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Abstract

We investigate optimal capital structure and debt maturity policies in the presence of fixed issuance costs. We identify the global-optimal policy that generates the highest values of equity across all states of nature consistent with limited liability. The optimal policy without commitment provides almost as much tax benefits to debt as does the global-optimal policy and, in the limit of vanishing issuance costs, allows firms to extract 100% of EBIT. This limiting case does not converge to the equilibrium of DeMarzo and He (2019), who report no tax benefits to debt when issuance costs are set to zero at the outset.

1 Introduction

A large and growing literature in corporate finance investigates the firm's optimal debt policy in a dynamic setting. Often, the focus has been on: (i) the optimal level of debt issuance, and (ii) the optimal debt maturity. Much of this literature has studied these questions through the lens of the tradeoff between the cost of financial distress and the benefit of tax deductibility of interest payments. For example, focusing on firms that are subject to issuance costs, Goldstein, Ju and Leland (GJL, 2001) show that management can increase shareholder value by issuing additional debt whenever leverage ratios become sufficiently low.[1] In their framework, the optimal debt issuance policy is characterized by a single state variable (i.e., inverse leverage) and an "(s, S)" policy, with a region of inaction bounded by a lower threshold at which bankruptcy occurs, and an upper threshold at which additional debt is issued.

More recently, DeMarzo and He (DH, 2019) investigate the optimal debt-issuance policy for the case in which issuance costs are zero. Restricting their attention to Markov perfect equilibria with *no commitment*, they demonstrate that the optimal debt issuance policy is characterized by a locally deterministic process in which new debt is issued in all states of nature, even when the firm is near default. Due to this aggressive issuance policy, DH show that such firms obtain no tax benefits to debt, regardless of the choice of debt maturity.

Although the scopes of the GJL and DH papers do not overlap—being separated by whether there are issuance costs, and whether management can commit to a particular debt policy—there is a crucial difference in their implications: The GJL framework predicts that tax benefits to debt are both positive and increasing as issuance costs *go to zero*. However, the DH framework predicts that, regardless of the debt maturity policy, there are no tax benefits to debt when issuance costs *are zero*. Moreover, GJL predict that debt is issued in discrete amounts, whereas DH predict that debt is issued continuously.

In this paper, we investigate the impact that issuance costs have on optimal debt issuance and maturity policies for the cases with and without commitment, paying particular attention

[1]Early studies of dynamic capital structure choice include Kane, Marcus, and McDonald (1984, 1985) and Fischer, Heinkel, and Zechner (1989).

to the case of vanishing issuance costs. We begin by generalizing the framework of GJL to finite maturity, non-callable debt. Debt issuance is subject to a fixed cost, the size of which is controlled by the parameter β. We investigate optimal policies both with and without commitment.[2] For both cases, and for any given value of β, we identify the optimal (s, S) policy in terms of a single state variable (inverse leverage (v_t)) and four policy parameters: the location of the default boundary (v_b), the location of the debt-issuance boundary (v_u), the size of the debt issuance (γ), and the (inverse) maturity of the debt issued (ξ). For the case with commitment, the manager can choose the values of the policy parameters (v_u, γ) for all future debt issuances. However, the choice of v_B is always subject to limited liability.

In contrast, for the case without commitment, the manager has control only over current actions, such as whether to issue additional debt today, and if so, the size of that issuance (i.e., this period's values of (v_u, γ)), taking the decisions regarding future debt issuances as exogenous. As in DH, when studying the no-commitment case, we restrict our attention to Markov-perfect equilibria.

Our paper makes three contributions. First, we demonstrate that for any given issuance cost parameter β and (inverse) maturity parameter ξ, the optimal policy for the case with commitment generates the highest values of equity compared to all other debt issuance policies (v_b, v_u, γ) consistent with limited liability. As such, we refer to this policy as the *global optimal policy*. Interestingly, for sufficiently high values of β and ξ, the optimal policy is to issue no debt, that is, $v_u \to \infty$. This occurs because debt associated with high values of ξ (i.e., short-maturity debt) needs to be "rolled over" more often, and high values of β imply that issuance costs are high. Hence, in this region, the present value of debt issuance costs outweighs the present value of tax benefits of debt net of bankruptcy costs.

Second, we demonstrate that for any exogenously specified issuance cost parameter β and

[2] At the risk of fighting over semantics, we follow the literature that uses the phrases "commitment" and "no-commitment" in a somewhat relative fashion. For example, DH refer to their equilibrium as one with no commitment in spite of the fact that they assume commitment with respect to future debt maturity decisions. Similarly, He and Milbradt (2016) also refer to their equilibrium solution as one of no commitment in spite of the fact that they assume firms commit to holding their future outstanding face value of debt constant. More accurately, we can refer to the two solutions we investigate as having "more" and "less" levels of commitment.

(inverse) maturity parameter ξ, the optimal policy for the case without commitment is characterized by three regions: (i) As in the case with commitment, for sufficiently high values of β and ξ, the optimal policy is to issue no debt (i.e., optimal $v_u = \infty$); (ii) for intermediate values, an optimal (s, S) policy exists; (iii) for sufficiently low values of β and ξ, no equilibrium (s, S) policy exists. That no equilibrium strategy exists in region (iii) underscores the inefficiency of myopic no-commitment policies, in contrast to the global-optimal policy that generates positive tax benefits to debt in this region.

Third, we investigate the optimal debt issuance policy when maturity is chosen optimally. As in DH, we restrict our attention to the case in which a firm commits to a particular amortization rate ξ.[3] Within this setting we demonstrate that, for both cases with and without commitment, there exists an optimal maturity $(1/\xi^*(\beta))$ that is an increasing function of β. This result is due to a tradeoff between tax benefits net of bankruptcy costs, and issuance costs: Issuing short-maturity debt has the advantage that the firm can choose relatively "high leverage" today, which creates a large tax shelter, while assuring that the firm's future debt obligations will be significantly lower. Hence, even if future earning before interest and taxes (EBIT) falls, the firm is more likely to remain solvent, in turn reducing the present value of bankruptcy costs. However, the disadvantage of shorter maturity debt is that it is associated with an increase in expected debt issuance costs. For decreasing values of the parameter β, the first effect becomes increasingly dominant, in turn decreasing optimal maturity. Indeed, as β goes to zero, optimal maturity also goes to zero.

Consistent with GJL, we confirm that these policies generate significant tax benefits to debt, and that these tax benefits increase as issuance costs drop. Moreover, we demonstrate that a firm's ability to choose an optimal debt maturity significantly amplifies the tax benefits to debt compared to the GJL infinite-maturity benchmark. Indeed, for both cases with and without commitment, we show that as issuance costs go to zero, the firm can extract 100% of the claim

[3]Studying optimal maturity policy under the assumption of no commitment when debt is non-callable potentially leads to an intractable framework in which the current face value of debt for each vintage becomes a state variable. As the focus of our paper is to understand the impact of issuance costs on debt policies in a framework as close as possible to that of DH, we leave such an extension to future research.

to EBIT by combining an optimal maturity policy (ξ) with an optimal capital structure policy (v_b, v_u, γ). The implication is that, in the presence of even tiny issuance costs, when firms can choose both their capital structure and debt maturity policies optimally, the question of whether or not a manager can commit to the global-optimal policy is of secondary importance in terms of available tax benefits to debt.

Importantly, these results imply that in the limiting case of vanishing issuance costs, the optimal Markov-perfect no-commitment (s, S) policy in our framework does *not* converge to the zero issuance cost solution of DH, who report no tax benefits to debt, regardless of maturity choice. The intuition for this result is the following: consider a discrete-time setting with N periods of length Δt such that $(N \times \Delta t)$ equals some finite time interval T. Then allow for the possibility that the optimal debt issuance policy is some combination of a locally deterministic policy as in DH (in which the amount of debt issued in each of the N periods is of order $\mathcal{O}(\Delta t)$), and the (s, S) policy considered here (in which a finite amount debt is issued whenever inverse leverage reaches some upper threshold). In the continuous-time limit $\Delta t \to 0$ (and thus $N = \left(\frac{T}{\Delta t}\right) \to \infty$), any arbitrarily small (but positive) fixed issuance cost parameter generates an infinite present value of issuance costs if debt is issued continuously as in DH. Therefore, the locally deterministic component of the debt policy must be set to zero, leaving only the (s, S) policy.[4]

The predictions of our model are in line with the empirical literature that investigates the capital structure and maturity decisions of firms. By specifying fixed issuance costs, our model predicts that debt will be issued in discrete (rather than continuous) amounts, consistent with observation. Moreover, our model generates both persistence in leverage and a negative correlation between profitability and leverage, consistent with the empirical literature (e.g., Titman and Wessels (1988) and Frank and Goyal (2014)). Intuitively, our model captures these

[4]At $\beta = 0$, there is a discontinuity in the nature of the optimal no-commitment policy. Indeed, in unreported results, we investigate the case in which we set $\beta = 0$ at the outset of the analysis, and allow for the possibility that the optimal debt issuance policy is a combination of a locally deterministic policy (as in DH) and the (s, S) policy considered here. We find that the optimal location of the upper boundary is ($v_u = \infty$), implying that there is no (s, S) component of optimal policy when $\beta = 0$. This result can be seen as a special case of the proof in DH, who show that their optimal smooth policy is the unique Markov-perfect no-commitment equilibrium when $\beta = 0$.

features because, when firms are in the inaction region, higher (lower) profitability increases equity values while debt outstanding remains constant, leading to lower (higher) leverage. Also consistent with our model's predictions are Van Binsbergen, Graham, and Yang (2010), and Blouin, Core, and Guay (2010), who document that firms are able to extract tax benefits to debt. In agreement with our model's prediction of an optimal maturity policy, Barclay and Smith (1995) and Stohs and Mauer (1996) find that firms are not indifferent toward debt maturity choice. We note that Fama and French (2002), Baker and Wurgler (2002), and Welch (2004) provide evidence that shocks to capital structures are persistent, and that Leary and Roberts (2005) attribute this persistence to the presence of adjustment costs, as opposed to indifference toward capital structure. Graham and Harvey (2001) report survey evidence that 45% of CFOs are concerned with the tax advantage of interest deductibility (Figure 5, p. 210 and Table 6, p. 212). That firms target specific minimum credit rating levels (e.g., Kisgen (2006, 2009)) is consistent with managers caring about the firm's reputation in the debt markets, given that debt issuance is a repeated game. This provides support for our claim that firms can credibly commit to our global optimal policy. Finally, calibrating our model to be consistent with empirical estimates of issuance costs in the range of 1-2% generates a predicted optimal maturity in the range of 3-5 years, in line with empirical observation.

Our paper builds upon the quickly evolving literature that examines optimal dynamic capital structure decisions of firms. There are only a few tractable frameworks in this literature, due to the difficulty of pricing assets in an economy in which their current prices depend on the future debt issuance policy of the firm.[5] Recent papers that investigate whether firms continue to issue debt in high leverage states, or whether they choose to repurchase outstanding debt in this situation, include Dangl and Zechner (2016), Admati et al (2018), Xu (2018), and Malenko and Tsoy (2020). Whereas DH note that their findings are reminiscent of the Coase conjecture for a durable-good monopolist (Coase (1972)), our findings are analogous to the literature

[5]See, for example, Titman and Tsyplakov (2007), Stebulaev (2007), Morellec, Nikolov and Schürhoff (2012), Décamps and Villeneuve (2014), Hugonnier, Malamud and Morellec (2015), Dangl and Zechner (2016), Benzoni, Garlappi and Goldstein (2019).

that identifies channels through which the Coase conjecture fails.[6] Indeed, our finding that an arbitrarily small debt issuance cost is sufficient to overturn the capital-structure and maturity irrelevance result of DH is analogous to the findings of McAfee and Wiseman (2008), who show that the presence of even a vanishingly small cost of capacity production is enough to counter the Coase conjecture. Our paper also adds to the literature that investigates optimal maturity structure.[7]

The rest of the paper is as follows. In Section 2 we present the model and derive expressions for the claims to EBIT. In Section 3, for an exogenous choice of (β, ξ), we derive the optimal debt issuance policies for both cases with and without commitment, in turn identifying regions of the parameter space (β, ξ) for which the optimal solution is characterized by an (s, S) policy. We also demonstrate that the optimal policy with commitment is in fact globally optimal. In Section 4, we identify the optimal maturity structure policies $\xi^*(\beta)$. For both cases with and without commitment, we show that as the debt issuance parameter β approaches zero, the optimal debt maturity also approaches zero, and moreover that the fraction of the claim to EBIT that firms can extract approaches 100%. Section 5 solves the model with vanishing issuance costs in the presence of commitment. Section 6 concludes. We relegate proofs to Appendix A, and discuss our claim that the optimal policy under commitment is in fact globally optimal in Appendix B. Appendix C identifies a supermartingale, which we argue demonstrates that if a firm finds itself with inverse leverage ratio above the debt restructuring boundary (i.e., $v_{(t-)} > v_u$), then it is optimal for that firm to immediately issue a sufficient amount of debt so that the firm's new inverse leverage jumps into the range $v_{(t+)} \in (v_b, v_u)$. In the Online Appendix we (i) investigate optimal capital structure and maturity choice in the presence of *proportional* costs; and (ii) investigate debt *repurchase* policies, and show that once realistic issuance costs are specified, a firm cannot credibly commit to repurchase its own debt.[8]

[6] See, for example, Bulow (1982), Kahn (1986), Ausubel and Deneckere (1989).

[7] See, for example, Leland and Toft (1996), Leland (1998), Brunnermeier and Yogo (2009), Brunnermeier and Oehmke (2013), He and Xiong (2012), Diamond and He (2014), He and Milbradt (2014), Abel (2016), DeMarzo, He and Tourre (2018), Chen, Xu and Yang (2019).

[8] Building on our framework, Malenko and Tsoy (2020) show that firms whose EBIT can jump downward also cannot credibly commit to repurchase their debt, even in the absence of issuance costs.

2 The Model

In this section, we investigate the optimal debt issuance policy for a firm that is subject to a fixed cost each time it issues debt, the magnitude of which is controlled by a parameter β. The cost is "fixed" in that it is independent of the size of the debt issuance. For tractability, however, we assume that issuance costs are a fraction β of the firm size. We generalize GJL to the case of non-callable debt with exponential maturity characterized by a constant amortization rate ξ. We investigate two cases: with and without commitment. These two cases are distinguished by whether or not the firm can commit to a particular debt issuance policy (v_u, γ) for all future periods, or whether these policy parameters can be chosen only for the current period, with future policy decisions made by future managers.

There are two state variables in the model. The first is EBIT (Y_t), whose risk-neutral dynamics are exogenously specified:

$$\frac{dY}{Y} \;=\; \mu\, dt + \sigma\, dB^{\mathbb{Q}}, \tag{1}$$

with $dB^{\mathbb{Q}}$ denoting increments of a standard Brownian motion under the risk neutral measure $\mathbb{Q}$. Given the parameter restriction $(r > \mu)$, the present value (V_t) of the claim to EBIT can be determined from the risk-neutral expectation:

$$V_t \;=\; \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^{\infty} ds\, e^{-r(s-t)}\, Y_s\right] = \frac{Y_t}{r-\mu}. \tag{2}$$

Since the claim to EBIT is proportional to EBIT itself, it inherits the same dynamics:

$$\frac{dV}{V} \;=\; \mu\, dt + \sigma\, dB^{\mathbb{Q}}. \tag{3}$$

As required to preclude arbitrage, these dynamics imply that the risk-neutral expected return on the claim to EBIT equals the risk free rate:

$$\frac{dV + Y\, dt}{V} \;=\; r\, dt + \sigma\, dB^{\mathbb{Q}}. \tag{4}$$

Due to the linear relation between Y_t and V_t in equation (2), we are free to choose either as the exogenous state variable. We find it more convenient to choose V_t.

The second state variable is the face value of outstanding debt F_t, which is characterized by a constant amortization rate ξ and coupon rate c. During the interval $(t, t+dt)$, $\xi F_t\, dt$ units of debt mature. Thus, over this interval, the sum of coupon and principal payments to debtholders is $(c+\xi)F_t\, dt$. The effective average maturity is $\left(\frac{1}{\xi}\right)$.

Here we conjecture (and later verify) that the optimal debt issuance policy can be characterized in terms of a single state variable $v_t = V_t/F_t$ that can be interpreted as inverse leverage.[9] There is a lower boundary v_b at which it is optimal for shareholders to default. We define the date of this event as τ_b. It therefore follows that $v_b = V_{\tau_b}/F_{\tau_b}$. There is also an upper boundary v_u at which it is optimal to issue additional debt so that total outstanding debt scales by a factor γ (the size of which is chosen optimally). We define the date of this event as τ_u. It therefore follows that $v_u = V_{\tau_u}/F_{\tau_u}$. Thus, at time τ_u, the level of outstanding debt jumps from F_{τ_u} to γF_{τ_u} $(\gamma > 1)$, and shareholders pay a fixed cost of $\beta V_{\tau_u} = \beta v_u F_{\tau_u}$. Due to the presence of fixed issuance costs, it is optimal for shareholders to follow an (s, S) policy in that no debt is issued when inverse leverage falls in the range $v \in (v_b, v_u)$.

In order to price assets in this economy, it is convenient to define "period-0" as an interval of time that includes the present moment t, and ends the first time either the lower or upper boundary is reached. We refer to the date that period-0 ends as $\tau = \min(\tau_b, \tau_u)$. Each period-$j$, $j > 0$, that follows period-0 begins at a time of debt issuance, and ends with either a subsequent issuance or a default.

Because there are no debt issuances (or repurchases) within period-0, it follows that the dynamics of F_t are driven only by its amortization rate:

$$dF_t = -\xi F_t\, dt, \tag{5}$$

[9]Note that V is not the sum of debt and equity values, but rather the claim to EBIT, which also includes the claims to taxes, bankruptcy costs, and issuance costs. Still, we find it convenient to refer to $v = \frac{V}{F}$ as inverse leverage.

with solution

$$F_s \;=\; F_t e^{-\xi(s-t)} \qquad \forall s \in (t,\tau), \tag{6}$$

for any t in period-0. Given the dynamics of V_t and F_t, Itô's lemma implies that the dynamics of $v_t = V_t/F_t$ follow:

$$\frac{dv_t}{v_t} \;=\; (\mu+\xi)\, dt + \sigma\, dB^{\mathbb{Q}}. \tag{7}$$

Note that the (inverse) maturity parameter ξ is chosen by management, and that by choosing $\xi \to \infty$, the firm can virtually guarantee that future inverse leverage will reach the upper boundary v_u prior to reaching the lower boundary v_b, in turn precluding default. As such, the present value of tax benefits to debt net of bankruptcy costs is increased by choosing large values of ξ (i.e., short maturity). However, such a policy also increases debt issuance costs. These two mechanisms capture a key tradeoff in our model, as we discuss below.

2.1 Valuation of claims to EBIT

In our framework, the cash flows to equity (i.e., dividends), government (i.e., taxes) and outstanding debt (i.e., principal and interest payments) during the interval $(t, t+dt)$ are:

$$\begin{aligned}
\text{Equity}: &\qquad \Big[Y_t - \pi(Y_t - cF_t) - (c+\xi)F_t\Big]\, dt \\
\text{Government}: &\qquad \pi(Y_t - cF_t)\, dt \\
\text{Debt}: &\qquad (c+\xi)F_t\, dt.
\end{aligned} \tag{8}$$

In the event of default, we assume that a proportion α of firm value is lost to bankruptcy costs. As such, at default, the claims are:

$$\begin{aligned}
\text{Equity}: &\qquad 0 \\
\text{Government}: &\qquad (1-\alpha)\pi V_{\tau_b} \\
\text{Debt}: &\qquad (1-\alpha)(1-\pi)V_{\tau_b} \\
\text{Bankruptcy Costs}: &\qquad \alpha V_{\tau_b}.
\end{aligned} \tag{9}$$

Below, we price the claims to five different assets: equity, debt, government, bankruptcy costs and issuance costs. Expressed in terms of risk-neutral discounted cash flows and using $Y_t = (r-\mu)V_t$ from equation (2), all of these claims take the form:

$$X(V_t, F_t) = \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^{\tau} ds\, e^{-r(s-t)}\,(h_F F_s + h_V V_s) + e^{-r(\tau_u - t)}\,\mathbf{1}_{(\tau_u<\tau_b)}\,H_u + \right.$$

$$\left. e^{-r(\tau_b - t)}\,\mathbf{1}_{(\tau_b<\tau_u)}\,H_b\right]. \tag{10}$$

The first term captures the present value of cash flows prior to the firm reaching either the upper or lower boundary, which are specified in equations (8). The second term captures the value of the asset when the upper boundary is reached. The third term captures recovery values conditional upon default, which are specified in equations (9). The five different assets we investigate differ only by their factor loadings (h_F, h_V, H_b, H_u), in which: (i) the loadings (h_F, h_V) are constants, and (ii) the loadings (H_b, H_u) are linear in either (V, F) or in the asset values themselves. Here we determine asset values in terms of these generic factor loadings, and then use this generic solution to price the five assets below.

Equation (10) implies that $\left(e^{-rt}X(V_t, F_t) + \int_0^t ds\, e^{-rs}\,(h_F F_s + h_V V_s)\right)$ is a $\mathbb{Q}$-martingale. Therefore, by Itô's lemma, the asset value satisfies the PDE:

$$0 = -rX + \mu V X_V + \frac{\sigma^2}{2}V^2 X_{VV} - \xi F X_F + h_F F + h_V V, \tag{11}$$

subject to the boundary conditions

$$X(v_b F_{\tau_b}, F_{\tau_b}) = H_b \tag{12}$$

$$X(v_u F_{\tau_u}, F_{\tau_u}) = H_u. \tag{13}$$

Because both the cash flows and state vector dynamics are linear in the state vector, it follows that asset values are homogeneous of degree one in the state vector. Thus, we define a single state variable that we refer to as inverse leverage $v_t = (V_t/F_t)$, and look for a solution of the form:

$$X(V, F) = F\, x(v)\big|_{v=(V/F)}, \tag{14}$$

where $x(v)$ denotes asset value per unit of face value F. It follows that the partial derivatives of $X(V,F)$ with respect to V and F can be expressed as:

$$\begin{aligned} X_V &= x_v && (15) \\ X_{VV} &= \left(\frac{1}{F}\right) x_{vv} && (16) \\ X_F &= x - v x_v. && (17) \end{aligned}$$

Substituting these expressions into the PDE (11), we obtain the following ODE:

$$0 = \frac{\sigma^2}{2} v^2 x_{vv} + (\mu + \xi)\, v x_v - (r+\xi) x + h_V v + h_F, \tag{18}$$

subject to the boundary conditions

$$\begin{aligned} x(v_b) &= \frac{H_b}{F_{\tau_b}} && (19) \\ x(v_u) &= \frac{H_u}{F_{\tau_u}}. && (20) \end{aligned}$$

The solution to this ODE is:

$$x(v) = M_x v^{\phi} + N_x v^{\omega} + \left(\frac{h_V}{r-\mu}\right) v + \left(\frac{h_F}{r+\xi}\right), \tag{21}$$

where the exponents (ϕ, ω) are given by

$$\begin{aligned} \phi &= \frac{\left(\frac{\sigma^2}{2} - (\mu+\xi)\right) + \sqrt{\left(\frac{\sigma^2}{2} - (\mu+\xi)\right)^2 + 2(r+\xi)\sigma^2}}{\sigma^2} > 1 && (22) \\ \omega &= \frac{\left(\frac{\sigma^2}{2} - (\mu+\xi)\right) - \sqrt{\left(\frac{\sigma^2}{2} - (\mu+\xi)\right)^2 + 2(r+\xi)\sigma^2}}{\sigma^2} < 0. && (23) \end{aligned}$$

The constants (M_x, N_x) are uniquely determined by the boundary conditions:

$$\begin{aligned} x(v_b) &= \left(\frac{H_b}{F_{\tau_b}}\right) && (24) \\ x(v_u) &= \left(\frac{H_u}{F_{\tau_u}}\right). && (25) \end{aligned}$$

With this general form of the solution, we price the five assets of interest:

Proposition 1 (Asset valuation) *Given a set of debt policy parameters* (v_b, v_u, γ, ξ)*:*

- *The value of debt* $d(v) = D(V,F)/F$ *per unit of face value* F *can be expressed as:*

$$d(v) = M_d v^{\phi} + N_d v^{\omega} + \frac{c+\xi}{r+\xi}, \tag{26}$$

where the constants (M_d, N_d) *are uniquely determined by the boundary conditions*

$$d(v_b) = (1-\alpha)(1-\pi)v_b \tag{27}$$

$$d(v_u) = d(v_u/\gamma). \tag{28}$$

- *The value of equity* $e(v) = E(V,F)/F$ *per unit of face value* F *can be expressed as:*

$$e(v) = M_e v^{\phi} + N_e v^{\omega} + (1-\pi)v - \frac{(c(1-\pi)+\xi)}{r+\xi}, \tag{29}$$

where the constants (M_e, N_e) *are uniquely determined by the boundary conditions*

$$e(v_b) = 0 \tag{30}$$

$$e(v_u) = \gamma e(v_u/\gamma) + (\gamma-1)d(v_u) - \beta v_u. \tag{31}$$

- *The value of the government claim* $g(v) = G(V,F)/F$ *per unit of face value* F *can be expressed as:*

$$g(v) = M_g v^{\phi} + N_g v^{\omega} + \pi v - \frac{c\pi}{r+\xi}, \tag{32}$$

where the constants (M_g, N_g) *are uniquely determined by the boundary conditions*

$$g(v_b) = (1-\alpha)\pi v_b \tag{33}$$

$$g(v_u) = \gamma g(v_u/\gamma). \tag{34}$$

- *The value of the claim to bankruptcy costs $b(v) = B(V,F)/F$ per unit of face value can be expressed as:*

$$b(v) = M_b v^{\phi} + N_b v^{\omega}, \tag{35}$$

where the constants (M_b, N_b) are uniquely determined by the boundary conditions

$$\begin{aligned} b(v_b) &= \alpha v_b & (36)\\ b(v_u) &= \gamma b(v_u/\gamma). & (37) \end{aligned}$$

- *The value of the claim to issuance costs $i(v) = I(V,F)/F$ per unit of face value can be expressed as:*

$$i(v) = M_i v^{\phi} + N_i v^{\omega}, \tag{38}$$

where the constants (M_i, N_i) are uniquely determined by the boundary conditions

$$\begin{aligned} i(v_b) &= 0 & (39)\\ i(v_u) &= \beta v_u + \gamma\, i(v_u/\gamma). & (40) \end{aligned}$$

Proof: See Appendix A.

3 Optimal Debt Policies

In the previous section, we determined asset values for any *given* set of policy parameters (v_b, v_u, γ, ξ). Here we investigate the optimal choice of these parameters. First, in Section 3.1 we fix the (inverse) maturity parameter ξ, and identify the optimal values of (v_b, v_u, γ) for both the cases with and without commitment. We also identify regions in the parameter space (β, ξ) for which the optimal policy is to not issue additional debt, and regions for which no (s, S) equilibrium exists. Second, in Section 3.2 we identify the optimal debt maturity policy parameter $\xi^*(\beta)$ as a function of β.

3.1 Optimal Debt Policies for a Fixed Debt Maturity

The asset value formulas derived in the previous section assumed that the policy parameters remain constant across all periods. This assumption is consistent with a scaling property inherent in our framework in which, at the beginning of each period, the firm looks the same except for its size. More generally, however, a firm's debt policy is characterized by a set of parameters $(v_{b,j}, v_{u,j}, \gamma_j)$, for *each* period $j \in [0, \infty)$. Ultimately, if the equilibrium optimal policy is characterized by a pure (s, S) strategy, then those parameters must be constant across periods. How those parameters are chosen by the firm, however, depends upon whether a manager can commit to a particular policy for all future periods.

In this context, it is natural to investigate two types of debt issuance policies: with and without commitment. When investigating debt issuance *with commitment*, we focus on policies in which the manager has full control over the debt issuance decisions for all future periods j. Thus, the optimal policy with commitment is determined by *first* imposing that the period-0 parameters are equal to those in all other periods: $(v_{u,j}, \gamma_j) = (v_u, \gamma)$, for all $j \geq 0$, and *then* choosing (v_u, γ) optimally. The location of the default boundary v_b is also chosen optimally, but must be consistent with limited liability. In this case, the optimal policy can be determined by directly using the asset valuations from the previous section.

In contrast, when studying debt issuance *without commitment*, we focus on myopic policies in which the manager in period-0 only gets to choose the policy parameters $(v_{u,0}, \gamma_0)$ for the current period, and takes the policy parameters $(v_{u,j}, \gamma_j)$ for all future periods $j > 0$ as exogenously specified. Only after imposing the first order conditions for optimality on $(v_{u,0}, \gamma_0)$ do we set $(v_{u,0}, \gamma_0) = (v_{u,j}, \gamma_j)$ for all $j > 0$ in order to identify the optimal policy parameters. Once again, the location of the default boundary $v_{b,0}$ is chosen optimally, but must be consistent with limited liability. This analysis requires that we rewrite the pricing equations in Proposition 1 to distinguish between claim valuations in period-0 and in subsequent periods, as we demonstrate in Section 3.1.2.

3.1.1 Optimal Debt Policy with Commitment

For the case with commitment, we specify the optimal debt issuance policy as follows:

Definition 1 (Global-optimal policy with commitment) *For a given set of model coefficients* $(\mu, \sigma, r, \pi, \alpha, \beta)$ *and an inverse maturity parameter* ξ*, the optimal debt issuance policy* (v_b^*, v_u^*, γ^*) *with commitment satisfies the following system of first-order conditions (FOCs)*

$$\left.\frac{\partial e}{\partial v_b}\right|_{v=v_b} = 0 \tag{41}$$

$$\left.\frac{\partial e}{\partial v_u}\right|_{v=v_u} = 0 \tag{42}$$

$$\left.\frac{\partial e}{\partial \gamma}\right|_{v=v_u} = 0. \tag{43}$$

Appendix B verifies numerically that this policy is globally optimal in that, compared to all other possible debt issuance policies (v_b, v_u, γ) consistent with limited liability, this solution generates the highest values of equity across *all* values of $v \in (v_b, v_u)$.

At first blush, there seems to be no reason that the policy generated from the first order conditions (FOC's) in Definition **??** should generate a global-optimal policy in that these FOC's are applied only at the locations where the manager takes action (either by defaulting at $v_t = v_b$ or by issuing debt at $v_t = v_u$). To provide intuition for our findings, it is convenient to use the boundary condition on equity at $v = v_b$ in equation (30) to eliminate the coefficient N_e in the equity valuation formula equation (29). This allows us to write the equity value as follows:

$$\begin{aligned} e(v) &= M_e v^{\phi}\left[1-\left(\frac{v}{v_b}\right)^{(\omega-\phi)}\right] + (1-\pi)v\left[1-\left(\frac{v}{v_b}\right)^{(\omega-1)}\right] \\ &\quad - \left(\frac{c(1-\pi)+\xi}{r+\xi}\right)\left[1-\left(\frac{v}{v_b}\right)^{\omega}\right]. \end{aligned} \tag{44}$$

Importantly, note that in equation (44), the policy parameters (v_u, γ) appear only in the coefficient M_e. (In contrast, v_b shows up explicitly in equation (44)). Thus, the FOC's with respect to (v_u, γ) for *any value* of $v = v_{any}$ are given by:

$$
\begin{aligned}
0 &= \left.\frac{\partial e}{\partial v_u}\right|_{v=v_{any}} \propto \frac{\partial M_e}{\partial v_u} && (45)\\
0 &= \left.\frac{\partial e}{\partial \gamma}\right|_{v=v_{any}} \propto \frac{\partial M_e}{\partial \gamma}. && (46)
\end{aligned}
$$

Hence, for any value $v = v_{any}$, equations (45)–(46) deliver the same optimal policy parameters (v_u^*, γ^*) as those identified by equations (42)–(43). This property explains the global optimal feature of our solution.

3.1.2 Optimal Debt Policy without Commitment

In Section 3.1.1, we assumed that a manager in period-0 had the power to commit to all present and future debt issuance policies $(v_{u,j}, \gamma_j)\ \forall j$. In this section, however, we investigate optimal policies when the manager does not have the ability to commit to future debt issuance policies. Specifically, we set $(v_{u,j}, \gamma_j) = (v_u, \gamma)$ for $j \geq 1$, which we distinguish, for now, from the period-0 controls $(v_{u,0}, \gamma_0)$. Under this restriction, we obtain the period-0 value of debt and equity.

For a given value of v_t and policy parameters (v_b, v_u, γ), the period-0 value of debt $d_0(v)$ is determined by first calculating the value of debt $d(v)$ for all future periods:

$$d(v) = M_d v^{\phi} + N_d v^{\omega} + \frac{c+\xi}{r+\xi}, \tag{47}$$

where the constants (M_d, N_d) are uniquely determined by the boundary conditions

$$
\begin{aligned}
d(v_b) &= (1-\alpha)(1-\pi)v_b && (48)\\
d(v_u) &= d(v_u/\gamma). && (49)
\end{aligned}
$$

We then determine the period-0, date-t value of debt from:

$$d_0(v_t) = M_{d,0} v_t^{\phi} + N_{d,0} v_t^{\omega} + \frac{c+\xi}{r+\xi}, \tag{50}$$

where the constants $(M_{d,0}, N_{d,0})$ are uniquely determined from the boundary conditions

$$d_0(v_{b,0}) = (1-\alpha)(1-\pi)v_{b,0} \tag{51}$$

$$d_0(v_{u,0}) = d(v_{u,0}/\gamma_0). \tag{52}$$

The parameters (v_b, v_u, γ) are outside the control of the period-0 manager, and impact the function $d(v)$ through the boundary conditions (48)–(49). The function $d(v)$, in turn, affects the value of the period-0 debt $d_0(v_t)$ through the boundary condition (52).

Analogously, for a given value of v_t and policy parameters (v_b, v_u, γ), we obtain the period-0 value of equity $e_0(v_t)$ by first determining the value of equity $e(v)$ for all future periods:

$$e(v) = M_e v^{\phi} + N_e v^{\omega} + (1-\pi)v - \frac{c(1-\pi)+\xi}{r+\xi}, \tag{53}$$

where the constants (M_e, N_e) are uniquely determined from the boundary conditions

$$e(v_b) = 0 \tag{54}$$

$$e(v_u) = \gamma e(v_u/\gamma) + \gamma d(v_u/\gamma) - d(v_u) - \beta v_u. \tag{55}$$

We then determine the period-0, date-t value of equity from:

$$e_0(v_t) = M_{e,0} v_t^{\phi} + N_{e,0} v_t^{\omega} + (1-\pi)v_t - \frac{c(1-\pi)+\xi}{r+\xi}, \tag{56}$$

where the constants $(M_{e,0}, N_{e,0})$ are uniquely determined from the boundary conditions

$$e_0(v_{b,0}) = 0 \tag{57}$$

$$e_0(v_{u,0}) = \gamma_0 e(v_{u,0}/\gamma_0) + \gamma_0 d(v_{u,0}/\gamma_0) - d_0(v_{u,0}) - \beta v_{u,0}. \tag{58}$$

The parameters (v_b, v_u, γ) impact the function $e(v)$ through the boundary conditions (54)–(55). The function $e(v)$, in turn, affects the value of the period-0 equity $e_0(v_t)$ through the boundary condition (58).

Equations (50)–(52) and (56)–(58) determine the value of period-0 debt and equity for a *given* set of policy parameters $(v_{b,0}, v_{u,0}, \gamma_0)$ and (v_b, v_u, γ). Due to the symmetry of our economy, ultimately, the optimal policy parameters will be the same across all periods. The period-0 policy is then chosen under the restriction that all future policies are identical to the one in period-0.

Definition 2 (Optimal policy without commitment) *For a given set of model coefficients* $(\mu,\ \sigma,\ r,\ \pi,\ \alpha,\ \beta)$ *and an inverse maturity parameter* ξ*, the optimal debt issuance policy* $(v_{b,0}, v_{u,0}, \gamma_0)$ *without commitment is the period-0 best response to future issuance policies* $(v_{b,j}, v_{u,j}, \gamma_j)$*,* $j \geq 1$*, that satisfies the following system of first-order conditions*

$$0 = \left.\frac{\partial e_0}{\partial v_{u,0}}\right|_{v=v_{u,0}} \tag{59}$$

$$0 = \left.\frac{\partial e_0}{\partial v_{b,0}}\right|_{v=v_{b,0}} \tag{60}$$

$$0 = \left.\frac{\partial e_0}{\partial \gamma_0}\right|_{v=v_{u,0}}, \tag{61}$$

together with the fixed-point condition $(v_{b,0}, v_{u,0}, \gamma_0) = (v_{b,j}, v_{u,j}, \gamma_j)$*, for all* $j \geq 1$*.*

3.1.3 Existence of Equilibrium Debt Policies: Commitment versus no Commitment

The FOC's (41)–(43) and (59)–(61) are necessary but not sufficient to identify an optimal issuance policy. Two problems can arise: First, the proposed solution may not satisfy the required second-order conditions, and thus might characterize a minimum or a saddle point. Second, the proposed solution might be only a local maximum. In this section, for given values of β and ξ, we identify regions for which solutions to the FOC's are in fact the optimal policy parameters.

Figure 1 shows that, for the case with commitment, the parameter space (β, ξ) consists of two regions separated by an indifference curve: For sufficiently high values of β and ξ (i.e., sufficiently low values of $(1/\xi)$), it is optimal for the firm to never issue debt, and therefore the optimal value of v_u is infinity. For lower values of β and ξ, there exists an optimal (s, S)

debt issuance policy with parameters (v_b, v_u, γ). The existence of these two regions can be understood as follows: For a given level of β, when debt maturity is low (i.e., when ξ is high), debt needs to be "rolled over" more often in order to maintain a level of leverage that generates significant tax benefits. Separately, for a given maturity $1/\xi$, when β is high, each debt issuance becomes more expensive. Therefore, when both β and ξ are sufficiently high, the present value of issuance costs becomes larger than the present value of tax benefits of debt net of bankruptcy costs. The line that separates these two regions represents the indifference curve along which the tax benefits of debt net of bankruptcy costs equal the present value of issuance costs. For reasons explained below, we refer to this line as the *optimal indifference curve.*

To provide support for this interpretation, in Figure 2 we hold ξ fixed, and plot the optimal values of $(v_b, v_u, v_u/\gamma)$ as a function of β. As β approaches its critical value (identified by the dashed vertical line), the optimal value of v_u approaches infinity, but both $\left(\frac{v_u}{\gamma}\right)$ and v_b remain finite. Moreover, the optimal value of v_b approaches the default boundary in a generalized version of Leland (1994) model in which debt has finite maturity. Taking advantage of these insights, in the online Appendix we show that the indifference curve can be approximated by the formula:

$$(\beta \times \xi)\Big|_{\text{no benefit to debt}} \approx r\pi(1-\pi). \tag{62}$$

Figure 1 shows that this approximate formula provides a good estimate for the actual location of the curve of critical values of $(\beta,\ \xi)$ along which tax benefits to debt net of bankruptcy costs equal debt issuance costs. Moreover, this approximation improves for smaller values of β and $1/\xi$, for which the underlying assumptions made to derive equation (62) are more justifiable.

In contrast, for the case with no commitment, Figure 3 shows that the parameter space (β, ξ) consists of three regions. As in the case with commitment, for sufficiently high values of β and ξ, it is optimal for the firm to never issue debt. This is for the same reason given above: In this region, the present value of issuance costs is larger than the present value of future tax benefits of debt net of bankruptcy costs. For intermediate values of β and ξ, there exists an optimal (s, S) policy characterized by the parameters (v_b, v_u, γ). As above, we refer to the line that separates these two regions as the optimal indifference curve. Interestingly, however, for sufficiently low values of β and ξ, we find that there is no equilibrium (s, S) strategy in that values for (v_b, v_u, γ) that satisfy the relevant first order conditions either do not satisfy the necessary second order conditions, or provide only a local maximum. The fact that there is no (s, S) equilibrium solution for sufficiently low values of $(\beta,\, \xi)$ underscores the inefficiency of myopic no-commitment policies, and stands in contrast to the global-optimal policy that, for the same values of $(\beta,\, \xi)$, generates positive tax benefits to debt. We refer to the line that separates this region from the region whose optimal policy is characterized by an (s, S) strategy as the *no equilibrium curve*.

3.2 Optimal Debt Maturity

In the previous section, we determined the optimal policy parameters $(v_b^*(\xi), v_u^*(\xi), \gamma^*(\xi))$ for a fixed inverse maturity parameter ξ. Here we choose the optimal maturity $1/\xi^*$ by maximizing the equity value at the time of issuance. The value of equity at the time of issuance is $E(v_u F, \gamma F) + D(v_u F, \gamma F) - D(v_u F, F) - \beta v_u F$. Using the scaling property $E(V, F) = F\, e(v)$, $D(V, F) = F\, d(v)$, the optimal value of ξ^* is given by:

$$\xi^* \quad = \quad \arg\max_{\xi} \left\{ \gamma\, e(v_u/\gamma) + \gamma\, d(v_u/\gamma) - d(v_u) - \beta v_u \right\}. \tag{63}$$

As in DH, we are assuming commitment with regard to maturity in that all past debt was, and all future debt will be, issued with inverse maturity ξ^*. This modeling choice allows us to compare our results more directly to DH.

4 Results

In this section, we analyze the properties of the optimal debt issuance policies derived from our model. Section 4.1 investigates debt and equity values. Section 4.2 analyzes the effect of the debt issuance cost β on optimal policies. In Section 4.3 we estimate the tax benefit to debt and in Section 4.4 we provide a decomposition of the claims to EBIT.

Table 1 reports the values of the coefficients that we use in our baseline model calibration. We set the annual risk-free rate $r = 4\%$, and the drift and volatility of the EBIT dynamics in equation (7) to $\mu = 0$ and $\sigma = 22\%$, respectively. We assume that corporate profits are taxed at a rate $\pi = 20\%$. Following DH, we set the bankruptcy cost parameter to $\alpha = 1$.[10] Below, we illustrate results for a range of values of the issuance cost parameter β. For each value of β, we choose the inverse maturity parameter ξ according to equation (63), and then determine the optimal parameters (v_b, v_u, γ) for these values of (β, ξ). Finally, we choose the coupon rate c so that the bond is priced at par at the time of issuance.

4.1 Debt and Equity Values

In Figure 4, we report the values of (scaled) equity $e(v)$ and debt $d(v)$ as a function of v for the case *with commitment*. We choose $\beta = 0.0036$ to reflect an issuance cost of 1% of the debt amount issued, consistent with empirical estimates:[11]

$$\frac{\beta v_u F_{\tau_u}}{\underbrace{D(v_u F_{\tau_u}, \gamma F_{\tau_u}) - D(v_u F_{\tau_u}, F_{\tau_u})}_{\text{Debt amount issued}}} = \frac{\beta v_u}{(\gamma - 1)\, d(v_u)} \overset{\text{set}}{=} 1\%, \tag{64}$$

where the first equality follows from the scaling property of the debt claim. Figure 4 shows that for values of $v > (v_u/\gamma)$, the value of debt is rather flat and remains close to the value of a risk-free bond with the same promised cash flows $d_{risk\,free} = \left(\frac{c+\xi}{r+\xi}\right)$. However, the value of debt drops quickly toward its recovery value (which, following DH, is zero in this parametrization) as v approaches the default boundary v_b. In contrast, equity exhibits positive convexity (consistent with the smooth pasting condition) for values of v near the default boundary v_b and is linear for larger values of v.

[10] DH mostly focus on the case $\alpha = 1$. For the case $\alpha < 1$, they find that the optimal policy includes issuing an infinite amount of face value of debt at the default boundary, which in turn reduces the recovery rate on debt as a fraction of face value to zero. This prediction is in stark contrast to empirical observation, in which a typical recovery rate is approximately 40% of the face value of debt. One way to avoid this counterfactual prediction in both our framework and theirs is to specify that covenants are in place that restrict debt issuance beyond some level of (inverse) leverage.

[11] See, for example, Altınkılıç and Hansen (2000), Hennessy and Whited (2007), Titman and Tsyplakov (2007), and Gamba and Triantis (2008).

In Figure 5, we plot the *difference* in equity values for the cases with and without commitment. The figure confirms that, for all values of v, equity values are higher for the case with commitment, consistent with this policy being globally optimal. More interesting, however, is that the differences between the equity prices with and without commitment is typically two- to three orders of magnitude smaller than the typical *level* of the claim's value reported in Figure 4.[12] As shown in more detail in Sections 4.3 and 4.4, this result suggests that managers who follow a no-commitment strategy can obtain *most* of the tax benefits afforded to the global-optimal strategy.[13]

4.2 Optimal Debt Policies as a Function of β

In the next several figures, we report results associated with the optimal policy controls $\big(v_b^*(\beta), v_u^*(\beta), \gamma^*(\beta), \xi^*(\beta)\big)$ as a function of β. For both cases with and without commitment, Figure 6 shows that the optimal average maturity is an increasing function of the issuance cost parameter β. This finding is consistent with the tradeoff between net tax benefits to debt and issuance costs. Specifically, issuing short-maturity debt increases tax benefits net of bankruptcy costs, whereas issuing long-maturity debt reduces issuance costs. As one lowers the issuance cost parameter β, the first channel becomes more dominant, leading to shorter optimal maturities. Interestingly, for all values of β, the optimal maturity for the case with commitment is slightly longer than the optimal maturity for the case without commitment.

In Figures 7 and 8, we present the optimal debt issuance size parameter $\gamma^*(\beta)$ and the optimal location of the upper boundary $v_u^*(\beta)$ as a function of the issuance cost parameter β. Both with and without commitment, the optimal location of the upper boundary and the size of the debt issuance increase monotonically with issuance costs. This is because, when

[12] In both cases with and without commitment, the equity value is close to zero when inverse-leverage v approaches the default boundary v_b. Next to that point, small deviations in the equity valuations are due to slight differences in the location of the default boundary for the policies with and without commitment.

[13] Because the global-optimal policy generates higher equity values in all states of nature compared to the no-commitment case, it can be supported in equilibrium if any deviation from the global optimal policy would imply all future debt issuances would be priced according to the no-commitment solution. Similarly, as we discuss in Section 5, the DH policy can serve as a punishment for the case of vanishing issuance costs. Such an equilibrium, however, would fall outside of the Markov-perfect class that DeMarzo and He (2019) restrict their attention to.

fixed issuance costs are high, it is optimal to reduce the present value of these costs by making issuances less frequent. This can be accomplished in two ways: First, by increasing v_u so that the inverse leverage of the firm (v_t) reaches the debt issuance boundary v_u less often. Second, by increasing debt issuance size so that, after each issuance date, the firm's inverse leverage $\left(\frac{v_u}{\gamma}\right)$ is further away from reaching the debt issuance boundary v_u again.

There are two important features to note from these figures. First, as can be seen from equation (8), the (scaled) tax flow to government can be expressed as $\pi\left[(r-\mu)v - c\right]$. Hence, taxes are zero when $v = \left(\frac{c}{r-\mu}\right)$. In the limit of $\beta \to 0$, Figure 8 shows that the optimal value for $v_u \to \left(\frac{c}{r-\mu}\right)$, implying that the present value of the government claim goes to zero in this limit.[14] Second, in the limit $\beta \to 0$, we find $\gamma \to 1$, implying that the size of new debt issuances becomes infinitesimally small. That is, in the limit of vanishing issuance costs, the upper boundary acts like a reflecting boundary. We will use this result in the next section when we formally investigate the limiting case $\beta \to 0$.

In Figure 9, we present the optimal location of the lower boundary $v_b^*(\beta)$ as a function of the issuance cost parameter β. We see that, for a given β, the case with commitment is always associated with a lower default boundary compared to the case without commitment. This is because the value of equity is always higher for the case with commitment, making the option to maintain ownership more valuable. Interestingly, however, $v_b^*(\beta)$ is not monotonic in β. This is due to the presence of two channels impacting this optimal policy function. The first (direct) channel is driven by issuance costs: *Ceteris paribus*, higher issuance costs lower equity valuations as well as the option to remain solvent. Hence, this channel alone would generate a monotonically increasing function $v_b^*(\beta)$. Indeed we find that the relation between v_b^* and β is monotonic when we hold ξ fixed. However, there is a second (indirect) channel due to the optimal maturity choice $\xi^*(\beta)$. It is evident from Figure 6 that optimal maturity is increasing in β. In those states of nature in which a firm is performing poorly, longer maturity (i.e., smaller

[14]More precisely, we can show that, in the limit $\beta \to 0$, the optimal values for both v_b and v_u go to $\left(\frac{c}{r-\mu}\right)$, which leads to the present value of the government claim going to zero, as $v \in (v_b, v_u)$ always holds.

values of ξ) implies a reduction in principal payments (ξF) currently due, and therefore less cash flow that shareholders need to raise to service outstanding debt. Hence, longer maturities are associated with higher equity valuations when inverse leverage is low, which in turn leads to a *lower* optimal default boundary. Figure 9 shows that this indirect channel dominates the direct channel for values of β greater than $\beta > 10^{-2.8}$.

Finally, Figure 10 shows the coupon rate c as a function of β, which is chosen so that the bond is priced at par at the time of issuance. As β is decreased, the optimal maturity $1/\xi^*$ shortens, and the bond becomes less risky. In the limit $\beta \to 0$, the par coupon rate converges to the riskfree rate r, set to 4% in our calibration.

4.3 Tax Benefits to Debt

We estimate tax benefits to debt as the ratio of the levered enterprise value to the value of an all-equity firm at the debt issuance boundary:

$$TB \;=\; \left(\frac{E(v_u F, F) + D(v_u F, F)}{(1-\pi) v_u F}\right) - 1 \;=\; \left\{\frac{e(v_u) + d(v_u)}{(1-\pi) v_u} - 1\right\}. \tag{65}$$

In Figure 11, we report the level of tax benefits in equation (65) as a function of the issuance cost parameter β for three cases: (i) the case with commitment and optimal maturity, (ii) the case without commitment and optimal maturity, and (iii) the case with commitment, but with the restriction $\xi = 0$. Three points are worth noting. First, in agreement with GJL, the tax benefits to debt increase as issuance costs decrease. Second, the tax benefits to debt are amplified considerably when firms have the right to choose optimal debt maturity. Indeed, as β goes to zero, for both the case with and without commitment, the optimal debt and maturity policy allows firms to extract 100% of the claim to EBIT, that is, $e(v_u) + d(v_u) \to v_u$, and tax benefits converge to $TB \to \left(\frac{v_u F}{(1-\pi) v_u F}\right) - 1 \;=\; \left(\frac{\pi}{1-\pi}\right) = 0.25$. Third, in contrast to both DH and DeMarzo (2019), the case with commitment generates *only slightly* higher tax benefits to debt than does the case without commitment. Hence, in the presence of even tiny issuance costs, the question of whether a manager can commit to the global-optimal policy is of secondary importance in terms of available tax benefits to debt when firms can choose their debt maturity optimally.

4.4 A Decomposition of Claims to EBIT

Figures 12 and 13 show the values of all claims to EBIT as a function of the issuance cost parameter β for the cases with and without commitment, respectively. As noted previously, tax benefits net of bankruptcy costs are maximized for debt with zero maturity ($\xi \to \infty$). In contrast, the present value of issuance costs are minimized for debt with infinite maturity ($\xi \to 0$). As $\beta \to 0$, the former channels dominates. As Figures 12 and 13 show, for both cases with and without commitment, the claims to issuance costs, bankruptcy costs, and government all go to zero as β vanishes, in turn allowing the firm to extract 100% of EBIT by following an optimal capital structure and maturity policy.

The fact that the present value of the claim to issuance costs vanishes with β is mostly intuitive, as the cash flows to this claim are linear in β.[15] As noted above, the government claim vanishes as $\beta \to 0$ because both v_b and v_u (and hence, all $v \in (v_b, v_u)$) approach the value $\left(\frac{c}{r-\mu}\right)$ in this limit, which in turn implies zero taxes paid to government. Finally, the claim to bankruptcy costs vanish because, as $\beta \to 0$, the optimal inverse maturity parameter $\xi \to \infty$, implying, via equation (7), that the drift of the stochastic process $(1/dt)\mathrm{E}\left[\frac{dv}{v}\right] = (\mu + \xi)$ goes to infinity and, hence, that the default probability vanishes.

5 The Case of Vanishing Issuance Costs

In Section 3.1.3, we focused on the larger values of β in Figure 2 to motivate our claim for the existence of a critical value β^* at which shareholders become indifferent to issuing additional debt. Here, we focus on the smaller values of β in Figure 2 to motivate our claim that, with commitment, in the limit of vanishing issuance costs (i.e., $\beta \to 0$), there exists a well-defined

[15]Admittedly, the frequency in which inverse leverage reaches the upper boundary becomes arbitrarily large in the $\beta \to 0$ limit. However, we find numerically that the product $(\beta\,\xi^*(\beta))$ converges to zero as β goes to zero.

limit for the policy parameters (v_b, v_u, γ) for any given value of ξ.[16] Indeed, Figures 14, 15, and 16 show optimal values of (v_b, v_u, γ) for extremely small values of β. Clearly, these policy parameters appear to converge to limiting values. In this section, we identify this limit more formally.

To begin, note that the general pricing ODE (equation (18)) and the lower boundary conditions for debt and equity (equations (27) and (30)) are independent of the parameters (v_u, γ). That is, these parameters appear only in the upper boundary conditions (equations (28) and (31)). Here we identify these upper boundary conditions for the case $\beta \to 0$.

Importantly, Figure 16 demonstrates that, as the issuance cost parameter $\beta \to 0$, the optimal debt issuance size parameter $\gamma \to 1$. Therefore, in order to identify the limiting case, we consider a Taylor series expansion around $\epsilon \equiv (\gamma - 1)$. In terms of ϵ, we can express the upper boundary condition for debt in equation (28) as:

$$(1+\epsilon)D(v_u F, F) \;=\; D(v_u F, F + \epsilon F) = D(v_u F, F) + \epsilon F D_F(V, F)\big|_{V=v_u F}. \tag{66}$$

Taking the limit $\epsilon \to 0$, this relation simplifies to

$$D(v_u F, F) \;\overset{\epsilon \to 0}{=}\; F D_F(V, F)\big|_{V=v_u F}. \tag{67}$$

Applying equations (14) and (17) to equation (67), we obtain a boundary condition that has a form typical of problems with a reflecting boundary:

$$d_v(v_u) \;=\; 0. \tag{68}$$

This equation can also be derived by taking a Taylor series expansion of equation (28). Equation (68) can be interpreted as precluding an arbitrage opportunity in that the price of debt per unit face value must be the same just prior to (i.e., at date τ_u^-) and just after (i.e., at date τ_u^+) the debt issuance.

A similar argument holds for the upper boundary condition on equity. Specifically, setting $\beta = 0$ in equation (A.10), and Taylor expanding about $\epsilon \equiv (\gamma - 1)$, this boundary condition for

[16] We restrict our attention to the case with commitment because, for the case with no commitment, as shown in Figure 3, for a fixed value of ξ, as we lower β, we enter a region in which no (s, S) equilibrium exists.

debt reduces to:

$$0 \;=\; \left(E_F(V,F) + D_F(V,F)\right)\Big|_{V=v_u F}. \tag{69}$$

Applying equations (17) and (68) to equation (69), we obtain

$$v_u e_v(v_u) \;=\; e(v_u) + d(v_u). \tag{70}$$

This condition can also be derived from equation (31) by setting $\beta = 0$, and then performing a Taylor series expansion on $\epsilon = (\gamma - 1)$. Moreover, note that, using equations (17) and (68), we can rewrite equation (69) as the first order condition that characterizes the no-commitment equilibrium of DH:

$$-E_F(V,F)\Big|_{V=v_u F} \;=\; d(v_u) \;=\; -\left[e(v_u) - v_u e_v(v_u)\right], \tag{71}$$

where the first equality follows from equation (67) and the last equality follows from equation (70). This expression captures the no-arbitrage condition that the firm keeps issuing debt until, at the margin, the decline in equity value $(-E_F)$ equals the dollar amount raised per unit of face value, $d(v_u)$. Note, however, that this condition holds *only* at the point $v = v_u$ for the case with commitment, whereas it holds for *all* values of v in the no-commitment policy studied by DH.

In Figure 17, we present the value of debt $d(v)$ as a function of inverse leverage v for the case $\beta \to 0$ and, as in DH, $\xi = 0.2$. For comparison, we also plot the value of debt for the no-commitment case of DH. This figure shows that, for all values of v, the value of debt is higher in our model with commitment than in the DH no-commitment equilibrium even in the case of vanishing issuance costs, $\beta \to 0$. This is because our debt policy is more conservative, which leads to lower future default probabilities, and thus to higher debt valuations for all values of v. On this figure, we also plot $-E_F(V,F) = -(e(v) - v e_v(v))$. Under the global-optimal solution, the value of debt is higher than the marginal value of equity for all values of v less than v_u. In contrast, as already mentioned, the condition $(-E_F(V,F) = d(v))$ holds for all values of v in the DH no-commitment equilibrium. A *myopic* manager would interpret $d(v) > -E_F(v)$

as an arbitrage opportunity to issue additional debt. However, such a manager would not be accounting for the fact that any deviation from the optimal policy would have significant impact on the pricing of debt, and on the optimal debt issuance policies for other values of the state variable v. Thus, the apparent arbitrage opportunity is in fact a mirage, and shareholder value is maximized by following the optimal policy that we have identified.

In Figure 18, we show the value of equity $e(v)$ as a function of inverse leverage v and we contrast it to that of the DH no-commitment solution. As previously noted, the value of equity in the case with commitment is higher across all states of nature: $e(v) > e_{DH}(v)$, for all v. Hence, there is no incentive for shareholders to ever deviate from the policy with commitment if the punishment is that, going forward, debt would be priced according to the DH no-commitment policy. Note that, because debtholders observe the size of the debt issuance and pay fair value for their claim, at the date of issuance, they are indifferent to the debt issuance policy chosen by the firm. Therefore, debtholders can credibly threaten to punish any deviation from the optimal policy by pricing debt according to the DH no-commitment equilibrium. As a result, shareholders would gain zero cash benefit at the date of the deviation,[17] and would be left with an equity claim that has lower valuation than under the optimal policy. In this sense, the global-optimal policy is the subgame perfect equilibrium, and the policy that value-maximizing managers will follow.

6 Conclusion

Within a standard tradeoff setting, we investigate the optimal dynamic capital structure and debt maturity policies of firms. We focus on two elements: (i) issuance costs, and (ii) optimal policies with and without commitment. For the case with commitment, we identify the global-optimal policy that maximizes shareholders' value across all policies consistent with limited liability. For the case without commitment, we show that the optimal policy generates almost as much tax benefits to debt as those obtained under the global-optimal policy. For both cases,

[17]Shareholders would obtain zero cash benefit because, as shown by DH, the optimal level of debt issuance at the deviation date is characterized by the locally deterministic process.

a reduction in issuance costs is associated with a decrease in the optimal maturity and an increase in the tax benefits to debt. Indeed, as issuance costs go to zero, the firm can extract 100% of the claim to EBIT. Hence, in the limiting case of vanishing issuance cost, our optimal policy does not converge to the optimal policy of either DeMarzo and He (2019) or DeMarzo (2019), whose models of no commitment predict that, regardless of debt maturity choice, there are no tax benefits to debt available to shareholders.

The predictions of our model are qualitatively consistent with the findings of the empirical capital structure literature. The fact that debt is issued in discrete amounts is a signature characteristic of fixed issuance costs. Our prediction that firms are able to extract tax benefits to debt is consistent with the empirical findings of van Binsbergen, Graham, and Yang (2010), and Blouin, Core, and Guay (2010). That firms are not indifferent toward debt maturity choice is consistent with the findings of Barclay and Smith (1995) and Stohs and Mauer (1996). Consistent with the findings of Titman and Wessels (1988) and Frank and Goyal (2014), our model generates a negative correlation between profitability and leverage, and predicts that leverage ratios are persistent. Intuitively, this is because when firms are in the inaction region, higher (lower) profitability increases equity values while debt outstanding remains constant, leading to lower (higher) leverage. That firms target specific minimum credit rating levels (e.g., Kisgen (2006, 2009)) is consistent with managers caring about the firm's reputation in the debt markets, given that debt issuance is a repeated game. This provides support for our claim that firms can credibly commit to our global optimal policy. Finally, when calibrated to estimated issuance costs, our model generates *quantitatively* accurate predictions for optimal debt maturity.

We acknowledge that our analysis abstracts from other important mechanisms that would allow our framework to better match observed debt issuance dynamics. Such mechanisms include investment opportunities, jumps in state vector dynamics, time-variation in debt market liquidity, asymmetric information between managers and debtholders, and other market imperfections. We leave the study of these important issues to future research.

A Proof of Proposition 1

Debt valuation. Expressed in terms of risk-neutral expected cash flows, the debt value is:

$$\begin{aligned} D(V_t, F_t) &= \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^{\tau} ds\, e^{-r(s-t)}\,(c+\xi)F_s\right] \\ &\quad + \mathbb{E}_t^{\mathbb{Q}}\left[e^{-r(\tau_u - t)}\,\mathbf{1}_{(\tau_u<\tau_b)}\,\left(\frac{1}{\gamma}\right) D(v_u F_{\tau_u}, \gamma F_{\tau_u})\right] \\ &\quad + \mathbb{E}_t^{\mathbb{Q}}\left[e^{-r(\tau_b - t)}\,\mathbf{1}_{(\tau_b<\tau_u)}\,(1-\alpha)(1-\pi)v_b F_{\tau_b}\right]. \end{aligned} \tag{A.1}$$

The first term captures the present value of cash flows prior to the firm reaching either the upper or lower boundary. The second term captures the fact that when the upper boundary is reached and the face value of debt jumps from F_{τ_u} to γF_{τ_u}, old bondholders have a claim to $\left(\frac{1}{\gamma}\right)$ of the total debt claim, as it is assumed that all debt is issued pari-passu. The third term captures recovery conditional upon default.

Comparing equation (10) and equation (A.1), the debt claim is described by the factor loadings:

$$h_F = (c+\xi) \tag{A.2}$$

$$h_V = 0 \tag{A.3}$$

$$H_b = (1-\alpha)(1-\pi)v_b F_{\tau_b} \tag{A.4}$$

$$H_u = \left(\frac{1}{\gamma}\right) D(v_u F_{\tau_u}, \gamma F_{\tau_u}). \tag{A.5}$$

It therefore follows from equation (21) that the bond price can be expressed as:

$$d(v) = M_d v^{\phi} + N_d v^{\omega} + \frac{c+\xi}{r+\xi}, \tag{A.6}$$

where the constants (M_d, N_d) are uniquely determined by the boundary conditions

$$d(v_b) = (1-\alpha)(1-\pi)v_b \tag{A.7}$$

$$d(v_u) = d(v_u/\gamma), \tag{A.8}$$

Equity valuation. The value of equity can be determined via the risk-neutral expectation:

$$
\begin{aligned}
E(V_t, F_t) &= \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^{\tau} ds\, e^{-r(s-t)} \left((1-\pi)Y_s - (c(1-\pi)+\xi)\, F_s\right)\right] \qquad \text{(A.9)}\\
&+\mathbb{E}_t^{\mathbb{Q}}\left[e^{-r(\tau_u-t)}\, \mathbf{1}_{(\tau_u<\tau_b)} \left(E(v_u F_{\tau_u}, \gamma F_{\tau_u}) + D(v_u F_{\tau_u}, \gamma F_{\tau_u}) - D(v_u F_{\tau_u}, F_{\tau_u}) - \beta v_u F_{\tau_u}\right)\right].
\end{aligned}
$$

The first term captures the present value of the claim to cash flows (i.e., dividends) throughout period-0, which ends at date $\tau = \min(\tau_b, \tau_u)$. The second term captures the fact that when the upper boundary is reached, shareholders still hold the entire equity claim, but now with the face value of debt scaled by a factor of γ. In addition, shareholders receive as dividend the value of the new debt issuance (net of issuance costs).

Comparing equation (10) and equation (A.9), the equity claim is described by the factor loadings:

$$
\begin{aligned}
h_F &= -(c(1-\pi)+\xi)\\
h_V &= (1-\pi)(r-\mu)\\
H_b &= 0\\
H_u &= E(v_u F_{\tau_u}, \gamma F_{\tau_u}) + D(v_u F_{\tau_u}, \gamma F_{\tau_u}) - D(v_u F_{\tau_u}, F_{\tau_u}) - \beta v_u F_{\tau_u}. \qquad \text{(A.10)}
\end{aligned}
$$

It therefore follows from equation (21) that the value of equity per unit face value can be expressed as:

$$
e(v) = M_e v^{\phi} + N_e v^{\omega} + (1-\pi)v - \frac{c(1-\pi)+\xi}{r+\xi}, \qquad \text{(A.11)}
$$

where the constants (M_e, N_e) are uniquely determined by the boundary conditions

$$
\begin{aligned}
e(v_b) &= 0 \qquad \text{(A.12)}\\
e(v_u) &= \gamma e(v_u/\gamma) + (\gamma-1)d(v_u) - \beta v_u, \qquad \text{(A.13)}
\end{aligned}
$$

Government Claim. The value of the government claim can be determined via the risk-neutral expectation:

$$G(V_t, F_t) = \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^{\tau} ds\, e^{-r(s-t)}\, \pi\,(Y_s - cF_s)\right] \tag{A.14}$$

$$+\mathbb{E}_t^{\mathbb{Q}}\left[e^{-r(\tau_u-t)}\, \mathbf{1}_{(\tau_u<\tau_b)}\; G(v_u F_{\tau_u}, \gamma F_{\tau_u})\right] + \mathbb{E}_t^{\mathbb{Q}}\left[e^{-r(\tau_b-t)}\, \mathbf{1}_{(\tau_b<\tau_u)}\; (1-\alpha)\pi v_b F_{\tau_b}\right].$$

The first term captures the present value of the claim to cash flows (i.e., taxes) throughout period-0, which ends at date $\tau = \min(\tau_b, \tau_u)$. The second term captures the fact that when the upper boundary is reached, the government still holds the entire tax claim, but now with the face value of debt scaled by a factor of γ. The third term captures recovery conditional upon default.

Comparing equation (10) and equation (A.14), the government claim is described by the factor loadings:

$$\begin{aligned} h_F &= -c\pi \\ h_V &= \pi(r-\mu) \\ H_b &= (1-\alpha)\pi v_b F_{\tau_b} \\ H_u &= G(v_u F_{\tau_u}, \gamma F_{\tau_u}). \end{aligned} \tag{A.15}$$

It therefore follows from equation (21) that the value of the government claim per unit face value can be expressed as:

$$g(v) = M_g v^{\phi} + N_g v^{\omega} + \pi v - \frac{c\pi}{r+\xi}, \tag{A.16}$$

where the constants (M_g, N_g) are uniquely determined by the boundary conditions

$$g(v_b) = (1-\alpha)\pi v_b \tag{A.17}$$

$$g(v_u) = \gamma g(v_u/\gamma). \tag{A.18}$$

Claim to Bankruptcy Costs. The value of the claim to bankruptcy costs can be determined via the risk-neutral expectation:

$$B(V_t, F_t) = \mathbb{E}_t^{\mathbb{Q}}\left[e^{-r(\tau_b-t)}\, \mathbf{1}_{(\tau_b<\tau_u)}\; \alpha v_b F_{\tau_b}\right] + \mathbb{E}_t^{\mathbb{Q}}\left[e^{-r(\tau_u-t)}\, \mathbf{1}_{(\tau_u<\tau_b)}\; B(v_u F_{\tau_b}, \gamma F_{\tau_b})\right], \tag{A.19}$$

which can be interpreted as the risk-neutral expected cash flows going to the bankruptcy cost claim conditional upon default.

Comparing equation (10) and equation (A.19), the claim to bankruptcy costs is described by the factor loadings:

$$\begin{aligned} h_F &= 0 \\ h_V &= 0 \\ H_b &= \alpha v_b F_{\tau_b} \\ H_u &= B(v_u F_{\tau_u}, \gamma F_{\tau_u}). \end{aligned} \tag{A.20}$$

It therefore follows from equation (21) that the value of the claim to bankruptcy costs per unit face value can be expressed as:

$$b(v) = M_b v^{\phi} + N_b v^{\omega}, \tag{A.21}$$

where the constants (M_b, N_b) are uniquely determined by the boundary conditions

$$b(v_b) = \alpha v_b \tag{A.22}$$

$$b(v_u) = \gamma b(v_u/\gamma)\,. \tag{A.23}$$

Claim to Issuance Costs. The value of the claim to issuance costs can be determined via the risk-neutral expectation:

$$I(V_t, F_t) = \mathbb{E}_t^{\mathbb{Q}}\left[e^{-r(\tau_u - t)}\,\mathbf{1}_{(\tau_u < \tau_b)}\ \left(\beta v_u F_{\tau_u} + I(v_u F_{\tau_u}, \gamma F_{\tau_u})\right)\right]. \tag{A.24}$$

This equation states that when the upper boundary is reached, the claim to issuance cost receives the cash flow $\beta v_u F_{\tau_u}$ in addition to a claim to future issuance costs after the debt issuance, which are captured recursively via the term $I(v_u F_{\tau_u}, \gamma F_{\tau_u})$.

Comparing equation (10) and equation (A.24), the claim to issuance costs is described by the factor loadings:

$$\begin{aligned} h_F &= 0 \\ h_V &= 0 \\ H_b &= 0 \\ H_u &= \beta v_u F_{\tau_u} + I(v_u F_{\tau_u}, \gamma F_{\tau_u}). \end{aligned} \tag{A.25}$$

It therefore follows from equation (21) that the value of the claim to issuance costs per unit face value can be expressed as:

$$i(v) = M_i v^{\phi} + N_i v^{\omega}, \tag{A.26}$$

where the constants (M_i, N_i) are uniquely determined by the boundary conditions

$$i(v_b) = 0 \tag{A.27}$$

$$i(v_u) = \beta v_u + \gamma\, i(v_u/\gamma)\,. \tag{A.28}$$

∎

B Global Optimality of the Commitment Policy in Definition 1

In this section we verify the global optimality of the policy (v_b^*, v_u^*, γ^*) for any given inverse maturity parameter ξ. We proceed in two steps. First, we show that, for any given v_b, the FOC's (42)–(43) determine the same optimal values for (v_u, γ) regardless of whether they are evaluated at v_u or at any value $v_t \in (v_b, v_u)$. Second, we show numerically that v_b^* is globally optimal under limited liability.

Step 1: For any given v_b, the lower boundary condition (30) implies:

$$N_e = \left[\frac{c\,(1-\pi)+\xi}{r+\xi} - (1-\pi)\,v_b - M_e v_b^{\phi}\right] v_b^{-\omega}. \tag{B.1}$$

Therefore, we can express the equity value in equation (29) as

$$e\left(v, M_e, v_b\right) \;=\; M_e v^{\phi}\left[1-\left(\frac{v}{v_b}\right)^{\omega-\phi}\right]+(1-\pi)\left[v-v_b\left(\frac{v}{v_b}\right)^{\omega}\right]$$
$$-\left(\frac{c\,(1-\pi)+\xi}{r+\xi}\right)\left[1-\left(\frac{v}{v_b}\right)^{\omega}\right]. \tag{B.2}$$

Note that the coefficient M_e in equation (B.2) is uniquely determined by the policy parameters (v_b, v_u, γ). Further note that, because (v_u, γ) appear in equation (B.2) only through M_e, it follows that if one could choose these parameters at any value of $v = v_{any} \in (v_b, v_u)$, one would solve the FOC:

$$0 \;=\; \left.\frac{\partial e}{\partial v_u}\right|_{v=v_{any}} = v_{any}^{\phi}\left[1-\left(\frac{v_{any}}{v_b}\right)^{\omega-\phi}\right]\frac{\partial M_e}{\partial v_u}. \tag{B.3}$$

$$0 \;=\; \left.\frac{\partial e}{\partial \gamma}\right|_{v=v_{any}} = v_{any}^{\phi}\left[1-\left(\frac{v_{any}}{v_b}\right)^{\omega-\phi}\right]\frac{\partial M_e}{\partial \gamma}. \tag{B.4}$$

Because $v_{any}^{\phi}\left[1-\left(\frac{v_{any}}{v_b}\right)^{\omega-\phi}\right] \neq 0, \forall v_{any} \in (v_b, v_u)$, it follows that these FOC's reduce to

$$\frac{\partial M_e}{\partial v_u} = 0 \;\text{ and }\; \frac{\partial M_e}{\partial \gamma} = 0. \tag{B.5}$$

This proves that the optimal policy parameters are independent of the current value of $v = v_{any}$.

Step 2: In the previous step, we demonstrated that the optimal values of $(v_u^*(v_b), \gamma^*(v_b))$ are independent of v, and functions only of v_b. As such, it is convenient to define

$$\widehat{M}_e(v_b) \;\equiv\; M_e(v_b, v_u^*(v_b), \gamma^*(v_b))$$
$$\widehat{e}(v, v_b) \;\equiv\; e\left(v, \widehat{M}_e(v_b), v_b\right). \tag{B.6}$$

Using the chain rule of calculus, we can write

$$\left.\frac{\partial \widehat{e}(v, v_b)}{\partial v_b}\right|_v \;\equiv\; \frac{\partial e\left(v, \widehat{M}_e(v_b), v_b\right)}{\partial \widehat{M}_e}\frac{\partial \widehat{M}_e}{\partial v_b}+\frac{\partial e\left(v, \widehat{M}_e, v_b\right)}{\partial v_b} \tag{B.7}$$

Now, because:[18]

$$\frac{\partial e\left(v, \widehat{M}_e(v_b), v_b\right)}{\partial \widehat{M}_e} = v^{\omega}\left[v^{\phi-\omega} - v_b^{\phi-\omega}\right] > 0, \tag{B.8}$$

$$\frac{\partial e\left(v, \widehat{M}_e, v_b\right)}{\partial v_b} = -\left(\frac{v}{v_b}\right)^{\omega}\left[(1-\bar{\pi})(1-\omega) + M_e(\phi-\omega)\, v_b^{\phi-1}\right] < 0, \tag{B.9}$$

it follows that, if $\frac{\partial \widehat{M}_e}{\partial v_b} < 0$, then $\left.\frac{\partial \widehat{e}(v, v_b)}{\partial v_b}\right|_v < 0$. This in turn would imply that the equity claim is maximized by choosing the smallest value of v_b consistent with limited liability, which is the value of v_b consistent with the FOC in equation (41). Hence, our problem is reduced to showing that $\frac{\partial \widehat{M}_e}{\partial v_b} < 0$ holds in our economy. Although the term M_e has a closed-form analytical expression in terms of the debt policy parameters (v_b, v_u, γ), it does not lend itself to an easy interpretation. Therefore, we resort to a numerical argument for the rest of this analysis, using the model coefficients in Table 1.

In Figure B.1 we plot $M_e\left(v_b, v_u^*(v_b), \gamma^*(v_b)\right)$ as a function of v_b, where $v_u^*(v_b)$ and $\gamma^*(v_b)$ are given by the FOC's (42)–(43). The plot shows that $M_e\left(v_b, v_u^*(v_b), \gamma^*(v_b)\right)$ is indeed decreasing in v_b. The black dot on the line marks the *highest* value of M_e for which the equity value is consistent with limited liability for all $v > v_b$. Furthermore, Figure B.2 reports the derivative of the equity value, $\left.\frac{\partial e}{\partial v}\right|_{v=v_b}$, as a function of v_b. As this figure shows, the value of v_b that gives the highest value of M_e consistent with limited liability satisfies the smooth pasting condition

$$\left.\frac{\partial e}{\partial v}\right|_{v=v_b} = 0. \tag{B.10}$$

Moreover the value v_b^* that satisfies condition (B.10) is the same as that obtained from the FOC's (41)–(43). Therefore, it follows that our policy generates the highest values of equity for $v_t > v_b^*$, since all other solutions for equity that satisfy limited liability will have an equity

[18] The proof of $\widehat{M}_e(v_b) \geq 0$ is available upon request.

valuation of zero at v_b^*. We conclude that the debt issuance policy (v_b^*, v_u^*, γ^*) that satisfies the FOC's (41)–(43) generates an equity valuation that is globally optimal.

C Optimal Policy for $v_t > v_u$

In the main text we identify optimal capital structure strategies characterized by an (s, S) policy with a single inaction region (v_b, v_u). In this Appendix, we provide numerical support for the claim that if a firm finds itself with an inverse leverage ratio $v_{(t-)} > v_u$, then it is optimal for that firm to immediately issue new debt so that its inverse leverage jumps into the region $v_{(t+)} \in (v_b, v_u)$.[19] Note that the condition $v_{(t-)} > v_u$ implies that the firm's inverse leverage is even higher than that value v_u for which the firm finds it optimal to pay the debt issuance cost in order to increase their tax benefits. Thus, it seems intuitive that if inverse leverage is higher than v_u, then the firm has even more incentive to issue debt immediately.

First, we identify the value of equity based on our proposed optimal policy. Consider a firm in a state (V_t, F_t) such that $(V_t/F_t) > v_u$. Under the proposed policy, the face value of debt will jump from F_t to $\hat{F}_t = (V_t/\hat{v})$, so that inverse leverage jumps from $v_{(t-)} = (V/F)$ to $v_{(t+)} = \hat{v}\left(v_{(t-)}\right)$, which is a function of $v_{(t-)}$. Under this policy, we have the relations:

$$\begin{aligned} \frac{D(V,F)}{F} &= d(\hat{v}) \\ E(V,F) &= \max_{\hat{F}} \left[E\left(V,\hat{F}\right) + \left(\hat{F} - F\right)\frac{D(V,\hat{F})}{\hat{F}} - \beta V \right]. \end{aligned} \tag{C.1}$$

The first equation captures a no-arbitrage condition in that the price per unit face value of debt just prior to the issuance must equal the price just after the issuance. The second equation also captures a no-arbitrage condition in that the value of the equity in state (V, F) should equal the sum of: (a) the value of equity in state $(V, \hat{F})$, plus (b) proceeds raised from the debt

[19] We thank Peter DeMarzo, Zhiguo He, and Fabrice Tourre for suggesting that we pursue this analysis.

issuance, minus (c) issuance costs. Proceeds raised from debt issuance equal the product of (i) the quantity $(\hat{F} - F)$ of debt issued, multiplied by (ii) the price per unit of debt—the latter is equal to $D(V, \hat{F})/\hat{F}$.

Looking for solutions of the form:

$$D(V, F) = Fd(v = (V/F)) \tag{C.2}$$

$$E(V, F) = Fe(v = (V/F)), \tag{C.3}$$

we can express these relations as:

$$d(v) = d(\hat{v}) \tag{C.4}$$

$$e(v) = \max_{\hat{v}} \left\{ (v/\hat{v})\, e(\hat{v}) + (v/\hat{v})\, d(\hat{v}) - d(\hat{v}) - \beta v \right\}. \tag{C.5}$$

The optimal $\hat{v}$ is determined by the following first order condition of equation (C.5):

$$\frac{\hat{v}e'(\hat{v}) - e(\hat{v}) + \hat{v}d'(\hat{v}) - d(\hat{v})}{\hat{v}^2} = \frac{d'(\hat{v})}{v}, \tag{C.6}$$

which implies the optimal $\hat{v}$ depends on v. We find that $\hat{v}(v) \in (v_b, v_u)$ for any $v > v_u$ under all β such that an (s, S) policy exists. Equation (C.5) gives the value of scaled equity for values of $v > v_u$ under our proposed optimal policy.

Next, we provide numerical support for our claim that this proposed strategy is indeed optimal. We do this two ways. First, we examine multiple strategies in which we posit additional inaction regions $(\underline{v}, \overline{v})$ for which, if the current value of firm's inverse leverage falls into the range $v_t \in (\underline{v}, \overline{v})$, no debt is issued. When reaching either boundary $(\underline{v}, \overline{v})$, we consider both possibilities that when these boundaries are reached, the firm issues sufficient debt so that inverse leverage immediately jumps into either: i) the region $(\underline{v}, \overline{v})$, or ii) into the region (v_b, v_u). Given any explicit strategy, we then determine the value of equity following such a strategy using the same types of calculations used above. In spite of investigating millions of different parameter choices for $(\underline{v}, \overline{v})$, we never identified a single example in which our proposed optimal policy did not generate a higher equity valuation.

In addition to investigating millions of specific cases, here we investigate a more general approach that provides additional evidence that, when the firm's inverse leverage is above the restructuring boundary ($v_{(t^-)} \geq v_u$), the optimal policy is to issue debt immediately so that the new inverse leverage jumps into the region $v_{(t^+)} \in (v_b, v_u)$. Specifically, we first identify the proposed optimal policy as described in equation (C.1). We then compare this equity value to that obtained if the manager waits a period dt, in turn receiving the dividend accrued over that period, and then restructures a period dt from now. The value of this alternative strategy is:

$$\begin{aligned} &\left[Y\left(1-\pi\right)-F\left(c(1-\pi)+\xi\right)\right]\,dt+(1-r\,dt)\mathrm{E}^{Q}\left[E(V+dV,F+dF)\right] \\ &\quad= \left(Y\left(1-\pi\right)-F\left(c(1-\pi)+\xi\right)\right)\,dt \\ &\qquad+(1-r\,dt)\left[E(V,F)+\mu V E_{V}\,dt+\frac{\sigma^2}{2}V^2E_{VV}\,dt-\xi F E_{F}\,dt\right]. \end{aligned} \tag{C.7}$$

Note that $E(V+dV,F+dF)$ is the value of equity provided that the firm restructures immediately at time $t+dt$.

Subtracting $E(V,F)$ from both sides, we study whether:

$$0 \overset{?}{>} \left(Y\left(1-\pi\right)-F\left(c(1-\pi)+\xi\right)\right)-rE(V,F)+\mu V E_{V}+\frac{\sigma^2}{2}V^2E_{VV}-\xi F E_{F}. \tag{C.8}$$

Basically, we are investigating whether or not we have identified a supermartingale, which would imply that issuing debt immediately dominates any strategy that involves waiting.

Using our scaling argument, this simplifies to

$$0 \overset{?}{>} v(r-\mu)(1-\pi)-c(1-\pi)-\xi-re+\mu v e_{v}+\frac{\sigma^2}{2}v^2e_{vv}-\xi(e-ve_{v}). \tag{C.9}$$

Numerically, after we set the grid on $\{v_i\}_{i=1}^{I} \in (v_u,\infty)$, we approximate the first and second order derivatives via the central differences:

$$e_v\left(v_i\right)\approx\frac{e\left(v_{i+1}\right)-e\left(v_{i-1}\right)}{2\triangle v};\; e_{vv}\left(v_i\right)\approx\frac{e\left(v_{i+1}\right)-2e\left(v_i\right)+e\left(v_{i-1}\right)}{\left(\triangle v\right)^2}$$

where $\triangle v = v_i - v_{i-1}$.

In addition to equation (C.9), to satisfy the verification theorem of the HJB equation, we check whether the following equation holds for all V:

$$0 = \max \left\{ \begin{array}{c} -E\left(V,F\right);\; \mathcal{M}E\left(V,F\right) - E\left(V,F\right); \\ \\ -rE\left(V,F\right) + Y\left(1-\pi\right) - F\left(c\left(1-\pi\right)+\xi\right) + \mathcal{L}E\left(V,F\right) \end{array} \right\}, \tag{C.10}$$

where $\mathcal{M}E\left(V,F\right)$ is defined in equation (C.1) and

$$\mathcal{L}E\left(V,F\right) = \mu V E_V\left(V,F\right) + \frac{\sigma^2}{2}V^2 E_{VV}\left(V,F\right) - \xi F E_F\left(V,F\right).$$

Dividing both sides of equation (C.10) by F we find,

$$0 = \max\left\{ \underbrace{-e\left(v\right)}_{\text{term } A};\; \underbrace{\mathcal{M}e\left(v\right) - e\left(v\right)}_{\text{term } B};\; \underbrace{-re\left(v\right) + v\left(r-\mu\right)\left(1-\pi\right) - \left(c\left(1-\pi\right)+\xi\right) + \mathcal{L}e\left(v\right)}_{\text{term } C} \right\} \tag{C.11}$$

where

$$\mathcal{L}e\left(v\right) = \mu v e_v\left(v\right) + \frac{\sigma^2}{2}v^2 e_{vv}\left(v\right) - \xi\left(e\left(v\right) - v_u e_v\left(v\right)\right).$$

Under both commitment and no-commitment, we verify numerically that equation (C.11) holds for all v and β for which an (s, S) policy exists.

Figure B.3 shows the value of all terms in equation (C.11) as a function of v for the economy without commitment. We interpret this result to imply that issuing debt immediately dominates any strategy that involves waiting, regardless of whether that waiting strategy involves waiting a particular amount of time, or whether it involves waiting to reach some pre-specified barrier before issuing debt.

Parameter	Symbol	Value
Annual risk-free rate	r	0.04
Annual asset drift	μ	0
Annual asset volatility	σ	0.22
Corporate tax rate	π	0.2
Loss given default	α	1

Table 1: **Baseline Model Coefficients.** The table shows the values of the model coefficients in the baseline calibration. We choose the coupon rate c so that the bond is priced at par at the time of issuance.

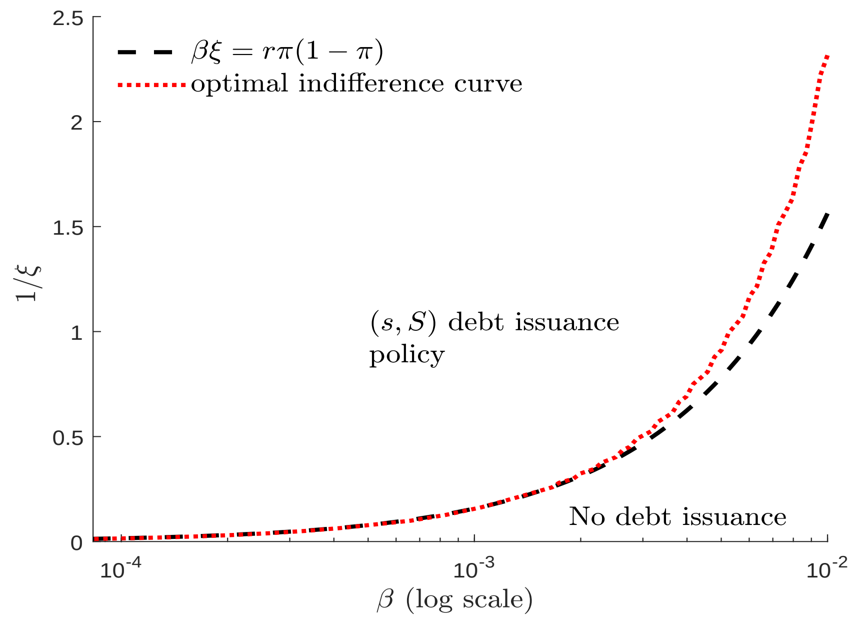

Figure 1: **Debt issuance with commitment.** For the case with commitment, this figure identifies the region in which it is optimal to issue additional debt according to the (s, S) policy with parameters (v_b, v_u, γ). The dashed line shows the approximate theoretical location of the debt issuance indifference curve, $\beta\xi = r\pi(1-\pi)$, that separates the (s, S) from the no-debt issuance regions.

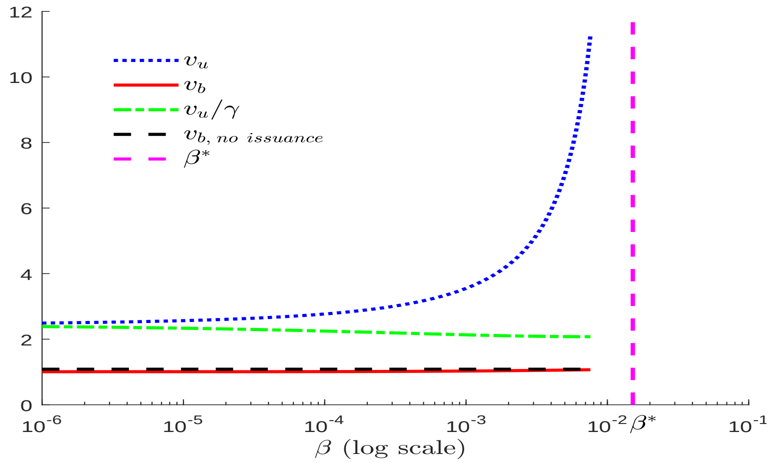

Figure 2: **Optimal policy parameters with commitment: Fixed maturity.** For $\xi = 0.2$, this figure shows the parameter values $(v_u, v_u/\gamma, v_b)$ as a function of the issuance cost parameter β. It also shows the default boundary $v_{\text{b,no issuance}}$ for a firm that is restricted from issuing debt in the future (equation (C.6) in the online Appendix). The vertical dashed line at β^* identifies the value of β for which optimal v_u goes to infinity. For this β^* value, there are no tax benefits to debt, and therefore no additional debt is issued.

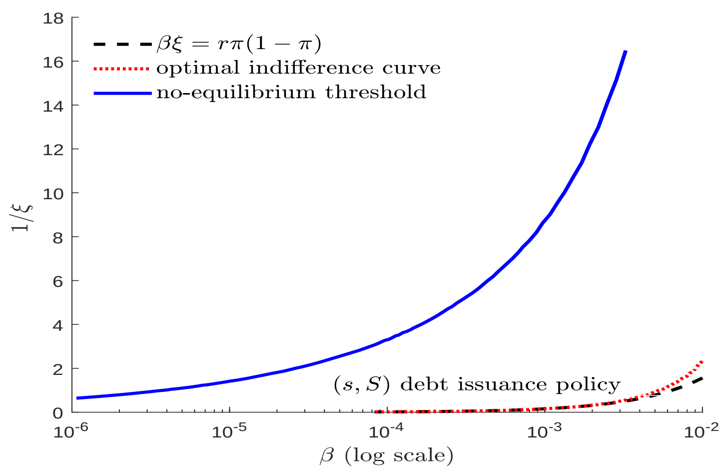

Figure 3: **Debt issuance without commitment.** For the case without commitment, this figure identifies three regions. The region that falls below the red dotted line refers to the case in which $v_u = \infty$, and thus it is never optimal to issue additional debt, as in Leland (1994). The region between the red dotted line and the blue solid line is characterized by an optimal (s, S) policy with parameters $(v_{b,0}, v_{u,0}, \gamma_0)$. Points in the region above the solid blue line do not possess a pure (s, S) strategy equilibrium. The dashed line shows the approximate theoretical location of the debt issuance indifference curve, $\beta\xi = r\pi(1-\pi)$, that separates the (s, S) region from the no-debt issuance region.

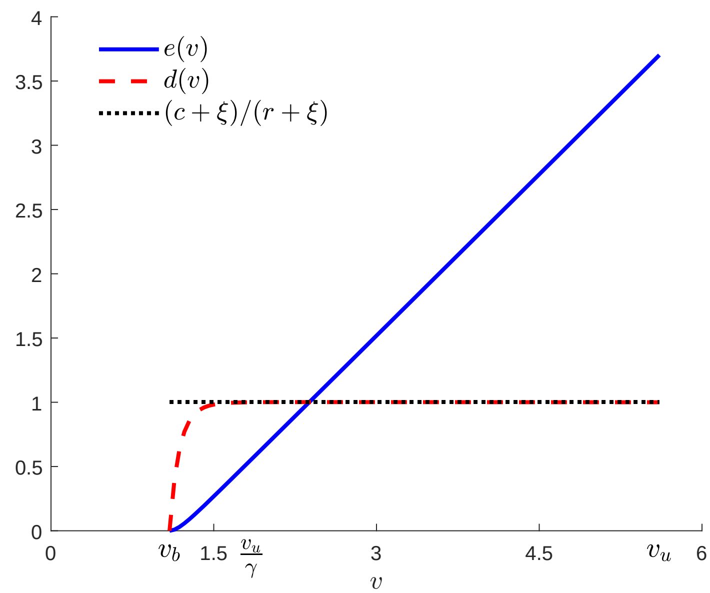

Figure 4: **Debt and equity values with commitment.** This figure shows debt and equity values for the case with commitment, equations (A.6) and (A.11), respectively, when $\beta = 0.0036$. This parameter value reflects a fractional issuance cost of 1%. The other parameter values are reported in Table 1.

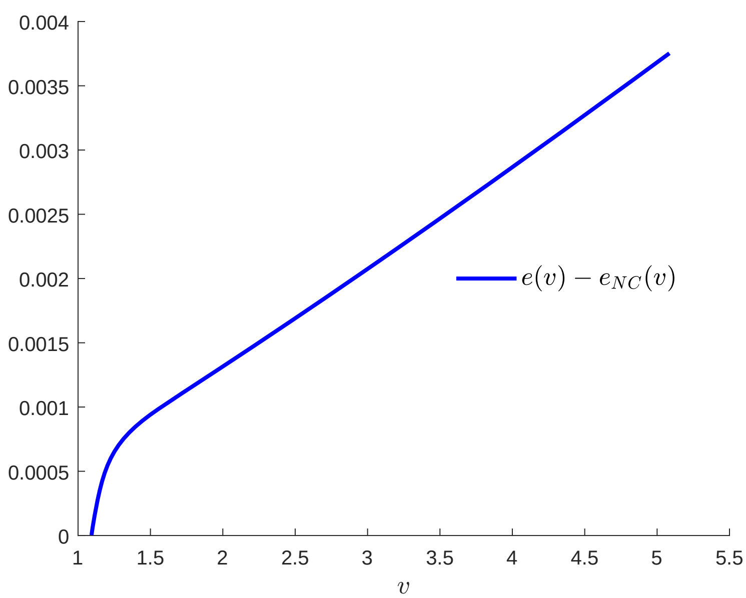

Figure 5: **Difference between equity values under commitment and no commitment.** This figure shows the difference in equity values for the cases with and without commitment, $(e(v) - e_{NC}(v))$, when $\beta = 0.0036$. This parameter value reflects a fractional issuance cost of 1% for the case with commitment. When calculating the difference $e(v) - e_{NC}(v)$, we use the coupon rate c and inverse maturity ξ from the optimal policy with commitment. The other parameter values are reported in Table 1.

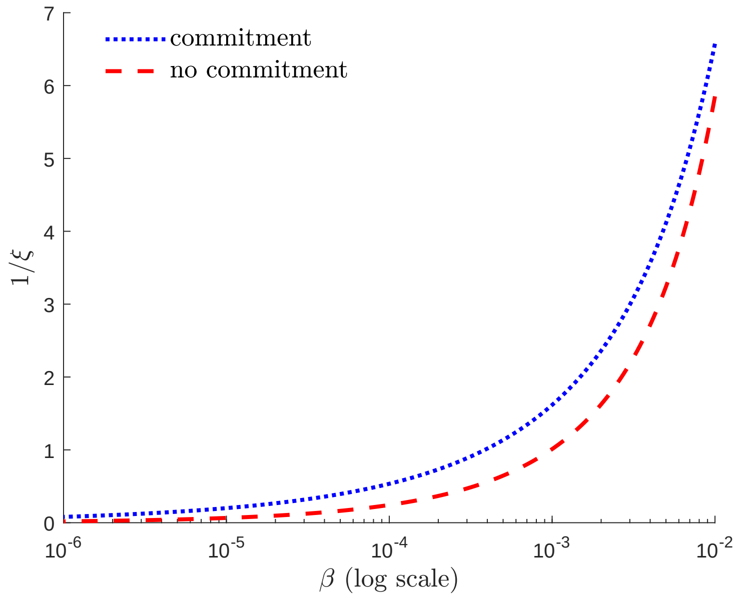

Figure 6: **Optimal maturity.** This figure shows the optimal debt maturity for the cases with and without commitment as a function of β. The other parameter values are in Table 1.

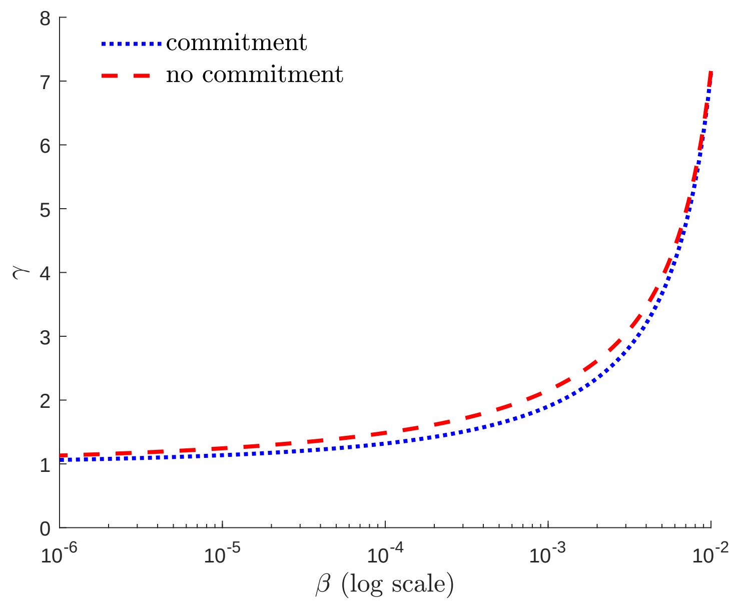

Figure 7: **Optimal debt issuance size.** This figure shows the optimal debt issuance parameter γ for the cases with and without commitment as a function of β. The other parameter values are in Table 1.

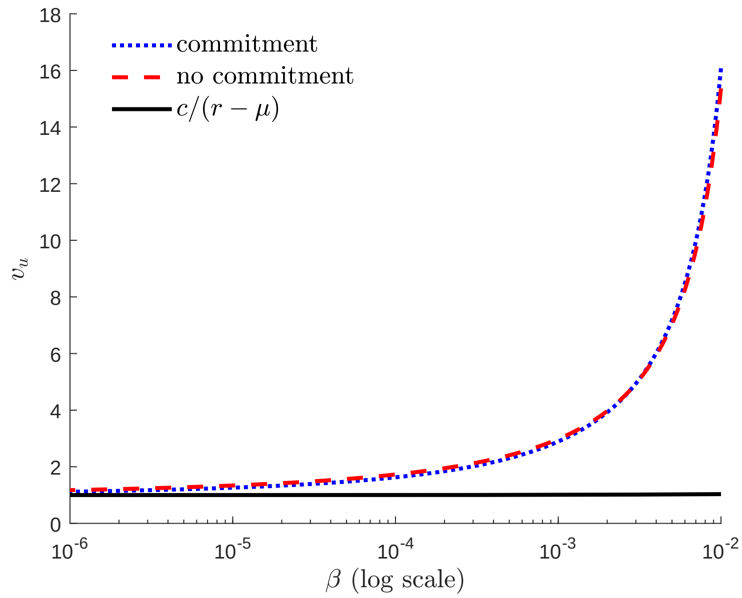

Figure 8: **Optimal debt issuance boundary.** This figure shows the location of the optimal debt issuance boundary v_u for the cases with and without commitment as a function of β. We also plot the function $c/(r-\mu)$ for the case with commitment. The coupon c is chosen so that debt is issued at par. As can be seen in Figure 10, the function $c/(r-\mu)$ for the no commitment case is very similar to that for the case with commitment, and therefore we omit it for clarity. The other parameter values are reported in Table 1.

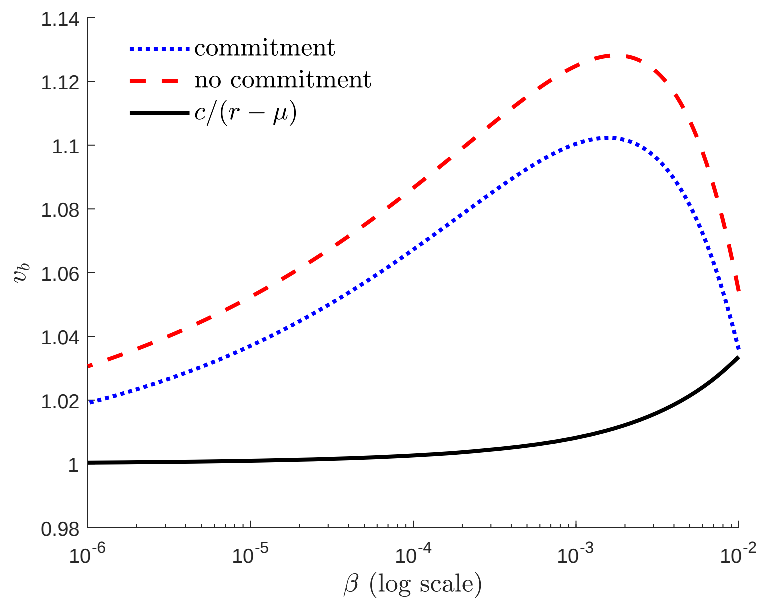

Figure 9: **Optimal default boundary.** This figure shows the location of the optimal default boundary v_b for the cases with and without commitment as a function of β. It also reports the function $c/(r-\mu)$ for the case with commitment. The coupon c is chosen so that debt is issued at par. As can be seen in Figure 10, the function $c/(r-\mu)$ for the no commitment case is very similar to that for the case with commitment, and therefore we omit it for clarity. The other parameter values are reported in Table 1.

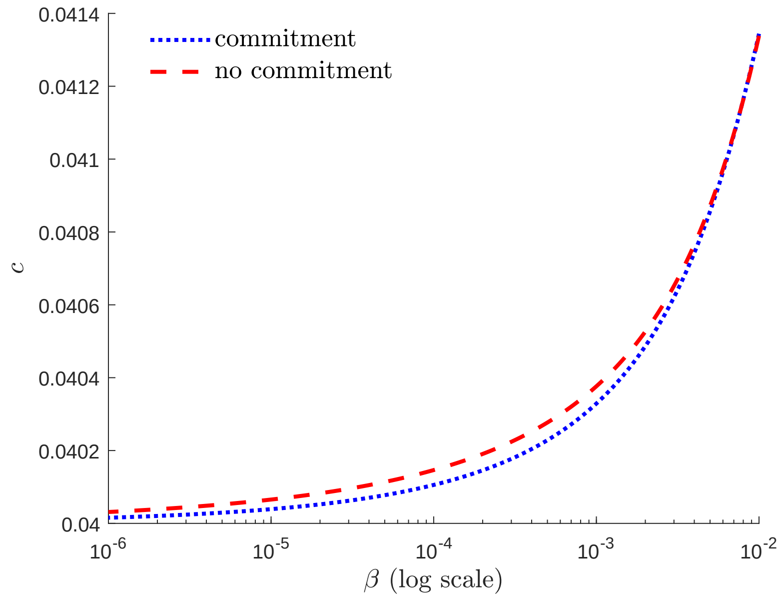

Figure 10: **Coupon rate.** This figure shows the coupon rate c such that the bond is priced at par at the time of issuance. The other parameter values are in Table 1.

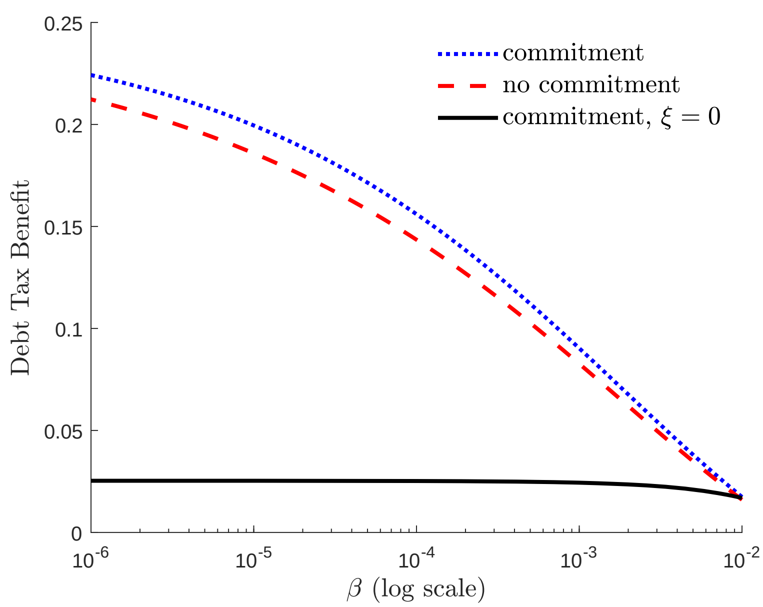

Figure 11: **Tax benefits to debt.** This figure reports the tax benefits to debt under the optimal policy parameters for the cases with and without commitment as a function of β. It also reports the tax benefits to debt for the case with commitment when the inverse maturity parameter is set to $\xi = 0$. The other parameter values are in Table 1.

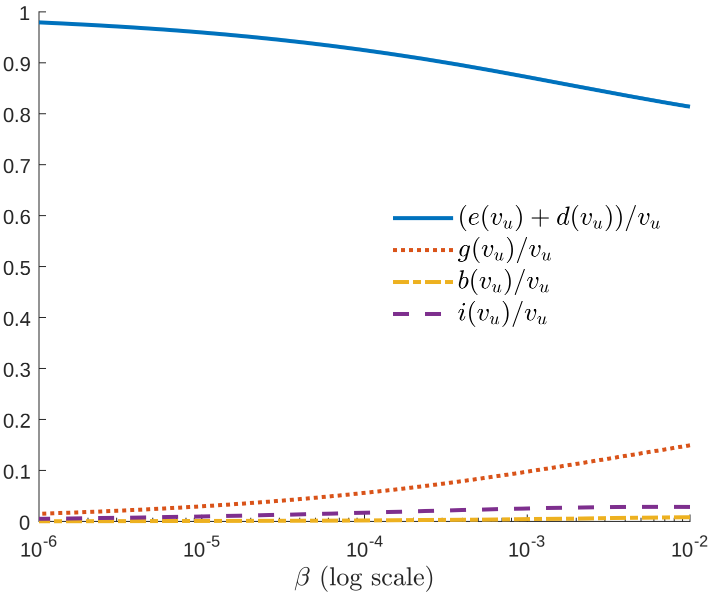

Figure 12: **Claim values with commitment.** For the model with commitment, this figure reports the value of the sum of debt and equity $(d(v_u)+e(v_u))$, bankruptcy costs claim $b(v_u)$, government claim $g(v_u)$, and issuance cost claim $i(v_u)$ at the debt issuance boundary v_u as a function of the debt issuance cost parameter β. The other parameter values are in Table 1.

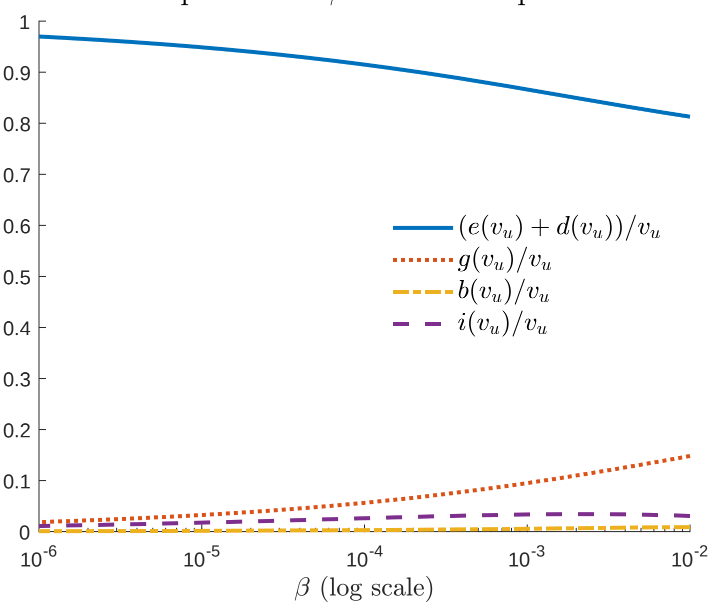

Figure 13: **Claim values without commitment.** For the model without commitment, this figure reports the value of the sum of debt and equity $(d(v_u)+e(v_u))$, bankruptcy costs claim $b(v_u)$, government claim $g(v_u)$, and issuance cost claim $i(v_u)$ at the debt issuance boundary v_u as a function of the debt issuance cost parameter β. The other parameter values are in Table 1.

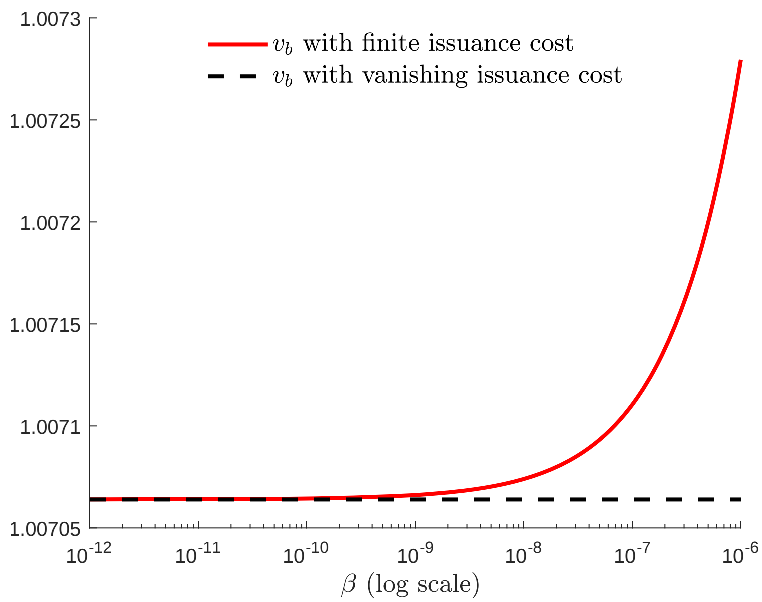

Figure 14: **Optimal lower boundary for fixed maturity.** This figure reports the optimal policy parameter v_b for $\beta < 10^{-6}$. The horizontal line is the optimal value of v_b calculated for the limiting case $\beta \to 0$ determined from equations in Section 5.

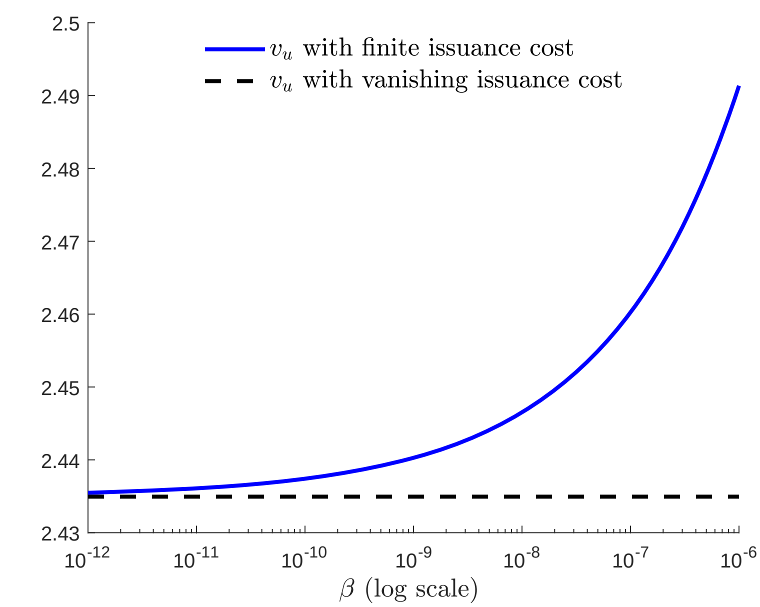

Figure 15: **Optimal upper boundary for fixed maturity.** This figure reports the optimal policy parameter v_u for $\beta < 10^{-6}$. The horizontal line is the optimal value of v_u calculated for the limiting case $\beta \to 0$ determined from equations in Section 5.

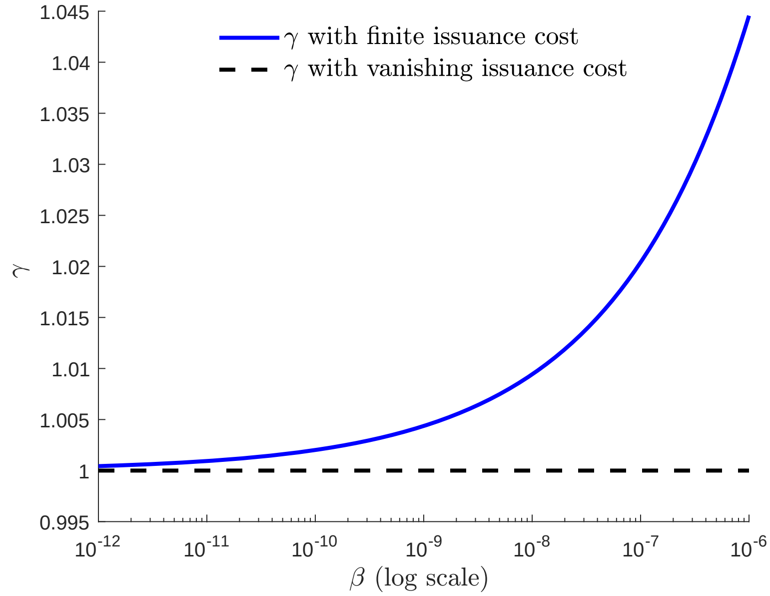

Figure 16: **Optimal debt issuance size for fixed maturity.** This figure reports the optimal policy parameter γ for $\beta < 10^{-6}$.

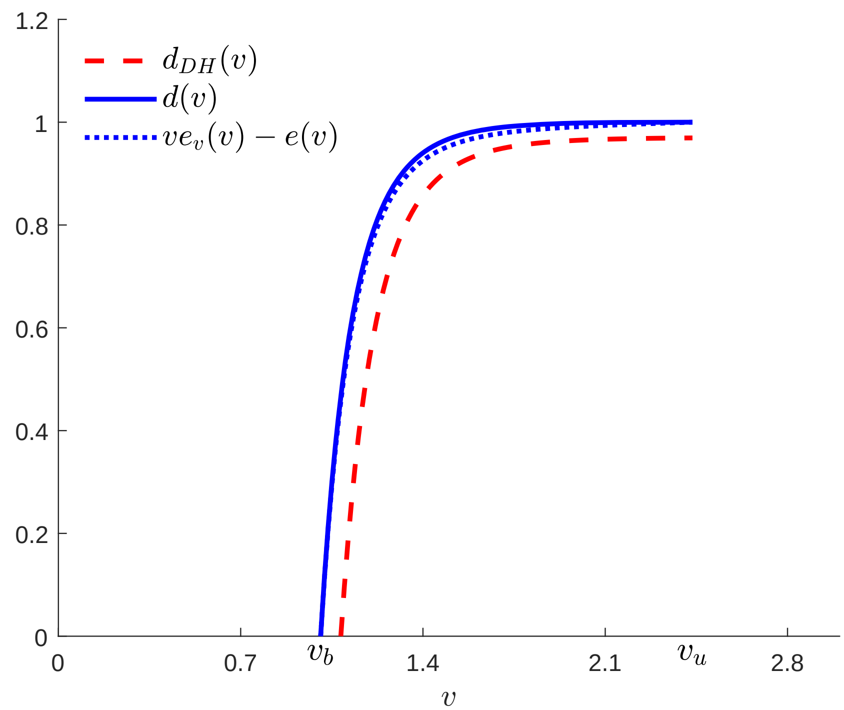

Figure 17: **Debt valuation in absence of issuance cost: commitment vs. no commitment.** For $\xi = 0.2$, this figure shows the value of the debt claim $d(v)$ for the case with commitment in the inaction region $v \in (v_b, v_u)$. We set the coupon rate to $c = 0.0407$ to guarantee that debt is priced at par at the time of issuance. The figure also shows the value of the debt claim for the no-commitment case studied by DH. Because the DH solution does not have an upper boundary v_u, the figure reports a truncated version of their function. The other parameter values are in Table 1.

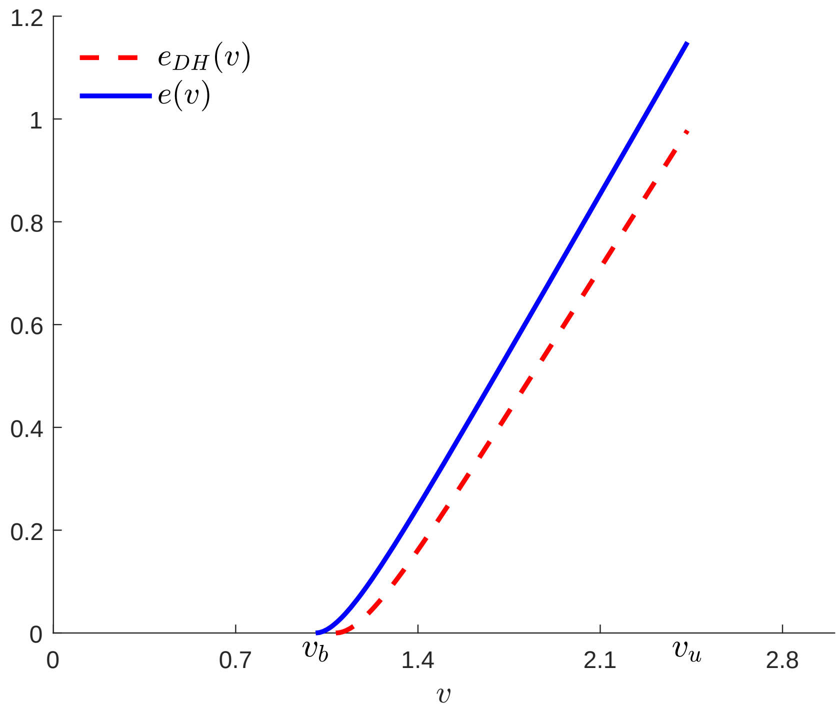

Figure 18: **Equity valuation in absence of issuance cost: commitment vs. no commitment.** For $\xi = 0.2$, this figure shows the value of the equity claim $e(v)$ for the case with commitment in the inaction region $v \in (v_b, v_u)$. We set the coupon rate to $c = 0.0407$ to guarantee that debt is priced at par at the time of issuance. The figure also shows the value of the equity claim for the no-commitment case studied by DH. Because the DH solution does not have an upper boundary v_u, the figure reports a truncated version of their function. The other parameter values are in Table 1.

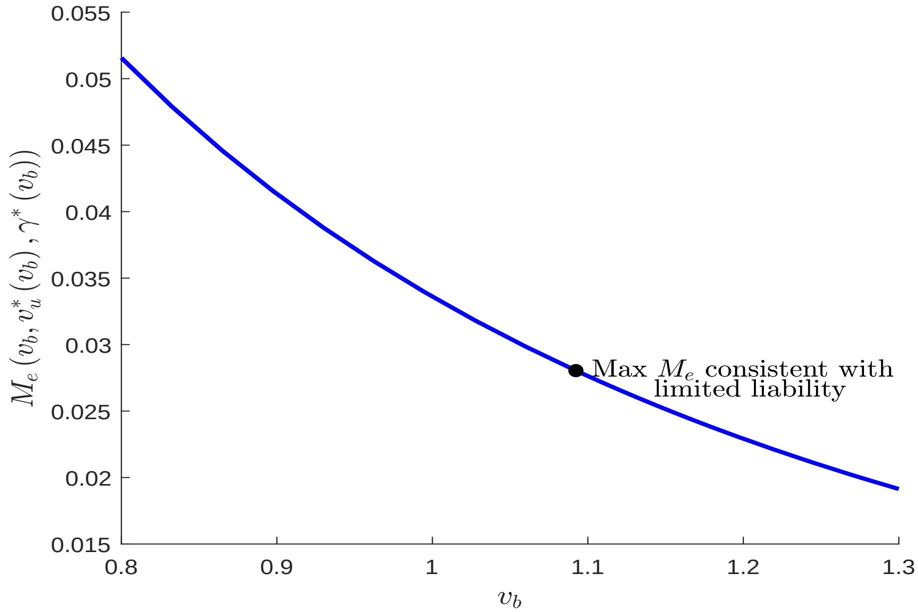

Figure B.1: **Optimality of the default boundary.** This figure shows $M_e\left(v_b, v_u^*\left(v_b\right), \gamma^*\left(v_b\right)\right)$ is decreasing in v_b, where $v_u^*\left(v_b\right), \gamma^*\left(v_b\right)$ are solved from the FOC approach for any given v_b. The issuance cost parameter is set to $\beta = 0.0036$, which is chosen to match a fractional debt issuance cost of 1% under the optimal maturity $\xi^* = 0.2976$. The other parameter values are reported in Table 1.

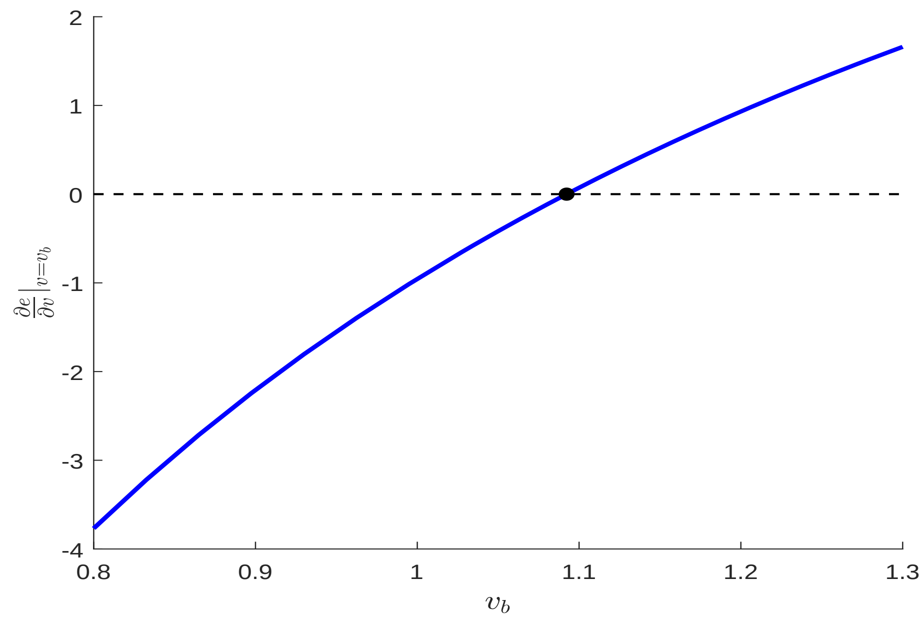

Figure B.2: **Slope of equity value $e(v)$ at v_b.** This figure shows the derivative of equity value $\frac{\partial e}{\partial v}\Big|_{v=v_b}$ as a function of v_b, with $M_e \equiv M_e\left(v_b, v_u^*\left(v_b\right), \gamma^*\left(v_b\right)\right)$ and $v_u^*\left(v_b\right), \gamma^*\left(v_b\right)$ are solved from the FOC approach for any given v_b. The issuance cost parameter is set to $\beta = 0.0036$, which is chosen to match a fractional debt issuance cost of 1% under the optimal maturity $\xi^* = 0.2976$. The other parameter values are reported in Table 1.

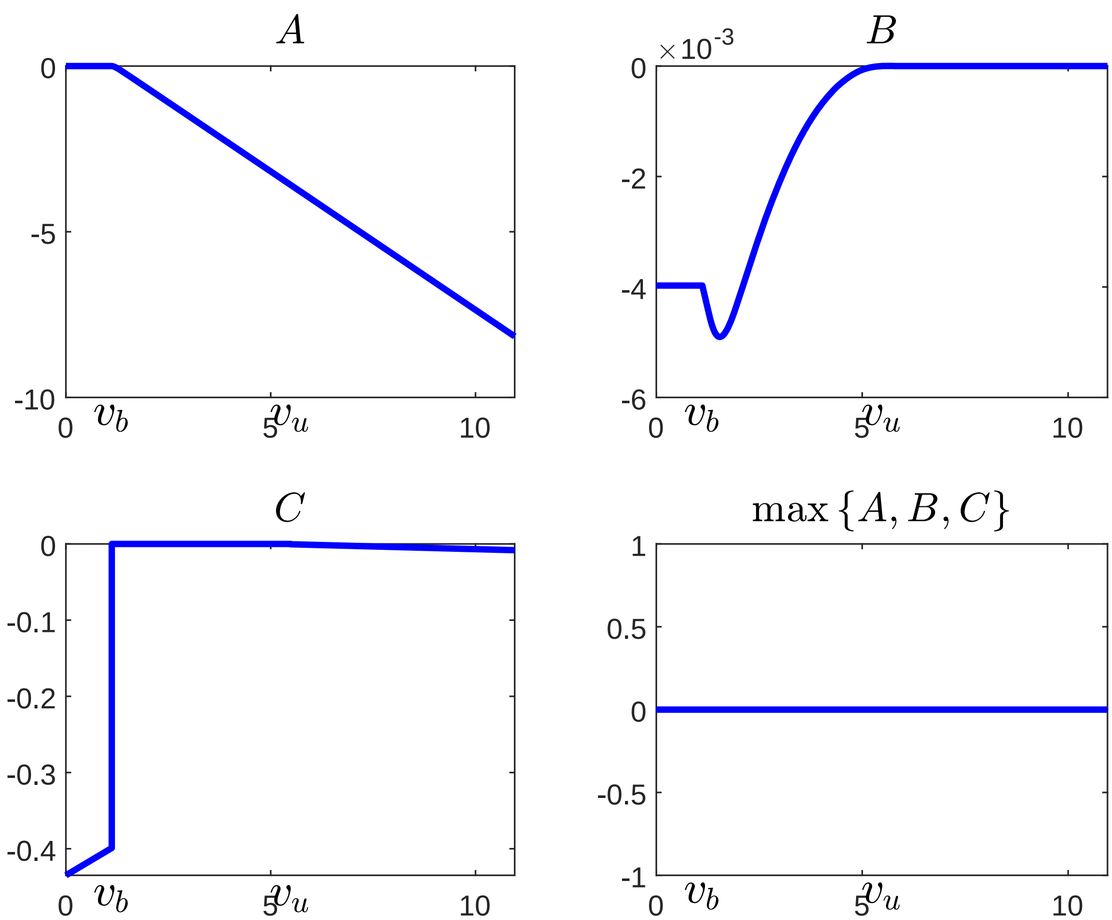

Figure B.3: **The verification of HJB equation.** This figure shows the value of each term of equation (C.11) when $\beta = 0.0036$ for the economy without commitment. The other parameter values are in Table 1 of the main text.

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Optimal Debt Dynamics, Issuance Costs, and Commitment

Internet Appendix

Luca Benzoni, Lorenzo Garlappi, Robert S. Goldstein, Julien Hugonnier and Chao Ying

This Online Appendix contains additional analysis to accompany the manuscript. Section A investigates the firm's optimal debt issuance policy in the presence of proportional, rather than fixed, debt issuance costs. Section B considers (s, S) policies in which the lower boundary is a debt-repurchase/equity-issuance boundary and compares them to the global-optimal policy of Section 3.1.1 of the main text. Section C derives an approximate formula for the location of the indifference boundary shown in Figure 1 that separates the regions where additional debt issuance is optimal, and where it is not.

A Optimal Debt Dynamics with Proportional Issuance Costs

In this section, we investigate the optimal debt issuance policy when the firm is subject to *proportional* debt issuance costs. The case with *no commitment* has been studied in earlier versions of DeMarzo and He (DH, 2019), but we add it here to the case *with commitment* in order to facilitate comparison between the two cases.

A.1 Economy without Commitment

Following DH, we conjecture that the optimal debt issuance policy in a setting with no commitment is locally deterministic. We therefore express the state variable dynamics in the form:

$$dV = \mu V\,dt + \sigma V\,dB \tag{A.1}$$

$$dF = (g(v) - \xi)\,F\,dt. \tag{A.2}$$

Using the definition of inverse leverage $v_t = (V_t/F_t)$, Itô's lemma yields

$$dv = (\mu + \xi - g(v))\,v\,dt + \sigma v\,dB. \tag{A.3}$$

In the presence of *proportional* costs, under the assumption that the debt issuance policy $g(v)$ is always (weakly) positive, the value of equity can be expressed as:

$$E(V_t, F_t) = \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^{\tau_b} ds\, e^{-r(s-t)}\left(Y_s(1-\pi) - F_s\left(c(1-\pi)+\xi\right) + (1-\beta)g(v_s)D(V_s, F_s)\right)\right]. \tag{A.4}$$

In equation (A.4), the term proportional to β represents the funds raised from the debt issuance that is lost to issuance costs. Below, we identify a constraint for the parameter β so that $g(v)$ is (weakly) positive for all values of v. In the special case $\beta = 0$, this framework reduces to the DH model. Equation (A.4) implies that

$$e^{-rt}E(V_t, F_t) + \int_0^t ds\, e^{-rs}\left(Y_s(1-\pi) - F_s\,(c(1-\pi)+\xi) + (1-\beta)g(v_s)D(V_s, F_s)\right)$$

is a $\mathbb{Q}$-martingale, hence the equity claim satisfies the PDE:

$$0 = \max_{g(v)}\left\{\begin{array}{l} -rE + \mu V E_V + \frac{\sigma^2}{2}V^2 E_{VV} + (g(v)-\xi)FE_F + \\ Y_t(1-\pi) - F_t\,(c(1-\pi)+\xi) + (1-\beta)g(v_t)D(V_t, F_t). \end{array}\right\}. \tag{A.5}$$

The first-order condition with respect to $g(v)$ yields:

$$0 = FE_F + (1-\beta)D(V_t, F_t). \tag{A.6}$$

Substituting this expression into equation (A.5), we find that the equity claim satisfies:

$$0 = -rE + \mu V E_V + \frac{\sigma^2}{2}V^2 E_{VV} - \xi F E_F + Y_t(1-\pi) - F_t\,(c(1-\pi)+\xi)\,. \tag{A.7}$$

As in the main text, the value of equity is homogeneous of degree one in its arguments. Thus, we look for a solution of the form:

$$E(V_t, F_t) = F_t\, e(v_t = V_t/F_t)\,. \tag{A.8}$$

Using the relations:

$$E_V = e_v \tag{A.9}$$

$$E_{VV} = \left(\frac{1}{F}\right) e_{vv} \tag{A.10}$$

$$E_F = e - v e_v, \tag{A.11}$$

and $Y_t = (r-\mu)V_t$, we can rewrite the PDE in equation (A.7) as the ODE

$$0 = \frac{\sigma^2}{2}v^2 e_{vv} + (\mu+\xi)v e_v - (r+\xi)e + (1-\pi)(r-\mu)v - (c(1-\pi)+\xi)\,. \tag{A.12}$$

Moreover, we can write the first order condition in equation (A.6) as

$$\frac{D(V,F)}{F} = -\left(\frac{1}{1-\beta}\right) E_{F} = \left(\frac{1}{1-\beta}\right)(ve_{v} - e). \tag{A.13}$$

In terms of the free coefficients (M_e, N_e), the solution of the ODE (A.12) is

$$e(v) = M_e v^{\phi} + N_e v^{\omega} + (1-\pi)v - \left(\frac{c(1-\pi)+\xi}{r+\xi}\right), \tag{A.14}$$

where the exponents (ϕ, ω) are given in equations (22)–(23) of the main text. Imposing the boundary conditions:

$$\begin{aligned} \lim_{v\to\infty} \frac{de}{dv} &= (1-\pi) \\ e(v_b) &= 0, \end{aligned} \tag{A.15}$$

and using the restrictions $\phi > 1$ and $\omega < 0$, we find that the scaled equity function is:

$$e(v) = (1-\pi)\left[v - v_b\left(\frac{v}{v_b}\right)^{\omega}\right] - \left(\frac{c(1-\pi)+\xi}{r+\xi}\right)\left[1 - \left(\frac{v}{v_b}\right)^{\omega}\right]. \tag{A.16}$$

Note that the value of equity in this economy is independent of the proportional issuance cost parameter β, and it is identical to the value of equity in the DH economy.

Smooth pasting or, equivalently, the first-order condition $\left.\frac{\partial e}{\partial v_b}\right|_{v=v_b} = 0$, identifies the location of the optimally-chosen default boundary:

$$v_b = \left(\frac{c(1-\pi)+\xi}{(1-\pi)(r+\xi)}\right)\left(\frac{\omega}{\omega-1}\right). \tag{A.17}$$

This allows us to rewrite scaled equity value as:

$$e(v) = (1-\pi)\left[v - v_b\left(1 - \frac{1}{\omega}\right) - \frac{v_b}{\omega}\left(\frac{v}{v_b}\right)^{\omega}\right]. \tag{A.18}$$

To determine the scaled debt value, we look for a solution of the form:

$$D(V_t, F_t) = F_t\, d(v_t = V_t/F_t). \tag{A.19}$$

Using the equity value in equation (A.16), we simplify the first order condition (A.13) to

$$d(v) \;=\; \left(\frac{1}{1-\beta}\right)\left[ve_{v} - e\right] = v_{b}\left(\frac{1-\pi}{1-\beta}\right)\left(1-\frac{1}{\omega}\right)\left[1-\left(\frac{v}{v_{b}}\right)^{\omega}\right]. \tag{A.20}$$

Except for the term $\left(\frac{1}{1-\beta}\right)$, this formula is identical to the one derived by DH. Thus, in contrast to the equity claim, which is independent of β, the debt claim is increasing in β. The intuition for this result is that, as we demonstrate below, higher issuance costs reduces the aggressiveness at which the firm issues new debt in the future. That is, $g(v)$ is a decreasing function of the issuance cost parameter β. As emphasized in equation (A.3), smaller values of $g(v)$ increase the drift in the dv dynamics, in turn reducing the probability that v reaches the default boundary v_{b} in the future. Lower default probabilities generate higher debt prices.

To determine the optimal policy $g(v)$, we follow DH in deriving the ODE for the debt claim. The debt price is given by the risk-neutral expectation:

$$d(v_{t}) \;=\; (c+\xi)\,\mathbb{E}^{\mathbb{Q}}_{t}\left[\int_{t}^{\tau_b} ds\, e^{-(r+\xi)(s-t)}\right]. \tag{A.21}$$

Because $e^{-(r+\xi)t}d(v_{t}) + (c+\xi)\int_0^t ds\, e^{-(r+\xi)s}$ is a $\mathbb{Q}$-martingale, it follows that the scaled debt function satisfies the ODE:

$$0 \;=\; \frac{\sigma^2}{2}v^2 d_{vv} + (\mu+\xi-g(v))vd_{v} - (r+\xi)d + (c+\xi). \tag{A.22}$$

Differentiating equation (A.7) with respect to F, and using the first-order condition $-E_{F} = (1-\beta)d$, we find:

$$0 \;=\; (1-\beta)\left[(r+\xi)d - (\mu+\xi)vd_{v} - \frac{\sigma^2}{2}v^2 d_{vv}\right] - (c(1-\pi)+\xi)\,. \tag{A.23}$$

Equations (A.22) and (A.23) yield the optimal debt issuance policy:

$$g(v) \;=\; \left(\frac{\pi c - \beta(c+\xi)}{(1-\beta)(vd_{v})}\right) = \left(\frac{\pi c - \beta(c+\xi)}{v_{b}(1-\pi)(1-\omega)}\right)\left(\frac{v}{v_{b}}\right)^{-\omega}. \tag{A.24}$$

As predicted above, $g(v)$ is decreasing in β, which explains why debt prices are increasing in β. In the limit of β going to zero, $g(v)$ in equation (A.24) converges to the optimal DH policy.

So far we have assumed that the debt issuance policy is (weakly) positive for all values of v. From the numerator of equation (A.24), this assumption holds when

$$\beta \;\leq\; \frac{\pi c}{c+\xi}. \tag{A.25}$$

This restriction is equivalent to

$$\left(\frac{1}{1-\beta}\right) \leq \left(\frac{c+\xi}{c(1-\pi)+\xi}\right), \tag{A.26}$$

or, using the expression for v_b in equation (A.17):

$$v_b \left(\frac{1-\pi}{1-\beta}\right)\left(1-\frac{1}{\omega}\right) \;\leq\; \left(\frac{c+\xi}{r+\xi}\right). \tag{A.27}$$

Using inequality (A.27) in the expression for debt value $d(v)$ in equation (A.20), we conclude that the bond price remains below the risk-free bond price, as required to preclude arbitrage opportunities:

$$d(v) \;\leq\; \left(\frac{c+\xi}{r+\xi}\right)\left[1-\left(\frac{v}{v_b}\right)^{\omega}\right] \;<\; \left(\frac{c+\xi}{r+\xi}\right). \tag{A.28}$$

A.2 Economy with Commitment

In the case of proportional issuance costs, the debt issuance size is infinitesimal. To capture this property, we conjecture that the optimal policy with commitment is characterized by an inaction region described by the four parameters (v_b, v_u, γ, ξ) and study its limit $\gamma \to 1^+$.

As in the main text, we define τ_b and τ_u to be the first times v_t reaches v_b and v_u, respectively, and $\tau = \min(\tau_b, \tau_u)$. For dates $t \in (0, \tau)$, there is no debt issuance. For a given debt issuance policy, the PDE and the boundary conditions for the debt claim are identical to those in the main text. From equation (26), the debt value is

$$d(v) \;=\; M_d v^{\phi} + N_d v^{\omega} + \frac{c+\xi}{r+\xi}, \tag{A.29}$$

with boundary conditions given by equations (27)–(28),

$$
\begin{aligned}
d(v_b) &= (1-\alpha)(1-\pi)v_b && \text{(A.30)}\\
d(v_u) &= d(v_u/\gamma). && \text{(A.31)}
\end{aligned}
$$

The value of equity can be determined via the risk-neutral expectation:

$$
\begin{aligned}
E(V_t,F_t) &= \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^{\tau} ds\, e^{-r(s-t)}\left((1-\pi)Y_s-\left(c(1-\pi)+\xi\right)F_s\right)\right] \qquad \text{(A.32)}\\
&+\mathbb{E}_t^{\mathbb{Q}}\left[e^{-r(\tau_u-t)}\,\mathbf{1}_{(\tau_u<\tau_b)}\left(E(v_uF_{\tau_u},\gamma F_{\tau_u})+(1-\beta)\left(D(v_uF_{\tau_u},\gamma F_{\tau_u})-D(v_uF_{\tau_u},F_{\tau_u})\right)\right)\right].
\end{aligned}
$$

The first term captures the present value of the claim to cash flows (i.e., dividends) throughout period-0, which ends at date $\tau=\min(\tau_b,\tau_u)$. The second term captures the fact that when the upper boundary is reached, shareholders still hold the entire equity claim, but now with the face value of debt scaled by a factor of γ. In addition, shareholders receive the value of the new debt issuance (net of issuance costs) as dividend.

Because $e^{-rt}E(V_t,F_t)+\int_0^t ds\, e^{-rs}\left((1-\pi)Y_s-\left(c(1-\pi)+\xi\right)F_s\right)$ is a $\mathbb{Q}$-martingale, the equity claim satisfies the PDE:

$$
0 = -rE+\mu VE_V+\frac{\sigma^2}{2}V^2E_{VV}-\xi FE_F+V(r-\mu)(1-\pi)-F\left(c(1-\pi)+\xi\right), \qquad \text{(A.33)}
$$

with boundary conditions:

$$
\begin{aligned}
E(v_bF,F) &= 0\\
E(v_uF,F) &= E(v_uF,\gamma F)+(1-\beta)\left(D(v_uF,\gamma F)-D(v_uF,F)\right). && \text{(A.34)}
\end{aligned}
$$

We look for a solution of the form $E(V,F)=F\,e\big(v=\frac{V}{F}\big)$ Using the relations in equations (A.9)–(A.11) we can rewrite the PDE as an ODE:

$$
0 = \frac{\sigma^2}{2}v^2e_{vv}+ve_v\left(\mu+\xi\right)-e(r+\xi)+v(r-\mu)(1-\pi)-\left(c(1-\pi)+\xi\right), \qquad \text{(A.35)}
$$

subject to the boundary conditions

$$
\begin{aligned}
e(v_b) &= 0 && \text{(A.36)}\\
e(v_u) &= \gamma e(v_u/\gamma)+(1-\beta)(\gamma-1)d(v_u), && \text{(A.37)}
\end{aligned}
$$

where we have used the relation $d(v_u) = d(v_u/\gamma)$ in the last equation.

The solution is:

$$e(v) = M_e v^{\phi} + N_e v^{\omega} + (1-\pi)v - \frac{(c(1-\pi)+\xi)}{r+\xi}, \tag{A.38}$$

where the constants (M_e, N_e) are uniquely determined by the boundary conditions in equations (A.36)–(A.37).

Under the conjecture that the size of the optimal debt issuance is infinitesimal, we set $\gamma = (1+\epsilon)$ and investigate the case $\epsilon \to 0^+$. Therefore, the boundary conditions for debt and equity at the upper boundary simplify to:

$$\begin{aligned} d_v(v_u) &= 0 && \text{(A.39)} \\ v_u e_v(v_u) &= e(v_u) + (1-\beta)d(v_u). && \text{(A.40)} \end{aligned}$$

The optimal debt issuance policy is determined as follows: for a given ξ, optimal boundaries are determined by the first-order conditions[1]

$$\begin{aligned} \left.\frac{\partial e}{\partial v_b}\right|_{v=v_b} &= 0 \\ \left.\frac{\partial e}{\partial v_u}\right|_{v=v_u} &= 0. && \text{(A.41)} \end{aligned}$$

We then identify the value of ξ that maximizes equity value $e(v)$ at time of issuance, where $e(v)$ satisfies the ODE (A.35) with boundary conditions (A.36) and (A.40). For any value of the proportional cost parameter β, we identify the optimal debt policy $\left(v_b^*(\beta), v_u^*(\beta), \xi^*(\beta)\right)$.

A.3 Results for the Cases with and without Commitment

In this section, we investigate model predictions for the cases with and without commitment under proportional costs. In the analysis that follows we set the coupon rate c so that debt is

[1] As in the fixed cost case, we can show that the solution with commitment corresponds to a global-optimal policy in that the solution to $0 = \left.\frac{\partial e}{\partial v_u}\right|_{v=v_{any};v_b}$ generates the same value for v_u^* independent of choice of v_{any}.

priced at par at the time of issuance. The other parameter values are in Table 1 of the main text.

A.3.1 Debt and Equity Values

Figures A.1 and A.2 show the debt and equity values with commitment for $\beta = 0.01$, a value consistent with empirical estimates of debt issuance costs. In both figures we fix $\xi^* = 0.2505$ to maximize the equity value at the time of issuance. Finally, we set $c = 0.0406$ to guarantee that debt is issued at par. For comparison, we also plot the value of debt and equity for the no-commitment case. Since, without commitment, ξ is undetermined, in this case we use the optimal value ξ^* of the commitment policy and its associated coupon c.

With commitment, the value of debt in Figure A.1 is higher than that without commitment for any inverse leverage v. The difference quantifies the value of committing to a less aggressive debt issuance policy that lowers future default probability and increases debt value. In the same figure, we also plot the marginal value of equity (adjusted for issuance cost), $-\frac{E_F}{1-\beta}$, which, by equation (A.13) is given by $\frac{ve_v(v)-e(v)}{1-\beta}$. We compute the functions $e(v)$ and $e_v(v)$ using the optimal policy with commitment. For all v values the debt value with commitment (the continuous blue line) is higher than the marginal value of equity (the dashed-blue line), except for $v = v_u$ where $-E_F(V, F) = (1-\beta)d(v_u)$. In contrast, consistent with DH, the value of debt without commitment (continuous red line) satisfies the condition $-E_F(V, F) = (1-\beta)d(v)$ for all values of v.

In Figure A.2, the value of equity with commitment exceeds the no commitment equity value for all v. Similar to the case of fixed issuance costs, shareholders would have no incentive to deviate from the commitment policy. Hence the no-commitment issuance policy can serve as a credible punishment to sustain the commitment equilibrium. Indeed, any deviation from the optimal policy would lead to future debt issues being priced according to the no-commitment equilibrium. Shareholders would gain zero cash benefit at the date of the deviation, and be left with an equity claim that has lower valuation than under the optimal policy.

A.3.2 Optimal Debt Policies as a Function of β

In Figures A.3–A.6, we report the optimal policy $\left(v_b^*(\beta), v_u^*(\beta), \xi^*(\beta)\right)$ and the coupon rate c as a function of β.

For the case with commitment, the optimal average maturity in Figure A.3 is an increasing function of the proportional cost parameter β. This results from the trade off between the tax benefits net of bankruptcy costs (which favor short-maturity debt) and debt issuance costs (which favor long-maturity debt). The effect of net tax benefits dominates for lower values of β, leading to shorter optimal maturities. For the case without commitment, regardless of maturity, the gain from additional tax shields is offset by increased default costs. Hence, consistent with the findings of DH, shareholders are indifferent toward the debt maturity structure. This implies that there is no optimal maturity in the economy with proportional cost under the no-commitment policy.

For the case with commitment, Figure A.4 shows that the upper boundary $v_u^*(\beta)$ increases monotonically with the issuance cost β. This is because, when issuance costs are high, it is optimal to reduce the present value of these costs by issuing less frequently. To this end, the firm increases the location of v_u so that the inverse leverage v_t reaches the upper boundary v_u less often. For the case with no commitment, the upper boundary is infinite for all values of β and therefore it does not appear in the figure.

In Figure A.5, we report the lower boundary $v_b^*(\beta)$ for both cases with and without commitment. As in Section A.3.1, in both cases, we use the optimal maturity $1/\xi^*$ and coupon c obtained under commitment. The default boundary with commitment is always lower than that without commitment. This is because commitment increases cash flows to shareholders, making the option to retain ownership more valuable. Interestingly, the functional dependence of the default boundary on β is rather different for the two cases. With commitment, $v_b^*(\beta)$ is a hump-shaped function of β, similar to what we find in the main text for the case of fixed issuance costs. Two channels affect the shape of the default boundary $v_b^*(\beta)$. The first (direct) channel is due to issuance costs. A higher β implies lower equity valuations *ceteris paribus*, making the option to remain solvent less valuable. Hence, this channel alone would generate a monotonically increasing relation for $v_b^*(\beta)$. However, there is a second (indirect) channel due to the optimal maturity choice $1/\xi^*(\beta)$, which is increasing in β. In those states of nature in which a firm is performing poorly, a longer maturity implies less cash flow that shareholders need to raise to service debt in place. Hence, the maturity channel leads to higher equity valuations in bad states of nature, leading to a *lower* optimal default boundary. Figure A.5 shows that, with commitment, this indirect channel dominates the direct channel for values of β greater than $\beta > 10^{-2.4}$. In contrast, without commitment $v_b^*(\beta)$ is a decreasing function of β. This is because for the case with no commitment, the equity value is independent of β. Hence the first (direct) channel has no impact and the second (indirect) channel dominates.[2]

Finally, Figure A.6 shows the coupon rate c as a function of β, chosen so that the bond is priced at par at the time of issuance. Similar to the case of fixed costs, a lower β is associated with a shorter optimal maturity $1/\xi^*$, which makes the bond less risky. In the limit $\beta \to 0$, the par coupon rate converges to the riskfree rate $r = 4\%$.

A.3.3 Tax Benefit of Debt

We define the tax benefits to debt as the ratio of the levered enterprise value at the debt issuance boundary and the value of an all-equity firm, that is:

$$TB = \frac{e(v_u) + d(v_u)}{(1-\pi)v_u} - 1. \tag{A.42}$$

In Figure A.7, we show the tax benefits from equation (A.42) as a function of the proportional cost coefficient β. The case with commitment always generates strictly positive tax benefits to debt, while, as previously mentioned, there are no tax benefits to debt under no commitment. Consistent with intuition, the tax benefit of debt under commitment increases as the proportional costs decrease. We also plot the level of tax benefits under the restriction $\xi = 0$ for the case with commitment. We get the similar conclusion as the fixed costs that the tax benefits to debt are amplified considerably when firms have the right to choose optimal debt maturity.

[2]Indeed, for a fixed ξ, the default boundary $v_b^*(\beta)$ in equation (A.17) is independent of β under the optimal no-commitment policy.

B Equity Valuation with Debt Repurchase

In Section 2 of the main text, we derived a global-optimal (s, S) policy in which the lower boundary is a default boundary, and compared its properties to those of optimal no-commitment policies that also involve a default boundary. In this section, instead, we investigate (s, S) policies in which the lower boundary is a debt-repurchase/equity-issuance boundary and compare their properties to the global-optimal policy of Section 2. Specifically, we consider the possibility that, in each period, there is a lower boundary v_ℓ at which the firm will scale down the level of outstanding debt by a factor $\psi \leq 1$. The implication is that debt becomes risk-free, as inverse leverage always stays within the region $v_t \in (v_\ell, v_u)$ for which equity values remain positive. The value of outstanding debt with coupon c and amortization rate ξ is therefore

$$D(V_t, F_t) \;\;=\;\; \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^\infty ds\, e^{-r(s-t)}\,(c+\xi)F_s\right] = \left(\frac{c+\xi}{r+\xi}\right)\, F_t. \tag{B.1}$$

In analogy with Section 2, we define τ_u as the time when the upper boundary is reached, τ_ℓ as the time when the lower boundary is reached, and $\tau = \min(\tau_\ell, \tau_u)$.

Up to this point, we have assumed full tax loss offset in that, when EBIT falls below interest payments ($Y_t < cF_t$), our specification for the dividend in equation (A.9) of the main text implies that government pays $\pi(cF_t - Y_t)$ to the firm.[3] In reality, firms do not receive cash infusions from government, but rather only a tax loss carryforward, whose present value may be significantly less than what this specification implies. Therefore, in this section, we investigate the situation in which the cash flow (i.e., dividends) to equity is specified as:

$$\mathrm{Div}(Y_t, F_t) \;\;=\;\; \begin{cases} Y_t - \pi(Y_t - cF_t) - (c+\xi)F_t & Y_t > cF_t \\ Y_t - (c+\xi)F_t & Y_t < cF_t. \end{cases} \tag{B.2}$$

With a slight abuse of notation, we rewrite this formula in terms of our state vector (V_t, F_t), where $V_t = (r-\mu)Y_t$ by equation (2) of the main text:

$$\mathrm{Div}(V_t, F_t) \;\;=\;\; \begin{cases} (1-\pi)(r-\mu)V_t - (c(1-\pi)+\xi)\,F_t & V_t > \frac{cF_t}{r-\mu} \\[2ex] (r-\mu)V_t - (c+\xi)\,F_t & V_t < \frac{cF_t}{r-\mu}. \end{cases} \tag{B.3}$$

[3] Note that, for the model investigated in the main text, EBIT never falls below interest payments, so the tax-loss offset constraint imposed here would not bind anyway.

As this specification implies no tax-loss carry-forward, the actual situation falls between this specification and the one that allows full tax loss offset. However, for the case we study below, we argue that this specification is the more relevant case. This is because, in what follows, the optimal policy will be to choose the location of the upper boundary such that the firm's EBIT always falls below coupon payments made to debtholders. As such, the company never pays taxes. Under this scenario, it is more realistic to assume no tax-loss carry-forward rather than a tax rebate from the government.

The value of equity can be determined as the risk-neutral expectation:

$$\begin{aligned} E(V_t, F_t) &= \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^{\tau} ds\, e^{-r(s-t)}\,\mathrm{Div}(Y_s, F_s)\right] \\ &+\mathbb{E}_t^{\mathbb{Q}}\left[e^{-r(\tau_u-t)}\,\mathbf{1}_{(\tau_u<\tau_\ell)}\left(E(v_u F_{\tau_u}, \gamma F_{\tau_u}) + (\gamma-1)\left(\frac{c+\xi}{r+\xi}\right)F_{\tau_u} - \beta v_u F_{\tau_u}\right)\right] \\ &+\mathbb{E}_t^{\mathbb{Q}}\left[e^{-r(\tau_\ell-t)}\,\mathbf{1}_{(\tau_\ell<\tau_u)}\left(E(v_\ell F_{\tau_\ell}, \psi F_{\tau_\ell}) - (1-\psi)\left(\frac{c+\xi}{r+\xi}\right)F_{\tau_\ell} - \beta_E v_\ell F_{\tau_\ell}\right)\right]. \end{aligned} \tag{B.4}$$

The first term captures the present value of the claim to dividends from the current date t until date $\tau = \min(\tau_\ell, \tau_u)$. The second term captures the fact that when the upper boundary is reached, shareholders still hold the entire equity claim, but now with the face value of debt scaled by a factor $\gamma \geq 1$. In addition, shareholders receive as dividend the value of the new debt issuance (net of issuance costs), which, because debt is risk-free, has price per unit face value of $\left(\frac{c+\xi}{r+\xi}\right)$. The third term captures the fact that when the lower boundary is reached, shareholders still hold the entire equity claim, but now with the face value of debt scaled by a factor $\psi \leq 1$. In addition, shareholders pay for the repurchase of the (risk-free) debt, and an equity issuance cost controlled by the parameter β_E.

Equation (B.4) implies that $\left[e^{-rt}\,E(V_t, F_t) + \int_0^t ds\, e^{-rs}\,\mathrm{Div}(V_s, F_s)\right]$ is a $\mathbb{Q}$-martingale, therefore, by Itô's Lemma, we have:

$$0 \;=\; -rE + \mu V E_V + \frac{\sigma^2}{2}V^2 E_{VV} - \xi F E_F + \mathrm{Div}(V, F). \tag{B.5}$$

As in Section 2, because both the cash flows to equity and state vector dynamics are linear in the state vector, the value of equity is homogeneous of degree one in the state vector. Thus, we look for a solution of the form:

$$E(V,F) \;=\; Fe\left(v=\frac{V}{F}\right). \tag{B.6}$$

This allows us to reexpress the PDE in equation (B.5) as the coupled ODEs:

$$\begin{aligned} 0 &= \tfrac{\sigma^2}{2}v^2 e_{vv} + ve_v\left(\mu+\xi\right) - e(r+\xi) + v(r-\mu) - \left(c+\xi\right), & v < \frac{c}{r-\mu}, \\ 0 &= \tfrac{\sigma^2}{2}v^2 e_{vv} + ve_v\left(\mu+\xi\right) - e(r+\xi) + v(r-\mu)(1-\pi) - \left(c(1-\pi)+\xi\right), & v > \frac{c}{r-\mu}, \end{aligned} \tag{B.7}$$

subject to the boundary conditions

$$e(v_\ell) \;=\; \psi e(v_\ell/\psi) - (1-\psi)\left(\frac{c+\xi}{r+\xi}\right) - \beta_E v_\ell \tag{B.8}$$

$$e(v_u) \;=\; \gamma e(v_u/\gamma) + (\gamma-1)\left(\frac{c+\xi}{r+\xi}\right) - \beta v_u. \tag{B.9}$$

Under this specification, the value of equity is given by:

$$e(v) \;=\; \begin{cases} M_e^{+}v^{\phi} + N_e^{+}v^{\omega} + (1-\pi)v - \frac{(c(1-\pi)+\xi)}{r+\xi} & v > \frac{c}{r-\mu} \\ M_e^{-}v^{\phi} + N_e^{-}v^{\omega} + v - \frac{c+\xi}{r+\xi} & v < \frac{c}{r-\mu}, \end{cases} \tag{B.10}$$

where the constants $(M_e^{+}, M_e^{-}, N_e^{+}, N_e^{-})$ are uniquely determined from the boundary conditions in equations (B.8)–(B.9), and the value- and slope-matching conditions[4]

$$\lim_{v\uparrow\frac{c}{r-\mu}} e\left(v\right) \;=\; \lim_{v\downarrow\frac{c}{r-\mu}} e\left(v\right) \tag{B.11}$$

$$\lim_{v\uparrow\frac{c}{r-\mu}} e_v\left(v\right) \;=\; \lim_{v\downarrow\frac{c}{r-\mu}} e_v\left(v\right). \tag{B.12}$$

We investigate this model under two different parameterizations: with and without issuance costs.

[4]These value- and slope-matching conditions assume that $v_\ell < \frac{c}{r-\mu}$ and $v_u > \frac{c}{r-\mu}$. If instead v_ℓ and v_u are both either above or both below $\frac{c}{r-\mu}$, then these additional conditions are not required.

B.1 Debt Repurchase in the Absence of Issuance Costs

For the case $\beta = 0$, $\beta_E = 0$, the value of equity possesses a simple analytic solution. Specifically, because (i) the claim to bankruptcy costs is zero when debt is risk-free, and (ii) the government's claim to taxes can be reduced to zero if the debt issuance threshold is chosen such that $v_u \leq \frac{c}{r-\mu}$, that is, coupon payments always exceed EBIT, then the value of equity is equal to the value of the EBIT claim minus the value of debt. Hence, under the assumption $v_u \leq \frac{c}{r-\mu}$, which implies that the firm never pays any taxes because coupon payments always exceed EBIT, we have

$$e_{repurchase}(v) \quad = \quad v - \left(\frac{c+\xi}{r+\xi}\right). \tag{B.13}$$

Now, for this equation to be compatible with limited liability, we must restrict the lower boundary v_ℓ such that:

$$0 \quad \leq \quad e_{repurchase}(v_\ell) = v_\ell - \left(\frac{c+\xi}{r+\xi}\right), \tag{B.14}$$

which implies:[5]

$$v_\ell \geq \left(\frac{c+\xi}{r+\xi}\right). \tag{B.15}$$

We then compare the value of the equity claim under the debt repurchase policy to the equity value under the global optimal policy for all $v \in (v_\ell, v_u)$. As the firm is free to choose v_ℓ arbitrarily close to v_u, ultimately, this is equivalent to comparing value of equity in the bond-repurchase case with its value under the global-optimal policy at $v_u = \left(\frac{c}{r-\mu}\right)$. That is, shareholders are better off by following a debt repurchase policy if

$$e_{repurchase}\left(\frac{c}{r-\mu}\right) = \left(\frac{c}{r-\mu}\right) - \left(\frac{c+\xi}{r+\xi}\right) > e_{global}\left(\frac{c}{r-\mu}\right). \tag{B.16}$$

We find that this condition holds for a range of parameters. As one example, Figure A.8 shows that this condition holds for the parametrization ($\psi = 1, \gamma = 1, \xi = 0.2, v_\ell = \frac{1}{2}\left(\frac{c+\xi}{r+\xi} + \frac{c}{r-\mu}\right), v_u =$

[5] More generally, we impose the following conditions: (i) $\frac{c+\xi}{r+\xi} \leq v_\ell \leq v_u \leq \frac{c}{r-\mu}$; (ii) $v_\ell \leq \frac{v_\ell}{\psi} \leq v_u$; and (iii) $v_\ell \leq \frac{v_u}{\gamma} \leq v_u$

$\frac{c}{r-\mu}$). Any value of v_ℓ that satisfies $e_{repurchase}(v_\ell) > e_{global}(v_\ell)$ guarantees that shareholders will be better off under the repurchase policy v_ℓ. The next section, however, shows that this result is not robust to the presence of realistic security issuance costs.

B.2 Debt Repurchase in the Presence of Issuance Costs

In the previous section, EBIT falls below the promised coupon payments. Hence, over each interval dt the firm must issue equity of order $\mathcal{O}(dt)$ in order to avoid default. Here we relax the counterfactual assumption that the firm can issue equity at no cost. For tractability, we impose that the firm incurs equity issuance costs only when issuing a finite amount of equity at v_ℓ, but not on issuances of infinitesimal size. Accounting for these additional equity issuance costs would serve only to reduce equity valuation even further, thus making equity value even lower under the debt-repurchase policy in the presence of realistic issuance costs.

Consistent with the empirical literature (e.g., Altınkılıç and Hansen (2000), Hennessy and Whited (2007), Titman and Tsyplakov (2007), and Gamba and Triantis (2008)), we calibrate the model so that equity issuance costs are approximately 7% and debt issuance costs are 1% of the amount raised. Calibrated to these issuance costs, we find that the model with debt repurchase generates values of equity at the lower boundary v_ℓ that are lower than equity values under the global optimal solution for all possible (s, S) debt issuance and repurchase policies $(v_\ell, v_u^{rep}, \gamma, \psi)$. It follows that the value of equity under the debt-repurchase policy will always be lower than under the global-optimal policy without debt repurchase. Figure A.9 provides an example. This result implies that, in the presence of realistic equity issuance costs, it is not in shareholders' best interest to repurchase debt at the v_ℓ boundary. Hence, debtholders would not be willing to purchase the firm's debt at risk-free prices.

This analysis shows that the presence of equity issuance costs provides one explanation for why firms are unable to issue risk-free debt. Many other mechanisms exist. For example, if the manager is more informed about the firm's asset value than creditors (e.g., Duffie and Lando (2001)), and the true asset value is sufficiently low, then a manager acting in the best interest of shareholders could not credibly commit to repurchasing debt according to a policy based on *public* information. As a second example, the possibility that firm value could jump well below the promised debt repurchase boundary may lead shareholders to renege on their promise.

C Derivation of the Approximate Debt Issuance Indifference Curve

We identify an approximate formula for the location of the indifference boundary shown in Figure 1 that separates the regions where additional debt issuance is optimal, and where it is not. Our approach is motivated by Figure 2, which shows how (v_b, v_u, γ) change as $\beta \to \beta^*$, where β^* is a specific point on the indifference boundary for a given value of ξ. Specifically, we find that $v_u \to \infty$, whereas v_b approaches the case with no debt issuance, and (v_u/γ) is only slightly larger than v_b (and hence, γ is only slightly less than (v_u/v_b)). The economic intuition for these findings is as follows: as we increase β, the optimal policy reduces issuance costs by making both v_u and γ larger, as increasing each reduces the number of times the debt issuance boundary is reached. The threshold v_b converges to the default boundary for the case of no debt issuance, $v_{\text{b, no issuance}}$, because, as $\beta \to \beta^*$, $v_u \to \infty$ and therefore the firm does not issue any further debt.

With these insights, we identify an approximate formula for the location of the indifference curve. Assume the firm is at a debt-issuance boundary $(V = v_u F, F)$, and is about to issue debt with face value ΔF. Thus, the state vector moves from $(v_u F, F)$ to $(v_u F, F + \Delta F)$, implying that inverse leverage jumps from v_u to $\left(\frac{v_u}{1+(\Delta F/F)}\right)$. Thus, we have identified the relation $\gamma = (1 + (\Delta F/F))$, or equivalently, $\Delta F = F(\gamma - 1)$

Now, if the debt issued is approximately risk-free, then the present value of bankruptcy costs is small, and therefore the net tax benefit of the new debt approximately equals its tax savings, which is a product of the tax rate π and the coupon payment paid at a date-t after the debt issuance date $c\,\Delta F\, e^{-\xi t}$:

$$\text{tax benefit} = \int_0^\infty dt\, e^{-rt}\, \pi c\, \Delta F\, e^{-\xi t} = \Delta F \left(\frac{c\pi}{r+\xi}\right) \approx F(\gamma - 1)\left(\frac{r\pi}{r+\xi}\right), \tag{C.1}$$

where this last line holds because $c \approx r$ when debt is approximately risk-free, and coupon is chosen so that debt is issued at par. Moreover, for all values of β that we examine, we have the condition $\xi \gg r$. Hence, we approximate further that

$$\text{tax benefit} \quad \approx \quad F(\gamma - 1)\left(\frac{r\pi}{\xi}\right). \tag{C.2}$$

As noted above, γ becomes large as we approach the critical value $\beta^*(\xi)$. Therefore, we approximate $(\gamma - 1) \approx \gamma$. Further, as we see in Figure 2, $\frac{v_u}{\gamma} \approx v_b$. Therefore, we approximate $\gamma \approx \left(\frac{v_u}{v_b}\right)$. Hence, we approximate tax benefits as:

$$\text{tax benefit} \quad \approx \quad F\left(\frac{v_u}{v_b}\right)\left(\frac{r\pi}{\xi}\right), \tag{C.3}$$

Now, the cost of this debt issuance is:

$$\text{issuance cost} \quad = \quad \beta v_u F. \tag{C.4}$$

Comparing equations (C.3) and (C.4), after dividing both sides by $v_u F$, we see that net tax benefits approximately equal issuance costs when:

$$\left(\frac{r\pi}{v_b \xi}\right) \quad = \quad \beta. \tag{C.5}$$

To get an estimate of v_b, we determine the location of the optimal default boundary under the assumption of no future debt issuance. Such a model is effectively that of Leland (1994) generalized to finite maturity $(1/\xi)$. As in Benzoni, Garlappi, and Goldstein (2018), we find:

$$v_{b,\text{no issuance}} \quad = \quad \left(\frac{\omega}{\omega - 1}\right)\left(\frac{1}{1-\pi}\right)\left(\frac{c\,(1-\pi)+\xi}{r+\xi}\right). \tag{C.6}$$

Again, using the approximation $\xi \gg r$ and noting that ω is large in magnitude for large ξ, the optimal location of the lower boundary simplifies to

$$v_b \quad \approx \quad \left(\frac{1}{1-\pi}\right). \tag{C.7}$$

Substituting the last expression into equation (C.5), we find an approximate solution for the location of the boundary at which the firm is indifferent to issuing additional debt or not:

$$\beta\xi \quad \approx \quad r\pi(1-\pi). \tag{C.8}$$

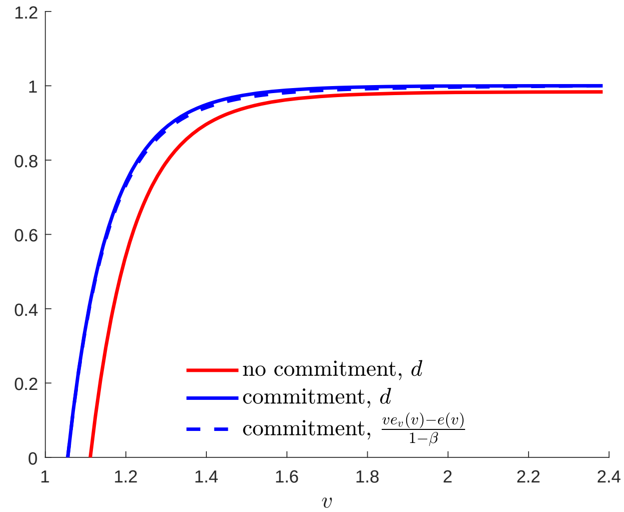

Figure A.1: **Debt valuation with proportional costs: commitment vs. no commitment.** This figure shows the value of the debt claim $d(v)$ for the cases with and without commitment and $v \in (v_b^*, v_u^*)$, with the proportional cost parameter $\beta = 0.01$ under the optimal inverse maturity $\xi^* = 0.2505$ and coupon rate $c = 0.0406$ to guarantee that debt is issued at par. The curve $\frac{ve(v)-e_v(v)}{1-\beta}$ is derived using the value of equity under commitment. The figure shows only a truncated version of the no-commitment solution because, without commitment, the optimal policy has no upper boundary v_u. The other parameter values are in Table 1 of the main text.

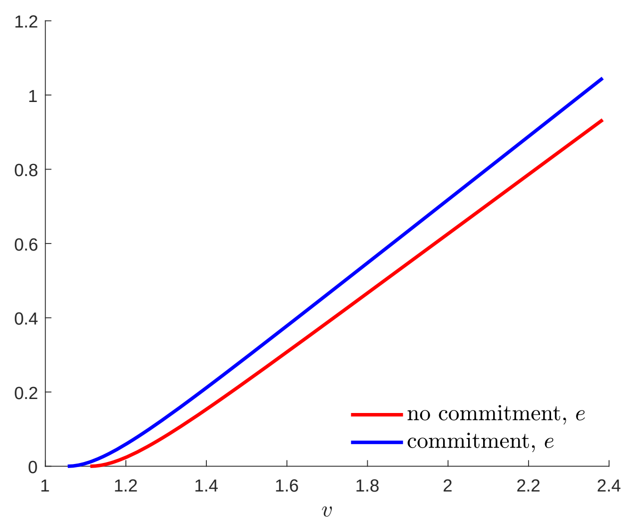

Figure A.2: **Equity valuation with proportional costs: commitment vs. no commitment.** This figure shows the value of the equity claim $e(v)$ for the cases with and without commitment and $v \in (v_b^*, v_u^*)$, with the proportional cost parameter $\beta = 0.01$ under the optimal inverse maturity $\xi^* = 0.2505$ and coupon rate $c = 0.0406$ to guarantee that debt is issued at par. The figure shows only a truncated version of the no-commitment solution because, without commitment, the optimal policy has no upper boundary v_u. The other parameter values are in Table 1 of the main text.

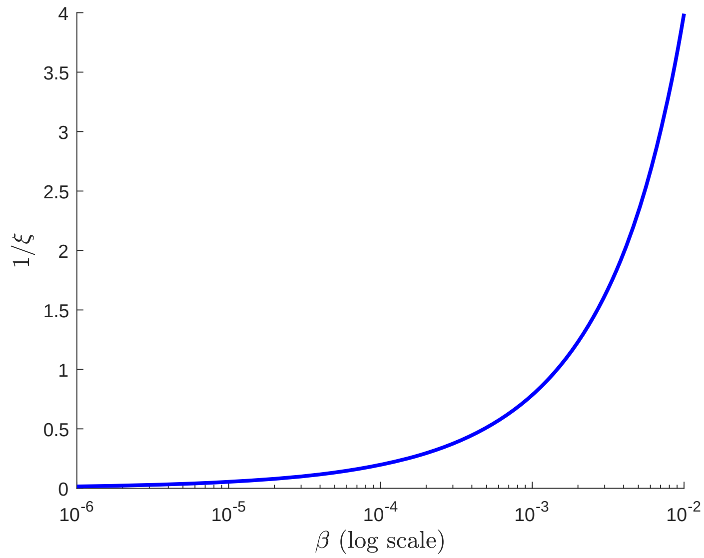

Figure A.3: **Optimal maturity.** This figure shows the optimal debt maturity $1/\xi$ with commitment as a function of the proportional cost parameter β. The other parameter values are in Table 1 of the main text.

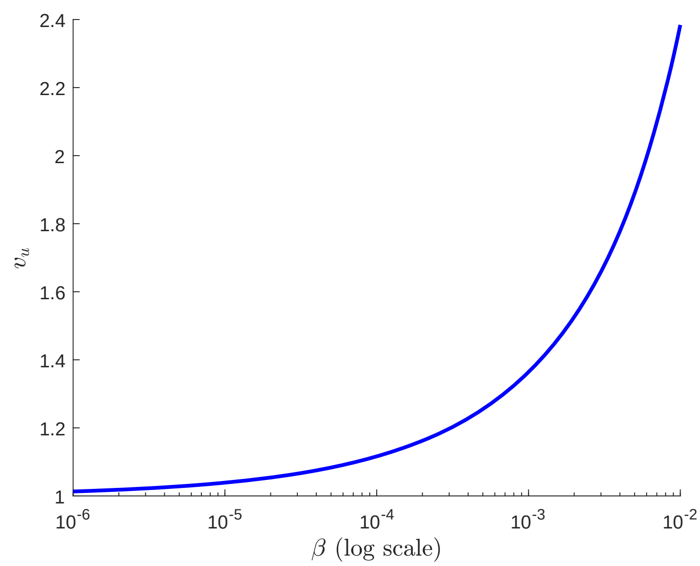

Figure A.4: **Optimal debt issuance boundary.** This figure shows the location of the optimal debt issuance boundary v_u with commitment as a function of the proportional cost parameter β. The other parameter values are in Table 1 of the main text.

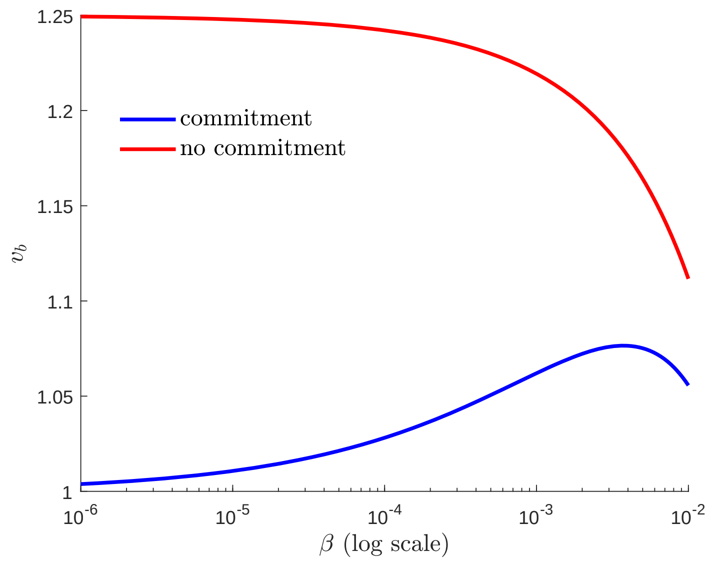

Figure A.5: **Optimal default boundary.** This figure shows the location of the optimal default boundary v_b for the cases with and without commitment as a function of the proportional cost parameter β. The other parameter values are in Table 1 of the main text.

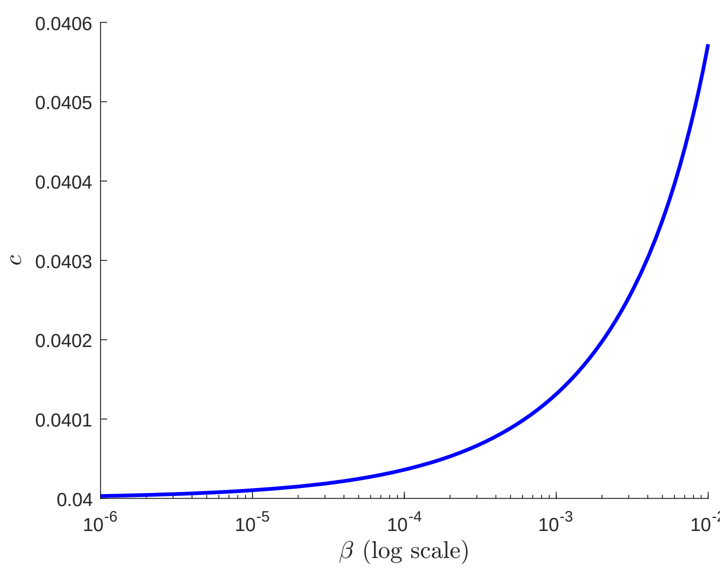

Figure A.6: **Coupon rate.** This figure shows the coupon rate c such that the bond is priced at par at the time of issuance. The other parameter values are in Table 1 of the main text.

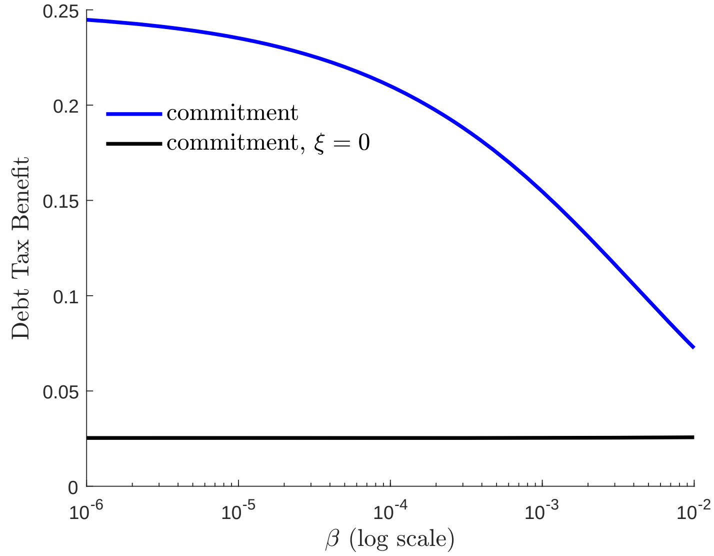

Figure A.7: **Tax benefits to debt.** This figure shows the tax benefits to debt under the optimal policy with commitment as a function of the proportional cost parameter β. It also reports the tax benefits to debt for the case with commitment when the inverse maturity parameter is set to $\xi = 0$. The other parameter values are in Table 1 of the main text.

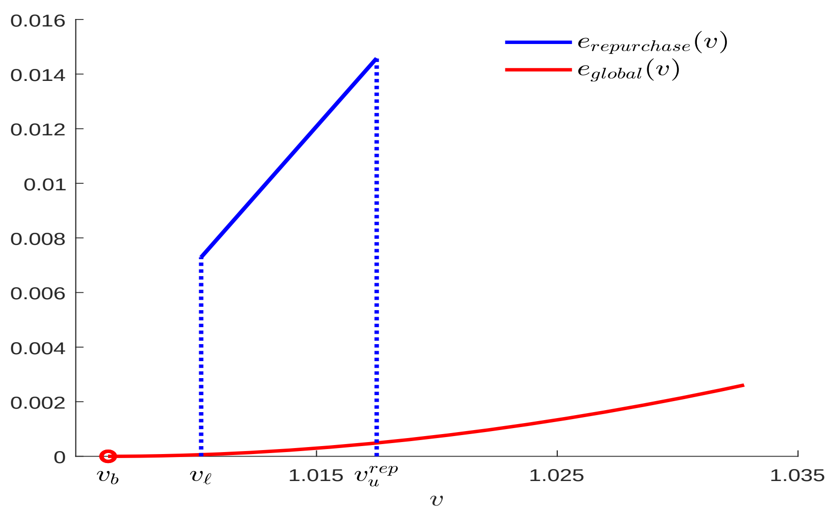

Figure A.8: **Debt repurchase and zero issuance cost.** This figure shows the value of equity under both the risk-free and defaultable debt cases, where both equity and debt issuance costs are zero. For the risk-free debt, we use the following parameters: $\psi = 1$, $\gamma = 1$, $\xi = 0.2$, $v_\ell = \frac{1}{2}\left(\frac{c+\xi}{r+\xi} + \frac{c}{r-\mu}\right)$, $v_u^{rep} = \frac{c}{r-\mu}$. We choose the coupon rate $c = 0.0407$ so that the bond is priced at par with defaultable debt. We only plot part of the equity value for defaultable debt, which has the upper boundary $v_u = 2.4358$. The other parameter values are in Table 1 of the main text.

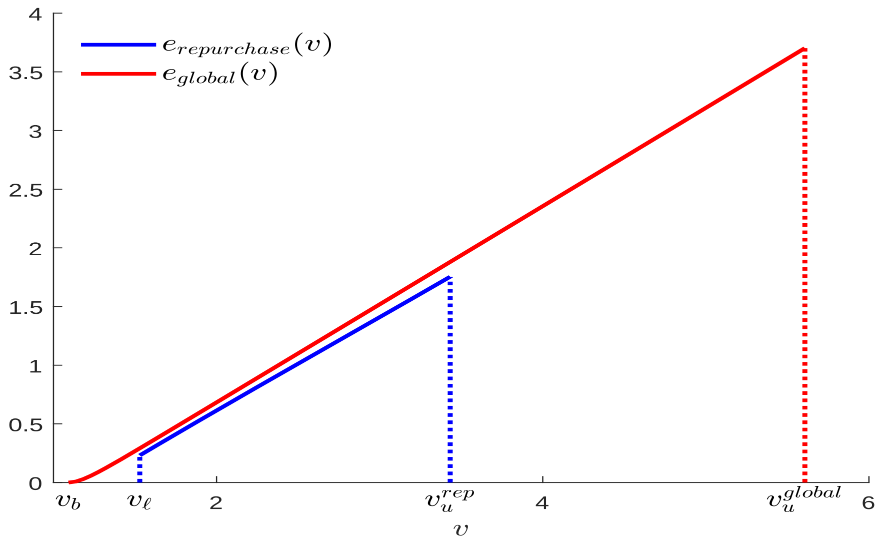

Figure A.9: **Debt repurchase and positive issuance cost.** This figure shows the value of equity under both the risk-free and defaultable debt cases, where both equity and debt issuance costs are positive. The issuance cost parameter is set to $\beta = 0.0036$, which is chosen to match a fractional debt issuance cost of 1% under the optimal maturity $\xi = 0.2976$. The equity issuance cost is set to $\beta_E = 0.001$, which is chosen to match a fractional equity issuance cost of 7%. For the risk-free debt, we choose the following parameters: $\psi = 0.9797$, $\gamma = 2.2437$, $v_\ell = 1.5292$, $v_u^{rep} = 3.4325$. The other parameter values are in Table 1 of the main text.