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Money under the Mattress: Inflation and Lending of Last Resort*

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Abstract

Central banks create money to lend during credit crunches, which might lead to inflation. We examine whether the two key functions of central banks— price stability and last-resort lending— conflict. We develop a nominal model of bank runs à la Diamond and Dybvig (1983) in which individuals can store the money they withdraw "under the mattress" or use it to buy assets. This feature allows for lending of last resort without creating inflation. Our analysis also provides a new rationale for the "Bagehot rule": High interest rates prevent inflation, rather than mitigate the risk of lending during credit crunches.

Keywords: Bagehot rule, price-level stability, financial stability, bank run

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1 Introduction

Historically, modern economies have looked to central banks to carry out two main functions: to act as lenders of last resort during credit crunches and to maintain price stability. However, these two goals can potentially conflict with one another. This is because when central banks act as lenders of last resort, they essentially create new money to lend to those in need of credit. But such an injection of liquidity could in principle generate inflation. Recent work by Schilling, Fernández-Villaverde and Uhlig (2021) reinforces the idea that the two may be in conflict. While they study a run on a digital currency issued by the central bank rather than a run on banks, one of their key results is that a central bank intent on avoiding a run may not be able to maintain a stable price level across all states of the world. More generally, Bordo (2018) has described the modern history of central banks in developed countries as a "varying evolution between monetary stability and financial stability," suggesting the two goals are not inherently compatible.[1]

This paper examines whether these two main goals of central banking are potentially compatible with one another. To examine this question, we develop a monetary version of a model of bank runs à la Diamond and Dybvig (1983) (henceforth DD). A feature of our model that departs from the existing literature is that agents are allowed to save the money that they withdraw from banks rather than being forced to spend it. In most of the paper, we let agents hoard the money they withdraw and store it "under the mattress." While this is a convenient modeling device, agents can more generally use the money they withdraw to buy financial assets other than cash, such as government debt. Indeed, we offer some examples in which agents use the money they withdraw to purchase other assets. We show that as long as such saving is permitted, then there does not have to be a conflict between the two aforementioned functions of the central bank: It can act as a lender of last resort without having to give up on price stability.

We further show that whether lending to banks is compatible with price stability depends crucially on the rate of interest that the central bank charges borrower banks. Specifically, saving the financial system and maintaining price-level stability requires

[1]Given that financial crises have historically been associated with disinflation, concern about inflation during a run might seem surprising. However, to the extent that a bailout can help prevent a run or prevent the run from being costly, disinflation should not be an issue, and the relevant question becomes whether the liquidity the central bank injects to avoid the run could end up creating inflation.

the central bank to charge a sufficiently high interest rate on its loans. Our finding thus offers a new justification for the well-known Bagehot rule, which holds that central banks should lend at a high rate of interest during a crisis. Bagehot argued that the high rate of interest would "operate as a heavy fine on unreasonable timidity, and will prevent the greatest number of applications by persons who do not require [credit]. The rate should be raised early in the panic, so that the fine may be paid early; that no one may borrow out of idle precaution without paying well for it; that the Banking reserve may be protected as far as possible" (Bagehot, 1873).[2] The argument for a high interest rate in our framework is not to screen out timid, idle, or otherwise unworthy borrowers, but because it helps promote price stability. The latter is somewhat related to Bagehot's argument for protecting the Banking reserve, since failure to protect the central bank from losses can potentially generate inflation.

The intuition for our results is as follows. If patient agents can save outside the banking system, then when they withdraw their deposits, they will simply replace one asset, a bank deposit, with another, such as cash or government debt. The creation of more money will not lead to higher spending or push up the price level. In addition, the money created by loans is essentially unwound when banks repay the loans they receive from the central bank. The higher the interest rate on these loans, the more money that will eventually be drained. There is thus no reason that the money the central bank lends to banks to pay their depositors has to be inflationary, as long as the central bank charges its borrowers a sufficiently high interest rate. More generally, a higher interest rate on these loans increases government revenues, which leads to a lower rate of inflation given the intertemporal government budget constraint.

Although we focus on the central bank as the lender of last resort, in principle it could be the fiscal authority that bails out banks. We illustrate such a bailout policy in our model, which is inspired by the Troubled Asset Relief Program (TARP) during the 2008-2009 financial crisis. In this scheme, the government purchases preferred shares from banks at a predetermined rate of return. Banks then use these injected funds to pay off running depositors. Since the fiscal authority cannot print money, it raises the appropriate funds through the sale of government bonds. Thus, at the end of the day, banks do not fail given the fiscal bailout, and running depositors end up saving in government bonds rather than cash. The same intuition as above holds regarding the

[2] Much of Bagehot's advice can be traced back to Thorton (1802).

relationship between the terms on the preferred shares purchased by the government during such a bailout and inflation. For a fiscal bailout not to be inflationary, the government must earn sufficient revenue on the preferred stock it acquires.

To illustrate the importance of letting agents save outside of the financial system, we revisit the Schilling, Fernández-Villaverde and Uhlig (2021) paper. While they focus on runs on a digital currency issued by the central bank rather than runs on commercial banks, we show that the two types of runs are closely related. One key difference is that agents who run on a digital currency do not wish to hold on to the currency, so there is no notion of storing the money an agent withdraws under the mattress as there is in the case of a bank run. However, we show that if agents who run on the digital currency could buy an alternative asset rather than having to spend their digital currency on goods to get rid of it, the central bank could discourage runs against the digital currency without having to give up on its commitment to price stability. The conflict between price stability and avoiding runs that Schilling, Fernández-Villaverde and Uhlig (2021) find relies on limits on the ability of agents who run to find some other means by which to save.

We proceed as follows. In the remainder of this section we discuss related literature. Section 2 lays out the basic model and characterizes the efficient allocation. Section 3 discusses implementation of the efficient allocation in a nominal setting, including the potential for a bank-run equilibrium. Section 4 discusses the tools at the central bank's disposal for saving the financial system during a bank run, and emphasizes the importance of the interest rate that the central bank charges as a lender of last resort in determining the overall price level. Section 4.4 shows the equivalence between the central bank acting as a lender of last resort at a given interest rate and a fiscal bailout of the commercial bank. Section 5 re-examines the Bagehot rule in the context of our model, and in particular emphasizes that Bagehot's "high rate of interest" can be seen as reconciling the central bank's two goals of financial stability and price-level stability. In Section 6 we first review the impossibility result in Schilling, Fernández-Villaverde and Uhlig (2021), before discussing how allowing agents to store money can resolve the issue. Section 7 concludes. All proofs are relegated to the Appendix.

1.1 Related Literature

This paper contributes to a growing literature on both nominal DD models and the general relationship between banking and inflation. Skeie (2008) proposes a model in which the central bank prints unlimited cash during a bank run, but only on an intra-day basis. In his model, running agents spend their cash on physical goods, driving up prices, rather than storing their cash as an asset for the future. Schilling, Fernández-Villaverde and Uhlig (2021) build on Skeie (2008) to derive conditions that generate a tension between the goals of the central bank. Andolfatto, Berentsen and Martin (2020) build a nominal DD model and analyze how inflation rates affect the relative advantages of saving through the banking system rather than participating in asset markets directly, but do not analyze the inflationary impact of the central bank injecting liquidity during a bank run as we do here. Antinolfi, Huybens and Keister (2001) argue that an elastic provision of currency can eliminate financial instability when there are aggregate risks in the economy, but that it introduces price-level indeterminacy. However, their analysis differs from ours, as they focus only on the "good equilibrium," in which agents do not lose faith in the financial system. By contrast, the primary objective of our analysis is to understand the impact of the central bank's intervention during a bank run. Moreover, unlike us, they do not assume that there is a fiscal backing to cash, and so in their setting the central bank cannot save the banking system if the agents lose confidence in it. Finally, a number of papers have argued that the central bank's actions do indeed affect price stability and risk sharing in the context of aggregate shocks, which are not studied here (e.g., Allen and Gale, 1998; Allen, Carletti and Gale, 2014).

This is not the first paper to study the interaction between cash and government debt in the context of banking. Diamond and Rajan (2006) build a model of the banking system in which money takes the greater of the following two values: the fiscal value, which is set by government finances, and the transactional value, which is set by the scarcity of a medium of exchange. Robatto (2019) builds a three-period DD model with imperfect information extracted from heterogeneous bank balance sheets, in which money is valued via the government balance sheet to study households' flight to quality during financial crises, such as the Great Depression.

Bordo (2018) provides an overview of the history of central banks in developed countries and emphasizes the evolution of the use of various tools, such as lender of last resort actions, that central banks have employed to achieve their goals, including (but not limited to) price-level stability and financial stability. Bordo (2014) gives a history of lender of last resort institutions in particular. He begins by describing the origins of the Bank of England acting as a lender of last resort. He then provides an institutional and policy history of the U.S. from 1791 to the aftermath of the 2008 Great Recession. Along the way, he emphasizes the role of the Bagehot rule in guiding policy decisions, and how policy makers in recent decades have deviated from Bagehot's advice. Similarly, Mishkin and White (2014) review central bank policy from France, the UK, and the U.S. from the late nineteenth century until the Great Recession. They also argue that central bankers deviated greatly from Bagehot's rule during the Great Recession, but argue that many of these deviations had historical precedents. We complement this literature by providing a new lens with which to view Bagehot's advice.

2 Model

We study a nominal version of Diamond and Dybvig's (1983) canonical model of bank runs. There are three periods, $t = 0, 1, 2$, and an economy with a continuum $[0, 1]$ of agents. Each agent is endowed with one unit of a real good, as well as cash—a form of government liability—with a nominal value of $M > 0$ dollars. Agents are risk-averse, and ex-ante symmetric. In period 1, agents privately observe their realized types: impatient, with probability $\lambda \in (0, 1)$, or patient, with probability $1 - \lambda$. Impatient agents wish to consume only in period 1, whereas patient agents wish to consume only in period 2. Each agent's utility from consumption (in the relevant period) is given by $u(\cdot)$, where u is strictly increasing, twice continuously differentiable, and has a relative risk aversion coefficient that is strictly greater than unity; that is, for all $c > 0$, $-cu''(c) > u'(c)$. In the economy there is a storage technology that enables agents to transfer consumption goods from one period to the next. This storage technology yields one unit of the good for each unit of the good stored in the previous period.

It is important to this model that people can save outside of the financial system. For this to make sense, cash must have value beyond its use for exchange. We follow the spirit of the fiscal theory of the price level in modeling cash as a claim on government resources. As mentioned above, the government has previously issued cash with a nominal value of M dollars. We assume that the government reclaims this cash against taxes that it raises in period 2. This (unmodeled) taxation provides the government with $T > 0$ units of the consumption good. Thus, cash derives its value by being a claim against T, and people will be willing to store it rather than simply spend. Alternatively, one can interpret our setup to mean that the government is endowed with T goods in period 2 that it is willing to sell for cash, and private agents who have no reason to hold money at the end of date 2 intend to spend it all on these goods.

In the economy there is a representative commercial bank that stands in for a continuum of competitive price-taking commercial banks. In period 0, agents can deposit their real and nominal endowments in a commercial bank. The bank offers a contract that enables the depositors to choose in period 1 (after the investors learn their types) between receiving an immediate nominal payout of d_1 (a deposit contract) or becoming an equity holder of the bank and receiving an equal share of its remaining assets in period 2. The commercial bank has access to an investment technology that yields $1 + R$ units of the consumption good in period 2, where $R > 0$, for each unit of the consumption good that is invested in period 0. The investment can be liquidated (at a cost) in period 1, in which case it generates only $(1 - \kappa)$ units of the good, where $\kappa \in [0, 1)$.

The commercial bank uses its deposits as follows: It stores an amount S of the consumption goods it receives and invests the remaining $1 - S$ units of goods according to the investment technology described above. We assume that in period 1, the commercial bank sells the consumption goods that it has stored at the market price, P_1, and then pays agents who withdraw their deposits from the proceeds of these sales.If these sales do not raise sufficient cash to cover the withdrawals, then the commercial bank will use its cash reserves to pay the depositors. If these reserves are insufficient, then the commercial bank will raise additional cash by liquidating investment assets (as needed) and selling them as consumption goods in the market.

Each agent learns her type at the beginning of period 1, and then decides whether to withdraw d_1 dollars or become an equity holder in the commercial bank. Since impatient agents value consumption only in period 1, they withdraw in period 1, use their withdrawal to purchase goods at price P_1, and consume. Patient agents face a choice between withdrawing in period 1 and waiting until period 2. If a patient agent chooses to withdraw in period 1, she must then decide whether to purchase goods in the market in period 1 at price P_1 (and store them for consumption in period 2), or to store the cash and use it to purchase goods at price P_2 in period 2. As anticipated in the Introduction, the option of storing either goods or cash is important for our results.

If too many patient agents withdraw their deposits in period 1, the commercial bank will be forced to liquidate investment assets to raise cash. This may induce a classic "run on the bank," in which it is optimal for patient agents to withdraw their deposits early, leading to further stress on the commercial bank and potentially even to bank failure.

To alleviate potential stress on the financial system, the central bank can intervene by acting as a lender of last resort. In particular, we assume that the central bank specifies the interest rate, i, at which the commercial bank can borrow. If $i \geq 0$, we assume that the central bank allows the commercial bank to borrow any amount at that interest rate. On the other hand, if $i < 0$, we assume that the central bank also specifies a maximal loan size that we denote by $\overline{Q}$.[3]

We denote the fraction of *patient* agents that withdraw their deposits in period 1 by $\mu \in [0, 1]$ and an *intervention policy* for the central bank by a pair $\langle i(\cdot), \overline{Q}(\cdot)\rangle$ that specifies, as a function of μ, the interest rate and maximal loan.[4] Given our previous assumption, we impose that $\overline{Q}(\mu) = \infty$ for all μ such that $i(\mu) \geq 0$. Note that for the commercial bank, the policy $\langle i, Q\rangle$ is equivalent to receiving a fiscal bailout of the commercial bank in which instead of taking out a loan, the bank sells Q worth of preferred shares to the government with a promised payment rate of i. We discuss this alternative intervention policy in Section 4.4.

2.1 Prices and Equilibrium

The price level in period 2 ensures that all of the money still held by private agents at the beginning of the period is equal to the value of the T goods the government chooses

[3]The cap on loan size is needed because the commercial bank can always buy goods and store them, and so would borrow without limit whenever the *real* interest rate is negative. The bound is not related to issues associated with a negative *nominal* interest rate. If we modified the model so the central bank aimed for a positive inflation rate, a similar cap would be needed for positive nominal interest rates.

[4]There is a small technical issue as to who owns the residual assets of the bank if all patient agents withdraw in period 1, but the intervention of the monetary authority prevents the bank's failure. One potential solution is to assume that, in such a case, the bank maximizes its profits for its own sake and whatever consumption goods that the bank owns are disposed of. Another solution is to assume that there is always an infinitesimal measure of patient agents who wait for period 2, even in the case of a bank run.

to sell in that period. In particular, if the commercial bank borrowed Q at interest rate i in period 1, then the amount of money held by the public at the start of period 2 is $M - Q \cdot i$. If this is equal to the value of the T goods sold by the government, then

$$P_2 = \frac{M - Q \cdot i}{T}. \tag{1}$$

As we noted above, if we interpret T as (unmodeled) taxes collected by the government, this is an application of the fiscal theory of the price level.[5]

The market price for the good in period 1 is set by the Walrasian market-clearing condition:

$$P_1 = \frac{D}{(1-\kappa)L + S}\,, \tag{2}$$

where L is the amount of investment assets that are liquidated by the commercial bank, and D is the amount of dollars used by agents to purchase goods in the market. To ease notation, we omit the explicit dependence of P_1, P_2 on the decisions made (by all players) in periods 0 and 1, which we denote by h_1.

Definition 1 (Equilibrium)**.** *Fix an intervention policy $\langle i(\cdot), \overline{Q}(\cdot)\rangle$. Strategies for the agents and the commercial bank, together with prices P_1, P_2, constitute an equilibrium if:*

1. *P_1 satisfies condition* (2) *for all h_1.*
2. *P_2 satisfies condition* (1) *for all h_1.*
3. *Agents behave optimally given the prices and contract offered by the commercial bank.*
4. *For every μ, the commercial bank chooses a loan size, $Q(\mu)$, that maximizes its value to shareholders.*

Note that the commercial bank's choice of how much assets to store, S, and what deposit to offer its investors, d_1, are not pinned down in equilibrium. We discuss how d_1 and S are determined in Section 3.

[5]Cochrane (2021) fully develops the fiscal theory of the price level, and discusses how the fiscal theory fits into many branches of the economics literature.

2.2 The Efficient Real Allocation

As a first step, we characterize the optimal policy of a social planner seeking to maximize welfare. The planner protects the agents from the risk that is associated with their uncertain time preferences. Denote the consumption of impatient and patient agents, respectively, by c_1 and c_2. The social planner's objective is to maximize

$$\lambda u(c_1) + (1-\lambda)u(c_2) \tag{3}$$

subject to the budget constraint

$$(1-\lambda)c_2 \leq (1-\lambda c_1)(1+R) + T \tag{4}$$

and the feasibility constraint

$$\lambda c_1 \leq 1. \tag{5}$$

Lemma 1. *Suppose that $T < (1-\lambda)/\lambda$. There exists a unique solution to the planner's problem in which* (4) *is binding,* (5) *is not binding, and*

$$u'(c_1) = (1+R)u'(c_2). \tag{6}$$

The solution to the planner's problem satisfies $c_2^* > c_1^* > 1$.

The assumption that $T < (1-\lambda)/\lambda$ implies that if the one unit of the consumption good with which the agents are endowed were given to the impatient agents and the T units of the consumption good with which the government is endowed with were given to the patient agents, then each impatient agent would consume more than each patient agent. Since $u(\cdot)$ is concave and there is a positive return on investment, such an allocation is clearly suboptimal. Thus, this assumption guarantees that the solution to the social planner's problem is interior and is characterized by condition (6). In the analysis that follows, we maintain this assumption and denote the solution to the planner's problem by (c_1^*, c_2^*).

3 Implementing the Efficient Allocation

In this section we study how the efficient allocation can be implemented without the intervention of the central bank; to do so, we characterize a contract that can be offered by the commercial bank for which there exists an equilibrium where consumption is efficient and prices are stable. Then, in Section 3.1, we characterize the inefficient equilibrium (a "bank run") that can also result from the same contract.

In an equilibrium that attains the efficient outcome, patient agents do not withdraw their deposits in period 1. Hence, in order to implement the efficient allocation, the commercial bank stores

$$S = \lambda c_1^*$$

and does not liquidate any assets in period 1. Moreover, patient agents receive an equal share of the commercial bank's remaining assets in period 2, which they use to finance their consumption. The commercial bank's assets in period 2 consist of $(1 - S)(1 + R)$ units of consumption goods and M units of cash.[6] Thus, to attain the efficient allocation, the dollar value of each patient agent's bank share in period 2 must satisfy

$$c_2^* P_2 = \frac{P_2(1 - \lambda c_1^*)(1 + R) + M}{1 - \lambda}. \tag{7}$$

In the absence of an intervention by the central bank, the price level in period 2 is

$$P_2 = \frac{M}{T}. \tag{8}$$

Plugging (8) into equation (7) yields the social planner's budget constraint (4), and hence the bank's behavior implements the efficient allocation for patient agents.

Impatient agents receive d_1 dollars in period 1. In order for these agents to purchase their efficient consumption of c_1^*, it must be the case that $d_1 = P_1 c_1^*$. Thus, the market price in period 1 is given by

$$P_1 = \frac{d_1}{c_1^*}. \tag{9}$$

[6]The impatient agents that withdraw in period 1 implicitly purchase the commercial bank's stored consumption goods. Thus, the commercial bank will hold all of the cash at the end of period 1.

There are potentially multiple contracts that implement the efficient allocation. To identify such contracts, note that there are three constraints on the choice of d_1. First, since the commercial bank maximizes its profits, it follows that if $(1-\kappa)P_1 > (1+R)P_2$, the commercial bank would liquidate its investment assets in period 1 in order to maximize its net value for its shareholders. In order for a contract to implement the efficient allocation, it must therefore be the case that $(1-\kappa)P_1 \leq (1+R)P_2$. Using (8) and (9), we therefore have that

$$d_1 \leq \frac{1+R}{1-\kappa} c_1^* \frac{M}{T}. \tag{IC_1}$$

Second, patient agents must weakly prefer not withdrawing in period 1 to withdrawing, storing cash, and purchasing goods in period 2 (purchasing goods in period 1 is suboptimal as $c_1^* < c_2^*$). The value of the commercial bank's assets in period 2 is

$$M + P_2(1-\lambda c_1^*)(1+R) = M\left(1 + \frac{1-\lambda c_1^*}{T}(1+R)\right).$$

This restriction requires that the nominal value of a patient agent's share in the commercial bank be at least d_1, i.e.,

$$d_1 \leq \frac{M}{1-\lambda}\left(\frac{1-\lambda c_1^*}{T}(1+R)+1\right). \tag{IC_2}$$

Finally, implementing the efficient allocation requires that agents find it optimal to deposit their endowment in the commercial bank in period 0. If d_1 is low, then P_1 is low as well. Hence, in period 0, an agent may benefit from privately storing her real endowment and using her cash to purchase consumption goods in the market in period 1. In this case, the agent consumes $1 + M/P_1$ if she is impatient, and $1 + M/(\min\{P_1, P_2\})$ if she is patient. In order for agents to find it optimal to deposit their endowment in the commercial bank in period 0, it must therefore be the case that

$$\lambda u\left(1+\frac{M}{d_1/c_1^*}\right) + (1-\lambda)u\left(1+\frac{M}{\min\{d_1/c_1^*, T/M\}}\right) \leq \lambda u(c_1^*) + (1-\lambda)u(c_2^*). \tag{IC_3}$$

Condition (IC_3) clearly holds if d_1, and hence P_1, are sufficiently high, whereas conditions (IC_1) and (IC_2) clearly hold if d_1, and hence P_1, are sufficiently low. However, it is unclear a priori whether all constraints are consistent with a stable price trajectory, that is, whether (IC_1),(IC_2), and (IC_3) can hold given the prices $P_1 = P_2 = M/T$. The following lemma shows that, under the aforementioned assumption that $T < \frac{1-\lambda}{\lambda}$, ($IC_1$), ($IC_2$), and ($IC_3$) can hold jointly while maintaining price stability.

Proposition 1. *The contract given by $d_1^* = c_1^* M/T$ satisfies constraints (IC_1), (IC_2), and (IC_3). Moreover, under this contract $P_1 = P_2 = M/T$.*

3.1 A "Bank Run" Equilibrium

The analysis in the previous section focused on the "good equilibrium," in which patient agents do not run on the commercial bank. However, if the central bank does not intervene, then there is an additional "bad equilibrium," or "bank-run equilibrium," where all agents attempt to withdraw their deposits in period 1, which leads to the full liquidation of the commercial bank's assets and to its eventual failure. As Diamond and Dybvig (1983) and some of the subsequent literature have emphasized, banks can in principle avoid this bad equilibrium by suspending convertibility if too many depositors show up, or more generally by replacing simple deposit contracts with state-contingent contracts. However, since we show that central bank lending can also help avoid these equilibria, banks would have no particular incentive to offer such contracts if they expected the central bank to bail them out during a run.

If a bank run occurs, then all agents attempt to withdraw their deposits in period 1. Thus, for a bank run to occur in equilibrium, the total cash demanded in period 1 must be greater than the sum of the commercial bank's reserves and the cash that it can obtain by liquidating all of its investment assets and selling them at the market price for the consumption good when a bank run occurs. We denote this market price by P_1^{run}.

If a bank run occurs, patient agents have a choice between using the cash they withdraw to buy the good in period 1 and storing the cash and using it to buy the government endowment in period 2. The following proposition establishes that, in equilibrium, patient agents must be indifferent between both options, which, in turn, pins down P_1^{run}. Moreover, it shows that the commercial bank will indeed fail if all agents attempt to withdraw their deposits in period 1 and T is sufficiently small.

Proposition 2. *If all agents demand their deposits in period 1, then $P_1^{run} = P_2 = M/T$. Moreover, there exists $\bar{T} > 0$, such that the commercial bank will fail during a run if $T < \bar{T}$.*

For the rest of the analysis, we assume that $T < \min\{\frac{1-\lambda}{\lambda}, \bar{T}\}$. The first bound implies that the social planner's problem has an interior solution, whereas the second implies that the commercial bank will collapse during a bank run. To see why the latter is needed, observe that during a bank run, the commercial bank must pay out cash that is worth $c_1^* > 1$ units of the good, but can sell only (at most) one unit of the good. Thus, whether or not the commercial bank fails depends on whether its cash reserve, M, is worth $c_1^* - 1$ units of the good at the market price of $\frac{M}{T}$. If T is low, then the price of the good is high and the commercial bank's reserves are insufficient to prevent its failure.

It is worth noting that a bank run in this economy does not result in deflation, which contrasts with the historical evidence on actual crises. One mechanism that can generate deflation, but is missing from this model, is that crises may be associated with a contraction in output as credit becomes scarce. In models with sticky prices, this would lower marginal costs. Although our model can generate lower consumption during the run given that assets are liquidated, there is no production in our model for the run to affect. Prices in our model are instead determined by the condition that patient agents must be willing to hold on to money.

4 Central Bank Intervention

The above analysis has shown that the economy is susceptible to bank runs that cause the commercial bank to liquidate its investment assets and lead to its eventual collapse. In practice, when such a threat arises, the central bank can act as a lender of last resort and allow the commercial bank to raise money without resorting to inefficient liquidation of assets. In this section, we analyze how interventions of this type impact the stability of the financial sector, prices, and the real allocation of resources in the economy.

4.1 Financial Stability

We begin our analysis by studying the effect of early withdrawal by some of the patient agents on the stability of the commercial bank. The commercial bank has reserves of M in period 1, and so if the fraction μ of patient agents that withdraw their deposits in period 1 is sufficiently small, the bank could pay those agents from its reserves and would not need to liquidate assets or borrow money from the central bank.[7] If the fraction of patient agents that withdraw early is higher, but not too high, then the commercial bank can remain solvent by liquidating assets and selling them on the market to raise funds even if the central bank elects not to intervene.

Proposition 3. *In the absence of intervention by the central bank, the commercial bank must liquidate assets in order to remain solvent if*

$$\mu > \mu^{\dagger} \equiv \frac{T}{(1-\lambda)c_1^*},$$

and will collapse if

$$\mu > \mu^{\dagger\dagger} \equiv \mu^{\dagger} + (1-\kappa)\frac{1-\lambda c_1^*}{c_1^* - \lambda c_1^*}.$$

When there is a risk of a bank run, the central bank may choose to intervene for two reasons. First, if $\mu \in (\mu^{\dagger}, \mu^{\dagger\dagger}]$, intervention can strengthen the commercial bank's balance sheet by allowing it to raise cash without having to inefficiently liquidate its assets. For the intervention to attain this goal, the commercial bank must raise enough cash under sufficiently favorable terms.

Proposition 4. *For every $\mu \in (\mu^{\dagger}, \mu^{\dagger\dagger}]$, there is an amount $Q^{\dagger}(\mu)$ that the bank must borrow to avoid having to liquidate any assets. Moreover, there is a maximum interest rate $i^{\dagger}(\mu)$ that ensures banks would be willing to borrow the amount $Q^{\dagger}(\mu)$ rather than liquidate.*

Second, if $\mu > \mu^{\dagger\dagger}$ then intervention by the central bank is necessary in order to prevent the commercial bank's collapse. Note that following an intervention, the requirements for the commercial bank to remain solvent are more stringent: the commercial bank not only has to pay agents that withdraw in period 1, but also has to be able to repay its loan to the central bank in period 2. In the following proposition we show that to prevent the collapse of the commercial bank, the terms of the central bank's intervention must, again, be sufficiently favorable. In particular, it must receive a loan that is strictly larger than the loan needed to avoid liquidation, at an interest rate that is no greater than the interest needed to avoid liquidation, $i^{\dagger}(\mu)$.

[7] The bank also sells the S units of consumption goods it stored to cover its obligations to impatient agents.

Proposition 5. *For every $\mu > \mu^{\dagger\dagger}$ there is an amount $Q^{\dagger\dagger}(\mu)$ that the bank must borrow in order to pay all depositors. Moreover, to avoid collapse the commercial bank must pay an interest rate of $i \leq i^{\dagger}(\mu)$ on the loan it receives.*

The details of how to derive the above thresholds appear in the proofs of the propositions. Moreover, it can be shown that as the fraction of the patient agents that withdraw early increases, the commercial bank must receive more favorable terms to avoid liquidation and prevent its collapse. Furthermore, note that the maximal interest rate for which the commercial bank will not liquidate assets is the same interest rate needed to prevent its collapse. This is the case since at this interest rate the commercial bank is indifferent between raising cash by liquidating assets or borrowing from the central bank. Hence, loans at an interest rate above this critical level will never be taken by the commercial bank.

As we are interested in avoiding inefficient liquidation of assets, for the remainder of the paper we will restrict attention to interventions that will not lead the commercial bank to liquidate assets. That is, we will assume that for every μ, the central bank's policy is such that $\overline{Q}(\mu) \geq Q^{\dagger}(\mu)$ and $i(\mu) \leq i^{\dagger}(\mu)$.

4.2 Price Dynamics

Since the price level in period 2 is determined by the government's obligations, the terms at which the central bank intervenes have an immediate implication for the price dynamics of the consumption good. In particular, the price of the consumption good in period 2 is given by the ratio of the amount of cash in circulation, $M - i \cdot Q$, and the amount of goods that the government obtains via taxation, T. Moreover, the price in the first period is determined by the period in which the patient agents that withdraw early choose to purchase the good, a choice that is itself impacted by the price in period 2.

In the following analysis, we analyze the impact of interventions on the price levels in periods 1 and 2. That is, we compare the price level in the model to the counterfactual price level without any intervention. Notice that the price level is M/T in both periods, in both the "good" and "bad" equilibria. Hence, the implication of policies for price level is independent of whichever equilibrium is used as a benchmark. Additionally, we study the inflationary implications of policy interventions, that is, how prices change over time in the model following an intervention.

The following proposition characterizes the connection between monetary intervention and price dynamics.

Proposition 6. *Suppose that the commercial bank takes a loan of Q at interest rate i from the central bank.*

1. *If $i > 0$, then monetary intervention does not impact the price level in period 1 ($P_1 = M/T$) and lowers the price level in period 2 to below M/T.*
2. *If $i = 0$, then the price level in both periods remains M/T.*
3. *If $i < 0$, then monetary intervention increases the price levels, relative to the no-intervention benchmark, in both periods. Moreover, if $|iQ| \leq M\frac{\mu(1-\lambda)}{\lambda}$, then $P_1 = P_2$, and otherwise $P_1 < P_2$.*

The intuition for this result is as follows. When the government makes profits from its open market operations (that is, charges positive interest rates), then government finances improve, yielding an increase in the value of cash (reducing the price level in period 2). The opposite is true if the government incurs a loss on its open market operations. In the latter case, inflation, to some extent, may be counterbalanced by changes in the choice of the time of purchase by patient agents that withdraw early.

The deterioration of the government's balance sheet when $i < 0$—and the resulting increase of prices in period 2—leads patient agents that withdraw early to purchase in period 1. The increase in period 1 expenditure creates inflationary pressure and leads to an increase in P_1. If μ is large relative to the government's losses from its open market operations, then this increase in expenditure can equate the prices between both periods. On the other hand, if μ is small, then even if all the patient agents that withdraw early purchase in period 1, P_1 will remain below P_2. When $i \geq 0$, the government's balance sheet improves due to the intervention, prices in period 2 decrease, and so the patient agents that withdraw early store their cash.

The above analysis indicates that intervention at positive interest rates yields deflation as well as a reduction in the period 2 price level. Such an intervention can be made consistent with price stability if the government either engages in free disposal or reduces government resources through a reduction in surpluses, such as a decrease in taxation or an increase in government spending (both of which are unmodeled here).

4.3 Incentives to Run

The terms of an intervention by the central bank not only determine the price dynamics and whether or not the financial system will collapse during a run, but also whether a bank run is even possible in equilibrium. For example, on the one hand, after an intervention at interest rate $i = i^{\dagger}$, the equity shares of the commercial bank are worthless, and so patient agents that withdraw their deposits in period 1 are strictly better off than those that withdraw in period 2. Hence, for this intervention policy, there exists an equilibrium in which a run on the bank occurs. On the other hand, after an intervention at interest rate $i = 0$, the equity value remains unchanged (relative to the case in which there is no run) but there are fewer patient agents that own a share in the bank. Since the patient agents that withdraw early consume $c_1^* < c_2^*$, it follows that, for such an intervention policy, patient agents that wait are better off than those that run, and so bank runs cannot occur in equilibrium.

Thus, whether or not a bank run can occur in equilibrium depends on the redistribution effects of the intervention policy. In particular, a bank run can occur only if patient agents that withdraw early consume more than those who wait. Since we are focusing on intervention polices that prevent the liquidation of assets, the commercial bank will borrow the minimal amount needed to avoid liquidation when the interest rate is positive and the maximal possible amount when the interest rate is negative. That is, the commercial bank will borrow

$$Q^*(\mu, i(\mu), \overline{Q}(\mu)) = \begin{cases} Q^{\dagger}(\mu) & \text{if } i(\mu) \geq 0 \\ \overline{Q}(\mu) & \text{if } i(\mu) < 0 \end{cases}.$$

The redistribution effects of an intervention policy depend on the profits/losses the central bank realizes from its open market operations. The central bank's profit from policy $\langle i(\mu), \overline{Q}(\mu) \rangle$ in state μ is given by

$$\pi^{CB}(\mu, i(\mu), \overline{Q}(\mu)) = i(\mu) Q^*(\mu, i, \overline{Q}).$$

In the following proposition, we show that the central bank's profit reduces the welfare of patient agents that do not withdraw early and increases the welfare of those that withdraw early. Intuitively, the central bank profits at the expense of the commercial bank, and so as π^{CB} increases the value of the commercial bank's equity decreases, and the patient agents that do not withdraw early become worse off. Moreover, we show that there is a critical level of profit at which patient agents consume c_2^* regardless of the time of their withdrawal.[8]

Proposition 7. *The utility of patient agents that withdraw early (wait) is weakly increasing (decreasing) in $\pi^{CB}(i, \overline{Q}, \mu)$, and their utility is strictly increasing if*

$$\pi^{CB}(i, \overline{Q}, \mu) \geq -M\frac{\mu(1-\lambda)}{\lambda}.$$

Moreover, there exists $\pi^*(\mu) > 0$, such that patient agents that withdraw early (wait) consume at least c_2^* if and only if $\pi^{CB} \geq (\leq) \pi^*(\mu)$.

An important implication of Proposition 7 is that if the the central bank's profit is less than $\pi^*(\mu)$, then bank runs cannot occur in equilibrium. Since the central bank's profits are increasing in the interest rate that it charges, it follows that bank runs can only occur if the central bank charges a sufficiently high positive interest rate.

Corollary 1. *There exists $i^{BR}(\mu) > 0$, such that a bank run cannot occur in equilibrium if $i(\mu) < i^{BR}(\mu)$.*

Note that Corollary 1 implies that that when $i = 0$, not only is there no liquidation, but there is also no equilibrium in which a bank run occurs.

4.4 Fiscal Bailout

So far we have focused on the traditional role of a central bank in financial crises, namely, that of a lender of last resort. However, this is not the only intervention policy that governments have used. In particular, on occasion, governments have saved distressed commercial banks using a fiscal bailout, such as the Troubled Asset Relief Program (TARP) during the financial crisis of 2008.[9]

This financial stabilization plan empowered the U.S. Treasury to purchase preferred shares (or similar instruments) from various banks. These shares came with a promised

[8]Recall that when the central bank makes a profit there is deflation, and so the intervention that generates this critical level of profits—and in so doing induces the ex-ante efficient allocation despite the early withdrawals—does not maintain price stability.

[9]See Mishkin and White (2014) for a history of Federal Reserve actions during financial crises.

rate of return equivalent to the interest rate modeled above. Legally, these shares were junior to demand deposits and senior to equity, meaning that the resources injected into the banks were used to pay off running depositors, but the government would get repaid before equity holders. Thus, if the interest rate chosen on these preferred shares is the same as that described above, the financial system may be saved while maintaining price level stability. Intuitively, if the government is not losing money on its actions, there should be no inflation.

The main difference between this intervention and that described above is the funding source. Here, it is the treasury injecting cash into the banks. Since the treasury cannot create cash, it must raise the appropriate funds by selling government bonds. Thus, the net effect of this program is that impatient agents who run hold government bonds, while the commercial banks owe money (through preferred shares) to the government. No new cash is created, and the money that patient agents withdraw is absorbed by the government in period 1 rather than being held by the private sector. Patient runners can save outside of the financial system without literally holding cash.

5 The Goals of Central Banks and the Bagehot Rule

The main result of this paper establishes that the two main goals of central banking, namely, maintaining price-level stability and acting as a lender of last resort (financial stability), are not mutually exclusive. Moreover, these goals can be attained without distorting the real allocation in the economy.[10] In Section 5.1 we formally establish this result. We then discuss in Section 5.2 how it allows us to interpret the Bagehot rule of allowing for unlimited loans during a financial crisis, but at a high interest rate.

5.1 Both Goals are Achievable

We begin by formally showing the lack of tension between the central bank's two goals.

Theorem 1. *Assume that the central bank sets $i(\mu) = 0$ for all μ. If the commercial bank offers the contract $d_1^* = c_1^* \frac{M}{T}$, then:*

[10] Ex-ante allocation efficiency (i.e., where all patient agents consume c_2^* and all impatient agents consume c_1^*) is obtained only on the equilibrium path. However, since some agents make dominated choices off the equilibrium path, it is not possible to offset such choices without endowing the central bank with additional tools (see Section 6.2).

1. *The price of the consumption good is $\frac{M}{T}$ in both periods, under any withdrawal strategy of the agents. That is, intervention has no effect on prices.*

2. *The commercial bank does not fail under any withdrawal strategy of the agents.*

3. *The equilibrium allocation is efficient, in the sense that agents that withdraw in period t consume c_t^*.*

The central banks attains its goals both on and off the equilibrium path. We make three observations to summarize how this is achieved.

1. In equilibrium, patient agents do not demand their deposits in period 1. Off equilibrium, some patient agents do demand their deposits in period 1. To accommodate this demand, the central bank prints money—which is government debt—that it lends to the commercial bank.

2. Off equilibrium, prices are unaffected by patient agents demanding their deposits. There is no upward pressure on prices in period 1 since the patient agents that demand their deposits store their cash and do not spend it. There is no upward pressure on prices in period 2 since government finances are not hurt: The money created by loans is essentially unwound when banks repay the loans they receive from the central bank.

3. Since the central bank prints money during a bank run, the commercial bank does not have to inefficiently liquidate its long-run investments.

Put differently, the issue with a bank run is not that patient agents want to consume in period 1, but that they want to trade their commercial bank deposits, which are implicitly an equity stake in the commercial bank, for a credible debt asset. This is because they no longer believe that their commercial bank equity will have value in period 2. The central bank allows them to do exactly this: It prints money, a form of government debt, in order to bail out the commercial bank. The commercial bank distributes the cash to the patient agents that run in period 1. Thus, the central bank implicitly provides patient agents that run with the option to trade their equity share in a commercial bank with a credible debt instrument, cash. What we have shown above is that if (i) the central bank offers inter-temporal loans of last resort that are at a zero real rate of interest and (ii) patient agents can store the cash that they receive, then this policy does not create inflation and prevents the collapse of the financial sector.

There are multiple ways of interpreting the intuition for our result. The first one, the idea we push hardest in this paper, is based on the idea that inflation is caused by too much money "chasing" too few goods. What is crucial to understand in this model is the importance of being able to store cash outside the banking system. When patient agents receive newly created cash during a bank run, they do not spend it (and thus it does not "chase" any goods). They save it for the time when they actually do want to spend. From their point of view, these patient agents have simply replaced one asset, a bank deposit, with another, cash, which is a form of government debt. There is no reason for spending, and hence no reason for prices to rise. The second way to interpret our results is through the unified government budget constraint. This constraint implies that a deterioration of government finances must lead to inflation. Since a loan at a high interest rate does not hurt government finances, there should be no inflation. If the monetary authority injects liquidity for lending of last resort purposes, then repayment of the loan (with interest) unwinds the injection of liquidity and drains it back out from the private sector. Hence, the two central bank goals of price-level stability and acting as a lender of last resort (financial stability) do not have to be in conflict.

5.2 A Reinterpretation of the Bagehot Rule

With the above analysis in hand, we now turn to consider the Bagehot rule, which states that during a banking crisis, the central bank should lend freely to solvent commercial banks against safe collateral and at a high rate of interest. By construction, the assets in this model are safe collateral, and thus the commercial banks are solvent unless they must liquidate too many of their investments. Our model provides a reason why the interest rate charged to banks must be high that differs from the screening arguments typically associated with the rule, and which Bagehot emphasized in his own writing. In particular, setting the interest rate too low is incompatible with price stability. A low interest would leave too much liquidity in period 2, leading to a higher price level P_2. Knowing this, patient agents in period 1 would have an incentive to buy goods in period 1 rather than hold on to cash. That, in turn, would generate an increase in P_1. The only way to ensure that prices do not rise is to ensure that banks are not left with large money balances in period 2, and this can be done by ensuring banks repay enough to the central bank.

The high rate of interest that emerges from our model is $i = 0$. However, this is not a deep feature of the model and is due to two key features. First, the fact that agents can store goods implies that the short-run real interest rate in our model is 0. If storage was productive, the real rate that the central bank would have to charge borrower banks to maintain price stability would be positive. Second, there is no background inflation in the model that would push the required nominal rate higher. If we introduced a force for inflation, e.g., the creation of additional money by the central bank between periods 1 and 2, the nominal rate the central bank would have to charge borrower banks to maintain price stability would likewise be positive.

6 The Importance of Saving Outside the Banking System

A key novelty of our framework as compared to the literature on nominal DD models is that we allow agents who withdraw their deposits to save outside of the banking system. We now show that this assumption is crucial for our results. We do so by revisiting recent work by Schilling, Fernández-Villaverde and Uhlig (2021) who, in the context of a central bank digital currency (CBDC), show that there exists a "monetary trilemma," whereby the central bank can choose up to two of the following three desirable properties: a run-free financial system, price stability, and efficient risk sharing. In this section, we explain that, even though the motivation behind Schilling, Fernández-Villaverde and Uhlig (2021) is quite different from ours, the models in fact have a fair amount in common. The main difference is that Schilling, Fernández-Villaverde and Uhlig (2021) assume agents who decide to run on the central bank get rid of their CBDC by buying goods. If we allowed these running agents to save with newly created government debt rather than buying goods, the conflict between objectives they find can be overcome.

6.1 Comparison of Models

We begin by describing the differences between the Schilling, Fernández-Villaverde and Uhlig (2021) model and ours. Although the papers explore different questions, both rely on a nominal DD model at the core of their analysis.

In Schilling, Fernández-Villaverde and Uhlig (2021), agents in period 0 are endowed only with goods. Agents can deposit these goods with a central bank, as opposed to a commercial bank, in an account that entitles them to M units of a digital currency in period 1 or a share of the central bank's resources in period 2, paid out in CBDC. The central bank then invests the goods it receives in long-run projects that yield a return of $1 + R$ in two periods or 1 unit if the project is liquidated after one period. Let n denote the fraction of the entire population that spends their CBDC in period 1.[11] After observing n, the central bank chooses the amount $y(n)$ of its long-run projects to liquidate and sell to agents. Patient agents who withdraw early spend their M units of the CBDC in period 1 and then store those goods and consume them in period 2.

Agents in the model buy their goods directly from the central bank. This is in contrast to our model, where agents buy goods from the commercial bank. Since all transactions in our model are conducted within the private sector, there is still money left in private hands after the commercial bank sells its goods, specifically the amount M net of any money banks use to pay their obligations to the central bank. The fact that this money will be used to buy T goods at the end of period 2 is what determines the price P_2 in our model. By contrast, prices in the Schilling, Fernández-Villaverde and Uhlig (2021) model are set by a Walrasian market to ensure that the spending by agents is equal to the value of the goods the central bank chooses to sell:

$$p_1 = \frac{nM}{y(n)}, \tag{10}$$

and

$$p_2 = \frac{(1-n)M}{(1-y(n))(1+R)}, \tag{11}$$

where $1 - y(n)$ is the amount of assets the central bank chooses not to liquidate and so earn the return $(1 + R)$.

Under this formulation, prices P_1 and P_2 are entirely determined by the fraction n of agents that spend in period 1 and the amount of projects that the central bank liquidates in period 1. For a given amount of liquidation, the more agents who spend, the higher are prices. For a given amount of expenditures, more liquidation decreases (increases) prices in period 1 (period 2). As observed by Schilling, Fernández-Villaverde and Uhlig

[11] Here we follow their notation, which stands in contrast to our convention of using μ to denote the fraction of *patient* agents that withdraw in period 1.

(2021), this is the quantity theory of money in action: The price level depends on the amount of money nM chasing product $y(n)$.

The values of n and $y(n)$ also determine the distribution of resources across agents. As the fraction of agents n that spend their CBDC in period 1 increases, holding $y(n)$ fixed, consumption in period 1 decreases (as a fixed amount of resources is divided among more agents) and consumption in period 2 increases (since the bank's resources are shared by fewer people). Likewise, for a given amount of expenditures nM in period 1, more liquidation increases consumption in period 1 and decreases it in period 2.

A key result in Schilling, Fernández-Villaverde and Uhlig (2021) is that the central bank in their model faces a trilemma: It cannot use its one policy tool $y(n)$ to ensure there is no run on the CBDC, maintain price stability in all states of the world, and achieve efficient risk sharing. To see why, suppose that a large number of agents ($n > \lambda$) spend their money in period 1. One of two things has to happen:

1. If the central bank liquidates $n \cdot c_1^*$ units of investment, it will have fewer resources at date 2. This means c_2 will fall below c_2^* and the optimal risk-sharing allocation will not be implemented. If c_2 falls enough so that $c_2 < c_1$, it will affirm the decision of agents to run on the digital currency.[12]

2. If the central bank liquidates less than $n \cdot c_1^*$ units of investment, P_1 must rise.

A credible liquidation policy that discourages patient agents from running on the CBDC must set $y(n)$ for any n to ensure that $c_1 \leq c_2$. Such a liquidation strategy would also implement the social optimum in equilibrium if $y(n) = \lambda c_1^*$ when $n = \lambda$. But to keep $c_1 < c_2$ for high values of n requires letting $y(n)$ rise by less than $n \cdot c_1^*$, which would inevitably drive up the price level P_1 for high n. This is the sense in which the monetary trifecta cannot be achieved: Either the central bank gives up on discouraging patient agents from running, it sets $y(n) < \lambda c_1^*$, or it allows P_1 to rise at high values of n.

Although the central bank policy tool in Schilling, Fernández-Villaverde and Uhlig (2021) involves an amount $y(n)$ to liquidate rather than an amount to lend to commercial banks, the two interventions in the two models are in fact closely related. This is because in our model, the reason the central bank lends to banks is to help them avoid liquidation. In choosing how much to lend to banks, the central bank in our model

[12] As explained in Schilling, Fernández-Villaverde and Uhlig (2021), this is not a run in the classic sense, as it is initiated by central bank policy.

is essentially choosing how much of the long-run assets held in private hands will be liquidated. The essential difference between the two models, then, setting aside the different motivations and the exact details of how prices are determined, is that when agents want to dump a CBDC, they have to spend it. There is no option to store the CBDC as in our model. This is intuitive—holding on to a CBDC that agents want to get rid of seems to violate the spirit of what a run on a CBDC is all about.

However, although storing a CBDC is contrary to the spirit of agents not wanting to hold on to their CBDC, agents who want to dump their CBDC could in principle opt to hold physical currency or other assets, like government bonds. We now describe one example of how allowing agents who want to dump the CBDC by purchasing other assets would allow the central bank to avoid a run on the CBDC without having to sacrifice price stability.

6.2 Suggested Policy Tool: Debt with a Special Purpose Vehicle

The way we allow agents to dump their CBDC without having to spend it involves the creation of a special purpose vehicle (SPV) by the central bank. That is, in period 1, the central bank offers everyone a newly created one-period debt against a newly formed special purpose vehicle (SPV) in exchange for their CBDC account. To back this debt, the central bank transfers $\frac{c_1^*}{1+R} \leq \tau \leq \frac{c_2^*}{1+R}$ units of investment from the general investment account funded by the CBDC to this SPV for every agent who requests to transfer funds. A transfer τ results in the patient agents who transfer their accounts consuming $c_2^\dagger = (1+R)\tau \in [c_1^*, c_2^*]$ in period 2. At the same time, we assume that the central bank follows the liquidation policy $y(n) = \lambda c_1^*$ for all n.

We make the following observations on the economy in which a SPV is offered:

1. For these values of $c_2^\dagger$, all patient agents at least weakly prefer the newly formed SPV to purchasing in period 1.

2. No patient agent (strictly) prefers this SPV to keeping the central bank digital currency. As a result, this remains an off-equilibrium strategy. In equilibrium, agents will choose not to run on the CBDC.

3. There is no excessive liquidation of investment. All impatient agents still consume c_1^*, and prices in period 1 are unaffected.

4. If $\tau = \frac{c_2^*}{1+R}$, then the central bank allows all patient agents to consume c_2^*, *regardless of the time they withdraw*.[13]

Such an SPV would not be without precedent. During the 2008–2009 financial crisis, the Federal Reserve set up a number of SPVs: The Fed wanted to engage asset markets, but was not legally allowed to do so directly. Instead, the Fed established SPVs, lent to the SPVs, and then through the SPVs engaged the financial markets. See Mishkin and White (2014) for a description of this policy, as well as a historical overview of similar policies in the past. Second, and more closely related to the problem at hand, the particular SPV described here behaves much like a clearinghouse certificate. During banking crises in the U.S. National Banking Era, depositors would be issued loan certificates backed by the assets of the entire banking system through clearinghouses. This solved the issue of depositors not knowing whether their individual banks were exposed to risk, which was a main cause of runs at the time. Indeed, Gorton (1985) argues that these clearinghouses were essentially acting as central banks, as "they issued money and provided a form of insurance during panics." These clearinghouse certificates, backed by bank assets, are akin to the SPV proposed in this paper.

The SPV described here allows the central bank to maintain financial stability, efficient allocations, and price stability by exchanging newly created government debt (cash or a share of the SPV) for bank deposits (in commercial banks or CBDC accounts). This SPV, in essence, enables agents to store cash outside of the financial system using newly created cash in a non-CBDC world.[14]

7 Conclusion

In this paper we build and analyze a nominal DD model in order to assess the price stability implications of a monetary intervention during a bank run. We find that the central bank goals of a stable financial system and price stability are not necessarily

[13]While we do not directly analyze the use of SPVs in our model, it can be shown that, under certain conditions, the central bank can offer a similar SPV to avoid a run without sacrificing price stability.

[14]Schilling, Fernández-Villaverde and Uhlig (2021) already note that it is theoretically possible to overcome the trilemma by decreasing the nominal value of deposits M during a run to prevent aggregate spending $nM(n)$ from increasing with n. However, they rightly dismiss the idea of issuing less money during a run as both odd and implausible. Allowing agents to save rather than spend allows the central bank to overcome the trilemma, even though it injects more liquidity during a run rather than less.

in conflict. We reinterpret the Bagehot rule as a guideline to achieving both of these, potentially conflicting, goals.

Central to our analysis is the idea that agents are able to store cash outside of the banking system during a run. We show this by revisiting the Schilling, Fernández-Villaverde and Uhlig (2021) model of a run on a central bank digital currency (CBDC). While they find a conflict between price-level stability and avoiding a run on the CBDC, we show that this stems from their assumption that agents who want to dump their CBDC can do so only by spending it. When agents who want to dump their CBDC can use them to buy other assets, it may be possible for the central bank to prevent a run on the CBDC while still committing to price stability.

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Appendix

Proof of Lemma 1. In optimum the budget constraint (4) is clearly binding. Moreover, as the objective is concave, it has a unique solution that satisfies the first-order condition (6). Assume by way of contradiction that the feasibility constraint is binding. In this case, $c_1 = 1/\lambda$, and by (4), $c_2 = T/(1-\lambda)$. The assumption on T implies that $c_1 \geq c_2$, which, by the concavity of $u(\cdot)$, implies that the first-order condition does not hold, yielding the contradiction.

To establish the final part of the lemma, assume by way of contradiction that $c_1 \leq 1$. If $c_1 = 1$ then, by (4), $c_2 = 1 + R + T/(1-\lambda)$. However,

$$
\begin{aligned}
(1+R)u'\left(1+R+\frac{T}{1-\lambda}\right) &< (1+R)u'\left(1+R\right) \\
&= u'(1) + \int_1^{1+R} \frac{\partial}{\partial c}\left(cu'(c)\right) dc \\
&= u'(1) + \int_1^{1+R} \left(u'(c) + cu''(c)\right) dc \\
&< u'(1),
\end{aligned}
$$

where the first inequality follows from the concavity of u and the second inequality follows from the assumption that the relative risk aversion coefficient is strictly greater than unity. Thus, the FOC do not hold. From the concavity of u and constraint (4), it follows that the FOC cannot be satisfied for any $c_1 < 1$. Hence, $c_2^* > c_1^* > 1$. ■

Proof of Proposition 1. It is straightforward to verify that (IC_1) and (IC_2) hold (with slack) when $d_1 = c_1^* M/T$ (i.e., under the contract that induces $P_1 = P_2$). Since agents cannot invest on their own, under this contract the utility of an agent that does not deposit her endowment in the commercial bank is

$$\lambda u\left(1+\frac{M}{P_1}\right) + (1-\lambda)u\left(1+\frac{M}{P_2}\right) = u(1+T).$$

On the other hand, the agent's expected utility from depositing in the bank is her expected utility in the solution to the social planner's problem. By the social planner's budget constraint (4), it follows that if $c_1 = 1 + T$ then

$$c_2 = 1 + T + R\big(1 - \frac{T\lambda}{1-\lambda}\big) > 1 + T,$$

where the last inequality follows from the assumption that $T < \frac{1-\lambda}{\lambda}$. Hence, the social planner provides agents with an expected utility that is strictly greater than $u(1+T)$. ■

Proof of Proposition 2. First, assume by way of contradiction that $P_1^{run} < P_2$. In this case, patient agents that withdrew their deposit would rather purchase in period 1 than in period 2. Thus, no agent is interested in holding cash in period 1, and neither is the bank, which cannot be the case in equilibrium.

Second, assume by way of contradiction that $P_1^{run} > P_2$. In this case, patient agents that withdrew their deposit strictly prefer storing cash and purchasing in period 2 to purchasing in period 1. Hence, only impatient agents spend, and they collectively purchase $S + (1-\kappa)L$ units of the good, where L is the amount investments assets liquidated by the commercial bank. The market-clearing condition therefore implies that $d_1^{run}\lambda = P_1^{run}(S + (1-\kappa)L)$, where d_1^{run} is the average payment the bank makes agents in case of a run. If the bank does not fail, then $d_1^{run} = d_1^* = \frac{M}{T}c_1^*$, and the market clearing condition becomes

$$\lambda\frac{M}{T}c_1^* = P_1^{run}(S + L(1-\kappa)).$$

However, since $S = \lambda c_1^*$ and $L(1-\kappa) \geq 0$, this condition cannot hold if $P_1^{run} > P_2 = \frac{M}{T}$. On the other hand, if the bank does fail, then it liquidates all of its investment assets, and so the market clearing condition becomes

$$\lambda d_1^{run} = P_1^{run}(S + (1-S)(1-\kappa)).$$

Since $d_1^{run} \leq d_1^*$ and $(S + (1-S)(1-\kappa)) \leq 1$, this conditions implies that

$$P_1^{run} \leq \lambda d_1^* = \lambda\frac{M}{T}c_1^* = \lambda\frac{M}{T}\frac{S}{\lambda} = P_2 S.$$

However, since $S < 1$, this contradicts the assumption that $P_1^{run} > P_2$.

Next, we must show that the commercial bank will fail in a run if T is sufficiently small. If all agents demand their deposit, then the commercial bank has an obligation to pay $d_1^* = c_1^* M/T$ units of cash. If the commercial bank liquidates all of its investment assets, it can secure

$$M + (S + (1-S)(1-\kappa))P_1^{run}$$

units of cash. Since $P_1^{run} = M/T$ and $S = \lambda c_1^*$, the bank will fail if

$$c_1^* > 1 + T - (1 - \lambda c_1^*)\kappa.$$

Denote by $c_1^*(T)$ the solution of the social planner's problem as a function of T. Since

$u(\cdot)$ has a relative risk aversion coefficient that is strictly greater than unity it follows that $c_1^*(0) > 1$, and since $u(\cdot)$ is concave it follows that $c_1^*(T)$ is increasing in T. As the social planner's problem is continuous in T, there exists $\bar{T} > 0$ (perhaps infinity) such that for all $T < \bar{T}$ it holds that

$$c_1^*(T) > 1 + T - (1 - \lambda c_1^*(T))\kappa.$$

■

Proof of Proposition 3. If the commercial bank pays early withdrawers from its reserves, and these early withdrawers store the cash that they receive, the prices $P_1 = P_2 = M/T$ remain an equilibrium. Hence, if $\mu(1-\lambda)d_1^* \leq M$, or equivalently,

$$\mu \leq \frac{T}{(1-\lambda)c_1^*},$$

then the commercial bank does not need to liquidate assets. If $\mu > \frac{T}{(1-\lambda)c_1^*}$, then the commercial bank cannot pay all early withdrawers from its reserves, and so, in the absence of intervention, it must liquidate assets.

From the same argument used to establish Proposition 2, it follows that so long as the central bank does not intervene, $P_1 = P_2 = M/T$. Thus, the commercial bank can avoid collapse, in the absence of intervention by the central bank, only if the sum of its reserves and the cash it can raise by liquidating and selling its $(1-\kappa)(1-S)$ units of assets at price M/T is less than the amount it must pay the patient agents that withdraw in period 1. That is, the commercial bank can avoid collapse only if

$$(1-\lambda)\mu d_1^* \leq M + (1-S)(1-\kappa)\frac{M}{T},$$

or, equivalently,

$$\mu \leq \frac{T + (1-\lambda c_1^*)(1-\kappa)}{(1-\lambda)c_1^*}.$$

■

Proof of Proposition 4. To avoid liquidating assets, the cash infusion that the commercial bank receives must be sufficiently high in order to guarantee that it can pay all early withdrawers. Since the bank can pay $\mu^\dagger$ of the patient agents from its reserves, it follows that to avoid liquidation of assets, it must be the case that $Q \geq (\mu - \mu^{\dagger})(1-\lambda)d_1^*$, or

$$Q \geq M\left(\mu(1-\lambda)\frac{c_1^*}{T} - 1\right) \equiv Q^{\dagger}(\mu).$$

Since $\mu \leq \mu^{\dagger\dagger}$ the commercial bank can remain solvent by liquidating assets. Thus, it remains to show that the bank would rather take the loan than liquidate assets in order to raise cash. Consider first the case in which $i(\mu) \leq 0$. In this case, the commercial bank will take a loan of size $\overline{Q}(\mu)$, and hence equation (1) implies that

$$P_2 = \frac{M - i(\mu)\overline{Q}(\mu)}{T} > \frac{M}{T}.$$

If $P_1 > P_2$ then only the impatient agents purchase in period 1, and so the market-clearing condition (10) would imply that $P_1 \leq M/T$, a contradiction. It follows that in this case there is no deflation, and hence the commercial bank will not liquidate assets.

Next, consider the case in which $i(\mu) > 0$. In this case, the commercial bank will take the minimal loan that it needs to pay early withdrawers, which, conditional on it not liquidating assets, is a loan of size $Q^{\dagger}(\mu)$. Moreover, equation (1) implies that $P_2 = \frac{M - i(\mu)Q^{\dagger}(\mu)}{T} < \frac{M}{T}$. Hence, the patient agents that withdrew early will store cash, and the price $P_1 = M/T$ clears the market in period 1. In order for the commercial bank to find it optimal not to liquidate assets, it must be the case that

$$i(\mu) \leq \frac{1+R}{1-\kappa}\frac{P_2}{P_1} = \frac{1+R}{1-\kappa}\frac{M - i(\mu)Q^{\dagger}(\mu)}{M},$$

or

$$i(\mu) \leq \frac{T(1+R)}{(1+R)(\frac{(1-\lambda)\mu}{T}c_1^* - 1) + T(1-\kappa)} \equiv i^{\dagger}(\mu).$$

■

Proof of Proposition 5. We begin by deriving the conditions under which the commercial bank remains solvent. If the commercial bank receives a loan of Q at interest rate i and liquidates L of its investments, it will remain solvent in period 1 if

$$(\lambda + (1-\lambda)\mu)d_1^* \leq (S + L(1-\kappa))P_1 + M + Q\,, \tag{12}$$

and it will remain solvent in period 2 if

$$Q \cdot i \leq (1 - S - L)(1 + R)P_2 + (S + L(1 - \kappa))P_1 + M - (\lambda + (1 - \lambda)\mu)d_1^*. \qquad (13)$$

Consider first the case in which $i \leq 0$. In this case, the bank will take a loan of $Q = \overline{Q}(\mu)$ and, by the same argument used in the proof of Proposition 4, it must be the case that $P_2 \geq P_1$. This, in turn, implies that the commercial bank will liquidate the minimal amount of assets that will allow it to remain solvent. Since the commercial bank takes the market prices as given, it follows that, for any intervention policy with $i < 0$, constraint (12) is binding. Moreover, for any fixed $i < 0$, increasing Q increases P_2 (see condition (1)), which, in turn, weakly increases the amount of patient agents that choose to purchase in period 1, and hence increases P_1. It follows that increasing Q enables the commercial bank to survive in period 1 while liquidating fewer assets. Moreover, since $i < 0$, increasing Q also relaxes constraint (13). Thus, for every $i < 0$, there is a minimal Q for which both constraints are satisfied.

Next, consider the case in which $i > 0$. Since we assumed that $\mu > \mu^{\dagger\dagger}$, the commercial bank must borrow from the central bank to survive. If $i \leq i^{\dagger}(\mu)$ then, by Proposition 4, the commercial bank can avoid collapse without liquidating assets. On the other hand, if $i > i^{\dagger}(\mu)$, the commercial bank must both borrow money from the central bank and liquidate assets. For this to be optimal, the cost of raising cash via loans, i, must equal the cost of raising cash by liquidating assets, $\frac{(1+R)P_2}{(1-\kappa)P_1}$. Such an equality can hold only if $P_2 < P_1$, but at such prices all the patient agents that withdraw early save their cash and purchase in period 2 and, hence, liquidating assets does not raise cash for the commercial bank. ■

Proof of Proposition 6. First, consider the case in which the central bank charges an interest rate of $i \geq 0$. In this case, equation (1) implies that $P_2 = \frac{M - i \cdot Q}{T} \leq \frac{M}{T}$. If $P_1 = \frac{M}{T}$, then it is optimal for all patient agents that withdraw early to store cash and purchase in period 2. Since we are focusing only on intervention under which there is no liquidation of assets, this price also satisfies the market-clearing condition (2).

Next consider the case where $i < 0$. Equation (1) implies that $P_2 = \frac{M - i \cdot Q}{T} > \frac{M}{T}$. Moreover, in the proof of Proposition 4 we established that in this case $P_1 \leq P_2$. Denote by X the expenditure of patient agents that withdraw early in period 1. Note that $X \in [0, \mu(1 - \lambda)d_1^*]$ and that $X < \mu(1 - \lambda)d_1^*$ only if $P_1 = P_2$. The market-clearing condition (10) implies that

$$P_1 = \frac{\lambda d_1^* + X}{\lambda c_1^*}.$$

If $|i \cdot Q|$ is sufficiently large, then even when X is at its upper bound there will be inflation. To see this, observe that $P_1 < P_2$ when X is at its upper bound is equivalent to

$$\frac{M - i \cdot Q}{T} > \frac{\lambda d_1^* + \mu(1-\lambda) d_1^*}{\lambda c_1^*}.$$

Plugging in the various expressions and simplifying yields that this condition is equivalent to

$$-i \cdot Q > M \frac{\mu(1-\lambda)}{\lambda}.$$

If $-i \cdot Q \leq M\frac{\mu(1-\lambda)}{\lambda}$, then by having the patient agents that withdrew early split their purchasing between both periods (at the appropriate proportion), prices will satisfy $P_1 = P_2$, which, in turn, makes it optimal for these agents to purchase in both periods. ■

Proof of Proposition 7. From equation (1) it follows that P_2 is decreasing in π^{CB}. First, consider the case in which $\pi^{CB}(\mu, i(\mu), \overline{Q}(\mu)) \geq 0$. In this case there is deflation (Proposition 6), and so all patient agents purchase in period 2, regardless of the time of their withdrawal. Moreover, for the nonnegative interest rates needed to have $\pi^{CB} \geq 0$, the size of the loan taken by the commercial bank is constant (as a function of μ), and so as the interest rate increases, the value of the commercial bank's equity decreases. As early withdrawers receive d_1^* regardless of the interest rate and no assets are liquidated, it follows that increasing i must increase the consumption of early withdrawers at the expense of the late withdrawers.

Next, consider the case in which $\pi^{CB}(\mu, i(\mu), \overline{Q}(\mu)) < 0$. In this case, prices rise in period 1 (Proposition 6), and, moreover, it is an optimal choice for patient agents that withdrew early to purchase in period 1. For $\pi^{CB}(\mu, i(\mu), \overline{Q}(\mu)) \geq -M\frac{\mu(1-\lambda)}{\lambda}$, it holds that $P_1 = P_2 = \frac{M-\pi^{CB}}{T}$. Hence, for this range of profits, the consumption of both the impatient agents and the patient agents that withdrew early is strictly decreasing in π^{CB}. Since there is no liquidation of assets, this implies that the consumption of patient agents that waited must increase. For lower levels of π^{CB}, the price in period 1 remains constant, and so changes in π^{CB} have no impact on the real allocation.

The second part of the proposition follows from the intermediate value theorem and the observation that if $\pi^{CB} = 0$, patient agents that withdraw late are better off than those that withdraw early, whereas if $i = i^{\dagger}(\mu)$, which is the policy that maximizes π^{CB}, the opposite is true. ■

Proof of Theorem 1. If $i(\mu) = 0$, then $P_2 = M/T$ regardless of Q. The arguments used to establish Proposition 6 imply that in such cases it will also be the case that $P_1 = M/T$. For this price dynamics, d_1^* is less than the nominal value of an equity share in the bank as $c_1^* < c_2^*$. Hence, if the commercial bank can get interest-free loans of last resort, it will remain solvent under any withdrawal strategy of the agents.

To see that for patient agents it is dominant to withdraw in period 2, note that since $P_1 = P_2$, the consumption of patient agents that withdraw early is c_1^* and, moreover, the patient agents that withdraw early purchase in period 2. Given the central bank's policy, no investment assets are liquidated, and hence the aggregate consumption in period 2 is given by $(1 - \lambda)c_2^*$. As this consumption is due to patient agents only, it follows that an agent that withdraws late receives at least $c_2^* > c_1^*$ units of the consumption good. ■