Federal Reserve Bank of Chicago

Macro Shocks and Firm Dynamics with Oligopolistic Financial Intermediaries

Alessandro T. Villa

REVISED
April 25, 2023

WP 2022-41

https://doi.org/10.21033/wp-2022-41

Macro Shocks and Firm Dynamics with Oligopolistic Financial Intermediaries

Alessandro T. Villa*

April 25, 2023

Abstract

Motivated by a secular increase in the concentration of the US banking industry, I develop a new macroeconomic model with oligopolistic financial intermediaries and heterogeneous firms. Imperfect competition allows banks to price discriminate and charge firm-specific markups, exerting a higher degree of market power on productive young firms that are more financially constrained. The time-varying effects of the cross-sectional dispersion of markups amplify the impact of macroeconomic shocks. During a crisis, banks exploit the higher number of financially constrained firms to extract higher markups, inducing a larger decline in real activity. When a big bank fails, the remaining banks use their increased market power to restrict the supply of credit, worsening and prolonging the downturn. The results suggest that bank market power should be an important concern when designing appropriate bail-out policies.

JEL Codes: D43, E44, G12, G21, L11.

Keywords: Dynamic Financial Oligopoly, Endogenous Financial Markups, Heterogeneous Firms, Firm Dynamics, Micro-Founded Financial Frictions, Price Discrimination.

---

*Federal Reserve Bank of Chicago. Mailing address: 230 S. La Salle St., Chicago, IL 60604, USA. Email: alessandro.villa@chi.frb.org. Acknowledgments: I am indebted to my advisors Andrea Lanteri, Lukas Schmid, Giuseppe Lopomo, Francesco Bianchi, and Pietro Peretto for their support and advice. I would also like to thank Joel David, François Gourio, Leonardo Melosi, Pablo D'Erasmo, Nicola Persico, Stijn G. Van Nieuwerburgh, Dean Corbae, Vincenzo Quadrini, Robert Barsky, Virgiliu Midrigan, Roberto Steri, Fabio Schiantarelli, Matthias Kehrig and Cosmin L. Ilut for their helpful comments and encouragements. I benefited from fruitful conversations with numerous seminar participants at Computing in Economics and Finance 2019, BFI Macro Finance Research Program 2020, Econometric Society/Bocconi World Congress 2020, University of Surrey, Duke University, University of Essex, Boston College, Chicago Fed, University of Washington, Federal Reserve Board, Penn State University, and University of Wisconsin-Madison. Disclaimer: The views expressed in this paper do not represent the views of the Chicago Fed or the Federal Reserve System. I declare that I have no relevant or material financial interests that relate to the research described in the paper. All errors are my own.

1 Introduction

The banking industry has become increasingly concentrated over the past two decades, with the asset market share of the five largest US banks rising from 26% in 1996 to 50% in 2018. Moreover, the Lerner index increased from 0.2 in 1996 to 0.33 in 2014, pointing to a sizable increase in markups.[1] A large and influential literature has studied the interactions between financial markets, firm, and aggregate dynamics but has typically assumed perfectly competitive financial intermediaries; thus, it does not speak to this trend of increasing banking sector consolidation and markups.[2] Moreover, an extensive empirical literature has highlighted that banks' market power has different effects on younger and older firms.[3]

In this paper, I study the role of imperfect competition in the financial intermediation sector for firm investment and financing dynamics, as well as for the transmission of macroeconomic shocks. First, I develop a dynamic general equilibrium model that incorporates an oligopolistic financial sector (including entry and exit decisions) with heterogeneous firms. The framework formalizes the idea that banks' market power has different effects along the life cycle of a firm. Imperfect competition enables financial intermediaries to charge firm-specific markups that depend on the idiosyncratic characteristics of the firms to which they lend. In particular, banks exert a higher degree of market power on firms that are more financially constrained and have a high marginal productivity of capital. These firms have worse outside options (e.g., a high cost of non-bank finance); hence, they exhibit a higher and less elastic demand for credit. This mechanism creates endogenous financial frictions that slow the growth of firms that operate in more concentrated credit markets. Second, I show that the time-varying effects of this mechanism have significant implications, not only for firm investment and financing dynamics, but also for the transmission of macroeconomic shocks. During a crisis, oligopolistic banks exploit the higher number of financially constrained firms to extract higher markups, inducing a larger misallocation of credits (hence, capital) and a larger decline in real activity. Notably, since my model features non-atomistic banks, I can study market structure changes in the intermediation sector (e.g., the failure of a large intermediary and a new bank entry). When a single big bank fails, surviving banks utilize their increased level of market power to restrict the supply of credit contributing to amplifying and prolonging the recession. The results suggest that banks' market power

[1] See Appendix C.2. See Corbae and D'Erasmo (2021) for related and more detailed evidence.

[2] See, for instance, Bernanke and Gertler (1989), Covas and den Haan (2011), Jermann and Quadrini (2012), Khan and Thomas (2013), and Midrigan and Xu (2014).

[3] See, for example, Petersen and Rajan (1995), Rajan and Zingales (1998), Cetorelli and Gambera (2001), Black and Strahan (2002), and Cetorelli and Strahan (2006). Firms reliant on external funding via bank loans, e.g. small and private firms, can become financially constrained when credit conditions tighten (Holmstrom and Tirole, 1997; Diamond and Rajan, 2005; and Chodorow-Reich, 2013). See Subsection 4.1.1 for details.

should be an important source of concern for policymakers deciding whether to bail out a large intermediary.

In summary, the paper makes three contributions. First, to the best of my knowledge, I am the first to develop a macroeconomic model that incorporates oligopolistic banks and heterogeneous firms that formalizes the idea that banks' market power has different effects along the life cycle of a firm. The model reveals a mechanism of endogenous financial frictions through which bank competition can play a role in shaping the speed at which firms grow, impacting aggregate productivity and output. More precisely, limited competition enables banks to price discriminate and charge firm-specific markups, exerting a higher degree of market power on productive young firms that are more financially constrained. The resulting dispersion of markups induces credit – and thus capital — misallocation reducing aggregate productivity. Second, the model suggests that the time-varying effects of the dispersion of markups play a potential significant role in amplifiying the impacts of macroeconomic shocks. Third, I make a methodological contribution by extending existing heterogeneous agents algorithms to solve for the stationary equilibrium and transitional dynamics in presence of a continuum of non-competitive markets and strategic interactions by exploiting generalized Euler equations.[4]

Succinctly, the model works as follows. Firms make optimal capital structure decisions by balancing equity and debt financing, generating an endogenous dynamic demand for loans. Banks are large (i.e., non-zero mass) players and firms are a continuum of followers in a Stackelberg fashion (i.e., each financial intermediary takes firms' dynamic demand for loans as given and competes to supply funding to each individual firm). Intermediaries make strategic decisions by internalizing the effect of their actions on present and future banks, firms' decisions, and on the aggregate economy. In such an environment, the bank's optimal equilibrium choice of loan supply is determined by solving the aforementioned generalized Euler equations. Each generalized Euler equation is an otherwise standard Euler equation, except that it contains a firm-specific elasticity that measures the sensitivity of the future interest rate with respect to the current supply of loans. The model generates firm-specific credit spreads that accrue to banks, which include default premia and markups.

My analysis proceeds in two main steps. First, I develop a stylized two-period model that I use to derive analytical insights on the role of oligopolistic intermediaries for firm dynamics and aggregate outcomes. Second, I build an infinite-horizon version of the model and discusses the impact of the main economic mechanism and its effects in greater details. I

[4] Note that the optimal fiscal policy literature uses generalized Euler equations and Markov perfect equilibria in macroeconomics (Klein and Ríos-Rull, 2003; Krusell, Martin, and Ríos-Rull, 2004; Klein, Krusell, and Ríos-Rull, 2008; and Clymo and Lanteri, 2020).

use the model to suggest that the cross-sectional and time-varying effects of the dispersion of financial markups play a potential significant role in amplyfing the impacts of macroeconomic shocks. I calibrate the model to match several financial and macroeconomic variables using Federal Deposit Insurance Corporation (FDIC) data. The calibrated model suggests that the cross-sectional effects of imperfect competition in the financial sector can have a sizable effect on short-run and long-run macroeconomic outcomes.

In particular, I use the model to investigate the role that banks' market power plays in the transmission of three unexpected aggregate shocks: (i) a credit quality deterioration (in the model, a decrease to aggregate TFP calibrated to be half the size of the Great Recession and not sufficiently large to induce a bank failure), (ii) a TFP shock calibrated to the Great Recession that is sufficiently large to induce a big bank failure (a sudden exit of an incumbent non-atomistic bank) with a subsequent new bank entry, and (iii) a TFP shock calibrated to the Great Recession combined with a permanent increase to the fixed entry cost in the credit market (that induces a permanent change to the bank market structure in the spirit of capturing the long run trend of consolidation in the banking sector).[5] I find that in each of these cases, the endogenous cross-sectional dispersion of markups plays a significant role in shaping — and in particular, amplifying – the economy's response to the exogenous shock.

A decrease to aggregate TFP, with an associated increase to the aggregate probability of firm default, induces a higher proportion of young, more financially constrained firms with a high marginal productivity of capital. In these conditions, a more concentrated banking sector can control the supply of credit more tightly by extracting higher markups from these firms, leading to higher interest rates. This mechanism allows banks to compensate for the larger losses due to defaults, but it leads to a larger decline in real activity, amplifying the recession. When this shock is also combined with a lower firm entry rate, then imperfect competition in the financial intermediation sector leads to a bigger and delayed amplification effect at the peak that fades away as new firms enter the production sector.[6]

When one large financial intermediary fails, the change in market structure lowers the supply of credit to firms, slowing down the economy. The surviving banks extend more credit to firms in order to capture the market share of the defaulted bank. However, the speed of this adjustment is dampened by the decreased level of competition. As a result of

[5] A valuable (and computational difficult) extension would be to allow for aggregate risk to study the interaction between risk-premia, firm dynamics, and banks' market power. Introducing non-bank-specific aggregate risk requires to extend the methodology proposed by Krusell and Smith (1998) with my approach (based on generalized Euler equations) to simultaneously solve for heterogeneous firms and strategic interactions among banks competing in a continuum of markets. Introducing bank-specific risk (which is a form of aggregate risk since banks have non-zero mass) requires to combine the algorithm proposed by Ifrach and Weintraub (2016) with my approach to deal also with banks heterogeneity.

[6] For instance, during the Great Recession we observed a decline of the firm entry rate.

both credit constraints and market power, the aggregate volume of credit drops sharply in the short run. In the long run, I analyze two different scenarios. First, I analyze a scenario where one bank reenters the credit market as the financial and economic conditions ease. Second, I analyze a scenario where no bank reenters (in the model, this is obtained through a permanent increase to the fixed entry cost in the credit market). In the former case, it takes several quarters for the economic conditions to ease sufficiently so that it is profitable for one bank to reenter. This persistently depresses investment, TFP and output. In the latter case, the resulting increase in banks' market power further amplifies and prolongs the recession and, in the long run, the economy stabilizes at a lower level of total credit, which results in permanent less investment, output, and productivity. The result suggests that banks' market power may be an important source of concern for policymakers deciding whether to bail out a big bank.

Related Literature This paper is mainly related to two strands of literature: (i) firm dynamics in the face of financing frictions and (ii) macroeconomics with financial intermediaries. Indeed, one of the paper's main contributions is to link the first literature, which typically assumes a perfectly competitive credit market, with the second, which typically abstracts from firm dynamics and the heterogeneous effects of imperfectly competitive financial intermediaries across firms with varying characteristics.

Credit Markets and Firm Dynamics. An important literature has studied the impact of credit market frictions (e.g., borrowing constraints) on firm and aggregate dynamics, but typically assumes that firms face a perfectly competitive credit market. My paper contributes to this line of work by jointly studying firms' financing and investment decisions in a credit market characterized by imperfectly competitive, non-atomistic banks that compete strategically and focusing on how banks' market power shapes the cross-sectional behavior of firms. Classical papers in this literature are Kocherlakota (2000); Gomes (2001); Cooley and Quadrini (2001); Cordoba and Ripoll (2004); Hennessy and Whited (2005); Hennessy and Whited (2007); Covas and den Haan (2011); and Jermann and Quadrini (2012). Khan and Thomas (2013) and Khan, Senga, and Thomas (2016) study models of heterogeneous firms in a dynamic stochastic general equilibrium environment in which firms can source their financing from a perfectly competitive intermediation sector.

Relatedly, the dynamic financial oligopoly I develop generates endogenous firm-level financial frictions that lead to time-varying second moments, such as the dispersion of loan rates (directly linked to the dispersion of marginal products of capital) and aggregate productivity. In agreement with other work and empirical findings (e.g., Lanteri, 2018 and

David, Schmid, and Zeke, 2022), the model generates an increasing dispersion of loan rates during recessions and hence, an increasing dispersion of marginal products of capital that shapes the dynamic behavior of aggregate productivity. Thus, the model uncovers a new channel for credit (hence, capital) misallocation linked to banks' market power.[7]

Burga and Céspedes (2022) empirically estimate the effect of changes in bank market power by exploiting a merger episode using a sample of small Peruvian firms and find that, in agreement with the predictions of the model, the change in bank market structure results in (i) a reduction of capital concentrated among small firms with a high marginal return and (ii) an increase in capital misallocation.[8] To conclude, another related literature in firm dynamics studies financing constraints and irreversibility (e.g., Caggese, 2007).

Macroeconomics with Financial Intermediaries. Several papers analyze the role of financial intermediaries in macroeconomics, either with a focus on banks' imperfect competition (e.g., Corbae and D'Erasmo, 2021) or focusing on the interaction between credit constraints and the financial intermediation sector (e.g., Elenev, Landvoigt, and Van Nieuwerburgh, 2021).

There has been increasing interest in macroeconomics in analyzing the role of market power.[9] Corbae and D'Erasmo (2021) are among the first to investigate the effects of imperfect competition in loan markets by building a rich quantitative model of banking industry dynamics to study the effects of financial regulations. My paper complements their seminal work by embedding an imperfectly competitive banking sector in a heterogeneous firm environment, in which each firm makes optimal capital structure decisions and each bank extracts endogenous firm-specific markups. Hence, the focus of my work is on the impact of bank market power on macroeconomic outcomes with endogenously evolving heterogeneity in borrower types.

Elenev, Landvoigt, and Van Nieuwerburgh (2021) investigate financial intermediaries' capital requirements in a model with both financially constrained firms and intermediaries. Similarly to my paper, their model focuses on the sudden and persistent fall in macroeconomic outcomes and credit supply during the financial crisis. My work complements their analysis by investigating the role of intermediary market power which, through time-varying

[7]Another literature studies constrained optimal dynamic contracts in partial equilibrium (e.g., Albuquerque and Hopenhayn, 2004; Clementi and Hopenhayn, 2006; Brusco, Lopomo, Ropero, and Villa, 2021).

[8]The empirical literature about relationship lending – Rajan and Zingales (1998); Black and Strahan (2002); Cetorelli and Gambera (2001); Cetorelli (2004); Cetorelli and Strahan (2006); and Cetorelli and Peretto (2012) (theoretically)– is discussed in detail in Subsection 4.1.1.

[9]For example, see Gutiérrez and Philippon (2016); Farhi and Gourio (2018); De Loecker, Eeckhout, and Unger (2020); Corhay, Kung, and Schmid (2020); Berger, Herkenhoff, and Mongey (2022); and Jamilov (2020).

endogenous firm-specific markups, leads to the amplification of macroeconomic shocks. As a direction for future research, it would be interesting to combine intermediaries' market power with collateral constraints in order to investigate the economic mechanism through which they interact and quantitatively disentangle the two joint effects.

Other salient contributions, such as Eisfeldt, Lustig, and Zhang (2017) and Atkeson, Eisfeldt, and Weill (2015), focus on other dimensions of heterogeneity. Eisfeldt, Lustig, and Zhang (2017) investigate the impact of asset complexity on the wealth distribution of complex asset investors in a heterogeneous agents model and Atkeson, Eisfeldt, and Weill (2015) propose a parsimonious model with heterogeneous financial intermediaries with entry and exit to study the equilibrium and socially optimal decisions to trade in over-the-counter markets. Another paper focusing on heterogeneous financial intermediaries is Jamilov and Monacelli (2021), who develop a rich quantitative model with heterogeneous monopolistic banks. I complement their work by focusing on the effects of banks' market power on firm financing and investing dynamics; hence, in an environment with heterogeneous firms (and not heterogeneous banks). Another strand of the literature investigates the impact of banks' market power on the transmission of monetary policy shock, such as Wang, Whited, Wu, and Xiao (2022), who analyze the impact of banks' deposit market power on the loan maturity structure and assess the relevance of banks' market power in both the deposit and loan markets. Differently from these papers, my focus is on firm dynamics with the salient distinctive feature that banks, in my framework, charge endogenous firm-specific markups, creating an equilibrium cross-sectional dispersion of interest rates, that I show is a relevant transmission channel of macroeconomic shocks. Note also that, Jamilov and Monacelli (2021) and Wang, Whited, Wu, and Xiao (2022) capture banks' market power via constant elasticity of substitution. In my environment, where I focus on a different dimension of heterogeneity (i.e., heterogeneous firms), the generalized Euler equations' approach allows for endogenous time-varying firm-specific markups which I would not be able to capture via constant elasticity of substitution. In this paper, I highlight how the endogenous time-varying dispersion of markups acts as a significant transmission channel of macroeconomic shocks.

Other papers studying the interaction between credit constraints and financial intermediation include Kiyotaki and Moore (1997) and Gertler and Karadi (2011).[10] He and Krishnamurthy (2013) introduce a stochastic model that explains how intermediary capital affects risk premia variation. Rampini and Viswanathan (2018) propose a dynamic model whereby financial intermediaries provide a superior collateralization service to households.

[10] Other relevant papers are He and Krishnamurthy (2013); Brunnermeier and Sannikov (2014); and Rampini and Viswanathan (2018).

In contrast to these papers, my focus is on the transmission of macroeconomic shocks when intermediaries have market power.

Outline. The rest of the paper is organized as follows. Section 2 presents a stylized version of the model aimed at delivering basic intuition about the proposed mechanism. Section 3 describes an infinite-horizon version of the model and discusses the main mechanism and its effects in detail. Section 4 explains the calibration and results of the oligopolistic stationary equilibrium. Section 5 illustrates the results of the three aforementioned macroeconomic shocks, such as the failure of one big bank and a new bank entry, together with their interaction with the production sector's default rate. Section 6 proposes an extension of the baseline model. Section 7 concludes.

2 Stylized Model

In this section, I analyze a two-period model designed to provide preliminary intuition for the infinite-horizon model presented in Section 3. An oligopolistic banking sector interacts with a continuum of heterogeneous firms in the presence of idiosyncratic total factor productivity (TFP) and default shocks. I provide analytical results on the effects of an increase of the number of banks B on several financial and macroeconomics variables of interest (including aggregate loans, interest rates, expected returns on equity, physical investment, aggregate leverage, dispersion of capital, dispersion of loan interest rates, dispersion of expected returns, and aggregate TFP). In the stylized version of the model, there are two dates denoted $t = 0, 1$.

Note that, in the stylized model, the number of banks is exogenous but in the infinite-horizon model is endogenous (banks make entry and exit decisions). Moreover, in the stylized model, I assume there are two different type of households: one who own the banks and one who owns the firms. Hence, in the stylized model the household who owns the banks cannot directly finance the firms. I also relax this assumption in the infinite-horizon model, which features one single type of households that can simultaneously trade shares of the banks and the firms.

Preferences. There are B identical banks, each owned by a continuum of identical savers; hence there are B representative savers. Each saver's preferences are represented by the following linear utility function:

$$C_{b,0} + \beta \cdot C_{b,1},$$

where $C_{b,t}$ is the saver's consumption at time t and $\beta \in (0,1)$ is the discount factor.[11]

There is a continuum of firms, each owned by a continuum of identical entrepreneurs; hence there is one representative entrepreneur. The entrepreneur is risk-neutral and has preferences represented by the utility function:

$$C_{E,0} + \beta \cdot C_{E,1},$$

where $C_{E,t}$ is the entrepreneur's aggregate consumption at time t.

Ownership Structure. Each representative saver owns a bank. In equilibrium, the saver is indifferent between financing the bank's loans with debt or equity. The representative entrepreneur owns the entire mass of firms $j \in [0,1]$. Each firm j is characterized by its state vector

$$x(j) \equiv \{\{l_b(j)\}_b^B, r_l(j), k(j), z(j), \mathcal{I}(j)\},$$

where $l_b(j)$ denotes the firm's loan by bank b, $r_l(j)$ (throughout the paper I will also refer to $R_l(j) = 1 + r_l(j)$) is the interest rate (charged by all banks), $k(j)$ is the firm's capital stock, $z(j)$ is the firm's productivity, and $\mathcal{I}(j)$ is an indicator function that takes value 1 if the firm has not defaulted. Let $\phi(x)$ denote the density function of firms in the economy. Each saver cannot own any firm's equity and thus needs to save through banks.

Markets. There are five markets in the economy: banks' debt, banks' equity, firms' loans, firms' equity, and the market for the representative good.

Banks' equity and debt markets. Each saver b invests in the production sector by supplying equity or debt to banks and faces the budget constraints:

$$C_{b,0} + p_b \cdot S_{b,1} + D_{b,1} = (p_b + \pi_{b,0}) \cdot S_{b,0}$$
$$C_{b,1} = \pi_{b,1} \cdot S_{b,1} + R_D \cdot D_{b,1},$$

where p_b, $S_{b,0}$, $S_{b,1}$, $D_{b,1}$, $\pi_{b,0}$, and $\pi_{b,1}$ are, respectively, the bank's share price, the share holdings at $t = 0, 1$, the debt holdings at $t = 1$, and the bank's profit at $t = 0, 1$. Bank b demands equity and debt from saver b, in order to finance loans to firms.

Firms' equity market. The entrepreneur invests in the production sector by supplying equity

[11] The risk-neutrality assumption in the stylized model is made to simplify the analysis and isolate effects that do not depend on the savers' risk-aversion. In the quantitative model of Section 3 the savers are risk-averse.

to the firms and faces budget constraints:

$$
\begin{aligned}
C_{E,0} + \int [\mathcal{I} \cdot p_0 \cdot S_1 + (1-\mathcal{I}) \cdot p_0 \cdot S_1] \, \mathrm{d}\Phi &= \int \mathcal{I} \cdot (p_0 + \tilde{d}_0) \cdot S_0 \, \mathrm{d}\Phi \\
C_{E,1} &= \int \mathcal{I} \cdot \tilde{d}_1 \cdot S_1 \, \mathrm{d}\Phi,
\end{aligned}
$$

where p_0, S_0, S_1, $\tilde{d}_0$, and $\tilde{d}_1$ are, respectively, the share price, share holdings at $t = 0, 1$, and the dividend of each firm (net of equity issuance cost) at $t = 0, 1$. Firms demand equity from, or distribute dividend to, the entrepreneur. If a firm decides to issue equity, it incurs a quadratic equity issuance cost at $t = 0$ (with λ_0 being a positive constant):

$$
\lambda(d_0) = \begin{cases} \lambda_0 \frac{d_0^2}{2} & \text{if } d_0 \leq 0 \\ 0 & \text{if } d_0 > 0 \end{cases},
$$

where d_0 is a firm dividend at $t = 0$, defined below. The convexity of $\lambda(.)$ captures the idea of increasing marginal underwriting cost, or the increasing threat posed by moral-hazard when a greater amount of equity is demanded.

Firms' loan market. A finite (and exogenous) number B of (identical) banks supply loans to a continuum of firms. Each bank $b = 1 \dots B$ can issue non state-contingent loans $l_{b,1}$ to each firm. Loans are due for repayment in the next period, unless the firm defaults. A firm j takes the interest rate $r_1(j)$ as given and chooses how much to invest and how much to borrow from each bank. Banks take each firm's demand schedule as given and compete à la Cournot, i.e, simultaneously and independently choose their loan portfolios. The Cournot model has the convenient property that the number of firms (banks in my case) is a sufficient statistic to determine market power, rendering it a parsimonious and tractable modeling choice. Alternatively, one could consider a Bertrand model where banks face capacity constraints, in order to rule out perfectly competitive outcomes. As shown by Kreps and Scheinkman (1983), quantity precommitment followed by Bertrand competition would induce Cournot outcomes.

Goods market. The representative entrepreneur and the B representative savers demand goods supplied by all firms.

Technology. In each period $t = 0, 1$, the output $y_t(j)$ produced by each firm $j \in [0, 1]$ is given by the production function $y_t(j) = z_t(j) \cdot k_t(j)^\alpha$, where $0 < \alpha < 1$.

Shocks. At time 0, firms are heterogeneous with respect to their capital stock k_0 and their idiosyncratic total factor productivity (TFP) z_0. At time 1, there are two types of idiosyncratic shocks: the firm can default, with exogenous probability $1-\rho$ and, if it survives, z_1 realizes according to $\log z_1 = \rho_z \log z_0 + \xi_1$ where $\xi_1 \sim \mathcal{N}(0, \sigma_\xi^2)$ and $0 \leq \rho_z < 1$.

Timing. All decisions are taken at $t = 0$. Given the initial distribution of firms with pdf $\phi(x_0)$ (and cdf $\Phi(x_0)$), the timing is as follows: (1) each firm produces output $y_0 = z_0 k_0^\alpha$; (2) each bank finances its supply of loans, $\int l_b(x_0)\, \mathrm{d}\Phi(x_0)$, by issuing equity and/or debt; (3) each firm takes the interest rate $R_1(x_0)$ as given and chooses how much to invest and the amount of loan to demand from each bank; (4) banks take each firm's demand schedule as given and compete with each others to supply the loans. The outcome is a contract establishing: loan amount $l_b(x_0)$, interest rate $R_1(x_0)$, and the new level of capital $k_1(x_0)$; and (5) firms distribute dividends $d_0 = z_0 k_0^\alpha + (1-\delta)k_0 - k_1 + \sum_b^B l_b$ to the entrepreneur. At $t = 1$, the $1-\rho$ mass of defaulting firms exits the market. For the surviving firms, z_1 is realized and: (1) firms produce output $y_1 = z_1 k_1^\alpha$; (2) firms repay their outstanding debt plus interest $R_1(x_0) \cdot \sum_b^B l_b(x_0)$; (3) each bank distributes its profit $\int \rho R_1(x_0) l_{1,b}(x_0)\, \mathrm{d}\Phi(x_0)$ to the saver; and (4) firms distribute dividend $d_1 = z_1 k_1^\alpha + (1-\delta)k_1 - R_1 \sum_b^B l_{1,b}$ to the entrepreneur.

2.1 Agents' Optimization Problems

The representative saver b maximizes its intertemporal utility subject to their budget constraint, yielding an Euler equation that pins down the price of banks' equity: $\forall b : \quad \beta \pi_{b,1} = p_{b,0}$. The representative entrepreneur maximizes its intertemporal utility subject to their budget constraint, yielding an Euler equation that pins down the price of each firm's equity:

$$\rho \beta \mathbb{E}_0 \left[\frac{d_1}{p_0} \right] = 1 - \lambda_d(d_0), \tag{1}$$

where p_0 is the price of the share of a firm at time 0. Firms maximize the net present value of dividends $d_0 + \beta \cdot \mathbb{E}_0 \left[\mathcal{I} \cdot d_1 \right]$, where dividends in each period are given by:

$$d_0 = z_0 k_0^\alpha + (1-\delta)k_0 - k_1 + \sum_b^B l_{1,b}, \quad d_1 = z_1 k_1^\alpha + (1-\delta)k_1 - R_1 \sum_b^B l_{1,b}.$$

The firm's optimality condition with respect to the loan requires that the discounted future expected interest rate be one net of the equity issuance cost:

$$\rho\beta R_1 = 1 - \lambda_d(d_0). \tag{2}$$

The firm's first-order condition with respect to capital requires that the future interest rate equals the expected marginal productivity of capital net of depreciation:

$$R_1 = \mathbb{E}_0\left[1 + \alpha z_1 k_1^{\alpha-1} - \delta\right]. \tag{3}$$

Generalized Euler Equation. Banks' strategies map firm characteristics (x_0) onto the current quantity and future interest rate of loans. Given the probability density function $\phi(x_0)$ (and cumulative distribution function $\Phi(x_0)$), each bank b chooses $l_{1,b}(x_0)$ to best respond to other banks' strategies $l_{1,-b}(x_0)$, such that

$$\max_{l_{1,b}(x_0)} \quad \pi = -\int l_{1,b}(x_0)\,\mathrm{d}\Phi(x_0) + \beta\int \rho R_1(x_0) l_{1,b}(x_0)\,\mathrm{d}\Phi(x_0),$$

subject to equations (1)-(3) for all firms in the distribution.
Each bank's best response is characterized by the following generalized Euler equation (GEE)

$$\forall x_0 : \; \frac{\partial \pi}{\partial l_{1,b}(x_0)} = -1 + \rho\beta\frac{\partial R_1(x_0)}{\partial l_{1,b}(x_0)} l_{1,b}(x_0) + \rho\beta R_1(x_0) = 0, \tag{4}$$

where $\frac{\partial R_1(x_0)}{\partial l_{1,b}(x_0)}$ can be determined by the implicit function theorem on equations (2) and (3). Equation (4) is a generalized Euler equation because it contains the derivative of the firm's policy functions. Each bank best responds by internalizing the effect of loans on the firms' capital choice $\frac{\partial k_1}{\partial l_{1,b}}$ as well.

GEE and Elasticity. For ease of notation, I drop the dependency of all optimal choices from x_0. The inverse elasticity implicitly contained in the GEE $\frac{\partial R_1}{\partial l_{1,b}}\frac{l_{1,b}}{R_1}$ requires the determination of the term $\frac{\partial R_1}{\partial l_{1,b}}$. From equation (3)

$$\frac{\partial R_1}{\partial l_{1,b}}\frac{l_{1,b}}{R_1} = \mathbb{E}_0\left[\underbrace{\alpha(\alpha-1)z_1 k_1^{\alpha-2}}_{\frac{\partial \mathrm{MPK}}{\partial k_1}} \cdot \frac{\partial k_1}{\partial l_{1,b}}\frac{l_{1,b}}{R_1}\right]. \tag{5}$$

The formula suggests that banks' market power depends on two components. First, banks extract higher markups out of firms that exhibit a lower (i.e., more negative) derivative of the marginal productivity of capital $\frac{\partial \text{MPK}}{\partial k_1}$.[12] These firms are characterized by a less elastic credit demand, since if banks restricted the supply of loans (hence, inducing less investment in physical capital) they would cause: (i) a greater missed production (these firms also exhibit a high MPK) and (ii) a higher interest rate (these firms' levels of capital correspond to a more concave point of the production function). Second, banks think strategically by internalizing the effects their actions have on the firms' investment decisions (i.e., they internalize the impact of their actions on the firms' policy functions). This second effect is captured by the cross-elasticity $\frac{\partial k_1}{\partial l_{1,b}} \frac{l_{1,b}}{R_1}$. Finally, note that expressions for the two cross-derivatives can be found jointly by taking the total derivatives of equations (2) and (3):

$$\frac{\partial R_1}{\partial l_{1,b}} = \frac{1-\rho\beta R_1}{\rho\beta l_{1,b}}, \quad \frac{\partial k_1}{\partial l_{1,b}} = \frac{1-\rho\beta R_1}{\rho\beta l_{1,b}} \cdot \frac{1}{\alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{\alpha-2}}.$$

In equilibrium, for the mass of financially constrained firms ($d_0 < 0$), the degree of imperfect competition (number of banks B) matters. For each firm, the equilibrium is a vector $(k_1^*, R_1^*, l_{1,b}^*, p_0^*)$ such that equations (1)-(4) hold. For the mass of firms that, in equilibrium, is not financially constrained, the degree of imperfect competition does not matter. For these firms, the solution is given by (k_1^*, R_1^*, p_0^*) such that equations (1)-(3) hold. For these firms, the Modigliani-Miller theorem holds; hence, $l_{1,b}^*$ is undetermined. Note that in the quantitative model there is a tax shield, so that an optimal capital structure is always well-defined and markups depend both on capital and debt as separate state variables.

2.2 Characterization of the Equilibrium

I now describe intuitively the main mechanism that drives the analytical results presented in this section. First, Equation (5) suggests that the higher the marginal productivity of capital (MPK) of a firm, the higher the marginal value of one unit of loan for that firm that translates in a lower inverse elasticity $\frac{\partial R_1}{\partial l_{1,b}} \cdot \frac{l_{1,b}}{R_1}$. Second, as explained above, the degree of imperfect competition (number of banks B) matters only for the mass of financially constrained firms. Hence, banks endogenously exert a higher degree of market power on firms that are financially constrained and with a high marginal productivity of capital. Intuitively, these firms have worse outside options (e.g., a high cost of non-bank finance) and one additional unit of investment in physical capital can contribute significantly to their future production; hence, they exhibit a less elastic demand for credit. An imperfectly competitive financial sector internalizes that the same financial resources are more valuable for this type of firm and for

[12]Note that the term $\alpha - 1$ is always negative; hence, $\frac{\partial \text{MPK}}{\partial k_1}$ and the inverse elasticity $\frac{\partial R_1}{\partial l_{1,b}} \cdot \frac{l_{1,b}}{R_1}$ are always negative. Banks exert higher market power when $\frac{\partial R_1}{\partial l_{1,b}} \cdot \frac{l_{1,b}}{R_1}$ is smaller.

their future growth paths; therefore, it can charge higher markups.

This mechanism leads financially constrained firms to grow slower in less competitive credit markets. As a result, the dispersion of marginal productivity of capital is higher when there are fewer banks B in the economy. At the same time, a lack of competition in the financial intermediation reduces aggregate productivity, since firms grow toward their efficient level of capital on slower trajectories. This intuitive mechanism is at the base of Proposition I. See Appendix B.2 for the proof.

Proposition I

Assume that the distribution $\phi(x_0)$ is such that there is a non-zero measure of financially constrained firms:[a]

$$\mathcal{P} = \int \mathbf{1}[d_0(x_0, k_1^*, l_{1,b}^*) \geq 0] \, \mathrm{d}\Phi(x_0) < 1.$$

A higher number of banks (i.e., a higher B) has the following effects:

1. aggregate loans per bank $\int l_b^* \, \mathrm{d}\Phi$ decreases;
2. average loan interest rate $\int R_l^* \, \mathrm{d}\Phi$ decreases;
3. aggregate physical investment $\int k_1^* - (1-\delta)k_0 \, \mathrm{d}\Phi$ increases;
4. aggregate expected returns $\int \mathbb{E}_0 \left[d_1^*\right] / p^* \, \mathrm{d}\Phi$ decreases;
5. aggregate loans $\int \sum_b^B l_b^* \, \mathrm{d}\Phi$ increases;
6. aggregate leverage $\int \sum_b^B l_b^* / k_1^* \, \mathrm{d}\Phi$ increases;
7. aggregate TFP $\int k_1^{*\alpha} \, \mathrm{d}\Phi / \left(\int k_1^* \, \mathrm{d}\Phi\right)^\alpha$ increases;
8. variance of capital $\int k_1^{*2} \, \mathrm{d}\Phi - (\int k_1^* \, \mathrm{d}\Phi)^2$ decreases;
9. variance of loan interest rates $\int R_l^{*2} \, \mathrm{d}\Phi - (\int R_l^* \, \mathrm{d}\Phi)^2$ decreases;
10. variance of expected returns $\int \left(\mathbb{E}_0 \left[d_1^*\right] / p^*\right)^2 \mathrm{d}\Phi - \left(\int \mathbb{E}_0 \left[d_1^*\right] / p^* \, \mathrm{d}\Phi\right)^2$ decreases.

[a]For subpoints 7, 8, 9 and 10, I assume that the mass of financially constrained firms $1 - \mathcal{P}$ are all ex-ante identical.

3 Model

In the two-period model, banks' choices are static. In the infinite-horizon model, each bank faces a dynamic problem that: (i) depends on the same bank's future strategies and other banks' current and future strategies, and (ii) is subject to all firms' dynamic demand for loans; also both the current and future distributions of firms matter. The equilibrium concept used in this section is a Markov perfect equilibrium (e.g., Maskin and Tirole, 2001). Specifically, I characterize the equilibrium using generalized Euler equations in a similar fashion to the optimal fiscal policy literature (see, for instance, Klein and Ríos-Rull, 2003; Krusell, Martin, and Ríos-Rull, 2004; Klein, Krusell, and Ríos-Rull, 2008; and Clymo and Lanteri, 2020).

In this section, I build a dynamic framework to study firms' financing-investment decisions when banks are big (i.e., non-atomistic), strategically interact with each others, and face idiosyncratic firms' default risk.[13] Households derive utility from a non-durable consumption good, own the shares of the banks, and supply deposits. Banks issue debt and use both their internal resources and debt to purchase firms' loans. Firms make investment decisions, taking into account the fact that debt provides a tax shield and issuing new equity is increasingly costly. The key feature of the framework is the simultaneous presence of strategic interactions among financial institutions, general equilibrium, macroeconomic shocks, and heterogeneous firms. Note that each firm stipulates an idiosyncratic contract with the banks: in equilibrium, banks have different degrees of market power on each single firm in function of its idiosyncratic characteristics. In my model, financial intermediaries can be thought not only as either commercial or investment banks but, more generally, also as lending non-bank institutions.[14]

I will now describe the model and proceed to define the stationary oligopolistic equilibrium, in which all aggregate quantities and prices are constant over time. I overcome the computational challenge by proposing GEE-based algorithms to solve for the oligopolistic stationary equilibrium and the related transitional dynamics in the presence of strategic interactions, general equilibrium, and heterogeneous firms. The algorithms are discussed in detail in Appendix A.

3.1 Environment

Time is discrete $t = 0, 1, \dots$ and the horizon is infinite. There is an endogenous number B of identical banks in equilibrium. Banks are owned by a continuum of identical and infinitely

[13] Specifically, I interpret the financial intermediation sector as a succession of decision makers – one at each date t – without commitment to future realized quantity of loans supplied.

[14] Hence, I explicitly do not model bank regulations or insured deposit.

lived households, equivalent to one representative household. The household also owns a continuum of firms.

Preferences. The household ranks stream of consumption C_t according to the following lifetime utility function:

$$\sum_{t=0}^{\infty} \beta^t \cdot u(C_t), \tag{6}$$

where $\beta \in (0,1)$ is the household's discount factor, and $u_c > 0$, $u_{cc} < 0$.

Ownership Structure. The household owns all the banks. In equilibrium, the household is indifferent between financing the bank's loans with debt or equity. The representative household also owns the entire mass of firms $j \in [0,1]$. Each firm j is characterized by its state vector

$$x_t(j) \equiv \{\{l_{b,t}(j)\}_b^{B_t}, r_{l,t}(j), k_t(j)\},$$

where $l_{b,t}(j)$ denotes the firm's loan by bank b, $r_{l,t}(j)$ is the interest rate (charged by all banks), and $k_t(j)$ is the firm's capital stock. Let $\phi_t(x_t)$ denote the density function of firms in the economy, with associated cumulative distribution $\Phi_t(x_t)$.
The household can save through the banks or through firm's equity, in this latter case it incurs a equity issuance cost.

Markets. There are six markets in the economy: banks' debt, banks' equity, firms' loans, firms' equity, interbank market, and the market for the representative good.
Banks' equity and debt markets. The household invests in the production sector by supplying equity or debt to banks and faces the budget constraint:

$$\begin{aligned} C_t + \sum_{b=1}^{B_{t+1}} (p_{b,t} \cdot S_{b,t+1} + D_{b,t+1}) + \int p_t \cdot S_{t+1} \, \mathrm{d}\Phi_t = \\ \sum_{b=1}^{B_t} ((p_{b,t} + \pi_{b,t}) \cdot S_{b,t} + R_{D,t} \cdot D_{b,t}) + \int \mathcal{I}_t \cdot (p_t + \tilde{d}_t) \cdot S_t \, \mathrm{d}\Phi_t, \end{aligned} \tag{7}$$

where $\mathcal{I}_t$, $p_{b,t}$, $S_{b,t}$, $S_{b,t+1}$, $D_{b,t}$, $D_{b,t+1}$, $R_{D,t}$, and $\pi_{b,t}$ are, respectively, an indicator function that takes value of one if the firm has not defaulted, the bank's share price, the share holdings at t and $t+1$, the bank's debt holdings at t and $t+1$, the interest rate on the bank's debt, and the bank's profit at t. Each bank b demands equity and debt from the representative household, in order to finance loans to firms.

Firms' equity market. The household invests in the production sector by supplying equity to the firms, and faces budget constraints (7) where p_t, S_t, S_{t+1}, and $\tilde{d}_t$ are, respectively, the share price, share holdings at t, and the dividend of each firm (net of equity issuance cost) at t. Firms demand equity from, or distribute dividends to, the household. If a firm decides to issue equity, it incurs a quadratic equity issuance cost $\lambda(d_t)$ (see Section 2), where d_t is a firm dividend at time t, defined below.

Firms' loan market. A finite (and endogenous) number B_t of (identical) banks supply loans to a continuum of firms. Each bank $b = 1 \ldots B_t$ can issue non-state-contingent loans $l_{b,t+1}$ to each firm. Loans are due for repayment in the next period, unless the firm defaults. A firm j takes the interest rate $r_{l,t+1}(j)$ as given and chooses how much to invest and how much to borrow from each bank. Banks take each firm's demand schedule as given and compete dynamically à la Cournot, i.e, simultaneously and independently choose their loan portfolios. As discussed previously, the Cournot model has the handy property that the number of firms (banks in my case) is a sufficient statistic to determine market power, rendering it a parsimonious and tractable modeling choice. Alternatively, one could consider a Bertrand model where banks face capacity constraints, in order to rule out perfectly competitive outcomes. As shown by Kreps and Scheinkman (1983), quantity precommitment followed by Bertrand competition would induce Cournot outcomes. The process determines the total amount of loans banks supply to each firm which, together with the firm's demand schedule, pins down the firm-specific interest rate $r_{l,t+1}(j)$. At time t, each bank and firm commit to such an interest rate.

Interbank market. A bank b can lend $M_{b,t}$ to other banks that will be repaid in the following period at rate $r_{M,t+1}$. Since all banks are identical, in equilibrium $\forall b: \; M_{b,t} = 0$.

Goods market. The representative household demands goods supplied by all firms.

Technology. In each period t, the output $y_t(j)$ produced by each firm $j \in [0,1]$ is given by the production function $y_t(j) = Z_t \cdot k_t(j)^\alpha$, where $0 < \alpha < 1$.

Shocks. At time t, $\mathcal{I}_t$ assume value of one with probability ρ_t, and value of zero with probability $1 - \rho_t$. A new mass of firms re-enters the economy with characteristics x_0 so that the total mass is constant over time. I relax this assumption in Appendix 5.2. Moreover, in Section 6, I also propose and solve a model with idionsycratic TFP shocks and firms endogenous default decisions. The results suggest that the baseline model adopted in this section, despite being fairly parsimonious, yields relatively conservative effects of banks' market power.[15]

Government. The government imposes proportional taxes τ on all firms' production. Firms can deduct loan interest and depreciated capital from their taxes. The government runs a balanced budget constraint. That is, the government uses the aggregate revenue from taxes $T_t = \tau \int \left(Z_t k_t^\alpha - \sum_{b=1}^{B_t} r_{l,t} l_{b,t} - \delta k_t \right) \mathrm{d}\Phi$ to finance an exogenous government expenditure that exactly balances T_t at each point in time.

Timing. The aggregate state space of the economy at time t is

$$X_t \equiv \{\{D_{b,t}\}_{b=1}^{B_t}, r_{D,t}, \{M_{b,t}\}_{b=1}^{B_t}, r_{M,t}, B_t, \rho_t, Z_t, \phi(x_t)\}.$$

Given X_t, the timing is as follows: (1) a mass $1 - \rho_t$ of firms defaults, (2) each surviving firm produces output $y_t = Z_t k_t^\alpha$ and repay its debt $\sum_b^{B_t} R_{l,t} l_{b,t}$ to all incumbent banks; (3) a mass $1 - \rho_t$ of firms enter the production sector with characteristics x_0; (4) each incumbent bank decides whether to exit; (5) potential banks entrants decide whether to enter, this requires an initial equity injection from the household; (6) each bank finances its supply of loans, $\int l_{b,t+1}(x_t)\, \mathrm{d}\Phi(x_t)$, by issuing equity and/or debt; (7) each firm takes the interest rate $r_{l,t+1}(x_t)$ as given and chooses how much to invest and the amount of loan to demand from each bank; (8) banks take each firm's demand schedule as given and compete with each others to supply the loans. The outcome is a contract establishing: loan amount $l_{b,t}(x_t)$, interest rate $r_{l,t+1}(x_t)$, and new level of capital $k_{t+1}(x_t)$; (9) firms distribute dividends $d_t = (1 - \tau) \left[Z_t k_t^\alpha - \sum_b^{B_t} r_{l,t} l_{b,t} \right] + \tau \delta k_t - \tilde{i}_t$ to the household, where $\tilde{i}_t = i_t + \sum_{b=1}^{B_t} l_{b,t} - \sum_{b=1}^{B_{t+1}} l_{b,t+1}$ and i denotes investment in physical capital; (10) bank b distributes profit to the household. To simplify notation, in the following subsections I avoid explicitly sub-scripting each variable with time t and $t + 1$, but it is understood that (x, X) refers to (x_t, X_t), and (x', X') refers to (x_{t+1}, X_{t+1}).

3.2 Household

I now describe the household's problem in recursive form. Let $V_H(X)$ be the value function of the household with banks' debt holdings $\mathbf{D} = [D_1 \dots D_B]$, banks' equity holdings $\mathbf{S} = [S_1 \dots S_B]$, and firms' shares holdings $S(x)$. This function satisfies the following functional

[15] Intuitively, when banks' market power interact with firms' default decisions, in equilibrium there is a higher mass of younger firms with a higher and less elastic demand for credit (from which banks can extract higher markups).

equation:

$$V_H(X) = \max_{\mathbf{S}',\mathbf{D}',S(\cdot)'} \quad u(C) + \beta \cdot V_H(X') \tag{8}$$

subject to the budget constraint (7). The left-hand side of budget equation (7) reports the household's expenditures: household aggregate consumption, banks b's equity and debt purchases, and firms' equity purchases. The right-hand side reports the household's resources: each bank b's equity holdings and debt, and firms' equity holdings with their corresponding dividends.

The household takes the future banks' debt market rate r'_D as given, together with future banks' profits, and purchases banks' debt and equity according to:

$$\forall b: \quad 1 = \mathcal{M}'(X, X') \cdot \frac{p'_b + \pi'_b}{p_b} \tag{9}$$

$$\forall b: \quad 1 = \mathcal{M}'(X, X') \cdot R'_D, \tag{10}$$

where $\mathcal{M}' \equiv \beta \frac{u_c(C')}{u_c(C)}$, and π_b is the profit of bank b distributed as a dividend to the household.

Each firm's share value is priced according to:

$$1 = \mathbb{E}\left[\mathcal{I}' \cdot \mathcal{M}'(X, X') \cdot \frac{p' + \tilde{d}'}{p} \mid (x, X)\right]. \tag{11}$$

3.3 Firms

I now characterize firm j's problem in recursive form. For convenience, I omit the index notation j. Let $V_F(x, X)$ be the value function of the firm j with loan holdings $\mathbf{l} = [l_1 \dots l_B]$ from each bank b and capital k. This function satisfies the following functional equation:

$$V_F(x, X) \;=\; \max_{\{l'_b\}_{b=1}^{B'}, k'} \; d - \lambda(d) + \mathbb{E}\left[\mathcal{I}' \cdot \mathcal{M}'(X, X') \cdot V_F(x', X') \mid (x, X)\right],$$

subject to

$$\begin{aligned}
k' &= k(1-\delta) + i \\
i &= \tilde{i} - \sum_{b=1}^{B} l_b + \sum_{b=1}^{B'} l'_b \\
d &= (1-\tau)\left[Zk^{\alpha} - \sum_{b=1}^{B} r_l l_b\right] + \tau\delta k - \tilde{i},
\end{aligned}$$

where $\mathcal{M}'$ is the discount factor of the household, as described in the previous subsection. Each firm takes the future loans' market rate r_l' as given and finances itself through internal financing (production and equity issuance) and external financing (loans from banks). The first-order condition with respect to k' is

$$1 - \lambda_d(d) = \mathbb{E}\left[\mathcal{I}' \cdot \mathcal{M}'(X, X') \cdot \left(1 + (1-\tau)\left(Z'\alpha k'^{\alpha-1} - \delta\right)\right) \cdot (1 - \lambda_d(d')) \mid (x, X)\right]. \tag{12}$$

The first-order condition with respect to l_b' is

$$1 - \lambda_d(d) = \mathbb{E}\left[\mathcal{I}' \cdot \mathcal{M}'(X, X') \cdot (1 + (1-\tau)r_l') \cdot (1 - \lambda_d(d')) \mid (x, X)\right]. \tag{13}$$

3.4 Banks

First, I present the decision problem of the incumbent banks. Then, I present the decision problem of the new potential entrants.

3.4.1 Incumbents

An incumbent bank b chooses the new level of debt to demand from the household and the new level of loans to offer to each firm. Formally, the strategy space is defined as:

$$\mathcal{S}_b'(x, X) \equiv \{D_b'(X), l_b'(x, X)\}.$$

The new amount of debt issued ($\Delta D_b' = D_b' - D_b$) and internal financing F is chosen to provide enough coverage for the change in interbank lending and aggregate loans:

$$F + \Delta D_b' = \Delta M_b' - \int (\mathcal{I} \cdot l_b - l_b'(x, X))\, \mathrm{d}\Phi. \tag{14}$$

I now describe the bank's problem in recursive form. Let $V_b(X)$ be the value function of a bank b. This function satisfies the following functional equation:

$$\tilde{V}_b(X) = \max_{\{D_b', r_D'\},\ M_b',\ \{l_b'(x,X), r_l'(x,X)\}} \quad \pi_b + \mathcal{M}'(X, X') \cdot V_b(X') \tag{15}$$

where each incumbent bank makes a exit decision

$$V_b(X) = \max\{0, \tilde{V}_b(X)\} \tag{16}$$

subject to: (i) equation (14), (ii) the household's interest rate-quantity schedule jointly defined by equation (9) and (10), (iii) each firm's interest rate-quantity schedule jointly defined by equations (12), (13). Bank b's profit π_b is given by:

$$\pi_b = \int \mathcal{I} \cdot r_l \cdot l_b \, \mathrm{d}\Phi + r_M M_b - r_D D_b - F. \tag{17}$$

Future market rates $r'_D(X)$ and $r'_l(x, X)$ adjust consistently with the interest rate-quantity schedules. Each bank b issues bank debt according to a generalized Euler equation:

$$1 = \mathcal{M}'(X, X') \cdot R'_D(X, X') \cdot (1 + \eta'_D(X, X')) \,, \tag{18}$$

where η'_D is the inverse elasticity $\frac{\partial R'_D}{\partial D'_b} \cdot \frac{D'_b}{R'_D}$ between debt and its rate.[16] In principle, equation (18) is a best response function that captures the trade-off that a bank faces issuing new debt. Every new unit of debt increases today financing capacity but needs to be repaid tomorrow at the contracted interest rate. Moreover, since η'_D is non-negative, when a bank issues new debt it is also increasing the market rate of deposits, incurring an additional future marginal cost. In equilibrium, η'_D is zero, as implied by equations (18) and (10). Without aggregate risk, households are indifferent to the financing structure of the banks.

A similar generalized Euler equation arises from the loans' first-order condition:

$$1 = \mathbb{E} \left[\mathcal{I}' \cdot \mathcal{M}'(X, X') \cdot R'_l(x, X, x', X') \cdot (1 + \eta'_l(x, X, x', X')) \mid (x, X) \right], \tag{19}$$

where $\eta'_l(x, X, x', X') \equiv \frac{\partial R'_l}{\partial l'_b} \cdot \frac{l'_b(x,X)}{R'_l(x,X,x',X')} < 0$ is the firm-specific inverse elasticity between loans and their rates. Note that equation (19) is a functional equation that depends on the idiosyncratic characteristics of each firm. See Appendix B.1 for details on how to calculate $\eta'_l(x, X, x', X')$. Equation (19) is a best response function that captures the trade-off that a bank faces issuing a new unit of loan to a specific firm. Every new unit of loan decreases the current bank's dividend but produces a marginal income tomorrow at the contracted interest rate. Moreover, since η'_l is non-positive, when banks issue new loans they are also decreasing the future market rate of loans, incurring a marginal loss in the future. Note that banks best respond internalizing the effects that their actions have on aggregate quantities and all firms' choices (e.g., if a bank changes the quantity of loan offered to a firm, that firm might decide to re-optimize and adopt a different capital structure as a function of the credit market conditions. Banks internalize all these effects in their decisions). Equation

[16] Note that since there is no aggregate risk in the model, problem 14 is differentiable since default will never occur in equilibrium. In order to induce default in equilibrium, I need to calibrate an unexpected shock that surprises the banks (but it is completely unforecast in the spirit of the MIT shocks).

(19), together with an Euler equation that regulates the banks' behavior on the interbank market

$$1 = \mathcal{M}'(X, X') \cdot R'_M(X, X'), \tag{20}$$

captures the decision making behavior of each bank. The outcome of the game played by the banks at time t is a contract that pins down the firm-specific intermediation margin $R'_l(x, X, x', X') - R'_D(X, X')$. In principle, this margin can be decomposed into: (i) firm-specific loan's intermediation margin $(R'_l(x, X, x', X') - R'_M(X, X'))$ and (ii) debt's intermediation margin $(R'_M(X, X') - R'_D(X, X'))$. Note that the debt intermediation margin is zero, since $\eta'_D = 0$, hence $R'_D(X, X') = R'_M(X, X')$.

3.4.2 New Entrants

A new bank b decides to entry if the following entry condition is satisfied

$$\mathcal{M}(X, X') \cdot V_b(X') \geq F_E, \tag{21}$$

where $B' = B + 1$ (recall that B' is contained in the vector X'). The entry of a new bank requires an initial capital injection from the household.

3.5 Oligopolistic Equilibrium

The government aggregate income from taxes is:

$$T = \tau \int \left(Zk^\alpha - \sum_{b=1}^{B} r_l(x, X) \cdot l_b(x, X) - \delta k \right) \mathrm{d}\Phi.$$

The aggregate resource constraint of the economy is:

$$C + \int i(x, X) + \lambda(x, X) \, \mathrm{d}\Phi + T = \int Zk^\alpha \, \mathrm{d}\Phi. \tag{22}$$

The total production of the economy on the right-hand side of equation (22) can: (i) be consumed by the household, (ii) be used for aggregate investment in physical capital (in case some dividends are negative, some resources are spent to pay the equity issuance cost λ), and (iii) or be paid in taxes. The inverse elasticity η'_D is zero and $\eta'_l(x, X, x', X')$ of equation (19), is calculated as described in Appendix B.1.

A formal definition of the notion of *Recursive Stationary Oligopolistic Equilibrium* is presented in Definition 3.1, and its extension to the dynamic case is discussed in Section 5.

Definition 3.1. A **Recursive Stationary Oligopolistic Equilibrium** is a Markov perfect equilibrium where i) the banks' debt holdings $\{D_b\}_{b=1}^B$ and the relative market rate R_D; ii) the banks' share holdings $\{S_b\}_{b=1}^B$ and the relative market prices $\{p_b\}_{b=1}^B$; iii) the interbank debt holdings $\{M_b\}_{b=1}^B$ and the relative market rate R_M; iv) the household's consumption C; v) the distribution $\phi(x)$; vi) the policy functions: $k'(x)$, $l'(x)$ and $R_l'(x)$; vii) the number of banks B are such that i) the household's problem is solved–i.e, equations (9)(10)(11) hold; ii) each firm's problem is solved–i.e, equations (12) and (13); iii) each incumbent bank is best responding to all other banks–i.e, equations (18), (19) and (20) hold; iv) there are no new banks who wish to enter the intermediation market–i.e, the inequality (21) is violated when evaluated with $B+1$ banks; v) and all markets clear: 1) the good market clears –i.e., equation (22) holds; 2) each bank's equity market clears –i.e., $\forall b : S_b = 1$; 3) each firm's equity market clears –i.e., $S(x) = 1$; 4) the interbank market clears –i.e., $\forall b : M_b = 0$.

4 Calibration

I now describe the choices of parameters values for the calibrated model, all values are summarized in Table I.

Table I. Parameter Values

		Parameter	Value	Target/Source
Household	Time Discount	β	0.9941	Match Deposit Rate
	Risk Aversion	γ	1	
Firms	Depreciation Rate	δ	0.03	
	Effective Capital Share	α	0.34	
	Corporate Tax Rate	τ	0.197	Effective Corp. Tax (OECD Tax Database)
	Default Rate	$1-\rho$	0.21%	Quarterly C&I Charge-off to Loan (FDIC)
	Equity Flotation Cost	λ_0	9	Internally calibrated (see Table II)
Banks	Fixed Cost to Entry	F_E	[0.427, 0.549]	Internally calibrated (see Table II)

Notes: The table reports the parameter values.

Household Preferences. A period in the model coincides with a quarter, consistent with the frequency in the data. I set $\beta = 0.9941$ to match a quarterly stationary bank's debt rate (or interbank debt rate) of 0.59%, which is consistent with the average of the 3-month T-bill rates calculated between 1997 and 2017, and reflects the prolonged period of low interest rates. I use a constant relative risk aversion utility function

$$u(C) = \begin{cases} \frac{C^{1-\gamma}}{1-\gamma} & \gamma \geq 0 \ \wedge \ \gamma \neq 1 \\ \log(C) & \gamma = 1 \end{cases},$$

where the parameter γ is the degree of relative risk aversion. I set γ equal to 1.

Firms. The quarterly depreciation rate δ and the effective capital share α are set to standard values of 3% and 0.34, respectively. Firms' income tax is set to 19.7%, which was the effective corporate income tax in the US in 2021. This taxation rate is also broadly consistent with the ratio between taxes on corporate income over corporate profit (both time series are retrievable from FRED) between 1997 to 2017. I use the the Commercial & Industrial Loans net charge-off rates to identify the risk of default, set to 0.21% (quarterly) consistent with the average of the time series between 1997 and 2017.[17] In Section 6, I propose and solve a model with idionsycratic TFP shocks and firms endogenous default decisions. I show that the baseline model adopted in this section, despite being fairly parsimonious, captures the effects of banks' market power in a relative conservative way.[18] The equity issuance cost λ_0 is calibrated to 9 to match an annualized frequency of equity issuance of 0.042. The frequency of equity issuance is computed from a sample of non-financial, unregulated firms from Compustat. I also fix a maximum age $\bar{N}$ in the life cycle of the firm. In particular, I consider an average lifespan of a company of 19.5 years, as calculated using the Standard and Poor's 500 Index.[19] Note that 19.5 years corresponds to 78 quarters; hence, $\bar{N}$ is such that:

$$\sum_{\text{age}=0}^{\bar{N}} \text{age} \cdot \rho^{\text{age}} = 78 \cdot \sum_{\text{age}=0}^{\bar{N}} \rho^{\text{age}}.$$

This yields a maximum age of $\bar{N} = 167$ quarters. In other words, the oldest firms in the model have an age of about 42 years. In each period, I also assume that a new mass of firms replaces the mass of firms that defaulted $1-\rho$, starting from age zero with zero capital and zero debt. I relax this assumption in Subsection 5.2.

[17] C&I charge-off rates are also broadly consistent with default rates reported by Moody's and Standard & Poor's.

[18] Intuitively, when banks' market power interact with firms' default decisions, in equilibrium, there is a higher mass of younger firms with a higher and less elastic demand for credit (from which banks can extract markups).

[19] This is consistent with an average of 19.6 years between 1997 and 2017 computed from the Standard and Poor's 500 Index available here: https://www.statista.com/statistics/1259275/average-company-lifespan/.

Banks. The fixed cost to entry the financial intermediation market F_E is calibrated to match the profitability of the banks, calculated as *net operating income* over *total interest income*, which can be obtained from the *Quarterly Income and Expense of FDIC-Insured Commercial Banks and Savings Institutions*.[20] In particular, I use the average of a quarterly time series from 1984 and 2018. In Table II, I report the corresponding aggregate stationary equilibrium moments. As explained, the first two moments are targeted by jointly controlling F_E and λ_0. More formally:

$$\min_{\{F_E,\ \lambda_0\}} \quad \left(\frac{\pi_b}{\int \mathcal{I} r_l(x,X) l_b d\Phi} - 0.1616\right)^2 + \left(4\int (d(x,X)<0)d\Phi - 0.042\right)^2,$$

where π_b is the bank's profit given by equation (17). All remaining moments are untargeted and reported for validation. Table II summarizes all targeted and untargeted moments. The calibrated model features, in equilibrium, an economy populated by 4 banks.

Table II. Stationary Equilibrium and Annualized Moments

Targeted	Description	Moment	Model	Data
Yes	Profit/Revenue	$\pi_b / \int \mathcal{I} r_l(x,X) l_b d\Phi$	15.15%	16.16%
Yes	Freq. of Equity Iss.	$4\int (d(x,X)<0)d\Phi$	4%	4.2%
No	Capital to GDP	$K/(4Y)$	2.1	2.2
No	Investment to K	$4I/K$	15%	16%
No	Debt Adjust. to K	$B\int 4\Delta l_b'(x,X)\,\mathrm{d}\Phi/K$	0.88%	0.62%
No	Market Leverage	$B\int l_b(x,X)/V_F(x,X)\,\mathrm{d}\Phi$	32.6%	34.3%
No	Num. Banks	B	4	-

Notes: This table reports the targeted and untargeted aggregated annualized moments. Capital and investment in the data are computed from current-cost net stock of private fixed assets, retrievable from the U.S. Bureau of Economic Analysis. Loans in the data are computed from the Commercial & Industrial Loans retrievable from FRED. Leverage in the data is computed as leverage of nonfinancial corporate business (debt as a percentage of the market value of corporate equities) also retrievable from retrievable from FRED. All moments in the data refers to the average of the respective yearly time series between 1997 and 2017.

Note that the banking sector features a dominant-fringe market structure. As an example, in 2018, the biggest four banks in terms of assets were: 1) JP Morgan Chase & Co with a market share of 14.45%, 2) Bank of America Corp. with a market share of 11.73%, 3) Wells Fargo & Company with a market share of 11.17%, and 4) Citigroup Inc. with a market share of 9.32%. The fifth bank was U.S. Bancorp with a significantly smaller market share of 3.02%, less than a third that of Citigroup Inc.[21]

[20] https://www.fdic.gov/analysis/quarterly-banking-profile/index.html.

[21] Not only the market for Commercial and Industrial Loans presents significantly high level of concentra-

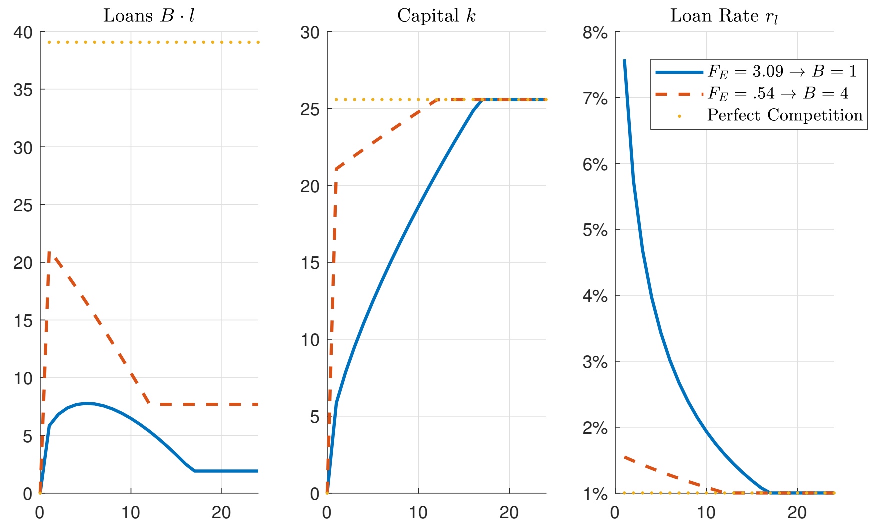
4.1 Firm Dynamics in the Stationary Equilibrium

I now describe the key properties of the stationary equilibrium of the calibrated model, with a greater focus on the role of strategic interactions. Figure 1 reports the life cycle of a firm in the stationary equilibrium. Firms can reach their capital objectives by (i) investing internal resources, (ii) issuing equity or (iii) demanding external financing resources on the loan market. A more concentrated banking sector reduces the credit availability in the economy. Firms with a high marginal productivity of capital and worse outside options, likely smaller or highly leveraged, exhibit a higher and less elastic credit demand. Therefore, this type of firm is more exposed to the negative effects of the lack of competition in the banking sector. This is the same intuition captured by equation (5) in the stylized model of Section 2.

Along the life cycle of firms, since markups are endogenous in the cross-section of firms, banks endogenously exert a higher degree of market power on firms with a high marginal productivity of capital and lower internal resources; hence, fewer outside options. These firms need banks' credits and would otherwise incur an equity issuance cost to finance their growth, in case their current production alone would not be sufficient to sustain the desired level of physical investment. An imperfectly competitive financial sector internalizes that the same financial resources are more valuable for firms with a higher marginal productivity of capital and fewer outside options (e.g., a higher equity issuance cost) and, therefore, can charge higher markups. This creates a mechanism of endogenous financial friction, as captured by the central panel of Figure 1. Through this mechanism, the lack of competition in the financial sector not only induces credit misallocation that forces firms to grow slower, but also induces lower aggregate productivity. This outcome is illustrated in Figure C1. The mechanism is consistent with the idea that the firms more reliant on bank credit are the most affected by the lack of competition, e.g. small and private firms (Holmstrom and Tirole, 1997, Diamond and Rajan, 2005, Chodorow-Reich, 2013, Saunders, Spina, Steffen, and Streitz, 2022), whereas firms with outside funding options, e.g. public bond markets, are less susceptible to bank credit market frictions (Chava and Purnanandam, 2011).

tion but also, among others, the market for Syndicated Loans. Although I do not model complex features of loan syndications, the reader can think to the big (non-atomistic) intermediaries introduced in my model as the arrangers of the syndication (whose negotiate with the firms quantities and prices).

Figure 1. Stationary Equilibrium and Firms' Life Cycle



Notes: This figure reports the equilibrium policies for loan quantity (left panel) and loan interest rate (right panel), along the life cycle of a firm in the stationary equilibrium. Younger firms need financing resources to reach their capital objectives, and since markups are endogenous in the cross-section, banks naturally extract higher markups from this type of firms, characterized by a higher marginal productivity of capital and low internal resources. This creates a mechanism of *endogenous financial friction*, as captured by the central panel. X-axes report the firms' age.

Figure C2 in Appendix C.1 reports the inverse elasticities $\eta_l'(x, X, x', X')$ contained in the generalized Euler equation (19), along the life cycle of a firm in the stationary equilibrium. These elasticities, which are endogenous and depend on current and future firm-level characteristics, are directly linked to the equilibrium trajectories of markups. A lower inverse elasticity translates into a higher financial markup. Furthermore, Figure C2 in Appendix C.1 shows that the higher the concentration of the banking sector, the lower the inverse elasticities and the longer it takes for a firm to reach its efficient level of capital. Under perfect competition, the inverse elasticity would be constant in the cross-section of firms (long-lived firms still experience a non-zero elasticity because of the tax shield). A more concentrated banking sector can extract higher rents out of financially constrained firms with a high marginal productivity of capital. This mechanism endogenously creates slower growth trajectories as a function of the banks' market structure, as shown in the central panel of Figure 1. The intensity of this mechanism is time-varying, acting as an amplification channel in the transition paths of the macroeconomic shocks reported in Section 5.

4.1.1 Relationship to the Empirical Literature

It is important to highlight that an extensive empirical literature has investigated the impact of bank competition on the cross-section of firms. Early seminal contributions include Rajan and Zingales (1998); Black and Strahan (2002); Cetorelli and Gambera (2001); and Cetorelli and Strahan (2006). Although evidence is mixed, the general consensus is that, consistent with my model, banks' market power reduces the total amount of credit available in the economy, but importantly this effect is not constant across firms. This same literature has suggested that two countervailing forces might be in play (potentially yielding mixed results).

On the one hand, younger firms exhibit higher credit demand and, therefore, are more exposed to the negative effects of lack of competition in the banking sector than established firms. This is the classic industrial organization perspective (see, Freixas and Rochet, 1997) and is fully captured by my model. On the other hand, a more concentrated banking sector has an incentive to sustain one of its established clients and refrain from extending credit to young firms. The less competitive the conditions in the credit market, the lower the incentive for lenders to finance newcomers as documented by Petersen and Rajan (1995) and Cetorelli and Strahan (2006). Although my model does not focus on capturing complex features of relationship banking, the desire for inter-temporal smoothing of banks' profits and households' consumption – embedded in the dynamic contract of equations (12), (13), and (19) – captures the fact that creditors, when contracting markups, take into account the expected stream of future dividends of the firms, as well as their own future profits; hence, in my model banks balance markups intertemporally in the spirit of relationship lending. This important economic force is consistent with Petersen and Rajan (1995) and Cetorelli and Strahan (2006).

Notably, Burga and Céspedes (2022) use a dataset of small firms from Peru to estimate the effect of banks' market power. In particular, they exploit a merger episode and find that the change in banks' market structure results in "a contraction of capital among small firms with high marginal returns, which increases capital misallocation," in agreement with the predictions of (i) the central panel of Figure 1 and (ii) the dispersion of marginal products of capital, $\sigma(r_L)$, reported on the left-axis of Figure C1 in Appendix C.1.

5 Aggregate Shocks

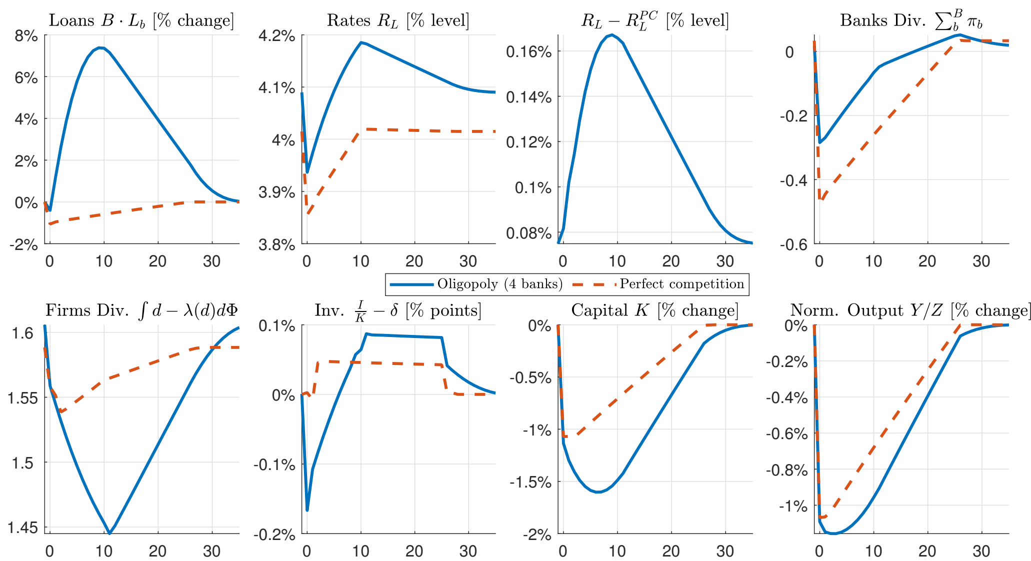
This section analyzes the role that banks' market power plays in the transmission of macroeconomic shocks. The section includes three unexpected aggregate shocks: (i) a credit quality deterioration (in the model, a decrease to aggregate TFP combined with an increase to the aggregate firms' default probability), (ii) a temporary change to the bank market structure (in the model, a decrease to aggregate TFP combined with an increase to the aggregate firms' default probability calibrated to push one bank to default), and (iii) a permanent change to the bank market structure (in the model, a decrease to aggregate TFP combined with an increase to the aggregate firms' default probability and a permanent increase to the fixed cost F_E to enter the banks' market, calibrated to push one bank to default without a subsequent new bank reentry on the equilibrium path).

I compute the transitional dynamics of the model initialized at the stationary equilibrium as defined in Section 4. Then, I hit the economy with the unexpected aggregate shocks and, depending on the type of shock, the economy converges to the old or a new stationary equilibrium in the long run. Several papers assume agents did not foresee the aggregate shocks of the Great Recession (e.g., Guerrieri and Lorenzoni, 2017). Along the transitional dynamics, after the shock, I assume all agents can perfectly foresee the paths of all aggregate variables. In order to compute the equilibrium dynamics, I find sequences of: (i) aggregate consumption $\{C_t\}_{t=0}^{T}$, and (ii) firms' distributions $\{\phi_t(x_t)\}_{t=0}^{T}$; such that households maximize utilities, all markets clear in each period and the firms' distributions evolve according to: (i) the firms' policy functions, (ii) the banks' generalized Euler equations (19) and (iii) the idiosyncratic default shocks. See Appendix A for additional computational details. The main computational challenge is to solve for an equilibrium path simultaneously characterized by general equilibrium, banks' strategic interactions, and heterogeneous firms: (i) firms' decisions are affected by other firms' decisions through the aggregate variables, (ii) banks' decisions are affected by each single firm's decisions (banks issue idiosyncratic firm-level optimal contract), aggregate variables (banks internalize the effects that their actions have on aggregate dividends), and other banks' decisions (each bank is best responding to other banks).

5.1 Credit Quality Deterioration

This section investigates the effects of banks' market power when the economy is hit at time $t = 0$ by an unexpected negative aggregate TFP shock of 1.25% (in correspondence of which the quarterly firms' survival probability ρ decreases by 0.35%) as shown in Figure C3 in Appendix C.1. The shock is calibrated to be half the size of the Great Recession (see Figure C3 in Appendix C.1 that reports both the dynamic of the shock in the model and the corresponding dynamic in the data during the Great Recession). Figure 2 reports the dynamic responses calculated (i) with the calibrated oligopolistic banking sector of Section 4 (solid line) and (ii) with the corresponding perfectly competitive banking sector (dashed line).

Figure 2. Credit Quality Shock, Financing, and Real Activity



Notes: This figure reports the transitional dynamics of: (i) the aggregate loans $B \int l_{b,t}(x_t, X_t)\, \mathrm{d}\Phi_t$, (ii) the annualized aggregate interest rate $\int r_{l,t}(x_t, X_t)\, \mathrm{d}\Phi_t$, (iii) the annualized aggregate interest rate spread calculated as the difference between the annualized aggregate interest rate $\int r_{l,t}(x_t, X_t)\, \mathrm{d}\Phi_t$ under the calibrated oligopoly and the annualized aggregate interest rate $\int r_{l,t}^{\mathrm{PC}}(x_t, X_t)\, \mathrm{d}\Phi_t^{\mathrm{PC}}$ under perfect competition, (iv) the aggregate banks dividend $\sum_{b=1}^{B_t} \pi_{b,t}$, (v) the annualized aggregate firms dividend $\int d_t(x_t, X_t) - \lambda(d_t(x_t, X_t))\, \mathrm{d}\Phi_t$, (vi) the aggregate physical investment $(\int i_t(x_t, X_t)\, \mathrm{d}\Phi_t)/(\int k_t\, \mathrm{d}\Phi_t)$ net of depreciation (quarterly), (vii) the aggregate capital $\int k_t\, \mathrm{d}\Phi_t$ and (viii) the aggregate output (normalized to remove the exogenous component Z_t to isolate the effects on the endogenous component) $\int k_t^{\alpha}(x_t, X_t)\, \mathrm{d}\Phi_t$, following the TFP shock reported in Figure C3 in Appendix C.1. X-axes report time t.

As discussed in Section 4.1, a more concentrated banking sector can extract higher rents out of the financially constrained firms with a high marginal productivity of capital. In the stationary equilibrium, this mechanism endogenously creates slower growth trajectories as a function of the banks' market structure. In the dynamics, this mechanism of endogenous financial frictions interacts with the higher density of financially constrained firms, generating a higher credit demand that drives up markups as shown in Figure 2.[22] Therefore, during the transitional dynamics, interest rates rise more under oligopolistic competition than under perfect competition, as shown in Figure 2. Moreover, total loans rise under oligopolistic

[22]This is consistent with the idea that firms more dependent on external funding via bank loans, such as small and private firms, can become more financially constrained when credit conditions tighten (Holmstrom and Tirole, 1997; Diamond and Rajan, 2005; Chodorow-Reich, 2013; Saunders, Spina, Steffen, and Streitz, 2022).

competition. Banks exploit their market power not only to extract higher markups, but also to prevent their profits from falling as much as in the perfectly competitive case.[23]

In summary, when credit quality deteriorates, a concentrated banking sector exploits its market power to extract higher interest rates as shown in Figure 2. This mechanism induces a larger decline in real activity in terms of aggregate investment, capital, and output. Finally, note that the decline in real activity captured by Figure 2 further constrains firms by restraining households' capacity to support firms with equity issuance (the aggregate firms' dividend declines). This mechanism creates a vicious cycle that further reduces the interest rate-quantity loan elasticities $\eta_l'(x, X, x', X')$ and boosts banks' interest rates. Throughout the paper, I assume that the defaulting mass of firms is replaced, at each date t, by an equal mass of new entrant firms. In Subsection 5.2, I relax this assumption.

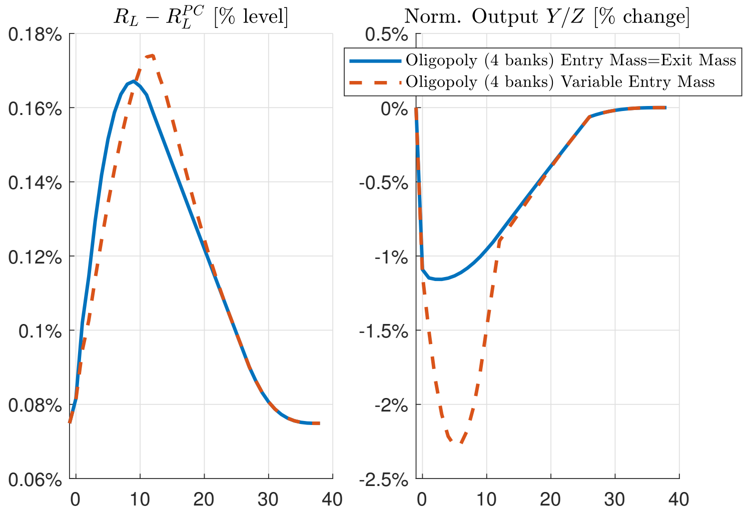
5.2 The Effects of the Firms' Entry Rate

During the Great Recession, the firms' default rate increases but not all firms immediately re-enter the market. In this section, I let the mass of exiting firms evolves according to the evolution of the default rate in the shock reported in Figure C3. Differently from before, I keep the entering mass of firms initially lower than the exit mass of firms and higher after 10 periods. The entry rate, along the shock, is calibrated so that the mass of firms in the production sector drops in the short-run by about 1 percent and returns to 1 in the long-run. In Figure 3, I refer to this type of shock with the label *Variable Entry Mass*. In contrast, the label *Entry Mass = Exit Mass* refers to the shock already analyzed in the previous Section 5.1, whose effects on real activity are reported in Figure 2.

Figure 3 suggests that, when the firms' default rate increases but not all firms immediately re-enter the market, then imperfect competition in the financial intermediation sector leads to a bigger and delayed amplification effect at the peak that fades away as new firms enter the production sector. The intuition is linked to the economic mechanism discussed in the Section 5.1. Banks can extract higher interest rates for longer as firms slowly re-enter the production sector (as shown by the left panel of Figure 3). A more concentrated banking sector extracts higher rents out of this financially constrained firms that slowly re-enter the market and need credits. Output eventually converges back to its $t = -1$'s level as the mass of producing firms reverts back to 1.

[23] When the shock hits, banks incur large losses and require financing resources from the household (i.e., a negative bank dividend). Note that the shock is calibrated in such a way that there is no bank default in equilibrium since $\forall b : \tilde{V}_b(X_0) > 0$.

Figure 3. Credit Quality Shock and Firms' Entry



Notes: This figure reports the transitional dynamics of (i) the aggregate interest rate spread and (ii) the aggregate output (normalized to remove the exogenous component Z_t to isolate the effects on the endogenous component) $\int k_t^{\alpha}(x_t, X_t)\,\mathrm{d}\Phi_t$ following the TFP shock reported in Figure C3 in Appendix C.1 with and without the assumption *Entry Mass = Exit Mass*. The evolution over time of the aggregate interest rate spread is calculated as the difference between (i) the annualized aggregate interest rate $\int r_{l,t}(x_t, X_t)\,\mathrm{d}\Phi_t$ under the calibrated oligopoly and (ii) the annualized aggregate interest rate $\int r_{l,t}^{\mathrm{PC}}(x_t, X_t)\,\mathrm{d}\Phi_t^{\mathrm{PC}}$ under perfect competition. X-axes report time t.

5.3 Changes in Bank Market Structure

One salient ingredient of my framework is the presence of non-atomistic financial intermediaries.[24] Thanks to this feature, I can study market structure changes, such as a bank failure, and a bank reentry. In this section I calibrate a TFP shock to the Great Recession. The economy is hit at time $t = 0$ by an unexpected negative aggregate TFP shock of 2.5% (in correspondence of which the quarterly firms survival probability ρ decreases by 0.7%) as shown in Figure C3 in Appendix C.1. Figure C3 in Appendix C.1 reports both the dynamic of the shock in the model and the corresponding dynamic in the data during the Great Recession. This same shock pushes one bank to default in the model at time $t = 0$. In Subsection 5.4, I report the resulting equilibrium path which includes a new bank entry at time $t = 15$. In Subsection 5.5, I combine the shock of Subsection 5.4 with a permanent increase in the fixed cost to entry F_E, so that it is not profitable for a new bank to enter the credit market along the equilibrium path.[25] For example, not only during the Great Reces-

[24] Through the lens of the model, a big bank is a financial firm with non-zero mass.

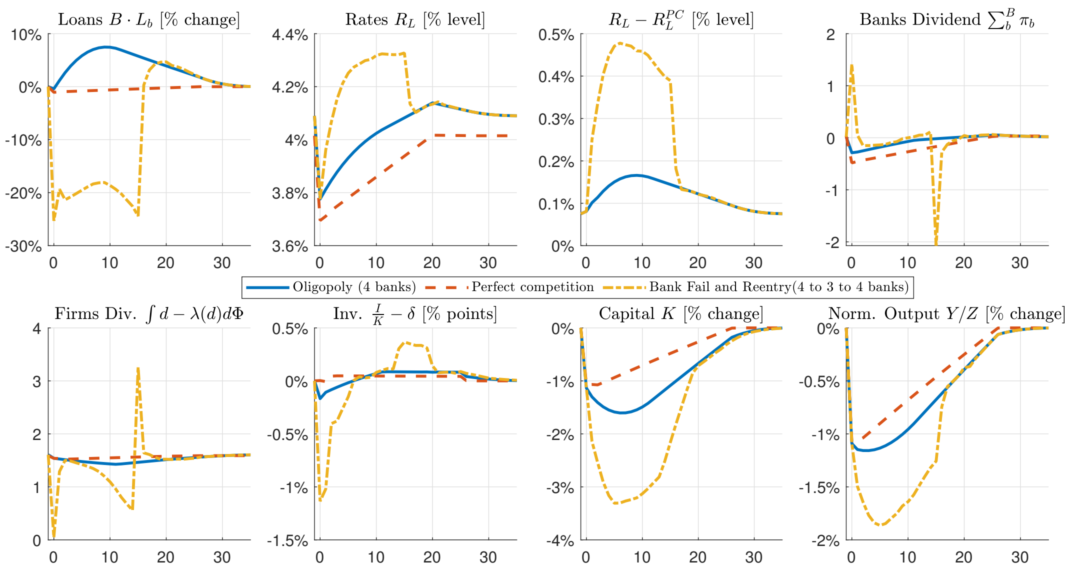
[25] The permanent increase to F_E is in the spirit to capture the long run trend of consolidation of the banking sector.

sion there were numerous bank failures, bank bailouts, and bank mergers (e.g., bankruptcy of Lehman Brothers, the collapse of Washington Mutual and the acquisition of Wachovia by Wells Fargo in 2008), but also a sharp collapse of bank entry since 2009.

5.4 Bank Failure and Banks' Market Power

In this subsection, the fixed cost to entry F_E is held fixed throughout the dynamics, allowing one bank to enter the credit market along the equilibrium path. At $t = 0$, when the shock hits, one incumbent bank decides to exit since $\tilde{V}_b(X_0) < 0$ and the market structure changes temporarily from 4 banks to 3 banks.

Figure 4. Bank Default, Reentry, Financing, and Real Activity



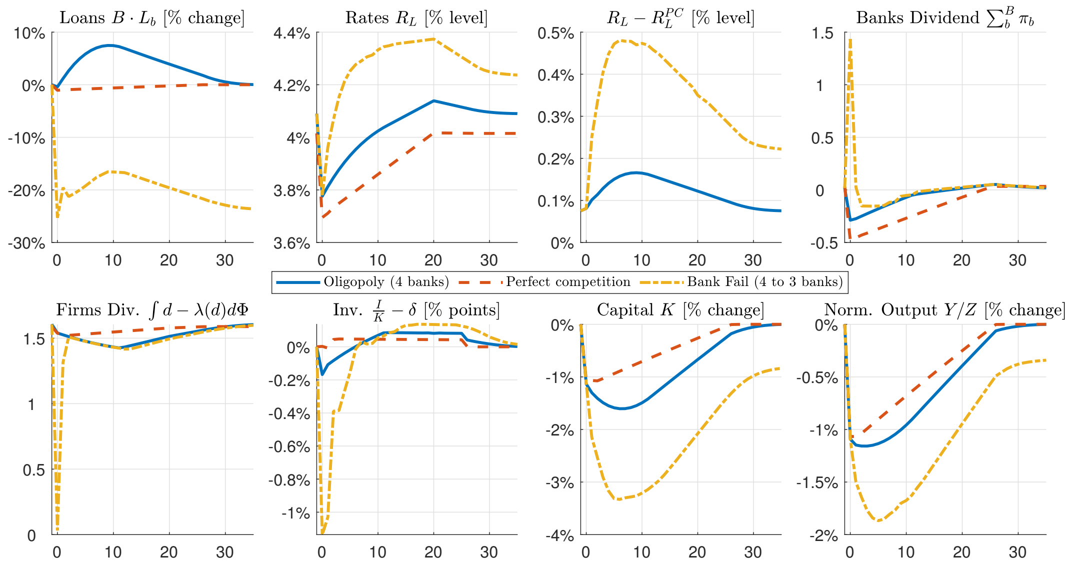
Notes: This figure reports the transitional dynamics of: (i) the aggregate loans $B \int l_{b,t}(x_t, X_t)\, \mathrm{d}\Phi_t$, (ii) the annualized aggregate interest rate $\int r_{l,t}(x_t, X_t)\, \mathrm{d}\Phi_t$, (iii) the annualized aggregate interest rate spread calculated as the difference between the annualized aggregate interest rate $\int r_{l,t}(x_t, X_t)\, \mathrm{d}\Phi_t$ under the calibrated oligopoly and the annualized aggregate interest rate $\int r^{\mathrm{PC}}_{l,t}(x_t, X_t)\, \mathrm{d}\Phi^{\mathrm{PC}}_t$ under perfect competition, (iv) the aggregate banks dividend $\sum_{b=1}^{B_t} \pi_{b,t}$, (v) the annualized aggregate firms dividend $\int d_t(x_t, X_t) - \lambda(d_t(x_t, X_t))\, \mathrm{d}\Phi_t$, (vi) the aggregate physical investment $(\int i_t(x_t, X_t)\, \mathrm{d}\Phi_t)/(\int k_t\, \mathrm{d}\Phi_t)$ net of depreciation (quarterly), (vii) the aggregate capital $\int k_t\, \mathrm{d}\Phi_t$ and (viii) the aggregate output (normalized to remove the exogenous component Z_t to isolate the effects on the endogenous component) $\int k^{\alpha}_t(x_t, X_t)\, \mathrm{d}\Phi_t$, following the TFP shock reported in Figure C4 in Appendix C.1. X-axes report time t. At $t = 0$, when the shock hits, one incumbent bank decides to exit since $\tilde{V}_b < 0$. At $t = 15$, it is profitable for one bank to enter the market given the fixed cost F_E held fixed at its stationary equilibrium value. Note that since the 4 banks are symmetric, it is immaterial for the equilibrium path which bank exits.

At $t = 15$, it is profitable for a new bank to enter the credit market given the fixed cost F_E and the market structure changes back from 3 banks to 4 banks. Figure 4 reports the dynamics. At the beginning of date t, when the market structure change occurs, firms that do not default repay their entire amount of loans outstanding to each bank. All loans are repaid before the market structure change, since agents trade one-period securities. Hence, there is a market structure change. After the change, all agents make decisions according to the new structure of the economy. When a big bank fails, the surviving bank starts to slowly extend more credit to firms in order to partially capture the market share of the defaulted bank. However, the speed of this adjustment is dampened by the decreased level of competition among surviving banks. The surviving banks' market power interacts with credit constraints, yielding a sharp drop in the aggregate volume of credit in the short run with interest rates that quickly increase after the initial drop. Moreover, banks lower credit supply in anticipation of the new bank's entry. The credit crunch propagates into the real economy, yielding a sharp and persistent drop in investment; hence physical capital and output. The new bank's entry requires an initial equity injection from the household (i.e., a negative bank's dividend). The entry of a new bank leads to a higher availability of credits and lower interest rates (as shown by Figure 4), which boost the firms' aggregate dividend. Following the new bank's entry there is a temporary boost in investment in physical capital, which push the dynamics of capital and output back to their respective pre-default levels.

5.5 A Permanent Change in Bank Market Structure

When the market structure of the banking sector changes permanently, the model captures two ideas: (i) in the short term, the effects of market power of the surviving bank contributes to lowering the supply of credit to firms, further slowing down the economy and (ii) in the long run, the heightened banks' market power further contributes to amplifying and prolonging the recession. Because of the general equilibrium effects and the reduced level of competition, in the long run, the economy stabilizes at a lower level of volume of credit, which results in less investment, capital, output, productivity, and more credit and capital misallocation (as shown by the Subsection 5.6). In this sense, my analysis suggests that banks' market power may be an important source of concern for policymakers deciding whether to bail out a big bank.

Figure 5. Bank Default, Financing, and Real Activity

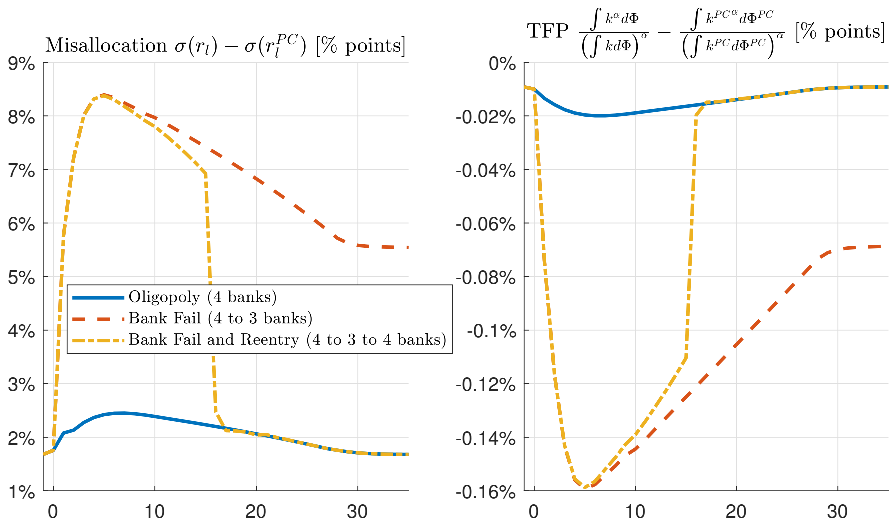


Notes: This figure reports the transitional dynamics of: (i) the aggregate loans $B \int l_{b,t}(x_t, X_t)\, \mathrm{d}\Phi_t$, (ii) the annualized aggregate interest rate $\int r_{l,t}(x_t, X_t)\, \mathrm{d}\Phi_t$, (iii) the annualized aggregate interest rate spread calculated as the difference between the annualized aggregate interest rate $\int r_{l,t}(x_t, X_t)\, \mathrm{d}\Phi_t$ under the calibrated oligopoly and the annualized aggregate interest rate $\int r_{l,t}^{\mathrm{PC}}(x_t, X_t)\, \mathrm{d}\Phi_t^{\mathrm{PC}}$ under perfect competition, (iv) the aggregate banks dividend $\sum_{b=1}^{B_t} \pi_{b,t}$, (v) the annualized aggregate firms dividend $\int d_t(x_t, X_t) - \lambda(d_t(x_t, X_t))\, \mathrm{d}\Phi_t$, (vi) the aggregate physical investment $(\int i_t(x_t, X_t)\, \mathrm{d}\Phi_t)/(\int k_t\, \mathrm{d}\Phi_t)$ net of depreciation (quarterly), (vii) the aggregate capital $\int k_t\, \mathrm{d}\Phi_t$ and (viii) the aggregate output (normalized to remove the exogenous component Z_t to isolate the effects on the endogenous component) $\int k_t^{\alpha}(x_t, X_t)\, \mathrm{d}\Phi_t$, following the TFP shock reported in Figure C4 in Appendix C.1. X-axes report time t. At $t = 0$, when the shock hits, one incumbent bank decides to exit since $\tilde{V}_b < 0$. Note that since the 4 banks are symmetric, it is immaterial for the equilibrium path which bank exits.

5.6 Dispersion of Loan Rates and Aggregate TFP

The dynamic financial oligopoly combined with heterogeneous firms generates firm-level endogenous financial frictions that create time-varying second moments, such as the dispersion of loan rates (directly linked to the dispersion of marginal products of capital) and aggregate TFP. The left panel of Figure 6 reports the dynamic of the standard deviation of loan rates, expressed in percentage levels and the right panel of Figure 6 reports the associated aggregate TFP (calculated as the residual of an aggregate production $Y_t = \mathrm{TFP}_t \cdot K_t^{\alpha}$). Both measures are calculated as difference from the perfectly competitive benchmark.

Figure 6. Market Structure Shock and Credit Misallocation



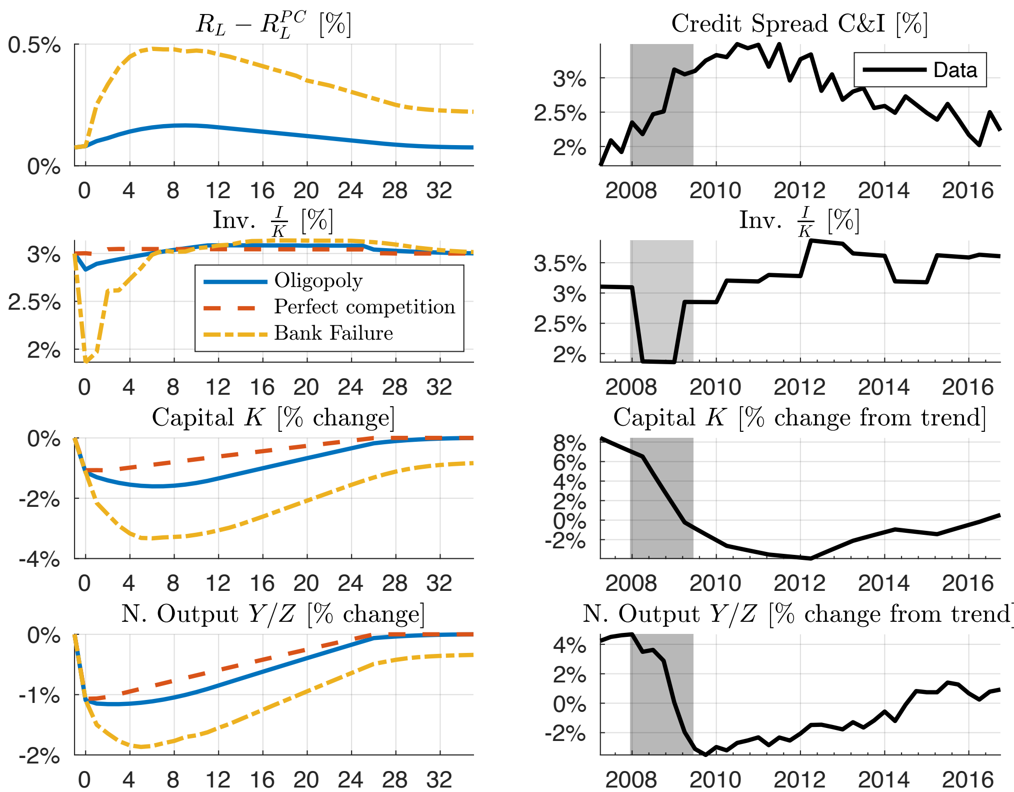
Notes: This figure reports the transitional dynamics of the dispersion of loans' interest rates, calculated as the square root of $\int r_{l,t}^2(x_t, X_t)\,\mathrm{d}\Phi_t - (\int r_{l,t}(x_t, X_t)\,\mathrm{d}\Phi_t)^2$, and aggregate TFP, calculated as $\int k_t^{\alpha}(x_t, X_t)\,\mathrm{d}\Phi_t / (\int k_t(x_t, X_t)\,\mathrm{d}\Phi_t)^{\alpha}$, following the unexpected shock reported in Figure C4 in Appendix C.1. Both measures are calculated as percentage points differences with the perfectly competitive benchmark. X-axes report time t.

In agreement with empirical evidence (e.g., Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry, 2018 and David, Schmid, and Zeke, 2022), the model produces a dynamic with an increasing dispersion of loan rates during recessions; hence, an increasing dispersion of marginal productivity of capital which, in turn, shapes the dynamic behavior of aggregate TFP. Figure 6 suggests that, after the failure of a large player, banks' market power contributes significantly to the misallocation of credits (hence, dispersion of marginal products of capital) and induces a persistent decline in aggregate TFP.

5.7 Comparison with the Great Recession

Figure 7 shows that the model dynamics of Figures 4 and 5 are broadly consistent with those of the Great Recession. For example, in the aftermath of the financial crisis C&I credit spreads increased at the peak by 1.5% (annualized). As shown by the top-left panel, the model suggests that, at the peak, approximately 0.18% and 0.50% of these credit spreads are attributable to financial markups (calculated with and without bank default respectively). Moreover, the model captures a drop in investment rate similar in magnitude to that of the Great Recession, and a persistent drop in capital (around 3% at the peak) and output (around 2% at the peak). The resulting increase in banks' market power amplifies and prolongs the recession, which is suggestive that banks' market power may have played a significant role in amplifiying and prolonging the crisis.

Figure 7. SHOCK II AND THE GREAT RECESSION



Notes: This picture compares the model dynamics of Section 5.5 with the data in proximity of the Great Recession. In particular, the right column reports: i) C&I credit spread, ii) quarterly investment rate, iii) capital (expressed in % deviation from the linear trend calculated from 1997:Q2 to 2017:Q2), and vi) normalized output (also expressed in % deviation from the linear trend calculated from 1997:Q2 to 2017:Q2). X-axes report time t, expressed in quarters for the model (left column).

6 Bank Market Power and Endogenous Firm Default

In this section, I consider an extension of the model in which banks' market power interacts with firms' default decisions. Banks strategically internalize the effects of their decisions on the current and future firms' default choices, while strategically interacting with the other banks. At a firm level, I introduce idiosyncratic TFP shock z so that each firm j is characterized by $x(j) \equiv \{\{l_b(j)\}_b^B, r_l(j), k(j), z(j)\}$. This problem is particularly challenging because it exhibits a non-differentiable dynamic demand for loans, rendering the Markov Perfect equilibrium non-differentiable and the generalized Euler equations approach not viable. In order to deal with the complexity and characterize the banks' behavior with generalized

Euler equations, I transform the firm's problem by approximating the max function implicitly contained in each firm's default choice $\mathcal{I}(x, X)$ with a smooth maximum function called softmax (therefore, the discrete choice $\mathcal{I}$ is approximated with a softargmax function, see for instance Goodfellow, Bengio, and Courville, 2016). In this section, I provide the expression for the generalized Euler equation that captures the interaction between banks' market power and firms' default decisions. Appendix A.3 discusses the solution algorithm in detail.

6.1 Firms

In each period t, the output $y_t(j)$ produced by each firm $j \in [0, 1]$ is given by the production function $y_t(j) = z_t(j) \cdot k_t(j)^\alpha$, where $\log z_t = \rho_z \log z_{t-1} + \xi_t$, $\xi_t \sim \mathcal{N}(0, \sigma_\xi^2)$, and $0 \le \rho_z < 1$. The value function $V_F(x, X)$ of a firm satisfies the following functional equation:

$$\tilde{V}_F(x, X) \; = \max_{\{l'_b\}_b^{B'}, k'} \; d - \lambda(d) + \mathbb{E}\left[\mathcal{M}' \cdot V_F(x', X')|(x, X)\right], \tag{23}$$

where, at the beginning of each period, each firm decides whether to default or not $\mathcal{I} = \{0, 1\}$, so that the function $V_F(x, X)$ satisfies the following functional equation

$$V_F(x, X) \; = \max\left\{0, \tilde{V}_F(x, X)\right\} = \max_{\mathcal{I}=\{0,1\}} \; \mathcal{I} \cdot \tilde{V}_F(x, X) + (1 - \mathcal{I}) \cdot 0. \tag{24}$$

All firms' constraints are identical to those specified in Section 3, with the exception of dividend, which contains a fixed exogenous cost χ calibrated to match the aggregate default rate. Note that the max function can be approximated by using the following differentiable function:

$$V_F(x, X) = \max(0, \tilde{V}_F(x, X)) \simeq \frac{1}{N} \ln\left(e^{N \cdot \tilde{V}_F(x, X)} + 1\right),$$

for a large N. The derivative of this smooth approximation of the maximum function (called softmax) yields the following approximation for $\mathcal{I}$ (called softargmax):[26]

$$\tilde{\mathcal{I}}(\tilde{V}_F(x, X)) = \frac{e^{N \cdot \tilde{V}_F(x, X)}}{e^{N \cdot \tilde{V}_F(x, X)} + 1}.$$

[26]This function is also called logistic or sigmoid function, typically used as activation functions in the neurons of a neural network. Note also that is similar in spirit to a logit.

The first-order condition with respect to k' is:

$$1-\lambda_d(d)=\mathbb{E}\left[\tilde{\mathcal{I}}(\tilde{V}_F')\cdot\mathcal{M}'(X,X')\cdot\Big(1+(1-\tau)(z'\alpha k'^{\alpha-1}-\delta)\Big)\cdot(1-\lambda_d(d'))|(x,X)\right]. \tag{25}$$

The first-order condition with respect to l_b' is:

$$1-\lambda_d(d)=\mathbb{E}\left[\tilde{\mathcal{I}}(\tilde{V}_F')\cdot\mathcal{M}'(X,X')\cdot(1+(1-\tau)r_l')\cdot(1-\lambda_d(d'))|(x,X)\right]. \tag{26}$$

The softargmax approach yields expressions (25) and (26), which are key to derive generalized Euler equations which, in turns, are essential to characterize the banks behavior in a computationally tractable way.

6.2 Banks

The decision problem for a new entrant is identical to that in the baseline model. The decision problem for an incumbent now leads to modified generalized Euler equations that take into account the sensitivity of banks' decisions on firms' default decisions. Similarly to the baseline model, given other banks contracts $\{D_{-b}', r_D'\}$ and $\{l_{-b}'(x,X), r_l'(x,X)\}$, a bank b best responds with a contract $\{D_b', r_D'\}, \{l_b'(x,X), r_l'(x,X)\}$ that satisfies the functional equations (15) and (16) subject to: (i) equation (14), (ii) the household's interest rate-quantity schedule jointly defined by equations (9) and (10), (iii) each firm's interest rate-quantity schedule which, in the extension, is jointly defined by equations (25) and (26).

Each bank's best response function satisfies the following generalized Euler equation:

$$1=\mathbb{E}\Bigg[\mathcal{M}'\cdot\left(\mathcal{D}(x,X,x',X')+\tilde{\mathcal{I}}(x',X')\cdot\left(1+\frac{l_b'}{R_l'}\frac{\partial R_l'}{\partial l_b'}\right)\right)\cdot R_l'|(x,X)\Bigg]. \tag{27}$$

This equation further generalizes equation (19). When firms' default decisions are endogenous, a bank internalizes the impact that an additional unit of loan has on each firm's default decision according to

$$\forall z':\quad \mathcal{D}(x,X,x',X')\equiv\frac{\mathrm{d}\tilde{\mathcal{I}}(\tilde{V}_F(x',X'))}{\mathrm{d}\tilde{V}_F}\cdot\left[\tilde{V}_{F,l_b}(x',X')\cdot l_b'+\tilde{V}_{F,R_l}(x',X')\frac{\partial R_l'}{\partial l_b'}\cdot l_b'\right].$$

Note that when firms' default is treated exogenously, the term $\frac{\mathrm{d}\tilde{\mathcal{I}}(\tilde{V}_F(x',X'))}{\mathrm{d}\tilde{V}_F}$ is zero and equation (27) collapses to equation (19). The term $\mathcal{D}(.)$ captures the strategic behavior of a bank that internalizes the marginal effects that an additional unit of loans has on the firm's future equity value and future default decisions. The model is calibrated as in Table I, with the addition of $\rho_z = 0.9$, $\sigma_\xi = 0.2$, and a fixed cost $\chi = 0.058$ calibrated to have in equilibrium a quarterly default rate of 0.21% (or 0.84% annualized). For the sake of comparison with the baseline model, I keep the level of competition fixed and I back out the fixed cost to enter the financial intermediation market so that there are 4 banks in equilibrium also in the extension. Table III reports the salient aggregate moments of the stationary equilibrium.

Table III. Stationary Equilibrium and Annualized Moments

Description	**Moment**	**Model**		**Data**
		Baseline	*Extension*	
Profit/Revenue	$\pi_b / \int \mathcal{I} r_l(x,X) l_b d\Phi$	15.15%	29.37%	16.16%
Freq. of Equity Iss.	$4 \int (d(x,X) < 0) d\Phi$	4%	6.8%	4.2%
Freq. of Default	$4(1\text{-}\int \mathcal{I}(x,X) d\Phi)$	0.84%	0.88%	0.84%
Capital to GDP	$K/(4Y)$	2.1	2.16	2.2
Investment to K	$4I/K$	15%	15%	16%
Debt Adjust. to K	$B \int 4\Delta l_b'(x,X)\, \mathrm{d}\Phi / K$	0.88%	0.99%	0.62%
Market Leverage	$B \int l_b(x,X)/V_F(x,X)\, \mathrm{d}\Phi$	32.6%	35.9%	34.3%
Num. Banks	B	4	4	-

Notes: This table compares the model with idionsycratic TFP shocks and enodgenous default with the baseline model.

Table III shows that the addition of idiosyncratic TFP shocks and endogenous default renders the banks more profitable (29.37% versus 15.15%), while keeping the level of competition fixed (4 banks in equilibrium). Moreover, firms tend to issue equity more frequently in the extension, which is a symptom of the higher level of market power of the financial intermediaries. In other words, firms prefer on average to pay a higher cost in equity issuance, instead of borrowing from the financial intermediaries. This is intuitive: in the extension younger firms are endogenously riskier hence, in equilibrium, there is a higher mass of younger firms with a high and less elastic demand for credit. This suggests that the relative parsimonious baseline model is fairly conservative in estimating the effects of banks' market power.

7 Conclusion

Motivated by a secular increase in the concentration and markups of the US banking industry, and by evidence that the effects of banks' market power are not constant across firms, this paper makes three contributions. First, I develop a new macroeconomic model that incorporates oligopolistic banks and heterogeneous firms that formalizes the idea that banks' market power has different effects along the life cycle of a firm. Through the lens of the model, I can study the equilibrium effects of banks' market power on firm dynamics revealing a mechanism of endogenous financial frictions through which bank competition plays a role in shaping the speed at which firms grow, impacting aggregate productivity and output. More precisely, limited competition enables banks to price discriminate and charge firm-specific markups, exerting a higher degree of market power on productive young firms that are more financially constrained. The resulting dispersion of markups induces credit – and thus capital — misallocation reducing aggregate productivity. Second, the model suggests that the time-varying effects of the dispersion of markups amplify the impacts of macroeconomic shocks. During a crisis, banks exploit the higher number of financially constrained firms to extract higher markups, inducing a larger decline in real activity. When a big bank fails, the remaining banks use their increased market power to restrict the supply of credit, worsening and prolonging the downturn. My analysis offers a different angle, i.e. the interaction between banks' market power and credit constraints, complimentary to typical arguments (e.g., related to the avoidance of a bank run), to explain why policymakers may want to bail out a big bank. Third, I make a methodological contribution by extending existing heterogeneous agents algorithms to solve for the stationary equilibrium and transitional dynamics in presence of a continuum of non-competitive markets and strategic interactions by exploiting generalized Euler equations. In an extension, I enrich the firm dynamics component to suggest that the baseline model, despite being relative parsimonious, is fairly conservative in estimating the effects of banks' market power.

References

Albuquerque, R., and H. A. Hopenhayn (2004): "Optimal Lending Contracts and Firm Dynamics," *The Review of Economic Studies*, 71(2), 285–315.

Atkeson, A. G., A. L. Eisfeldt, and P.-O. Weill (2015): "Entry And Exit in OTC Derivatives Markets," *Econometrica*, 83(6), 2231–2292.

Berger, D., K. Herkenhoff, and S. Mongey (2022): "Labor Market Power," *American Economic Review*, 112(4), 1147–93.

Bernanke, B., and M. Gertler (1989): "Agency Costs, Net Worth, and Business Fluctuations," *American Economic Review*, 79(1), 14–31.

Black, S. E., and P. E. Strahan (2002): "Entrepreneurship and Bank Credit Availability," *The Journal of Finance*, 57(6), 2807–2833.

Bloom, N., M. Floetotto, N. Jaimovich, I. Saporta-Eksten, and S. J. Terry (2018): "Really Uncertain Business Cycles," *Econometrica*, 86(3), 1031–1065.

Brunnermeier, M. K., and Y. Sannikov (2014): "A Macroeconomic Model with a Financial Sector," *American Economic Review*, 104(2), 379–421.

Brusco, S., G. Lopomo, E. Ropero, and A. T. Villa (2021): "Optimal financial contracting and the effects of firm's size," *The RAND Journal of Economics*, 52(2), 446–467.

Burga, C., and N. Céspedes (2022): "Bank Competition, Capital Misallocation, and Industry Concentration: Evidence from Peru," *Working Paper*.

Caggese, A. (2007): "Financing constraints, irreversibility, and investment dynamics," *Journal of Monetary Economics*, 54(7), 2102–2130.

Cetorelli, N. (2004): "Real Effects of Bank Competition," *Journal of Money, Credit and Banking*, 36(3), 543–558.

Cetorelli, N., and M. Gambera (2001): "Banking Market Structure, Financial Dependence and Growth: International Evidence from Industry Data," *The Journal of Finance*, 56(2), 617–648.

Cetorelli, N., and P. F. Peretto (2012): "Credit Quantity and Credit Quality: Bank Competition and Capital Accumulation," *Journal of Economic Theory*, 147(3), 967–998.

Cetorelli, N., and P. E. Strahan (2006): "Finance as a Barrier to Entry: Bank Competition and Industry Structure in Local U.S. Markets," *The Journal of Finance*, 61(1), 437–461.

Chava, S., and A. Purnanandam (2011): "The effect of banking crisis on bank-dependent borrowers," *Journal of Financial Economics*, 99(1), 116–135.

Chodorow-Reich, G. (2013): " The Employment Effects of Credit Market Disruptions: Firm-level Evidence from the 2008–9 Financial Crisis *," *The Quarterly Journal of Economics*, 129(1), 1–59.

Clementi, G. L., and H. A. Hopenhayn (2006): "A Theory of Financing Constraints and Firm Dynamics," *The Quarterly Journal of Economics*, 121(1), 229–265.

Clymo, A., and A. Lanteri (2020): "Fiscal Policy with Limited-Time Commitment," *The Economic Journal*, 130(627), 623–652.

Cooley, T. F., and V. Quadrini (2001): "Financial Markets and Firm Dynamics," *American Economic Review*, 91(5), 1286–1310.

Corbae, D., and P. D'Erasmo (2021): "Capital Buffers in a Quantitative Model of Banking Industry Dynamics," *Econometrica*, 89(6), 2975–3023.

Cordoba, J.-C., and M. Ripoll (2004): "Credit Cycles Redux," *International Economic Review*, 45(4), 1011–1046.

Corhay, A., H. Kung, and L. Schmid (2020): "Competition, Markups, and Predictable Returns," *The Review of Financial Studies*, 33(12), 5906–5939.

Covas, F., and W. J. den Haan (2011): "The Cyclical Behavior of Debt and Equity Finance," *American Economic Review*, 101(2), 877–99.

David, J. M., L. Schmid, and D. Zeke (2022): "Risk-adjusted capital allocation and misallocation," *Journal of Financial Economics*, 145(3), 684–705.

De Loecker, J., J. Eeckhout, and G. Unger (2020): "The Rise of Market Power and the Macroeconomic Implications*," *The Quarterly Journal of Economics*, 135(2), 561–644.

Diamond, D. W., and R. G. Rajan (2005): "Liquidity Shortages and Banking Crises," *The Journal of Finance*, 60(2), 615–647.

Eisfeldt, A. L., H. Lustig, and L. Zhang (2017): "Complex Asset Markets," (23476).

Elenev, V., T. Landvoigt, and S. Van Nieuwerburgh (2021): "A Macroeconomic Model With Financially Constrained Producers and Intermediaries," *Econometrica*, 89(3), 1361–1418.

Farhi, E., and F. Gourio (2018): "Accounting for Macro-Finance Trends: Market Power, Intangibles, and Risk Premia," *Brookings Papers on Economic Activity*, Fall, 147–250.

Freixas, X., and J. Rochet (1997): "Microeconomics of Banking, Volume 1," *The MIT Press, 1 edition.*

Gertler, M., and P. Karadi (2011): "A model of unconventional monetary policy," *Journal of Monetary Economics*, 58(1), 17–34, Carnegie-Rochester Conference Series on Public Policy: The Future of Central Banking April 16-17, 2010.

Gomes, J. F. (2001): "Financing Investment," *American Economic Review*, 91(5), 1263–1285.

Goodfellow, I., Y. Bengio, and A. Courville (2016): "Softmax Units for Multinoulli Output Distributions," *Deep Learning. MIT Press. pp. 180–184. ISBN 978-0-26203561-3.*

Guerrieri, V., and G. Lorenzoni (2017): "Credit Crises, Precautionary Savings, and the Liquidity Trap*," *The Quarterly Journal of Economics*, 132(3), 1427–1467.

Gutiérrez, G., and T. Philippon (2016): "Investment-less Growth: An Empirical Investigation," (22897).

He, Z., and A. Krishnamurthy (2013): "Intermediary Asset Pricing," *American Economic Review*, 103(2), 732–70.

Hennessy, C. A., and T. M. Whited (2005): "Debt Dynamics," *The Journal of Finance*, 60(3), 1129–1165.

——— (2007): "How Costly Is External Financing? Evidence from a Structural Estimation," *The Journal of Finance*, 62(4), 1705–1745.

Holmstrom, B., and J. Tirole (1997): "Financial Intermediation, Loanable Funds, and the Real Sector," *The Quarterly Journal of Economics*, 112(3), 663–691.

Ifrach, B., and G. Y. Weintraub (2016): "A Framework for Dynamic Oligopoly in Concentrated Industries," *The Review of Economic Studies*, 84(3), 1106–1150.

Jamilov, R. (2020): "Credit Market Power: Branch-level Evidence from the Great Financial Crisis," *Working Paper.*

Jamilov, R., and T. Monacelli (2021): "Bewley Banks," *Working Paper.*

Jermann, U., and V. Quadrini (2012): "Macroeconomic Effects of Financial Shocks," *American Economic Review*, 102(1), 238–71.

Khan, A., T. Senga, and J. K. Thomas (2016): "Default Risk and Aggregate Fluctuations in an Economy with Production Heterogeneity," *Working paper.*

Khan, A., and J. K. Thomas (2013): "Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity," *Journal of Political Economy*, 121(6), 1055–1107.

Kiyotaki, N., and J. Moore (1997): "Credit Cycles," *Journal of Political Economy*, 105(2), 211–248.

Klein, P., P. Krusell, and J.-V. Ríos-Rull (2008): "Time-Consistent Public Policy," *The Review of Economic Studies*, 75(3), 789–808.

Klein, P., and J.-V. Ríos-Rull (2003): "Time-Consistent Optimal Fiscal Policy," *International Economic Review*, 44(4), 1217–1245.

Kocherlakota, N. R. (2000): "Creating Business Cycles through Credit Constraints," *Federal Reserve Bank of Minneapolis Quarterly Review, Volume 24, No. 3*.

Kreps, D. M., and J. A. Scheinkman (1983): "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes," *The Bell Journal of Economics*, 14(2), 326–337.

Krusell, P., F. Martin, and J.-V. Ríos-Rull (2004): "On the Determination of Government Debt," *https://www.sas.upenn.edu/ vr0j/slides/humboldt.pdf*.

Krusell, P., and A. Smith, Jr. (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 106(5), 867–896.

Lanteri, A. (2018): "The Market for Used Capital: Endogenous Irreversibility and Reallocation over the Business Cycle," *American Economic Review*, 108(9), 2383–2419.

Maskin, E., and J. Tirole (2001): "Markov Perfect Equilibrium: I. Observable Actions," *Journal of Economic Theory*, 100(2), 191–219.

Midrigan, V., and D. Y. Xu (2014): "Finance and Misallocation: Evidence from Plant-Level Data," *American Economic Review*, 104(2), 422–58.

Petersen, M. A., and R. G. Rajan (1995): "The Effect of Credit Market Competition on Lending Relationships," *The Quarterly Journal of Economics*, 110(2), 407–443.

Rajan, R. G., and L. Zingales (1998): "Financial Dependence and Growth," *American Economic Review*, 88(3), 559–586.

Rampini, A. A., and S. Viswanathan (2018): "Financial Intermediary Capital," *The Review of Economic Studies*, 86(1), 413–455.

Saunders, A., A. Spina, S. Steffen, and D. Streitz (2022): "Corporate Loan Spreads and Economic Activity," *Working Paper*.

Tauchen, G. (1986): "Finite state markov-chain approximations to univariate and vector autoregressions," *Economics Letters*, 20(2), 177–181.

Wang, Y., T. M. Whited, Y. Wu, and K. Xiao (2022): "Bank Market Power and Monetary Policy Transmission: Evidence from a Structural Estimation," *The Journal of Finance*, 77(4), 2093–2141.

Young, E. R. (2010): "Solving the incomplete markets model with aggregate uncertainty using the Krusell–Smith algorithm and non-stochastic simulations," *Journal of Economic Dynamics and Control*, 34(1), 36–41, Computational Suite of Models with Heterogeneous Agents: Incomplete Markets and Aggregate Uncertainty.

APPENDIX (For Online Publication)

This online appendix is organized as follows. First, Appendix A is the computational appendix that contains details about the algorithms to solve for (i) the stationary equilibrium (ii) the transitional dynamics and (iii) the stationary equilibrium in the extension. Second, Appendix B contains mathematical details. Third, Appendix C contains additional material.

A Computational Appendix (For Online Publication)

In this section, I describe the algorithm to solve both the stationary equilibrium and the dynamics with the MIT shocks. I highlight the novel methodology used to solve for both General Equilibrium and strategic interactions. Since banks optimize over the optimal choices of the firms, solving this problem using value function iteration would require to nesting two value function iterations inside each other and iterating on the nested value function system given guesses for the aggregate dynamics. Moreover, accounting for strategic interactions with value function iteration would require to solving this system of two nested VFIs given other banks' strategies and finding the fixed point of the resulting policies. This brute force approach is clearly not viable. To avoid this, I use projection methods jointly on the generalized Euler equations (19) and the loan firms optimality conditions (12) and (13). Hence, I leverage the fact that the elasticities can be calculated applying the implicit function theorem as described in Appendix B.1. Note that the aggregate quantities are not only contained in the discount factors, but also in the elasticities of the generalized Euler equations (see Appendix B.1). In order to account for strategic interactions and solve the generalized Euler equations (which can also be interpreted as best response functions), I impose ex-post symmetric strategies between banks after calculating the elasticity as described in Section B.1; hence, I proceed to calculate the root of the resulting equation with time iteration and projection methods, as any other Euler equation.[27] Note also that the projection step with time iteration can be efficiently computed parallelizing the calculation of the policy functions, fixing state variables. In particular, on a grid of $\mathcal{K} \times \mathcal{L}$, such that $\mathcal{K} = [0, k_1, ..., k_{\bar{K}}]$ and $\mathcal{L} = [0, l_{b,1}, ..., l_{b,\bar{L}}]$, this corresponds to have $\bar{K} \times \bar{L}$ parallel subproblems at each step of the time iteration. The code is written in C/C++, each subproblem can be efficiently parallelized using the OpenMP API specification for parallel programming. Moreover, each subproblem is solved using the Levenberg-Marquardt algorithm contained in ALGLIB.

[27] This is equivalent to finding the fixed-point of the banks' strategies.

A.1 Oligopolistic Stationary Equilibrium

Here are the main steps to solve for the *oligopolistic stationary equilibrium* (see Definition 3.1). Create grids $\mathcal{K} = [0, k_1, ..., \bar{k}]$ and $\mathcal{L} = [0, l_{b,1}, ..., l_{b,\bar{L}}]$. Initialize the policy functions for investment and loan to the solution of the corresponding steady-state model without firms heterogeneity; i.e., $\forall (k, l_b) \in \mathcal{K} \times \mathcal{L}, \quad l'_b(k, l_b) = l^*_b$ and $k'(k, l_b) = k^*$. Create an iterator j and set $j = 0$; hence, proceed as follow.

1. Guess a number of banks $B^j = 1$, an aggregate consumption $C^j = \int \tilde{d}\, \mathrm{d}\Phi + \sum_b^{B^j} \pi_b$ (e.g., use the steady-state consumption calculated without firm heterogeneity).[28]

2. Create an iterator w and set $w = 0$.

   (a) Fork the program in $\bar{k} \times \bar{L}$ parallel threads. Each thread solves the subproblem associated with a fixed pair $(k, l_b) \in \mathcal{K} \times \mathcal{L}$.[29]

   (b) Each thread uses Levenberg-Marquardt to jointly solve for ${k'}^{w+1}$, ${l'}_b^{w+1}$, ${R'}_l^{w+1}$ such that the firms' first-order conditions (12) and (13) hold, and such that the generalized Euler equation (19) hold, given future policy functions ${k''}^{w}({k'}^{w+1}, {l'}_b^{w+1})$, ${l''}_b^{w}({k'}^{w+1}, {l'}_b^{w+1})$, and ${R''}_l^{w}({k'}^{w+1}, {l'}_b^{w+1})$.[30] The elasticity η'_l of equation (19) is calculated according to equations (28), (29), (30), (31) and the condition of symmetry among bank's strategies ${l'}_1^{w+1} = ... = {l'}_b^{w+1} = ... = {l'}_B^{w+1}$, which is imposed ex-post.

   (c) Wait for all parallel threads to complete their tasks and update the policy functions accordingly $\forall (k, l_b) \in \mathcal{K} \times \mathcal{L}, \quad k'(k, l_b) = {k'}^{w+1}, \ l'_b(k, l_b) = {l'}_b^{w+1}, \ R'_l(k, l_b) = {R'}_l^{w+1}$.[31]

   (d) If the policy functions converged (i.e., $\max(\sup |{k'}^{w+1} - {k'}^{w}|, \sup |{l'}_b^{w+1} - {l'}_b^{w}|) < \epsilon$) proceed to step 3. Otherwise, set $w = w + 1$ and restart from step 2.

3. The probability density function over age is given by

$$\phi(\text{age}) = \frac{\rho^{\text{age}}}{\sum_{\text{age}=0}^{\bar{N}} \rho^{\text{age}}}.$$

   Start from age $= 0$ and simulate the policy functions up to age $\bar{N}$. This yields a mapping between $\phi(\text{age})$ and $\phi(x)$.

[28] Note that the aggregate dividend $\tilde{D}^j$ is contained in the elasticity η_l of equation (19).

[29] This can be achieved either by using POSIX Threads directly or, at a higher level, using OpenMP API.

[30] I use $\bar{k} = 10$, $\bar{L} = 10$ and perform piece-wise bilinear interpolation.

[31] The update requires dampening. For example, I update the policy function for capital according to ${k'}^{w+1} = (1 - \omega) \cdot {k'}^{w} + \omega \cdot {k'}^{w+1}$, with $\omega = 0.01$.

4. Compute the implied aggregate consumption C^{j+1} according to equation (7).

5. If the aggregate consumption converged (i.e., $|C^{j+1} - C^j| < \epsilon$) and the number of banks B^j is such that there is no incentive for an additional bank to enter the market as per equation (21), the program terminates. Otherwise, set $j = j+1$, update $B^{j+1} = B^j + 1$ if equation (21) is satisfied and restart from step 2. Use a quasi-Newton method to correct the guess of the aggregate consumption C^j, given the implied aggregate dividend C^{j+1}.[32]

A.2 Transitional Dynamics

The economy is initially in its stationary equilibrium when all agents discover a sudden change in a model parameter at $t = 0$. In order to compute the equilibrium dynamics, I need to find sequences of: (i) aggregate consumption $\{C_t\}_{t=0}^T$, and (ii) firms distributions $\{\phi_t(x)\}_{t=0}^T$; such that the representative household maximizes its utility, all markets clear in each period and the firms distributions evolve according to: (i) the firms' policy functions, (ii) the incumbent banks generalized Euler equations and (iii) the idiosyncratic default shocks. First, compute the two stationary equilibria associated with the configuration of parameters before and after the shock, as described previously.[33] Second, create an iterator j and set $j = 0$; hence, proceed as follow. Note that in order to solve the transitional dynamics with new potential entrants, I repeat this procedure several times till I find the date t that satisfies the entry condition (21). In principle, this requires to guess a sequence $\{B_t\}_{t=0}^T$. In practice, since I know that only one entry will occurs on the equilibrium path (given the fixed entry cost held at its stationary equilibrium value), I can solve the transitional dynamics several times till I find the new bank entry's date.

1. Guess sequences of: (i) aggregate consumption $\{C_t^j\}_{t=0}^T$.[34]

2. Create an iterator t and set $t = T - 1$. Hence, use projection with backward time iteration from $t = T - 1$ to $t = 0$. The policy functions at $t = T$, are the ones associated with the ending stationary equilibrium, previously calculated. At each time t proceed similarly to before.

   (a) Fork the program in $\bar{k} \times \bar{L}$ parallel threads. Each thread solves the subproblem associated with a fixed pair $(k, l_b) \in \mathcal{K} \times \mathcal{L}$.

[32]The update of consumption also requires dampening, similarly to the update of the policy functions.

[33]If there are not permanent change to the parameters, the two stationary equilibria coincides.

[34]T should be long enough, so that after the shock the economy converges to its long-run stationary equilibrium. In this paper, I use T=40 quarters.

(b) Each thread uses Levenberg-Marquardt to jointly solve for k^{t+1}, l_b^{t+1}, R_l^{t+1} such that the firms' first-order conditions (12) and (13) hold, and such that the generalized Euler equation (19) hold, given future policy functions $k^{t+2}(k^{t+1}, l_b^{t+1})$, $l_b^{t+2}(k^{t+1}, l_b^{t+1})$, and $R_l^{t+2}(k^{t+1}, l_b^{t+1})$, always performing piece-wise bilinear interpolation when needed. The elasticity η_l' of equation (19) is calculated according to equations (28), (29), (30), (31) and the condition of symmetry among bank's strategies $l_1^{t+1} = ... = l_b^{t+1} = ... = l_B^{t+1}$, which is imposed ex-post.

(c) Wait for all parallel threads to complete their tasks and update the policy functions accordingly $\forall (k, l_b) \in \mathcal{K} \times \mathcal{L}, \quad k^{t+1}(k, l_b) = k^{t+1}$, $l_b^{t+1}(k, l_b) = l_b^{t+1}$, $R_l^{t+1}(k, l_b) = R_l^{t+1}$.

3. Now, start from $t = 0$ and iterate forward up to $t = T$. The probability density function over age is now time-varying

$$\phi_t(\text{age}) = \frac{\rho_t^{\text{age}}}{\sum_{\text{age}=0}^{\bar{N}} \rho_t^{\text{age}}}.$$

At each time t, start from age $= 0$ and simulate the time t policy functions up to age $\bar{N}$. This yields a mapping between $\phi_t(\text{age})$ and $\phi_t(x)$.

4. For each time t, compute the implied aggregate consumption C_t^{j+1} according to equation (7).

5. If the sequences for aggregate consumption converged; i.e.,

$$\sup\{|C_t^{j+1} - C_t^j|\}_{t=0}^T < \epsilon,$$

the program terminates. Otherwise, set $j = j+1$ and restart from step 2, after heaving updated the sequences with a heavy dampening parameter.

A.3 Oligopolistic Stationary Equilibrium with Banks' Market Power Interacting with the Firms' Endogenous Default Decisions

Here are the main steps to solve for the *oligopolistic stationary equilibrium* when firms make endogenous default decisions (see Section 6). Start with discretizing the AR(1) process for the idionsycratic TFP shock z using Tauchen (1986).[35] This yields a grid $\mathcal{Z} = [z_1, ..., \bar{z}]$ and an associated transitional probability matrix P_z. Create grids $\mathcal{K} = [0, k_1, ..., \bar{k}]$, $\mathcal{L} =$

[35] In particular, I use three unconditional standard deviations and 11 nodes.

$[0, l_{b,1}, ..., l_{b,\bar{L}}]$. Initialize the policy functions for investment and loan to the solution of the corresponding steady-state model without firms heterogeneity; i.e., $\forall(k, l_b, z^-, z) \in \mathcal{K} \times \mathcal{L} \times \mathcal{Z} \times \mathcal{Z}, \quad l_b'(k, l_b, z^-, z) = l_b^*$ and $k'(k, l_b, z^-, z) = k^*$. Note that I keep track of z^- instead of $r_{l,t}$ (which is in principle a state variable). Through z^- it is possible to reconstruct $r_{l,t}$ through the calculation of the previous period expected marginal productivity. Create an iterator j and set $j = 0$; hence, proceed as follow.

1. Guess a number of banks $B^j = 1$, an aggregate consumption $C^j = \int \tilde{d}\, \mathrm{d}\Phi + \sum_b^{B^j} \pi_b$ (e.g., use the steady-state consumption calculated without firm heterogeneity).[36]

2. Create an iterator w and set $w = 0$.

   (a) Fork the program in $\bar{k} \times \bar{L} \times \bar{Z} \times \bar{Z}$ parallel threads. Each thread solves the subproblem associated with a fixed pair $(k, l_b, z^-, z) \in \mathcal{K} \times \mathcal{L} \times \bar{Z} \times \bar{Z}$.[37]

   (b) Each thread uses Levenberg-Marquardt to jointly solve for k'^{w+1}, $l_b'^{w+1}$, $R_l'^{w+1}$ such that the firms' first-order conditions (25) and (26) hold, and such that the generalized Euler equation (27) hold, given guesses for the future policy functions $k''^w(k'^{w+1}, l_b'^{w+1}, z, z')$, $l_b''^w(k'^{w+1}, l_b'^{w+1}, z, z')$, and $R_l''^w(k'^{w+1}, l_b'^{w+1}, z, z')$ and future value functions $V_F'^w(k'^{w+1}, l_b'^{w+1}, z, z')$ and $\tilde{V}_F'^w(k'^{w+1}, l_b'^{w+1}, z, z')$.[38] The elasticity η_l' of equation (27) is calculated applying to implicit function theorem on equations (25) and (26) in a similar fashion to the case of the baseline model captured by equations (28), (29), (30), (31) and imposing ex-post the condition of symmetry among bank's strategies $l_1'^{w+1} = ... = l_b'^{w+1} = ... = l_B'^{w+1}$.

   (c) Update the guess for the value functions such that V_F^{w+1} and $\tilde{V}_F^{w+1}$ are consistent with the new policies k'^{w+1}, $l_b'^{w+1}$, $R_l'^{w+1}$ and with the functional equations (23) and (24).

   (d) Wait for all parallel threads to complete their tasks and update the policy functions accordingly $\forall(k, l_b, z^-, z) \in \mathcal{K} \times \mathcal{L} \times \mathcal{Z} \times \mathcal{Z}, \quad k'(k, l_b, z^-, z) = k'^{w+1}$, $l_b'(k, l_b, z^-, z) = l_b'^{w+1}$, $R_l'(k, l_b, z^-, z) = R_l'^{w+1}$ and firm's value functions $V_F(k, l_b, z^-, z) = V_F^{w+1}$ and $\tilde{V}_F(k, l_b, z^-, z) = \tilde{V}_F^{w+1}$.[39]

   (e) If the policy functions converged (i.e., $\max(\sup|k'^{w+1} - k'^w|, \sup|l_b'^{w+1} - l_b'^w|) < \epsilon$) proceed to step 3. Otherwise, set $w = w + 1$ and restart from step 2.

[36] Note that the aggregate dividend $\tilde{D}^j$ is contained in the elasticity η_l of equation (27).

[37] This can be achieved either by using POSIX Threads directly or, at a higher level, using OpenMP API.

[38] I use $\bar{k} = 10$, $\bar{L} = 10$ and perform piece-wise bilinear interpolation.

[39] The update requires dampening. For example, I update the policy function for capital according to $k'^{w+1} = (1 - \omega) \cdot k'^w + \omega \cdot k'^{w+1}$, with $\omega = 0.01$.

3. Given the policy functions and the firm's value functions (which can be used to back out the firm's default decisions), I simulate using the Young (2010) lottery method to find the invariant distribution of firms $\phi(k, l_b, z^-, z)$.

4. Compute the implied aggregate consumption C^{j+1} according to equation (7).

5. If the aggregate consumption converged (i.e., $|C^{j+1} - C^j| < \epsilon$) and the number of banks B^j is such that there is no incentive for an additional bank to enter the market as per equation (21), the program terminates. Otherwise, set $j = j+1$, update $B^{j+1} = B^j + 1$ if equation (21) is satisfied and restart from step 2. Use a quasi-Newton method to correct the guess of the aggregate consumption C^j, given the implied aggregate dividend C^{j+1}.[40]

B Mathematical Appendix (For Online Publication)

This section contains: (i) the calculation of the elasticity η_l in the generalized Euler equation (19), and (ii) the proofs of the statements in the proposition of Section 2.

B.1 Firm-Specific Inverse Elasticity η_l

This subsection contains the calculation of the elasticity in the generalized Euler equation (19). Combine the firms' first-order conditions (12) and (13) to define functions

$$\begin{aligned} f\left(x, X, x', X'\right) &\equiv \pi_k(k') - \delta - R_l' + 1 = 0, \\ g\left(x, X, x', X'\right) &\equiv \rho \cdot \mathcal{M}' \cdot (1 - \lambda_d') \left((1-\tau)(R_l' - 1) + 1\right) - 1 + \lambda_d = 0. \end{aligned}$$

Compute the total derivatives of these two functions with respect to k' and l_b'. Hence, solve the resulting linear system to get the following expression:[41]

$$\frac{\partial R_l'}{\partial l_b'} = \frac{f_{k'} \cdot g_{l_b'} - f_{l_b'} \cdot g_{k'}}{f_{R_L'} \cdot g_{k'} - f_{k'} \cdot g_{R_L'}}.$$

Note that $f_{l_b'} = 0$, $f_{R_l'} = -1$, and $f_{k'} = \pi_{kk}(k')$. Therefore, the elasticity η_l' is given by

$$\eta_l' = \frac{\partial R_l'}{\partial l_b'} \frac{l_b'}{R_l'} = -\frac{f_{k'} \cdot g_{l_b'}}{g_{k'} + f_{k'} \cdot g_{R_l'}} \frac{l_b'}{R_l'}, \tag{28}$$

[40] The update of consumption also requires dampening, similarly to the update of the policy functions.
[41] This is equivalent to apply the implicit function theorem.

where the partial derivatives of the g function are

$$g_{k'} = \rho \frac{\partial \mathcal{M}'}{\partial k'} (1 - \lambda_d(d')) \left((1-\tau) {r_l}' + 1 \right) - \rho \mathcal{M}' \frac{\partial \lambda_d(d')}{\partial k'} \left((1-\tau) r_l' + 1 \right) + \frac{\partial \lambda_d(d)}{\partial k'}, \tag{29}$$

$$g_{l'_b} = \rho \frac{\partial \mathcal{M}'}{\partial l'_b} (1 - \lambda_d(d')) \left((1-\tau) {r_l}' + 1 \right) - \rho \mathcal{M}' \frac{\partial \lambda_d(d')}{\partial l'_b} \left((1-\tau) {r_l}' + 1 \right) + \frac{\partial \lambda_d(d)}{\partial l'_b}, \tag{30}$$

$$g_{R'_l} = \rho \frac{\partial \mathcal{M}'}{\partial R'_l} (1 - \lambda_d(d')) \left((1-\tau) {r_l}' + 1 \right) - \rho \mathcal{M}' \frac{\partial \lambda_d(d')}{\partial R'_l} \left((1-\tau) {r_l}' + 1 \right) + \rho \mathcal{M}' (1-\tau) + \frac{\partial \lambda_d(d)}{\partial R'_l}. \tag{31}$$

Note that equations (29), (30), and (31) contain all other banks strategies $\mathbf{l}_{-b} = [l_1, ..., l_B] \backslash \{l_b\}$ in the discount factor $\mathcal{M}'$, in the marginal equity issuance costs $\lambda_d(d)$ and $\lambda_d(d')$.

B.2 Proofs

This section contains the proofs of the statements 1-10 in the main proposition of Section 2.

Proof. Statements 1,2 and 3. Note that the number of banks matters only for the financially constrained firms, so that each integral can be rewritten as follow

$$\frac{\partial}{\partial B} \int x^* \, \mathrm{d}\Phi = \int \frac{\partial x^*}{\partial B} \cdot \mathbf{1}[d_1^* < 0] \, \mathrm{d}\Phi + \int \frac{\partial x^*}{\partial B} \cdot \mathbf{1}[d_1^* \geq 0] \, \mathrm{d}\Phi = \int \frac{\partial x^*}{\partial B} \cdot \mathbf{1}[d_1^* < 0] \, \mathrm{d}\Phi,$$

where x^* is a place holder for $l_{1,b}^*$, R_1^*, p_0^* and k_1^*. Hence, a sufficient condition to establish the sign of $\frac{\partial}{\partial B} \int x^* \, \mathrm{d}\Phi$ is to determine the sign of $\frac{\partial x^*}{\partial B} \cdot \mathbf{1}[d_1^* < 0]$. Total differentiation of the optimality conditions (1), (3) and (2) of the financially constrained firms yields the following linear system

$$\begin{bmatrix} \kappa_1 & \kappa_2 & \frac{\partial R_1^*}{\partial l_{1,b}} \cdot \kappa_2 \\ -\alpha(\alpha-1)\mathbb{E}_0[z_1] k_1^{*\alpha-2} & 1 & 0 \\ -1 & \rho\beta & B \end{bmatrix} \begin{bmatrix} \frac{\partial k_1^*}{\partial B} \\ \frac{\partial R_1^*}{\partial B} \\ \frac{\partial l_{1,b}^*}{\partial B} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -l_{1,b}^* \end{bmatrix}.$$

Note first that equation (3), for firms with $d_0 < 0$, implies $R_1^* = \frac{1-\lambda_0 d_0}{\rho\beta} > \frac{1}{\rho\beta}$ for $\lambda_0 > 0$. Hence, the GEE implies $\frac{\partial R_1^*}{\partial l_{1,b}} < 0$ for financially constrained firms. Hence, by concavity of the production function and since $0 < \alpha < 1$, $\kappa_1 = \frac{\partial R_1^*}{\partial l_{1,b}} \frac{\lambda_0(\alpha-2)}{\alpha(\alpha-1)\mathbb{E}_0[z_1] k_1^{*\alpha-1}} < 0$. It also follows that $\kappa_2 = \frac{1}{l_{1,b}^*} \left(\frac{\lambda_0}{\alpha(\alpha-1)\mathbb{E}_0[z_1] k_1^{*\alpha-2}} - \rho\beta \right) < 0$. The determinant of the matrix is therefore:

$$\mathcal{D} = \frac{\partial R_1^*}{\partial l_{1,b}} \kappa_2 + \kappa_1 B + \kappa_2 \alpha(\alpha-1)\mathbb{E}_0[z_1] k_1^{*\alpha-2} B - \frac{\partial R_1^*}{\partial l_{1,b}} \kappa_2 \alpha(\alpha-1)\mathbb{E}_0[z_1] k_1^{*\alpha-2} \rho\beta.$$

Direct inversion yields:

$$\begin{bmatrix} \frac{\partial k_1^*}{\partial B} \\ \frac{\partial R_1^*}{\partial B} \\ \frac{\partial l_{1,b}^*}{\partial B} \end{bmatrix} = \begin{bmatrix} \frac{\partial R_1^*}{\partial l_{1,b}} \kappa_2 l_{1,b}^* \\ \frac{\partial R_1^*}{\partial l_{1,b}} \kappa_2 \alpha(\alpha-1)\mathbb{E}_0[z_1] k_1^{*\alpha-2} l_{1,b}^* \\ -l_{1,b}^*(\kappa_1 + \kappa_2\alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-2}) \end{bmatrix} \cdot \mathcal{D}^{-1}.$$

Note that if $\kappa_1 + \kappa_2\alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-2} > 0$, then we can conclude that: $\frac{\partial k_1^*}{\partial B} > 0$, $\frac{\partial R_1^*}{\partial B} < 0$ and $\frac{\partial l_{1,b}^*}{\partial B} < 0$. This is equivalent to showing:

$$\frac{1-\rho\beta R_1^*}{\rho\beta l_{1,b}^*} \frac{\lambda_0(\alpha-2)}{\alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-1}} + \frac{\lambda_0}{l_{1,b}^*} - \frac{1}{l_{1,b}^*}\rho\beta\alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-2} > 0.$$

Note that equation (1) of the optimalities of the constrained firm can be rewritten as

$$\lambda_0 \frac{1-\rho\beta R_1^*}{\rho\beta l_{1,b}^*} \frac{1}{\alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-1}k_1^{*-1}} = \rho\beta\frac{1-\rho\beta R_1^*}{\rho\beta l_{1,b}^*} + \lambda_0.$$

Using this equivalence *the want to show* can be rewritten as

$$\begin{aligned} &\frac{1-\rho\beta R_1^*}{\rho\beta l_{1,b}^*} \frac{\lambda_0(\alpha-2)}{\alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-1}} + \frac{\lambda_0}{l_{1,b}^*} - \frac{1}{l_{1,b}^*}\rho\beta\alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-2} \\ &= \rho\beta\frac{1-\rho\beta R_1^*}{\rho\beta l_{1,b}^*}(\alpha-2)k_1^{*-1} + \lambda_0(\alpha-2)k_1^{*-1} + \frac{\lambda_0}{l_{1,b}^*} - \frac{1}{l_{1,b}^*}\rho\beta\alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-2} > 0. \end{aligned}$$

Multiply everything by $l_{1,b}^* > 0$, to get:

$$(1-\rho\beta R_1^*)(\alpha-2)k_1^{*-1} + \lambda_0(\alpha-2)\frac{l_1^*}{k_1^*} + \lambda_0 - \rho\beta\alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-2} > 0.$$

Hence, use equation (2) of the optimalities of the constrained firms to back out an expression for $l_{1,b}^*$ in function of R_1^* and k_1^*, and rewrite

$$(1-\rho\beta R_1^*)(\alpha-2)k_1^{*-1} + (\alpha-2)\frac{1-\rho\beta R_1^* - \lambda_0(z_0k_0^\alpha + (1-\delta)k_0 - k_1^*)}{Bk_1^*} + \lambda_0 - \rho\beta\alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-2} > 0.$$

The left-hand side can be rearranged as

$$\begin{aligned} &(\alpha-2)\frac{(1-\rho\beta R_1^*)(B+1) - \lambda_0(z_0k_0^\alpha + (1-\delta)k_0 - k_1^*)}{Bk_1^*} + \lambda_0 - \rho\beta\alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-2} \\ &= (\alpha-2)\left[(1-\rho\beta R_1^*)(B+1) - \lambda_0(z_0k_0^\alpha + (1-\delta)k_0 - k_1^*)\right] + \lambda_0 Bk_1^* - \rho\beta\alpha(\alpha-1)\mathbb{E}_0[z_1]Bk_1^{*\alpha-1}. \end{aligned}$$

Divide by $(\alpha-2) < 0$ (changing sign because it is always negative), the previous *want to* *show* is equivalent to show

$$\underbrace{(1-\rho\beta R_1^*)(B+1)}_{<0 \text{ when } d_0<0} - \underbrace{\lambda_0(z_0k_0^\alpha+(1-\delta)k_0)}_{>0} + \lambda_0 k_1^* \underbrace{\frac{\alpha-2+B}{\alpha-2}}_{<0 \text{ if } B>1} - \underbrace{\rho\beta\alpha\frac{\alpha-1}{\alpha-2}\mathbb{E}_0[z_1]Bk_1^{*\alpha-1}}_{>0} < 0.$$

Consider two cases. If $B>1$ (oligopoly), this last inequality is always satisfied. For $B=1$ (monopoly), the inequality collapses to

$$\underbrace{(1-\rho\beta R_1^*)2}_{<0 \text{ when } d_0<0} - \underbrace{\lambda_0(z_0k_0^\alpha+(1-\delta)k_0)}_{>0} + \underbrace{(\lambda_0k_1^*-\rho\beta R_1^*)}_{<0}\underbrace{\frac{\alpha-1}{\alpha-2}}_{>0} < 0.$$

Finally, rearrange the Euler $\rho\beta R_1^* = 1-\lambda_0 d_0$ to get

$$\lambda_0k_1^*-\rho\beta R_1^* = \lambda_0(z_0k_0^\alpha+(1-\delta)k_0+l_{1,b}^*)-1,$$

which yields the result

$$\underbrace{(1-\rho\beta R_1^*)2}_{<0 \text{ when } d_0<0} + \left(\frac{\alpha-1}{\alpha-2}-1\right)\underbrace{\lambda_0(z_0k_0^\alpha+(1-\delta)k_0)}_{>0} + \underbrace{\left(\lambda_0l_{1,b}^*-1\right)}_{<0}\underbrace{\frac{\alpha-1}{\alpha-2}}_{>0} < 0.$$

□

Proof. Statements 4, 5 and 6. Following a similar logic as the one of equation (5) the proof focuses in studying the signs of the optimal choices of the financially constrained firms. Hence, all equations that follow refer to those firms such that $d_0(k_0,z_0,k_1^*,l_{1,b}^*)<0$.

For the expected return of the shares, equating the Eulers for loans and the price of shares provides the following non-arbitrage condition $\mathbb{E}_0\left[\frac{d_1^*}{p_0^*}\right]=R_1^*$. Hence, for financially constrained firms $\frac{\partial}{\partial B}\mathbb{E}_0\left[\frac{d_1^*}{p_0^*}\right]=\frac{\partial R_1^*}{\partial B}<0$, which is always negative by previous result.

Before studying the effect of the number of banks on leverage, first note that the effect on total debt is ambiguous:

$$\frac{\partial}{\partial B}B\cdot l_{1,b}^* = B\frac{\partial l_{1,b}^*}{\partial B}+l_{1,b}^*.$$

As shown previously, as the number of banks increases $l_{1,b}$ decreases. Plugging the formula for $\frac{\partial l^*_{1,b}}{\partial B}$ found previously can resolve this ambiguity:

$$\frac{\partial}{\partial B}B \cdot l^*_{1,b} = l^*_{1,b}\left(1 - B\frac{\kappa_1 + \kappa_2\alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-2}}{\mathcal{D}}\right) = l^*_{1,b}\left(1 - \frac{1}{1 + \frac{\frac{\partial R_1^*}{\partial l_{1,b}}\kappa_2\left(1-\alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-2}\rho\beta\right)}{B\kappa_1 + B\kappa_2\alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-2}}}\right),$$

since $l^*_{1,b} > 0$ and $\frac{\frac{\partial R_1^*}{\partial l_{1,b}}\kappa_2\left(1-\alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-2}\rho\beta\right)}{B\kappa_1 + B\kappa_2\alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-2}} > 0 \implies \frac{\partial}{\partial B}B \cdot l^*_{1,b} > 0$.

In order to prove that the leverage increases with the number of banks, it remains to show that the following inequality is always satisfied for the financially constrained firms:

$$\begin{aligned}\frac{\partial}{\partial B}\frac{B \cdot l^*_{1,b}}{k_1^*} &= \left(B\frac{\partial l^*_{1,b}}{\partial B} + l^*_{1,b}\right)\frac{1}{k_1^*} - \frac{B \cdot l_{1,b}}{k_1^{*2}} \cdot \frac{\partial k_1^*}{\partial B} \\ &= \frac{l^*_{1,b}}{k_1^*}\left(1 - \frac{B\kappa_1 + B\kappa_2\alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-2} - B\frac{l^*_{1,b}}{k_1^*}\kappa_2\frac{\partial R_1^*}{\partial l_{1,b}}}{\mathcal{D}}\right) > 0.\end{aligned}$$

Since $l^*_{1,b}/k_1^* > 0$, $\mathcal{D} > 0$ and $\kappa_2\frac{\partial R_1^*}{\partial l_{1,b}} > 0$, this is equivalent to show:

$$-B\frac{l^*_{1,b}}{k_1^*}\kappa_2\frac{\partial R_1^*}{\partial l_{1,b}} < \frac{\partial R_1^*}{\partial l_{1,b}}\kappa_2 - \frac{\partial R_1^*}{\partial l_{1,b}}\kappa_2\alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-2}\rho\beta \iff -B\frac{l^*_{1,b}}{k_1^*} < 1 - \alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-2}\rho\beta,$$

which is always true since $-B\frac{l^*_{1,b}}{k_1^*}$ is always negative and $1-\alpha(\alpha-1)\mathbb{E}_0[z_1]k_1^{*\alpha-2}\rho\beta$ is always positive.

□

Proof. Statements 7, 8, 9 and 10. For statements 7, 8, 9 and 10, I assume that the mass of financially constrained firms $1-\mathcal{P}$ are all ex-ante identical. For TFP, the *want to show* is

$$\frac{\partial}{\partial B}\frac{\mathbb{E}\left[k_1^{*\alpha}\right]}{\left(\mathbb{E}\left[k_1^*\right]\right)^\alpha} = \frac{\alpha\mathbb{E}\left[k_1^{*\alpha-1}\frac{\partial k_1^*}{\partial B}\right]}{\left(\mathbb{E}\left[k_1^*\right]\right)^\alpha} - \alpha\frac{\mathbb{E}[k_1^{*\alpha}]\mathbb{E}\left[\frac{\partial k_1^*}{\partial B}\right]}{\left(\mathbb{E}\left[k_1^*\right]\right)^{\alpha+1}} > 0.$$

This is equivalent to show that

$$\mathbb{E}\left[k_1^{*\alpha-1}\frac{\partial k_1^*}{\partial B}\right]\mathbb{E}\left[k_1^*\right] - \mathbb{E}[k_1^{*\alpha}]\mathbb{E}\left[\frac{\partial k_1^*}{\partial B}\right] > 0.$$

Which is again equivalent to

$$
\begin{aligned}
& k_1^{*\alpha-1}\frac{\partial k_1^*}{\partial B}(1-\mathcal{P})(k_1^*(1-\mathcal{P})+\bar{k}\mathcal{P})-(k_1^{*\alpha}(1-\mathcal{P})+\bar{k}^\alpha\mathcal{P})\frac{\partial k_1^*}{\partial B}(1-\mathcal{P})>0 \\
\iff\ & k_1^{*\alpha-1}(k_1^*(1-\mathcal{P})+\bar{k}\mathcal{P})-(k_1^{*\alpha}(1-\mathcal{P})+\bar{k}^\alpha\mathcal{P})>0 \\
\iff\ & k_1^{*\alpha}(1-\mathcal{P})+k_1^{*\alpha-1}\bar{k}\mathcal{P}-k_1^{*\alpha}(1-\mathcal{P}-\bar{k}^\alpha\mathcal{P}>0 \\
\iff\ & k_1^{*\alpha-1}\bar{k}\mathcal{P}-\bar{k}^\alpha\mathcal{P}>0 \\
\iff\ & k_1^{*\alpha-1}>\bar{k}^{\alpha-1}.
\end{aligned}
$$

Since $k_1^* < \bar{k}$, the last inequality is always verified.

For the dispersion of capital, the *want to show* is

$$
\frac{\partial}{\partial B}\mathbb{E}\left[(k_1^*-\mathbb{E}\left[k_1^*\right])^2\right]=\mathbb{E}\left[\frac{\partial}{\partial B}(k_1^*-\mathbb{E}\left[k_1^*\right])^2|d_0<0\right](1-\mathcal{P})+\mathbb{E}\left[\frac{\partial}{\partial B}(\bar{k}_1-\mathbb{E}\left[k_1^*\right])^2|d_0\geq 0\right]\mathcal{P}<0,
$$

where $\mathcal{P}$ is the mass of the firms not financially constrained and $\bar{\kappa}$ is the optimal choice of capital of the non financially constrained firms. Hence:

$$
\begin{aligned}
\frac{\partial}{\partial B}\mathbb{E}\left[(k_1^*-\mathbb{E}\left[k_1^*\right])^2|d_0<0\right] &= 2(k_1^*-k_1^*(1-\mathcal{P})-\bar{k}\mathcal{P})\left(\frac{\partial k_1^*}{\partial B}-\frac{\partial k_1^*}{\partial B}(1-\mathcal{P})-\underbrace{\frac{\partial \bar{k}}{\partial B}}_{=0}\mathcal{P}\right) \\
&= 2\mathcal{P}(k_1^*-\bar{k})\frac{\partial k_1^*}{\partial B}\mathcal{P}<0.
\end{aligned}
$$

Note that the last inequality follows from the fact that $k_1^* < \bar{k}$, otherwise the mass of firms $1-\mathcal{P}$ would not be financially constrained. $\frac{\partial k_1^*}{\partial B} > 0$ is positive from the previous proof. Note that the second term is always negative

$$
\mathbb{E}\left[\frac{\partial}{\partial B}(\bar{k}_1-\mathbb{E}\left[k_1^*\right])^2|d_0\geq 0\right]=2\mathbb{E}\left[\underbrace{(\bar{k}_1-\mathbb{E}\left[k_1^*\right])}_{>0}\left(\underbrace{\frac{\partial \bar{k}_1}{\partial B}}_{=0}-\underbrace{\frac{\partial}{\partial B}\mathbb{E}\left[k_1^*\right]}_{>0}\right)|d_0\geq 0\right]<0.
$$

Furthermore, note that $R_1^* = \mathbb{E}_0[1+\alpha z_1 k_1^{*\alpha-1}-\delta]$ and

$$
\sigma(R_1^*)=\sigma^2\left(1+\alpha\mathbb{E}_0[z_1]k_1^{*\alpha-1}-\delta\right)=\alpha^2\mathbb{E}_0^2[z_1]\sigma\left(k_1^{*\alpha-1}\right).
$$

Hence:

$$\frac{\partial \sigma^2(R_1^*)}{\partial B} = \alpha^2 \mathbb{E}_0^2[z_1] \frac{\partial \sigma^2\left(k_1^{*\alpha-1}\right)}{\partial B}$$

$$= \alpha^2 \mathbb{E}_0^2[z_1] \left(2\mathcal{P} \underbrace{\left(k_1^{*\alpha-1} - \bar{k}^{\alpha-1}\right)}_{>0} \underbrace{(\alpha-1)}_{<0} k_1^{*\alpha-2} \frac{\partial k_1^*}{\partial B} \mathcal{P} + 2\mathbb{E}\left[\underbrace{\left(\bar{k}_1^{\alpha-1} - \mathbb{E}\left[k_1^{*\alpha-1}\right]\right)}_{<0} \left(\underbrace{\frac{\partial \bar{k}_1^{\alpha-1}}{\partial B}}_{=0} - \underbrace{\frac{\partial \mathbb{E}\left[k_1^{*\alpha-1}\right]}{\partial B}}_{<0} \right) |d_0 \geq 0 \right] \right) < 0.$$

Equating the two Euler equations for the price of the shares of the firms and the price of the bonds yields: $\frac{\partial}{\partial B} \mathbb{E}\left[\frac{d_1^*}{p_0^*}\right] = \frac{\partial R_1^*}{\partial B} < 0$.

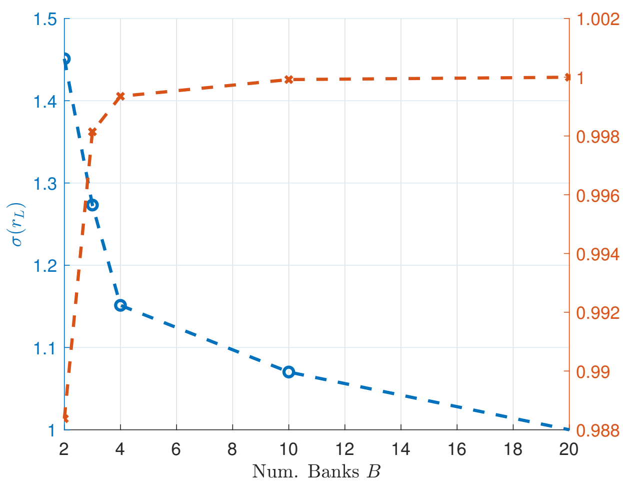
□

C Additional Material (For Online Publication)

This section is divided in two parts: (i) the first part contains additional figures, and (ii) the second part contains additional empirics.

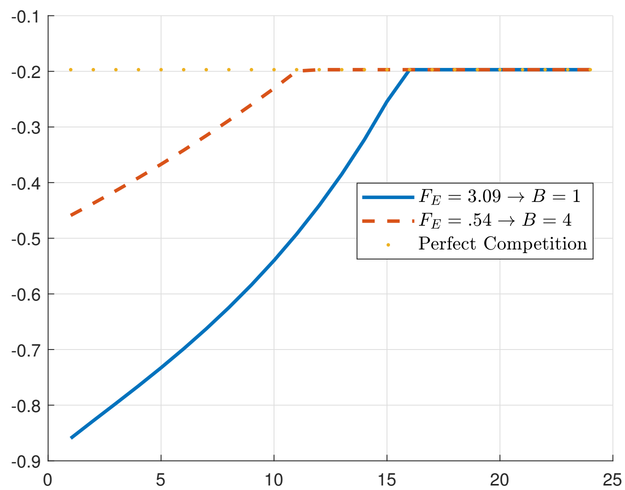
C.1 Additional Figures

Figure C1. Stationary Equilibrium and Credit Misallocation



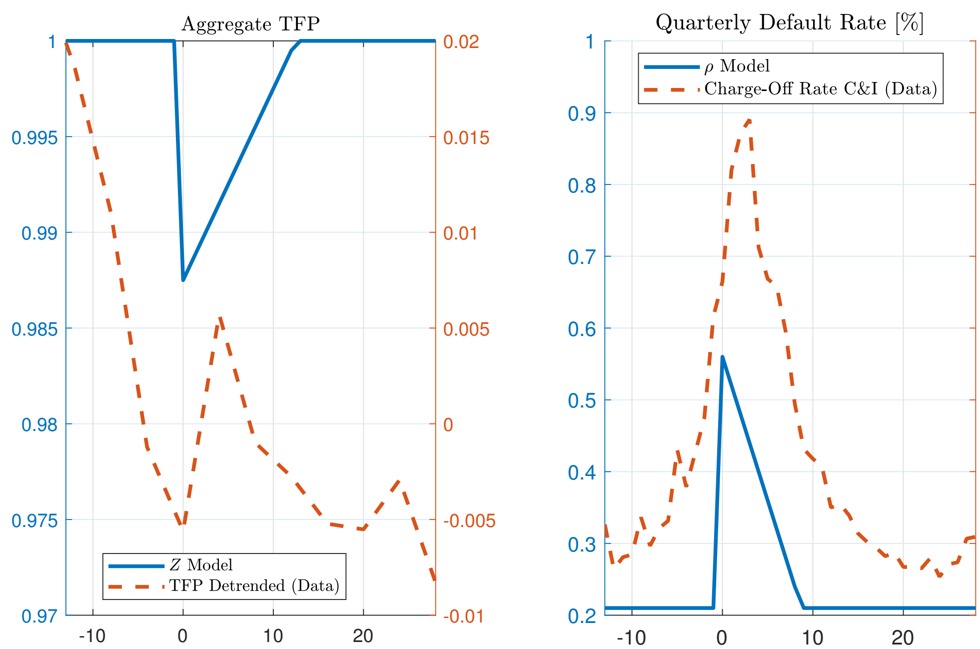
Notes: This figure reports the standard deviation of interest rates and TFP, both calculated as a ratio to those obtained in an economy with 20 banks. I vary the fixed cost to entry F_E and gradually let more banks enter the financial intermediation market. The figure illustrates the mechanism: lack of competition in the financial sector induces misallocation of credits, which is linked to misallocation of capital and a reduced productivity.

Figure C2. Stationary Equilibrium and Cross-Sectional Markups



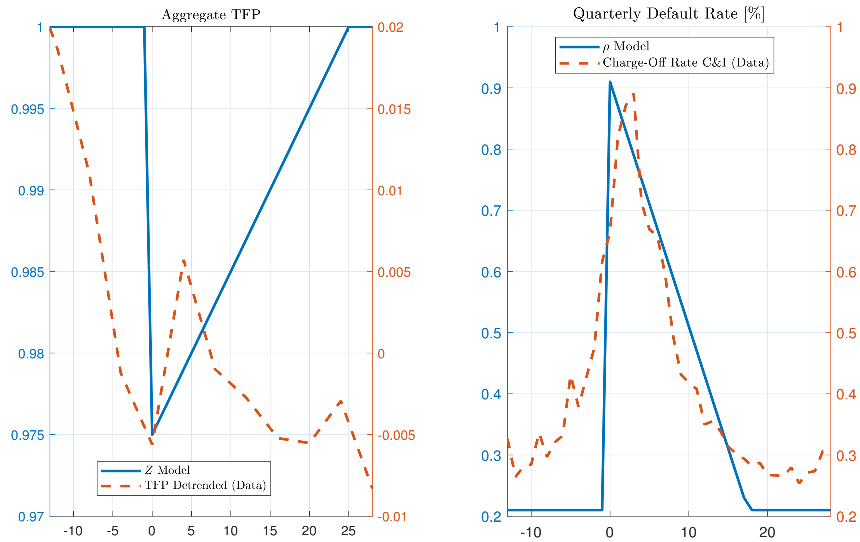
Notes: This figure reports the inverse elasticities $\eta'_L(x, X, x', X')$ contained in the generalized Euler equations (19) along the life cycle of a firm in the stationary equilibrium. These are the elasticities of the future loans' interest rate with respect to loans' quantity. The X-axes reports the firms' age.

Figure C3. Shock I



Notes: This shock is calibrated to be half the size that of the Great Recession. The X-axis reports time t, expressed in quarters both in the model and in the data. The data reported in the graph are from 2005:Q1 till 2014:Q3. TFP has been linearly detrended using data from 1997:Q2 to 2017:Q2. Following a sudden unexpected decrease in the aggregate TFP (the firms default probability $1-\rho$ at time $t = 0$ decreases) the economy mean-reverts to its original level. After the unexpected shock, all agents can perfectly forecast the mean-reversion path.

Figure C4. Shock II

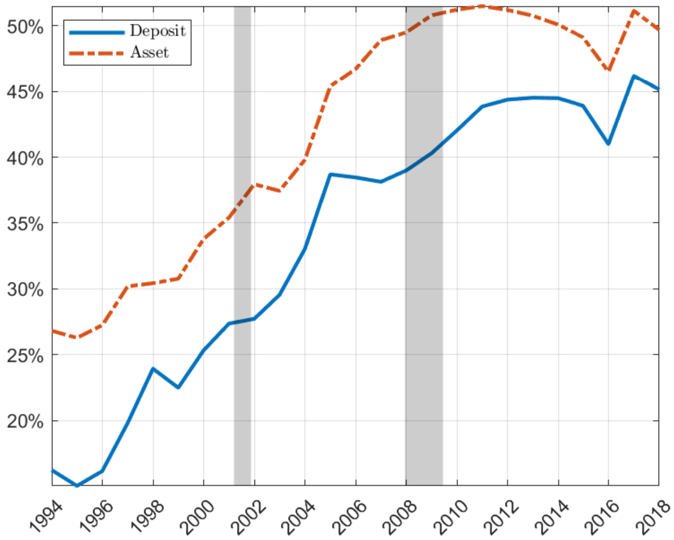


Notes: This shock is calibrated to a similar magnitude to that of the Great Recession. The X-axis reports time t, expressed in quarters both in the model and in the data. The data reported in the graph are from 2005:Q1 till 2014:Q3. TFP has been linearly detrended using data from 1997:Q2 to 2017:Q2. This same shock pushes one bank to default in the calibrated model. Following a sudden unexpected decrease in the aggregate TFP (the firms default probability $1-\rho$ at time $t=0$ decreases) the economy mean-reverts to its original level. After the unexpected shock, all agents can perfectly forecast the mean-reversion path.

C.2 Additional Empirics

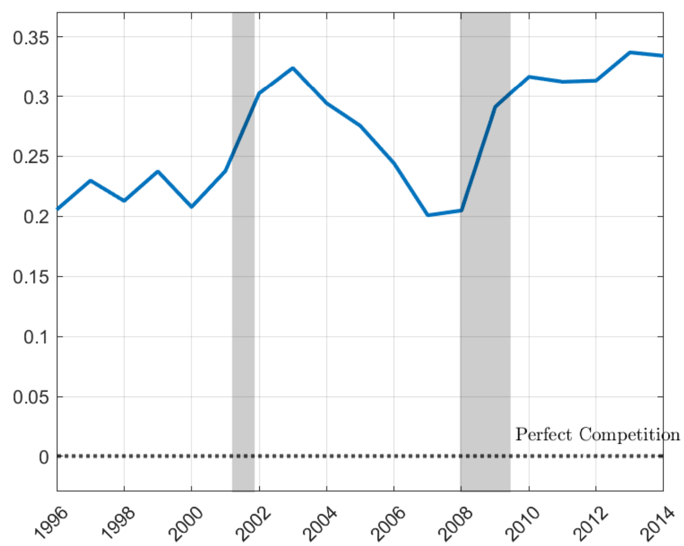
This section contains additional empirical evidences of imperfect competition in the banking sector. Figure C5 shows the time evolution of the 5-Bank Asset Concentration for the United States, which has been increasing both in terms of deposits and assets since 1995. This data are obtained using the Federal Deposit Insurance Corporation (FDIC) summary of deposits survey of branch from 1994 to 2018, from bank's level data. Banks deposits and assets are aggregated at the level of the holding bank and divided by the deposits and assets of the entire industry. As an example, in 2018, the biggest five banks in terms of assets were: (1) JP Morgan Chase & Co (14.45%), (2) Bank of America Corp. (11.73%), (3) Wells Fargo & Company (11.17%), (4) Citigroup Inc. (9.32%) and (5) U.S. Bancorp (3.02%).

Figure C5. Market Share of the Top 5 US Banks



Notes: The figure shows the evolution over time of the deposit and asset market share of the top 5 US banks. The source of the data is the FDIC summary of deposits survey of branch.

Figure C6. Lerner Index (US Banks)



Notes: This figure reports the Lerner index, which is the difference between output prices and marginal costs divided by prices: 0 indicates perfect competition and 1 indicates monopoly. Output prices are calculated as total bank revenue over assets. Marginal costs are calculated as estimated translog cost function with respect to output. The source of the data is the World Bank Global Financial Development Database.

Another evidence of imperfect competition in the banking sector is provided by the Lerner Index. As shown by Figure C6, the Lerner index has been increasing. Moreover, it is significantly different from zero, where zero is the perfect competition benchmark.

Another relevant statistics in this context is the Rosse-Panzar H index, also reported in the World Bank Global Financial Development Database. The value has fluctuated around 0.45 between 2010 and 2015. It is a measure of the elasticity of bank revenues relative to input prices: 1 indicates perfect competition, 0 (or less) indicates monopoly. Overall, these evidences suggest that there is a significant degree of imperfect competition in the banking sector.

To conclude the analysis, I now turn the attention to the data used to calibrate the model. Table C1 reports the effects that the market share of the top 5 US banks and the net charge-off rates have on C&I credit spreads. As a reminder, credit spreads are calculated as the difference between the weighted-average effective loan rate for all C&I Loans (R_L) and 3-Month T-bill rates (R_M). Two other measures are added to the analysis: (i) outstanding quantity of C&I Loans ($tn) and (ii) the weighted-average maturity for all C&I Loans. Each period t is a quarter between 1997Q2 and 2017Q2. The results of the following regression

$$R_{L,t} - R_{M,t} = \beta_0 + \beta_1 \times C_{5,t} + \beta_2 \times (1 - \rho_t) + \beta_3 \times L_t + \beta_4 \times M_t,$$

are reports in Table C1.

Table C1. Credit spread and Banks Market Concentration

	Dependent variable:		
	Commercial & Industrial Loan Rates Spreads over intended federal funds rate		
	(1)	(2)	(3)
Market share of top 5 banks (%)	0.040***	0.053***	0.056***
	(0.004)	(0.006)	(0.007)
Net Charge-Off Rate (%)	0.337***	0.295***	0.272***
	(0.051)	(0.051)	(0.059)
C&I Loans ($tn)		−0.391***	−0.345**
		(0.139)	(0.152)
Maturity			−0.121
			(0.157)
Constant	0.434**	0.384**	0.406**
	(0.183)	(0.177)	(0.179)
Observations	81	81	81
R^2	0.644	0.677	0.680
Adjusted R^2	0.635	0.664	0.663
Residual Std. Error	0.291 (df = 78)	0.279 (df = 77)	0.280 (df = 76)
F Statistic	70.500*** (df = 2; 78)	53.802*** (df = 3; 77)	40.292*** (df = 4; 76)

Note: *p<0.1; **p<0.05; ***p<0.01

Notes: There is a strong significant positive correlation between credit spread, banks market concentration, and net charge-off rate. The correlation with the quantity of C&I Loans outstanding is significant and negative, consistently with the model (the elasticity between loan and loan rate is negative).