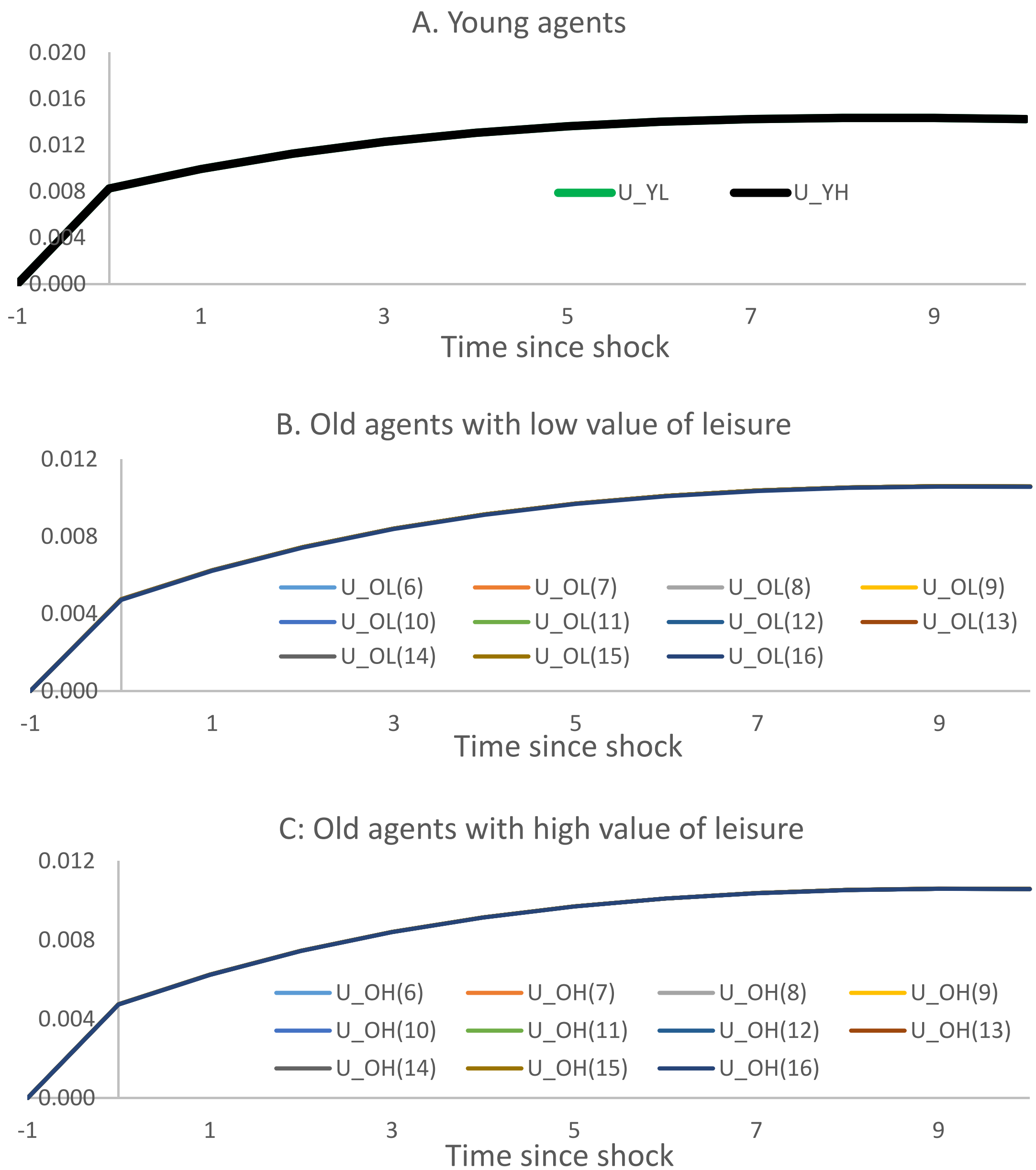


Figure 3: Impulse responses of utility of leisure

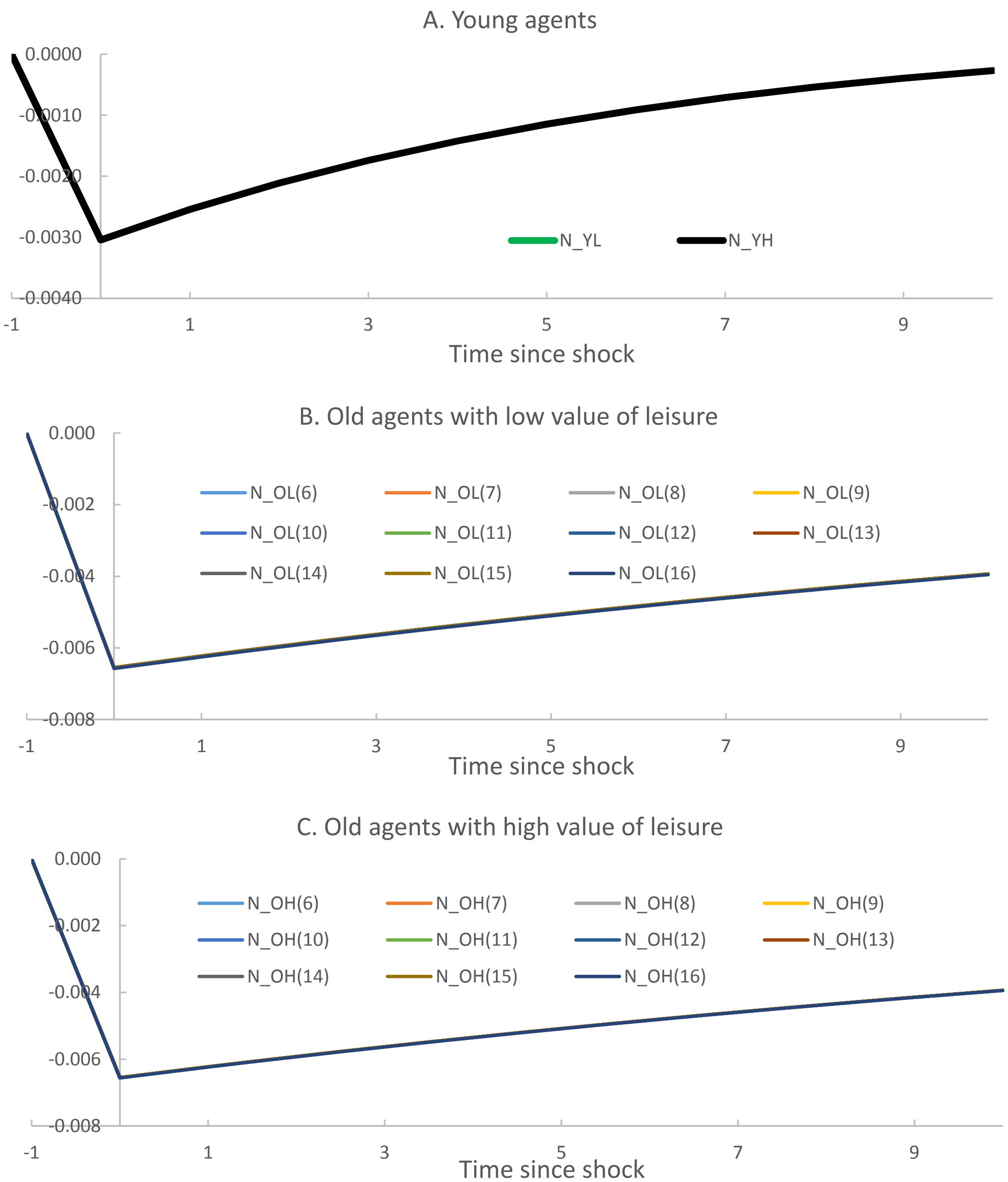


Figure 4: Impulse responses of promised values

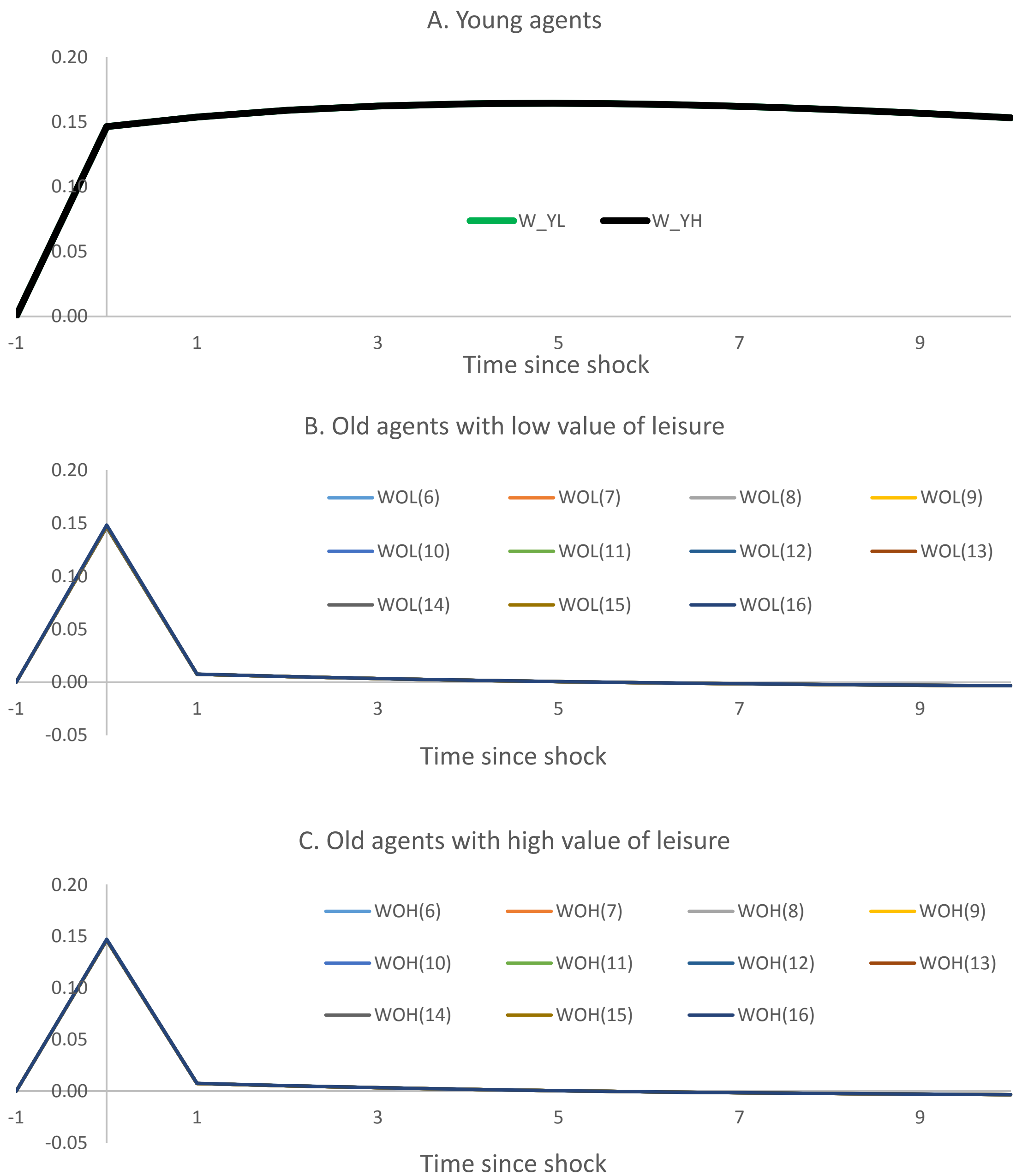


Figure 5: Cross-sectional distributions and macro variables

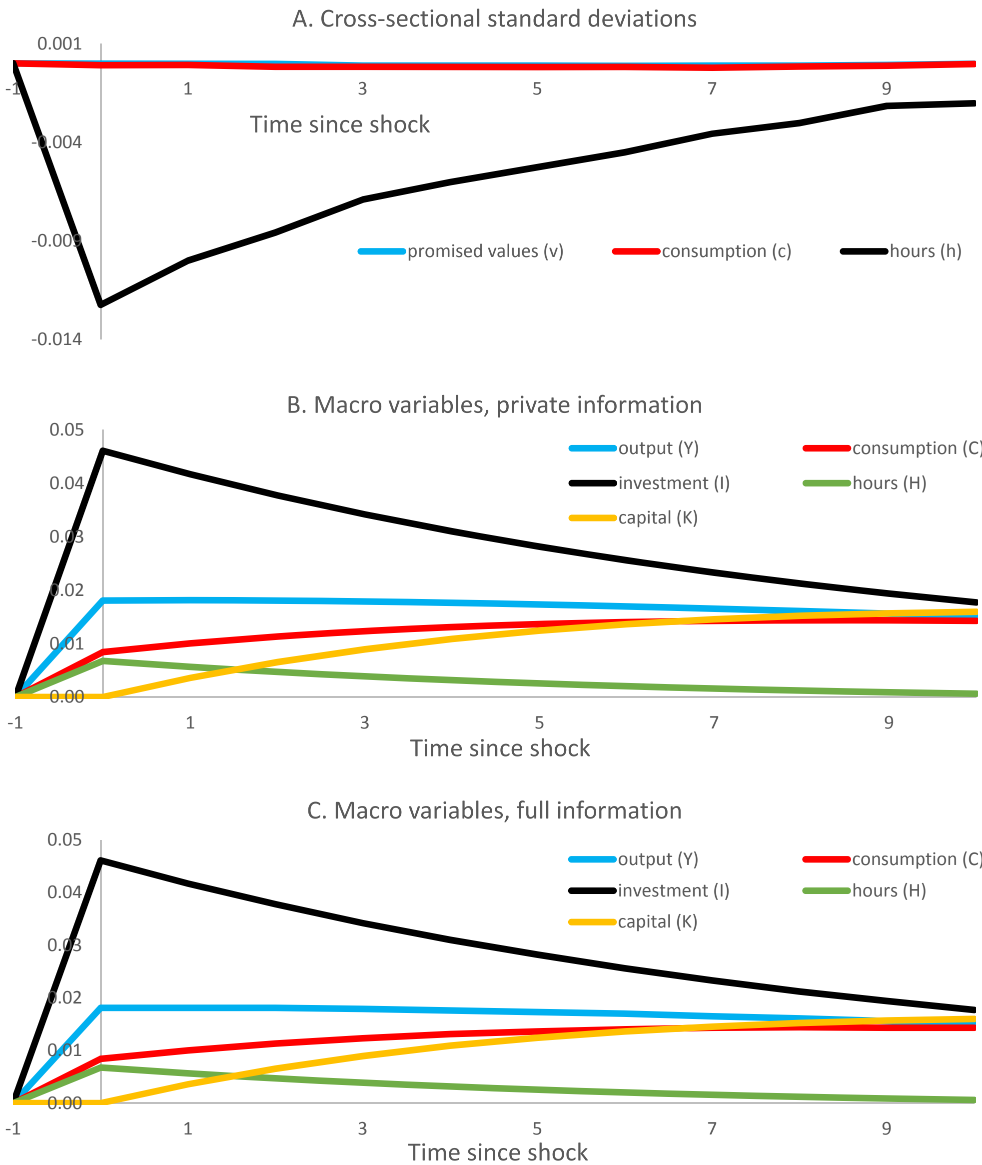
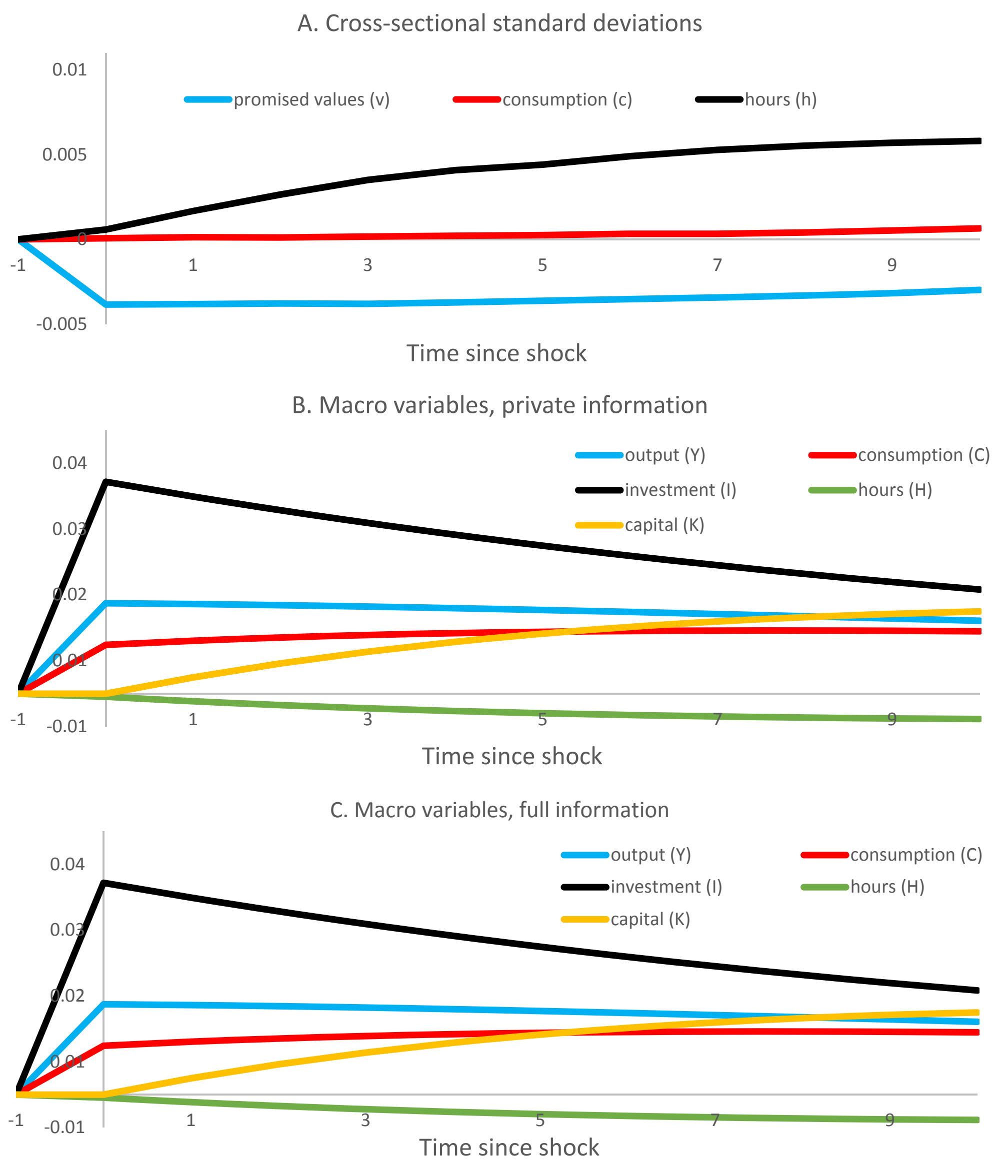


Figure 6: Cross-sectional distributions and macro variables



## Working Paper Series

A series of research studies on regional economic issues relating to the Seventh Federal Reserve District, and on financial and economic topics.


Corporate Average Fuel Economy Standards and the Market for New Vehicles **WP-11-01**
*Thomas Klier and Joshua Linn*

The Role of Securitization in Mortgage Renegotiation **WP-11-02**
*Sumit Agarwal, Gene Amromin, Itzhak Ben-David, Souphala Chomsisengphet, and Douglas D. Evanoff*

Market-Based Loss Mitigation Practices for Troubled Mortgages Following the Financial Crisis **WP-11-03**
*Sumit Agarwal, Gene Amromin, Itzhak Ben-David, Souphala Chomsisengphet, and Douglas D. Evanoff*

Federal Reserve Policies and Financial Market Conditions During the Crisis **WP-11-04**
*Scott A. Brave and Hesna Genay*

The Financial Labor Supply Accelerator **WP-11-05**
*Jeffrey R. Campbell and Zvi Hercowitz*

Survival and long-run dynamics with heterogeneous beliefs under recursive preferences **WP-11-06**
*Jaroslav Borovička*

A Leverage-based Model of Speculative Bubbles (Revised) **WP-11-07**
*Gadi Barlevy*

Estimation of Panel Data Regression Models with Two-Sided Censoring or Truncation **WP-11-08**
*Sule Alan, Bo E. Honoré, Luojia Hu, and Søren Leth–Petersen*

Fertility Transitions Along the Extensive and Intensive Margins **WP-11-09**
*Daniel Aaronson, Fabian Lange, and Bhashkar Mazumder*

Black-White Differences in Intergenerational Economic Mobility in the US **WP-11-10**
*Bhashkar Mazumder*

Can Standard Preferences Explain the Prices of Out-of-the-Money S&P 500 Put Options? **WP-11-11**
*Luca Benzoni, Pierre Collin-Dufresne, and Robert S. Goldstein*

Business Networks, Production Chains, and Productivity: A Theory of Input-Output Architecture **WP-11-12**
*Ezra Oberfield*

Equilibrium Bank Runs Revisited **WP-11-13**
*Ed Nosal*

Are Covered Bonds a Substitute for Mortgage-Backed Securities? **WP-11-14**
*Santiago Carbó-Valverde, Richard J. Rosen, and Francisco Rodríguez-Fernández*

The Cost of Banking Panics in an Age before “Too Big to Fail” **WP-11-15**
*Benjamin Chabot*

## **Working Paper Series** *(continued)*


Import Protection, Business Cycles, and Exchange Rates:
Evidence from the Great Recession **WP-11-16**
*Chad P. Bown and Meredith A. Crowley*

Examining Macroeconomic Models through the Lens of Asset Pricing **WP-12-01**
*Jaroslav Borovička and Lars Peter Hansen*

The Chicago Fed DSGE Model **WP-12-02**
*Scott A. Brave, Jeffrey R. Campbell, Jonas D.M. Fisher, and Alejandro Justiniano*

Macroeconomic Effects of Federal Reserve Forward Guidance **WP-12-03**
*Jeffrey R. Campbell, Charles L. Evans, Jonas D.M. Fisher, and Alejandro Justiniano*

Modeling Credit Contagion via the Updating of Fragile Beliefs **WP-12-04**
*Luca Benzoni, Pierre Collin-Dufresne, Robert S. Goldstein, and Jean Helwege*

Signaling Effects of Monetary Policy **WP-12-05**
*Leonardo Melosi*

Empirical Research on Sovereign Debt and Default **WP-12-06**
*Michael Tomz and Mark L. J. Wright*

Credit Risk and Disaster Risk **WP-12-07**
*François Gourio*

From the Horse's Mouth: How do Investor Expectations of Risk and Return
Vary with Economic Conditions? **WP-12-08**
*Gene Amromin and Steven A. Sharpe*

Using Vehicle Taxes To Reduce Carbon Dioxide Emissions Rates of
New Passenger Vehicles: Evidence from France, Germany, and Sweden **WP-12-09**
*Thomas Klier and Joshua Linn*

Spending Responses to State Sales Tax Holidays **WP-12-10**
*Sumit Agarwal and Leslie McGranahan*

Micro Data and Macro Technology **WP-12-11**
*Ezra Oberfield and Devesh Raval*

The Effect of Disability Insurance Receipt on Labor Supply: A Dynamic Analysis **WP-12-12**
*Eric French and Jae Song*

Medicaid Insurance in Old Age **WP-12-13**
*Mariacristina De Nardi, Eric French, and John Bailey Jones*

Fetal Origins and Parental Responses **WP-12-14**
*Douglas Almond and Bhashkar Mazumder*

## **Working Paper Series** *(continued)*

Repos, Fire Sales, and Bankruptcy Policy **WP-12-15**
*Gaetano Antinolfi, Francesca Carapella, Charles Kahn, Antoine Martin, David Mills, and Ed Nosal*

Speculative Runs on Interest Rate Pegs
The Frictionless Case **WP-12-16**
*Marco Bassetto and Christopher Phelan*

Institutions, the Cost of Capital, and Long-Run Economic Growth:
Evidence from the 19th Century Capital Market **WP-12-17**
*Ron Alquist and Ben Chabot*

Emerging Economies, Trade Policy, and Macroeconomic Shocks **WP-12-18**
*Chad P. Bown and Meredith A. Crowley*

The Urban Density Premium across Establishments **WP-13-01**
*R. Jason Faberman and Matthew Freedman*

Why Do Borrowers Make Mortgage Refinancing Mistakes? **WP-13-02**
*Sumit Agarwal, Richard J. Rosen, and Vincent Yao*

Bank Panics, Government Guarantees, and the Long-Run Size of the Financial Sector:
Evidence from Free-Banking America **WP-13-03**
*Benjamin Chabot and Charles C. Moul*

Fiscal Consequences of Paying Interest on Reserves **WP-13-04**
*Marco Bassetto and Todd Messer*

Properties of the Vacancy Statistic in the Discrete Circle Covering Problem **WP-13-05**
*Gadi Barlevy and H. N. Nagaraja*

Credit Crunches and Credit Allocation in a Model of Entrepreneurship **WP-13-06**
*Marco Bassetto, Marco Cagetti, and Mariacristina De Nardi*

Financial Incentives and Educational Investment:
The Impact of Performance-Based Scholarships on Student Time Use **WP-13-07**
*Lisa Barrow and Cecilia Elena Rouse*

The Global Welfare Impact of China: Trade Integration and Technological Change **WP-13-08**
*Julian di Giovanni, Andrei A. Levchenko, and Jing Zhang*

Structural Change in an Open Economy **WP-13-09**
*Timothy Uy, Kei-Mu Yi, and Jing Zhang*

The Global Labor Market Impact of Emerging Giants: a Quantitative Assessment **WP-13-10**
*Andrei A. Levchenko and Jing Zhang*

## **Working Paper Series** *(continued)*


Size-Dependent Regulations, Firm Size Distribution, and Reallocation **WP-13-11**
*François Gourio and Nicolas Roys*

Modeling the Evolution of Expectations and Uncertainty in General Equilibrium **WP-13-12**
*Francesco Bianchi and Leonardo Melosi*

Rushing into American Dream? House Prices, Timing of Homeownership,
and Adjustment of Consumer Credit **WP-13-13**
*Sumit Agarwal, Luojia Hu, and Xing Huang*

The Earned Income Tax Credit and Food Consumption Patterns **WP-13-14**
*Leslie McGranahan and Diane W. Schanzenbach*

Agglomeration in the European automobile supplier industry **WP-13-15**
*Thomas Klier and Dan McMillen*

Human Capital and Long-Run Labor Income Risk **WP-13-16**
*Luca Benzoni and Olena Chyruk*

The Effects of the Saving and Banking Glut on the U.S. Economy **WP-13-17**
*Alejandro Justiniano, Giorgio E. Primiceri, and Andrea Tambalotti*

A Portfolio-Balance Approach to the Nominal Term Structure **WP-13-18**
*Thomas B. King*

Gross Migration, Housing and Urban Population Dynamics **WP-13-19**
*Morris A. Davis, Jonas D.M. Fisher, and Marcelo Veracierto*

Very Simple Markov-Perfect Industry Dynamics **WP-13-20**
*Jaap H. Abbring, Jeffrey R. Campbell, Jan Tilly, and Nan Yang*

Bubbles and Leverage: A Simple and Unified Approach **WP-13-21**
*Robert Barsky and Theodore Bogusz*

The scarcity value of Treasury collateral:
Repo market effects of security-specific supply and demand factors **WP-13-22**
*Stefania D'Amico, Roger Fan, and Yuriy Kitsul*

Gambling for Dollars: Strategic Hedge Fund Manager Investment **WP-13-23**
*Dan Bernhardt and Ed Nosal*

Cash-in-the-Market Pricing in a Model with Money and
Over-the-Counter Financial Markets **WP-13-24**
*Fabrizio Mattesini and Ed Nosal*

An Interview with Neil Wallace **WP-13-25**
*David Altig and Ed Nosal*

## **Working Paper Series** *(continued)*


Firm Dynamics and the Minimum Wage: A Putty-Clay Approach **WP-13-26**
*Daniel Aaronson, Eric French, and Isaac Sorkin*

Policy Intervention in Debt Renegotiation:
Evidence from the Home Affordable Modification Program **WP-13-27**
*Sumit Agarwal, Gene Amromin, Itzhak Ben-David, Souphala Chomsisengphet, Tomasz Piskorski, and Amit Seru*

The Effects of the Massachusetts Health Reform on Financial Distress **WP-14-01**
*Bhashkar Mazumder and Sarah Miller*

Can Intangible Capital Explain Cyclical Movements in the Labor Wedge? **WP-14-02**
*François Gourio and Leena Rudanko*

Early Public Banks **WP-14-03**
*William Roberds and François R. Velde*

Mandatory Disclosure and Financial Contagion **WP-14-04**
*Fernando Alvarez and Gadi Barlevy*

The Stock of External Sovereign Debt: Can We Take the Data at ‘Face Value’? **WP-14-05**
*Daniel A. Dias, Christine Richmond, and Mark L. J. Wright*

Interpreting the *Pari Passu* Clause in Sovereign Bond Contracts:
It’s All Hebrew (and Aramaic) to Me **WP-14-06**
*Mark L. J. Wright*

AIG in Hindsight **WP-14-07**
*Robert McDonald and Anna Paulson*

On the Structural Interpretation of the Smets-Wouters “Risk Premium” Shock **WP-14-08**
*Jonas D.M. Fisher*

Human Capital Risk, Contract Enforcement, and the Macroeconomy **WP-14-09**
*Tom Krebs, Moritz Kuhn, and Mark L. J. Wright*

Adverse Selection, Risk Sharing and Business Cycles **WP-14-10**
*Marcelo Veracierto*