

Money under the Mattress:

Inflation and Lending of Last Resort

Gadi Barlevy, Daniel Bird, Daniel Fershtman,
and David Weiss

REVISED
August 23, 2023

WP 2022-14

<https://doi.org/10.21033/wp-2022-14>

FEDERAL RESERVE BANK *of* CHICAGO

*Working papers are not edited, and all opinions are the responsibility of the author(s). The views expressed do not necessarily reflect the views of the Federal Reserve Bank of Chicago or the Federal Reserve System.

MONEY UNDER THE MATTRESS: INFLATION AND LENDING OF LAST RESORT*

Gadi Barlevy[†]

Daniel Bird[‡]

Daniel Fershtman[§]

David Weiss[¶]

August 23, 2023

Abstract

This paper examines whether the two key functions of central banks—ensuring price stability and lending during crises—must be in conflict. We develop a nominal model of bank runs à la Diamond and Dybvig (1983) in which individuals can store the money they withdraw “under the mattress” or use it to buy assets. In this setting, lending of last resort does not need be inflationary. Whether it is inflationary depends on the interest rates the central bank charges. Our results suggest central banks should not charge low rates to ensure price stability, and should charge a high rate to robustly attain the ex-ante efficient outcome. These results are in line with the Bagehot rule of charging high interest rates on loans during a crisis.

Keywords: Bagehot rule, price-level stability, financial stability, bank run

JEL Codes: E31, E50, G01

*We thank Bob Barsky, Marco Bassetto, Nittai Bergman, Jeremy Greenwood, Moshe Hazan, Ed Nosal, Jonathan Rose, Harald Uhlig, and Eran Yashiv for invaluable comments. We also thank the editor and three anonymous referees. The views expressed in this paper do not reflect the views of the Federal Reserve Bank of Chicago or the Federal Reserve System. Bird and Weiss acknowledge financial support from the Sapir Center for Economic Development.

[†]Federal Reserve Bank of Chicago; email: gbarlevy@frbchi.org

[‡]Eitan Berglas School of Economics, Tel Aviv University; email: dbird@tauex.tau.ac.il

[§]Eitan Berglas School of Economics, Tel Aviv University; email: danielfer@tauex.tau.ac.il

[¶]CORRESPONDING AUTHOR Eitan Berglas School of Economics, Tel Aviv University; email: davidweiss@tauex.tau.ac.il; phone: +972584833163

1 Introduction

Historically, modern economies have looked to central banks to carry out two main functions: to act as lenders of last resort during credit crunches and to maintain price stability. An important question is whether these two goals conflict with one another: Central banks that act as lenders of last resort create money to lend to distressed banks, and such liquidity injections can potentially be inflationary. Mishkin (2001) captures this view in his discussion of an international lender of last resort, where he notes that “Injecting reserves, either through open market operations or by lending to the banking sector, causes the money supply to increase, which in turn leads to a higher price level.”

Consistent with this view, proposals to bail out distressed financial intermediaries during the Global Financial Crisis of 2007-8 generated widespread concern about impending inflation.¹ Similar concerns were voiced when central banks were forced to deal with financial stability issues in the midst of their efforts to contain inflation in the wake of the Covid pandemic. This includes the shift by the Bank of England from selling to buying gilts in September 2022 during the Fiscal Tantrum that threatened pension funds and the creation of the Bank Term Funding Program by the Federal Reserve in March 2023 that lent to banks after the collapse of Silicon Valley Bank.²

Academic research has also suggested that bank bailouts may be inflationary. For example, recent work by Schilling, Fernández-Villaverde and Uhlig (2022) reinforces the idea that financial stability and price stability may be in conflict. While their paper studies a run on a digital currency issued by the central bank rather than a run on banks, and does not consider bailout policies, one of their key results is that a central bank intent on avoiding a run will not be able to maintain a stable price level across all states of the world. More generally, Bordo (2018) has described the modern history of central banks in developed countries as a “varying evolution between monetary stability and financial stability,” suggesting the two goals are inherently incompatible.

This paper argues that the two goals of central banks do not need to conflict: It is

¹For real-time examples of these concerns, see “A Wider U.S. Financial Bailout Could Fuel Inflation,” *New York Times*, March 20, 2008 and “Deep Pockets,” *The Economist*, October 9, 2008.

²See, for example, Eshe Nelson on the tensions for the Bank of England in “A Central Bank on the High Wire,” *New York Times*, October 13, 2022, and Greg Ip on the tension for the Federal Reserve in “Fed Walks Tightrope Between Inflation and Bank Turmoil,” *Wall Street Journal*, March 22, 2023.

possible for a central bank to bail out banks during a panic without having to deviate from its commitment to price stability. This does not mean that there is no potential for conflict between the two roles. Achieving the two goals depends on whether the central bank appropriately sets the interest rate on loan offers to banks. It also requires that depositors who run on banks be able to save outside of the banking system after they withdraw their money rather than spend it and bid up prices.

Intuitively, the reason bailouts need not be inflationary is that the households who run on banks during a panic do not necessarily want to spend the money they withdraw. These households are patient and inclined to save but have lost faith in financial institutions. When offered a substitute for bank deposits — be it hoarding cash “under the mattress” or buying a different financial asset like government bonds — they may prefer saving to spending. If the central bank sets the interest rate on its loans in a way that keeps long-run prices in check, it will encourage patient households to hold money, which will in turn keep the short-run price level in check. If it lets long-run prices rise, patient households might opt to spend, driving up the short-run price level as well.

We establish this result using a monetary version of the quintessential model of bank runs by Diamond and Dybvig (1983). In this model, the central bank can avoid raising the price level by setting the interest rate on its loans to banks above some threshold. This includes a zero interest rate, which drains out the exact amount of liquidity the central bank injected. However, we show that positive rates would be equally effective at avoiding a higher price level while liquidity is high, and even negative interest rates might be consistent with price stability in the short run. Nevertheless, we show there is a limit on how low interest rates can go without triggering a rise in the price level.

We further show that if the central bank was not only concerned about short-run price stability but also about ensuring ex-ante efficiency even in the event of a run, it would need to charge a high interest rate on loans. If it charges too low of a rate, households that choose not to run can consume more than the ex-ante efficient amount at the expense of patient households who do run. The only way all households receive the ex-ante efficient levels of consumption even in the event of a run is to ensure that banks do not profit too much from borrowing from the central bank. At the same time, ensuring households receive the ex-ante efficient amount whether they withdraw early or not means there is nothing to discourage households from running. Moreover,

charging banks a high interest rate would drain money from circulation and result in a lower long-run price level, even though it would not affect the short-run price level while the liquidity it injects still circulates. Our results that central banks should avoid charging interest rates that are too low to maintain price stability and should charge high rates to attain ex-ante efficiency accord with the well-known Bagehot (1873) dictum that central banks should lend to banks at high interest rates during a crisis.

While we focus on central bank loans, the same logic would apply to fiscal bailouts financed by debt. A fiscal authority could lend to banks to allow them to pay their depositors. Patient households who withdraw could then use the cash they withdraw to buy newly issued debt. If households return the cash they withdraw to the Treasury by buying public debt, no new money would need to be created. The Troubled Asset Relief Program (TARP) during the 2008-9 Financial Crisis is an example of such a program: The government purchased preferred shares from banks at a predetermined rate of return, financing these purchases with bonds. A fiscal bailout can thus replicate lending of last resort. The return on preferred shares cannot be too low to maintain price stability, just as is true for the interest rate on bailout loans.

A fiscal bailout of this type is akin to a sterilized monetary intervention. Suppose a central bank created money to lend to banks during a run, but simultaneously sold government bonds to the public to drain the liquidity it added. The end result would be identical to a fiscal bailout: Patient households could be satisfied without needing to increase the money supply. Our results demonstrate that unsterilized monetary interventions, sterilized monetary interventions, and fiscal interventions can all be used to avoid inefficient liquidation without raising the short-run price level.

We proceed as follows. The remainder of this section discusses the related literature. Section 2 lays out the basic model and characterizes the efficient allocation. Section 3 discusses implementation of the efficient allocation in a nominal setting, including the potential for a bank-run equilibrium that features deflation and a fire sale. Section 4 discusses the tools at the central bank's disposal for saving the financial system during a bank run, and emphasizes the importance of the interest rate that the central bank charges banks in determining both the price level and welfare. Section 5 compares our basic, unsterilized monetary intervention to both a fiscal intervention and a sterilized monetary intervention. Section 6 concludes. All proofs are contained in an Appendix.

1.1 Related Literature

This paper contributes to a growing literature that incorporates money into Diamond and Dybvig models. While some results in this literature are related to ours, we offer novel insights on the tradeoff between financial stability and inflation, and particularly how this tradeoff depends on the terms at which the central bank lends to banks.

One branch of this literature involves models in which any money that the central bank creates is drained early on, and it is up to banks to create money in response to higher than expected demand for liquidity, e.g., if there is a run on banks or a larger number of impatient households than expected. Examples include Skeie (2008), Schilling, Fernández-Villaverde and Uhlig (2022), and Allen, Carletti and Gale (2014).

In Skeie (2008), agents who withdraw money must use it to buy goods from banks. In this setting, banks never run out of money, even during a run: Any money agents withdraw will simply return to the bank as spending on goods. While runs are possible, there is no need for the central bank to bail out banks when a run occurs.

Schilling, Fernández-Villaverde and Uhlig (2022) apply Skeie's setting to study the implications of issuing a central bank currency (CBDC). They argue that it will not be possible to achieve efficiency, avoid runs on the currency, and maintain price stability in all states of the world. If agents who run on the digital currency must buy goods, spending on goods would rise during a run. Should the central bank succeed in avoiding inefficient liquidation, the amount of goods sold would remain fixed. As a result, the short-run price level would rise. By contrast, agents who run (on banks) in our model can store cash rather than be forced to spend it on goods. The price level would therefore not have to rise if inefficient liquidation was averted.

In Allen, Carletti and Gale (2014), the fraction of impatient consumers and the return on long-run investments are both random. The socially efficient outcome requires patient agents to consume more when few impatient agents show up or when the realized return on long-run investments is high. Allowing agents to trade money can help to attain the socially efficient outcome. Allen, Carletti and Gale show one can implement the socially efficient outcome without inflation. While related to our result, the questions they study differ substantially from ours. Efficiency in our model involves preventing financially distressed banks from liquidating long-run investments, a feature

absent from their framework. Moreover, while money in their model allows agents to trade, it is otherwise “super-neutral” and has no effect on outcomes. In our setting, money is decidedly not neutral given that more of it can help prevent costly liquidation.

A separate branch of the literature assumes banks cannot create money and must instead acquire money that is created by the central bank. Examples of such models include Antinolfi, Huybens and Keister (2001), Carapella (2012), Robatto (2019), Andolfatto, Berentsen and Martin (2020), and Altermatt, van Buggenum and Voellmy (2022). Our model is closer to these papers: Banks in our model also hold a mix of money and long-run real investments, providing money to impatient agents who need liquidity and the proceeds from real investments to those who are patient. While some of our results mirror those in these papers, others are new to our setup.

Antinolfi, Huybens and Keister (2001) consider an infinite horizon setting in which a random fraction of agents need liquidity each period. Banks choose how much liquidity to hold before knowing how many agents need liquidity and can end up with too little liquidity. Antinolfi, Huybens and Keister show the central bank can achieve the first-best outcome without inflation by extending zero interest rate loans to banks when demand for liquidity is high. However, such loans admit additional equilibria with inflation. In our model, loans to banks serve to prevent inefficient liquidation rather than maintain the consumption of agents who withdraw early, and multiplicity is not an issue. More importantly, we consider various interest rates on loans rather than just zero interest rate loans.

Andolfatto, Berentsen and Martin (2020) develop a framework in which agents can trade money and long-run assets directly or deposit money in banks that invest in long-run assets. Their motivation is to compare asset trading and banking and to study the effects of inflation under these two cases (as well as when both coexist). While they have a section about bank runs, they only consider runs on a zero-measure set of banks that leaves the aggregate price level unchanged. By contrast, we consider a systemic bank run that can affect the aggregate price level. We also consider loans to banks with different interest rates, while they only discuss zero-interest rate loans.

Carapella (2012) and Robatto (2019) develop models in which banks experience financial distress that is similar to bank runs in our setting. They show that distress leads to deflation, as is true in our model. Robatto (2019) further shows that a monetary

injection that is subsequently withdrawn can attain the efficient outcome without raising the price level above its initial level. Such an intervention is equivalent to a zero-interest rate loan. However, in our model an intervention prevents liquidation, which combined with an injection of money could affect the price level in a way that does not occur in these papers. We also study different interest rates on loans rather than just zero.

Altermatt, van Buggenum and Voellmy (2022) also consider an economy in which impatient agents want money while patient agents seek returns on long-term investments. In their setup, a run does not affect the price level of goods but does affect the fraction of entrepreneurs who produce. A run in their model cannot affect the price level by design, and providing emergency liquidity does not produce inflation. In our model, a run does lead to deflation, and emergency liquidity can produce inflation, depending on the interest rate that the central bank charges banks.

Additional work by Martin (2006) and Keister (2016) considers bailout policies without incorporating money. These papers thus have nothing to say about the implications of bailouts for price stability.

Finally, our work is related to a large literature on lending of last resort and the Bagehot rule. A nice overview is provided in Bordo (2014) and Bordo (2018). We complement this literature by providing justifications for the Bagehot rule based on short-run price stability and ensuring ex-ante efficiency even in case of a run.

2 Model

We study a monetary version of Diamond and Dybvig's (1983) canonical model of bank runs that allows us to analyze price stability. The model features three periods, $t = 0, 1, 2$, and three types of agents: households, banks, and a government.

Households are risk-averse and ex-ante symmetric. The mass of households is normalized to 1. Each household is endowed with 1 unit of a good and M units of cash (issued to it by the government) at date 0. With probability λ a household is impatient and values consumption only in period 1, and with probability $1 - \lambda$ it is patient and values consumption only in period 2. Utility from consumption in the relevant period is given by a utility function $u(\cdot)$ that is strictly increasing, twice continuously

differentiable, and features a relative risk aversion coefficient that is strictly greater than unity; that is, for all $c > 0$, $-cu''(c) > u'(c)$. Households privately learn their type at the beginning of period 1. In terms of technology, households can store goods and money across periods but do not have direct access to productive investments.

We assume that the mass of banks is also equal to 1. Banks can offer households two key financial services. First, we assume that banks can access a technology that converts 1 unit of goods invested in period 0 into $1 + R$ units of goods in period 2 for some $R > 0$. This allows banks to invest on behalf of households.³ Banks can liquidate these long-run investments in period 1. If so, they will receive $1 - \kappa$ units of goods per unit invested for some $\kappa \in (0, 1)$. Since $\kappa < 1$, liquidating investments can provide a bank with resources in period 1 should it need them. Since $\kappa > 0$ and households can store goods, it will be inefficient for banks to invest all goods and then liquidate them to provide consumption as needed in period 1.

Second, banks can pool resources and insure households against being impatient. A household will tend to be better off if they are patient given investment only pays off in the long run when patient households consume. Risk-averse households would therefore be willing to give up some of the returns on investment they could consume in period 2 if they turned out to be patient in exchange for higher consumption in period 1 if they turned out to be impatient. Pooling allows banks to insure impatient households by committing to give less to patient households. That is, banks will not invest all of the deposits they raise and will instead try to provide impatient households with more consumption than the latter can get on their own. However, we assume that banks do not have the physical capacity to store goods. Banks can only store money to give to impatient households so they can go and buy goods elsewhere.

Finally, the government issues and reclaims money. With a finite horizon, households need a reason to hold cash until period 2. We assume that in period 2 the government is endowed with T goods, which it commits to sell for cash at the end of period 2. Money thus serves as a claim to buy goods from the government at date 2.⁴ As will become clear below, we will need to impose restrictions on the values T can assume.

³ We could allow households to have access to the same technology, but in line with Jacklin (1987), we would need to prevent households from trading equity shares in their investment to attain the first best allocation. We find it easier to assume only banks can make investments and households enter into exclusive contracts with banks that cannot be sold to others.

⁴Our approach is similar to the fiscal theory of the price level in which households must pay a real

2.1 The Efficient Real Allocation

We begin by characterizing the social planner's problem for this economy. The planner allocates goods to maximize the ex-ante welfare of the representative household. Denote the consumption of impatient and patient households by c_1 and c_2 , respectively. The social planner's objective is to maximize:

$$\lambda u(c_1) + (1 - \lambda)u(c_2), \quad (1)$$

subject to the budget constraint:

$$(1 - \lambda)c_2 \leq (1 - \lambda c_1)(1 + R) + T, \quad (2)$$

and the feasibility constraint:

$$\lambda c_1 \leq 1. \quad (3)$$

When $T = 0$, as in Diamond and Dybvig (1983), the solution to the planner's problem is interior, i.e., $\lambda c_1^* < 1$ and banks undertake some investment. When $T > 0$, we must restrict T to ensure an interior solution, as described in the next lemma.

Lemma 1. *There exists a cutoff $T^* > \frac{1-\lambda}{\lambda}$ such that if $T < T^*$, the planner's problem has a unique solution in which $\lambda c_1^* < 1$. This solution satisfies*

$$u'(c_1^*) = (1 + R)u'(c_2^*). \quad (4)$$

Moreover, under this solution, $c_1^* > 1$ and $1 < \frac{c_2^*}{c_1^*} < 1 + R$.

For large values of T , the planner would give all of the goods at date 0 to impatient households and let patient households consume the T goods that impatient households cannot access. To see why the bound on T exceeds $\frac{1-\lambda}{\lambda}$, suppose $T = \frac{1-\lambda}{\lambda}$. If we assigned all of the goods at date 0 to impatient households, impatient households would consume the same amount as patient ones. Since $u(\cdot)$ is concave and $R > 0$, such

tax obligation of T at date 2 using cash. See Cochrane (2021) for an extensive treatment. Diamond and Rajan (2006) and Robatto (2019) also use the fiscal theory to model money and banks in a finite horizon environment. We find it easier to assume the government sells goods rather than collects taxes to avoid the question of what households would do if they could not afford their tax liability.

an allocation would be suboptimal. In what follows, we assume $T < T^*$ and denote the solution to the planner's problem by (c_1^*, c_2^*) .

3 Decentralizing the Efficient Allocation

There are various ways to decentralize trade in this economy in order to attain the social optimum in Lemma 1. Given our interest in studying the connection between bank bailouts and inflation, we focus on a decentralization that allows bank runs.

Previous work that incorporated money into the Diamond and Dybvig framework by Skeie (2008) considered a decentralization in which banks buy all goods from households in period 0, storing some and investing others. Skeie further assumed households had to spend any cash they withdrew and could not store it. Every dollar households withdraw from banks thus returns to banks as payment for goods, and so banks never run out of cash and would never need to be bailed out.

We consider a different decentralization in which banks can fail. First, we do not require households who withdraw from banks to spend their cash; they can store it if they want. Second, as mentioned above, we assume banks are not equipped to store goods. They can only store money and undertake investments. Although undertaking investments requires banks to buy goods and use them as inputs into investment, we do not think of this investment as something that requires banks to physically house goods. Consistent with this, we assume that if banks liquidate investments in period 1, they cannot store the goods released but must sell them. Households, by contrast, can store goods (as well as money). Impatient households will use money to buy goods from other households rather than banks, and the money they spend will not flow back to banks. This decentralization is similar to Antinolfi, Huybens and Keister (2001), Robatto (2019), and Andolfatto, Berentsen and Martin (2020) in the way banks hold a mix of money and real long-term investments. Our novelty lies not in the decentralization but in using it to study how bailouts can affect price stability and efficiency.

We first describe the nature of trade. We then show there exists an equilibrium that implements the efficient allocation when all agents expect that only patient households withdraw early. We then argue that for the same equilibrium, patient households could

unexpectedly coordinate on a continuation equilibrium in which patient households withdraw early and banks liquidate investments.

3.1 Markets and Equilibrium

The trade between banks and households unfolds as follows:

Period 0: Households choose a bank to deposit their cash endowment. In exchange, they will be entitled to a share of the bank's equity in period 2, which they can choose to give up in period 1 for a cash payment of d from the bank. Since banks cannot store goods and liquidating investment is costly, banks will prefer to pay households in period 1 with money.

Contracts with banks are exclusive and non-transferable: Households cannot sell their claim to a bank's equity to others. Competition among banks will lead them to pledge all of their equity in period 2. The only relevant parameter of the contract is thus the amount of cash d that households receive for withdrawing early. The value of a bank's equity depends on the bank's investment decisions in period 0 and on how much investment it liquidates in period 1. For ease of exposition, we leave the details of how a bank's equity depends on these choices to Online Appendix B.

After households deposit money with banks, a Walrasian market opens in which goods are traded for cash. At this point, banks hold all of the cash and households hold all goods. If there is trade, it will be banks that buy goods from households. Banks will need goods if they want to undertake long-term investments.

Let P_0 denote the price of goods in terms of money in the Walrasian market. Since households are symmetric, we assume their trades (and those of banks) are identical. Let m_0 denote a household's cash holdings once the market closes. At the end of period 0, each household holds $s_0 = 1 - \frac{m_0}{P_0}$ units of the good and m_0 units of cash, while banks invest the $I = 1 - s_0$ units of goods they acquired and hold $M - m_0$ units of cash.⁵

Period 1: At the beginning of period 1, households learn their type and simultaneously announce whether they intend to withdraw early. Unlike the original Diamond

⁵Households cannot deposit cash at the end of period 0 with banks as at the beginning period 0, since at this point banks have already made investment decisions and cannot make use of new deposits.

and Dybvig model, there is no sequential service constraint in our model. All economic outcomes are contingent on the *total* fraction of patient households who withdraw early.

Impatient households will want to consume the s_0 goods they have and withdraw early so that they can spend $m_0 + d$ units of cash to buy more goods. Patient households must decide whether to store their s_0 goods or sell them, whether to withdraw cash or wait to receive an equity payment, and — if they withdraw — whether to store that money to buy goods in period 2 or use it to buy goods in period 1 and store them.

Let $\mu \in [0, 1]$ denote the fraction of patient households that withdraw early. If the requested amount of withdrawals $(\lambda + (1 - \lambda)\mu) d$ is below a bank's cash holding $M - m_0$, the bank will have enough cash to give d units of money to each household. After that, a Walrasian market opens in which goods can trade for cash. Let $P_1(\mu)$ denote the price of goods in this market. Impatient households in this market will want to buy goods with their cash. These goods must be supplied by patient households.⁶

If $(\lambda + (1 - \lambda)\mu) d > M - m_0$ and a bank cannot pay all of its demanded deposits, it will have to liquidate some of its investments to raise cash. That is, if demand for cash exceeds $M - m_0$, there will be a preliminary liquidation market in which banks can convert their investments to goods and sell them to households for the cash. Let $P_1^L(\mu)$ denote the price in this market and L the amount of investments banks liquidate. The most a bank can liquidate is its original investment of $1 - s_0$. If this doesn't cover the bank's obligations, it will pay all depositors whatever money it has after liquidation on a pro-rata basis. After this, the regular Walrasian market opens in which goods can be traded for cash at a price of $P_1(\mu)$.⁷

By the end of period 1, impatient households will have spent all of their money and consumed all their goods. Patient households will be left with some combination of cash and goods that they carry over into period 2.

Period 2: At the beginning of period 2, returns are realized on any investments the banks did not liquidate. Banks distribute any goods and cash they have on a pro-rata

⁶In principle, banks can also supply goods by liquidating investments. In equilibrium, however, banks would never liquidate investments if they had enough cash to pay their obligations in period 1.

⁷In principle, the price P_1 could depend on what transpired in the liquidation market. Our notation merely highlights that the price depends on μ rather than implying it is the only relevant variable.

period. We skip these details here to facilitate the exposition. While we chose not to include the amount of goods I that banks invest in period 0 or the amount of projects $L \leq I$ they liquidate in period 1 in our definition of equilibrium given they can be inferred from s_0 and s_1 , we will refer to these variables below.

3.2 An Efficient Equilibrium

We now look for an equilibrium that implements the planner's solution. We first look for such an equilibrium where there are no runs, i.e., where only impatient households withdraw in period 1 while patient households wait. We then argue this is the unique efficient equilibrium, i.e., the only way to implement the planner's solution is if $\mu = 0$. We will refer to it as the efficient equilibrium, anticipating that there is an additional continuation equilibrium not associated with the planner's solution in which at least some patient households unexpectedly run on banks in period 1.

Any equilibrium that implements the social planner's solution must satisfy the following four conditions.

First, banks invest the same amount as the planner. This implies

$$I = 1 - s_0 = 1 - \lambda c_1^*. \quad (5)$$

Second, since banks cannot physically store goods and will only buy goods to invest, the households' money holdings m_0 at the end of period 0 must equal the value of goods banks purchased to invest. That is, at date 0 we have

$$m_0 = P_0(1 - \lambda c_1^*). \quad (6)$$

Third, each impatient household consumes c_1^* in period 1. Since impatient households will consume the s_0 goods they carry over from the end of period 0 and use the money balances m_0 they carry over plus the d they withdraw to buy goods, this implies

$$s_0 + \frac{m_0 + d}{P_1} = c_1^*. \quad (7)$$

Fourth, no investment projects should be liquidated, i.e.,

$$L = 0. \tag{8}$$

Next, equilibrium prices must satisfy the following three restrictions.

First, households cannot strictly prefer either money or goods in period 0. The efficient allocation requires $\lambda c_1^* \in (0, 1)$ goods be stored in period 0, which only households can do. At the same time, households must hold the cash that they receive from selling goods to the banks. Hence, the return to holding money between period 0 and period 1 must equal the return on storage, i.e.,

$$P_0 = P_1. \tag{9}$$

Second, for the Walrasian market to clear in period 1, patient households must be willing to hold money. For these households to be willing to hold money into period 2, the return on money must at least equal the return on storage, i.e.,

$$P_1 \geq P_2. \tag{10}$$

Third, for the Walrasian market to clear in period 2, households must spend all of their money holdings to buy goods from the government. This implies

$$P_2 = \frac{M}{T}. \tag{11}$$

Finally, we use the equilibrium conditions above to derive one final condition that governs any no-run efficient equilibrium. Given equation (10), if a patient household withdraws in period 1, they would find it at least weakly optimal to hold their period-1 wealth in cash (as opposed to goods). From equation (7), we know their wealth if they withdraw early in period 1 will be $P_1 c_1^*$. In an efficient no-run equilibrium, this amount cannot buy more than c_2^* goods in period 2. If it did, a patient household could consume more than the socially optimal level, which is a contradiction. This implies

$$\frac{P_1 c_1^*}{P_2} \leq c_2^* \tag{12}$$

From Lemma 1, we know that $\frac{c_2^*}{c_1^*} < 1 + R$. Equation (12) can therefore only be satisfied if $\frac{P_1}{P_2} < 1 + R$. Since $P_0 = P_1$ from equation (9), holding cash from period 0 to period 2 has a real return that is less than investing between period 0 and 2. Since $\kappa > 0$ and $P_0 = P_1$, investing in a long-run project in period 0 with the intention of liquidating in period 1 is dominated by holding cash. Banks should thus hold the minimal amount of cash needed to pay depositors in period 1 and use the rest to buy goods to invest. In a no-run equilibrium, the minimal amount of cash banks will need is λd . Hence,

$$M - m_0 = \lambda d. \quad (13)$$

We can use equations (6)- (9), and (13) to obtain unique values for P_0, P_1, P_2, m_0 , and d , which we denote with a star:

$$P_0^* = P_1^* = \frac{M}{1 - \lambda}, \quad P_2^* = \frac{M}{T}, \quad m_0^* = \frac{1 - \lambda c_1^*}{1 - \lambda} M, \quad d^* = \frac{c_1^* - 1}{1 - \lambda} M. \quad (14)$$

We can solve for s_0^* using the fact that it coincides with the planner's solution:

$$s_0^* = 1 - I = \lambda c_1^*. \quad (15)$$

We can use the value of s_0^* to solve for the amount of money patient households hold at the end of period 1 in the no-run equilibrium:

$$m_1^* = m_0^* + P_1^* s_0^* = \frac{M}{1 - \lambda}, \quad (16)$$

Finally, we can solve for the amount of goods that patient households hold at the end of period 1:

$$s_1^* = s_0^* - \frac{m_1^* - m_0^*}{P_1} = 0. \quad (17)$$

In a no-run equilibrium that implements the planner's solution, patient households sell all of the goods they stored back in period 0 to impatient households. This leaves patient households with no goods, i.e., $s_1^* = 0$. Patient households also end up with the entire stock of money at the end of period 1, which they will use to buy goods from the government in period 2. Our first result shows that the above will indeed be an equilibrium when T assumes a particular range of values:

Proposition 1. *There exists a $T^{**} \in (1 - \lambda, T^*]$ such that if $1 - \lambda \leq T \leq T^{**}$, a no-run efficient equilibrium exists and is uniquely defined by (14) - (17). For such values of T , $\mu = 0$ in any efficient equilibrium, i.e., efficiency implies no early withdrawals by patient households.*

A no-run equilibrium that implements the first best exists if T is neither too small nor too large. In what follows, we will assume that $1 - \lambda \leq T \leq T^{**}$.

Intuitively, when T is small, money will not be sufficiently valuable to sustain trade in period 1. Since households can always store goods and consume them next period, the money they receive for selling goods in period 1 must compensate them for the goods they give up that period. This requires $\frac{P_1}{P_2} \geq 1$, i.e., households must be able to buy at least one good in period 2 for each good they give up in period 1. Given $P_1^* = \frac{M}{1-\lambda}$ and $P_2^* = \frac{M}{T}$, this is only possible if $T \geq 1 - \lambda$. For smaller values of T , a no-run equilibrium would still exist, but it would not implement the efficient allocation.

Conversely, when T is large, the value of cash is high and patient households prefer withdrawing in period 1 to waiting. A no-run equilibrium consistent with the planner's solution would be unsustainable.

Finally, the reason there can be no run in an efficient equilibrium is that banks will not liquidate any investments and will therefore have positive equity in period 2. When equity is positive, the payoff to waiting as $\mu \rightarrow 1$ would tend to ∞ as equity is divided over a vanishing mass of households. If not all households withdraw early, the only possible equilibrium is where patient households are indifferent between withdrawing in period 1 and waiting for period 2. But as we show in the Appendix, in equilibrium patient households cannot be indifferent between withdrawing early and waiting.

Note that P_1^* and P_2^* denote equilibrium prices for the efficient equilibrium. These prices will serve as benchmarks to compare equilibrium prices if there is a run and, later on, if the central bank provides loans to bail out banks during a run.

3.3 A Bank Run Continuation Equilibrium

We now show that the equilibrium in Proposition 1 that supports the efficient allocation can allow for a continuation equilibrium in which some (or all) of the patient households

choose to withdraw in period 1, contrary to the expectations in period 0. We will refer to any situation in which $\mu > 0$ as a run.

In the efficient equilibrium, only impatient households will withdraw early and banks hold just enough money to meet their demand for cash. Any value of $\mu > 0$ would require banks to liquidate investments to pay depositors. The reason there may be a continuation equilibrium with $\mu > 0$ is that withdrawing early is a strategic complement: If a household withdraws and forces a bank to liquidate, the value of equity decreases for other households. In the original Diamond and Dybvig (1983) model, this complementarity implied $\mu = 1$ was always a continuation equilibrium. If all other households ran, no equity would remain, and so a household would be better off withdrawing early and receiving something. In our model, we need an additional condition on the cost of liquidation κ to ensure that $\mu = 1$ is a continuation equilibrium:

Proposition 2. *For every $\mu > 0$ there exists $\bar{\kappa}(\mu) \in [0, 1)$ such that a bank will have 0 equity if a fraction μ of patient households withdraw in period 1 and $\kappa \geq \bar{\kappa}(\mu)$. Hence, $\mu = 1$ is an equilibrium if $\kappa > \bar{\kappa}(1)$.*

To understand the role of κ , recall that households in our model hold positive amounts of goods ($s_0^* > 0$) and cash ($m_0^* > 0$) going into period 1. They thus do not rely on the cash they withdraw (d^*) to pay for their entire consumption, in contrast to the original Diamond and Dybvig model. While it is still true in our model that $c_1^* > 1$, the consumption value of the cash amount d^* promised to households in period 1 may be less than 1. If that were the case, obtaining one good per each good invested back in period 0 may suffice to cover the banks' obligations even if all patient households withdraw. Only if banks receive less than one good per unit of goods invested — as would be the case when $\kappa > 0$, would banks run out of equity. Of course, if the equilibrium real value of d^* exceeded 1, then $\mu = 1$ would leave banks with zero equity, just as in the original Diamond and Dybvig model.⁹

Proposition 2 concerns the existence of a continuation equilibrium in which all patient households run. This can only be an equilibrium if banks are left with zero

⁹ As a numerical example, suppose $\lambda = \frac{1}{2}$, $R = 1$, $T = \frac{3}{4}$ and households have CRRA utility $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. We can show that $\kappa^*(1) = 0$ iff $\gamma > \frac{\log(2)}{\log(27/14)} > 1$. That is, if there were no liquidation costs ($\kappa = 0$) and γ was not too large, banks would remain solvent even if there was a full run.

equity when all households run: As we noted above, with positive equity, the payoff to waiting as $\mu \rightarrow 1$ would tend to $+\infty$, and waiting would be better than withdrawing.

If bank equity were positive when $\mu = 1$, there could still be an equilibrium in which $0 < \mu < 1$. In such an equilibrium, patient households would be indifferent between d dollars in period 1 and a share of remaining bank equity in period 2 after liquidation. While such a run would not cause the bank to fail, it would still force inefficient liquidation. The logic behind bailouts does not hinge on bank failures but on the way inefficient liquidation cuts into what patient households can consume.

More generally, even if there were no continuation equilibrium with $\mu > 0$, we could still ask what would happen off the equilibrium path if a fraction $\mu > 0$ of patient households happened to withdraw. Would that force the central bank to choose between avoiding liquidation and price stability? In what follows, we mostly consider what happens if a fraction $\mu > 0$ of patient households withdraw in period 1 without asking whether this μ is a continuation equilibrium. As with any well-specified dynamic game, we can determine prices $P_1(\mu)$ and $P_2(\mu)$ given a value of μ . We will refer to these as equilibrium prices even for values of μ that are not themselves equilibria. We reserve the term *continuation equilibrium* to refer to values of μ that are consistent with patient households optimally choosing whether to withdraw, as well as the equilibrium prices given such μ . We will eventually return to the question of whether values of $\mu > 0$ are in fact continuation equilibria. In particular, we will ask whether bailouts work by avoiding liquidation when runs occur or by preventing runs from even occurring.

Bank Runs, Fire Sales, and Deflation

While our primary interest is in whether avoiding liquidation interferes with price stability, we briefly discuss what would happen absent any intervention if a positive fraction $\mu > 0$ of patient households did run. Banks would need to liquidate investments in the liquidation market to cover their obligations. Since households can spend at most m_0^* in the liquidation market, the equilibrium price given μ in the liquidation market for an amount of liquidation L is bounded above, i.e., $P_1^L(\mu) \leq \frac{m_0^*}{(1-\kappa)L}$.¹⁰

¹⁰Note that L and $P_1^L(\mu)$ may not be uniquely determined for a given μ . Intuitively, if $P_1^L(\mu)$ is low, banks will need to liquidate more to cover their obligations, and the higher L will depress the price $P_1^L(\mu)$. If $P_1^L(\mu)$ is high, banks do not need to liquidate as much, and the lower L supports a higher price $P_1^L(\mu)$.

In principle, households can also sell some of the s_0^* goods they stored back in period 0 in the liquidation market. Unlike banks, households can wait for the second market in period 1 to sell these stored goods. This ensures $P_1(\mu) \geq P_1^L(\mu)$. If this inequality were reversed, no goods would be sold in the Walrasian market at the end of period 1. But this cannot be an equilibrium given impatient households would try to buy goods with the cash they withdraw from banks after liquidation. Formally, we have

Lemma 2. $P_1^L(\mu) \leq P_1(\mu)$ for any $\mu > 0$. That is, the equilibrium price of goods in the liquidation market given μ cannot exceed the equilibrium price of goods in the Walrasian market.

The inequality in Lemma 2 can be strict, i.e., $P_1^L(\mu) < P_1(\mu)$. If that were the case, patient households would wait to sell goods in the Walrasian market. Banks would also want to wait, but they need cash to pay depositors. Impatient households would prefer to buy goods in the liquidation market than in the Walrasian market, but they each have only m_0^* units of cash. Patient households would buy goods for a low price in the liquidation market and sell them to impatient households for a higher price in the Walrasian market, effectively front-running impatient households. Once impatient households withdrew money from their bank, they would buy goods in the Walrasian market from patient households — both the s_0^* goods they stored and the goods they purchased from the bank in the liquidation market. This suggests a strict inequality could be possible if d^* was large, which would push up $P_1(\mu)$, and m_0^* was small, which would push $P_1^L(\mu)$ down. From (14), this will be true for large c_1^* . In that case, banks would be forced to liquidate their investments in what is effectively a fire sale.

Although banks liquidate their investments in a separate market, this liquidation will tend to dampen the price of goods in the Walrasian market in period 1 as compared to the equilibrium without a run. We confirm this in the next proposition.

Proposition 3. *The equilibrium price $P_1(\mu) \leq P_1^*$ for any $\mu > 0$. More precisely,*

1. *If $T \in (1 - \lambda, T^{**}]$, goods prices given μ satisfy $\frac{M}{T} = P_2(\mu) \leq P_1(\mu) < P_1^* = \frac{M}{1-\lambda}$.*
2. *If $T = 1 - \lambda$, then $P_1(\mu) = P_2(\mu) = \frac{M}{T}$ for all μ .*

Hence, the price of goods in period 1 is weakly lower if there is a run than if there is no run.

Proposition 3 shows that unless $T = 1 - \lambda$, a bank run will coincide with a fall in the price level. This finding is similar to Carapella (2012) and Robatto (2019), who show that financial distress is associated with a lower price level. It is also consistent with empirical evidence on inflation during financial crises. However, the reason the price level is lower during a run in our model is a bit different. In our model, a bank run is associated with liquidation: More goods are up for sale than when there is no run, even though the economy is in some sense poorer. At the same time, the amount of money spent on goods is either unchanged or falls. Households may receive less than d^* if their bank collapses. While more households hold cash during a run, in equilibrium patient households who run on the bank will store money rather than spend it. Consistent with this, $P_1(\mu) \geq P_2(\mu)$, so there is no reason for households to buy goods in period 1. Since the same or even a smaller amount of money will chase after more goods, the price level falls. In Carapella (2012), the causality runs in the opposite way: A fall in the price level causes financial distress because banks earn less on the goods they sell. In Robatto (2019), financial intermediaries redirect money towards those with a higher propensity to spend. Financial distress leads to a decline in the amount of money spent, as in our model. But there is no sense that more goods are up for sale as in our model.

4 Intervention

So far, we have shown that the efficient equilibrium in our economy is susceptible to bank runs that can force banks to liquidate investments inefficiently. It is well understood that there are various ways to either avoid a run or avoid liquidation if a run occurs. Diamond and Dybvig (1983) explain how suspension of convertibility and deposit insurance can prevent runs. Keister (2016) studies fiscal bailouts using real resources, while Andolfatto, Berentsen and Martin (2020) discuss how central banks can lend money to banks during a run. In this section, we also analyze lending by the central banks during a run. Our contribution is not to show bailouts work but to analyze how the terms on loans to banks affect price stability and allocative efficiency.

To incorporate bailouts, note that banks will need $\mu(1 - \lambda)d^*$ to avoid liquidation if a fraction $\mu > 0$ of patient households withdraw in period 1. We assume that after μ is revealed in period 1, the central bank offers to lend banks exactly this amount. Banks

that accept these loans will be charged a nominal interest rate $i(\mu)$. For notational ease, we will mostly suppress the dependence on μ and refer to the interest rate as i . Banks must repay the monetary authority $(1+i)\mu(1-\lambda)d^*$ in period 2 in cash. We assume debts to the monetary authority are senior to all other obligations.

Note that for banks to borrow from the monetary authority, the interest rate i cannot be too large. First, banks must be willing to borrow rather than liquidate. The cost to borrow a unit of money, $1+i$, cannot exceed the forgone investment from raising one unit of money by liquidation, i.e., $\frac{P_2(1+R)}{P_1(1-\kappa)}$. Second, banks must be able to repay the central bank in full. The amount they would have to pay if they borrowed is $(1+i)\mu(1-\lambda)d^*$. The most they could pay is $P_2(1+R)I$. Hence, $1+i$ would also have to be below $\frac{P_2(1+R)I}{\mu(1-\lambda)d^*}$. Let $1+\bar{i}(\mu)$ denote the minimum of these two rates.¹¹ We henceforth focus on $i \leq \bar{i}(\mu)$.

Since banks will need to raise cash to repay the monetary authority before they can pay their other liabilities, we need a market in period 2 in which banks can sell some of the proceeds of their investments for cash before distributing them to investors. We assume banks can sell the goods they earned from their investments in a preliminary market before the government sells its goods. Let $P_2^L(\mu)$ denote the price in this market. Banks must raise $(1+i)\mu(1-\lambda)d^*$ in cash to repay their debt. They could in principle raise more than this amount in cash. Whether they do so is irrelevant: Households that buy goods in the liquidation market will simply buy fewer goods from the government. Any excess cash banks raise in the liquidation market returns to equity holders as cash, so the money available to buy goods from the government in the next market is unchanged. Without loss of generality, we assume banks sell the minimal amount they need. Banks will then distribute their equity to remaining depositors on a pro-rata basis.

Finally, period 2 still ends with a Walrasian market in which the government sells its endowment of T goods. We continue to denote the price in the latter market by $P_2(\mu)$, although the price will also depend on i . Since banks can avoid liquidation by borrowing from the central bank, there will not be a need for a liquidation market in period 1 if banks borrow. The equilibrium path of prices given μ when the monetary authority intervenes is thus given by $P_1(\mu)$, $P_2^L(\mu)$, and $P_2(\mu)$.

¹¹As we show below, P_2 depends on i . The upper bound $\bar{i}(\mu)$ is thus defined implicitly. Either consideration could be binding. As $\kappa \rightarrow 1$, liquidation becomes infinitely costly and affordability becomes binding. As $\mu \rightarrow 0$, affordability becomes moot and cost considerations become binding.

4.1 Price Dynamics

We now consider whether avoiding inefficient liquidation requires the monetary authority to sacrifice its commitment to price stability. While there is no explicit reason in our model for the monetary authority to care about price stability, there are well known arguments in favor of price stability that our setup ignores. One example is heterogeneity in price stickiness across producers. In that case, a departure from price stability would lead to misallocation. We abstract from reasons for the monetary authority to care about price stability and only ask whether intervention is consistent with price stability.

To determine whether there is a tradeoff between avoiding liquidation and price stability, we need to solve for the equilibrium prices given $\mu > 0$ when banks can borrow from the monetary authority. We begin with the following observation:

Lemma 3. *If $1 + i > 0$, in any equilibrium in which banks borrow from the monetary authority, prices in periods 2 are given by $P_2^L(\mu) = P_2(\mu) = \frac{M - i\mu(1-\lambda)d^*}{T}$.*

As long as $1 + i > 0$ so that banks must pay the central bank a positive amount of cash, the price of goods in the liquidation market will be the same as in the subsequent Walrasian market. This is in contrast to the liquidation market in period 1, in which banks might be forced to liquidate investments at a fire-sale price. The difference is that in period 1, households buying in the liquidation market can only access part of the money they are entitled to. Even if the price in that market were low, they could not shift all of their demand to buy at the lower price. This is not the case here: Patient households buy goods with cash, and they start period 2 with all of the cash in the economy. We can thus refer to a single price in period 2. Since we focus on loans that allow banks to avoid liquidation, there is also a single price $P_1(\mu)$ in period 1. We will therefore refer to the effects of interventions only on the two prices $P_1(\mu)$ and $P_2(\mu)$.

The price $P_2(\mu)$ is equal to the ratio of the cash households bring into the Walrasian market and the goods the government sells. The logic is the same as without interventions: Households have no use for money at the end of period 2 and will spend it all to buy the T goods the government sells. However, the amount of cash households have by the end of the period now depends on the interest rate i .

Previous work on bank bailouts such as Antinolfi, Huybens and Keister (2001), Robatto (2019), and Andolfatto, Berentsen and Martin (2020) focused on the case of $i = 0$. A zero interest rate ensures that the monetary authority removes from circulation the exact amount it lent out to banks. This is a necessary and sufficient condition for bailouts to have no effect on long-run price stability. Consistent with this, $i = 0$ is the unique interest rate on loans to banks in our model that ensures $P_2(\mu) = P_2^* = \frac{M}{T}$, i.e., the price level if there were no run. If the monetary authority wants the long-run price after its intervention to be the same as in the efficient equilibrium, it would need to charge banks an interest rate of $i = 0$.

However, the concerns of policymakers and academics about threats to price stability stem from the short-run effects of injecting liquidity while this money is circulating, not the long-run effects of loans that are eventually repaid. As we show below, ensuring that $P_2(\mu)$ lines up with the price level in the efficient equilibrium is not equivalent to ensuring that $P_1(\mu)$ lines up with the price level in the efficient equilibrium.¹²

The price of goods in period 1 depends on what patient households who run on the bank do with the money they withdraw: Do they buy goods and store them, bidding up the price of goods, or do they store money so that demand for goods is the same as when there is no run? That decision depends on how the price $P_1(\mu)$ compares with $P_2(\mu)$: Patient households will buy goods when they are cheapest. Whether $P_2(\mu)$ exceeds $P_1(\mu)$ in turn depends on the interest rate i that the monetary authority charges banks. For $P_1(\mu)$ to equal P_1^* in period 1, the price level in period 2 cannot exceed P_1^* . Since $P_2(\mu)$ is decreasing in i , the interest rate on loans i must be sufficiently high to ensure that the price level $P_2(\mu)$ in period 2 does not exceed P_1^* .

Formally, we know from Lemma 3 that $P_2(\mu) = \frac{M - i\mu(1-\lambda)d^*}{T}$. Given the value of d^* in (14), for $P_2(\mu)$ to be at least as high as $P_1^* = \frac{M}{1-\lambda}$ requires that

$$i \geq \frac{1 - \lambda - T}{\mu(1 - \lambda)(c_1^* - 1)} \equiv i^\dagger(\mu). \quad (18)$$

Our next result shows that the equilibrium $P_1(\mu)$ will equal P_1^* when $i \geq i^\dagger(\mu)$ and will

¹² Relatedly, Berentsen and Waller (2011) show how monetary policy interventions that will be undone after one period can be non-neutral in the short run even though they are neutral in the long run. We are similarly interested in the short-run impact of a policy on prices in period 1, which may be different from the long-run impact on prices in period 2.

exceed P_1^* if $i < i^\dagger(\mu)$.

Proposition 4. *Suppose $\mu > 0$ and that banks borrow $\mu(1 - \lambda)d^*$ at interest rate i from the monetary authority to avoid liquidation.*

1. *If $i \geq i^\dagger(\mu)$, then $P_1(\mu) = P_1^* = \frac{M}{1-\lambda}$ and $P_2(\mu) = \frac{M-i\mu(1-\lambda)d^*}{T}$. Any interest rate above the cutoff ensures that the short-run price level is the same as if there were no run.*
2. *If $i < i^\dagger(\mu)$, then under a bailout $P_1(\mu) = P_2(\mu) = \frac{M-i\mu(1-\lambda)d^*}{T} > \frac{M}{T}$. The price level is higher in both periods than it would be if there was no run.*

When $P_2(\mu)$ exceeds the value P_1^* , the price $P_1(\mu)$ will also have to exceed P_1^* . If this were not the case, the fraction μ of patient households who withdrew early would buy goods in period 1 given the price in period 1 was lower. With more money being spent on the same amount of goods (due to the banks not having to liquidate any goods), we would counterfactually get $P_1(\mu) > P_1^*$. Setting i high enough will push $P_2(\mu)$ below P_1^* . At that point, the fraction μ of patient households who withdraw early will store money and wait to buy goods in period 2. But then the price level in period 1 would be the same as with no run, i.e., $P_1(\mu) = P_1^*$.

Note that the cutoff $i^\dagger(\mu) \leq 0$. When $T = 1 - \lambda$, which recall is the lowest value of T we allow, the cutoff interest rate $i^\dagger(\mu)$ equals zero. Ensuring short-run price stability in this case requires the monetary authority to remove all of the liquidity it injected. For $T \in (1 - \lambda, T^{**}]$, the cutoff interest rate $i^\dagger(\mu)$ is strictly negative. The central bank can maintain short-run price stability even without removing all of the liquidity it injected. For these values of T , the equilibrium price $P_1^* = \frac{M}{1-\lambda} > \frac{M}{T} = P_2^*$, and so even if we raise $P_2(\mu)$ a little above P_2^* , patient households would choose to store the cash they withdraw. Stabilizing the price level in period 1 can thus be consistent with allowing the long-run price level $P_2(\mu)$ to exceed, equal, or fall below its level in the absence of a run, $P_2^* = \frac{M}{T}$. To stabilize the price level in the short run, the central bank must not set i too low rather than set $i = 0$ and promise to drain out exactly the same amount of liquidity it previously injected in the bailout.

4.2 Welfare and Redistribution

So far, we have shown that a monetary authority that wants to ensure its liquidity injection does not increase the short-run price level $P_1(\mu)$ can do so by avoiding setting the interest rate too low. The fact that short-run price stability is achievable under a wide range of interest rates suggests that the monetary authority may have some freedom in setting the interest rate on loans to achieve other objectives. A natural objective beyond avoiding inefficient liquidation and ensuring price stability is to ensure that the welfare of households is equal to the ex-ante utility they are due under the optimal allocation.

When banks are able to avoid liquidation, changing the interest rate i that banks must pay the monetary authority has no effect on the quantity of goods available for consumption in each period. Instead, changes in i only lead to a zero-sum reallocation of goods between three types of households: impatient, patient who withdraw early, and patient who wait. This reallocation will occur through changes to market prices.

When $i \geq i^\dagger(\mu)$, patient households who withdraw early will store money and wait to buy goods in period 2. Since the price level $P_2(\mu)$ in period 2 is decreasing in i , patient households who withdraw early will benefit from higher i . Since total consumption in period 2 is constant and patient households who withdrew early are better off with a higher i , those patient households who waited must be worse off with a higher i . Finally, since the price $P_1(\mu)$ does not change with i when $i \geq i^\dagger(\mu)$, impatient households will be unaffected by a higher i . Increasing i above the cutoff $i^\dagger(\mu)$ redistributes resources from patient households that wait to patient households that withdraw early.

When $i < i^\dagger(\mu)$, Proposition 4 establishes that both $P_1(\mu)$ and $P_2(\mu)$ are decreasing in i . In this case, both impatient households and patient households who withdraw early benefit from a higher i , while patient households who wait will be worse off. This is because a higher i leaves fewer goods for patient households who hold bank equity.

Since $P_1(\mu) = P_1^*$ for all $i \geq i^\dagger(\mu)$, impatient households get to consume c_1^* if the central bank sets $i \geq i^\dagger(\mu)$ for all μ : The bailout ensures impatient households are paid the full amount d^* promised to them by the bank even when there is a run, and since the price coincides with the price under the efficient equilibrium, the amount impatient households consume, $s_0^* + \frac{m_0^* + d^*}{P_1^*}$, will be the same as in that equilibrium.

Setting $i \geq i^+(\mu)$ thus ensures impatient households receive the utility they are due under the social optimum. Is there also a way to set interest rates so that even with a run, all patient households consume c_2^* whether they withdraw or not? The next proposition summarizes the effect of changing i on welfare for the different types of households and shows that there indeed exists an interest rate $i^{++}(\mu) \geq 0$ that ensures patient households consume the same amount whether they withdraw early or wait.

Proposition 5. *Suppose $\mu > 0$ and that banks borrow $\mu(1 - \lambda)d^*$ at rate i .*

1. *When $i < i^+(\mu)$, raising i increases the utility of impatient households. When $i \geq i^+(\mu)$, raising i does not change the utility of impatient households.*
2. *Raising the interest rate i decreases the utility of patient households that do not withdraw early and increases the utility of those that withdraw early.*
3. *There exists an interest rate $i^{++}(\mu) \geq 0$ for every $\mu > 0$ at which the utility for patient households is the same whether they withdraw early or wait.*

The last part of Proposition 5 states that the interest rate that equates utility across patient households is nonnegative. Since $i^{++}(\mu) \geq 0 \geq i^+(\mu)$, setting $i = i^{++}(\mu)$ leads impatient households to consume c_1^* . Since all patient households consume the same amount, it follows that for this rate they consume c_2^* .

To see why the interest rate $i^{++}(\mu)$ that equates the consumption of all patient households must be nonnegative, observe that the wealth of patient households who run is equal to $P_1(\mu)c_1$, where c_1 is the amount impatient households consume (and could in principle differ from c_1^*). The only way to ensure that patient households with this wealth consume c_2^* is to induce patient households who withdraw to store money and then set the price $P_2(\mu)$ so that $P_1(\mu)c_1 = P_2(\mu)c_2^*$. If patient households who run store money, then $P_1(\mu) = P_1^*$ and $c_1 = c_1^*$. In that case, we would need

$$P_1^*c_1^* = P_2(\mu)c_2^* \tag{19}$$

Recall that a necessary condition for the efficient equilibrium to feature no runs is equation (12) which stipulates that

$$P_1^*c_1^* \leq P_2^*c_2^*$$

Ensuring that patient households who run consume c_2^* thus requires setting $P_2(\mu) \leq P_2^*$, so the amount of money in circulation in period 2 must stay fixed or fall.¹³

The observation that getting all patient households to consume c_2^* may require reducing the money stock in period 2 has important implications. First, while ensuring ex-ante efficiency implies $P_1(\mu) = P_1^*$ and is thus compatible with short-run price stability, it would conflict with long-run price stability if it required $P_2(\mu) < P_2^*$.

Second, it may not always be possible to remove the necessary amount of money in period 2 by charging banks $i = i^{++}(\mu)$. Equation (19) implies that to achieve the ex-ante efficient outcome requires setting $P_2(\mu) = P_1^* \frac{c_1^*}{c_2^*}$, which is independent of μ . The net amount of money removed from circulation due to loan repayments is $i\mu(1 - \lambda)d^*$. This amount tends to 0 as $\mu \rightarrow 0$ unless $i \rightarrow \infty$. When few households run, the central bank cannot charge banks $i^{++}(\mu)$ and get them to agree to borrow. In that case, policymakers could still ensure the ex-ante efficient outcome by setting a tax $\tau(\mu)$ on banks that banks must pay in cash in period 2. For example, banks might be asked to contribute more to a deposit insurance fund following a run. This observation highlights that while high interest rates can help to ensure ex-ante efficient consumption, any intervention that increases government revenue can achieve the same result.

4.3 Implications for Lender of Last Resort Policy

We now collect our results to summarize the policy implications of our analysis. A bailout rule specifies an interest rate $i(\mu)$ on loans to banks as a function of the fraction μ of patient households that withdraws early. The role of this bailout rule is as follows:

Theorem 1. *Suppose that the monetary authority lends the amount $\mu(1 - \lambda)d^*$ to banks at rate $i(\mu) \leq \bar{i}(\mu)$ whenever $\mu > 0$.*

1. **Financial Stability:** *Since $i(\mu) \leq \bar{i}(\mu)$, there is no inefficient liquidation, no bank failure, and no fire sales.*

¹³For $T \in [1 - \lambda, T^{**})$, ensuring patient households consume c_2^* would require setting P_2 strictly below P_2^* , i.e., the central bank would have to reduce the amount of money in circulation.

2. **Price Level Stability:** If $i(\mu) \geq i^+(\mu)$ for all μ , then $P_1(\mu) = P_1^*$ for all μ and prices are stable in the short run regardless of how many households run. If $i(\mu) = 0$ for all μ , then $P_2(\mu) = P_2^*$ for all μ and prices are also stable in the long run.
3. **Efficiency:** Whether consumption is ex-ante efficient depends on the rate $i(\mu)$:
 - If $i(\mu) \geq i^+(\mu)$ for all μ , then impatient households consume c_1^* .
 - If $i(\mu) = i^{++}(\mu)$ for some μ , then patient households consume c_2^* regardless of whether they withdraw or wait, when a fraction μ of patient households run.
4. **Equilibrium Runs:** Whether $\mu > 0$ is a continuation equilibrium depends on $i(\mu)$:
 - If $i(\mu) < i^{++}(\mu)$ for all μ , the only continuation equilibrium is $\mu = 0$ (no run).
 - If $i(\mu) = i^{++}(\mu) \leq \bar{i}(\mu)$ for some μ , then there exists a continuation equilibrium in which a fraction μ of patient households run.

Figure 2 illustrates the key insights from Theorem 1 graphically. It shows the range of interest rates that are consistent with price level stability in the short and long run, as well as how interest rates affect consumption.

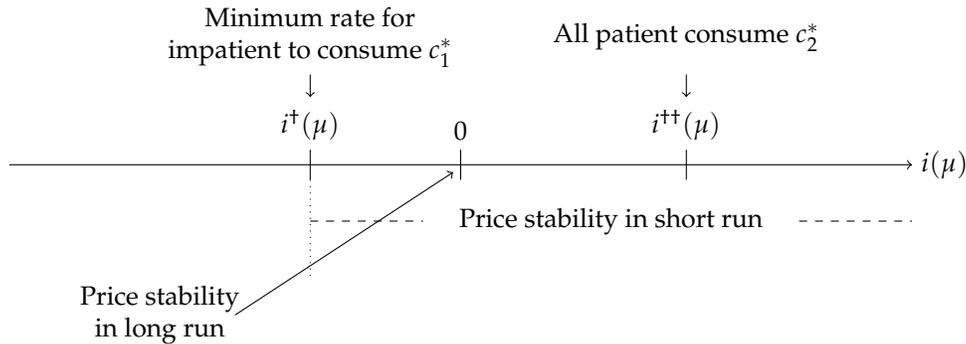


Figure 2: Interest Rates, Price Stability, and Consumption

The monetary authority can avoid inefficient liquidation and bank failures by lending at rate $i(\mu) \leq \bar{i}(\mu)$. Although lending can result in higher price levels, it does not have to result in higher price levels. Ensuring price stability in both the short run and long requires setting $i(\mu) = 0$ for all μ . If the monetary authority is only concerned

with price stability in the short run when it injects liquidity, it can avoid liquidation and maintain short-run price stability by setting $i(\mu) \geq i^+(\mu)$ for all μ .

Within the range of interest rates that avoid liquidation and ensure $P_1(\mu) = P_1^*$, the choice of interest rate will affect whether the allocation will coincide with the ex-ante efficient allocation if there is a run. Impatient households will consume the ex-ante efficient level c_1^* for any $i \geq i^+(\mu)$, regardless of the realization of μ . To ensure patient households also consume the ex-ante efficient level c_2^* for any realization of μ , the monetary authority would have to set $i(\mu) = i^{++}(\mu)$. For any other interest rate, patient households who withdraw early will consume a different amount from patient households who hold on to bank equity. In particular, those who withdraw early will consume less than those who wait if $i(\mu) < i^{++}(\mu)$ and will consume more than those who wait if $i(\mu) > i^{++}(\mu)$. A monetary authority can avoid inefficient liquidation, maintain short-run price stability, and ensure all patient households consume the ex-ante efficient amount c_2^* , but only by making loans sufficiently costly to banks.

Part (4) of Theorem 1 establishes that if $i(\mu) < i^{++}(\mu)$ for all μ , patient households will not run: Given any $\mu > 0$, patient households would be better off waiting. If the central bank only wanted to maintain price stability and ensure the ex-ante efficient outcome for all continuation equilibria in period 1, including continuation equilibria that were not expected in period 0, it could set $i < i^{++}(\mu)$ and know that $\mu = 0$ is the unique continuation equilibrium. But if the central bank wanted to further guard against off-equilibrium scenarios for μ , it would need to set $i(\mu) = i^{++}(\mu)$ to ensure all patient households consume c_2^* . For example, the central bank might worry that households are aware that it intends to bail out banks. It might then worry that some fraction μ of patient households could still run. Setting $i(\mu) = i^{++}(\mu)$ should ensure that all patient households consume c_2^* even if a fraction μ of patient households run, but with two important caveats. First, this would only be effective if $i^{++}(\mu) \leq \bar{i}(\mu)$, which is not true for very small values of μ . Second, part (4) of Theorem 1 also tells us that by guarding against the scenario where a fraction μ run, the central bank allows that run to become a continuation equilibrium. Intuitively, if patient households receive c_2^* regardless of what they or others do, nothing will discourage them from running. Safeguarding against off-equilibrium scenarios can turn them into equilibrium scenarios.

Relationship to the Bagehot Rule

Our results on the type of interest rates on loans that achieve particular outcomes is reminiscent of the so-called Bagehot rule, which states that the central bank should lend freely to banks during a crisis at a high rate of interest. Our model provides various reasons to charge banks a high interest rate. On the one hand, charging a low interest rate conflicts with the goal of (short-run) price stability. Separately, charging a high interest rate robustly ensures ex-ante efficiency even if there is a run.

Various authors have offered interpretations for the original Bagehot (1873) rule. One often-cited explanation is that a central bank should charge a high rate to discourage unprofitable borrowing and to screen risky borrowers to reduce losses for the central bank. Others, such as Goodhart (1999), have argued that the purpose of discouraging borrowing is to avoid an excessive expansion of the money stock, which is in line with our observations on price stability. Bignon, Flandreau and Ugolini (2011) cite a related observation that focuses on stabilizing the exchange rate. Our model suggests an additional motive for a high interest rate based on ex-ante efficiency.

5 Fiscal Bailout and Sterilization

While our discussion so far has focused on liquidity injections by the central bank, the same logic would apply if the bailout was done by a fiscal authority. Intuitively, when government finances decline and the government collects less money from the private sector, it would have to collect more money when it sells the T goods for its budget to remain in balance. More generous loans to banks would thus require a higher long-run price level in period 2, which can in turn lead to a higher price level in period 1.

There are various ways for a government to bail out banks in case of a run. We briefly discuss two such interventions: a fiscal bailout and a sterilized intervention.

Under a fiscal bailout, governments provide resources to banks in exchange for some future promise of payment. This was the logic behind the Troubled Asset Relief Program (TARP) during the financial crisis of 2008, which empowered the U.S. Treasury to purchase preferred shares from banks. These shares came with a promised rate of return

and were junior to demand deposits but senior to equity, meaning the government would get repaid before equity holders.

Since the fiscal authority cannot create cash as the monetary authority can, a fiscal bailout requires the government to raise funds by selling government bonds for cash. Specifically, the government would sell bonds to patient households for the cash they withdrew from banks. The money that patient households withdraw is thus absorbed by the government in period 1 rather than held by the private sector. Patient households can save outside of the banking system without storing cash. Beyond this difference, a fiscal intervention is equivalent to a monetary authority lending money to banks. As long as the promised payment on preferred shares is high enough, the bailout would not increase P_1 , similarly to how we showed a high i ensures price stability.

We can interpret a fiscal bailout as giving cash to banks to pay off depositors while simultaneously removing the newly generated cash from the economy by issuing debt. If the central bank owned government debt, it could similarly lend to banks and then sell bonds to soak up the newly generated cash. This is known as a sterilized intervention. By contrast, creating money to lend to banks that is then held by the public as in the model above corresponds to an unsterilized intervention. Since it makes no difference if such an action is taken by the treasury or monetary authority, a fiscal bailout and sterilized intervention are equivalent in our model.

It may seem counterintuitive that an unsterilized intervention yields the same implications for financial stability and price level stability as either a fiscal bailout or sterilized intervention. Our model illustrates that the key is that patient households are in principle willing to save under the mattress, or outside of the financial system, in lieu of an account at a bank. Just because they have lost faith in banks does not mean that they feel compelled to spend any money they have on hand. Since patient households in our model are happy to store money under the mattress, they effectively sterilize the monetary intervention themselves, removing it from active circulation.¹⁴

¹⁴Humphrey (1989) discusses the importance of sterilizing bank bailouts, attributing the idea to the original work of Thorton (1802) and Bagehot (1873).

6 Conclusion

This paper investigated the view advanced by some policymakers and researchers that there is an inherent conflict between the central bank's role as a guarantor of price stability and its role as lender of last resort during the crisis. Our analysis does not deny that the two goals could conflict, but suggests that the conflict is not inexorable as some of the discussion suggests. The terms at which the central bank lends during a bank run affect what households who run on a bank do with the cash they withdraw, which determines what happens to the price level following a bailout. If the central bank avoids setting a rate that is too low, lending to banks should not lead to higher prices. Our setup further suggests that by setting a high rate, the central bank can ensure both short-run price stability and ex-ante efficiency even in the case of a run.

We conclude with two observations. First, as we discuss in the paper, there are various ways to prevent inefficient liquidation during a run, including ways that also ensure short-run price stability. We specifically cited fiscal bailouts and sterilized interventions as examples. In our framework, these are equivalent to lender-of-last-resort loans. However, there may be reasons outside our model to prefer one approach over the other. This could involve ease of communicating policy to the public or features that our model ignores that can break the equivalence between these different policies in our model. These issues are something for future work to explore.

Second, while our results suggest that the lender-of-last-resort role does not have to conflict with price stability, they do not speak to potential conflicts between price stability and financial stability more broadly. The crisis prompted by the collapse of Silicon Valley Bank helps illustrate this point. On the one hand, the Federal Reserve created a facility to lend to troubled banks and mitigate the fallout from a run. Our results suggest this can be done without threatening price stability. A separate question is whether policies designed to rein in inflation, such as increasing the short-term interest rate, might encourage depositors to run on banks as they appeared to do in the case of Silicon Valley Bank. Whether policies aimed at promoting price stability might encourage runs is a separate question from whether bailouts once a run has occurred must be inflationary. Exploring the connection between policies that promote price stability and the possibility of runs is also best left for future work.

References

- Allen, Franklin, Elena Carletti, and Douglas Gale.** 2014. "Money, Financial Stability and Efficiency." *Journal of Economic Theory*, 149: 100–127.
- Altermatt, Lukas, Hugo van Buggenum, and Lukas Voellmy.** 2022. "Systemic Bank Runs without Aggregate Risk: How a Misallocation of Liquidity May Trigger a Solvency Crisis." University of Essex, ETH Zurich, and Swiss National Bank.
- Andolfatto, David, Aleksander Berentsen, and Fernando M. Martin.** 2020. "Money, Banking and Financial Markets." *Review of Economic Studies*, 87(5): 2049–2086.
- Antinolfi, Gaetano, Elisabeth Huybens, and Todd Keister.** 2001. "Monetary Stability and Liquidity Crises: The Role of the Lender of Last Resort." *Journal of Economic Theory*, 99(1–2): 187–219.
- Bagehot, Walter.** 1873. *Lombard Street: A Description of the Money Market*. Henry S. King and Co.
- Berentsen, Aleksander, and Christopher Waller.** 2011. "Price-Level Targeting and Stabilization Policy." *Journal of Money, Credit, and Banking*, 43(7): 559–580.
- Bignon, Vincent, Marc Flandreau, and Stefano Ugolini.** 2011. "Bagehot for beginners: the making of lender-of-last-resort operations in the mid-nineteenth century." *The Economic History Review*, 1–29.
- Bordo, Michael D.** 2014. "Rules for a Lender of Last Resort: An Historical Perspective." Hoover Institute: Central Banking in the Next Century: A Policy Conference.
- Bordo, Michael D.** 2018. "An Historical Perspective on the Quest for Financial Stability and the Monetary Policy Regime." *Journal of Economic History*, 78(2): 319–357.
- Carapella, Francesca.** 2012. "Banking Panics and Deflation in Dynamic General Equilibrium." Federal Reserve Board.
- Cochrane, John.** 2021. *The Fiscal Theory of the Price Level*. Draft Book.
- Diamond, Douglas W., and Philip H. Dybvig.** 1983. "Bank Runs, Deposit Insurance, and Liquidity." *Journal of Political Economy*, 91(3): 401–419.

- Diamond, Douglas W., and Raghuram G. Rajan.** 2006. "Money in a Theory of Banking." *American Economic Review*, 96(1): 30–53.
- Goodhart, C.A.E.** 1999. "Myths about the Lender of Last Resort." *International Finance*, 2(3): 339–360.
- Humphrey, Thomas M.** 1989. "Lender of Last Resort: the Concept in History." *Economic Review*, 8–16.
- Jacklin, Charles.** 1987. "Demand Deposits, Trading Restrictions, and Risk Sharing." In *Contractual Arrangements for Intertemporal Trade.*, ed. Edward Prescott and Neil Wallace, 26–47. University of Minnesota Press.
- Keister, Todd.** 2016. "Bailouts and Financial Fragility." *Review of Economic Studies*, 83(2): 704–736.
- Martin, Antonie.** 2006. "Liquidity Provision vs Deposit Insurance: Preventing Bank Panics without Moral Hazard." *Economic Theory*, 28: 197–211.
- Mishkin, Frederic S.** 2001. "The International Lender of Last Resort: What are the Issues?" In *The World's New Financial Landscape: Challenges for Economic Policy.*, ed. Horst Siebert, 291–312. Springer-Verlag.
- Robatto, Roberto.** 2019. "Systemic Banking Panics, Liquidity Risk, and Monetary Policy." *Review of Economic Dynamics*, 34: 20–42.
- Schilling, Linda, Jesús Fernández-Villaverde, and Harald Uhlig.** 2022. "Central Bank Digital Currency: When Price and Bank Stability Collide." University of Pennsylvania working paper.
- Skeie, David R.** 2008. "Banking with Nominal Deposits and Inside Money." *Journal of Financial Intermediation*, 17(4): 562–584.
- Thorton, Henry.** 1802. *An Enquiry Into the Nature and Effects of the Paper Credit of Great Britain.* James Humphreys.

A Appendix

Proof of Lemma 1. In optimum the budget constraint (2) is clearly binding. If the feasibility constraint (3) is binding, then $c_1 = 1/\lambda$, and by (2), $c_2 = T/(1 - \lambda)$. If it were the case that $T \leq (1 - \lambda)/\lambda$, then $c_1 \geq c_2$, which, by the concavity of $u(\cdot)$, would violate the first-order condition of Equation (1). Furthermore, there exists a unique $T^* > (1 - \lambda)/\lambda$ for which the first-order conditions hold when the feasibility constraint is binding. It follows that for any $T < T^*$, the solution to the social planner's problem is interior. As the objective is concave, the solution is unique and given by the solution of Condition (4).

Next, we show that $1 < \frac{c_2^*}{c_1^*} < 1 + R$ and $c_1^* > 1$. The fact that $1 < \frac{c_2^*}{c_1^*}$ follows immediately from the concavity of $u(\cdot)$ and Equation (4). Assume by way of contradiction that $\frac{c_2}{c_1} \geq 1 + R$. Then,

$$\begin{aligned} (1 + R)u'(c_2) &\leq (1 + R)u'((1 + R)c_1) \\ &= u'(c_1) + \frac{1}{c_1} \int_{c_1}^{(1+R)c_1} \frac{\partial}{\partial c} (cu'(c)) dc \\ &= u'(c_1) + \frac{1}{c_1} \int_{c_1}^{(1+R)c_1} (u'(c) + cu''(c)) dc \\ &< u'(c_1), \end{aligned}$$

where the first inequality follows from the concavity of u and the second inequality follows from the assumption that the relative risk aversion coefficient is strictly greater than unity. Thus, the FOC does not hold, a contradiction. Therefore, $\frac{c_2^*}{c_1^*} < 1 + R$.

Finally, if it were the case that $c_1 \leq 1$, then by (2), $c_2 \geq 1 + R + T/(1 - \lambda)$. This, in turn, would imply that $\frac{c_2}{c_1} > 1 + R$, a contradiction. Thus, $c_1^* > 1$. ■

Proof of Proposition 1. We begin by fully characterizing the equilibrium candidate derived in the main text. We then verify that this is indeed an equilibrium. Finally, we show that there is no equilibrium that implements the efficient allocation with $\mu > 0$.

The market clearing condition in the goods market in period 1 implies that banks do not liquidate, $L = 0$. Moreover, market clearing in the money market implies that

the cash holding of banks at the end of periods 0 and 1 are given, respectively, by $m_0^B = M - m_0^*$ and $m_1^b = 0$.

To satisfy Condition (10) requires that $\frac{M}{T} \leq \frac{M}{1-\lambda}$. This implies that for an equilibrium that implements the ex-ante efficient allocation to exist, we need

$$T \geq 1 - \lambda.$$

Next, we show that Condition (12) holds if T is not too large. From the solution of the planner's problem, we have that $c_1^*(T) < c_2^*(T)$ for any T , where $c_i^*(T)$ denotes the solution of the social planner's problem as a function of T . Suppose that $T = 1 - \lambda$. Then $T = 1 - \lambda < (1 - \lambda) \frac{c_2^*(T)}{c_1^*(T)}$. The social planner's problem is continuous in T , implying $c_1^*(T), c_2^*(T)$ are also continuous in T . By continuity, there exists a maximal $T^{**} \in (1 - \lambda, T^*]$ such that $T \leq (1 - \lambda) \frac{c_2^*(T)}{c_1^*(T)}$ for all $T \in [1 - \lambda, T^{**}]$. Thus, constraint (12) is satisfied if $T \leq T^{**}$.

Finally, we need to verify that the equilibrium is consistent with households and banks acting optimally. Since $P_0^* = P_1^*$, households are willing to store both cash and goods between periods 0 and 1. Since $P_2^* \leq P_1^*$, patient households would rather store cash in period 1. Finally, since Condition (12) holds, patient households do not want to withdraw early.

Next, consider the banks. For a bank's decision to avoid liquidation to be optimal, P_1^L needs to be low enough. Since liquidation is an off-path event, we are free to specify what this price would be if a liquidation market opened. Banks are already providing households with the maximal expected utility, so there is no better contract a bank could offer to steal depositors away. Since $P_0^* = P_1^*$, it is optimal to pay deposits from stored money (rather than by liquidating investments). Moreover, Condition (12) implies that $\frac{P_0^*}{P_2^*} < 1 + R$. Thus, a bank maximizes the value of its equity by investing all the money it does not need to pay out deposits.

We now show that for $T < T^{**}$ the efficient allocation cannot be implemented by some other equilibrium in which $\mu > 0$. Suppose by way of contradiction that there were an equilibrium that implemented the efficient allocation in which $\mu > 0$. For impatient households to obtain the efficient allocation, their budget in period 1 must equal $P_1 c_1^*$. If it is optimal for patient households to withdraw early, then it must be the

case that $\frac{P_1 c_1^*}{P_2} \geq c_2^*$. Since the equilibrium implements the efficient allocation, patient households must consume c_2^* , and so

$$\frac{P_1 c_1^*}{P_2} = c_2^*. \quad (20)$$

As households hold both cash and goods between period 0 and 1, we must have $P_0 = P_1$. In combination with the fact that there is no liquidation and banks cannot store goods, under such an equilibrium we must have that

$$\lambda d + \mu(1 - \lambda)d = M - m_0. \quad (6')$$

By an analogous argument to the one used in the main text, such an equilibrium would be given by the solution of (6'), (7), (9), and (13) and the added constraint (20). Solving this system of equations yields

$$\mu = \frac{c_2^*(1 - \lambda) - c_1^*T}{(c_1^* - 1)c_2^*(\lambda - 1)}.$$

Since $c_2^* > c_1^* > 1$ and $\lambda \in (0, 1)$, we can only have $\mu > 0$ if $c_2^*(1 - \lambda) < c_1^*T$ or

$$\frac{c_2^*}{c_1^*}(1 - \lambda) < T.$$

However, by the definition of T^{**} this inequality does not hold for any $T < T^{**}$. ■

Proof of Proposition 2. If μ patient households demand their deposit, then banks must raise $\mu(1 - \lambda)d^*$ units of cash by liquidating assets. Proposition 3 below shows that the price in the liquidation market is at most P_1^* . Therefore, an upper bound on what banks can raise by liquidation is $(1 - s_0^*)(1 - \kappa)P_1^*$. A bank will fail if

$$(1 - s_0^*)(1 - \kappa)P_1^* < \mu(1 - \lambda)d^*.$$

Define $\bar{\kappa}(\mu)$ as the solution to

$$(1 - s_0^*)(1 - \kappa(\mu))P_1^* = \mu(1 - \lambda)d^*,$$

Clearly, $\bar{\kappa}(\mu) < 1$. ■

Proof of Lemma 2. Suppose towards a contradiction that $P_1^L(\mu) > P_1(\mu)$. In this case, no household would be willing to buy goods in the liquidation market. Since banks must sell goods in the liquidation market in order to raise cash, the liquidation market would not clear. ■

Proof of Proposition 3. In period 2, households spend all available money to buy goods from the government, implying that $P_2(\mu) = \frac{M}{T}$.

Suppose that $P_1(\mu) \geq P_1^*$. From Proposition 1, we know that $P_1^* = \frac{M}{1-\lambda} \geq \frac{M}{T} = P_2^*$. If $T > 1 - \lambda$, then $P_1(\mu) \geq P_1^* > P_2(\mu)$, and only impatient households buy goods in period 1, then patient households would strictly prefer money in period 1.

We now consider two different cases, depending on whether $P_1^L(\mu) = P_1(\mu)$ or $P_1^L(\mu) < P_1(\mu)$ (by Lemma 2, these are the only two possible cases). If $P_1^L(\mu) = P_1(\mu)$, the price level that clears both markets (the regular market and the liquidation market) in period 1 will be given by

$$P_1(\mu) = \frac{\lambda(\tilde{d} + m_0)}{(1-\lambda)s_0^* + (1-\kappa)\mu(1-\lambda)L'}$$

where \tilde{d} is the deposit paid out to households, and $L > 0$ is the amount of assets liquidated by banks. Note that $L > 0$ as banks do not hold excess cash reserves and therefore must liquidate some assets whenever $\mu > 0$. Recall that $P_1^* = \frac{\lambda(d^* + m_0^*)}{(1-\lambda)s_0^*}$. Since $\tilde{d} \leq d^*$ and $L > 0$, it follows that $P_1(\mu) < P_1^*$; in contradiction to the assumption that $P_1(\mu) \geq P_1^*$.

Next, suppose that $P_1^L(\mu) < P_1(\mu)$. All households will spend all of their cash in the liquidation market (since there is an "arbitrage" between the liquidation and regular market) to buy the $(1-\kappa)L$ goods supplied by the banks. In the regular market, patient households will sell both the goods that they stored and those they purchased in the liquidation market (since $P_1(\mu) > P_2$). Furthermore, impatient households will purchase goods with only \tilde{d} units of cash (as they spend their stored cash in the liquidation market). It follows that the market clearing price in the regular market is $P_1(\mu) = \frac{\tilde{d}}{(1-\lambda)(s_0^* + L(1-\kappa))} < P_1^*$. This contradicts the assumption that $P_1(\mu) \geq P_1^*$.

Finally, were it the case that $P_1(\mu) < \frac{M}{T}$, no household would hold money at the end of period 1, which cannot be an equilibrium. This ensures that $P_1(\mu) \geq \frac{M}{T}$.

If $T = 1 - \lambda$, we can use the arguments above to establish that $\frac{M}{T} \leq P_1(\mu) \leq P_1^* = \frac{M}{1-\lambda} = \frac{M}{T}$. Hence, if $T = 1 - \lambda$, the equilibrium price $P_1(\mu) = \frac{M}{T}$, as claimed. ■

Proof of Lemma 3: Since impatient households will spend money to buy goods in period 1, patient households must hold a positive amount of money at the end of period 1, i.e., $m_1 > 0$. Patient households can choose whether to spend their money holdings m_1 to buy goods from banks at a price $P_2^L(\mu)$ in the liquidation market or from the government at a price $P_2(\mu)$ in the Walrasian market. If $P_2^L(\mu) \neq P_2(\mu)$, households will only buy goods in the market where goods are cheaper. That means there exists one market in which demand for goods is zero but supply is positive: The government sells T goods, while banks will need to sell goods whenever their obligation to the monetary authority $(1+i)\mu(1-\lambda)d^*$ is strictly positive. The latter expression is strictly positive since $i > -1$.

Finally, the market clearing price in period 2 is given by $P_2(\mu) = \frac{M-i\mu(1-\lambda)d^*}{T}$ as the government sells its endowment of T against the outstanding cash in circulation $M - i\mu(1-\lambda)d^*$.

Proof of Proposition 4.

Part 1: Consider the case where $i \geq i^\dagger(\mu)$. Since banks do not liquidate investments or fail, impatient households receive their full deposit. Hence, their demand for goods in period 1 is given by $\frac{d^*+m_0}{P_1(i,\mu)}$, where $P_1(i,\mu)$ is the period 1 price when μ patient households withdraw early, and the monetary authority sets the interest rate at i . The amount of goods that impatient households purchase is at most $s_0^*(1-\lambda)$. It follows that the market clearing price is at least $\frac{d^*+m_0}{s_0^*(1-\lambda)}$.

If $i > i^\dagger(\mu)$, by the definition of i^\dagger , it holds that $P_1(i,\mu) > P_2(i,\mu)$, and so patient households would want to sell stored goods in period 1 and hold cash, regardless of whether they withdrew their deposit. It follows that the market clearing price must be

$$P_1(i,\mu) = \frac{d^*+m_0}{s_0^*(1-\lambda)} = P_1^*.$$

If, on the other hand, $i = i^\dagger(\mu)$, then patient households are indifferent between storing goods and money. However, should a strictly positive measure of them choose to store goods, the market clearing price in period 1 would exceed P_1^* , which, by the definition of $i^\dagger(\mu)$, would imply that patient households would strictly prefer to store money.

Part 2: Next, consider the case where $i < i^\dagger(\mu)$. From the definition of i^\dagger , in this case $P_2(i, \mu) > \frac{M}{1-\lambda}$. For the cash market to clear in period 1, it must be the case that $P_1(i, \mu) \geq P_2(i, \mu)$, as otherwise no one would hold cash. If $P_1(i, \mu) > P_2(i, \mu)$, only impatient households will want to buy goods, setting the market-clearing price to P_1^* , which, in this case, is below $P_2(i, \mu)$. Hence, clearing the markets requires that $P_1(i, \mu) = P_2(i, \mu) > \frac{M}{1-\lambda}$. ■

Proof of Proposition 5. Consider the case where $i < i^\dagger(\mu)$. In this case, by Proposition 4, the price levels are the same in both periods, and, moreover, the price levels are decreasing in i . Hence, the consumption of impatient households and patient household who withdraw early is increasing in i . Consequently, the consumption of patient households who do not withdraw must decrease in i .

Next, consider the case where $i \geq i^\dagger(\mu)$. From Proposition 4, we know $P_1(\mu) = P_1^*$ and $P_2(\mu) = \frac{M - i\mu(1-\lambda)d}{T}$. Therefore, for such cases, increasing the interest rate does not change the utility of impatient households, establishing part 1 of the proposition.

From Proposition 4 it follows that, in this case, patient households weakly prefer storing cash to storing goods in period 1. Hence, to compare the utilities of patient households who withdraw early and those who do not, it is sufficient to compare the nominal value of the assets that each type of household has in period 2. Households who withdraw early have: $d^* + m_0^* + P_1^*s_0^*$, whereas those who wait have $\pi(\mu, i) + m_0^* + P_1^*s_0^*$, where $\pi(\mu, i)$ represents the nominal value of an equity share in a bank.

The value of a bank's stored goods is

$$(1 - s_0^*)(1 + R)P_2(i, \mu) = (1 - s_0^*)(1 + R)\frac{M - i(1 - \lambda)\mu d^*}{T}.$$

As it must also repay the loan, its overall equity is worth

$$(1 - s_0^*)(1 + R)\frac{M - i(1 - \lambda)\mu d^*}{T} - (1 + i)(1 - \lambda)\mu d^*.$$

Hence, patient households that wait are better off than those who withdraw early if and only if $\pi(\mu, i) > d^*$, or:

$$\frac{(1 - s_0^*)(1 + R) \frac{M - i(1 - \lambda)\mu d^*}{T} - (1 + i)(1 - \lambda)\mu d^*}{(1 - \mu)(1 - \lambda)} > M \frac{c_1^* - 1}{1 - \lambda}.$$

The left hand side of this expression is decreasing in i , whereas the right hand side is constant in i . Since there is no liquidation (and impatient households consume c_1^* for any interest level in this case), it follows that as i increases patient households that did not withdraw become worse off, whereas those that did withdraw become better off. This establishes part 2 of the proposition.

To establish part 3 of the proposition, note that if $i = 0$, then by Proposition 4 the price levels in both periods are the same as in the efficient equilibrium. Hence, patient households who withdraw early will consume $c_1^* \frac{P_1^*}{P_2(\mu)}$. If $T < T^{**}$, this level of consumption is strictly less than c_2^* . That is, if $i = 0$ the consumption of patient households who withdraw early is less than their consumption in the efficient equilibrium. Since there is no liquidation and the consumption of impatient households remains the same, it follows that for $i = 0$ the nominal value of an equity share is greater than the nominal value of a deposit. Since the former is linear and decreasing in i , whereas the latter is independent of i , it follows that there exists $i^{++}(\mu) > 0$ for which the nominal value of a deposit is exactly equal to the nominal value of an equity share. In which case, patient households' consumption does not depend on their time of withdrawal. ■

Proof of Theorem 1. The only new part is part (4). By definition, the utility of households that withdraw early is lower than those who wait for $i < i^{++}(\mu)$. Hence, if $i < i^{++}(\mu)$, there cannot be a continuation equilibrium in which $\mu > 0$. If $i = i^{++}(\mu) \leq \bar{i}(\mu)$, then for the equilibrium prices given μ and the intervention, patient households are indifferent between waiting and withdrawing. This confirms μ is a continuation equilibrium. ■

B Online Appendix

In this appendix we formally derive the optimization problems of the banks and households, as well as the equilibrium conditions.

Banks' optimization problem.

A (representative) bank takes as given the market prices $\mathbb{P} = \langle P_0, P_1^L, P_1, P_2 \rangle$. Its objective is to provide the contract that maximizes the expected utility its depositors can attain (in combination with the depositors own trades). In this section, we derive the feasible contracts $\langle d, \pi \rangle$ a bank can offer, where d is the money households receive if they withdraw early and π is the expected real value of an equity share in the bank they will receive if they wait. We then analyze the households' problem, and derive their preferences over contracts of the form $\langle d, \pi \rangle$ given prices \mathbb{P} . Note that a bank's optimization problem depends on its beliefs about households' withdrawal strategies. Let η denote the measure of households the bank believes will withdraw in period 1.

A bank begins period 0 with M units of cash. In period 0, it must choose how many goods to purchase. We denote the amount of goods the bank purchases and invests in period 0 by I .

In period 1, a bank chooses how many goods to liquidate in the liquidation market, L . Denote the cash holdings of the bank at the end of period 0 by m_0^B . The bank's budget constraint in period 0 is

$$m_0^B = M - P_0 I. \quad (\text{BB})$$

Fix \mathbb{P} and η . A contract $\langle d, \pi \rangle$ is feasible if there exist $I \in [0, \frac{M}{P_0}]$ and $L \in [0, I]$ such that:

$$\eta d \leq m_0^B + P_1^L L (1 - \kappa), \quad (\text{Feasibility-1})$$

$$(1 - \eta)\pi = (1 + R)(I - L) + \frac{m_0^B + P_1^L (1 - \kappa)L - \eta d}{P_2}. \quad (\text{Feasibility-2})$$

The first constraint states that the bank will have enough cash to pay the η households that demand their deposits, whereas the second constraint equates the aggregate value of the equity shares to the real value of the bank's equity. If the demand for deposits is

$\tilde{\eta} > \eta$, we assume that the bank must either choose L such that $\tilde{\eta}d \leq m_0^B + P_1^L(1 - \kappa)L$ or else set $L = I$. That is, a bank must continue to liquidate as long as it falls short of the cash it is obligated to pay households who withdraw early.

Next, we show how a bank will maximize its equity value for a given set of prices. We begin with how the bank obtains the cash to pay out deposits. The nominal cost of carrying cash from period 0 to period 1 is 1. Alternatively, the bank can purchase a good, invest it in period 0, and sell it in the liquidation market. The cost of the latter option is $\frac{P_0}{P_1^L(1-\kappa)}$. Thus, if $(1 - \kappa) \frac{P_1^L}{P_0} < 1$, the bank will store money to pay deposits, while if $(1 - \kappa) \frac{P_1^L}{P_0} > 1$, the bank will not store money at the end of period 0.

Second, consider how the bank will obtain the promised equity π . There are three ways in which the bank can generate real goods in period 2: (i) it can purchase goods in period 0, invest them, and wait for the investment to mature. The nominal time-0 cost of obtaining a (marginal) unit of goods in period 2 in this way is $\frac{P_0}{1+R}$. (ii) It can hold cash from period 0 to period 2. The nominal time-0 cost of obtaining a (marginal) unit of goods in period 2 in this way is P_2 . (iii) It can purchase goods in period 0, invest them, liquidate them, and give the cash to the households. The nominal time-0 cost of obtaining a (marginal) unit of goods in period 2 in this way is $\frac{P_0}{P_1^L(1-\kappa)}P_2$. The bank will implement the feasible contract by choosing the cheapest of these three options.

Households' optimization problem.

The households take \mathbb{P} as given as well as the contract offered by their bank: namely, a nominal deposit d , and the expected real value of their equity share (in real goods) π . Let $\theta = \langle \mathbb{P}, d, \pi \rangle$ denote the state variables that are relevant for a household's decisions. It is convenient to describe and solve households' utility maximization problem using backward induction, starting from period 2.

In period 2, households consume all the goods that they have stored or received from banks and use all of their cash to purchase goods from the government. The household's period-2 budget constraint is

$$m_1 + P_2(\pi \cdot \mathbb{1}_{equity} + s_1) = P_2c_2, \quad (BC_2)$$

where m_1 (s_1) is the cash (real good) holding of a patient household at the end of period 1, and $\mathbb{1}_{equity}$ is an indicator for whether the household has an equity share of a bank. Hence, a patient household's utility in period 2 is

$$V_2^p(m_1, s_1; \theta) = u\left(\pi \cdot \mathbb{1}_{equity} + s_1 + \frac{m_1}{P_2}\right).$$

Next, consider period 1. A household that learns it is impatient will withdraw its deposit, use all of its cash to purchase goods, and consume all of its goods (stored or purchased). An impatient household's period-1 budget constraint is

$$m_0 + d + P_1 s_0 = P_1 c_1, \quad (BC_1 - impatient)$$

and so such a household has the following continuation utility:

$$V_1^I(m_0, s_0; \theta) = u\left(s_0 + \frac{d + m_0}{P_1}\right).$$

A household that learns that it is patient must decide how to allocate its resources between goods and cash. In particular, such a household's continuation utility, given its choice of whether to withdraw or not, is given by the solution to the following maximization problem:

$$W(m_0, s_0; \theta) = \max_{m_1, s_1} V_2^p(m_1, s_1; \theta)$$

s.t.

$$P_1 s_1 + m_1 = P_1 s_0 + m_0 + (1 - \mathbb{1}_{equity})d. \quad (BC_1 - patient)$$

Denote the values of m_1 and s_1 that solve this problem by $m^*(m_0, s_0, \mathbb{1}_{equity}; \theta)$ and $s^*(m_0, s_0, \mathbb{1}_{equity}; \theta)$, respectively.

This solution is "bang-bang:" if $P_1 < P_2$ the household will store only goods ($m^* = 0$) and if $P_1 > P_2$ the household will store only cash ($s^* = 0$). Hence, the value of a patient household at period 1, as a function of its resources at the end of period 0, is equal to

$$V_1^p(m_0, s_0; \theta) = \max\{W(m_0 + d, s_0; \theta), V_2^p(m^*(m_0, s_0; \theta), s^*(m_0, s_0; \theta) + \pi; \theta)\}.$$

Note that a patient household withdraws if

$$V_1^p(m_0 + d, s_0; \theta) > V_2^p(m^*(m_0, s_0; \theta), s^*(m_0, s_0; \theta) + \pi; \theta).$$

In period 0, the household's problem boils down to:

$$\begin{aligned} \max_{m_0, s_0} \lambda V_1^l(m_0 + d, s_0; \theta) + (1 - \lambda) V_1^p(m_0, s_0; \theta), \\ \text{s.t. } P_0 s_0 + m_0 = P_0. \end{aligned} \quad (BC_0)$$

Note that since there is another market in period 1, the solution of this problem is again "bang-bang:" if $P_0 < P_1$ the households will store only goods ($m_0 = 0$), whereas if $P_0 > P_1$ the household will store only cash ($s_0 = 0$).

Finally, consider the household's problem should a liquidation market be opened. The household's budget constraint in this case is given by

$$\text{s.t. } P_1^L s_1^L + m_1^L = P_1^L s_0 + m_0, \quad (BC_1 - liq)$$

where m_1^L and s_1^L are, respectively, the household's cash and goods holdings when the liquidation market closes. Since the liquidation market is followed by a regular market, the objective of the households in the liquidation market is to maximize their net wealth in the regular market, regardless of their type. Hence, if $P_1^L < P_1$ the households store only goods ($m_1^L = 0$), whereas if $P_1^L > P_1$ the households store only cash ($s_1^L = 0$).

Equilibrium

An equilibrium consists of the following objects:

- A price vector: $\langle P_0, P_1^L(\mu), P_1(\mu), P_2 \rangle$.
- A strategy for the bank: $\langle m_B, d, I, L(\mu) \rangle$.
- A strategy for the households $\langle m_0, s_0, m_1(\mathbb{1}_{equity}, \mu), s_1(\mathbb{1}_{equity}, \mu), m_1^L(\mu), s_1^L(\mu) \rangle$, where μ represents the probability with which patient households choose to withdraw in period 1, and the proportion of households that choose to do so.

These objects consist an equilibrium if the following conditions hold:

- The induced contract $\langle d, \pi \rangle$ is feasible and maximizes the households' expected utility over all feasible contracts.
- The banks' strategy is optimal given the prices.
- The households' strategy is optimal given the prices.
- All budget constraints are satisfied.
- The goods market clears in every period, that is,

$$s_0 + I = 1, \quad (MC_0)$$

$$s_1^L(\mu) = s_0 + (1 - \kappa)L(\mu), \quad (MC_1^L)$$

$$\lambda c_1(\mu) + (1 - \lambda)\mu s_1(0, \mu) + (1 - \lambda)(1 - \mu)s_1(1, \mu) = s_0 + (1 - \kappa)L(\mu), \quad (MC_1)$$

$$(1 - \lambda)\mu c_2(0, \mu) + (1 - \lambda)(1 - \mu)c_2(1, \mu) = (I - L(\mu))(1 + R) + T + (1 - \lambda)\mu s_1(0, \mu) + (1 - \lambda)(1 - \mu)s_1(1, \mu). \quad (MC_2)$$