Federal Reserve Bank of Chicago

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August 3, 2022

WP 2022-35

https://doi.org/10.21033/wp-2022-35

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Abstract

This paper examines whether tasking central banks with leaning against asset booms can conflict with their existing mandates to stabilize goods prices and output. The paper embeds the Harrison and Kreps (1978) model of speculative booms in a monetary model based on Rocheteau, Weill, and Wong (2018). In the model, a speculation shock that generates an asset boom is associated with higher output but a lower price level, unlike aggregate demand shocks that raise both output and prices. This creates a trilemma for central banks in that contemporaneous monetary policy cannot simultaneously stabilize output, the price level, and real asset prices. Stabilizing all three requires alternative policies.

*I thank Fernando Arce, Robert Barsky, Marco Bassetto, Björn Brügemann, Ricardo Caballero, Jeff Campbell, Larry Christiano, John Conlon, John Geanakoplos, Francois Gourio, Tomohiro Hirano, Ryo Jinnai, Arvind Krishnamurthy, Guillaume Rocheteau, Alp Simsek, Naoki Takayama, Iichiro Uesugi, and Burak Uras for comments. I also thank Helen Burkhardt and Craig Epstein for research assistance. The views here do not represent those of the Federal Reserve Bank of Chicago or the Federal Reserve System.

Introduction

The Global Financial Crisis has led to growing calls for policymakers to act against asset booms to avoid the potential consequences should these booms become busts as happened during the crisis. These interventions are often framed as part of a proposed financial stability mandate for central banks. Since central banks already face mandates to stabilize aggregate output and prices, an important question before tasking central banks with responding to asset booms is whether leaning against asset prices is compatible with their existing mandates. Is financial stability inherently complementary to what central banks already do, or would central banks be forced to decide between mandates if asked to take on additional responsibilities?

In an influential paper, Bernanke and Gertler (1999) considered this question and argued that "central banks should view price stability and financial stability as highly complementary and mutually consistent objectives" (p78). They reached this conclusion by introducing an exogenous wedge between the price of capital and the present discounted value of capital income in an otherwise standard macroeconomic model and showing that the wedge leads to higher output and higher inflation. This suggests there is no conflict between aligning asset prices with fundamentals and stabilizing prices and output. It also suggests little benefit from letting monetary policy respond directly to asset prices, since responding to inflation and output would already lead central banks to effectively respond to an asset boom should one arise.

This paper revisits the question of whether financial stability is consistent with the goals of stabilizing prices and output. Rather than assume an exogenous wedge between the price of an asset and its fundamental value as in Bernanke and Gertler (1999), I allow for an endogenous asset boom as in Harrison and Kreps (1978). In that model, time-varying beliefs combined with short-selling constraints give rise to a speculative boom in which the asset price can exceed what any agent expects the asset will pay out in dividends. To study the implications of such a speculative boom on output and prices, I embed a version of their model into a tractable monetary framework recently proposed by Rocheteau, Weill, and Wong (2018). I find that a shock that induces a sufficiently large speculative boom will be associated with higher output but a *lower* price level. A speculative boom driven by shocks to beliefs may not resemble the aggregate demand shocks that central banks are already tasked with offsetting.

The logic behind this result is due to the interaction between speculation and savings. Agents in the model buy assets intending to sell them to others later. These other agents must save to be able to buy assets in the future. The return these future buyers expect to earn must not be too high, or else they will only want to save and the goods market will not clear. In some circumstances, this requires a low price level to ensure the *real* price for the asset is sufficiently high to keep the expected return on the asset in check. Basically, the high expected return to speculation during a boom induces some agents to save, at least in part by hoarding money. This depresses the price of goods and raises the real price of the asset until the market for savings clears.Caballero (2006) conjectured an asset boom might lead to lower inflation for similar reasons, although he did not formally model this phenomenon.

Since asset booms need not coincide with aggregate demand shocks, a financial stability mandate may conflict with stabilizing output and prices. Indeed, I show that my model implies a trilemma: It is impossible to use contemporaneous monetary policy to stabilize asset prices, goods prices, and output in the face of a speculation shock. Increasing liquidity can offset the lower price level during the boom, but it will amplify either the asset boom or the output boom, depending on which agents receive this liquidity. Essentially, the asset boom is driven by optimism. Monetary policy cannot cure agents of their optimism; it can only prevent them from acting on it to buy assets. If agents remain optimistic, their incentive to work and save will remain high even if they can't buy the asset. Monetary policy may therefore fail to stabilize output or the price level even if it prevents optimists from buying the asset. Effective stabilization requires offsetting agents' optimism, e.g., imposing a financial transactions tax that makes speculation less attractive.

The implication of my model that asset booms may be associated with lower inflationary pressures accords with empirical observations. Okina, Shirakawa, and Shiratsuka (2001) describe the challenge for the Bank of Japan in the mid 1980s when it faced asset and output booms with little sign of inflation. The Bank of Japan reported a similar tension just before the Global Financial Crisis.[1] As I discuss in more detail below, Bordo and Wheelock (2007) and Christiano et al. (2010) identify historical stock market booms in various countries and find that they tend to be associated with lower rather than higher inflation. In line with the key mechanism in my model, I provide suggestive evidence from the 1980s boom that some of the entities that engaged in speculation, namely large Japanese corporations, maintained larger cash holdings.

Beyond the insights on asset booms, my model suggests new factors that can affect inflation. New Keynesian models emphasize the role of demand shocks in shaping inflation through their effect on marginal costs: When goods prices are rigid, lower demand for goods can lead to lower output, which decreases the marginal costs producers expect to face now and in the near future. By contrast, prices in my model are flexible, and what drives the price level are shocks in asset markets. In that sense, my model is closer in spirit to Brunnermeier and Sannikov (2016) and Piazzesi and Schneider (2018) who also study the effect of shocks originating in the asset market on the price level in flexible price models. These papers consider shocks that lower the value of risky assets and increase demand for safe money just when financial intermediaries create fewer money-like assets. That makes money more valuable, implying a fall in the price level. In my model, a speculation shock *increases* asset valuations but still leads to a fall in the price level. This is because future asset buyers save more, including by holding liquid assets they would have spent on goods.

The paper is organized as follows. The remainder of this section reviews the related literature. Section 1 lays out a purely monetary version of the Rocheteau, Weill, and Wong (2018) model. I then add an illiquid asset in Section 2 and allow for time-varying beliefs about its dividends in Section 3. Here I show that an asset boom will be associated with a lower price level. In Section 4, I endogenize output and show that the asset boom is also associated with an output boom. Section 5 considers policy interventions and illustrates the trilemma for monetary policy. Section 6 reviews the evidence on asset booms, inflation, and liquidity hoarding. Section 7 concludes. Proofs of all propositions are in an Appendix.

[1] See, for example, the Bank of Japan Outlook for Economic Activity and Prices, April 27, 2007.

Related Literature. As noted above, Bernanke and Gertler (1999) already examined whether financial stability is compatible with stabilizing output and prices. Using the financial accelerator model of Bernanke, Gertler, and Gilchrist (1999), they find that a wedge between the price of capital and the present discounted value of the income from capital acts like an aggregate demand shock. While they motivate the wedge as something that could reflect excessive optimism, they introduce the wedge exogenously. As such, their model ignores the possibility that during a boom agents might save to buy the asset in the future as in my model. By the same token, my model abstracts from important features in their model such as sticky goods prices and the role of assets as collateral against which agents can borrow.

Gilchrist and Leahy (2002), using the same Bernanke, Gertler, and Gilchrist (1999) model, consider the effects of two types of shocks. First, they consider a news shock about future productivity, which like the wedge in Bernanke and Gertler (1999) increases the price of capital without affecting current productivity. They find that a central bank which targets inflation but sets a higher real interest rate in response to higher future productivity comes close to replicating the first-best outcome. This suggests no conflict between price stability and responding to a shock that gives rise to asset booms. Gilchrist and Leahy (2002) then consider a shock to the net worth of the producers who own capital. They confirm that this shock increases both inflation and output in the benchmark version of the model, and that aggressive inflation targeting can help lower inflation, output, and asset prices. At the same time, they observe that a net-worth shock is similar to a cost-push shock in that it reduces the cost of borrowing for firms, implying it will be impossible for a central bank to *fully* stabilize both inflation and output. Although I consider a different shock and a different model, my result that some shocks that lead to asset booms create a conflict between different stabilization mandates is similar in spirit to this finding.

Christiano et al. (2010) also consider the effect of a news shock about future productivity as in Gilchrist and Leahy (2002). However, they assume the central bank does not aim for a higher real interest rate in response to this shock. They find that when the central bank is aggressive in targeting inflation without aiming for a higher real rate, a news shock will be associated with an asset boom, lower inflation, and an output boom. This is similar to the effect of a speculation shock in my model, although the mechanism is different. In addition, the main insight from Christiano et al. (2010) is that a central bank should aim for a higher real rate in response to a news shock about future productivity, which would also help dampen asset prices. In my model, by contrast, contemporaneous monetary policy cannot simultaneously stabilize asset prices, output, and goods prices in the midst of a speculation shock.

There is also work on the compatibility of financial stability and price and output stability in models with financial crises rather than asset booms. Examples include Svensson (2017), Gourio, Kashyap, and Sim (2018), and Boissay et al. (2021). In principle, asset booms and financial crises can be related. Christiano et al. (2010) show that asset booms are associated with credit growth both in their model and in the data and may lead to financial crises once the boom ends. Geanakoplos (2010) and Simsek (2013) show that when agents hold heterogeneous beliefs about an asset, they would naturally want to trade in credit. This would be true in my setting as well, which could generate a financial crisis if agents default. While I abstract from credit to simplify the analysis, I could add limited borrowing without changing any key results.

My analysis draws on two particular literatures. The first concerns models of beliefs-driven asset booms. Miller (1977) first argued that the combination of heterogeneous beliefs and short-sales constraints can lead to high asset prices. I follow Harrison and Kreps (1978) in studying a model with *time-varying* beliefs that can give rise to speculation. Harris and Raviv (1993) and Scheinkman and Xiong (2003) show how time-varying beliefs can emerge endogenously when agents are overconfident in their own signals. Both emphasize that models with time-varying beliefs can generate asset booms that feature realistically high trading volumes. For this reason, such models have become a popular framework for studying asset booms. Simsek (2021) offers a recent survey of various macroeconomic applications of these models. Among these, perhaps the most closely related paper is Bigio and Zilberman (2020). Their model also implies that beliefs-driven asset booms will be associated with higher output. In their setup, this is because employers must hire workers before they see realized productivity, and so more optimism implies more hiring. This is similar to how agents in my model produce more when they expect to earn a higher return on the income they earn from producing. However, the motive to save that is central in my model does not play a key role in their setup, and Bigio and Zilberman (2020) abstract from both money and monetary policy.

A related strand of the literature on heterogeneous beliefs concerns consumption and savings decisions, e.g., Guzman and Stiglitz (2016) and Iachan, Nenov, and Simsek (2021). These papers show that whether savings rise when agents hold different beliefs depends on whether the intertemporal elasticity of substitution for agents exceeds one. I follow Iachan, Nenov, and Simsek (2021) in assuming preferences that imply that agents want to save more when they expect the return on the assets they hold will be higher.

The other literature I draw on are monetary models that explore the interaction between money and asset prices. For a review of the broad literature, see Williamson and Wright (2010). The framework I use draws on Rocheteau, Weill, and Wong (2018) and Herrenbrueck (2019). Within this literature, the paper that comes closest to the issues I explore is Lagos and Zhang (2019). They also develop a model with money and a dividend-bearing asset that agents value differently. However, they focus on the opposite direction of causality, i.e., on how changes in the growth rate of money (and thus inflation) affect asset prices. The mechanism they emphasize is that inflation discourages agents from holding money, which discourages asset trading and lowers the value of the asset given it may not be held by the agents who value it the most. By contrast, I examine how a shock to beliefs about the asset affects goods prices, and the key mechanism is that optimism can lead agents to hoard liquidity.

Finally, an important feature of my model is that when agents are sufficiently optimistic, the price of the asset will be pinned down by the wealth agents have to spend on assets rather than their expectations of dividends. This corresponds to the notion of "cash-in-the-market" asset pricing first discussed in Allen and Gale (1994). In their setting, cash-in-the-market pricing arises when agents in need of liquidity try to unload their illiquid assets but total available liquidity is limited, leading the asset price to fall. My model is closer to Bolton, Santos, and Scheinkman (2021). Like in my model, they assume some agents expect to profit more than others from buying assets. In their model this is because some agents can identify underpriced assets. The agents who stand to profit from buying these assets are borrowing constrained, just as optimists are in my setup, and asset prices depend on how much wealth these agents have. Closer still is the work of Caballero and Farhi (2017) on demand for safe assets. They also assume some agents have a stronger preference for an asset than others, although in their case it is because of differences in risk-aversion rather than beliefs. My assumption that pessimists avoid the asset is equivalent to their assumption that infinitely risk-averse agents refuse to hold the asset. Cash-in-the-market pricing in my model corresponds to the constrained regime equilibrium in their setup.

1 Preliminaries: A Purely Monetary Economy

As anticipated in the Introduction, I embed a model of speculative trade as in Harrison and Kreps (1978) into a monetary setting. I begin by setting up the monetary model, which is based on Rocheteau, Weill, and Wong (2018). This section describes the model starting with the case in which agents can only hold money. The next section adds a dividend-bearing asset in addition to money. Section 3 then allows agents to hold different and time-varying beliefs about the asset as in Harrison and Kreps (1978).

The economy is populated by a mass 1 of infinitely-lived agents. Time is continuous. Each agent is endowed with a constant flow of y units of a non-storable consumption good per instant. For now, the supply of goods is exogenously fixed. Later on, I consider a production economy where agents are endowed with inputs and choose how much to produce.

Agents derive utility from consumption only at individual-specific random dates $\{t_n\}_{n=1}^{\infty}$, where the gap between urges is distributed exponentially with rate λ. Agents thus experience occasional idiosyncratic urges to consume. I assume the law of large numbers holds, meaning that a constant flow λ of agents chosen at random will have an urge to consume at each instant.

For ease of notation, I omit reference to a household's identity. Let c_t denote the amount an agent consumes at date t. Agents are risk-neutral with respect to consumption when they have an urge to consume and discount the future at rate ρ, i.e., their utility is given by

$$U \equiv \sum_{n=1}^{\infty} e^{-\rho t_n} c_{t_n} \tag{1}$$

Strict risk-neutrality is not essential for my main results, although it greatly simplifies the analysis.[2] That said, linear preferences imply an infinite elasticity of intertemporal substitution. For agents to save more when they expect a higher return on their savings, the elasticity must exceed unity. See Iachan, Nenov, and Simsek (2021) for a discussion in a related context.

[2] Rocheteau, Weill, and Wong (2018) show how to solve the general case in which (1) is replaced by $\sum e^{-\rho t_n} u(c_{t_n})$ for some concave function $u(\cdot)$. Herrenbrueck (2019) similarly focuses on the linear case to simplify the analysis.

At any instant, consumption should only be allocated to the flow of agents with an urge to consume. Following Rocheteau, Weill, and Wong (2018), I assume agents cannot rely on intertemporal trade, e.g. because of limited tools to enforce contracts. However, agents are endowed with money that they can exchange for goods. The model is thus essentially a continuous-time version of Bewley (1980).[3]

The supply of money is fixed at an amount $\overline{M}$ at all dates. Allowing the money supply to grow over time would lead to similar results but requires more notation. Let $F_t(M)$ denote the fraction of households whose money holdings at date t is less than or equal to M for any value $M \in [0, \infty)$. The initial distribution $F_0(M)$ at $t = 0$ is assumed to contain no mass points.

Let P_t denote the price of goods in terms of money at date t. An equilibrium constitutes paths $\{P_t, F_t(M)\}_{t\geq 0}$ for the price of goods and the distribution of money holdings such that the market between goods and money clears at all dates when households choose their spending optimally. As usual in monetary models, many equilibria are possible. Following Rocheteau, Weill, and Wong (2018), I restrict attention to *stationary monetary* equilibria. A *monetary* equilibrium is one in which $P_t < \infty$ for all t so money always has value. A *stationary* equilibrium is one in which the price of goods grows at the same rate as money. For a constant money supply, this amounts to equilibria in which $\frac{\dot{P}_t}{P_t} = 0$.

In any monetary equilibrium, agents prefer to sell their entire endowment y when they don't have an urge to consume: Consuming the endowment yields no utility, while selling it allows agents to increase their consumption when they do have an urge to consume. In between urges to consume, then, agents accumulate money by selling their endowment. Each individual's money holdings evolve according to

$$\dot{M}_t = P_t y \tag{2}$$

At dates $\{t_n\}_{n=1}^{\infty}$ in which the household has an urge to consume, it must choose how much of its money holdings to spend. In a stationary equilibrium, a household with linear preferences would choose to spend all of its money holdings: When the price level P_t is constant, waiting to consume will not result in a better price. Since the marginal utility from consumption is constant, discounting implies the household should consume immediately rather than wait to spend its money.[4]

Given these optimal decisions, solving for the price level P_t in a stationary monetary equilibrium is straightforward. At any instant, a flow of λ agents chosen at random will have an urge to consume and spend all of their money holdings. Under the law of large numbers, the average money holdings of these

[3] More precisely, Bewley (1980) assumed agents have random time-varying endowments and a fixed concave utility. Here, agents have a fixed endowment and linear utility with random time-varying marginal utility. Both specifications imply agents' marginal utilities would fluctuate over time in autarky, encouraging trade.

[4] If I let the money stock $\overline{M}_t$ grow at rate μ, a stationary monetary equilbrium would feature $\frac{\dot{P}_t}{P_t} = \mu$. As long as $\mu > -\rho$, i.e., as long as prices do not fall enough to overcome discounting, agents would prefer to spend all of their money holdings rather than hold on to money. If $\mu > 0$, agents have more reason to spend since the prices they face will be higher if they wait.

agents will be the same as the average money holdings of all agents, which is given by

$$\int_0^\infty M dF_t(M) = \overline{M} \tag{3}$$

Total spending on goods at each instant will equal $\lambda\overline{M}$ regardless of how money holdings are distributed across agents. Since only a measure zero of agents will want to consume at any instant while all other agents sell their endowment, the total value of goods up for sale at each instant is $P_t y$. Equating the two yields a unique value for the price level P consistent with a stationary monetary equilibrium:

$$P_t = \frac{\lambda\overline{M}}{y} \tag{4}$$

What makes the Rocheteau, Weill, and Wong (2018) setup convenient is that spending on goods is always equal to $\lambda\overline{M}$ regardless of how money holdings are distributed. Even though the money holdings of any individual household rise and fall over time, there is no need to track the distribution of money holdings across households to determine the price level P_t. Nothing prevents P_t from jumping to its steady-state value immediately starting from any initial distribution of money holdings across households.

To recap, in the stationary monetary equilibrium, agents accumulate money in between urges to consume and spend down all of their money holdings when hit with an urge to consume. Since money is the only asset agents can hold, they have no choice of what to accumulate in between urges. The next section introduces a dividend-bearing asset that agents can hold in addition to money.

2 Assets with Homogeneous Beliefs

Suppose now that in addition to a stock of money $\overline{M}$, agents are endowed with one unit of an asset that yields a constant flow of D consumption goods per instant. Let p_t denote the price of the asset and a_t denote a household's asset holdings at date t. The holdings of all households integrate up to 1. Rocheteau, Weill, and Wong (2018) study a similar setup in which agents can hold either money or a dividend-bearing asset, although they consider a nominal government bond that pays out money rather than goods.

When there is a dividend-bearing asset, money would have to offer some advantage such as liquidity for agents to hold it. I follow Rocheteau, Weill, and Wong (2018) in assuming agents can exchange money for goods and money for assets, but not assets for goods. Technically, neither an agent's money nor her asset holdings at date t are measurable with respect to whether she has an urge to consume at t. Agents must allocate their wealth before knowing if they have an urge to consume. Once they learn if they want to consume, they can buy goods with money. Agents can exchange assets for money within an infinitesimally short period of time and then buy goods, but by then the urge to consume would have lapsed. Money allows agents to buy goods if they have an urge to consume, but only by forgoing earning a return on their wealth.

Formally, let W_t denote an agent's nominal wealth at date t, where

$$W_t = M_t + p_t a_t \tag{5}$$

Since agents cannot trade goods for assets, their consumption must satisfy

$$P_t c_t \leq M_t \tag{6}$$

I continue to assume households cannot engage in intertemporal trade. This means agents cannot sell assets short, i.e. they cannot borrow assets and promise to deliver similar assets to their lender in the future. Household asset holdings must therefore be non-negative and cannot exceed a household's wealth, i.e.,

$$0 \leq p_t a_t \leq W_t \tag{7}$$

An equilibrium is a path of prices $\{P_t, p_t\}_{t\geq 0}$ for goods and assets, respectively, together with a path for the distribution of nominal wealth holdings $F_t(W)$ such that the market between goods and money and the market between assets and money clear at all dates. I again restrict attention to stationary monetary equilibria. Given the path of dividends is the same from any starting date, a stationary equilibrium now requires that both P_t and p_t grow at the same rate as money, i.e., $\frac{\dot{P}_t}{P_t} = \frac{\dot{p}_t}{p_t} = 0$.

As in the purely monetary economy, in any monetary equilibrium households prefer to sell any goods they have in between urges to consume. This now includes their endowment y and the Da_t goods they earn as dividends on their assets. In between urges to consume, then, wealth W_t evolves according to

$$\dot{W}_t = P_t y + (P_t D + \dot{p}_t)\, a_t \tag{8}$$

In any stationary equilibrium, an agent will once again prefer to spend her entire money holdings M_t on goods when hit with an urge to consume. To see this, note that the instantaneous return to holding an asset in a stationary equilibrium is $\frac{D+\dot{p}_t}{p_t} = \frac{D}{p_t} > 0$. Since money offers a zero nominal return, forgoing a positive return can only be optimal if the agent intends to spend her money on goods whenever she has an urge to consume; holding money for the next instant would have been a waste otherwise. The total amount spent on goods at each instant thus remains $\lambda\overline{M}$, while the value of goods supplied at any instant is $P_t(y + D)$. The equilibrium price level in a stationary monetary equilibrium is then

$$P_t = \frac{\lambda\overline{M}}{y + D} \tag{9}$$

I next turn to the asset. Agents have linear utility, so the tradeoff between money and assets will not depend on wealth. Since all agents trade off money and the asset equally, they must be indifferent between the two in equilibrium. When agents earn money from selling goods, they don't care if they hold on to it or to use it to buy assets. In particular, agents will find it optimal to keep what assets they have and to accumulate any additional wealth they earn from selling goods in the form of money, waiting until the next consumption urge to spend it all.

I can use the requirement that agents be indifferent between money and the asset to pin down the equilibrium price p. The utility from holding the asset for a small instant and then selling it must be the same as the utility from holding money.[5] Holding the asset for an instant dt increases an agent's nominal wealth by $PD \cdot dt$. Let v_t denote the marginal value to an agent of a unit of nominal wealth at date t, measured in utility. The utility gain from buying an asset and holding it for an instant is $(PD \cdot dt) \times v_t$. Before comparing this to the utility gain from holding money over the same instant, let me first derive an expression for v_t. Since agents are always indifferent between money and the asset in equilibrium, they would be willing to hold their wealth as money and spend it at the next urge to consume. Let T denote the time of the next urge. The value v_t of an additional unit of money at date t is just the utility of consuming $\frac{1}{P}$ at date T where T is exponentially distributed with rate λ, i.e.,

$$v_t = \int_t^{\infty} \lambda e^{-(\rho+\lambda)(T-t)} \frac{1}{P} dT = \frac{\lambda}{\lambda+\rho} \frac{1}{P} \tag{10}$$

The gain from buying an asset for an instant dt is thus $(PD \cdot dt) \times v_t = \frac{\lambda D}{\rho+\lambda} \cdot dt$. If the agent instead held the amount p as money over the next instant, her nominal wealth wouldn't change. But there is a λdt chance she would have an urge to consume over the next instant. In that case, she could buy $\frac{p}{P}$ goods immediately rather than wait for the next urge. The value of waiting until the next urge to consume is $v_t = \frac{\lambda}{\rho+\lambda} \frac{1}{P}$. Hence, the expected gain from holding money is equal to $\lambda dt \left[1 - \frac{\lambda}{\rho+\lambda}\right] \frac{p}{P} + (1 - \lambda dt) \cdot 0$. Equating the two gains implies

$$\frac{p}{P} = \frac{D}{\rho} \tag{11}$$

Steady state equilibrium prices are thus given by (9) and (11). Adding an asset that agents perceive and value equally does not qualitatively change the equilibrium from the pure monetary economy, since agents have no reason to trade the asset in this setup.

In the next section, I introduce a motive for agents to trade the asset by assuming they hold different beliefs about its payoff. This is not the only way to introduce a motive for trade. For example, Rocheteau, Weill, and Wong (2018) allow agents to have strictly concave utility over consumption. In that case, agents who go a long time without an urge to consume amass significant wealth and may not want to spend it all when they next have an urge to consume given diminishing returns. This would lead them to buy illiquid assets from agents who just experienced an urge to consume and prefer to hold more liquid assets. Herrenbrueck (2019) generates a motive for trade by assuming households are more likely to have an urge to consume at certain times than at others. Households who are more likely to have an urge to consume would sell their assets to those who are less likely to have an urge to consume.

[5] To be precise, the payoff to holding an asset for a period of length Δ and taking the limit as $\Delta \to 0$ must be the same as holding the asset. The limit corresponds to the heuristic argument in the text.

3 Heterogeneous Time-Varying Beliefs: Steady State

I now incorporate heterogeneous time-varying beliefs about the asset as in Harrison and Kreps (1978). Suppose that in addition to the flow dividend D, agents now believe the asset might pay discrete lump-sum amounts of Δ_t per share if and when certain events occur. However, agents track different aspects of the asset and do not perceive the same payoff-relevant events. For example, some agents might follow a firm's patenting activity while others follow its merger activity. As a result, agents differ on which events they believe will trigger the next payment Δ_t and what they expect that payment Δ_t to be. Agents may also be unaware of events that they don't follow and which have no effect on dividends, such as a patent or a merger that didn't pan out. In what follows, I impose assumptions on realized payments Δ_t to ensure dividend payments don't reveal information to agents that they do not already track.

Formally, suppose an agent observes events at dates $\{\tau_n\}_{n=1}^{\infty}$. These dates are idiosyncratic and independent across agents, reflecting the idiosyncratic nature of the news that different agents follow. For each agent, events arrive at rate α. Assuming the law of large numbers, only a flow α of households believe there was news about the asset at each instant. Define $\mathcal{T} \equiv \min\{\tau_n : \tau_n \geq t\}$ as the date of the next news event an agent sees, and let $\Delta \equiv \Delta_{\mathcal{T}}$ denote the lump-sum dividend payment at date $\mathcal{T}$. For all agents, $\mathcal{T}$ is exponentially distributed with rate α. However, agents differ in their expectations of Δ at the next arrival date. Specifically, any agent can be either optimistic or pessimistic at a given point in time. While optimistic, the agent expects Δ will equal $\frac{\Delta^+}{q} > 0$ with probability q and 0 with probability $1-q$, for an average payoff of Δ^+. While pessimistic, the agent expects to incur a lump-sum cost to maintain the asset that is equal to $\frac{\Delta^-}{q}$ with probability q and a cost of 0 with probability $1-q$, for an average of Δ^-. Following Harrison and Kreps (1978) and Scheinkman and Xiong (2003), beliefs vary over time. In particular, agents believe payoffs alternate sequentially across events. If they expect the next payoff to be Δ^+, they expect the payoff in the event after that to be Δ^-, and vice versa. Agents who believe a payoff event just occurred switch between optimism and pessimism. Half the agents start as optimists at date 0 and half as pessimists. By symmetry, the share of agents who are optimistic always remains at $\frac{1}{2}$.

To ensure no agent observes something they believe is impossible, I assume the realized payments Δ_t always equal 0. Agents who believe no news occurred are thus never proven wrong. However, this assumption implies agents are motivated by payoffs that never materialize. I could add payoff events that all agents observe and which involve non-zero Δ_t, complicating the analysis without affecting my key results. One might also worry that agents should doubt their beliefs if Δ_t keeps coming up 0. I assume agents hold dogmatic beliefs, or alternatively that they view events as distinct enough that each time can be treated as different. More realistically, I could allow disagreement to go away at a constant a rate before such doubts fester. However, it is easier to model disagreement as permanent and focus on steady states.

In short, at any given instant, all agents believe a payoff event could happen over the next instant. Half believe the payoff will be positive (on average) and half believe it will be negative (on average). Only a flow α actually believe an event does happen. Those who are optimistic will be disappointed to see no windfall and turn pessimistic, while those who are pessimistic will be relieved to see no loss and turn optimistic.[6] Assuming agents oscillate between optimism and pessimism as in the original Harrison and Kreps (1978) model is convenient. However, my main results only require that those who are currently pessimistic expect to turn optimistic in the future and save to buy assets later. It is not essential that optimists who turn pessimistic intend to buy the asset back again, and I could have equally assumed optimists leave the market after they sell and are replaced by new entrants who buy the asset immediately (if they are optimistic) or eventually (if they are pessimistic).

Agents continue to allocate their wealth between money and assets before learning whether they have an urge to consume. Optimistic agents expect the return on the asset over the next instant to be

$$P_t\left(D+\alpha\Delta^{+}\right)+\dot{p}_t$$

Pessimistic agents expect the return to be

$$P_t\left(D-\alpha\Delta^{-}\right)+\dot{p}_t$$

When $\Delta^{+}=\Delta^{-}=0$, we are back to case where agents value the asset equally as in Section 2. The model thus nests homogeneous beliefs as a special case.

Suppose the economy starts at $\Delta^{+}=\Delta^{-}=0$ but then unexpectedly shifts to positive values for Δ^{+} and Δ^{-}. This shock is often described as a *disagreement* shock. In the current context, it is more aptly described as a *speculation* shock, since for my results it is not so important that agents disagree as it is that they all want to buy the asset, possibly only in the future, with the intent to eventually sell it. I assume

$$\alpha\Delta^{-}>D \tag{12}$$

This implies pessimists expect a negative nominal return on the asset over the next instant and would prefer to hold money. The exact value of Δ^{-} will not matter in a stationary equilibrium, since it will be optimists who own the asset in equilibrium and the price of the asset will reflect their views. Pessimism can be viewed as a stand-in for other reasons agents wait to buy assets, e.g., down-payment requirements that force agents to save or information frictions that lead agents to wait for buying opportunities for assets they know.

Define W_t^{+} as the total wealth of optimists at date t and W_t^{-} as the wealth of pessimists at date t. These must add up to total available wealth, i.e.,

$$W_t^{+}+W_t^{-}=\overline{M}+p_t \tag{13}$$

In a stationary equilibrium, (12) implies only optimists hold the asset. If the initial wealth of optimists

[6] This pattern is reminiscent of the model of diagnostic beliefs in Bordalo, Gennaioli, and Shleifer (2018) in which agents form beliefs based on salient recent experiences such as surprises and disapointments. However, agents in my model switch between optimism and pessimism regardless of how the realization of Δ_t compares to their expectation.

W_0^+ is too low, optimists cannot buy up all available assets at the price that clears the market. In that case, the economy cannot jump immediately to the stationary equilibrium as in the previous two sections. Instead, the total wealth of optimists acts as a state variable that governs equilibrium prices, even as the distribution of wealth among optimists (or among pessimists) remains irrelevant for equilibrium prices. Define an *asymptotically stationary monetary equilibrium* as a monetary equilibrium that is stationary in the limit as $t \to \infty$. With a fixed supply of money, that means equilibria in which $\lim_{t\to\infty} \frac{\dot{P}_t}{P_t} = \lim_{t\to\infty} \frac{\dot{p}_t}{p_t} = 0$. In this section, I focus on the long-run steady states of these equilibria, i.e., when p_t, P_t, and W_t^+ have had enough time to converge to their long-run levels. I return to transitional dynamics in Section 5.

In both the purely monetary economy and the economy with a dividend-bearing asset that agents value equally, an optimal rule for agents was to accumulate money in between urges to consume and to spend down any money holdings when they have an urge to consume. With heterogeneous time-varying beliefs, the optimal rule will deviate from this prescription when the degree of optimism Δ^+ is sufficiently large. First, in between urges to consume, optimistic agents may strictly prefer to buy assets rather than hold money. They will forgo a chance to satisfy their consumption urges to earn potentially high returns. Second, pessimistic agents might no longer spend all of their money on goods when they have an urge to consume. Instead, they may choose to hold on to money to buy assets when they next expect a high return on the asset (which will happen after the next payoff event they observe).

The implied law of motion for W_t^+ when agents face a constant path for prices is as follows:

$$\dot{W}_t^+ = \frac{\alpha}{2}\left(W_t^- - W_t^+\right) + P\left(\frac{y}{2} + D\right) - \frac{\lambda}{2}\left(W_t^+ - p\right) \tag{14}$$

The first term denotes the change in the total wealth of optimists that is due to changes in beliefs. A flow $\frac{\alpha}{2}$ of optimists chosen at random turn pessimists in the next instant, and their wealth would be subtracted from the total wealth of optimists. At the same time, a flow $\frac{\alpha}{2}$ of pessimists chosen at random become optimists, and their wealth would be added to the total wealth of optimists. The second term denotes the income optimists earn from selling their goods, both those they are endowed with and those they earn as dividends. Since half of the agents are optimists, they account for half of the total endowment. But under (12), optimists own all of the asset in steady state and receive all of its endowment. Finally, a flow $\frac{\lambda}{2}$ of optimists chosen at random will have an urge to consume. They will spend any money they hold, since the only reason for optimists to hold money is to buy goods if they have an urge to consume. The money holdings of optimists equal the difference between their total wealth W_t^+ and the value p of the asset.

Setting $\dot{W}_t^+ = 0$ and substituting in for W_t^- from (13), we can solve for the steady-state level of wealth of optimists $\overline{W}^+$ at which $\dot{W}_t^+ = 0$:

$$\overline{W}^+ = \frac{P\left(y + 2D\right)}{2\alpha + \lambda} + \frac{\alpha}{2\alpha + \lambda}\overline{M} + \frac{\alpha + \lambda}{2\alpha + \lambda}p \tag{15}$$

The steady-state wealth of optimists depends on the price level P and the asset price p. While total wealth $\overline{M} + p$ increases one-for-one with p, the steady-state wealth of optimists $\overline{W}^+$ rises *less* than one-for-one with p. Intuitively, a fraction of the asset at any instant is held by agents who just turned pessimist, so a higher asset price benefits more than just optimists. This has an important implication: If optimists are more bullish on the asset, say because Δ^+ is high, and they bid up the asset price, their wealth will not rise by the same amount as the asset price. When optimists are very bullish, they might not be able to bid up asset prices in line with their beliefs. This leads to what Allen and Gale (1994) define as *cash-in-the-market pricing*, when the price of the asset is determined by the resources agents have to buy it rather than its (perceived) fundamentals. In Allen and Gale (1994), such pricing occurs because of unexpectedly high demand for liquid assets. Here, such pricing occurs when borrowing constraints limit demand for illiquid assets, similarly to cash-in-the-market pricing in Bolton, Santos, and Scheinkman (2021) or to pricing in the constrained regime in Caballero and Farhi (2017).

Proposition 1 below fully characterizes the stationary equilibrium prices P and p associated with the steady state wealth of optimists $\overline{W}^+$. But I begin with a heuristic description of this equilibrium. Recall from Section 2 that in a stationary equilibrium, agents are indifferent between money and the asset if and only if they expect the nominal return on the asset is ρ. In a stationary equilibrium, optimists expect the return to be $P\left(D+\alpha\Delta^+\right)/p$, so the price that ensures this return equals ρ is

$$p^* = \frac{P\left(D+\alpha\Delta^+\right)}{\rho} \tag{16}$$

If $\overline{W}^+ > p^*$, optimists have enough steady state wealth to buy the asset at the price p^* that ensures they expect a return of ρ from the asset. In that case, optimists will bid up the asset price to p^* and still have additional wealth to hold as money. If $\overline{W}^+ < p^*$, optimists will not be able to buy the asset if it were priced to ensure an expected return of ρ. In that case, the price of the asset p will be $\overline{W}^+$ rather than p^*. Optimists expect to earn a return above ρ from the asset and will strictly prefer the asset to money.

When $\Delta^+ = 0$, the price $p^* = \frac{PD}{\rho}$ will be the same as in the case of homogeneous beliefs. To ensure that agents can afford to buy the asset at this price requires an additional assumption. This was not an issue when all agents had the same beliefs: In that case, all agents were willing to buy the asset, and their collective wealth is $W = \overline{M} + p > p$ always allowed them to afford the asset. By contrast, if $\Delta^- > D/\alpha$, half of all agents refuse to hold the asset, and so we need to make sure that $\overline{W}^+ > \frac{PD}{\rho}$. A sufficient condition that ensures optimists can afford to buy the asset when $\Delta^+ = 0$ is

$$\frac{y+D}{\lambda} > \frac{D}{\rho} \tag{17}$$

Intuitively, (17) stipulates that the average real income agents earn in between urges to consume exceeds the real value of the asset, allowing agents to amass enough wealth to afford the asset.

Since p^* is increasing in the degree of optimism Δ^+, and since the expression for $\overline{W}^+$ in (15) rises less than one-for-one with p, then for sufficiently high Δ^+, optimists will be constrained and assets will be priced according to cash-in-the-market pricing. This is illustrated graphically in Figure 1. The gray lines correspond to the equilibrium real asset price p/P and the return optimists expect when $\Delta^+ = \Delta^- = 0$ and beliefs are homogeneous as in Section 2. The black lines trace the equilibrium with heterogeneous and time-varying beliefs for different values of Δ^+ and assuming $\Delta^- > D/\alpha$. When Δ^+ is small, optimists have sufficient wealth to buy the asset at the price p^*. This price, recall, ensures optimists expect to earn a return ρ on the asset. The real price p^*/P rises linearly with the degree of optimism Δ^+. Intuitively, to keep the expected return at ρ even though agents expect a bigger windfall Δ^+ requires the asset to be more expensive. Eventually, for sufficiently high Δ^+, the price p^* needed to ensure that optimists expect a return of ρ would exceed $\overline{W}^+$. In this case, the asset price $p = \overline{W}^+$. Since Δ^+ only affects the expected return but not the actual wealth of optimists, the asset price p is invariant to Δ^+ in this range. By contrast, the real price p/P rises with Δ^+ when Δ^+ is large, for reasons I explain shortly and which have to do with changes in the price of goods P rather than the asset price p. With a fixed asset price, a higher Δ^+ implies optimists should expect to earn more than ρ from the asset.

Comparing asset prices and returns when Δ^+ and Δ^- are positive to their corresponding values when $\Delta^+ = \Delta^- = 0$ suggests that a speculation shock should lead to a higher real asset price (an asset boom) and, if Δ^+ is sufficiently large, a higher expected return on the asset among optimists.

The reason that the real price of the asset, p/P, starts rising again for very high levels of Δ^+, as depicted in Figure 1, is because of how the price level P varies with Δ^+. This is illustrated in Figure 2. The gray line once again shows the case where $\Delta^+ = \Delta^-$ and beliefs are homogeneous. When agents value the asset equally, they will spend down their money holdings when they are hit with an urge to consume. Hence, total spending on goods is equal to $\lambda\overline{M}$. Since these are traded for $y + D$ goods, the price level is $P = \frac{\lambda\overline{M}}{y+D}$. When Δ^+ is not too large, agents will continue to spend down their money holdings when they are hit with an urge to consume, and the price level will be the same. However, this becomes unsustainable for very high degrees of optimism. If both the price level P remained constant at $\frac{\lambda\overline{M}}{y+D}$ and the nominal asset price p remained constant at $\overline{W}^+$, the expected return to holding the asset $P\left(D + \alpha\Delta^+\right)/p$ would rise linearly with Δ^+. Eventually, the return to holding the asset would be high enough that pessimists would prefer to hold on to their money and buy assets when they expect the return on the asset to turn high. But if nobody wants to consume, the goods market will not clear. Hence, for very large degrees of optimism, the price level P must fall to ensure pessimists are indifferent between consuming and saving to buy assets in the future. If we let v_t^- denote the value of an additional unit of nominal wealth for pessimists, the price P must adjust in equilibrium so that v_t^- does not exceed $1/P$ and holding on to money becomes more valuable than consumption. For the price level P to fall, pessimists must hoard some of the cash they hold rather than spend it all when they have an urge to consume. As P falls, the real price of the asset p/P rises, and the return $P\left(D + \alpha\Delta^+\right)/p$ that optimists expect falls. If pessimists hoard just the right amount, v_t^- will equal $1/P$ and pessimists would be indifferent between hoarding money and spending it on consumption. Essentially, a large degree of optimism Δ^+ induces pessimists who want to wait to buy the asset to save enough money until they are just indifferent between saving and buying goods.

The way equilibrium prices P and p vary with Δ^+, as shown in Figures 1 and 2, can be summarized thus:

Proposition 1: *Suppose $\alpha\Delta^- > D > 0$ and condition (17) holds. There exists a unique steady state monetary equilibrium. Moreover, there exist cutoffs Δ^* and Δ^{**} where $0 < \Delta^* < \Delta^{**}$ such that*

1. *If $\Delta^+ < \Delta^*$, the steady state equilibrium is $\frac{p}{P} = \frac{D+\alpha\Delta^+}{\rho}$ and $P = \frac{\lambda\overline{M}}{y+D}$. In this equilibrium, optimists hold both assets and money, pessimists only hold money, and both optimists and pessimists spend down their money balances when they have an urge to consume.*

2. *If $\Delta^+ \in [\Delta^*, \Delta^{**}]$, the steady state equilibrium is $\frac{p}{P} = \frac{y+D}{\lambda} + \frac{y+2D}{\alpha}$ and $P = \frac{\lambda\overline{M}}{y+D}$. In this equilibrium, optimists only hold assets, pessimists only hold money, and pessimists spend all of their money balances when they have an urge to consume.*

3. *If $\Delta^+ \geq \Delta^{**}$, the steady state equilibrium is $\frac{p}{P} = \frac{(\alpha+\rho)D+\alpha^2\Delta^+}{\rho(2\alpha+\rho)}$ and $P = g(\Delta^+)\overline{M}$ for some function g where $g'(\cdot) < 0$. In this equilibrium, optimists only hold assets, pessimists only hold money, and pessimists hoard some of their money to buy assets in the future.*

In equilibrium, agents buy assets when they are optimistic and sell them when they turn pessimistic. If the boom is small, they will consume whether they hold assets or not. If the boom is large, they will consume only when they are pessimistic. A speculation shock leads to a higher real asset price and, when the asset boom is sufficiently large, to a higher expected return on savings. Very large booms will be associated with a lower price level as agents hoard liquid assets to speculate on assets in the future. In Section 6, I discuss evidence on low inflation and liquidity hoarding during asset booms.

I conclude this section with a few remarks. First, the model predicts asset booms should feature higher savings. Empirically, asset booms tend to be associated with consumption booms. The two things are in fact compatible. Since my model assumes an endowment economy with a fixed supply of nondurable goods, the increase in desired savings is not associated with lower consumption. Instead, *real* savings rise when the price level falls. When I allow for production in the next section, I show that a speculation shock leads to higher production of nondurable goods and thus higher consumption. A speculation shock can thus increase both consumption *and* real savings. Indeed, work on the recent U.S. housing boom has highlighted the role of higher savings. For example, Bernanke (2005) argued the housing boom was associated with a global savings glut. Caballero (2006) argued this boom was associated with a shortage of assets that were in high demand and points out that the demand for assets may have led to lower inflation. Bates, Kahle, and Stulz (2009) documented an increase in cash holdings by corporations during the dot com and housing booms.

Second, a potential issue with my model is whether its predictions about the price level would remain if agents could save using something other than money as they wait to buy assets. Suppose I added a third asset into the model that is also illiquid but which agents agree on. As in Section 2, in equilibrium agents will be indifferent between money and illiquid assets they agree on. With a sufficiently large degree of optimism Δ^+ for the asset agents disagree on, optimists hold the asset about which agents disagree while pessimists would hold money and the illiquid asset on which agents agree. Agents who save to buy assets in the future would thus be willing to save using illiquid assets rather than money. However, equilibrium still requires that the price level P adjusts to ensure pessimists don't just want to save. In other words, the price level P falls in asset booms not because agents are forced to hold money in order to save, but because equilibrium requires the price level to fall, and so agents must end up hoarding money.

Finally, one can ask whether the asset boom in the model can be viewed as a bubble. Harrison and Kreps (1978) originally pointed out that the equilibrium price of the asset may exceed what any agent believes the asset will pay out as dividends. In particular, as long as the price of the asset is not determined according to cash-in-the-market pricing, the asset will be priced as if the expected payoff at each payoff date is Δ^{+}. But no agent believes all payoff events have this payoff. Scheinkman and Xiong (2003) and others have interpreted this pattern to mean the asset can be viewed as a bubble. Barlevy (2015) argues it is hard to argue the asset is objectively overpriced simply because agents disagree. Regardless, when the price of the asset is determined by cash-in-the-market pricing, there is no guarantee that the price will necessarily exceed what all agents believe the asset will pay out. Moreover, whether the asset price exceeds what agents believe about discounted dividends does not play an important role for my results. Instead, the key feature behind my results is that agents to expect a higher return on the asset during a boom. In line with this, Greenwood and Shleifer (2014) show empirically that high realized asset price growth is often associated with higher expected returns.[7] Higher expected returns are what lead agents in the model to hoard money, and, as we shall see in the next section, are also what lead agents to produce more.

4 Endogenous Output

Up to now, I considered an endowment economy in which the amount of goods available for consumption was exogenously fixed. I now endogenize the quantity of goods by letting agents decide how much to produce. This allows me to examine what happens to output during an asset boom.

I introduce production as in Rocheteau, Weill, and Wong (2018) and Herrenbrueck (2019). Rather than being endowed with a fixed amount of goods y, agents are endowed with a productive input, e.g. labor. At each instant, they choose the amount of input n_t to use to produce. Agents produce on their own rather than sell their services in a labor market. In other words, households choose how much to produce given the price P_t rather than how much to work at a given wage.

I assume a linear production technology for a household in which $y_t = n_t$. Each household incurs a cost of using its input that is given by a differentiable function $\phi(n)$ with $\phi'(n) \geq 0$, $\phi''(n) > 0$, and

[7] Realized returns on the asset will be high (technically infinite) on impact when the disagreement shock hits and the asset price jumps. Thereafter, the direction is ambiguous. Section 5 shows that if the initial wealth of optimists W_0^{+} is below the steady state $\overline{W}^{+}$, the asset price p_t will rise along the transition and the realized return on the asset can remain high. However, since the true $\Delta_t = 0$ for all t, realized returns in steady state are lower than before the disagreement shock.

$\lim_{n\to\infty} \phi'(n) = \infty$. Households choose production n_t, consumption c_t, and asset holdings a_t to maximize

$$E\left[\sum_{n=1}^{\infty} e^{-\rho t_n} c_{t_n} - \int_0^{\infty} e^{-\rho t}\phi(n_t)\,dt\right] \tag{18}$$

given their initial wealth W_0 and subject to the budget constraint

$$\dot{W}_t = P_t n_t + (P_t(D+\Delta_t) + \dot{p}_t)\,a_t \tag{19}$$

and the constraints that stem from the lack of intertemporal trade:

$$0 \le p_t a_t \le W_t \tag{20}$$

$$P_t c_t \le W_t - p_t a_t \tag{21}$$

Agents continue to believe Δ_t follows a Poisson arrival process where $E[\Delta_t]$ alternates between Δ^+ and Δ^-, although I consider a realization in which $\Delta_t = 0$ for all t.

Let v_t^+ and v_t^- denote the utility value of a unit of nominal wealth at date t for optimists and pessimists, respectively. The value of a marginal unit of effort that produces a unit of the good is thus $P_t v_t$, the value of the money the household earns from the additional good it sells. Agents will choose n_t to satisfy

$$\phi'(n_t) = P_t v_t$$

I continue to focus on the steady state of the asymptotically stationary monetary equilibrium. When $\Delta^+ = \Delta^- = 0$ and agents hold the same beliefs, we know from (10) that $v_t = \frac{\lambda}{\lambda+\rho}\frac{1}{P}$. In that case, $Pv = \frac{\lambda}{\lambda+\rho}$, and so workers will choose n according to

$$n_t^+ = n_t^- = \phi'^{-1}\left(\tfrac{\lambda}{\lambda+\rho}\right) \equiv n^* \tag{22}$$

Total output will be constant and equal to n^*.

Next, suppose the economy is hit with a speculation shock so that Δ^+ and Δ^- are both positive and $\alpha\Delta^- > D$ as before. In the endowment economy, I imposed a regularity condition (17) to ensure that when $\Delta^+ = 0$, optimists had enough wealth to buy the asset at the real price $\frac{D}{\rho}$. The analogous condition in the production economy is given by

$$\frac{n^* + D}{\lambda} > \frac{D}{\rho} \tag{23}$$

where n^* is defined in (22). Condition (23) ensures that the steady state wealth of optimists $\overline{W}^+$ exceeds $\frac{PD}{\rho}$. Hence, as $\Delta^+ \to 0$, the wealth of optimists would exceed the price of the asset. This means that optimists must be indifferent between money and the asset for low values of Δ^+. By the same argument I used to derive (10), this indifference implies $v_t^+ = v_t^- = \frac{\lambda}{\lambda+\rho}\frac{1}{P}$: Since agents always remain indifferent between money and the asset, the value of a unit of money is just the utility value of waiting until the next urge to consume to spend it. If Δ^+ is small, both optimists and pessimists will continue to produce n^* just as when there is no speculation. This is illustrated in Figure 3, where the thick black and gray lines depict the output of optimists and pessimists, respectively, for different Δ^+.

Once the degree of optimism Δ^+ exceeds some cutoff Δ^*, the wealth of optimists who produce n^* will no longer suffice to buy the asset at the price $p^* = \frac{P\left(D+\alpha\Delta^+\right)}{\rho}$ that ensures optimists expect to earn a return of ρ on the asset. Unlike the endowment economy, optimists can now produce more, increase their wealth, and spend more on the asset. But they would only be willing to produce more if Pv_t^+ exceeded $\frac{\lambda}{\lambda+\rho}\frac{1}{P}$. The formal analysis of this case is summarized in Proposition 2 below. As seen in Figure 3, optimists in equilibrium do end up working more when $\Delta^+ > \Delta^*$. However, they will only agree to work more if they are rewarded more for each additional unit they produce. What happens in equilibrium is thus similar to what happens in the endowment economy: For higher values of Δ^+, the real asset price will not rise by the amount needed to keep the return to holding the asset at ρ. Optimists therefore prefer to hold the asset, which implies their marginal value of nominal wealth v_t^+ increases with Δ^+. Essentially, for higher Δ^+, optimists view speculation as more profitable than holding money and are therefore willing to produce more. One might have expected pessimists to produce less given they believe the asset is overvalued. However, since they do not hold the asset, the low return does not discourage them from producing. Moreover, since pessimists expect a high return the future, they will also choose to produce more, just not as much as optimists. This is why, as shown in Figure 3, pessimists also produce more when $\Delta^+ > \Delta^*$.

Finally, just as in the endowment economy, once Δ^+ exceeds a still higher cutoff Δ^{**}, the return that optimists would expect from holding the asset if the price level P remained constant would be sufficiently high that pessimists would prefer to hoard money than spend it on consumption. Once again, hoarding would lower the price level P until the return to holding the asset made pessimists indifferent about consuming. That is, the marginal value of nominal wealth for pessimists v_t^- must equal the marginal utility of using nominal wealth to consume when they have an urge. The latter is $1/P$, so in equilibrium $Pv^- = 1$, and pessimists produce $n^- = \phi'^{-1}(1)$. The formal analysis in the Appendix shows that if $Pv^- = 1$ then $Pv^+ = 1 + \frac{\rho}{\alpha}$, so optimists produce $n^+ = \phi'^{-1}\left(1 + \frac{\rho}{\alpha}\right)$. For $\Delta^+ > \Delta^{**}$, then, output no longer increases with the degree of optimism Δ^+. The next proposition formalizes these results.

Proposition 2: *Suppose $\alpha\Delta^- > D$ and condition (23) holds. There exists a unique steady state equilibrium. Moreover, there exist cutoffs Δ^* and Δ^{**} where $0 < \Delta^* < \Delta^{**}$ such that*

1. *If $\Delta^+ < \Delta^*$, optimists and pessimists produce a constant amount in equilibrium for all levels of Δ^+. In particular, $n^+ = n^- = n^* \equiv \phi'^{-1}\left(\frac{\lambda}{\lambda+\rho}\right)$*
2. *If $\Delta^+ \in [\Delta^*, \Delta^{**}]$, optimists produce more than pessimists in equilibrium, and the amount both groups produce increases in Δ^+, i.e., $n^+ > n^- > n^*$.*
3. *If $\Delta^+ > \Delta^{**}$, optimists produce more than pessimists in equilibrium, and the amount both produce is the same for all Δ^+. In particular, $n^+ = \phi'^{-1}\left(1 + \frac{\rho}{\alpha}\right) > n^*$ and $n^- = \phi'^{-1}(1) > n^*$.*

An implication of Proposition 2 is that a speculation shock will have a limited effect on the macroeconomy when the degree of optimism Δ^+ is small: Optimists will bid up the price of the asset, but the shock will have no effect on what optimists expect to earn on the asset, output y, or the price level P. When Δ^+ is large, a speculation shock will have macroeconomic effects: Optimists bid up the price of the asset and expect to earn a higher return, output will rise, and the price level will fall. A speculation shock will not produce movements in output and goods prices that accord with a standard aggregate demand shock.

For intermediate values of Δ^+ between the cutoffs Δ^* and Δ^{**}, a speculation shock will induce agents to produce more but will not induce pessimists to hoard money. The price level in this case is given by

$$P = \frac{\lambda \overline{M}}{\frac{1}{2}n^+ + \frac{1}{2}n^- + D}$$

Since production increases while the money supply $\overline{M}$ is fixed, the price level falls with Δ^+ in this region. Intuitively, the same amount of money must buy more goods, so the price level falls. The central bank could increase $\overline{M}$ in line with output to ensure a fixed price level. By contrast, when Δ^+ exceeds Δ^{**}, pessimists hoard money. In that case, the money supply $\overline{M}$ would have to rise more than output to keep the price level stable. This highlights the distinct forces affecting the price level with endogenous output: The decision by agents to produce more following a speculation shock acts like an aggregate supply shock, while the decision by pessimists to hoard liquid assets causes the price level to change even if output were fixed. These forces matter not only for how a speculation shock affects the price level, but for what the central bank must do if it wants to stabilize output and prices. The next section considers these issues.

5 Trilemma and Transitional Dynamics

I now consider the effects of monetary policy. My focus is not what a central bank *should* do but what it *can* do, i.e., on what is feasible rather than on what is optimal. This analysis requires going beyond steady state equilibria, since a monetary intervention can have different effects in the short and long run.

To fix ideas, suppose that when the speculation shock hits at date 0, the economy immediately transitions to the steady-state equilibrium described in Propositions 1 and 2 for the new values for Δ^+ and Δ^- and the money supply $\overline{M}$. Since agents are indifferent between assets and money when $\Delta^+ = \Delta^- = 0$, any distribution of asset holdings can be equilibrium before the shock hits. That includes the one in which optimists hold the same mix of assets and money they would in the new steady state. If that were case, the wealth of optimists would jump to $\overline{W}^+$ immediately after the speculation shock hit. Denote the steady-state prices after the speculation shock hits by $\widehat{p}$ and $\widehat{P}$.

What would happen if the central bank also intervened by at date 0, increasing the money supply from $\overline{M}$ to $(1 + \mu)\,\overline{M}$ just when the speculation shock hit? Since changing the money supply may take the economy to a new steady state, I need to solve for the equilibrium along the transition. While I frame the discussion in terms of the equilibrium path following a change in the money supply, the same logic governs the equilibrium path after a speculation shock if the initial wealth of optimists W_0^+ after the shock differed from the steady state value $\overline{W}^+$ but the money supply was kept at $\overline{M}$. Injecting liquidity is equivalent to changing the initial distribution of wealth between optimists and pessimists.

In the steady state described in Propositions 1 and 2, equilibrium prices p and P are proportional to $\overline{M}$. Increasing the money supply from $\overline{M}$ to $(1+\mu)\,\overline{M}$ would thus lead to new steady-state equilibrium prices equal to $(1+\mu)\,\widehat{p}$ and $(1+\mu)\,\widehat{P}$, respectively. The new steady-state real price of the asset will equal $\widehat{p}/\widehat{P}$, which is the same as if there was no change in the money supply. Likewise, the new steady-state return that optimists expect will equal $\frac{\widehat{P}(D+\alpha\Delta^+)}{\widehat{p}}$, just as without intervention. As in most standard monetary models, money is neutral with respect to real quantities in the long run. However, a one-time injection can have real short-run effects, depending on how it is distributed between optimists and pessimists.

I begin with a short-run neutrality result. Suppose the additional $\mu\overline{M}$ of liquidity injected at date 0 was distributed between optimists and pessimists in proportion to their initial money holdings. That is, if we denote the original money holdings of optimists and pessimists by M_0^+ and M_0^-, respectively, then the amounts of additional liquidity the two groups receive would be μM_0^+ and μM_0^-, respectively. In this case, the intervention can leave the asset market unaffected at all dates.[8]

Proposition 3: *Suppose the central bank injects $\mu\overline{M}$ worth of liquidity to optimists and pessimists in proportion to their money holdings at date* 0 *when the economy is at its original steady state. Then there exists an asymptotically stationary equilibrium in which $p_t = (1+\mu)\,\widehat{p}$ and $P_t = (1+\mu)\,\widehat{P}$ for all t.*

Proposition 3 establishes that there is a way to inject liquidity following a speculation shock that has no impact on the real economy. Such an injection can still be used to offset the effect of the speculation shock on the price level P by choosing μ to match the original price level before the speculation shock. But it would not help to stabilize asset prices or output, even temporarily.

Since short-run neutrality rests on a particular allocation between optimists and pessimists, there may be scope for using liquidity policy to stabilize output and asset prices by directing the additional liquidity to favor some agents over others. To restore both asset prices and output to their levels before the speculation shock, the central bank would need to direct liquidity in a way that dampens the price of the asset and, to discourage agents from producing more, also dampens the expected return on the asset. I now argue that this will not be possible; a central bank that tries to use immediate injections to stabilize output, asset prices, and the price level will face a trilemma. Intuitively, temporarily depressing the price of the asset increases the expected return to holding the asset: A lower price implies a higher dividend yield and, to the extent the fall in price is temporary, higher future capital gains. Directing liquidity in a way that

[8] I write that the intervention *can* rather than *must* be neutral because I have been unable to formally prove this transition path is the unique continuous asymptotically stationary equilibrium. I conjecture that it is unique. The price paths are governed by a system of differential equation, and if these equations are well behaved, they should admit a unique solution.

temporarily increases the relative wealth of either optimists or pessimists will dampen asset prices or the expected return on the asset, but not both.

For analytical tractability, I establish the impossibility of stabilizing both the asset price and the expected return of optimists in the endowment economy. The same logic would carry over to the production economy, but in that case I can only confirm the results numerically.

Consider first an injection that favors optimists so that optimists receive more than μM_0^+ and pessimists receive less than μM_0^-. By assumption, before the liquidity injection, the initial wealth of optimists W_0^+ equals the steady state $\overline{W}^+$. When the degree of optimism $\Delta^+ \leq \Delta^*$, the steady state wealth $\overline{W}^+$ exceeds $\widehat{p}$. In this case, giving optimists more resources should not matter for asset prices; if optimists wanted to spend more on the asset, they could have done so already. When $\Delta^+ > \Delta^*$, optimists are wealth constrained in the steady sate before we inject liquidity. In that case, giving optimists more resources allows them to spend more on the asset and temporarily bid up its price. This is confirmed in Proposition 4 below.

Figure 4 illustrates the effects of a liquidity injection at date 0 that favors optimists when $\Delta^+ > \Delta^{**}$. It traces the paths of optimists' real wealth $\frac{W_t^+}{P_t}$, the real price of the asset $\frac{p_t}{P_t}$, the return optimists expect on the asset, and the price level P_t following the liquidity injection. As evident in the figure, the injection makes optimists temporarily wealthier in real terms relative to steady state. This will lead them to spend more on the asset, although the initial price of the asset p_0 rises less than the wealth of optimists W_0^+. Optimists thus hold both money and the asset during the transition, implying the expected return on the asset during the transition is ρ. By contrast, the steady state return on the asset will exceed ρ. Directing liquidity to optimists thus temporarily drives up the real asset price and temporarily depresses the expected return on the asset below its steady state level. Figure 4 also shows that the price level overshoots $(1+\mu)\,\widehat{P}$. Essentially, a continuous path for the price level when $\Delta^+ > \Delta^{**}$ requires that pessimists hoard money before the economy reaches steady state. Deflation makes holding money more attractive and encourages pessimists to hoard it during the transition. Formally, we have

Proposition 4: *Suppose the central bank injects $\mu\overline{M}$ worth of liquidity at date 0 in the endowment economy in a way that gives optimists more than their original share of the money supply. Then*

1. *If $\Delta^+ \leq \Delta^*$, there exists an asymptotically stationary equilibrium in which $p_t = (1+\mu)\,\widehat{p}$ and $P_t = (1+\mu)\,\widehat{P}$ for all t, i.e., prices jump to their steady state values immediately.*
2. *If $\Delta^+ > \Delta^*$, there exists an asymptotically stationary equilibrium in which there is some date $T < \infty$ such that $W_t^+ = p_t = (1+\mu)\,\widehat{p}$ for all $t \geq T$. Before T, the real asset price $p_t/P_t > \widehat{p}/\widehat{P}$, optimists expect a return of $\rho < \frac{\widehat{P}\left(D+\alpha\Delta^+\right)}{\widehat{p}}$, and, if $\Delta^+ > \Delta^{**}$, the price level $P_t > (1+\mu)\,\widehat{P}$.*

When the degree of optimism Δ^+ is small, a liquidity injection that favors optimists has no effect on the asset market. Recall that in this case, the speculation shock itself has no effect on the price level, output, or the return optimists expect to earn. But when the degree of optimism Δ^+ is large and the asset price is set according to cash-in-the-market pricing, a liquidity injection temporarily raises asset prices and depresses expected returns. In a production economy, a lower expected return on savings would discourage production, so the central bank could push output towards its level before the speculation shock. But this policy would drive the real asset price further away from its value before the speculation shock. The latter is intuitive: Giving constrained optimists more resources will lead them to spend more on assets.

What about a liquidity injection that favors pessimists so that they receive more than μM_0^- and optimists receive less than μM_0^+? That depends on whether the injection leaves optimists with enough wealth to still afford the asset at date 0. When $\Delta^+ < \Delta^*$, optimists would have held both money and the asset in the original steady state, and giving more liquidity to pessimists would reduce the relative wealth of optimists but could still leave them with enough wealth to afford the asset. In that case, the intervention would be neutral. When $\Delta^+ > \Delta^*$, optimists would have only held the asset in the original steady state. For a liquidity injection to favor pessimists, it must involve a negative injection to optimists, forcing optimists to sell some of their asset holdings. Following such an intervention, the asset price would temporarily fall until optimists amassed enough wealth to afford to repurchase all of the asset.

Figure 5 illustrates the effects of a liquidity injection at date 0 that favors pessimists when $\Delta^+ > \Delta^{**}$. It traces the paths of optimists' real wealth $\frac{W_t^+}{P_t}$, the real price of the asset $\frac{p_t}{P_t}$, the return optimists expect on the asset, and the price level P_t following the liquidity injection. As evident from the figure, the injection temporarily depresses the real asset price. Since the wealth of optimists W_t^+ initially falls below p_t, pessimists must hold some of the asset. They would only hold the asset if they anticipated a large capital gain, so p_t must appreciate when they hold the asset. The price level will once again overshoot $(1+\mu)\widehat{P}$ if $\Delta^+ > \Delta^{**}$, for the same reason as with an injection that favored optimists. Formally, we have

Proposition 5: *Suppose the central bank injects $\mu\overline{M}$ worth of liquidity at date 0 in the endowment economy in a way that gives pessimists more than their original share of the money supply. Then*

1. *If $W_0^+ \geq (1+\mu)\widehat{p}$ there exists an asymptotically stationary equilibrium in which $p_t = (1+\mu)\widehat{p}$ and $P_t = (1+\mu)\widehat{P}$ for all t.*
2. *If $W_0^+ < (1+\mu)\widehat{p}$, there exists an asymptotically stationary equilibrium in which there is some date $T < \infty$ such that $W_t^+ = p_t = (1+\mu)\widehat{p}$ for all $t \geq T$. Before T, the real asset price $p_t/P_t < \widehat{p}/\widehat{P}$, optimists expect a return that exceeds $\frac{\widehat{P}\left(D+\alpha\Delta^+\right)}{\widehat{p}}$, and, if $\Delta^+ > \Delta^{**}$, the price level $P_t > (1+\mu)\widehat{P}$.*

An injection that favors pessimists in response to a speculation shock will either have no real effects or dampen the asset boom. If the injection dampens the asset price, it will also increase the expected return on the asset. By shifting wealth away from optimists, the injection prevents them from being able to afford all of the asset and forces pessimists to hold some of the asset for a time. But pessimists would only agree to hold the asset if the expected return on the asset was higher than what they expect to earn in steady state. Specifically, they must anticipate the price of the asset will grow enough to offset the expected loss from a negative payoff event. Optimists would also expect a higher return on the asset. The same pattern would arise in a production economy. An intervention that directed liquidity to pessimists would then make production more attractive by increasing the return on savings that agents could earn. But that would amplify the output boom following a speculation shock in a production economy. While the central bank could temporarily rein in asset prices, it can do so only by stimulating output. The central bank thus faces a trilemma with its liquidity injections: To stabilize the price level and asset prices, it needs to inject liquidity and direct it to pessimists. But that boosts the expected return on the asset and stimulates output.

I conclude this section with a few remarks. First, for an intervention to undo the effects of a speculation shock, it should ideally address the underlying optimism driving these effects. This is not what liquidity injections do. Injections merely shift wealth between optimists and pessimists. Directing resources away from optimists when a speculation shock hits prevents them from acting on their optimism and will temporarily depress asset prices. But it will not discourage agents who expect high returns in the future from hoarding cash, nor will it discourage agents from producing more in expectation of a higher return on their savings, either now or in the future. To offset a speculation shock requires something that makes speculation less attractive. A financial transactions tax could make speculation less attractive by depressing the returns to buying the asset even if it doesn't change agents' beliefs about returns. An intervention that directly lowers the wealth of optimists only depresses the price of the asset, not the return it offers.

In principle, policymakers might be able to use the entire time path of liquidity rather than just a one-time liquidity injection to stabilize all three variables. The trilemma I illustrate for monetary policy arises because the central bank is tasked with three targets while a one-time injection involves only two tools, the amount of liquidity and how to direct that liquidity. If the central bank could promise to intervene in case of a windfall in a way that would hurt asset owners, it could make the asset less attractive just as a financial transactions tax would. Allen, Barlevy, and Gale (2021) similarly argue that promises by the monetary authority to intervene in the future may be more effective against an asset boom than direct intervention. However, designing an entire path is more complicated and may be prone to time inconsistency problems. The current model is also not well suited for exploring such policies.[9]

While I focus on liquidity injections, a similar logic should apply to macroprudential policy. In my model, agents cannot engage in intertemporal trade. Geanakoplos (2010) and Simsek (2013) point out that when agents hold different beliefs, pessimists would have an incentive to lend to optimists. If agents can borrow only a small amount, my main results would still go through: Optimists could still be wealth constrained and asset prices would be set according to cash-in-the-market pricing. But I could then examine the effects of restricting credit. For example, a permanent ban on credit would presumably dampen asset prices by restricting the amount optimists can spend on the asset. But lowering the price of the asset would only

[9] To discourage optimists, the monetary authority could promise to transfer resources to pessimists after a windfall to lower the price of the asset and reduce the capital gains. The problem is that the model does not pin down what pessimists or agents who don't believe an event occurred would do in that case. Modelling such promises requires a different set of beliefs.

make the asset more attractive. More generally, the type of intervention needed to stabilize both asset prices and output would lower both the price of the asset and its expected return. The issue with liquidity injections and credit restrictions is that they drive down the price of the asset directly rather than lowering the return on the asset holding the price of the asset fixed.

A second remark is that if the central bank injected liquidity in a way that favors pessimists, the asset price p_t would fall immediately and then rise during the transition back to steady state. As p_t rises, the expected dividend yield on the asset falls. For pessimists to still hold the asset requires that p_t grow even faster. The asset price will thus grow at an accelerating rate during the transition, up until optimists buy all of the asset and the asset price growth stops abruptly. While directing liquidity toward pessimists serves to depress the real price of the asset, it also fuels explosive asset price growth. This is reminiscent of a result in Galí (2014) that a monetary intervention designed to dampen an asset boom may lead to faster asset price growth. In Galí's model, the central bank raises the real interest rate rather than shifts wealth towards pessimists. But in his model, just like here, the agents who hold the asset demand a higher return as a result, which requires that the asset price grows more rapidly.

Finally, I have ignored the question of whether stabilization is desirable. This is because the model as specified is not well suited for welfare analysis. First, there is nothing in the model that makes stabilizing the price level desirable, unlike models with price rigidity where stabilizing prices can mitigate misallocation when some prices are rigid and others are not. It is also not obvious that stabilizing asset prices is desirable. Brunnermeier, Simsek, and Xiong (2014) discuss the difficulty of doing welfare analysis when agents hold different beliefs. They offer a welfare criterion which suggests that trade rooted in disagreement should be discouraged. But their argument implies we should discourage trade, not stabilize asset prices. Moreover, the case for discouraging trade is controversial. My model is mathematically equivalent to a model in which agents differ in how productively they can deploy the asset rather than in what they expect the asset to pay out. With heterogeneous productivity, efficiency dictates the assets should be held by those who deploy the asset most productively. Letting agents trade is beneficial and ensures the agents who derive the most from the asset use it. One can analogously argue that when agents hold different beliefs, they can achieve gains from trade by ensuring the asset is held by those who value it the most.

A distinct reason to worry about asset booms in this environment involves externalities that could arise if agents were allowed to borrow to buy assets. Caballero and Simsek (2020) argue that when agents can borrow to buy assets and goods prices are sticky, there may be a reason to discourage optimists from borrowing to buy assets out of concern that they will be forced to sell their assets to pessimists if and when there is a negative shock. In that case, the asset price would be depressed following a negative shock. A lower asset price would in turn depress aggregate demand when goods prices are sticky as agents become less wealthy. In this case, dampening asset prices during boom by restricting borrowing can help boost asset prices when the boom ends and mitigating the recession that would occur in this case.[10] Allen, Barlevy,

[10] Farhi and Werning (2020) make a similar argument about a potential role for policy when agents have diagnostic beliefs that depend on past price growth rather than time-varying heterogeneous beliefs.

and Gale (2021) consider a different externality that arises when lenders cannot monitor what borrowers do and borrowers ignore the default costs they impose on their lenders. In that case, too, acting against an asset boom can mitigate losses when the boom ends. A more reasonable setting to analyze the desirability of stabilization policy would thus feature price rigidity, credit and credit frictions, and information frictions.

6 Empirical Evidence

I now turn to available evidence on the key implications of the model. I first review the evidence on inflation and asset booms, which shows that various historical asset booms were associated with lower rather than higher inflation. The discrepancy between these episodes and the predictions of Bernanke and Gertler (1999) suggests that a model in which asset booms manifest as aggregate demand shocks do not adequately describe many historical episodes. My model offers one potential explanation for why asset booms might be associated with reduced price pressures. Since this explanation relies on the notion that agents hoard liquid assets during asset booms, I then turn to evidence on liquidity hoarding during asset booms.

I begin with the evidence on inflation during asset booms. Recall that in the model, speculation shocks associated with small degrees of optimism have little effect on the macroeconomy. That is, when Δ^+ is small, a speculation shock will lead to higher asset prices but will have no effect on the price level or output. Hence, the prediction that inflation should be lower when asset prices are high applies for large asset booms rather than small changes in asset prices.[11] Previous work has has already sought to identify large asset booms and characterize their associated macroeconomic conditions. For example, Bordo and Wheelock (2007) use statistical methods to identify asset booms as increases in real stock prices that precede isolated peaks in real stock prices. Their analysis covers 10 developed countries starting in 1900: Australia, Canada, France, Germany, Italy, Japan, Netherlands, Sweden, United Kingdom, and the United States. They summarize their results as follows:

> Stock market booms typically arose when output growth exceeded its long-run average and when inflation was below its long-run average... We find less variation in the association of booms with low inflation than we do in the association of booms with rapid output or productivity growth. (p115)

In subsequent work, Christiano et al. (2010) identify stock market booms in the United States going back to the 1800s. First, they look for historical episodes from 1800 until just before World War I (but excluding the Civil War) that were commonly described as financial panics. All of these occurred before the creation of the Federal Reserve, and so before monetary policy could respond to any shocks. Here, they

[11] Historically, inflation has tended to be negatively correlated with stock returns, as prominently documented in Fama and Schwert (1977) and Modigliani and Cohn (1979). That is, inflation tends to be low when stock prices rise, just as the model predicts for large asset booms. However, Gourio and Ngo (2020) find that the correlation unrestricted to asset booms turned positive around 2008.

follow a narrative approach. In each case, they define the boom as the run-up in stock prices just before the respective panic occurred. They summarize their results for this period as follows:

> In virtually every stock market boom, the price level actually declined. Moreover, in no case did the price level rise more than its average in the non-boom, non-Civil War periods. (p93)

The fall in the price level during these episodes lines up with what happens in my model in response to a speculation shock when the supply of money is fixed. Christiano et al. (2010) then turn to stock market booms after World War I but excluding the World War II period. They identify these using statistical methods. They summarize their findings as follows:

> As in the earlier data set, each boom episode is a time of non-accelerating inflation. In several cases, inflation actually slowed noticeably from the earlier period. (p95)

When monetary policy responds to shocks, stock market booms may not be associated with outright deflation, although the logic suggests we would likely see lower inflation for the same monetary policy rule. Finally, Christiano et al. (2010) look at Japan between 1960 and 2010, and find the same pattern for the stock market boom in the mid 1980s:

> CPI inflation is significantly positive before the start of the 1980s stock market boom, and it then slows significantly as the boom proceeds. Inflation even falls below zero a few times in the second half of the 1980s. (p95)

The results above concern equity booms. However, the logic of the model should in principle apply to any dividend-bearing asset that is subject to a speculation shock. A natural candidate to look at are housing booms. Here, the evidence suggests that some but not all housing booms were associated with low inflation. For example, Laidler (2003) cites examples related to both housing and equity in his discussion of price stability and financial stability:

> Even at the end of the 1980s, real estate bubbles occurred in some economies without being accompanied by any obvious general inflationary pressures. The Nordic countries provide a notable example here. Furthermore, the high-tech bubble that shocked North American and European markets in the late 1990s occurred in markets where monetary policy was aimed at domestic goals and inflation remained low. (p1)

Piazzesi and Schneider (2008) point out that the housing boom in the United States starting around 2003 occurred during a period of low inflation. At the same time, they argue that this is not a universal pattern, citing the (considerably smaller) housing boom that occurred in the U.S. in the 1970s during a period of high inflation. Brunnermeier and Julliard (2008) look at housing data in the United Kingdom between 1966 and 2004 and find that inflation tends to predict a lower price-rent ratio. This suggests low inflation is associated with higher house prices, although their approach does not try to make a distinction between large and small housing booms. Indeed, Brunnermeier and Julliard (2008) interpret their evidence to mean that inflation affects house prices rather than responds to a shock that affects house prices. That said, the large housing booms of the mid 2000s in the U.S. and elsewhere occurred in a period widely acknowledged to be associated with relatively low inflation, and when Fed officials were publicly expressing concern about the prospect of price deflation.[12]

Given the difficulty in identifying and even defining large asset booms, I do not wish to argue that asset booms are systematically associated with lower inflation. However, the evidence does suggest that there are several large asset booms that do not conform to the prediction of the Bernanke and Gertler (1999) model that asset booms correspond to aggregate demand shocks. My model offers one explanation for this pattern, based on the idea that asset booms encourage agents to save in order to buy assets in the future, including through liquid assets that would have otherwise been spent on consumption. I now examine whether there is any evidence of liquidity hoarding during asset booms.

Documenting the presence of liquidity hoarding during asset booms turns out to be somewhat subtle. In the model, a speculation shock induces agents to desire more liquid assets. However, the total supply of liquid assets $\overline{M}$ is exogenously fixed by assumption. The higher demand for liquid assets in the model results in a lower price level, which increases *real* money balances $\overline{M}/P$. But the behavior of real balances would reflect the price level, and so would not provide independent evidence on the channel through which the price level adjusts. An alternative approach would be to expand the model to consider the money holdings of individuals. Suppose that in addition to optimists and pessimists, I introduced a third type of agent into the model who believed that $\Delta^+ = 0$ for all t. Such agents would not hold the asset in the steady state following a speculation shock: Optimists would bid up the asset price enough that these agents would expect to earn a return below ρ from the asset and would prefer to hold money. At the same time, these agents would see no reason to hoard money given they do not expect to earn high returns from the asset in the future. Their share of money holdings should therefore fall relative to the agents who alternate between optimism and pessimism. By contrast, agents who intend to speculate would hold more liquidity. This suggests looking for groups that engage in speculation during asset booms and looking at whether they hoard more liquidity during the boom.

One candidate group for this analysis is corporate entities. Ordinarily, corporations have an incentive to distribute their cash flow to shareholders or use it to repay debt rather than hold on to it. But in some countries and in some periods, firms do appear to hold large amounts of cash. A substantial literature has emerged that tries to answer why this might be the case.[13] Among the various reasons for why firms might hold on to cash is a precautionary motive that allows firms to undertake investment or speculative

[12] See, for example, Bernanke (2003).

[13] For a comprehensive survey of the work on corporate cash holdings, see Ferreira da Cruz, Kimura, and Sobreiro (2019).

opportunities when these become available.[14] To the extent that firms actively speculate during asset booms, we can look at whether firms also hoard liquidity in line with the view that speculation should be associated with liquidity hoarding.

Bates, Kahle, and Stulz (2009) find that corporate cash holdings in the U.S. increased in the period of the dot com and housing booms, specifically that "the average cash ratio of S&P 500 firms roughly doubled from 1998 to 2006." (p1992) While they argue that the precautionary motive for holding cash played an important role in this increase, they do not provide evidence that specifically relates the increase in cash holding of U.S. corporations to a desire to speculate on assets. By contrast, there is some evidence that Japanese firms engaged in speculation during the Japanese stock market and real estate boom of the mid 1980s. Kester (1991) described the phenomenon in which firms began to engage in *zaiteku*, or financial engineering, as follows:

> With financial emancipation has come the deployment of cash in ways that are of dubious value. Much corporate free cash flow in Japan is being used in *zaiteku* operations – essentially speculation on the stock market and other types of financial risk taking. (p65)

To see if these corporations held more cash in the 1980s boom, I look at data from the Financial Statements Statistics of Corporations reported by the Japanese Ministry of Finance. The data are depicted in Figure 6. I focus on large corporations, or firms with capital holdings of at least 1 billion yen. This is because Pinkowitz and Williamson (2001) argue that cash holdings at smaller firms were distorted by the demands of monopolistic Japanese banks, who pressured their borrowers to hold a large amount of deposits with their lender banks. Pinkowitz and Williamson (2001) argue that this practice can explain why Japanese firms hold considerably more cash than comparable firms in other countries. Since the market power of Japanese banks began to wane in the late 1980s, the cash holdings of smaller companies may have changed at this time for reasons unrelated to the stock market boom. By contrast, large corporations were less reliant on banks and thus less subject to such pressures. The red line in Figure 6 reports the ratio of cash and deposits to total assets among large Japanese corporations, while the blue reports the ratio of stocks to total assets for these same corporations. The black line corresponds to the Nikkei 225 index. The data suggest that from the mid 1960s to the mid 1980s, large corporations in Japan held about 10% of their assets in cash and deposits. They increased their cash holdings to 13% in the early 1970s, during an earlier smaller stock boom.[15] They then increased their cash holdings again, to 15%, by the late 1980s. In the latter case, the data on stock holdings confirm that these firms as a whole were buying publicly traded stock. There is therefore some evidence that the same Japanese corporations that as a whole purchased stocks in the 1980s boom also held significantly more cash. They also seemed to hold more cash back in the stock market boom

[14] For an example of a model where firms hold liquidity for speculative reasons, see Gale and Yorulmazer (2013).

[15] French and Poterba (1991) show that stock boom in the early 1970s corresponded to a high real stock price as well as a high stock price relative to earnings as compared with the rest of the 1970s and early 1980s.

of the early 1970s, although data on stock holdings for this period are unavailable.[16]

To be sure, the evidence in Figure 6 is only suggestive. A more definitive analysis would attempt to control for other motives to hold cash among Japanese firms that may have varied at the time. One would also need to account for the higher revenues that Japanese firms were earning at this time when output was booming and firms were actively borrowing. In principle, firms may have faced difficulties in deploying the earnings they amassed towards investment or dividend payments as opposed to hoarding cash for speculative purposes. Nor is it clear whether the additional cash holdings were significant enough to have much impact on the overall price level. Still, it is noteworthy that at a time when asset prices were surging in Japan while inflation was low, there were agents that increased their stock holding and appeared to be engaged in speculation that at the same time also increased their cash holdings.

7 Conclusion

This paper explored the implications of the Harrison and Kreps (1978) model, which has become a common framework for studying asset booms, in a monetary setting. The analysis revealed that an asset boom fueled by optimism about asset returns can be associated with an output boom, higher expected returns, and a lower price level for goods and services. This result offers a contrast to previous work, most notably Bernanke and Gertler (1999), which argued that an asset boom should operate similarly to an aggregate demand shock. The implications of my results are noteworthy for two reasons. First, it offers a potential explanation for the empirical pattern described in Section 6 that quite a few asset booms in practice occurred during periods of lower inflation. The explanation this paper offers is that a speculation shock that leads to an asset boom also encourages agents to hoard liquidity, which other things equal would drive the price level down. This is a contrast to models such as Lagos and Zhang (2019) which emphasize the opposite, but mutually compatible, direction of causality in a decision by the monetary authority to lower inflation encourages agents to trade the asset, improving the allocation of the asset and driving up its value.

The model also sheds some insight on why the Bank of Japan might have expressed frustration in dealing with asset and output booms that are not accompanied by inflation. This reaction suggests that the low inflation prevalent during the asset boom was not engineered by the central bank, but was instead something the Bank of Japan was reacting to and baffled by. The model suggests central banks might face lower inflation if and when the economy is hit with a speculation shock. In that case, the central bank will be unable to use liquidity injections (or contractions) to stabilize asset prices, the price level, and output in the face of speculation shocks. While a central bank could in principle withdraw liquidity from those who are actively trading the asset to dampen its price, that should dampen the real asset price but increase what agents expect to earn from speculation. This could lead to an even larger output boom as well as

[16] In the early 1970s, corporations in all three size bins reported by the Japanese Ministry of Finance increased their cash holdings. By contrast, the cash holdings of smaller firms in the late 1980s were flat or declining.

more rapid asset price growth if the agents who end up holding the asset are less optimistic and require higher price appreciation to be willing to hold the asset. Tasking central banks to use monetary policy to stabilize multiple targets may be therefore be asking the impossible. Policies that lower the expected return on speculation for a fixed asset price are a better way to offset speculation shocks than interventions that aim to inhibit the agents most inclined to speculate.

To the extent that there is a tradeoff between financial stability and stabilizing the price level and output, say because financial transactions taxes are not feasible, one would need a richer framework than the one I present to explore these issues. The framework I use has the advantage of simplicity, which highlights the potential conflicts between financial stability and price and output stability. But my model features none of the elements that make price stability or financial stability desirable to evaluate potential tradeoffs between the two. The typical argument for why price stability is desirable is that in a world where some prices are rigid, changes in the price level can lead to misallocation. One of the arguments in favor intervening against an asset boom is that reducing the price during the boom may lead to a less severe recession once the boom ends due to aggregate demand externalities or default costs that are proportional to the amount agents borrow during the boom. An important direction for future research is to extend the model here to incorporate price rigidity and credit. Both of these elements also figure prominently in the Bernanke and Gertler (1999) analysis, and so incorporating these elements should also help to assess how the incentive for speculators to save compares with the considerations highlighted in their framework.

Appendix

Proof of Proposition 1: Let p and P denote the constant price of the asset and of goods in a steady state equilibrium. Since $D < \alpha\Delta^-$, pessimists strictly prefer money in a steady state where prices are constant since the return on money 0 while the return on the asset is negative. Hence, in any stationary equilibrium, optimists own all assets and pessimists only hold money.

Given optimists hold the entire stock of assets in a stationary equilibrium, they will earn all of the dividends D. As described in the text, when p and P are constant, this means the wealth of optimists W^+ evolves as follows:

$$\dot{W}_t^+ = \frac{\alpha}{2}\left(W_t^- - W_t^+\right) + P\left(\frac{y}{2} + D\right) - \frac{\lambda}{2}\left(W_t^+ - p\right) \tag{A.1}$$

Using the fact that $W_t^+ + W_t^- = \overline{M} + p$, we can rewrite this law of motion in terms of W_t^+ alone:

$$\dot{W}_t^+ = \frac{\alpha}{2}\overline{M} + \left(\frac{\alpha+\lambda}{2}\right)p + P\left(\frac{y}{2} + D\right) - \left(\alpha + \frac{\lambda}{2}\right)W_t^+$$

Setting $\dot{W}_t^+ = 0$ allows us to solve for the steady state value $\overline{W}^+$:

$$\overline{W}^+ = \frac{\alpha\overline{M}}{2\alpha+\lambda} + \frac{P\left(y+2D\right)}{2\alpha+\lambda} + \frac{\alpha+\lambda}{2\alpha+\lambda}p \tag{A.2}$$

Given the steady state level for $\overline{W}^+$ in (A.2), we have that $\overline{W}^+ > p$ iff

$$\overline{M} + \frac{P\left(y+2D\right)}{\alpha} > p$$

Optimists would be willing to hold the asset in steady state only if $p \leq \frac{P\left(D+\alpha\Delta^+\right)}{\rho}$, i.e., when the instantaneous return on the asset is at least ρ. Since optimists must hold the asset in equilibrium, a necessary condition for the existence of an equilibrium in which the wealth of optimists $\overline{W}^+$ strictly exceeds the asset price p is

$$\frac{\overline{M}}{P} + \frac{y+2D}{\alpha} > \frac{D+\alpha\Delta^+}{\rho} \tag{A.3}$$

If (A.3) holds for some value P, it will also hold for any lower P. Since $P \leq \frac{\lambda\overline{M}}{y+D}$ in equilibrium, inequality (A.3) holds in *any* stationary equilibrium if it holds at $P = \frac{\lambda\overline{M}}{y+D}$, i.e., if

$$\frac{y+D}{\lambda} + \frac{y+2D}{\alpha} > \frac{D+\alpha\Delta^+}{\rho} \tag{A.4}$$

Condition (A.4) ensures that $\overline{W}^+$ exceeds p in any steady state equilibrium. If (A.4) holds, optimists must hold both money and the asset and so must be indifferent between the two. Optimists would never hold money they don't intend to spend if they have an urge to consume given they can earn a positive return on the asset. Pessimists have no reason to hoard money to buy assets in the future, since they anticipate they will be indifferent between money for assets as optimists. Hence, all agents spend all of their money balances when they have an urge to consume. To ensure optimists hold both the asset and money, they must expect the return on the asset $\frac{P\left(D+\alpha\Delta^+\right)}{p}$ to equal ρ.

The equilibrium prices are thus given by

$$\begin{aligned} P &= \frac{\lambda\overline{M}}{y+D} \\ p &= \frac{\lambda\overline{M}}{y+D}\frac{D+\alpha\Delta^{+}}{\rho} \end{aligned} \tag{A.5}$$

In sum, under condition (A.4), the steady state equilibrium prices P and p are given by (A.5). Let Δ^{*} denote the highest value of Δ^{+} for which (A.4) holds, i.e.

$$\Delta^{*} \equiv \frac{\rho}{\alpha}\left[\frac{y+D}{\lambda}+\frac{y+2D}{\alpha}-\frac{D}{\rho}\right]$$

Condition (17) ensures $\Delta^{*}>0$ for any $\alpha \geq 0$. Hence, for $\Delta^{+}<\Delta^{*}$, the unique steady state equilibrium is given by (A.5). This establishes the first part of proposition 1.

Next, suppose $\Delta^{+}>\Delta^{*}$, i.e.,

$$\frac{y+D}{\lambda}+\frac{y+2D}{\alpha} \leq \frac{D+\alpha\Delta^{+}}{\rho} \tag{A.6}$$

In this case, there can be no stationary equilibrium in which $\overline{W}^{+}>p$. For suppose there were such an equilibrium. Then p would have to equal $\frac{P\left(D+\alpha\Delta^{+}\right)}{\rho}$ to ensure optimists are willing to hold both money and assets. The fact that optimists are indifferent between money and the asset would imply that both optimists and pessimists spend all of their money holdings when they have an urge to consume. This implies $P=\frac{\lambda\overline{M}}{y+D}$, which contradicts our assumption that $\overline{W}^{+}>p$. Instead, in any steady state equilibrium optimists only hold the asset, i.e.,

$$p=\overline{W}^{+}$$

Using (A.2), the equilibrium price p is given by

$$p=\overline{M}+\frac{P\left(y+2D\right)}{\alpha} \tag{A.7}$$

I next solve for the price level P. Since $W^{+}=p$, all money is held by pessimists, and the price level P depends on their spending. If pessimists spend all of their money balances when they have an urge to consume, the equilibrium price would equal $P=\frac{\lambda\overline{M}}{y+D}$. Otherwise, the equilibrium price P will be below $\frac{\lambda\overline{M}}{y+D}$. To determine what pessimists do with their money holdings, let v^{-} denote the value of a unit of nominal wealth for an agent who is currently a pessimist, and v^{+} the value of a unit of nominal wealth for an optimist. Whether pessimists spend all of their money holdings when they have an urge to consume thus depends on how v^{-} compares with the marginal utility from spending a unit of nominal wealth, which is $1/P$.

In equilibrium, v^{-} cannot exceed $1/P$. Otherwise, agents would never consume and the goods market cannot clear. This means an agent would always be willing to spend her money holdings when faced with an urge to consume as a pessimist, either out of indifference if $v^{-}=1/P$ or because she strictly prefers to consume if $v^{-}=1/P$. We can therefore characterize v^{-} using a Bellman equation that assumes the agent consumes the next time they have an urge to consume if she is still a pessimist:

$$\rho v^{-}=\lambda\left(\frac{1}{P}-v^{-}\right)+\alpha\left(v^{+}-v^{-}\right) \tag{A.8}$$

The value of a unit of nominal wealth v^- changes to $1/P$ if she has an urge to consume while still a pessimist, and changes to v^+ if an event payoff occurred. Next, consider the value of nominal wealth v^+ for an optimist. Since $\Delta > \Delta^*$, the optimist invests any nominal wealth in assets. The wealth of the agent then grows due to dividend payments from the asset, i.e., $\dot{v}^+ = \frac{PD}{p} v^+$. If a payoff event occurred, the optimist would also realize a windfall dividend of Δ^+. Hence, v^+ satisfies the Bellman equation

$$\rho v^+ = \left(\frac{PD}{p}\right) v^+ + \alpha \left(\left(1 + \frac{P\Delta^+}{p}\right) v^- - v^+\right) \tag{A.9}$$

Solving the system of equations given by (A.8) and (A.9) yields

$$\begin{aligned} v^+ &= \frac{1}{P} \frac{\alpha\lambda\left(1 + \frac{P\Delta^+}{p}\right)}{(\rho+\alpha+\lambda)\left(\rho+\alpha-\frac{PD}{p}\right) - \alpha^2\left(1+\frac{P\Delta^+}{p}\right)} && \text{(A.10)} \\ v^- &= \frac{1}{P} \frac{\lambda\left(\rho+\alpha-\frac{PD}{p}\right)}{(\rho+\alpha+\lambda)\left(\rho+\alpha-\frac{PD}{p}\right) - \alpha^2\left(1+\frac{P\Delta^+}{p}\right)} && \text{(A.11)} \end{aligned}$$

Pessimists will prefer to spend their money balances when $\dfrac{1}{P} > v^-$. Using the value of v^-, this condition implies

$$\frac{p}{P} > \frac{(\alpha+\rho)\,D + \alpha^2\Delta^+}{\rho\,(2\alpha+\rho)} \tag{A.12}$$

Recall that under (A.6), the equilibrium asset price is given by $p = \overline{M} + \frac{P}{\alpha}(y + 2D)$. Using the fact that in equilibrium $P \leq \frac{\lambda\overline{M}}{y+D}$, we have

$$\frac{p}{P} = \frac{\overline{M}}{P} + \frac{y+2D}{\alpha} \geq \frac{y+D}{\lambda} + \frac{y+2D}{\alpha}$$

Hence, if

$$\frac{D+\alpha\Delta^+}{\rho} > \frac{y+D}{\lambda} + \frac{y+2D}{2\alpha} > \frac{(\alpha+\rho)\,D+\alpha^2\Delta^+}{\rho\,(2\alpha+\rho)} \tag{A.13}$$

the unique equilibrium is one where pessimists spend all of their money holdings when they have an urge to consume, and equilibrium prices are given by

$$\begin{aligned} P &= \frac{\lambda\overline{M}}{y+D} \\ p &= \left[1 + \frac{1}{\alpha}\frac{y+2D}{y+D}\right]\lambda\overline{M} && \text{(A.14)} \end{aligned}$$

Let Δ^{**} denote the highest value of Δ^+ for which the second inequality in (A.13) holds, i.e.,

$$\Delta^{**} = \left(2 + \frac{\rho}{\alpha}\right)\frac{\rho}{\alpha}\left[\frac{y+D}{\lambda} + \frac{y+2D}{\alpha} - \frac{(\alpha+\rho)\,D}{(2\alpha+\rho)\,\rho}\right]$$

The unique equilibrium when $\Delta^+ \in (\Delta^*, \Delta^{**})$ is given by (A.14), establishing the second part of Proposition 1.

Finally, we turn to the case where $\Delta^+ > \Delta^{**}$, we have

$$\frac{y+D}{\lambda} + \frac{y+2D}{\alpha} < \frac{(\alpha+\rho)\,D+\alpha^2\Delta^+}{\rho\,(2\alpha+\rho)} \tag{A.15}$$

In this case, there cannot be an equilibrium in which pessimists spend all of their money balances when they have an urge to consume, since they would strictly prefer to hold on to money. Instead, we need pessimists to be indifferent between holding money and spending it, i.e., $v^{-} = 1/P$. This requires

$$\frac{p}{P} = \frac{(\alpha+\rho)\,D + \alpha^2\Delta^{+}}{\rho\,(2\alpha+\rho)} \tag{A.16}$$

Since

$$\frac{y+D}{\lambda} + \frac{y+2D}{\alpha} < \frac{(\alpha+\rho)\,D + \alpha^2\Delta^{+}}{\rho\,(2\alpha+\rho)} < \frac{D+\alpha\Delta^{+}}{\rho}$$

it follows that p is equal to $\overline{W}^{+}$, and so

$$p = \overline{M} + \frac{P}{\alpha}\,(y+2D) \tag{A.17}$$

Solving the system of equations given by (A.16) and (A.17) yields

$$\begin{aligned} P &= \left(\frac{(\alpha+\rho)\,D + \alpha^2\Delta^{+}}{\rho\,(2\alpha+\rho)}\left[1 - \frac{y+2D}{\alpha}\right]\right)^{-1}\overline{M} \\ p &= \overline{M} + \frac{P}{\alpha}\,(y+2D) \end{aligned} \tag{A.18}$$

This establishes the third part of Proposition 1. Essentially, the stationary equilibrium depends on how the expression $\frac{y+D}{\lambda} + \frac{y+2D}{\alpha}$ falls between two cutoffs, $\frac{(\alpha+\rho)D+\alpha^2\Delta^{+}}{\rho(2\alpha+\rho)}$ and $\frac{D+\alpha\Delta^{+}}{\rho}$. This can be reinterpreted as how the degree of optimism Δ^{+} compares to two cutoffs, Δ^{*} and Δ^{**}. ■

Proof of Proposition 2: Part of the proof mirrors the proof of Proposition 1. In any stationary equilibrium, the assumption that $D < \alpha\Delta^{-}$ implies pessimists will not hold the asset. The steady state wealth of optimists must therefore be enough to buy the asset, i.e., $\overline{W}^{+} \geq p$. I first look for a steady state equilibrium in which $W^{+} > p$. In any such equilibrium, optimists hold both assets and money. As in the endowment economy, we can solve for the steady state wealth of optimists as

$$\overline{W}^{+} = \frac{\alpha\overline{M}}{2\alpha+\lambda} + \frac{P\left(n^{+}+2D\right)}{2\alpha+\lambda} + \frac{\alpha+\lambda}{2\alpha+\lambda}p \tag{A.19}$$

This expression for $\overline{W}^{+}$ exceeds p iff

$$\frac{\overline{M}}{P} + \frac{n^{+}+2D}{\alpha} > \frac{p}{P}$$

However, in a stationary equilibrium, optimists will only hold the asset if $\frac{p}{P} < \frac{D+\alpha\Delta^{+}}{\rho}$ so that the return to holding the asset is at least ρ. Hence, a sufficient condition for a stationary equilibrium in which $\overline{W}^{+} > p$ is if

$$\frac{\overline{M}}{P} + \frac{n^{+}+2D}{\alpha} > \frac{D+\alpha\Delta^{+}}{\rho}$$

Optimists will be indifferent between money and assets if the expected return on the asset equals ρ, i.e., $\frac{p}{P} = \frac{D+\alpha\Delta^{+}}{\rho}$. We can use this to solve for how optimists and pessimists value a marginal unit of nominal wealth. The equations for v^{+} and v^{-} are the same as in the endowment economy, i.e., (A.10) and (A.11). When $\frac{p}{P} = \frac{D+\alpha\Delta^{+}}{\rho}$, these expressions imply $v^{+} = v^{-} = \frac{1}{P}\frac{\lambda}{\lambda+\rho}$. Since agents choose n to solve $\phi'(n) = Pv$, optimists and pessimists produce the same amount, i.e., $n^{+} = n^{-} = \phi'^{-1}\left(\frac{\lambda}{\rho+\lambda}\right) \equiv n^{*}$. If $\overline{W}^{+} > p$, then, output per instant is $n^{*} + D$, and the price of goods will equal $P^* = \frac{\lambda \overline{M}}{n^*+D}$. Thus, as long as

$$\frac{n^*+D}{\lambda} + \frac{n^*+2D}{\alpha} > \frac{D+\alpha\Delta^+}{\rho} \tag{A.20}$$

the equilibrium must satisfy

$$\begin{aligned} n^+ &= n^- = n^* \\ P^* &= \frac{\lambda \overline{M}}{n^*+D} \\ p^* &= \frac{\lambda \overline{M}}{n^*+D}\frac{D+\alpha\Delta^+}{\rho} \end{aligned} \tag{A.21}$$

Condition (A.20) defines a cutoff Δ^* such that (A.21) is an equilibrium, and (23) ensures $\Delta^* > 0$. To show there is no equilibrium in which $\overline{W}^+ = p$ when $\Delta^+ < \Delta^*$, suppose there was such an equilibrium. Then we would have

$$\frac{p}{P} = \frac{\overline{M}}{P} + \frac{n^+ + 2D}{\alpha}$$

Since the most agents spend on goods is $\lambda\overline{M}$, we know that $P \leq \frac{\lambda\overline{M}}{\frac{1}{2}n^+ + \frac{1}{2}n^- + D}$. Substituting in this inequality implies

$$\frac{p}{P} \geq \frac{\frac{1}{2}n^+ + \frac{1}{2}n^- + D}{\lambda} + \frac{n^+ + 2D}{\alpha}$$

In any equilibrium, $\frac{p}{P} \leq \frac{D+\alpha\Delta^+}{\rho}$ to ensure optimists are willing to hold the asset. Suppose $\frac{p}{P} = \frac{D+\alpha\Delta^+}{\rho}$. Then $v^+ = v^- = \frac{\lambda}{\rho+\lambda}\frac{1}{P}$, which implies $n^+ = n^- = n^*$ and so

$$\begin{aligned} \frac{p}{P} &= \frac{\frac{1}{2}n^+ + \frac{1}{2}n^- + D}{\lambda} + \frac{n^+ + 2D}{\alpha} \\ &= \frac{n^*+D}{\lambda} + \frac{n^*+2D}{\alpha} \\ &> \frac{D+\alpha\Delta^+}{\rho} \end{aligned}$$

where the last inequality comes from the fact that $\Delta^+ < \Delta^*$. But this contradicts our original supposition that $\frac{p}{P} = \frac{D+\alpha\Delta^+}{\rho}$. Suppose instead that $\frac{p}{P} < \frac{D+\alpha\Delta^+}{\rho}$. From the solutions above, we know Pv^+ and Pv^- are both increasing in P/p. Hence, we would have $n^+ > n^*$ and $n^- > n^*$, in which case

$$\begin{aligned} \frac{\frac{1}{2}n^+ + \frac{1}{2}n^- + D}{\lambda} + \frac{n^+ + 2D}{\alpha} &> \frac{n^*+D}{\lambda} + \frac{n^*+2D}{\alpha} \\ &> \frac{D+\alpha\Delta^+}{\rho} \end{aligned}$$

where again the second inequality follows from the fact that $\Delta^+ < \Delta^*$. But this implies $\overline{W}^+ > p$, which is a contradiction. So when $\Delta^+ < \Delta^*$, there cannot be a stationary equilibrium other than (A.21).

I next turn to the case where $\Delta^+ > \Delta^*$. I first argue that the real price of the asset $\frac{p}{P} \geq \frac{(\alpha+\rho)D+\alpha^2\Delta^+}{\rho(2\alpha+\rho)}$. For suppose this inequality was violated. Since $\frac{(\alpha+\rho)D+\alpha^2\Delta^+}{\rho(2\alpha+\rho)} < \frac{D+\alpha\Delta^+}{\rho}$, optimists would prefer assets over money. At the same time, the equation for v^- would imply a value below $\frac{1}{P}$, and so pessimists would refuse to spend money when they have in urge to consume. But then the market for goods would not clear.

Consider the case where $\frac{p}{P}=\frac{(\alpha+\rho)D+\alpha^2\Delta^+}{\rho(2\alpha+\rho)}$. Substituting in for this value in the expressions for v^+ and v^- yields

$$Pv^+ = 1+\frac{\rho}{\alpha} \tag{A.22}$$

$$Pv^- = 1 \tag{A.23}$$

In this case, we have

$$\begin{aligned} n^+ &= \phi'^{-1}\left(1+\frac{\rho}{\alpha}\right) \equiv (n^{**})^+ \\ n^- &= \phi'^{-1}(1) \equiv (n^{**})^- \end{aligned}$$

The steady state equilibrium in this case is as follows. The real price of the asset is given by

$$\frac{p}{P}=\frac{(\alpha+\rho)\,D+\alpha^2\Delta^+}{\rho\,(2\alpha+\rho)} \tag{A.24}$$

Optimists and pessimists choose labor optimally, i.e.

$$n^+ = (n^{**})^+ \tag{A.25}$$

$$n^- = (n^{**})^- \tag{A.26}$$

Finally, since $\Delta^+>\Delta^*$, the price $p=\overline{W}^+$, i.e.,

$$\frac{p}{P}=\frac{\overline{M}}{P}+\frac{\frac{1}{2}(n^{**})^+ + D}{\alpha} \tag{A.27}$$

I can solve P using (A.24) and (A.27). Since $P\le\frac{\lambda\overline{M}}{\frac{1}{2}(n^{**})^+ + \frac{1}{2}(n^{**})^- + D}$, this can only be an equilibrium if

$$\frac{\frac{1}{2}(n^{**})^+ + \frac{1}{2}(n^{**})^- + D}{\lambda}+\frac{\frac{1}{2}(n^{**})^+ + D}{\alpha}\le\frac{(\alpha+\rho)\,D+\alpha^2\Delta^+}{\rho\,(2\alpha+\rho)} \tag{A.28}$$

Hence, there exists a cutoff Δ^{**} such that (A.28) holds iff $\Delta^+>\Delta^{**}$. Moreover, since $(n^{**})^+$ and $(n^{**})^-$ both exceed n^*, it follows that $\Delta^{**}>\Delta^*$. Hence, (A.25)-(A.27) constitute an equilibrium whenever $\Delta^+>\Delta^{**}$.

To show that (A.24) - (A.27) is the unique equilibrium when $\Delta^+>\Delta^{**}$, recall that when $\Delta^+>\Delta^{**}>\Delta^*$, we must have $\overline{W}^+=p^*$. Equilibrium requires that $\frac{p}{P}\ge\frac{(\alpha+\rho)D+\alpha^2\Delta^+}{\rho(2\alpha+\rho)}$ to ensure pessimists are willing to spend some money when hit with an urge to consume. If $\frac{p}{P}=\frac{(\alpha+\rho)D+\alpha^2\Delta^+}{\rho(2\alpha+\rho)}$, the equilibrium will correspond to (A.24) - (A.27). So, for the equilibrium to be distinct requires that $\frac{p}{P}>\frac{(\alpha+\rho)D+\alpha^2\Delta^+}{\rho(2\alpha+\rho)}$. Since Pv^+ and Pv^- are are both increasing in P/p and $\phi'(\cdot)>0$, it follows that $n^+<(n^{**})^+$ and $n^-<(n^{**})^-$. Moreover, since v^- is decreasing in p/P, pessimists will prefer to spend all of their money balances when they have an urge to consume. This implies $P=\frac{\lambda\overline{M}}{\frac{1}{2}n^+ + \frac{1}{2}n^- + D}$. At the same time, since $\Delta^+>\Delta^*$, we must have $W^+=p$, which implies $\frac{p}{P}=\frac{\overline{M}}{P}+\frac{\frac{1}{2}n^+ + D}{\alpha}$. It follows that

$$\begin{aligned} \frac{p}{P} &= \frac{\frac{1}{2}n^+ + \frac{1}{2}n^- + D}{\lambda}+\frac{\frac{1}{2}n^+ + D}{\alpha} \\ &< \frac{\frac{1}{2}(n^{**})^+ + \frac{1}{2}(n^{**})^- + D}{\lambda}+\frac{\frac{1}{2}(n^{**})^+ + D}{\alpha}=\frac{(\alpha+\rho)\,D+\alpha^2\Delta^+}{\rho\,(2\alpha+\rho)} \end{aligned}$$

But this contradicts our presumption that $\frac{p}{P} > \frac{(\alpha+\rho)D+\alpha^2\Delta^+}{\rho(2\alpha+\rho)}$. So there can be no other stationary equilibrium when $\Delta^+ > \Delta^{**}$.

Finally, I turn to the case where $\Delta^+ \in (\Delta^*, \Delta^{**})$. Since $\Delta^+ > \Delta^*$, optimists only hold assets and so $\overline{W}^+ = p$. At the same time, since $\Delta^+ < \Delta^{**}$, pessimists prefer to spend all of their available money holdings when they have an urge to spend. These two imply

$$
\begin{aligned}
\frac{p}{P} &= \frac{\overline{M}}{P} + \frac{\frac{1}{2}n^+ + D}{\alpha} \\
P &= \frac{\lambda\overline{M}}{\frac{1}{2}n^+ + \frac{1}{2}n^- + D}
\end{aligned}
$$

Substituting in for P in the equation for p yields

$$
\frac{p}{P} = \frac{\frac{1}{2}n^+ + \frac{1}{2}n^- + D}{\lambda} + \frac{\frac{1}{2}n^+ + D}{\alpha} \tag{A.29}
$$

which is increasing in n^+ and n^-. The labor supplies n^+ and n^- in turn satisfy

$$
\begin{aligned}
\phi'\left(n^+\right) &= Pv^+\left(\frac{p}{P}, \Delta^+\right) \\
\phi'\left(n^-\right) &= Pv^-\left(\frac{p}{P}, \Delta^+\right)
\end{aligned}
$$

A higher value of $\frac{p}{P}$ holding all other terms fixed decreases Pv^+ and Pv^-. It follows that for any fixed Δ^+, there can be at most one value of $\left(n^+, n^-\right)$ that solves the above system, and this solution is increasing in Δ^+. ■

Proof of Proposition 3: The proof proceeds in steps. I first show that the new steady state prices are given by $p = (1+\mu)\,\widehat{p}$ and $P = (1+\mu)\,\widehat{P}$. This is immediate for the endowment economy given the steady state p and P are proportional to $\overline{M}$ per Proposition 1. But the argument for the production economy is a bit more subtle since agents choose how much to produce. I then argue that if $\widehat{W}^+$denotes the steady state wealth of optimists before the liquidity injection, then steady state wealth of optimists after the liquidity injection is given by $\overline{W}^+ = (1+\mu)\,\widehat{W}^+$. Finally, I show that a liquidity injection that is proportional to the original money holdings leaves optimists with a wealth equal to $\overline{W}^+$ at date 0, which implies that $p_t = (1+\mu)\,\widehat{p}$ and $P_t = (1+\mu)\,\widehat{P}$ for all t is an equilibrium.

I begin with the equations for the steady state values of v^+ and v^- of the value of marginal wealth for optimists and pessimists, respectively. These are given by (A.10) and (A.11). Multiplying these by P reveals that the steady state expressions Pv^+ and Pv^- can be expressed solely as a function of the steady state real asset price p/P. From the first order condition, we have

$$
\begin{aligned}
n^+ &= \phi'^{-1}\left(Pv^+\right) \\
n^- &= \phi'^{-1}\left(Pv^-\right)
\end{aligned}
$$

Hence, n^+ and n^- can also be expressed solely as a function of the real asset price p/P. Consider the prices $(1+\mu)\,\widehat{p}$ and $(1+\mu)\,\widehat{P}$. The ratio of these prices is equal to $\widehat{p}/\widehat{P}$, which is the same as the original steady state before the liquidity injection. If we let $\widehat{n}^+$ and $\widehat{n}^-$ denote the production levels in the original steady state, then if $p_t = (1+\mu)\,\widehat{p}$

and $P_t = (1+\mu)\,\widehat{P}$ for all t, optimists and pessimists would optimally choose to produce the same quantities as in the original steady state before the liquidity injection, i.e., $n^+ = \widehat{n}^+$ and $n^- = \widehat{n}^-$.

Next, from the proof of Proposition 2, we know that holding n^+ and n^- fixed, the steady state prices P and p will be proportional the $\overline{M}$. That is, holding production fixed, increasing liquidity by a factor of $1+\mu$ would increase the steady state equilibrium prices to $p = (1+\mu)\,\widehat{p}$ and $P = (1+\mu)\,\widehat{P}$. Together, these results imply that $p = (1+\mu)\,\widehat{p}$, $P = (1+\mu)\,\widehat{P}$, $n^+ = \widehat{n}^+$ and $n^- = \widehat{n}^-$ is a steady state equilibrium when the money supply is $(1+\mu)\overline{M}$ given that $\widehat{p}$, $\widehat{P}$, $\widehat{n}^+$, and $\widehat{n}^-$is a steady state when the money supply is $\overline{M}$. From the proof of Proposition 2, we know this must be the unique steady state.

From the proof of Proposition 2, the steady-state wealth of optimists is given by

$$\overline{W}^+ = \frac{\alpha\overline{M}}{2\alpha+\lambda} + \frac{P\left(n^+ + 2D\right)}{2\alpha+\lambda} + \frac{\alpha+\lambda}{2\alpha+\lambda}p$$

It follows that

$$\overline{W}^+ = (1+\mu)\,\widehat{W}^+ \tag{A.30}$$

Finally, let $\widehat{M}^+$ and $\widehat{M}^-$ denote the money holdings of optimists and pessimists in the original steady state before the liquidity injection. Optimists hold all of the asset in steady state, so their original steady state wealth is given by $\widehat{W}^+ = \widehat{M}^+ + \widehat{p}$. If the price of the asset increased to $(1+\mu)\widehat{p}$ and the money holdings of optimists is increased to $(1+\mu)\,\widehat{M}^+$, then the wealth of optimists would immediately jump to $\overline{W}^+(1+\mu)\widehat{W}^+$. As such, jumping to the new steady state immediately is an equilibrium. ■

Proof of Proposition 4: The proof of Proposition 3 establishes that in the new steady state after a liquidity injection features $p = (1+\mu)\,\widehat{p}$, $P = (1+\mu)\,\widehat{P}$ and $\overline{W}^+ = (1+\mu)\,\widehat{W}^+$. A liquidity injection that favors optimists would give them more than $\mu\widehat{M}^+$, and so their initial wealth at date 0 amounts to $W_0^+ > (1+\mu)\,\widehat{M}^+ + p_t$. I now consider three cases in turn, depending on the value of Δ^+.

Case (a): $\Delta^+ \leq \Delta^*$

I show there exists an equilibrium in which $p_t = (1+\mu)\,\widehat{p}$ and $P_t = (1+\mu)\,\widehat{P}$ for all t. Given prices p_t and P_t, the wealth of optimists W_t^+ evolves as

$$\begin{aligned}\dot{W}_t^+ &= \alpha\left((1+\mu)\,\overline{M} + (1+\mu)\,\widehat{p} - 2W_t^+\right) + (1+\mu)\,\widehat{P}\left(\frac{y}{2} + D\right) - \lambda\left(W_t^+ - (1+\mu)\,\widehat{p}\right) \\ &= (2\alpha+\lambda)\left(\overline{W}^+ - W_t^+\right)\end{aligned}$$

where $\overline{W}^+ = (1+\mu)\left[\frac{\alpha\overline{M}}{2\alpha+\lambda} + \frac{P\left(n^+ + 2D\right)}{2\alpha+\lambda} + \frac{\alpha+\lambda}{2\alpha+\lambda}p\right] = (1+\mu)\,\widehat{W}^+$. This implies

$$W_t^+ = \overline{W}^+ + \left[W_0^+ - \overline{W}^+\right]e^{-(2\alpha+\lambda)t} \tag{A.31}$$

Since $p_t = (1+\mu)\widehat{p}$, an initial injection that favors optimists would leave optimists with an initial wealth that exceeds the new steady state, since

$$\begin{aligned} W_0^+ &> (1+\mu) M_0^+ + p_t \\ &= (1+\mu) M_0^+ + (1+\mu)\widehat{p} = \overline{W}_0^+ \end{aligned} \tag{A.32}$$

It follows that $W_t^+ > \overline{W}^+ \geq (1+\mu)\widehat{p} = p_t$ for all t, so at these prices, optimists always have sufficient wealth to buy the asset. Given it was optimal for optimists to hold the asset in the original steady state, it will be optimal for them to hold the asset when the prices $p_t = (1+\mu)\widehat{p}$ and $P_t = (1+\mu)\widehat{P}$ for all t when the expected return on the asset is unchanged. It will also be optimal for agents to spend their money holdings when they have an urge to consume given these prices, and so $P_t = \frac{\lambda(1+\mu)\overline{M}}{y+D}$ is a market clearing price at each date t.

Case (b.1): $\Delta^+ \in (\Delta^*, \Delta^{**})$

I show there exists an equilibrium in which there is some finite date $T < \infty$ where $W_t^+ > p_t > (1+\mu)\widehat{p}$ for $t < T$ and $W_t^+ = p_t = (1+\mu)\widehat{p}$ for $t \geq T$, while the price level $P_t = (1+\mu)\widehat{P} = \frac{\lambda(1+\mu)\overline{M}}{y+D}$ for all t.

First, if the wealth of optimists $W_t^+ > p_t$ for all $t < T$, optimists must be willing to hold both money and assets prior to date T. Let v_t^+ denote the value of marginal wealth for an agent at date t. The instantaneous utility return to holding money is ρv_t^+ while the instantaneous return to holding the asset over the next instant is $\frac{P_t\left(D+\alpha\Delta^+\right)+\dot{p}_t}{p_t} v_t^+$. Optimists are indifferent only if the latter return is equal ρ. That is, for $t < T$, for the asset price p_t at $t < T$ to be an equilibrium as specified, it must satisfy the differential equation

$$P_t\left(D + \alpha\Delta^+\right) + \dot{p}_t = \rho p_t \tag{A.33}$$

Given a value for T and a path for the price level P_t, we can solve this differential equation forward to obtain

$$p_t = \int_t^\infty e^{-\rho(s-t)} P_s\left(D+\alpha\Delta^+\right) ds + K e^{\rho t} \tag{A.34}$$

where the constant K is determined by the boundary condition that the price p_t at $t = T$ must equal $(1+\mu)\widehat{p}$, i.e.,

$$K = e^{-\rho T}(1+\mu)\widehat{p} - \int_T^\infty e^{-\rho s} P_s\left(D+\alpha\Delta^+\right) ds \tag{A.35}$$

If the price level is $P_s = (1+\mu)\widehat{P}$ for all s as conjectured, the constant is given by

$$K = \int_T^\infty e^{-\rho s} P_s\left(D+\alpha\Delta^+\right) ds = (1+\mu)e^{-\rho T}\left[\widehat{p} - \frac{\widehat{P}\left(D+\alpha\Delta^+\right)}{\rho}\right] < 0$$

where the last inequality uses Proposition 1 and the fact that $\Delta^+ > \Delta^*$.

Turning to the wealth of optimists, given a path of prices p_t and P_t, the law of motion for W_t^+ for $t < T$ when agents act optimally is given by

$$\dot{W}_t^+ = \alpha(1+\mu)\overline{M} + (\alpha+\lambda)p_t + P_t\left(\frac{y}{2}+D\right) - (2\alpha+\lambda)W_t^+ + \dot{p}_t \tag{A.36}$$

Given a value for T and paths for P_t and p_t, we can solve the differential equation backward to obtain

$$W_t^+ = \int_0^t e^{-(2\alpha+\lambda)s} \left[\alpha (1+\mu) \overline{M} + P_s \left(\frac{y}{2} + D\right) + (\alpha+\lambda) p_s + \dot{p}_s\right] ds + Ke^{-(2\alpha+\lambda)t} \tag{A.37}$$

where now $K = W_0^+$ in the initial wealth that is determined by p_0 and the liquidity injection.

Turning to the price level P_t, we know that from date T on, optimists and pessimists would want to spend all of the money holdings given the steady state asset price if the price level was also constant. This implies $P_t = \frac{\lambda(1+\mu)\overline{M}}{y+D}$ for all $t \geq T$ is an equilibrium. Before date T, optimists must hold money if $W_t^+ > p_t$. But since they can earn a positive return of ρ from holding the asset, they would only agree to hold money at a given instant if they intended to spend it. So optimists would spend all of their money holdings before T as well. Pessimists expect the return on the asset to be highest at $t \geq T$. But at this return, they prefer to spend all of their cash when they have an urge to consume. It follows that they would also prefer to spend all of their cash when they have an urge to consume at earlier dates, when the return to holding the asset is lower and waiting will give then at best the return at date T. If all agents spend their available cash, the market clearing price would be given by $P_t = \frac{\lambda(1+\mu)\overline{M}}{y+D}$.

Finally, I need to confirm that $W_t^+ > p_t$ for $t < T$ and show how to solve for T. I first argue that the date T in which $p_t > W_t^+ > (1+\mu)\widehat{p}$ is finite. For suppose $T = \infty$. In that case, we know from (A.35) that $K = 0$. Given the price level P_t, the price of the asset in (A.34) would asymptotically converge to $(1+\mu)\frac{\widehat{P}\left(D+\alpha\Delta^+\right)}{\rho} > (1+\mu)\widehat{p}$. But this contradicts Proposition 1 which establishes that there is a unique steady state equilibrium in which the asset price is $(1+\mu)\widehat{p}$. Hence, there exists a finite date T at which $p_T = W_T^+$. By contrast, at date 0, the wealth of optimists exceeds the price of the asset by the amount of the liquidity injection optimists receive at that date. Let T' denote the first date at which the money holdings of optimists are equal to 0, i.e., $T' = \inf\left\{t : M_t^+ = 0\right\}$. Since the liquidity injection at date 0 favors optimists, and since the money holdings of optimists in the original steady state is 0, it follows that $M_0^+ > 0$ after the injection. Hence, $T' > 0$. By construction, $T' \leq T < \infty$. Since the wealth of optimists is given by $W_t^+ = M_t^+ + p_t$, then $W_t^+ - p_t = M_t^+ > 0$ for all $t \in [0, T')$. Hence, there exists a finite date T' such that $W_t^+ > p_t$ for $t < T'$ as desired. That is, $T = T'$, so T is the first date at which money holdings of optimists are equal to 0. To solve for T, we choose the value of T that ensures W_t^+ at $t = T$ is equal to $(1+\mu)\widehat{p}$. In particular, p_t in (A.34) depends on T through the coefficient K, so we can write p_t^T to indicate that it is a function of T. We then solve for T from the equation

$$\int_0^T e^{-(2\alpha+\lambda)s} \left[\alpha (1+\mu) \overline{M} + (1+\mu) \widehat{P} \left(\frac{y}{2} + D\right) + (\alpha+\lambda) p_s^T + \dot{p}_s^T\right] ds + \left(M_0^+ + p_0^T\right) e^{-(2\alpha+\lambda)T} = (1+\mu)\widehat{p} \tag{A.38}$$

The existence of a solution follows from the existence of T'. We can thus find a date $T < \infty$ where $W_t^+ > p_t > (1+\mu)\widehat{p}$ for $t < T$ and $W_t^+ = p_t = (1+\mu)\widehat{p}$ for $t \geq T$ that is consistent with optimization and market clearing.

Case (b.2): $\Delta^+ > \Delta^{**}$

Once again, I construct an equilibrium in which there is some finite date $T < \infty$ where $W_t^+ > p_t > (1+\mu)\widehat{p}$ for $t < T$ and $W_t^+ = p_t = (1+\mu)\widehat{p}$ for $t \geq T$ while the price level $P_t = (1+\mu)\widehat{P}$ for all t.

The path for the asset price p_t is once again given by (A.34), and the path for the total wealth of optimists W_t^+

is given by (A.37). Both are governed by the path for for the price level, P_t. I now look for a path for the price level in which $P_t > (1+\mu)\widehat{P}$ for $t < T$ and $P_t = (1+\mu)\widehat{P}$ for $t \geq T$. That is, the price level attains its steady state level at the same date as when the wealth of optimists is equal to the price of the asset.

For $\Delta^+ > \Delta^{**}$, we know from Proposition 1 that $\widehat{P} < \frac{\lambda\overline{M}}{y+D}$. Since the path for the price level is continuous, this means there exists an interval $(T_0, T]$ where $0 \leq T_0 < T$ in which the price level is close to $(1+\mu)\widehat{P}$ and thus below $\frac{\lambda(1+\mu)\overline{M}}{y+D}$. This can only be an equilibrium if not all agents want to spend their money holdings for $t \in (T_0, T]$. Since optimists hold both money and the asset before date T, and the nominal return on the asset is positive, they will only hold money if they intend to spend it if they have an urge to consume. This suggests pessimists are either indifferent between holding money and spending it, or else strictly prefer to hold their money.

I begin with the case in which pessimists are indifferent between holding and spending money. Once again, let v_t^+ denote the value of a unit of money at date t for an optimist and v_t^- denote the analogous value for a pessimist. The proof of Proposition 1 implies that the steady state values of v_t^+ and v_t^- from date T on are given by

$$v^+ = \frac{1+\rho/\alpha}{(1+\mu)\widehat{P}} \tag{A.39}$$

$$v^- = \frac{1}{(1+\mu)\widehat{P}} \tag{A.40}$$

Before date T, if pessimists are indifferent between holding money and spending it, it would be optimal for them to spend their money at the first urge to consume that occurred before date T regardless of their type at the time. I can therefore compute v_t^- assuming they spend their money holdings if they have an urge to consume before date T, and if they don't have an urge to consume by date T their payoffs will be given by (A.39) and (A.40). That is, v_t^- will satisfy the integral equation

$$v_t^- = \int_t^T \frac{\lambda e^{-(\lambda+\rho)(s-t)}}{P_s} ds + \frac{e^{-(\rho+\lambda)(T-t)}}{P_T}\left[\Pr\left(v_T = v^- | v_t = v^-\right) + \Pr\left(v_T = v^+ | v_t = v^-\right)\frac{\alpha+\rho}{\alpha}\right] \tag{A.41}$$

For a two-state switching model, we know that

$$\Pr\left(v_T = v^- | v_t = v^-\right) = \frac{1+e^{-2\alpha(T-t)}}{2}$$

$$\Pr\left(v_T = v^+ | v_t = v^-\right) = \frac{1-e^{-2\alpha(T-t)}}{2}$$

Substituting in for these probabilities yields

$$v_t^- = \int_t^T \frac{\lambda e^{-(\lambda+\rho)(s-t)}}{P_s} ds + \frac{e^{-(\lambda+\rho)(T-t)}}{P_T}\left(1 + \frac{\rho}{2\alpha}\left[1 - e^{-2\alpha(T-t)}\right]\right) \tag{A.42}$$

If pessimists are indifferent between spending and holding money from date T_0 on, we must have $v_t^- = \frac{1}{P_t}$ for all $t > T_0$. Equating v_t^- above with P_t thus translates into the integral equation for the price P_t:

$$\int_t^T \frac{\lambda e^{-(\lambda+\rho)(s-t)}}{P_s} ds = \frac{1}{P_t} - \frac{e^{-(\lambda+\rho)(T-t)}}{P_T}\left(1 + \frac{\rho}{2\alpha}\left[1 - e^{-2\alpha(T-t)}\right]\right)$$

This is a linear Volterra integral equation of the second kind, i.e. it has the form

$$y(t) - \int_t^T y(s)\,k(s-t)\,ds = f(t)$$

where $y(t) = \frac{P}{P_t}$, $k(s-t) = \lambda e^{-(\lambda+\rho)(s-t)}$, and $f(t) = \frac{e^{-(\lambda+\rho)(T-t)}}{P^*}\left(1 + \frac{\rho}{2\alpha}\left[1 - e^{-2\alpha(T-t)}\right]\right)$. When $k(s-t)$ is an exponential, as is the case here, the Volterra equation has a closed form solution, i.e.,

$$\frac{1}{P_t} = \frac{1}{P_T}\left[\frac{\rho+\lambda+2\alpha}{\lambda+2\alpha}e^{-\rho(T-t)} - \frac{\rho}{\lambda+2\alpha}e^{-(\rho+\lambda+2\alpha)(T-t)}\right]$$

Rearranging yields

$$P_t = \frac{(\lambda+2\alpha)\,e^{-\rho(t-T)}P_T}{\lambda+2\alpha+\rho\left[1-e^{-(\lambda+2\alpha)(t-T)}\right]} \tag{A.43}$$

This path exceeds $P_T = (1+\mu)\widehat{P}$ for $t < T$, and reaches $(1+\mu)\widehat{P}$ when $t = T$. Since the price level cannot exceed $\frac{\lambda(1+\mu)\overline{M}}{y+D}$, this path is only feasible for $t \geq T_0$ where T_0 solves

$$\frac{(\lambda+2\alpha)\,e^{-\rho(T_0-T)}P_T}{\lambda+2\alpha+\rho\left[1-e^{-(\lambda+2\alpha)(T_0-T)}\right]} = \frac{\lambda\overline{M}}{y+D}$$

If $T_0 > 0$, then for $0 \leq t \leq T_0$, the price level $P_t = \frac{\lambda\overline{M}}{y+D}$ for $t < T_0$. At these dates, we have $v_t^- < 1/P$ and pessimists would strictly prefer to spend all of their money holdings for $t \in [0, T_0)$.

Although the path in (A.43) ensures pessimists are indifferent between holding money and spending it when they have an urge to consume, the fact that optimists always want to spend any money they have implies that the price level is bounded below given their spending, i.e.,

$$P_t > \frac{\lambda M_t^+}{y+D} \tag{A.44}$$

Although we know that the money holdings $M_t^+ \to 0$ as $t \to T$, so that this condition will hold close to the terminal date, we cannot be sure that it also holds earlier. But I now argue that

$$P_t = \max\left\{\frac{\lambda M_t^+}{y+D}, \frac{(\lambda+2\alpha)\,e^{-\rho(t-T)}P_T}{\lambda+2\alpha+\rho\left[1-e^{-(\lambda+2\alpha)(t-T)}\right]}\right\} \text{ for } t > T_0 \tag{A.45}$$

is an equilibrium. In particular, at any date in which $\frac{\lambda M_t}{y+D} > \frac{(\lambda+2\alpha)e^{-\rho(T_0-T)}P_T}{\lambda+2\alpha+\rho\left[1-e^{-(\lambda+2\alpha)(T_0-T)}\right]}$ and the continuation path weakly exceeds the path that leaves pessimists indifferent, it will be optimal for pessimists to hold on to their cash, since the return to holding the asset is higher while the utility value from spending is lower. But in that case, only optimists spend, and the equilibrium price will be given by $\frac{\lambda M_t}{y+D}$. In short, the price level in this case is given by

$$P_t = \begin{cases} \frac{\lambda(1+\mu)\overline{M}}{y+D} & \text{if } t \leq T_0 \\ \max\left\{\frac{\lambda M_t^+}{y+D}, \frac{(\lambda+2\alpha)e^{-\rho(t-T)}(1+\mu)\widehat{P}}{\lambda+2\alpha+\rho\left[1-e^{(\lambda+2\alpha)(t-T)}\right]}\right\} & \text{if } t \in (T_0, T) \\ (1+\mu)\widehat{P} & \text{if } t \geq T \end{cases}$$

The final step is to confirm that $W_t^+ > p_t$ for $t < T$ and show how to solve for T. The argument here is analogous to case (b.1) above. First, we know that T is finite. This means the money holdings of optimists reaches 0 in finite time. But then the path between date 0 and when optimists first run down their cash satisfies all of these conditions. This completes the proof. ■

Proof of Proposition 5: From the proof of Proposition 3, the new steady state after a liquidity injection is $p = (1+\mu)\,\widehat{p}$, $P = (1+\mu)\,\widehat{P}$ and $\overline{W}^{+} = (1+\mu)\,\widehat{\overline{W}}^{+}$. A liquidity injection that favors pessimists at date 0 leaves optimists with an initial wealth of $W_0^+ < (1+\mu)\,\widehat{M}^{+} + p_t$. I consider three cases in turn, depending on how the initial wealth of optimists W_0^+ compares with the new steady state asset price $(1+\mu)\,p$ and on Δ^+.

Case (a): $W_0^+ \geq (1+\mu)\,\widehat{p}$

I show there exists an equilibrium in which $p_t = (1+\mu)\,\widehat{p}$ and $P_t = (1+\mu)\,\widehat{P}$ for all t. If $W_0^+ \geq (1+\mu)\,\widehat{p}$, the same logic as in case (a) in the proof of Proposition 4 applies. The only difference is that now we have

$$\begin{aligned} W_0^+ &< (1+\mu)\,M_0^+ + p_t \\ &= (1+\mu)\,M_0^+ + (1+\mu)\,\widehat{p} = \overline{W}_0^+ \end{aligned} \tag{A.46}$$

The wealth of optimists increases over time rather than decreases. The time it takes the wealth of optimists W_t^+ to reach the new steady state price $(1+\mu)\,\widehat{p}$ is infinite.

Case (b.1): $W_0^+ < (1+\mu)\,\widehat{p}$ and $\Delta^+ \leq \Delta^{**}$

I show there exists an equilibrium in which there is a finite date $T < \infty$ where $W_t^+ < p_t < (1+\mu)\,\widehat{p}$ when $t < T$ and $W_t^+ = p_t = (1+\mu)\,\widehat{p}$ for $t \geq T$. The price level $P_t = \frac{\lambda(1+\mu)\overline{M}}{y+D}$ for $t \in [0,T)$. If $\Delta^+ \in (\Delta^*, \Delta^{**}]$, the price level $P_t = \frac{\lambda(1+\mu)\overline{M}}{y+D}\,(1+\mu)\,\widehat{P}$ for $t > T$ as well.

Since $p_t > W_t^+$ for all $t < T$, pessimists must own at least some of the asset given optimists can't afford to buy all of it. Since optimists expect a higher return than pessimists, if pessimists are willing to hold the asset, optimists would as well. Hence, before date T, pessimists must be indifferent between the asset and money and optimists must prefer the asset. Otherwise, nobody would hold money. Using the same argument as in the proof of Proposition 4, pessimists are indifferent between the asset and money only if they expect the return on the asset to equal ρ. This implies that for $t \in [0,T]$, the asset price must satisfy the condition

$$P_t\left(D - \alpha\Delta^-\right) + \dot{p}_t = \rho p_t \tag{A.47}$$

Solving this differential equation forward as in the proof of Proposition 4, we have

$$p_t = \int_t^\infty e^{-\rho(s-t)} P_s\left(D - \alpha\Delta^-\right) ds + K e^{\rho t} \tag{A.48}$$

where K is determined by the boundary condition that the price p_t at $t = T$ must equal $(1+\mu)\,\widehat{p}$, i.e.,

$$K = e^{-\rho T}\,(1+\mu)\,\widehat{p} - \int_T^\infty e^{-\rho s} P_s\left(D - \alpha\Delta^+\right) ds \tag{A.49}$$

If the price level is $P_s = (1+\mu)\widehat{P}$ for all s as conjectured, the constant is given by

$$K = \int_T^\infty e^{-\rho s} P_s \left(D - \alpha\Delta^+\right) ds = (1+\mu)\, e^{-\rho T}\left[\widehat{p} - \frac{\widehat{P}\left(D-\alpha\Delta^+\right)}{\rho}\right] < 0$$

Since pessimists expect to earn ρ, optimists expect to earn a return $\frac{P_t\left(D+\alpha\Delta^+\right)+\dot{p}_t}{p_t}$ that exceeds ρ.

The law of motion for the wealth of optimists is given by

$$\dot{W}_t^+ = P_t\left(\frac{y}{2} + D\frac{W_t^+}{p_t}\right) + \alpha\left(\overline{M} + p_t - 2W_t^+\right) + \frac{W_t^+}{p_t}\dot{p}_t \tag{A.50}$$

The coefficient on W_t^+ is now a function of p_t^+, in contrast to the proof of Proposition 4 for an injection that favors optimists. Given a path for p_t and P_t, we can solve for this differential equation.

Turning to the price level P_t, we know that from date T on, only pessimists hold money. From Proposition 1, we know that as long as $\Delta^+ < \Delta^{**}$, pessimists would want to spend all of their money holdings when they have an urge to consume. This implies $P_t = \frac{\lambda(1+\mu)\overline{M}}{y+D}$ for all $t \geq T$. Before date T, the fact that $W_t^+ < p_t$ implies pessimists must be indifferent between the money and the asset while optimists, who expect a higher return, strictly prefer the asset. Hence, pessimists hold all money. Since they are indifferent between money and the asset, they would want to spend all of their money holdings if they have an urge to consume, since they could have earned a positive return by holding the asset. If the agents who hold cash spend it all when they have an urge to consume, the market clearing price would be given by $P_t = \frac{\lambda(1+\mu)\overline{M}}{y+D}$ at these dates as well.

Finally, I need to confirm that $W_t^+ < p_t$ for $t < T$ and show how to solve for T. The argument is analogous to the proof in Proposition 4. First, I argue that the date T in which $p_t < W_t^+$ is finite. For suppose $T = \infty$. In that case, we know from (A.35) that $K = 0$. Given the price level P_t, the price of the asset in (A.34) would asymptotically converge to $(1+\mu)\frac{\widehat{P}\left(D-\alpha\Delta^+\right)}{\rho} < (1+\mu)\widehat{p}$. But this contradicts Proposition 1 which establishes that there is a unique steady state equilibrium in which the asset price is $(1+\mu)\widehat{p}$. Hence, there exists a finite date T at which $p_T = W_T^+$. Let T' denote the first date at which have enough wealth to buy the entire asset, i.e., $T' = \inf\left\{t : W_t^+ - p_t = 0\right\}$. By construction, $T' \leq T < \infty$. Hence, there exists a finite date T' such that $W_t^+ < p_t$ for $t < T'$ as desired. That is, $T = T'$, so T is the first date at which money holdings of optimists are equal to 0. To solve for T, we choose the value of T that ensures W_t^+ at $t = T$ is equal to $(1+\mu)\widehat{p}$, analogously to the proof of Proposition 4.

Case (b.2): $W_0^+ < (1+\mu)\widehat{p}$ and $\Delta^+ > \Delta^{**}$

Once again, I construct an equilibrium in which there is some finite date $T < \infty$ where $W_t^+ > p_t > (1+\mu)\widehat{p}$ for $t < T$ and $W_t^+ = p_t = (1+\mu)\widehat{p}$ for $t \geq T$. The difference from case (b.1) above is the price level P_t.

The path for the asset price p_t is once again given by (A.48), and the path for the total wealth of optimists W_t^+ is governed by the differential equation in (A.50).

I now turn to the equilibrium price level P_t. Before date T, the fact that $W_t^+ < p_t$ implies pessimists must be indifferent between the money and the asset while optimists, who expect a higher return, strictly prefer the asset. Hence, pessimists hold all money. Since they are indifferent between money and the asset, they would want to spend all of their money holdings if they have an urge to consume, since they could have earned a positive return by holding the asset. If the agents who hold cash spend it all when they have an urge to consume, the market clearing price would be given by $P_t = \frac{\lambda(1+\mu)\overline{M}}{y+D}$. From Proposition 1, we know that when $\Delta^+ > \Delta^{**}$, the original steady-state equilibrium price level $\widehat{P} < \frac{\lambda\overline{M}}{y+D}$. Hence, $P_t > (1+\mu)\,\widehat{P}$ when $t < T$.

After date T, the price level must converge towards its new steady state level $(1+\mu)\,\widehat{P}$. Throughout this transition, when the price $P_t < \frac{\lambda(1+\mu)\overline{M}}{y+D}$, pessimists must be indifferent between spending and holding on their cash. This is because optimists do not hold cash at all. For pessimists to be indifferent, the value of marginal wealth for them, v_t^-, must equal $1/P_t$. Define $\eta_t^+ = P_t v_t^+$ and $\eta_t^- = P_t v_t^-$. Then equilibrium requires that $\eta_t^- = 1$ for all $t > T$. Using the laws of motion for v_t, we have

$$\begin{aligned}
\rho\eta_t^+ &= \frac{P_t\left(D+\alpha\Delta^+\right)}{(1+\mu)\,\widehat{p}}\eta_t^+ + \alpha\left(\eta_t^- - \eta_t^+\right) + \dot{\eta}_t^+ \\
\rho\eta_t^- &= \lambda\left(1-\eta_t^-\right) + \alpha\left(\eta_t^+ - \eta_t^-\right)
\end{aligned}$$

The second equation implies

$$\eta_t^- = \frac{\lambda + \alpha\eta_t^+}{\rho+\alpha+\lambda}$$

We can substitute this into the first equation to get a single differential equation for η_t^+, i.e.,

$$\rho\eta_t^+ = \frac{P_t\left(D+\alpha\Delta^+\right)}{(1+\mu)\,\widehat{p}}\eta_t^+ + \alpha\left(\frac{\lambda+\alpha\eta_t^+}{\rho+\alpha+\lambda} - \eta_t^+\right) + \dot{\eta}_t^+ \tag{A.51}$$

The boundary condition for η_t^+ is that there exists some date T_1 such that $\eta_t^+ = 1 + \rho/\alpha$ at $t = T_1$. The equation for η_t^+ depends on the path of P_t. At the same time, we have the equilibrium condition $\eta_t^- = 1$, or

$$\frac{\lambda+\alpha\eta_t^+}{\rho+\alpha+\lambda} = 1 \tag{A.52}$$

Combining (A.51) and (A.52) yields an integral equation for P_t that is analogous to the one in the proof of Proposition 4. To pin down T_1, we have the boundary condition that $P_T = \frac{\lambda(1+\mu)\overline{M}}{y+D}$. This equation characterizes the path of the price level from date T on. Along this path, pessimists are indifferent and would be willing to spend the amount necessary needed to ensure the price level P_t that corresponds to η_t^+. The equilibrium is consistent with all the characterizations in the statement of the proposition. ■

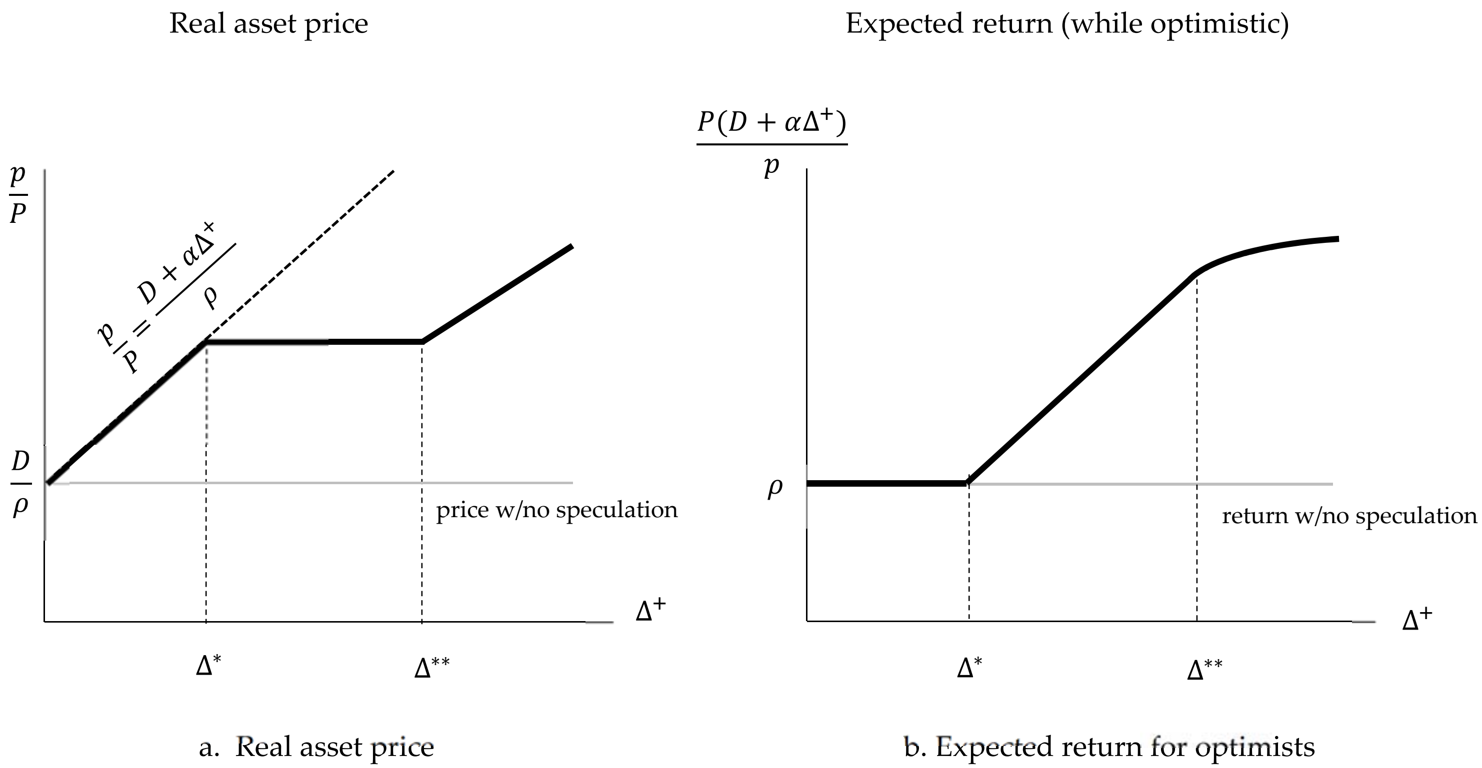

Figure 1: Equilibrium real asset price and expected return on asset as a function of the degree of optimism Δ^+

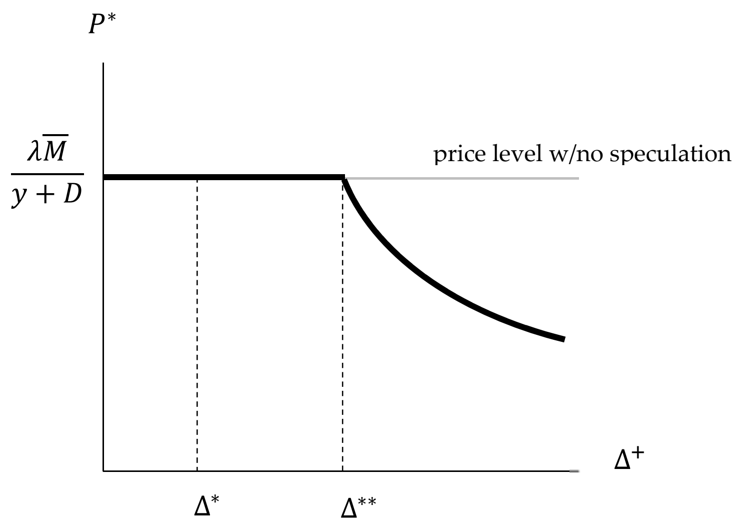

Figure 2: Steady state price level P as a function of the degree of optimism Δ^+

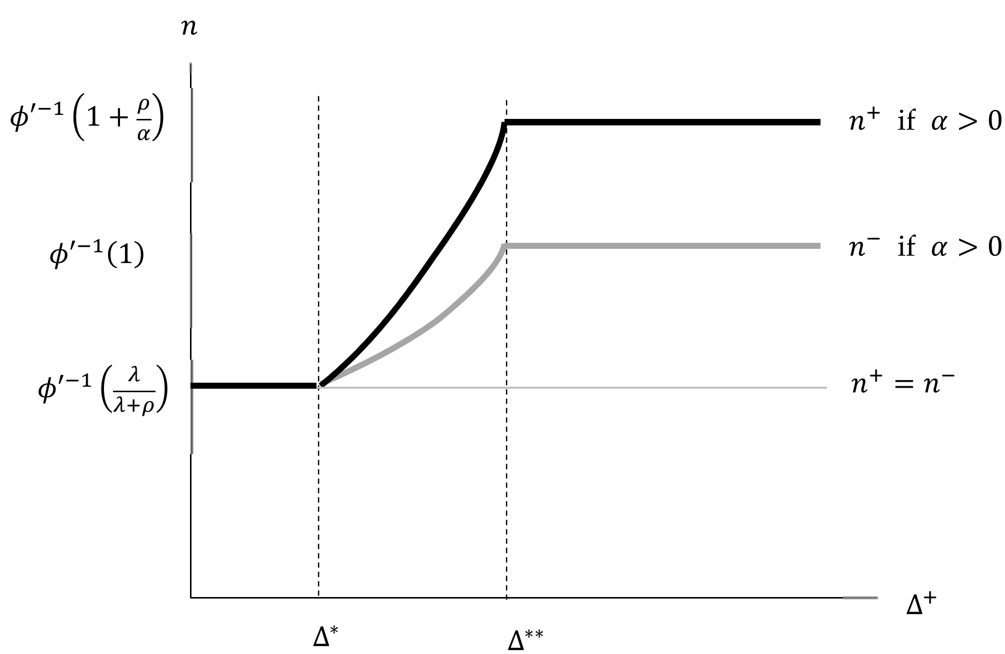

Figure 3: Production choices of optimists and pessimists as a function of the degree of optimism Δ^+

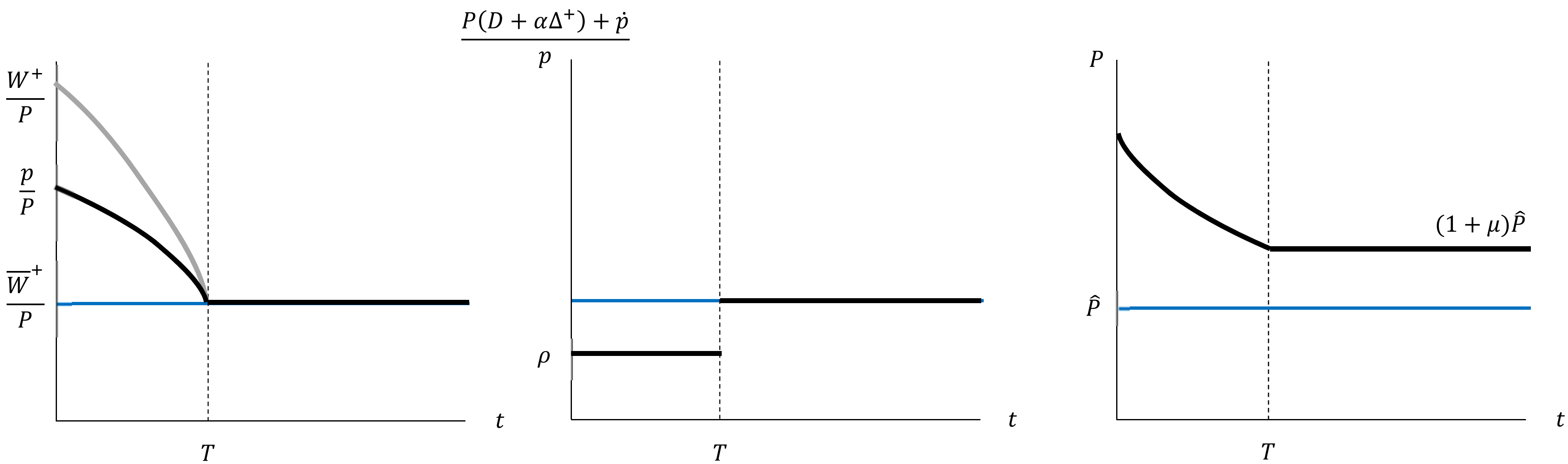

Figure 4: The effect of a liquidity injection that favors optimists when $\Delta^+ > \Delta^{**}$ (blue line = original steady state)

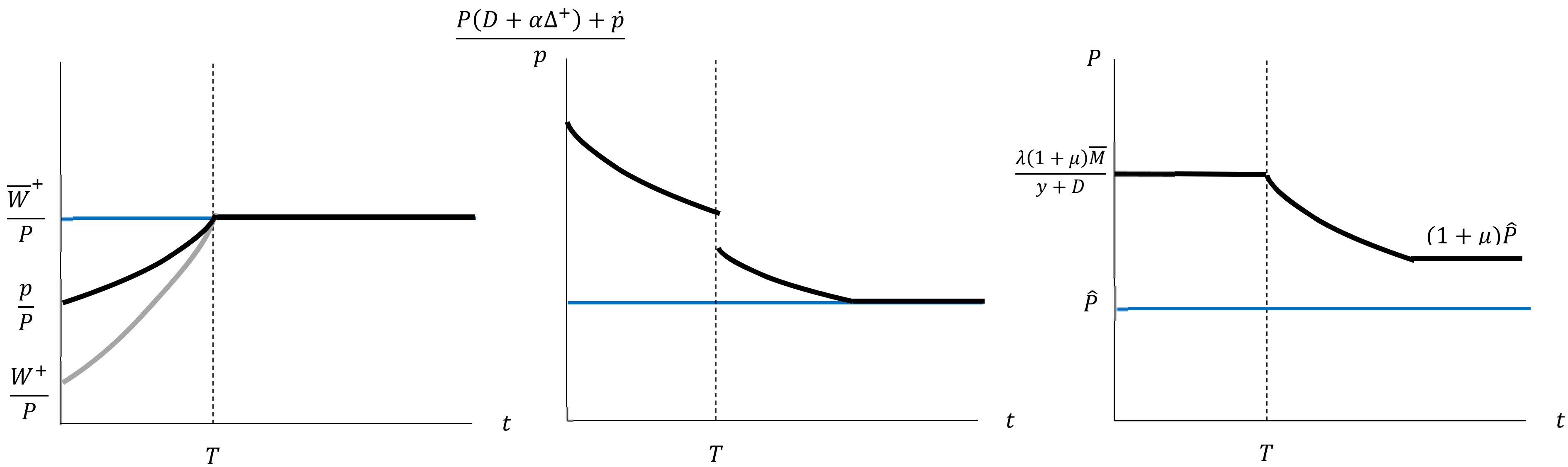

Figure 5: The effect of a liquidity injection that favors pessimists when $\Delta^+ > \Delta^{**}$ (blue line = original steady state)

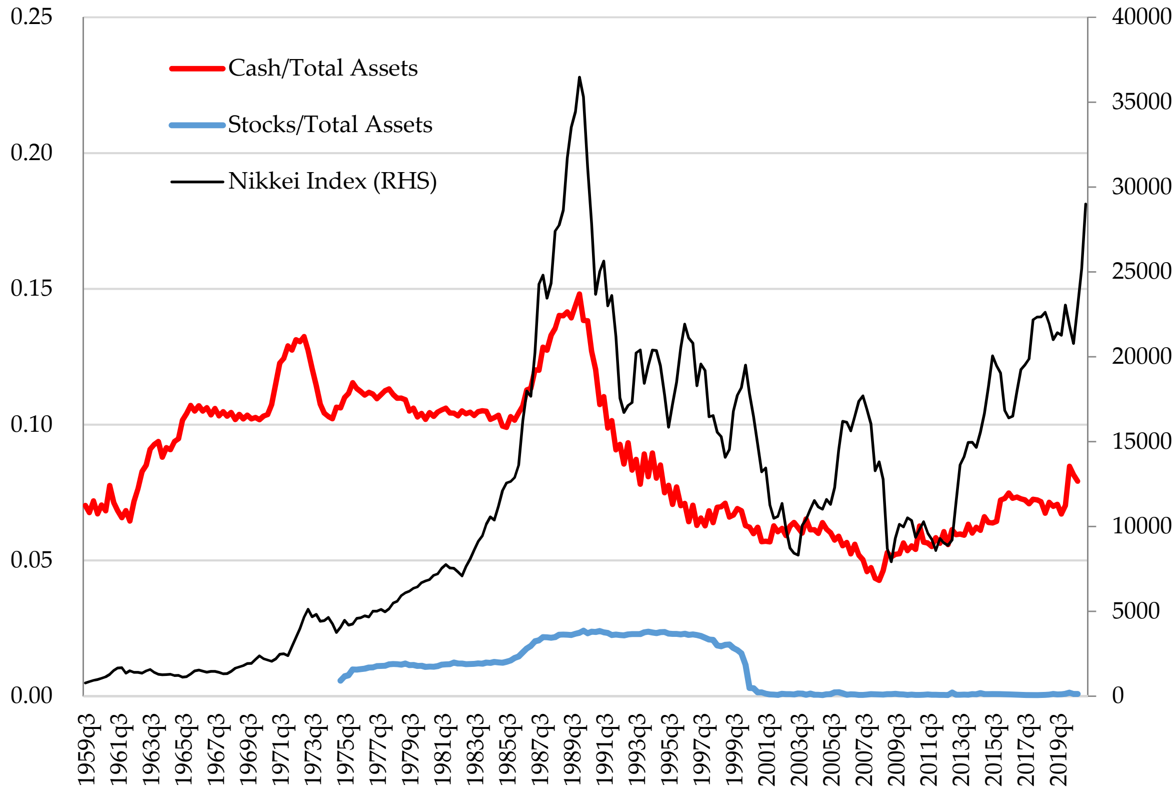

Figure 6: Asset Holdings of Large Non-Financial Corporations in Japan

The red and blue line denote the ratio of cash and deposits to total assets and the ratio of stocks to total assets of large Japanese corporations (with at least 1 billion yen in capital). The black line denotes the quarterly average of the Nikkei 225 Index. Source: Financial Statements Statistics of Corporations by Industry, Japanese Ministry of Finance for ratios and Federal Reserve Economic Data for the Nikkei 225 Index.

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