Optimal Fiscal Policy Under Endogenous Disaster Risk:

How to Avoid Wars?

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March 2025

WP 2025-13

https://doi.org/10.21033/wp-2025-13

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Optimal Fiscal Policy under Endogenous Disaster Risk: How to Avoid Wars? *

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March 2025

Abstract

We examine the role of government investment in defense capital as a deterrence tool. Using an optimal fiscal policy framework with endogenous disaster risk, we allow for an endogenous determination of geopolitical risk and defense capacity, which we discipline using the Geopolitical Risk Index. We show both analytically and quantitatively that financing defense primarily through debt, rather than taxation, is optimal. Debt issuance mitigates present tax distortions but exacerbates them in the future, especially in wartime. However, since additional defense capital deters future wars, the expected tax distortions decline as well, making debt financing a welfare-improving strategy. Quantitatively, the optimal defense financing in the presence of heightened risk involves a twice higher share of debt and backloading of tax distortions compared to other types of government spending.

JEL Classification: E62, D52, E60.

Keywords: Optimal Fiscal Policy, Incomplete Markets, Endogenous Disaster Risk.

*We thank Anastasios Karantounias, Leonardo Melosi, Axelle Ferriere, François Gourio, Ricardo Nunes, Ethan Ilzetzki, as well as seminar participants at the Bank of Lithuania, University of Surrey, EEA-ESEM, and ES Winter Meeting, University of Manchester, Kiel-CEPR European Defence Economics Initiative. Disclaimer: The views expressed in this paper do not represent the views of the Federal Reserve Bank of Chicago or the Federal Reserve System. Declaration of conflicts of interest: none. All errors are our own.

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1 Introduction

How should fiscal policy optimally manage disaster risks? In this paper, we answer this question by means of a Ramsey problem where the planner faces the risk of war, which we model as an economic disaster. The planner can preemptively invest in defense capital stock to mitigate both the disaster probability and its damage. We find the question interesting from both theoretical and policy points of view. Theoretically, disaster risk models have demonstrated major success in explaining stock market moments (e.g., Rietz, 1988; Barro, 2006) and the business cycles, as in Gourio (2012). Yet, there is little theoretical guidance on how disaster risk affects optimal policy and how policymakers should manage these risks. Policy-wise, increasingly common calls to boost defense spending by Western policymakers beg the question, how should it be financed? Intuitively, higher defense spending acts as deterrence and creates future benefits by making disasters less likely at the cost of increasing tax distortions in the present. Optimal policy calls for spreading the benefits and smoothing taxes over time. Borrowing helps to achieve these objectives by reducing present tax distortions while, at the same time, making the disaster state distortions exceedingly costly as debt needs to be repaid in either state. The optimal debt level is determined by the balance between the current tax smoothing benefits against the expected cost of distortions in the disaster state, while taking into account that government investment in defense capital reduces the probability of disaster. We find that the optimal defense financing mix is heavily tilted toward debt compared to other types of government spending.

We study the Ramsey problem where the planner can issue non-state contingent debt and levy distortionary labor taxes, building on Aiyagari, Marcet, Sargent, and Seppala (2002). Besides standard stochastic expenditure shocks, the planner faces time-varying disaster risk in the form of exceedingly high expenditure needs and large drops in productivity. In addition to these standard policy instruments, the planner can choose to spend on defense to replenish an additional stock variable – namely, defense capital stock– that allows the planner to mitigate the disaster risk. Defense capital is valuable for two purposes. First, in its deterrence role, it mitigates the disaster probability. Second, defense capital stock and defense investment can be used to absorb some of the spending needs in the disaster state. We refer to the former as the *deterrence*, or the disaster risk management, motive, and to the latter as the *insurance* motive. The insurance benefits of defense capital confer to defense capital stock similar properties to those of a state-contingent asset. The key novelty of the paper is to study the optimal policy when the planner can affect the disaster probability, along with the use of standard policy tools. The analysis proceeds in two steps.

First, after laying out the full infinite-horizon quantitative model, we study the two-period sub-case of the model to gain analytical insights in Section 4. Such simplification allows to highlight the key trade-offs and to isolate the role of each channel separately. We show that borrowing to finance defense investment has different implications for bond prices than borrowing to finance standard government expenditure. In particular, both *deterrence* and *insurance* motives exert a negative pressure on bond prices. Abstracting from these price effects, we then study what defense spending implies for tax smoothing. By accumulating assets, the planner builds a cushion in case a disaster occurs in the future. Such reserves enable the planner to smooth taxes in all future states but require to increase taxes in the current period. Hence, such policy implies tax smoothing across states. In contrast, investment in defense, if financed through borrowing, does not require to increase current taxes and allows to mitigate the disaster risk. Yet, the debt needs to be repaid independently of the state realizing in the future, which means that smoothing of distortions in the disaster state needs to be sacrificed. We show that optimal financing of defense spending sacrifices smoothing across states to favor smoothing over time. In other words, defense spending through borrowing minimizes the disaster probability but makes the disaster more severe for the households. Finally, we show that the optimal financing mix of defense spending involves more borrowing than financing of other types of government expenditure. The intuition comes from the optimality condition that equates the excess burden of taxation today to the expected excess burden of taxation tomorrow. Debt issuance increases the expected excess burden of future taxation, as higher future taxes will be needed to repay debt. However, because of the *deterrence* channel, defense spending makes the disaster state less likely, and the expected excess burden of taxation increases by less than when debt is issued to finance other types of government spending. Consequently, through the optimality condition of the planner, the current excess burden of taxation increases less, implying lower taxes and higher debt than when debt is used to finance other types of government spending.

Second, we analyze optimal defense spending dynamics in an infinite-horizon model solved nonlinearly and globally using a neural network-based algorithm proposed in Valaitis and Villa (2024). Specifically, in Section 5, we study our baseline model dynamics comparing it to counterfactual models, where we first switch off the *insurance* channel and then, in addition, we also switch off the *deterrence* channel. In this case, the problem collapses to a standard policy problem under incomplete asset markets as in Aiyagari, Marcet, Sargent, and Seppala (2002), with the only difference that our setting contains uncertainty shocks and disasters still happen but are outside of the planner's control. To quantify the importance of defense capital, we estimate the *deterrence* channel using the threats and acts series of the Geopolitical Risk Index by Caldara and Iacoviello (2022). We also verify that the calibrated model fares well in terms of untargeted moments and in terms of responses to spending shocks, which we compare to the local projection estimates using a similar methodology as in Ramey and Zubairy (2018). The quantitative model allows us to test whether the analytical predictions from the two-period model survive in the infinite-horizon setting as well as to gain additional insights.

Even though the planner can invest in defense to mitigate disaster risk, ex-ante, it is not obvious if it is optimal to do so and which of the channels is more relevant. Instead, the planner may simply accumulate assets to be able to smooth tax distortions arising from spending needs in the disaster state. Long-run averages show that, in our baseline calibration, it is optimal to invest significant amounts of output into defense. Once we switch off the insurance channel, the average defense spending increases further as the planner has higher incentives to prevent disasters, which are now more severe. Both channels are also relevant when determining the optimal levels of debt. While in the baseline the average debt is around 86% of GDP, without the insurance channel it drops. On the one hand, the planner borrows significantly to finance defense, on the other hand, it lowers the war frequency and consequently, does not need to borrow during war episodes. Because of this, the average debt falls to 58% of GDP. When both of the channels are absent and the average war frequency is the same as in the baseline, average debt is around 75% of GDP.

We present generalized impulse responses to both government spending and uncertainty shocks. The response to the spending shock allows us to ask if it is optimal to cut defense spending when other spending needs arise and why. Results show that, under such circumstances, it is indeed optimal to cut investment in defense. Moreover, such cuts reduce the total spending needs, but falling defense stocks increase household precautionary saving motives, making debt financing cheaper and allowing to smooth tax distortions. We then study the dynamics in response to uncertainty shocks. In the standard model, the planner responds by accumulating assets, which is accompanied by a modest increase in taxes and a fall in consumption. In our baseline model and the benchmark without the *insurance* motive, the planner responds by issuing large amounts of debt to finance defense investments. This entails a large fall in household consumption and utility, which the planner finds optimal as the policy allows to minimize risks of even larger shocks. This differential response of debt to uncertainty shocks is also the driving force behind the higher average debt levels in the model with endogenous disasters.

In light of these findings, in Section 5.3 we then investigate the implications of the *deterrence* channel for tax smoothing by solving the model with an increasingly stronger *deterrence* motive. As this motive becomes stronger, consistent with our theoretical results, debt becomes more volatile and there is more borrowing in normal states, which is used to finance an increasingly relevant defense investment. As a consequence, wars tend to occur when debt levels are already high, which inhibits tax smoothing across the normal and the war states. Nevertheless, by investing more in defense, the planner reduces the war probability and, in this way, it still achieves greater tax smoothing over time. Hence, the optimal disaster risk management policy trades-off tax smoothing across states for tax smoothing over time.

In the last section, we consider two policy applications to understand the role of budget deficit rules for defense financing. First, we compare the response of debt and taxes to a government spending shock and to an uncertainty shock that induces the same response in defense investment. Results show that financing defense entails twice as large deficits and significant backloading of tax distortions. Second, we compare the role of deficit constraints for the long-run dynamics by resolving our model with an exogenously imposed 3% of GDP constraint on government primary deficits, to mimic the budget constraint rule of the EU Maastricht treaty. Results show that such deficit constraint leads to over-insurance through the accumulation of defense capital that consequently is related to a lower average war probability and average debt levels. When the deficit constraint only applies in the normal states, we find that the optimal response is to run larger surpluses in normal states and simultaneously to borrow more in war states. This effect appears to be quantitatively small in the baseline model but large in the standard model without defense investment. The difference is driven by the *deterrence* motive, prescribing larger borrowing in normal states.

Related Literature

This paper builds on the optimal fiscal policy literature (Lucas and Stokey, 1983; Barro, 1979). Specifically, it considers the Ramsey problem without state-contingent debt and under Full Commitment, following Aiyagari, Marcet, Sargent, and Seppala (2002). We contribute to the literature by merging the optimal fiscal policy approach with the disaster risk literature (Rietz, 1988; Barro, 2006, 2009) by allowing the Ramsey planner to invest in the stock variable that mitigates the disaster risk. Our framework nests Aiyagari, Marcet, Sargent, and Seppala (2002) as a special case, when the planner cannot affect the disaster probability.

The paper is not the first to study optimal policies when the planner uses policy tools to affect the actual (or perceived) event probabilities. A series of papers have considered optimal fiscal policy design under ambiguity-averse agents (Karantounias, 2013; Ferriere and Karantounias, 2019; Karantounias, 2023;

Michelacci and Paciello, 2019; Benigno and Paciello, 2014), in which case the planner has incentives to use policy tools to affect the perceived worst-case belief of the agents. The latter two papers (Michelacci and Paciello, 2019; Benigno and Paciello, 2014) study how monetary policy is affected by ambiguity-averse agents who endogenously form worst-case beliefs. Our focus is on fiscal policy. Karantounias (2013) considers a setting where agents have doubts about the probability model of government expenditures and a planner who trusts the model and acts paternalistically using contingent taxes to manage the endogenous probability of a particular state to affect prices of contingent claims. Karantounias (2023) considers a more general setting where both the agents and the planner have doubts about the true model. Ferriere and Karantounias (2019) instead consider a setting with ambiguity-averse agents and endogenous government expenditure. They uncover a crucial role of the elasticity of intertemporal substitution. The planner uses state-contingent taxes to affect the agents' perceived probability distribution and, consequently, the prices of state-contingent bonds. The correlation between government expenditure, taxes, and prices depends crucially on whether income and substitution effects dominate, as determined by the elasticity of intertemporal substitution. These papers consider how the planner can use standard policy tools to manipulate agents' beliefs in a setting with state-contingent debt, building up on Lucas and Stokey (1983). We, instead, consider the setting with non-state contingent debt following Aiyagari, Marcet, Sargent, and Seppala (2002) and ask whether the planner should resort to the standard policy tools or invest in the stock variable that affects the disaster probability.

Another closely related paper is Niemann and Pichler (2011), who consider optimal fiscal policies under disaster risk and when the planner issues non-state contingent debt. They compare and contrast policies under full commitment and no commitment to future policies. They show that, under full commitment, the planner mainly uses debt while, under no commitment, an increase in debt leads to rising inflation expectations, rendering debt issuance costly. Consequently, the planner issues little debt and resorts to using distortionary taxes. In their paper, disaster risk is completely exogenous, and the planner has no policy tools to affect its probability. Hence, our setting nests the full commitment case of Niemann and Pichler (2011) as a limiting case when the *deterrence* channel is absent.

The analysis of policies where the planner's choices endogenously affect the future size of the economy and the financing needs shares similarities to the carbon taxation literature, which uses a seminal Dynamic Integrated Climate-Economy (DICE) framework (Nordhaus, 2008). In these settings, carbon taxation affects the private sector incentives to use energy, leading to lower emissions, lower stock of carbon dioxide, and lower damage to future output. Additionally, higher emissions increase the likelihood of reaching climate tipping points, which refer to a critical threshold at which a tiny perturbation can qualitatively alter the state of a climate system. A typical approach is to consider Pigouvian taxes that correct for this climate externality, as in Golosov, Hassler, Krusell, and Tsyvinski (2014) or Cai and Lontzek (2019). Hong, Wang, and Yang (2023) study the first-best policies in a model where climate change manifests itself as an increase in the likelihood of climate disasters. They study the first-best policies to adapt to these events, which has parallels to our insurance property of defense capital. At the same time, they treat disaster probabilities as fixed. Notable exceptions of the application of the Ramsey taxation to DICE economy include Barrage (2019) and Douenne, Hummel, and Pedroni (2022). Barrage (2019) asks how carbon taxes affect the use of other distortionary taxes, namely labor and capital. Douenne, Hummel, and Pedroni (2022) extend Barrage (2019)'s analysis to include household heterogeneity and study whether inequalities and redistributive taxation call for more or less ambitious environmental policies. Both papers consider deterministic settings, while our paper is about how the planner should manage risks in an incomplete market setting. Hence, it is possible to see our work as more general and applicable to answer other questions, such as the optimal financing of green technologies.

The paper is also related to the recent work on the economics of wars (Federle, Rohner, and Schularick, 2025; Levine and Ohanian, 2024; Pflueger and Yared, 2024). Federle, Rohner, and Schularick (2025) empirically establish a causal link between exogenous changes in government's financial resources and the odds of winning military conflicts. Levine and Ohanian (2024), in a game-theoretic setting, characterize the equilibrium dynamics of deterrence and appeasement. Similarly to us, Pflueger and Yared (2024) propose a model where geopolitical risk drives the interaction between military investment and bond prices. In their setting, wars occur with exogenous probability and military investment affects the likelihood of wining the war and consequently the default spreads. We, instead, focus on the deterrence channel of military spending and how this determines the optimal financing mix between debt and distortionary taxes.

The paper is organized as follows. Section 2 provides empirical context. Section 3 presents the full infinite-horizon model. Section 4 provides analytical results from a two-period model. Section 5 contains the quantitative results. Section 6 presents two policy applications. Section 7 concludes.

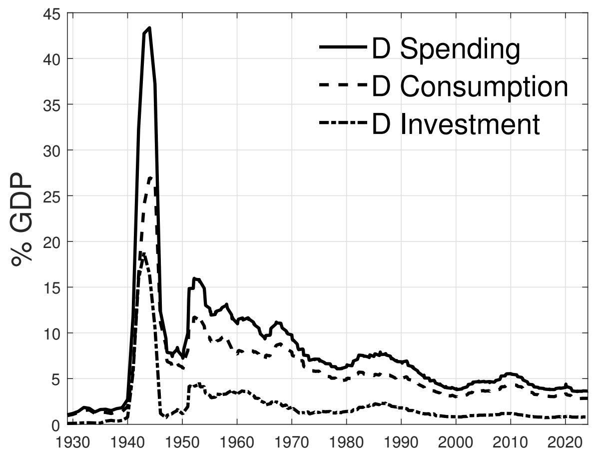
2 Context

In this section, we provide empirical evidence that we later use as guidance to motivate certain modeling choices. First, we present the relationship between geopolitical risk, defense capital, and expected real rate on US bonds. Using the geopolitical risk index data from Caldara and Iacoviello (2022), we show that the expected real rate on US bonds typically falls when the ex-ante geopolitical risk rises, consistent with the household saving behavior in presence of disaster risk. We then show that increases in the difference between ex-post realizations of geopolitical risk and its ex-ante probability are followed by declines in the US military capital investment. Second, we provide evidence of key government variables from WWII, which we consider a representative definition of a disaster. We show that disaster entails a huge increase in government spending needs that consist in large part of defense spending.

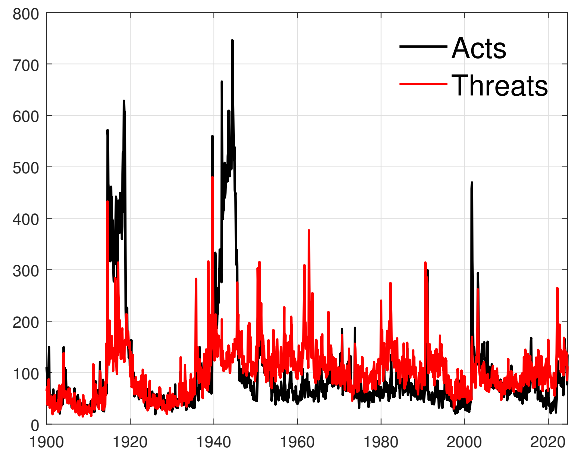
2.1 Geopolitical Risk, Defense Spending and Yields

To provide context, Figure 1 shows the evolution of the US defense capital and geopolitical risk. The left panel of Figure 1 shows the US defense spending as a share of GDP and splits it into consumption and investment components. One can think of the consumption component as salaries and other operating expenditures, and the investment component as the purchase of structures and equipment. We view the investment component as a contributing factor to the overall stock of defense capabilities, an asset that takes years to build, and that depreciates over time absent new investment. This stock also gets depleted during war episodes. The right panel shows the geopolitical risk (GPR) index from Caldara and Iacoviello (2022). The index attempts to capture geopolitical risk, defined as *the threat, realization, and escalation of adverse events associated with wars, terrorism, and any tensions among states and political actors that affect the peaceful course of international relations.* The index is constructed using a machine learning algorithm that computes the share of articles mentioning adverse geopolitical events in leading newspapers published in the United States, the United Kingdom, and Canada. While the aggregate index interprets risk to include both threats and realized events, such as wars, the authors provide separate indices for threats (red line) and acts (black line). We can see that the acts index peaks during the two world wars and the September 11 episode. The threats index is intended to capture ex-ante geopolitical risks and tends to be more stable. Notably, it steadily increased in the 1930s leading to World War II, and it was above the threats index throughout the Cold War period. Simultaneously, US defense spending was meager in the 1930s, increased dramatically during World War II, then remained above 10% of GDP during the weight of the Cold War and has been on a declining trend since then.

Figure 1: Geopolitical Risk and Defense Spending.



(a) US Defense Spending, % of GDP

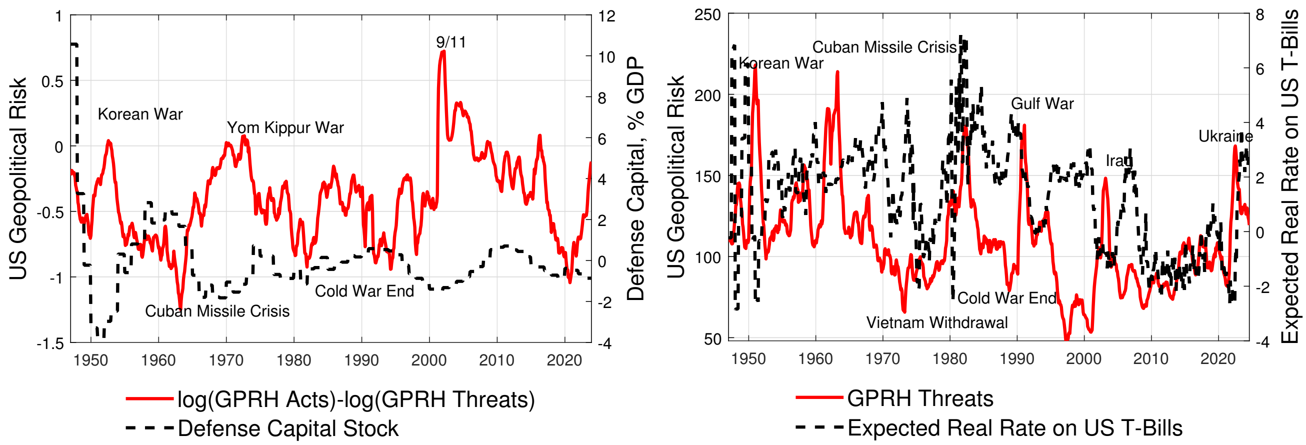


(b) Geopolitical Risk Index

Notes: The left panel shows various US annual defense spending measures, all expressed as a share of GDP. The solid line shows total defense spending. The dashed line shows defense spending that goes to consumption. The dot-dashed line shows the defense spending that goes to investment in equipment and structures. Data are sourced from the NIPA Table 3.9.5. The right panel shows the historical geopolitical risk index constructed by Caldara and Iacoviello (2022). The black line shows the geopolitical acts index. The red line shows the geopolitical threats index.

The left panel of Figure 2 presents the relation between the US defense capital stock and the geopolitical risk more systematically. The black-dashed line plots the evolution of the US defense capital stock, as a share of GDP and the red line shows the log difference between the geopolitical risk acts and threats indices. The figure reveals a negative correlation between the two. In the postwar sample, the log difference between acts and threats peaks during the September 11 episode, when the event occurred without a prior increase in underlying geopolitical tensions. It also peaks during the war episodes, such as the Korean and Yom Kippur wars. The same difference hits the lowest point during the Cuban missile crisis when no actual disaster followed after the rise in geopolitical tensions. The figure shows that such increases in the difference between acts and threats tend to follow gradual declines in the US defense capital. In contrast, the cases with large underlying risks but no geopolitical acts, such as the Cuban missile crisis, then occur when the US defense capital stock is relatively high. We attribute this to the deterrence role of military investment. The right panel shows that the expected real interest rate on US treasury bills tends to fall in periods when the ex-ante geopolitical risk is high. Notably, this can be seen during the Korean War, Cuban Missile Crisis, Gulf War, and the second Iraq invasion whereas the rate went up when the geopolitical risk was at the lowest - around the time of US withdrawal from Vietnam and during the period of relative stability of the late 90s. A similar finding has been documented in Barro (2006), which shows that US bond returns tend to fall during disaster episodes, and is consistent with Hirshleifer, Mai, and Pukthuanthong (2024), showing that wars negatively predict returns on government bonds. This is also consistent with the household precautionary saving effect in the asset pricing models with time-varying disaster risk, e.g. Wachter (2013) and Gourio (2012).

Figure 2: Defense Capital, Yields and Geopolitical Risk.



(a) Defense Capital and Geopolitical Risk

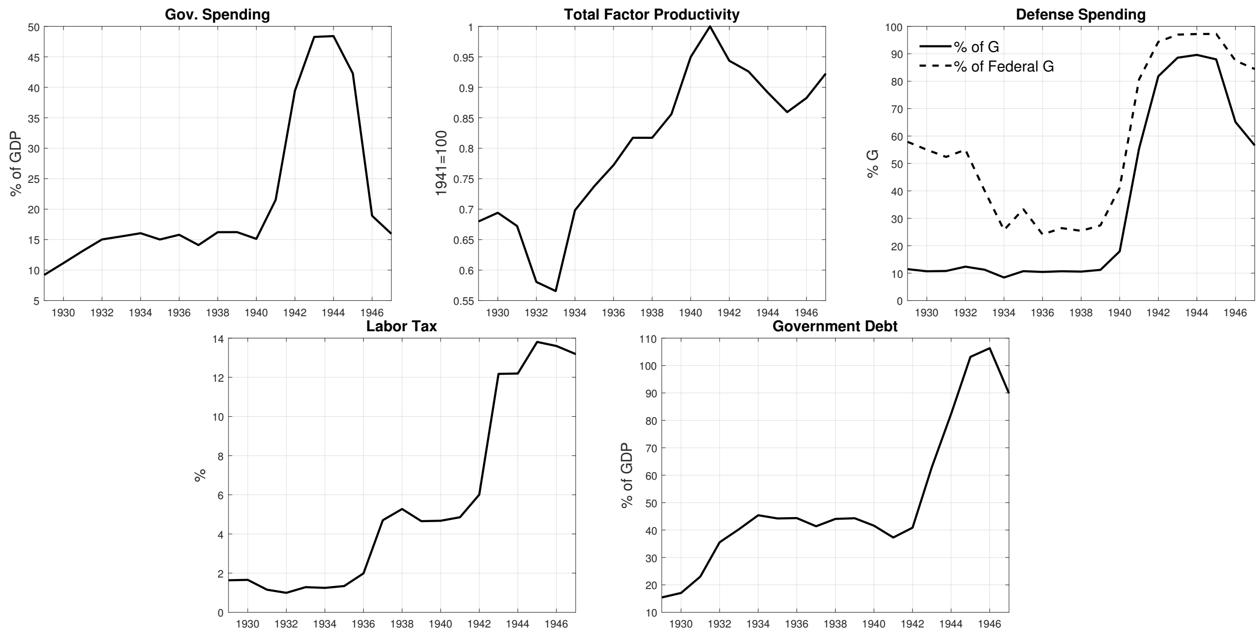
(b) US Real Rate and Geopolitical Risk

Notes: The right panel shows the Geopolitical risk threats index (GPRHT) (red line) and the expected rate on the US treasury bills. We take the 12-month moving average of the GPRHT series to remove the short-term volatility. Nominal T-bill rate is the 3-Month Treasury Bill Secondary Market Rate from FRED. For 1947-1982 expected inflation is sourced from the Livingston Survey. For 1982-2024, expected inflation is the 1-year ahead expected inflation from the Cleveland Fed. The left panel shows the log difference between the geopolitical risk acts (GPRHA) and threats (GPRHT) indices (red line) and the US defense capital stock. We take the 12-month moving average of both GPRHA and GPRHT series. US defense capital stock series is detrended by subtracting the 20-year moving average. Series comes from the NIPA Fixed Asset Table 7.1.

2.2 Disaster Dynamics

Next, we look into economic dynamics during disaster episodes. In particular, we are interested in the macroeconomic dynamics during World War II, which we consider a representative definition of a disaster considered in this paper. Figure 3 shows the dynamics of government spending, total factor productivity, government debt, labor tax rate, and defense spending.

Figure 3: Disaster state dynamics.



Notes: The figure shows the dynamics of US government spending, total factor productivity, debt-to-GDP ratio, and labor tax rate around World War II. Government spending, debt, GDP, and labor tax rate come from NIPA tables. A detailed explanation of the data construction for Government spending, debt, GDP, and labor tax rate can be found in Appendix B.3. Total factor productivity is the series estimated in Field (2023). Defense spending data comes from the NIPA Table 3.9.5.

Top panels show that disaster manifests as a simultaneous rise in government expenditure and a drop in total factor productivity. In this particular case, government spending increased by 30% of GDP up from 15% of GDP and total factor productivity fell by around 15% relative to the initial level in 1941. As Field (2023) argues, this was due to wartime supply chain disruptions, capital and manpower shortages, and the need to adapt production plants for the production of wartime goods. As is natural during wartime, most of the government spending increase was going to defense. The middle-left panel shows that defense spending – expressed as a share of total government spending – went up from 10% to 90% and almost to a 100% share of the federal government spending. The last two panels show that this increase was financed with a mix of taxes and debt. Notably, US did not default on its World War II debt in the postwar period, which is in line with the observation that postwar defaults have been unlikely in countries where active hostilities did not take place (Barro, 2006).

The next section introduces the infinite-horizon model that is consistent with the above-documented empirical facts. Most importantly, we allow defense spending to negatively affect the disaster probability and assume that the majority of the disaster spending needs can be met through defense spending.

3 Model

In this section, we describe an infinite-horizon model where the Ramsey planner levies distortionary taxes and issues non-state contingent bonds to finance an exogenous stream of government expenditures. Additionally, it invests in defense capabilities that are useful for *deterrence* and *insurance* purposes. This setting allows us to understand how the optimal defense financing mix differs from financing government expenditures and enables direct comparison with the classic Aiyagari, Marcet, Sargent, and Seppala (2002) results.

Environment

Time is discrete and infinite, $t = 0, 1, 2, \ldots$. We consider an economy consisting of a home country and a foreign country. The foreign country chooses whether to engage in conflict after observing the actions of the home government. The home country is a closed economy populated by a continuum of identical households, a continuum of identical firms, and a government. The economy is driven by two stochastic processes: (i) government expenditure in the home country, and (ii) the foreign government's preference for conflict. Allocations and policy variables in the home country are determined by a Ramsey planner who represents the home government. The planner is subject to the constraints imposed by the competitive equilibrium in the home country and the strategic responses of the foreign country. In this sense, the home government, acting through the Ramsey planner, is a Stackelberg leader, while the home country's households and the foreign country are Stackelberg followers. All agents discount the future at a common rate $\beta \in (0, 1)$.

Preferences. Households in the home country rank streams of consumption c_t and leisure l_t according to the following utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + v(l_t)\right], \tag{1}$$

where $\beta \in (0, 1)$ is the discount factor, and $u(\cdot)$ and $v(\cdot)$ are differentiable functions such that $u_c > 0$, $u_{cc} < 0$, $v_l > 0$, $v_{ll} < 0$.

Technology. Output in the home country is produced by a continuum of measure one of competitive firms with a linear production function $F(z_t, h_t) = z_t \cdot h_t$, where hours worked h are the only input and z is the exogenous labor productivity. Hence, aggregate output is given by $Y_t = F(z_t, h_t)$.

Shocks. There are two types of shocks. First, in the home country, there are government expenditure shocks denoted as g_t, which we assume to be a continuously distributed $AR(1)$ process in logs, i.e. $\ln g_t = \mu_g + \rho_g \ln g_{t-1} + \epsilon_t^g$. Second, we introduce an exogenous variable that governs the foreign country's policy preference for conflict, which in turn determines the risk of war. We label it as ξ_t and we assume that it follows another continuously distributed AR(1) process in logs, i.e. $\ln \xi_t = \mu_\xi + \rho_\xi \ln \xi_t + \epsilon_t^\xi$. We use $g^t \equiv \{g_0, g_1, ..., g_t\}$ and $\xi^t \equiv \{\xi_0, \xi_1, ..., \xi_t\}$ to denote the histories of the shock realizations up until time t. To simplify notation, we avoid explicitly denoting allocations as functions of histories g^t and ξ^t but it is understood that c_t, l_t and other allocations are measurable with respect to g^t and ξ^t.

Foreign Country. Consider a foreign country that chooses whether or not to engage in war. The decision is made in the end of the period after observing the realization of the shocks and the home country's planner policies. If a war realizes, it occurs at the beginning of the following period when the foreign country receives the war payoff. We assume that the war payoff depends on the home country's defense capital stock DS and on the foreign government's preference for conflicts ξ. Given our timing assumption, the payoff at period t depends on DS_{t-1} and ξ_{t-1} through a payoff function $V(DS_{t-1}, \xi_{t-1})$, such that $\frac{\partial V(\cdot)}{\partial DS} < 0$ and $\frac{\partial V(\cdot)}{\partial \xi} > 0$. In addition to the deterministic component $V(\cdot)$, the decision is affected by an idiosyncratic shock ϵ_t drawn from a standard logistic distribution with cumulative density function $F(\epsilon_t) = (1 + e^{-\epsilon_t})^{-1}$. The foreign country chooses to engage in a war if the total payoff is nonnegative:

$$V(DS_t, \xi_t) + \epsilon_t \geq 0.$$

Note that this condition can be rewritten as $\epsilon_t \geq -V(DS_t, \xi_t)$. Hence, given the logistic distribution of ϵ_t, the probability that the foreign country engages in a war is

$$P(\mathcal{I}_{t+1} = 1) \equiv \Pr(\epsilon_t \geq -V(DS_t, \xi_t)) = 1 - F\big(-V(DS_t, \xi_t)\big),$$

where the indicator variable $\mathcal{I}_{t+1}$ assumes the value of 1 in the war state and the value of 0 in the peace state. Using the symmetry property of the standard logistic cumulative distribution, we obtain

$$P(\mathcal{I}_{t+1} = 1) = \frac{1}{1 + e^{-V(DS_t, \xi_t)}}. \tag{2}$$

This is the logistic function that maps the defense stock DS_t and the exogenous variable ξ_t into a probability of war that lies strictly between 0 and 1.

Wars. Following the US World War II experience, we interpret wars as extreme events that happen outside of the home country and are marked by a productivity drop and an additional expenditure need. We denote the realization of government expenditure during the war state as g_t^W and assume that $g_t^W = g_t + g^e$, with g^e being a positive constant. Additionally, wartime productivity falls to a level z^W with $z^W << z$, where the productivity z during the normal state is constant.

Deterrence. The planner can choose to invest in the defense capital stock DS_t that, together with the exogenous process ξ_t, determines the disaster probability through equation (2). Indeed, given the foreign country's problem, the probability of a war occurring at time t is given by:

$$P(\mathcal{I}_{t+1} = 1) = \frac{1}{1 + e^{-V(DS_t, \xi_t)}} = P(DS_t, \xi_t),$$

where it is trivial to show that $\frac{\partial P(DS_t, \xi_t)}{\partial DS_t} < 0$ and $\frac{\partial P(DS_t, \xi_t)}{\partial \xi_t} > 0$, given our assumptions about the gradient of the foreign country's payoff.[1] The planner's incentive to invest in DS_t to affect the probability of the disaster state is at the core of our *deterrence* channel.

Insurance. We assume that a fraction ϕ of the additional government expenditure needed during war g^e can be met by depleting the defense stock. Hence, the additional spending need during the war state, g^e, is related to the undepreciated defense stock, $DS_{t-1}(1 - \delta) + D_t$ — where D_t denotes current defense

[1] Note that all our analytical results only rely on $\frac{\partial P(DS_t, \xi_t)}{\partial DS_t} < 0$ and $\frac{\partial P(DS_t, \xi_t)}{\partial \xi_t} > 0$ and not on a specific functional form for $P(DS_t, \xi_t)$.

investment — through a function $\mathcal{S}(DS_{t-1}(1-\delta)+D_t, \phi g^e)$, which captures the idea that at most a fraction ϕg^e of the additional wartime spending can be met through the undepreciated defense capital stock. In principle, this function corresponds to a min operator. Throughout the paper, $\mathcal{S}(\cdot)$ takes the form of a differentiable version of the min operator, which we specify and calibrate in Section 5.1. Since a portion ϕ of the additional financing needs in the war state can be covered using the undepreciated defense stock, the net financing needs in the war state are equal to $g_t^W - \mathcal{S}(DS_{t-1}(1-\delta)+D_t, \phi g^e)$. The *insurance* channel has a natural interpretation. Typically, a large share of wartime expenditure is defense-related and can be purchased in advance, e.g. ammunition stockpiles. If accumulated in advance, it can be used during wartime without incurring additional costs. At the same time, the stockpiles keeps depreciating if a war state does not realize and it is useful only for *deterrence* motives.

Defense Stock. Defense stock is an endogenous state variable that depreciates at a rate δ and is built up through defense investment (D_t). The stock may also get depleted during the war episodes if the insurance channel is strong, in the sense that ϕ is close to 1. More formally, defense stock evolves according to the following law of motion:

$$DS_t = DS_{t-1}(1-\delta) + D_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e), \tag{3}$$

where $\mathcal{S}(DS_{t-1}(1-\delta)+D_t, \phi g^e)$ is the portion of the defense stock used in wartime, when $\mathcal{I}_t = 1$, through the *insurance* channel.

Resources. The resource constraint of the economy is given by

$$c_t + D_t + g_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e) = Y_t = z_t h_t. \tag{4}$$

Note that we normalize the household's time endowment to one, therefore $h_t = 1 - l_t$.

Household Optimality

Households demand consumption goods, supply labor, and trade real non-contingent government bonds denoted as b_t, respectively. To simplify notation, we avoid explicitly denoting bonds as functions of histories s^{t-1} where $s^t \equiv \{g^{t-1}, \xi^{t-1}\}$, but it is understood that b_t is measurable with respect to s^{t-1}. The household budget constraint reads

$$q_t b_{t+1} + c_t \leq w_t h_t (1 - \tau_t) + b_t,$$

where w_t is the wage rate, τ_t is the proportional labor tax, and q_t is the bond's price.
Households rationality yields the following standard private sector optimality conditions:

$$q_t = \beta \mathbb{E}_t \frac{u_c(c_{t+1})}{u_c(c_t)}, \tag{5}$$

$$\tau_t = 1 - \frac{v_l(l_t)}{w_t u_c(c_t)}. \tag{6}$$

Government

The government needs to finance the exogenous stream of government spending g_t and the endogenously chosen defense spending D_t using labor income tax and bonds, subject to the following constraint:

$$g_t + D_t + b_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e) = \tau_t w_t h_t + q_t b_{t+1}. \tag{7}$$

At date t, the government chooses current tax rate τ_t, D_t, and current bonds b_{t+1}, which are measurable with respect to $\{g^t, \xi^t\}$.

Implementability Constraints

We now derive the implementability constraint of the government problem and follow Lucas and Stokey (1983) by taking the primal approach, which allows to substitute away bond prices and taxes with policy instruments.

The government budget constraint (7) can be combined with the private sector's first-order conditions (5) and (6) to obtain a sequence of recursive implementability constraints for $t = 0, 1, ...$ that reads:

$$\forall t : b_t = s_t + \mathbb{E}_t \left[\beta \frac{u_c(c_{t+1})}{u_c(c_t)} \cdot b_{t+1} \right], \tag{8}$$

where $u_c(c_t)s_t \equiv u_c(c_t)c_t - v_l(l_t)h_t$ denotes the government's surplus in marginal utility terms, and wage w_t is equal to z_t. Besides, we substitute out leisure and labor everywhere using the resource constraint (4). Also, the notation is such that $b > 0$ indicates a positive amount of government debt and $b < 0$ corresponds to government lending to households. We follow the literature on optimal fiscal policy under incomplete markets (e.g., Aiyagari, Marcet, Sargent, and Seppala, 2002; Faraglia, Marcet, Oikonomou, and Scott, 2019) and we assume that exogenous debt limits $b_t \in [\underline{b}, \overline{b}]$ are in place to prevent Ponzi schemes.

Alternatively, the implementability constraint (8) can be formulated to express the government's liabilities b_t as an expected net present value of surpluses.

$$\forall t : \; b_t = \mathbb{E}_t \left[\sum_{j=0}^{\infty} \beta^j \frac{u_c(c_{t+j})}{u_c(c_t)} \cdot s_{t+j} \right], \tag{9}$$

provided that a tangentiality condition $\lim_{t\to\infty} b_{t+1} = 0$ is in place.

3.1 The Ramsey Problem under Incomplete Markets

In this subsection, we solve for the time-inconsistent Ramsey plan under incomplete debt markets, following Aiyagari, Marcet, Sargent, and Seppala (2002). Such a problem is nonrecursive as the planner needs to keep track of all the past promises made when deciding on policies at time t. To make the problem recursive, we follow Marcet and Marimon (2019) by introducing an additional co-state variable that summarizes the previous commitments made by the planner. The Ramsey planner seeks to maximize household utility (1) by choosing the sequences of plans $\{c_t(g^t, \xi^t), l_t(g^t, \xi^t), b_{t+1}(g^t, \xi^t), D_t(g^t, \xi^t), DS_{t+1}(g^t, \xi^t)\}_{t=0}^{T}$ subject to the implementability constraint (8) with multiplier μ_t, law of motion for the defense stock (3) with multiplier μ_t^D, taking into account that the defense stock affects the probability of war $P(DS,\xi)$ through equation (2), and given initial conditions $\mu_0 = b_0 = 0$ and DS_0 set such that the probability of war at time 0 is 10%. More formally, the recursive Lagrangian of the planner reads:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ u(c_t) + v(l_t) + \mu_t(\Omega_t + \beta \mathbb{E}_t u_c(c_{t+1}) b_{t+1} - u_c(c_t) b_t) + \\ \mu_t^D (DS_{t-1}(1-\delta) + D_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e) - DS_t) + \lambda_t^D D_t \Big\},$$

where $\Omega_t \equiv s_t u_c(c_t) = u_c(c_t)c_t - v_l(l_t)h_t$ is the government primary surplus in marginal utility terms. We use the resource constraint (4) to substitute out l_t. The optimality conditions for DS_t, D_t, and b_{t+1} illustrate the key key planner's trade-offs.[2] The first-order condition with respect to bonds is:

$$\mu_t = \mathbb{E}_t(n_{t+1}\mu_{t+1}), \quad \text{where} \quad n_t \equiv \frac{u_c(c_t)}{\mathbb{E}_{t-1} u_c(c_t)}. \tag{10}$$

Equation (10) corresponds to the standard result of Aiyagari, Marcet, Sargent, and Seppala (2002) that the recursive multiplier μ_t is a risk-adjusted martingale sequence. This is because of the non-state contingent nature of government debt, past promises matter for current policies, which introduces persistence in the tax rates and debt. μ_t can also be interpreted as the excess burden of taxation. Condition (10) captures that the planner uses tax and debt policies to smooth distortions on average. Using recursive notation we can expand the expression to highlight the role of deterrence in smoothing tax distortions through DS, as shown in equation (11):[3]

$$\begin{aligned} \mu = & P(DS,\xi)\mathbb{E}_{g'|g}[\mathbb{E}_{\xi'|\xi}[n(g',\xi',\mu,b',DS,1)\mu(g',\xi',\mu,b',DS,1)]] + \\ & (1-P(DS,\xi))\mathbb{E}_{g'|g}[\mathbb{E}_{\xi'|\xi}[n(g',\xi',\mu,b',DS,0)\mu(g',\xi',\mu,b',DS,0)]]. \end{aligned} \tag{11}$$

In the standard model, the planner has no control over the probabilities of future states and uses tax and debt policies to influence the value of the excess burden of taxation (μ_{t+1}) state by state so that, in expectation, the excess burden of taxation at $t+1$ is the same as at t. We define this as *smoothing across states*. Deterrence through DS introduces an additional channel to achieve the same tax smoothing properties by influencing the probabilities of certain states realizing at $t+1$. We define this policy as *smoothing over time*. While ex-ante it is not obvious which type of smoothing is preferred, in Section 4 we investigate analytically how investment in DS affects the excess burden of taxation in various states.

The optimality conditions with respect to D_t is

$$\underbrace{\lambda_t^D + \mu_t^D \left(1 - \mathcal{I}_t \frac{\partial \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e)}{\partial D_t}\right)}_{\text{Marginal Benefit}} = \underbrace{-\mu_t \frac{\partial \Omega_t}{\partial D_t} - v_l(l_t)\frac{\partial l_t}{\partial D_t}}_{\text{Marginal Cost}}, \tag{12}$$

[2]In Appendix A.2 we report all optimality conditions.

[3]The vector of state variables X_t at time t is $X_t \equiv \{g_t, \xi_t, \mu_{t-1}, b_t, DS_{t-1}, \mathcal{I}_t\}$, where $\mathcal{I}_t \in \{0,1\}$ indicates whether the economy is in the state of war at time t. $\mathcal{I}_t = 0$ stands for the peace state and $\mathcal{I}_t = 1$ stands for the war state.

and the optimality condition with respect to DS_t is

$$\mu_t^D = \underbrace{\beta \frac{\partial P(DS_t, \xi_t)}{\partial DS_t} \mathbb{E}_t^{g,\xi} \left(U(c_{t+1}^W, l_{t+1}^W) - U(c_{t+1}^N, l_{t+1}^N) \right)}_{\text{Deterrence}} + \underbrace{\beta \mathbb{E}_t \left(\mu_{t+1} \frac{\partial \Omega_{t+1}}{\partial DS_t} + v_l(l_{t+1}) \frac{\partial l_{t+1}}{\partial DS_t} \right)}_{\text{Insurance}} +$$

$$\underbrace{\beta \mathbb{E}_t \left(\mu_{t+1}^D (1-\delta) - \mu_{t+1}^D \frac{\mathcal{I}_t \partial \mathcal{S}(DS_t(1-\delta) + D_{t+1}, \phi g^e))}{\partial DS_t} \right)}_{\text{Future Terms}}. \tag{13}$$

Naturally, these conditions highlight that the optimal investment in DS weights in marginal costs and marginal benefits. Marginal costs are contemporaneous and come from the fact that higher D_t decreases the government primary surpluses and makes the implementability constraint more binding. Additionally, the last term in equation (12) captures the resource costs of allocating more labor hours for D_t.

The marginal benefits are shifted in the future, as shown by equation (13). The first is the *deterrence* term, which captures the idea that a higher DS makes the disaster state less likely and, consequently, helps to smooth household consumption. The quantitative importance of this term depends on the degree to which DS can affect the disaster probability, as captured by the gradient of P with respect to DS. Additionally, the term becomes more important if households cannot insure against the disaster, captured by the difference in their utility in normal and disaster states. The second term captures the *insurance* channel. It consists of two terms. The first one captures the idea that higher DS can help to alleviate some of the spending needs in the disaster state and, therefore, it also helps to reduce the future excess burden of taxation. The second one is the potential saving of aggregate resources due to higher DS stock at period $t+1$ in the case of disaster. The last term captures the benefits from the same two motives beyond $t+1$. Note that *deterrence* becomes irrelevant if we make P invariant to DS and the *insurance* term becomes irrelevant if we set ϕ to 0.

Proposition 1 states the conditions under which the Ramsey policy for D_t coincides with the first-best policy, i.e. the social planner policy. As we show in Appendix A.3, the social planner's trade-offs for choosing D_t and DS_t do not involve terms related to the effects of D_t and DS_t on the government primary surpluses, Ω_t, but are otherwise similar to the Ramsey policies. Therefore, the two Ramsey policies for D_t and DS_t achieve the first-best in two cases. First when, trivially, the Ramsey planner accumulates enough assets so that it does not rely on distortionary taxes and $\mu_t = \mu_{t+1} = 0$. Second, when the distortionary effect of an additional unit of D_t is equal in expectation to the reduced need for the distortionary taxation in period $t+1$ as a consequence of a higher DS_t. Hence, the planner can potentially achieve the first-best policy for D_t by accumulating assets or by managing debt and taxes in a way that equates the current tax distortions arising from defense spending to the net expected benefit of lower future tax distortions due to a higher DS_t.

Proposition 1 ***Defense spending and the First-Best policy.***
The Ramsey and the first-best policy for D_t and DS_t coincide iff $\mu_t \frac{\partial \Omega_t}{\partial D_t} = \beta \mathbb{E}_t \left(\mu_{t+1} \frac{\partial \Omega_{t+1}}{\partial DS_t} \right)$.

Proof. See Appendix A.3. ■

Note that when the *insurance* channel is absent ($\phi = 0$), $\frac{\partial \Omega_{t+1}}{\partial DS_t}$ is equal to zero as the current investment in defense stock has no effect of future surpluses. In that case, the planner can only achieve the first-best policy for D_t by accumulating assets to the point where the implementability constraint does not bind.

As explained in Appendix A.1.1, by combining the optimality conditions for consumption and leisure, we can express the optimal tax rate as a function of elasticities and, importantly, multipliers and debt levels:

$$\tau_t = \frac{\mu_t(\epsilon_{cc} + \epsilon_{hh})}{1 + \mu_t(1 + \epsilon_{hh})} - \frac{b_t}{c_t}\epsilon_{cc}\frac{(\mu_t - \mu_{t-1})}{1 + \mu_t(1 + \epsilon_{hh})}. \tag{14}$$

Equation (14) highlights how taxes depend on past multipliers and the levels of outstanding debt.[4] While it is the same as in the standard model with exogenous disasters, it still offers insights on how the optimal optimal debt management relates to taxes. Under complete markets, $\mu_t = \mu_{t-1} = \mu$ and debt levels become irrelevant. In the standard incomplete markets setting, as in Aiyagari, Marcet, Sargent, and Seppala (2002), the planner is issuing debt to achieve smoothing across states and, therefore, μ_t is a near random walk meaning that μ_t and μ_{t-1} are typically close. This means that tax volatility is not impacted by outstanding debt levels. Under endogenous disaster management, the planner may opt for tax smoothing *over time*, which would aim to reduce the probability of bad states, while allowing the multiplier in those states to be higher than otherwise. Such policy would allow for occasional large differences between μ_t and μ_{t-1}. If the planner simultaneously issues large levels of debt, the policy of smoothing over time then allows for large changes in taxes in some periods.

4 Analysis

To isolate the mechanisms driving the policy choices in anticipation of wars, we consider a two-period version of the model, with dates denoted as t = 0, 1, along with other simplifying assumptions, as discussed below. This streamlined setting allows us to analytically characterize the behavior of multipliers, taxes, and debt in the models with and without *deterrence* and *insurance* channels.

Assumptions. We make the following four assumptions. 1. The economy consists of two periods, denoted as $t = 0, 1$. At $t = 0$, the economy is in a normal state, while the state at $t = 1$ is uncertain. 2. We assume that $\sigma(\epsilon^g) = \sigma(\epsilon^\xi) = 0$, such that $\xi_t = \exp(\mu_\xi/(1 - \rho_\xi))$ and g_t is either $\exp(\mu_g/(1 - \rho_g))$ or $\exp(\mu_g^W/(1-\rho_g))$ in the normal and the war state, respectively. 3. Household preferences are time-separable with constant Frisch elasticity of labor supply. 4. The economy does not experience a productivity drop in the disaster state, thus $z_t = 1$ for $t \in \{0, 1\}$. Relaxing any of these assumptions still allows for an analytical characterization of the planner's trade-offs but the expressions become too involved thereby offering little additional insights.

Notation. Under these assumptions, there are two states of the world in period 1, namely, war and normal. We use superscripts W and N to denote period 1 variables in the war and normal states,

[4] We thank Anastasios Karantounias for showing this tax formula. See Karantounias (2024) for a detailed discussion.

respectively. The planner's implementability constraints then read

$$\tau_0 h_0 + Q_0 b_1 = g_0 + D_1 + b_0, \text{ at } t = 0,$$
$$\tau^W h^W = g^W + b_1 - \mathcal{S}((1-\delta)D_1, \phi g^e), \text{ at } t = 1 \text{ and war,}$$
$$\tau^N h^N = g^N + b_1, \text{ at } t = 1 \text{ and peace,}$$

and the bond's optimality condition (10) simplifies to

$$\mu_0 = P(D_1)\mu^W + (1 - P(D_1))\mu^N \tag{15}$$

and, similarly, the bond's price is

$$Q_0 = \beta \left(P(D_1)\frac{u_c(c^W)}{u_c(c_0)} + (1 - P(D_1))\frac{u_c(c^U)}{u_c(c_0)} \right).$$

We begin by analyzing the planner's trade-offs. We consider hypothetical scenarios where all the planner's constraints hold – i.e., we are in a feasible competitive equilibrium – but the economy is not necessarily at the Ramsey equilibrium. First, it is instructive to compare and contrast how defense spending affects bond prices. For a benchmark, consider an increase in g_0 financed with period 0's debt issuance that is to be repaid in period 1. Equation (16) decomposes the effect on bond prices:

$$\frac{\partial Q_0}{\partial g_0} = \underbrace{\frac{\partial Q_0}{\partial c_0}\frac{\partial c_0}{\partial g_0}}_{\text{GE effect}} + \underbrace{P(D_1)\beta\frac{u_{cc}(c^W)}{u_c(c_0)}\left(\frac{\partial c^W}{\partial \tau^W}\frac{\partial \tau^W}{\partial g_0}\right) + (1 - P(D_1))\beta\frac{u_{cc}(c^N)}{u_c(c_0)}\left(\frac{\partial c^N}{\partial \tau^N}\frac{\partial \tau^N}{\partial g_0}\right)}_{\text{Higher } \tau_1}. \tag{16}$$

The first term captures the general equilibrium effect through consumption and labor supply. Through the resource constraint, higher government expenditure requires either a fall in consumption, or higher hours worked, or both. To the extent that both consumption and leisure are normal goods, this term is negative, as lower current consumption means higher marginal utility and hence lower price. The second term captures the effect of higher taxes in period 1. Higher taxes are associated with lower consumption; hence, higher future marginal utility and higher prices. This standard household intertemporal consumption smoothing channel implies that bond prices increase following debt-financed increase in government spending.

Now consider an analogous debt-financed increase in defense spending D_1. Equation (17) decomposes the effect on bond prices:

$$\frac{\partial Q_0}{\partial D_1} = \underbrace{\frac{\partial Q_0}{\partial c_0}\frac{\partial c_0}{\partial D_1}}_{\text{GE effect}} + \underbrace{P(D_1)\beta\frac{u_{cc}(c^W)}{u_c(c_0)}\left(\frac{\partial c^W}{\partial \tau^W}\frac{\partial \tau^W}{\partial D_1}\right) + (1 - P(D_1))\beta\frac{u_{cc}(c^N)}{u_c(c_0)}\left(\frac{\partial c^N}{\partial \tau^N}\frac{\partial \tau^N}{\partial D_1}\right)}_{\text{Higher } \tau_1} +$$
$$\underbrace{\beta P'(D_1)\frac{u_c(c^W) - u_c(c^N)}{u_c(c_0)}}_{\text{Deterrence}} + \underbrace{\beta P(D_1)\frac{u_{cc}(c^W)}{u_c(c_0)}\frac{\partial c^W}{\partial D_1}}_{\text{Insurance}}. \tag{17}$$

In addition to the terms in equation (16), equation (17) contains both the *deterrence* and *insurance* channels contributing to affect the bond's price.

The *deterrence* channel contained in equation (17) has a natural interpretation. D_1 makes the disaster state less likely, i.e. $P'(D_1) < 0$. Indeed, investment in defense reduces the household's precautionary saving motives and lowers the bond's price. The importance of this term depends on the gradient of disaster probability with respect to D_1 and the difference between marginal utilities of consumption in W and N states. The last term captures the *insurance* channel. Investing in D_1 alleviates spending needs in the war state. This, in turn, lowers τ^W resulting in a larger c^W and, through household intertemporal substitution motive, a lower price. These considerations lead us to formulate the following proposition, which formalizes the differential effect of debt-financed spending on bond prices.

Proposition 2 ***Defense Spending and the Bond's Price.***
Assume that the planner decides to finance an increase spending by issuing debt. Debt-financed defense spending D_1 exerts a higher negative pressure on the bond's price compared to debt-financed government spending g_0; i.e., $\frac{\partial Q_0}{\partial D_1} < \frac{\partial Q_0}{\partial g_0}$.

Proof. See Appendix A.1.2. ■

Under quasilinear preferences and assuming that b_0 is equal to 0, the optimal tax formula (14) simplifies to

$$\mu^W = \frac{\tau^W}{\epsilon_{hh} - \tau^W(1 + \epsilon_{hh})},$$

with analogous expressions for the N state and period 0 variables. It states that the multipliers in a particular state become solely an increasing function of tax rates in that state. This mapping allows to study the model behavior when both debt and taxes are allowed to adjust optimally. The planner can respond to disaster risks by either accumulating assets or by investing in D_1. If the planner chooses to invest in D_1, it can be done by either issuing debt or using current taxes. Financing through current taxes front-loads tax distortions and, importantly, allows to avoid the risk of exceedingly high tax distortions in the disaster state. In this sense, the policy allows cross-state tax smoothing in period 1 by sacrificing the smoothing between period 0 and period 1. Debt financing does the opposite. Since debt needs to be repaid in period 1 regardless of the state of the world, such financing sacrifices cross-state tax smoothing, while allowing to smooth tax distortions over time. In such a case, there is little change in tax distortions in period 0, while the expected distortions in period 1 move in an ambiguous way as higher D_1 reduces the disaster probability. The other option for the planner is to ignore the *deterrence* motive and to accumulate assets that can be used to smooth tax distortions in the disaster state. Note that if the *deterrence* motive is absent, the planner always insures by accumulating assets rather than investing in D_1 for insurance. The reason is that assets pay off in both states of the world, while D_1 only in one, hence it has a lower expected return as an investment.
Proposition 3 states that, in absence of insurance motives, the optimal mix of defense financing is such that cross-state smoothing of distortions is sacrificed. In other words, it is optimal to finance D_1 with a mix of taxes and debt. The fact that D_1 is financed with debt also means that simultaneous disaster risk management through D_1 and accumulation of assets are not optimal.

Proposition 3 ***Optimal Financing of Defense Spending.***
Assume quasilinear preferences in consumption and no insurance motive, i.e. $\phi = 0$. Optimal financing

of defense spending is such that following the increase in D_1, excess burden of taxation increases by more in the disaster state than in the normal state, i.e. $\frac{\partial \mu^W}{\partial D_1} > \frac{\partial \mu^N}{\partial D_1}$. *Optimal financing of defense spending sacrifices smoothing across states to smoothing over time.*

Proof. See Appendix A.1.3. ■

Proposition 3 assumes no insurance motive. In this case, cross-state smoothing of distortions is sacrificed whenever expenditure financing involves some debt issuance. Hence, this holds for both D_1 and g_0. Proposition 4 then highlights that the optimal mix of D_1 financing involves a larger share of borrowing and more backloading of tax distortions compared to financing g_0. The economics can be understood through the optimality condition for debt (15). Both debt-financed g_0 and D_1 increase the expected excess burden of taxation in period 1, $\mathbb{E}_0(\mu_1)$. However, because deterrence through D_1 makes the disaster state less likely, $\mathbb{E}_0(\mu_1)$ increases by less in response to debt-financed D_1 than debt-financed g_0. Consequently, through the bond optimality condition, optimal μ_0 also increases by less, meaning there is a smaller increase in current taxes and a larger increase in debt. Intuitively, optimal policy calls for spreading the benefits and smoothing taxes over time. Borrowing helps to achieve that by reducing present tax distortions, while at the same time, making the disaster state distortions exceedingly costly as debt needs to be repaid in either state. Optimal borrowing to finance defense is then determined by the balance between the current tax smoothing benefits against the expected cost of distortions in the disaster state. As defense spending minimizes the probability of the disaster states, the expected cost of borrowing in terms of future tax distortions is lower compared to borrowing to finance g_0. In this sense, Proposition 4 states that, in the absence of the *insurance* motive, the optimal financing mix of D_1 sacrifices cross-state tax smoothing by more than the optimal financing mix of government expenditure g_0.

Proposition 4 ***Debt Levels and Defense Spending.***
Assume quasilinear preferences in consumption and no insurance motive, i.e. $\phi = 0$. *Optimal level of debt responds more strongly to changes in defense spending than to changes in government expenditure; i.e.,* $\frac{\partial b_1}{\partial D_1} > \frac{\partial b_1}{\partial g_0}$.

Proof. See Appendix A.1.4. ■

The propositions above analyze the underlying mechanisms of the model when only some of the choice variables are chosen optimally while others are treated as exogenous. It is informative to see how the optimal allocations in the endogenous disasters model compare to the standard model. One way to see the role of debt is to split the Ramsey problem into two steps. In the first step, the planner chooses $\{\tau_0, \tau^N, \tau^W, D_1\}$ given an arbitrary b_1 and in the second step it optimizes b_1 taking into account the optimal responses of taxes and D_1 through a set of implementability constraints and optimality conditions. The second stage optimization would give the following expression:

$$\frac{\partial U_0}{\partial \tau_0}\frac{\partial \tau_0}{\partial b_1} + \beta P(D_1)\frac{\partial U^W}{\partial \tau^W}\frac{\partial \tau^W}{\partial b_1} + \beta(1-P(D_1))\frac{\partial U^N}{\partial \tau^N}\frac{\partial \tau^N}{\partial b_1} + \left(\frac{\partial U_0}{\partial D_1} + \underbrace{\beta\frac{\partial P}{\partial D_1}(U^W - U^N)}_{\text{Deterrence}} + \underbrace{\beta\frac{\partial U^W}{\partial D_1}}_{\text{Insurance}}\right)\frac{\partial D_1}{\partial b_1} = 0. \tag{18}$$

The first three terms capture the standard tax-smoothing motives of debt management. In doing this the planner weights the consumption smoothing benefits of lower taxes in period 0 against the costs of higher taxes in period 1. Specifically, the planner weighs the fact that debt needs to be repaid in any state of the world and the large and negative effects of higher τ^W reduce the planner's incentives to issue more debt. The last term captures all the effects related to the optimal response of D_1. It contains the marginal costs of forgone leisure in period 0 and marginal benefits due to the *deterrence* and *insurance* motives. The term is generally positive as long as the optimal D_1 is positive and increasing in b_1. Proposition 5 states that the optimal debt in the endogenous disasters model is at least as high as in the standard model and is strictly higher when the optimal D_1 is greater than zero.

Proposition 5 ***Optimal Debt Level.***
Assume quasilinear preferences and no insurance motive, i.e. $\phi = 0$. Denote with b_1 the optimal debt level and b_1^* the optimal debt level when $\frac{\partial P}{\partial D_1} = 0$. At the Ramsey optimum, $b_1 \geq b_1^*$ and $b_1 > b_1^*$ if $D_1 > 0$.

Proof. See Appendix A.1.5. ■

Note that Proposition 5 assumes that the insurance motive is absent. We consider this assumption as a restriction since adding more benefits makes it more likely that D_1 will be positive, and the optimal debt should be even higher in this more general case. Taking stock, using a two-period model we have shown that (i) debt-financed defense spending exerts negative pressure on bond prices (Proposition 2), (ii) optimal mix of defense financing involves debt (Proposition 3) and (iii) the model with deterrence channel should have more debt (Propositions 4 and 5). In the next section, we quantitatively evaluate the two-period model insights in a calibrated infinite-horizon model.

5 Quantitative Results

This section presents the quantitative results using the calibrated infinite-horizon model. We show that the qualitative results from the two-period model of Section 4 hold in the infinite-horizon model, in absence of the assumptions of the two-period model. In addition to that, quantitative results allow us to gain insights that are inaccessible analytically. We first discuss the calibration strategy and then turn to analyzing the results.

5.1 Calibration

The model frequency is annual with $\beta = 0.96$. We parameterize the utility function as follows: $u(c) = log(c)$ and $v(l) = -B\frac{(1-l)^{1+\eta}}{1+\eta}$, with $\eta = 1$ to have a Frisch elasticity $1/\eta = 1$, which is in line with estimates in the literature. We set $B = 16.99$ to match average hours worked of 1/3 of the time endowment in the first-best case of the N state. The production function is linear $F(z, h) = zh$, where z^N is normalized to a unit value and z^W is calibrated as described in the "Disasters" paragraph.

We calibrate the peacetime government expenditure process using US government expenditure data from 1947 to 2023. We define g_t to include government consumption and investment net of federal defense investment, both reported in the NIPA Table 3.9.5. The stochastic properties of g_t, ρ^g and σ^g, are estimated using the linearly detrended and deflated data series. We then set μ_g so that the model implied government share of output is equal to 13.12%, consistent with the data given our definition of g_t.

Our empirical counterpart of D_t is the US federal defense investment, which includes investment in military equipment, structures and intellectual property products. Consequently, the empirical counterpart of DS_t is the stock of the sum of these categories. We use information from the NIPA Fixed Asset Tables from 1929 to 2023 to obtain the annual depreciation rate and the stock of disaggregated DS_t categories in order to calculate the depreciation rate of each of these categories separately.[5] Calibrated δ is then the weighted average of depreciation rates by each D_t category. This procedure gives an annual depreciation rate of 9.31%.

Disasters. To calibrate parameters related to disasters, we exploit the information contained in the Caldara and Iacoviello (2022) Geopolitical Risk Index. The index can be split into two sub-indices, denoted as *threats* and *acts* indices. The *threats* index captures the underlying geopolitical risk relevant to the US without the actual events realizing. The *acts* index, on the other hand, captures the realizations of actual geopolitical events and higher index values are related to higher event intensity. To identify disasters in our sample, we normalize the *acts* index to be between 0 and 1. We interpret this as disaster probability and we classify as disasters those events where the value of the normalized index is above 0.5. Given this definition, two episodes classify as disasters: World War II and the September 11 episode. To calibrate g^e we compare the US federal government spending in the year before the disaster with the peak of government spending during the disaster. Specifically, in the World War II case, we compare the US federal government spending as a share of GDP in 1940 with the highest value during years $1941-1945$. Similarly, for the September 11 episode, we compare the US Federal government spending in 2001 with the highest value in years $2002-2003$, when the US was involved in wars in Iraq and Afghanistan as a consequence of the September 11 acts. To get g^e we then take the average difference of the two episodes. This gives the value of 0.0568 or 17.5 percent of GDP. We calibrate z^W similarly. We compare the US TFP in a year following the disaster episode with the year just before the episode. Specifically, we compare the US TFP in the year 1946 with 1941 and, similarly, the TFP in 2003 with 2001. The calibrated z^W is then the average of the two differences, which is 0.9653. To obtain the TFP data for the pre-World War II as well as for the World War II period we rely on the estimates from Field (2023).

Insurance. To discipline the insurance motive of DS, we use the two disaster episodes defined when calibrating g^e and check how much of the increase in g^e was accounted for by an increase in federal defense spending relative to GDP. If there were no increase in defense spending, this would imply $\phi = 0$; conversely, if the entire increase in federal spending were due to defense, this would imply $\phi = 1$. Averaging over the two disaster episodes yields a value of $\phi = 0.98$.

Additionally, solving the model via the system of optimality conditions requires taking the derivative of the function $\mathcal{S}(DS, \phi g^e)$, which is, in principle, a non-differentiable min function. We approximate it using the LogSumExp function:

$$\mathcal{S}(DS, \phi g^e) = \frac{1}{\alpha} \log\left(e^{\alpha DS} + e^{\alpha \phi g^e}\right) \text{ with } \alpha \leq 0,$$

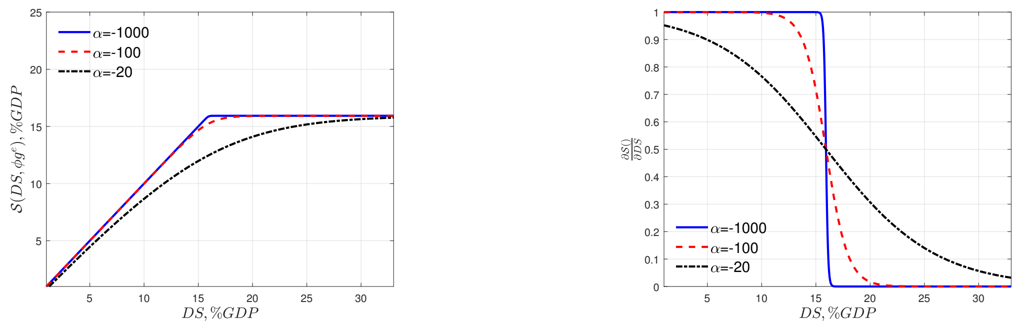
where

$$\lim_{\alpha \to -\infty} \mathcal{S}(DS, \phi g^e) = \min(DS, \phi g^e) \quad \text{and} \quad \lim_{\alpha \to 0} \mathcal{S}(DS, \phi g^e) = \frac{DS + \phi g^e}{2}.$$

[5] The NIPA Fixed Asset Tables split the defense capital stock in the following categories: (i) Equipment (Aircraft, Missiles, Ships, Vehicles, Electronics, Other equipment), (ii) Structures (Buildings, Military Facilities), and (iii) Intellectual property products (Software, Research and Development).

A key advantage of LogSumExp over other smooth maximum/minimum operators is that it yields monotonic first derivatives, which is beneficial for numerical implementation. Figure 4 illustrates the function and its derivative for different values of α.

Figure 4: LogSumExp at different values of α.



(a) $\mathcal{S}(DS, \phi g^e)$. (b) $\frac{\partial \mathcal{S}(DS,\phi g^e)}{\partial DS}$.

Deterrence. To estimate the parameters of the ξ_t process, we exploit the *threats* component of the Caldara and Iacoviello (2022) Geopolitical Risk Index. According to the authors, the *threats* index captures developments outside US control—such as war threats, military buildups, or terrorist activity—that elevate the risk of actual events. This aligns closely with our definition of ξ_t. We estimate the AR(1) parameters of $\log(\xi_t)$ directly from the *threats* index data, yielding values of -0.2528, 0.8483, and 0.2446 for μ_ξ, ρ_ξ, and σ_ξ, respectively.

We then use the Caldara and Iacoviello (2022) Geopolitical Risk Index and U.S. defense capital stock data to estimate the *deterrence* motive. Specifically, we parameterize the foreign country's payoff as $V(DS, \xi) = \beta_1 + \beta_2 \log(DS) + \beta_3 \log(\xi)$. In this case, the war probability, given by equation (2), takes the following form:

$$P(DS, \xi) = \frac{1}{1 + e^{-\beta_1 - \beta_2 \log(DS) - \beta_3 \log(\xi)}}. \tag{19}$$

We estimate the parameters $\beta_1, \beta_2, \beta_3$ using nonlinear least squares applied to equation (19). We use the normalized *acts* index as a proxy for $P(DS, \xi)$, the *threats* index as a proxy for ξ, and obtain DS data from the NIPA Fixed Asset Tables. This procedure yields estimates of $\beta_1 = -2.99$, $\beta_2 = -0.76$, and $\beta_3 = 0.87$, where β_1 governs the baseline disaster probability when $DS = 0$, and β_2 captures the sensitivity to changes in defense capital.[6] Note that a negative estimate for β_2 and a positive estimate for β_3 are consistent with our assumptions on the foreign country's payoff, i.e. $\frac{\partial V(\cdot)}{\partial DS} < 0$ and $\frac{\partial V(\cdot)}{\partial \xi} > 0$.

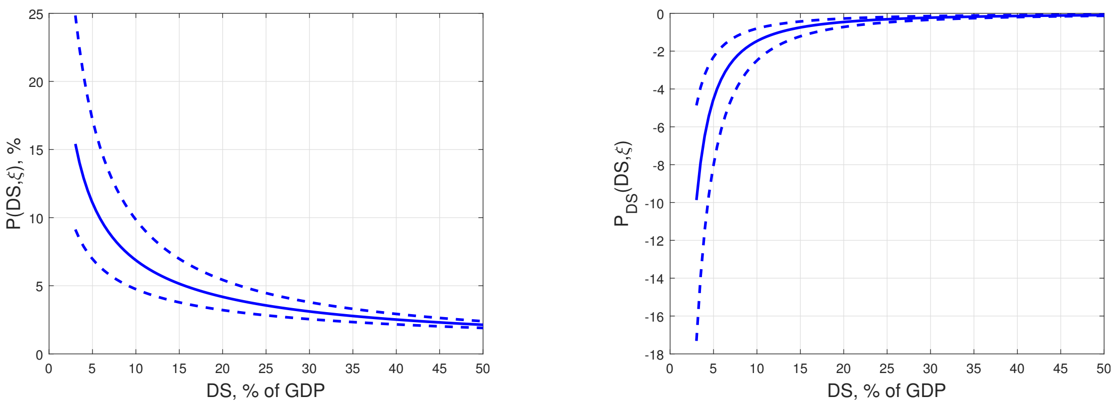
Figure 5 plots $P(DS, \xi)$ and its derivative at the estimated parameter values, as well as 10% confidence bounds for $\hat{\beta}_2$, evaluated at the average value of ξ. The left panel shows that a defense capital stock equal to 5% of GDP corresponds to an 11% disaster probability, while increasing the stock to 15% reduces this probability to below 5%. Table 1 summarizes all parameter estimates.

We solve the model using an algorithm similar in spirit to the Parameterized Expectations Algorithm proposed by den Haan and Marcet (1990). Details are provided in Appendix A.4. The method relies on stochastic simulation and uses an artificial neural network to approximate forward-looking terms in the

[6] In Appendix B.2 we provide a detailed explanation of the estimation procedure and the related data transformations. We also explore alternative specifications, which yield robust results.

optimality conditions as functions of the state vector, as proposed in Valaitis and Villa (2024). Stochastic simulation is necessary because the model features six state variables, making grid-based methods computationally infeasible. The presence of disasters introduces highly nonlinear dynamics with large deviations from average values, making the flexible, nonparametric structure of the neural network particularly useful.

Figure 5: $P(DS, \xi)$ and $P_{DS}(DS, \xi)$ at the estimated values of β_2.



Notes: The figure shows the $P(DS, \xi)$ and $P_{DS}(DS, \xi)$ as functions of DS measured as a share of GDP. Solid lines show the functions at the estimated values for β_2 and dashed lines show $P(DS, \xi)$ and $P_{DS}(DS, \xi)$ evaluated at 10% confidence bounds for β_2.

Table 1: Parameter values.

Parameter	Value	Description	Source/target
β	0.96	Discount factor	Annual frequency
γ	1	RRA	Literature
η	1	Inverse Frisch elasticity	Literature
B	16.99	Relative labor disutility	Hours 1/3 of time endowment
$\beta_1, \beta_2, \beta_3$	-2.99, -0.76, 0.87	$P(DS, \xi)$ parameters	Own estimates
δ	0.093	Depreciation rate of military capital	NIPA
$z^W - z^N$	0.0347	Output loss during disasters	Own estimates
g^e	0.0568	Govt. disaster spending, 17.5% of GDP	Own estimates
μ_g, ρ_g, σ_g	$-0.1362, 0.9558, 0.0486$	g^N parameters	US govt. spending net of defense investment
$\mu_\xi, \rho_\xi, \sigma_\xi$	$-0.2528, 0.8483, 0.2446$	ξ_t parameters	Geopolitical risk *threats* index
ϕ	0.98	DS and g substitutability	Own estimate
α	-1000	$\mathcal{S}(DS, g^e)$ parameter	High to mimic a hard min

External Validation. Next, we evaluate how the model compares with US data in terms of untargeted moments, as shown in Table 2. The model-implied averages align closely with the relevant empirical counterparts. For instance, the model implies an average defense investment of 2.26% of GDP, slightly above the 2.13% observed in the data. Unsurprisingly, the model's average government spending share $(g_t + D_t)$ is also very close to the data, at 16.02% versus 15.95%.

Moreover, the model implies a disaster probability of approximately 10.47%, which is somewhat higher than in the data, given the level of defense investment. The results also show that the planner tends to maintain positive levels of debt, averaging 86.57% of GDP—above the postwar US average of 64%, but more in line with recent levels. Importantly, these outcomes are not hardwired into the model: the planner could choose not to invest in defense and instead accumulate assets to cover disaster-time expenditures.

The model-implied defense capital stock averages 5.63% of GDP, which is significantly lower than the historical US average since 1929. This discrepancy reflects an implication of the model's *insurance* motive: disaster-time defense spending substitutes for emergency fiscal needs but does not contribute to the future defense capital stock unless D_t in the disaster state exceeds g^e, as shown in equation (3). In contrast, US data suggest that much of the military capital accumulated during World War II was retained. This explains why the model matches the defense investment rate but falls short in reproducing the average defense capital stock. Nevertheless, our number is in the ballpark of the current US defense capital stock of 8% of the GDP.

Table 2: Data and Model Untargeted Moments.

Description	Moments	Model	Data
Defense Stock to GDP, %	$\mathbb{E}(DS_t/Y_t)$	5.63	18.50
Defense Investment to GDP, %	$\mathbb{E}(D_t/Y_t)$	2.26	2.13
Gov. Spending to GDP, %	$\mathbb{E}(g_t + D_t)$	16.02	15.95
Disaster Probability, %	$\mathbb{E}(P(DS_{t-1}(1-\delta) + D_t, \xi_t))$	10.47	7.30
Debt to GDP, %	$\mathbb{E}(b_t/Y_t)$	86.57	64.02
Tax Rate, %	$\mathbb{E}(\tau_t)$	20.44	17.21

Notes: The table shows the model implied moments and the US empirical counterparts. Model moments are obtained from a simulations of 5000 periods. US data moments use years 1929-2023. Debt to GDP data uses years 1939-2023. Government spending and defense data come from the BEA NIPA Table 3.9.5 and the BEA Fixed Asset Table 7.1. The disaster probability follows the calibration procedure exploiting the Caldara and Iacoviello (2022) geopolitical risk index. The average tax rate uses the labor tax rate calculated in Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015).

In order to assess how the model performs in terms of the dynamic responses of endogenous variables, we compute impulse responses to military spending shocks in US data using the local-projection method of Òscar Jordà (2005), as implemented by Ramey and Zubairy (2018). The authors use a narrative approach to identify news about future changes in military spending, which are plausibly exogenous to the current state of the economy. These shocks are then used to isolate exogenous changes in government spending.

We use the same sample period as Ramey and Zubairy (2018), from 1929 to 2015, and estimate impulse responses via local projections including four lags of the endogenous variables as controls.[7] We focus on the empirical responses of labor taxes, debt, and government spending, shown in Figure 6. By regressing changes in these variables on the military news shock, we estimate their impulse responses to exogenous government spending changes.

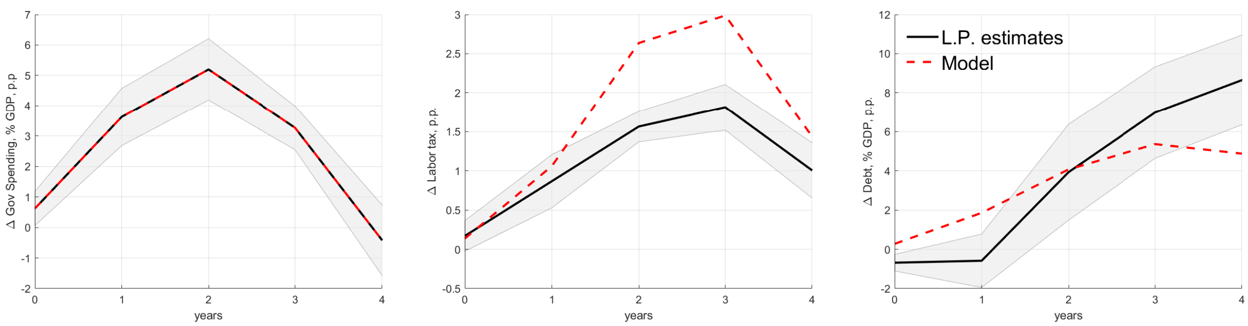
To compare the model with the data, we scale the military news shocks so that the peak response of government spending is approximately 5% of GDP, corresponding to two standard deviations of the

[7] Appendix B.4 provides a detailed description of the estimation procedure.

estimated AR(1) government spending process used in the calibration. We then solve the model for the sequence of exogenous government spending shocks that replicates the empirical government spending response.

Figure 6 plots the empirical impulse responses (solid black line) alongside the model-implied responses (dashed red line). The model replicates the qualitative patterns observed in the data: the tax response is hump-shaped, while the debt response is smaller in magnitude and more persistent. Although the model generates a somewhat larger tax increase, we view the overall alignment as satisfactory, especially since these responses were not directly targeted in the calibration.

Figure 6: Data and Model Responses.



Notes: The figure shows responses to exogenous government spending shocks in the model and in the data. Solid black lines display the results of a local projection estimation on yearly US data from 1929 to 2015, giving the impulse responses to a military spending news shock. Solid lines give point estimates and the ranges are 95% confidence intervals. Dashed-red lines display the model impulse responses to the same government expenditure shock as the local projection response in the data.

5.2 Quantitative Analysis

We now turn to the quantitative analysis using the infinite-horizon model. In addition to helping understand the quantitative importance of *deterrence* and *insurance* channels in the face of disaster risk, the infinite-horizon model allows us to understand the wartime dynamics as well as how debt is decumulated in the aftermath of wars. We proceed in three main steps. First, we present long-run dynamics obtained over multiple realizations of exogenous processes. Our main focus is on the long-run distributions of endogenous variables. Second, we analyze the dynamics during war episodes, again, obtained through long stochastic simulations over multiple realizations of exogenous processes. Third, we analyze generalized impulse responses to both g_t and ξ_t shocks.

In all these experiments, we compare the baseline model against two benchmarks. In the first benchmark, we switch off the *insurance* motive by setting $\phi = 0$ and label this as the *No Insurance* benchmark. The purpose is to identify which of the two motives drives the results and to enable direct comparison with Propositions 3 and 4, that also assume $\phi = 0$. In the second benchmark, we switch off both channels to render investment in D ineffective and label this as the *No D* benchmark. In this case, the economy still experiences ξ_t shocks but the planner can only respond to such uncertainty shocks using standard policy tools, such as taxes and debt. In this sense, it can be thought as the standard optimal policy problem in incomplete markets (Aiyagari, Marcet, Sargent, and Seppala, 2002) extended with uncertainty shocks, where disasters still happen but are outside of the planner's control. We also refer to this as the "standard model." In this benchmark, we readjust the mean of the ξ_t process so that the average disaster probability is the same as in the baseline. It would also be possible to solve a version of the model where we switch off only the *deterrence* channel. However, in this case defense capital becomes equivalent to a state-contingent asset that only pays off in the war state. In such a scenario it will always be more worthwhile to simply accumulate assets in the form of b_t.

We show that the analytical results from Section 4 hold in the infinite-horizon model. Specifically, regarding Proposition 2, we show in Subsection 5.2.3 that an increase in D_t is indeed related to a lower bond's price compared to the second benchmark. Regarding Propositions 4 and 5, we show in Subsection 5.2.1 that the baseline model has higher levels of debt. In Section 5.2.3 we show that debt responds more strongly to both shocks when *deterrence* motives are present. In Subsection 5.3, we perform an additional exercise, where we resolve the *No Insurance* benchmark by increasing the strength of the *deterrence* motive. This allows us to understand how investment in D_t affects the smoothing of distortions over time and across states. The quantitative results from this exercise are consistent with Proposition 3 suggesting that the optimal policy favors smoothing distortions over time versus across states.

Besides confirming the analytical results, the quantitative results offer additional insights. The main takeaway is that most of the peacetime investment in DS is due to *deterrence* motives. This channel is also the main force driving the differential effects of bond prices in response to both g_t and ξ_t shocks. On the other hand, both channels are relevant in determining the average debt levels. We proceed by discussing these results in greater detail.

5.2.1 Long-Run Moments

We start by comparing long-run moments across the three models. To allow for a fair comparison, in the *No D* benchmark, we set μ^g so that the peace time's average government spending $\mathbb{E}(g_t^N)$ matches the average total spending $\mathbb{E}(g_t^N + D_t)$ of the baseline model. Since g^N is linear in logs, we also adjust σ^g so that the standard deviation of g^N does not change. Furthermore, we tune μ_ξ so that the average disaster probability in the *No D* benchmark is the same as in the baseline specification.

Table 3 shows the relevant moments. Comparison across the models highlights how the *deterrence* and *insurance* channels shape the dynamics of debt, taxes, defense investment, and geopolitical risk. To isolate the role of each channel separately, we first compare the *No Insurance* and the *No D* benchmarks to isolate the role of *deterrence* and, second, we compare the baseline with *No Insurance* to isolate the role of the *insurance* channel. Finally, we compare the baseline model with the *No D* model.

First, the comparison of the *No Insurance* model (column two) with the *No D* model (column three) model highlights the role of the *deterrence* channel. The optimal level of debt is shaped by two forces. On the one hand, when the planner has the option to invest in defense, it invests significant amounts in it, and, as shown in Proposition 3, it mainly finances it with borrowing. On the other hand, through *deterrence*, the planner is able to minimize the average disaster frequency and the variation in disaster risk. As the planner also tends to borrow significantly during war episodes, all else equal, lower war frequency implies less average debt. We observe that as a consequence of endogenously lower war frequency, this *No Insurance* benchmark has lower average debt levels, which are nevertheless more volatile, consistently with

Proposition 4.

Second, the comparison of the baseline model (column one) with the *No Insurance* model (column two) highlights the role of the *insurance* channel. The option to deplete the existing DS stock as a substitute for wartime spending effectively makes wars less dramatic episodes from the tax smoothing point of view. This creates an extra incentive to maintain a large stock of DS, but at the same time makes the *deterrence* channel quantitatively less relevant, since the term $\mathbb{E}_t^{g,\xi}\left(U(c_{t+1}^W, l_{t+1}^W) - U(c_{t+1}^N, l_{t+1}^N)\right)$ in equation (13) shrinks. We observe that the second effect dominates and the planner endogenously invests less (5.63% versus 8.35% of GDP) in defense and allows for more frequent disasters (10.47% versus 7.86%). As a consequence of more frequent wars, the planner accumulates more debt on average, although it exhibits a lower volatility than that observed in the *No Insurance* benchmark.

Finally, the comparison between the baseline model (column one) and the *No D* model (column three) shows that, once we control for the average war probability, the model with endogenous disasters features more debt, consistently with Proposition 5. At the same time, even though we match the average war probability, the baseline model has endogenously lower disaster risk volatility (3.35 versus 3.95), which allows for greater tax smoothing.

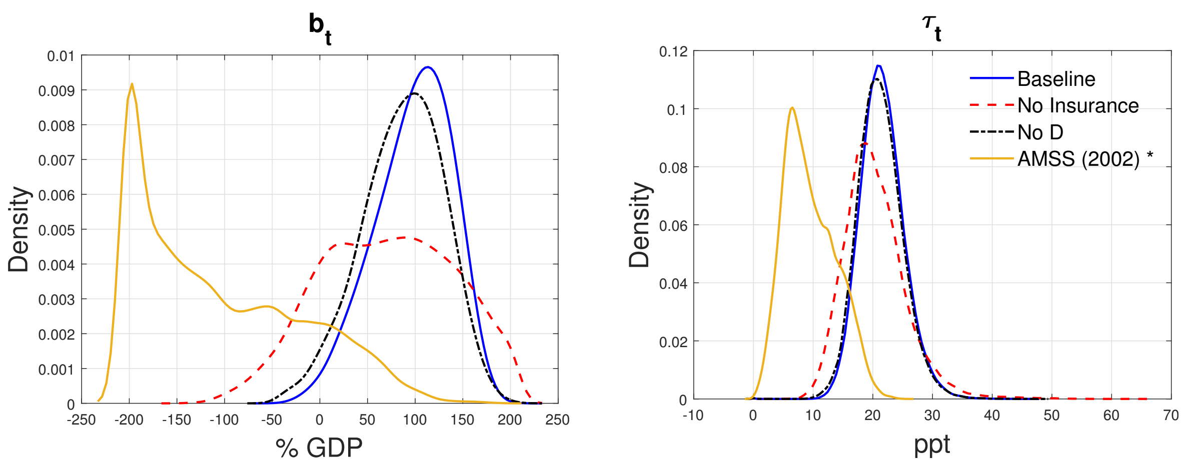
Table 3: Counterfactual Outcomes.

Moments	Baseline	No Insurance	No D
Debt, % GDP			
$\mathbb{E}(b_t/Y_t)$	86.57	57.71	74.91
$\sigma(b_t/Y_t)$	49.68	66.85	52.38
$\rho(b_t/Y_t, b_{t-1}/Y_{t-1})$	0.9910	0.9958	0.9901
Taxes, ppt			
$\mathbb{E}(\tau_t)$	20.17	18.72	19.97
$\sigma(\tau_t)$	3.79	4.05	3.86
$\rho(\tau_t, \tau_{t-1})$	0.98	0.99	0.98
Defense, % GDP			
$\mathbb{E}(DS_t/Y_t)$	5.63	8.35	-
$\mathbb{E}(D_t/Y_t)$	2.26	0.78	-
$\mathbb{E}(D_t/Y_t \vert \mathcal{I}_t = 0)$	0.73	0.85	-
War Probability, %			
$\mathbb{E}(Pr)$	10.47	7.86	10.47
$\sigma(Pr)$	3.35	2.04	3.95

Notes: The table shows the salient long-run moments across the three models. In the *No D* benchmark we adjust the average level of peace-time government expenditure to make it equal to the total peace-time government expenditure $(g + D)$ in the baseline. Moments come from 50 simulations for 5000 periods. Bond limits are set to $\pm 200\%$ of GDP.

Figure 7 shows the long-run distributions of debt and taxes, providing a more complete picture beyond the first two moments. The left panel shows that, compared to the standard model, debt in the baseline is not only higher on average but also more skewed toward higher values. In contrast, debt in the *No Insurance* benchmark is more volatile. The right panel shows that higher debt levels in the baseline do not necessarily lead to higher taxes, as the debt is used to finance defense spending, which in turn facilitates tax smoothing. For reference, we also present long-run distributions of debt and taxes of the standard optimal policy problem without ξ_t shocks and wars, as studied in Aiyagari, Marcet, Sargent, and Seppala (2002).[8] Our results confirm the underlying knowledge that the planner tends to accumulate assets and minimize tax distortions in the long-run in that case.

Figure 7: Long-run Distributions of Debt and Taxes.



Notes: The figure shows the long-run distributions of taxes and debt. Specifically, it shows fitted kernel densities using data from 50 simulations of 5000 periods each. Bond limits are set to ±200% of GDP. "AMSS (2002)*" refers to the Aiyagari, Marcet, Sargent, and Seppala (2002) model with the exception that we do not allow lump-sum transfers.

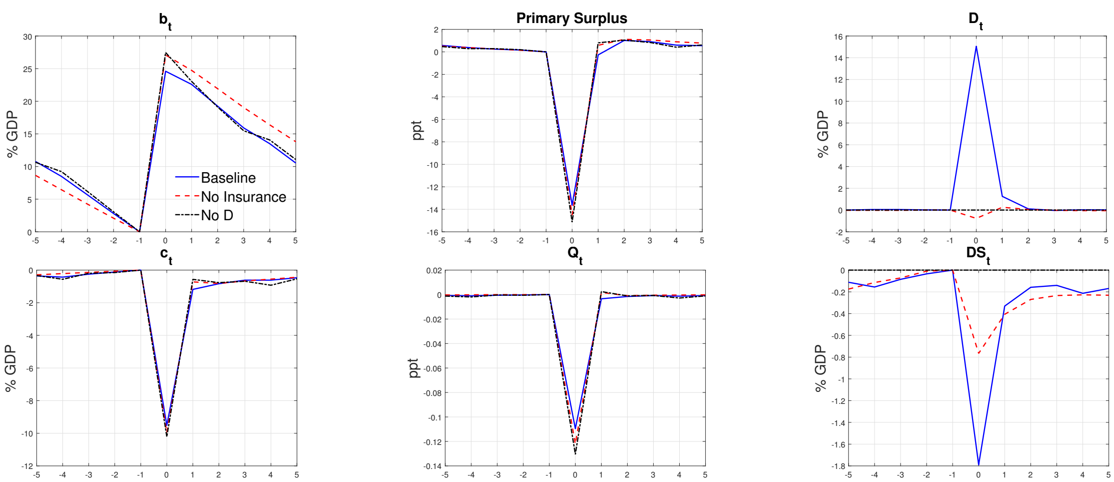
5.2.2 War Episodes

As shown above, it is optimal to engage in deterrence by building up defense stock, while simultaneously financing it with debt instead of accumulating assets. Next, we look at how the *deterrence* and *insurance* channels shape the wartime dynamics. Figure 8 shows the median dynamics around war episodes in all three models. The top left panel reports debt dynamics, which are qualitatively similar in all three models. The peacetime dynamics are marked by the gradual decumulation of wartime debt, while during the war the planner borrows heavily. In the standard model, the planner borrows heavily and runs a primary deficit of 15% of GDP, accompanied by a 10% of GDP drop in consumption. Next, consider the *No Insurance* benchmark. A small cut in defense spending increases the probability of consecutive wars, which creates upward pressure on bond prices through household precautionary saving motives. Such policy allows to run smaller deficits and is also associated with a smaller fall in consumption relative to the standard model. At the same time, the mechanism relies on a counterfactual finding that defense spending falls during wars.

[8] The only difference from the original Aiyagari, Marcet, Sargent, and Seppala (2002) paper and this benchmark is that we do not allow for lump-sum transfers from government to households. Parameterization is kept the same as in the baseline specification.

The dynamics of debt and surpluses are further dampened in the baseline model. While the planner now increases the defense spending by 14% of GDP, this is still less than the total wartime spending needs and the defense capital stock falls significantly more than in the *No Insurance* model, creating a further upward pressure on bond prices allowing for cheaper borrowing. This, together with the insurance role of DS, allows to run smaller deficits and experience smaller drops in consumption. Overall, the total primary deficits in the baseline are around 2% of GDP smaller than in the standard model.

Figure 8: Median dynamics during war episodes.



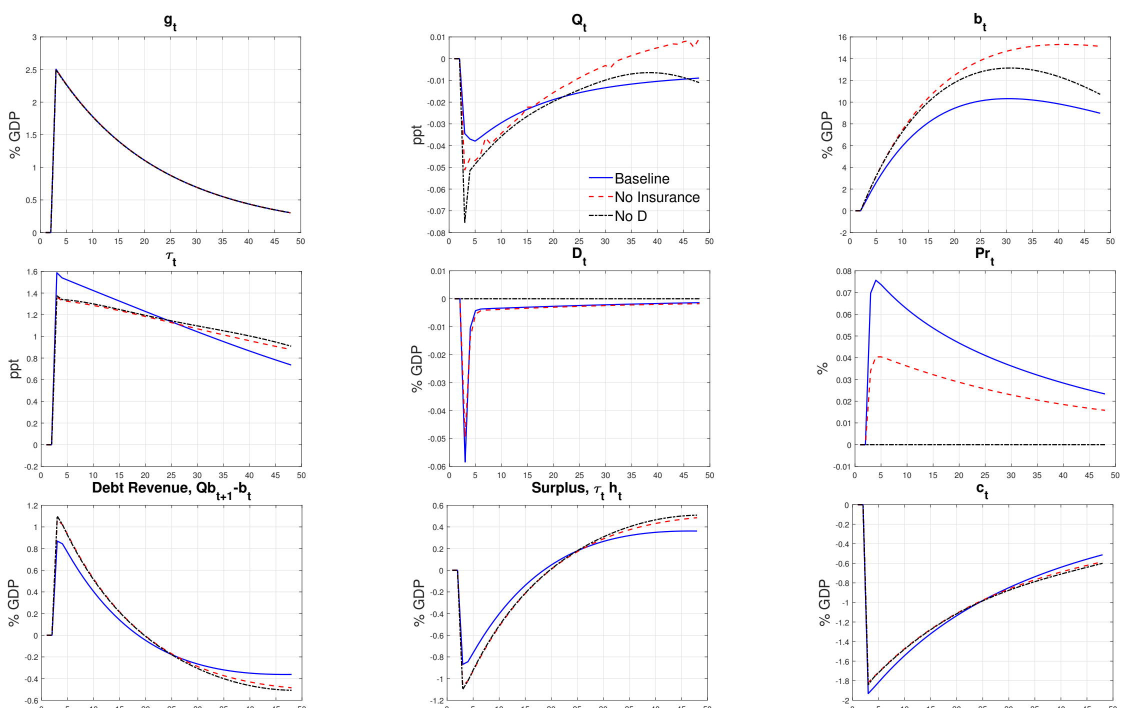
Notes: The figure shows the median model dynamics around war events. The solid blue line represents the baseline model. The dashed red line indicates the model without insurance motives. The dot-dashed black line reports the benchmark where D is ineffective. Simulated data comes from 50 shock realizations of 5000 periods each. In total, this gives 19773 war episodes in the baseline model.

5.2.3 Generalized Impulse-Responses to g_t and ξ_t shocks

We now investigate the model responses to a one standard deviation positive shock to g_t and ξ_t. The first exercise allows us to ask whether it is optimal to decrease investment in DS when other spending needs arise and what are the effects on prices and debt issuance. This allows for a neat comparison as the responses to g_t in the *No D* benchmark are the same as in Aiyagari, Marcet, Sargent, and Seppala (2002). Figure 9 shows that the response in the standard model (dot-dashed black line) is to finance an increase in spending needs with a mix of taxes and debt. An initial increase in g_t by 2.5% of GDP leads to a ~ 1.4 percentage point increase in taxes, a primary deficits of 1% of GDP, and a subsequent increase in debt by roughly 12% of GDP. This is accompanied by an immediate drop in the bond's price as falling consumption in the shock period reduces household inter-temporal motives to save. The responses of the baseline model in the blue line show that it is optimal to cut defense spending. On impact, it falls by 0.06% of output or by 8.2% of the average peace-time D spending. Such a cut has two effects. First, it allows to redirect labor tax income to financing g_t, easing the pressure to increase taxes. Second, it leads to falling defense stocks and, consequently, rising disaster probabilities. Such endogenous rise in risk increases the household's precautionary saving motives and alleviates a fall in bond prices relative to the standard model. Turning to the *No Insurance* benchmark, the relative drop in the defense investment compared to the baseline depends on two effects. On the one hand, without the *insurance* channel, disasters are more extreme events from the tax smoothing perspective, and hence, the planner would be less willing to cut D and to allow an increase in disaster probability. On the other hand, because disasters are more painful events, the planner on average maintains a large stock of defense capital and due to a nonlinear nature of the calibrated $P(D, \xi)$ function, the effect of D_t cuts on disaster probability decreases when there is more defense capital. We observe that the second effect is highly relevant. A small difference in the cut for D results in a large difference in the rise in disaster probability between the baseline and the *No Insurance* benchmark. Consequently, larger precautionary saving motives mean that the bond price in the baseline fall by less.

Defense investment cuts allow the planner to run smaller primary deficits, while the response of debt is determined by both the financing needs after accounting for a reduction in D_t as well as by how debt attractiveness is shaped by higher price and elevated disaster risks. While higher prices due to household precautionary concerns make borrowing more attractive, tax smoothing concerns call for less borrowing in the presence of elevated risk, as debt needs to be repaid in either state. We find that the second effect dominates in the baseline, while the first effect is more relevant in the *No Insurance* benchmark.

Figure 9: Generalized impulse-response to a government expenditure shock.

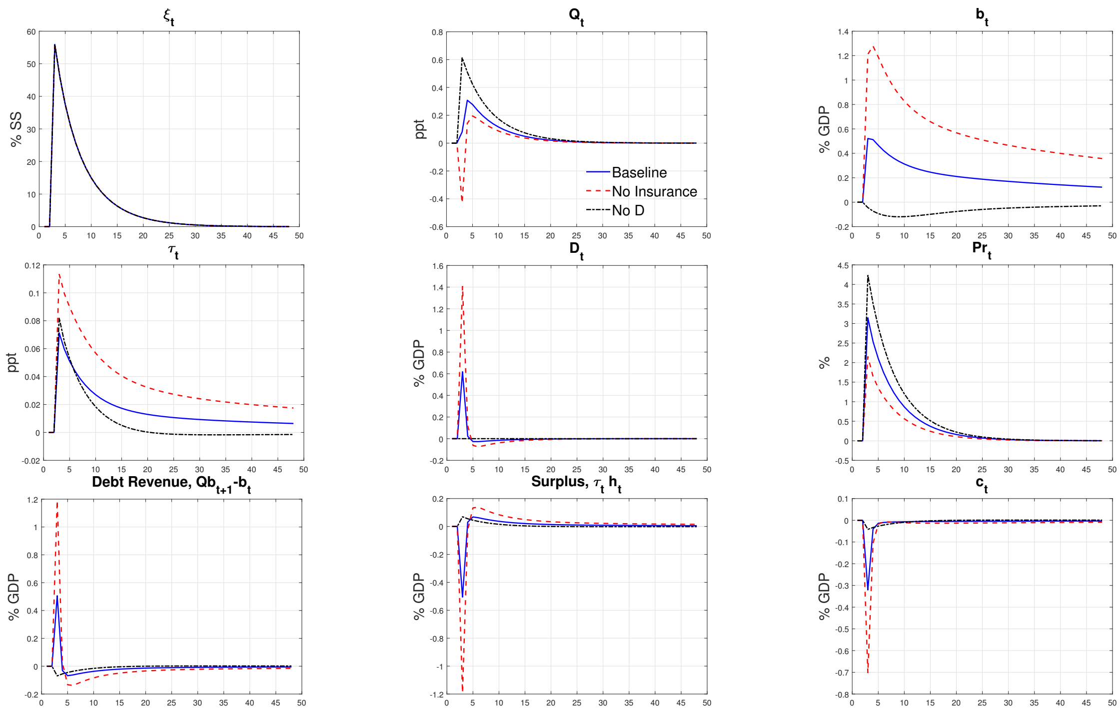


Notes: The figure shows the generalized impulse responses to a one standard deviation shock to ϵ_t^g. The solid blue line represents the baseline model. The dashed red line indicates the model without insurance motives. The dot-dashed black line reports the benchmark where D is ineffective.

Next, we consider a one standard deviation shock to ϵ_t^{ξ}, or equivalently, a 4.1% increase in disaster probability as shown by the dot-dashed black line in the middle right panel. Since keeping DS fixed, an increase in ξ rises the disaster probability, we can interpret ϵ_t^{ξ} as an uncertainty shock. In the standard model, where D is ineffective, the planner responds to an increased risk by accumulating assets financed by an increase in taxes. Essentially, the planner uses non-contingent assets to create insurance against the disaster state. Such asset purchases are associated with a minor fall in consumption and an increase in the bond's price, as the household's precautionary motives also increase in the presence of higher uncertainty. In our baseline model, as well as in the *No insurance* benchmark, the *deterrence* motives are quantitatively strong and the planner forgoes insurance motives and responds by investing in DS, which is financed by a mix of taxes and debt. Comparing the baseline with the standard model, taxes increase by a similar amount but are used for different purposes. In the baseline, the planner responds by increasing defense investment by around 0.6% of GDP for deterrence motives as it allows to mitigate the increase in the disaster probability to 3%. Overall, in responding to uncertainty shocks, the planner weighs the benefits of building up reserves to meet the disaster state versus investing in DS to mitigate the disaster's risk, at the expense of a large fall in current consumption. The analysis shows that a fall in current consumption is optimal.

Comparison between the baseline and the *No Insurance* benchmark highlights the role of the *insurance* channel. In the absence of insurance, the incentives to prevent disasters are large and therefore, the planner responds by increasing defense investment more than twice compared to the baseline, which mitigates the rise in probability to 2%. This is accompanied by larger increases in both taxes and debt. Yet, an increase in debt (1.2% versus 0.5% of GDP) relative to the baseline is disproportionately larger than the tax rate increase (0.11 ppt versus 0.7 ppt). Response of bond prices is determined by the interplay of two effects - falling current consumption and rising precautionary motives. We see that in the *No Insurance* benchmark the first effect dominates initially followed by the price increase after several periods once consumption stabilizes. Debt revenue and surplus panels at the bottom are informative about the relative financing mix. The surplus plot shows that a 0.6% of GDP increase in defense investment is accompanied by a primary deficit of 0.45% of GDP, suggesting that it is optimal to finance defense investment in the wake of increasing geopolitical risk mainly with debt, despite the implied negative pressure on bond prices specific to defense investment.

Figure 10: Generalized impulse-response to a ξ shock.



Notes: The figure shows the generalized impulse responses to a one standard deviation shock to ϵ_t^g. The solid blue line represents the baseline model. The dashed red line indicates the model without insurance motives. The dot-dashed black line the benchmark model where D is ineffective.

5.3 Smoothing taxes across states or over time?

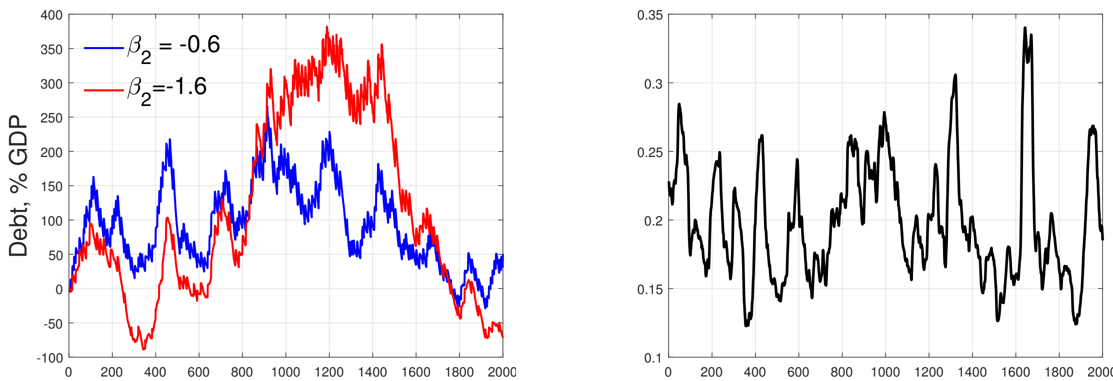
In this exercise, we are interested in understanding how the planner achieves tax smoothing, whether by minimizing the variance of tax distortions over time or across states. The two-period model intuition states that the planner optimally issues more debt to finance defense spending in the presence of increased risks (Propositions 4 and 5). This implies that in the second period, the planner ends up with higher liabilities than in the standard model and consequently sacrifices tax smoothing across war and normal states while minimizing the war probability. We are interested in how this intuition carries through to the infinite horizon. In this setting, dynamics become more involved as the planner is able to borrow and cut defense spending in war states, helping to alleviate the tax distortions. Additionally, the recursive multiplier now captures not just the tax distortions arising from the current shocks but also the history of previous shocks as planners' policies become time-dependent.

We answer these questions by resolving the *No Insurance* benchmark with different values of the marginal effect of DS on war probability, as captured by $\frac{\partial P(DS,\xi)}{\partial DS}$. We do so by changing β_2, while simultaneously adjusting β_1 such that given the defense capital stock of 8.32%, the war probability is 7.98%, consistent with our calibration, reported in Table 3.[9] Figure 11 illustrates how varying β_2 affect debt dynamics. It shows

[9] In this section, we solve the model with debt constraints at ±400% of GDP, to ensure that debt never hits these limits. This is to ensure that the behavior of taxes and multipliers is not affected by these constraints. Also, differently from Section 5.2.1, here we do not readjust average wasteful peacetime government spending g_t^N when

an excerpt from the stochastic simulation, with ξ_t series in the right panel and debt to GDP series in the left panel, which we show for high and low values of β_2. When the *deterrence* channel is strong (red line), the debt is more persistent and has a greater unconditional variance, which is consistent with Proposition 4. Essentially, in the economy with $\beta_2 = -1.6$, debt responds more to uncertainty shocks, which can be seen most clearly between periods 800 and 1000 when there is a prolonged increase in the uncertainty shock ξ_t. The increase in debt levels as well as the decline in the aftermath of the elevated uncertainty interval around period 1400 is considerably more pronounced in the economy with $\beta_2 = -1.6$. As wars are more likely to occur when uncertainty is high, it happens that in the economy with strong *deterrence* motives, wars tend to occur when the planner is already constrained by high debt levels accumulated to finance defense spending. Hence, the planner optimally accumulates more debt in response to increased uncertainty knowing that it will be ex-post more constrained when the wars occur.

Figure 11: Debt sample paths with high and low β_2.



Notes: The figure shows an excerpt from stochastic simulation from models with $\beta_2 = -0.6$ and $\beta = -1.6$. This amounts to setting it to $\pm 400\%$ of GDP.

Next, Table 4 provides a more complete picture of what these differences in debt dynamics imply for relevant policy moments and the smoothing of tax distortions. It shows selected moments from long simulations for several β_2 specifications, where more negative values are associated with the more negative marginal effect of DS and thus a stronger *deterrence* motive. As β_2 becomes more negative, the planner invests more in defense and reduces that war probability from 8.3% to 4.8% by increasing the average defense capital stock from around 5% to above 10% of GDP, which is qualitatively not surprising. The average level of debt, reported in the middle panel is shaped by two effects. As the *deterrence* channel becomes stronger, the planner borrows more to finance defense spending. This first channel captures the ex-ante borrowing motive in the presence of geopolitical risks and is associated with higher levels of debt. The planner also borrows extensively during the war episodes, therefore, an economy with lower war probability should have less debt all else equal. The middle panel shows that when the *deterrence* channel is relatively weak, the first effect dominates, and average debt increases. Then gradually the second effect begins to dominate and the average debt levels decline. Consistently with this, we notice the same hump-shaped pattern in average taxes, multipliers, and the average total government spending.

adjusting β_1.

Next, we look at the implications for smoothing of tax distortions. The debt optimality condition (11) states that planners' tax smoothing objectives imply that the recursive multiplier μ_t is a risk-adjusted marginal sequence. Hence, the closer its autocorrelation is to one, the better the planner is able to achieve the tax smoothing objectives. Results show that autocorrelations of the multiplier, taxes, and debt monotonically increase towards one as the *deterrence* channel becomes quantitatively more important. Effectively, by investing in defense, the planner minimizes the frequency of disaster shocks, which allows to smooth tax distortions over time. Such investments are mainly financed by debt, which becomes more volatile and more persistent. As the planner accumulates debt in the presence of high uncertainty, it optimally sacrifices the smoothing of distortions across states, as wars also tend to occur mostly in high uncertainty periods. Consequently, the difference between the average multiplier values in the war and normal states increases as β_2 becomes more negative. In this sense, the *deterrence* motive shapes the policy towards smoothing distortions over time and away from smoothing across states.

Table 4: Role of $\frac{\partial P(DS,\xi)}{\partial DS}$, selected moments.

Moments	$\beta_2 = -0.6$	$\beta_2 = -0.8$	$\beta_2 = -1$	$\beta_2 = -1.2$	$\beta_2 = -1.4$	$\beta_2 = -1.6$
Debt, % GDP						
$\mathbb{E}(b_t/Y_t)$	96.23	98.56	105.74	80.85	70.02	65.73
$\sigma(b_t/Y_t)$	71.66	77.51	120.28	140.51	145.10	162.17
$\rho(b_t/Y_t, b_{t-1}/Y_{t-1})$	0.9951	0.9960	0.9983	0.9990	0.9991	0.9993
Taxes and multipliers						
$\mathbb{E}(\tau_t)$	20.58	20.73	20.88	19.70	19.22	18.94
$\sigma(\tau_t)$	4.27	4.50	6.18	6.97	7.26	7.84
$\rho(\tau_t, \tau_{t-1})$	0.9815	0.9850	0.9865	0.9897	0.9891	0.9907
$\mathbb{E}(\mu_t^W) - \mathbb{E}(\mu_t^U)$	0.0043	0.0043	0.0051	0.0062	0.0066	0.0072
$\rho(\mu_t, \mu_{t-1})$	0.9898	0.9909	0.9954	0.9967	0.9968	0.9976
Other						
$\mathbb{E}((g_t + D_t)/Y_t)$	15.09	15.13	15.02	14.95	14.92	14.86
$\mathbb{E}(DS_t/Y_t)$	6.42	7.67	8.64	9.36	9.86	10.21
$\mathbb{E}(P(DS_t, \xi_t))$	8.33	7.91	6.72	5.99	5.38	4.85

Notes: The table shows selected moments from the *No Insurance* model solutions with different values of β_2. When changing β_2, we adjust β_1 so that the disaster probability is equal to 7.98% when defense stock DS is equal to 8.32% of GDP. These are the values obtained in a model solution with the estimated β_1 and β_2 parameters, reported in Table 3. To enable a clean comparison of the behavior of multipliers, in this exercise, we make debt constraints wide enough so that they never bind. This amounts to setting it to ±400% of GDP.

6 Policy Applications

In the last section, we study the role of constraints on primary surpluses in financing defense investment. In doing this, we aim to contribute to an ongoing discussion as to whether the EU should lift the budget deficit rules when the borrowing is used to finance military spending. While historical examples show that such restrictions should be removed in times of heightened geopolitical risk (Marzian and Trebesch, 2015), theoretical guidance is still scarce. We contribute to this debate by conducting two policy exercises. In the first exercise, we ask how the optimal policy mix depends on the type of shock by considering the economy's responses to g_t and D_t shocks of the same size. In the second exercise, we compare and contrast our model with the version where we impose an exogenous constraint on primary deficits.

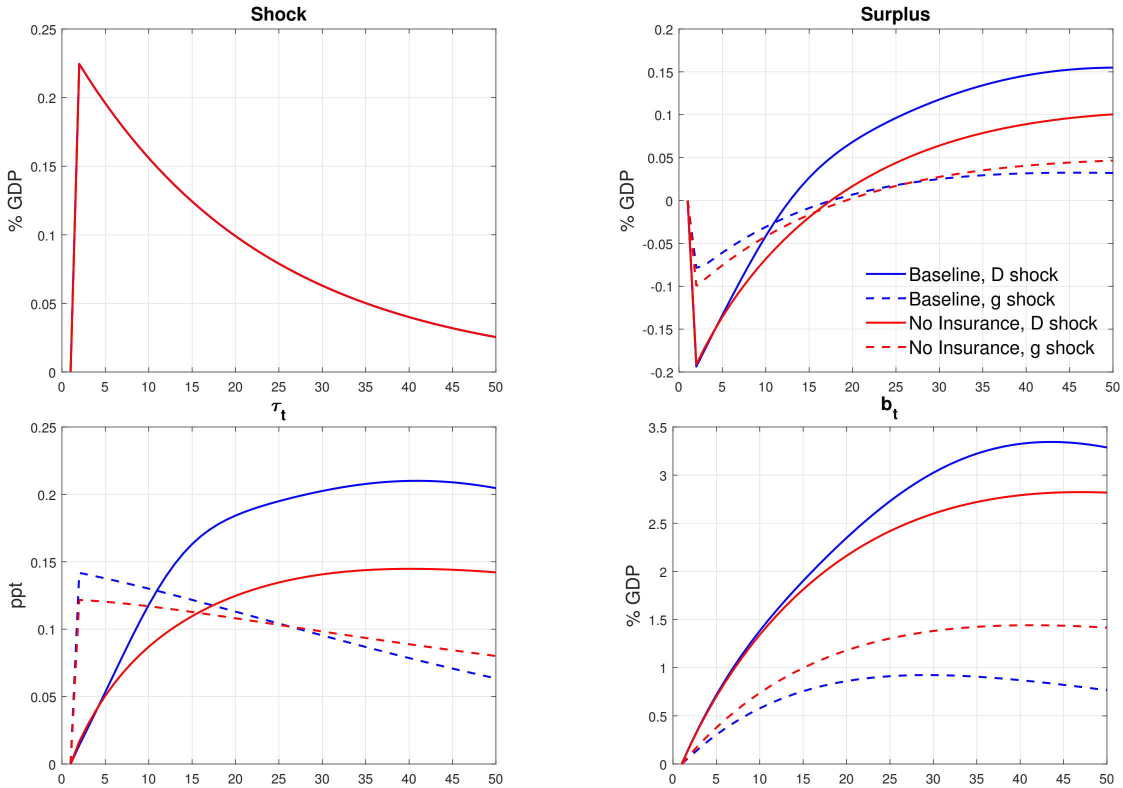
Optimal Mix of Defense Financing

One of the underlying questions behind this is whether defense investment is any different from other expenditure needs. We use our model to answer how much more the planner should borrow to finance defense compared to the government expenditure shock of the same magnitude. Specifically, we compare the model responses to a 1/3 standard deviation g_t shock and then look for the values of ξ_t shock such that the implied increase in defense investment exactly matches the government expenditure sequence.

Figure 12 shows the results. Dashed lines indicate responses to a g_t shock and solid lines - responses to an analogous increase in defense spending. Baseline model responses are depicted in blue and *No Insurance* benchmark in red. The main observation is the striking difference in responses to these two shocks. In both models, responses to a defense spending shock equivalent to an initial increase of 0.22% of GDP entail a primary deficit of almost the same size -0.18% of GDP in the baseline and in the *No Insurance* benchmark, for instance. This is followed by larger surpluses in the aftermath of the shock. This is almost twice as much as the response to a g_t shock. The timing of debt issuance and taxes also differs markedly. While the response to g_t shock implies an immediate increase in taxes with a gradual decline in the aftermath, the response to a defense spending shock entails backloading of tax distortions while most of the initial spending increase is financed with debt. Consequently, at the peak, debt increases by more than twice and the difference in debt response is more pronounced in the baseline model and is mainly driven by more muted response of debt to g_t shock, as shown in Section 5.2.3.

Both the *insurance* and the *deterrence* channels contributed to this difference. First, consider an *No Insurance* benchmark. D_t spending helps to mitigate the disaster risk. As disasters are states, where the planner's implementability constraint is likely to bind heavily, the deterrence affects the perceived disaster probability and therefore, lowers the value of the expected future multiplier on the implemantability constraint. Tax smoothing condition then dictates that the current multiplier should also fall, which entails more borrowing. This is the mechanism identified in Proposition 4, which states that borrowing enables the planner to bring the future benefits of defense investment into the present. The *insurance* channel then makes the disasters endogenously less severe in terms of tax distortions, which means that the future benefits of D_t through *deterrence* become quantitatively smaller. Therefore, the response to defense spending shock in the baseline model features slightly smaller deficits and less backloading of taxes. Yet, quantitatively, the *deterrence* channel dominates and the responses in both models are not markedly different.

Figure 12: Differential responses to D_t and g_t expenditure shocks.



Notes: The figure shows the generalized impulse-responses to a government expenditure shock (dashed lines) and to the ξ_t shock that induces the same path in D_t (solid lines). Blue lines represent the baseline model, red lines - the *No Insurance* benchmark.

Defense Spending and Fiscal Deficits

As shown above, optimal defense financing involves a policy mix heavily tilted towards debt, implying potentially large budget deficits when D_t needs to increase. At the same time, it is not uncommon for countries to face an exogenous primary deficit constraint, such as the 3% budget deficit rule inscribed in the EU Maastricht Treaty. Such constraints are meant to reduce the time inconsistency problem, which is not present in our Ramsey setting, where the planner commits to the whole path of policies from time zero. Hence, such constraints would be unambiguously welfare decreasing from the time zero perspective. We can nevertheless try to understand how harmful such budget deficit constraints are in our model as opposed to the standard model without defense investment. To answer this, we resolve the baseline model by imposing a 3% of GDP constraint on primary surpluses. We then consider a 0% of GDP primary deficit constraint that is only relevant in the normal state. We consider this constraint in the baseline as well as in the *No D* benchmark.

Table 5: Role of primary deficit constraints.

	Baseline	Baseline with constr.	Baseline with constr. excl. war	*No D*	*No D* with constr. excl. war
Debt and Taxes, % GDP					
$\mathbb{E}(b_t/Y_t)$	86.57	78.40	92.30	74.91	87.73
$\mathbb{E}(\tau_t)$	20.17	19.70	20.40	19.97	20.57
$\mathbb{E}(\Delta b_{t+1}/Y_t \vert \mathcal{I}_t = 1)$	24.34	19.26	24.82	26.76	28.28
$\mathbb{E}(\Delta b_{t+1}/Y_t \vert \mathcal{I}_t = 0)$	-2.68	-2.08	-2.73	-3	-3.16
Defense, % GDP					
$\mathbb{E}(DS_t/Y_t)$	5.63	5.78	5.63	-	-
$\mathbb{E}(D_t/Y_t \vert war_t = 0)$	0.73	0.80	0.74	-	-
War Probability, %					
$\mathbb{E}(Pr)$	10.47	10.30	10.48	10.47	10.48
$\sigma(Pr)$	3.35	3.43	3.33	3.95	3.95

Notes: The table shows selected moments for the baseline and the *No D* benchmark with and without the 3% budget deficit constraints. Data comes from 50 shock realizations of 5000 periods. Bond limits are set to $\pm 200\%$ of GDP. In the *No D* benchmark with the constraint, we readjust the average disaster probability to be the same as the baseline model with the same deficit constraint. "with constr." is a model with 3% of GDP primary surplus constraint that is relevant in both war and normal states. "with constr. excl. war" is a model with 3% of GDP primary surplus constraint that is relevant only in the normal state.

Table 5 shows the results from long-run simulations using the baseline and the *No D* benchmark with and without the budget deficit constraints. A comparison of the first two columns highlights the role of the constraint in the baseline model. The constraint inhibits the optimal debt response to uncertainty shocks as well as during the war episodes. This second force effectively makes the war episodes more severe in terms of tax distortions and consequently, creates greater incentives for the planner to avoid them. In other words, it amplifies the quantitative importance of the *deterrence* motive. This can be seen clearly in the third row, showing the average increase in debt in the war periods, which goes down from 24.34 to 19.26 percent of GDP. An optimal response to such constraint is a slightly higher average defense investment and slightly lower resulting war probability. The third column shows results from the baseline model with 0% deficit constraint that only applies in the normal state. For this case, we consider a tighter constraint as the planner tends to decumulate debt in normal times and the previous 3% constraint only binds in the war states. The planner responds to such constraint by borrowing more in war states as the constraint forces her to decumulate wartime debt faster than otherwise. This policy leads to slightly higher average debt levels and marginally higher war probability.

Next, columns four and five show the role of the same peace-time constraint in the *No D* benchmark. The effect of the constraint is qualitatively similar to the baseline model. The planner begins to borrow more in war states and decumulates debt faster in normal states, as shown in rows three and four. Yet, the quantitative importance is much larger in this standard model. The reason is that in the baseline, the unconstrained planner is willing to run more frequent deficits, which are used to borrow in response to uncertainty shocks. Consequently, even under the 0% deficit constraint, it decumulates debt more slowly than in the standard model. It is then optimal to also increase the wartime borrowing by less than in the standard model. Overall, the peacetime deficit constraint restricts the planners ability to borrow responding to uncertainty shocks and the optimal response to this is to borrow more in the war states. Yet, the quantitatively, the constraint has large much larger effect in the standard model.

7 Conclusion

This paper studies the Ramsey policy, where the planner optimally manages disaster risks where war is the disaster. The planner optimally invests in the defense capital stock responding to time-varying geopolitical risk, besides using standard policy tools, such as taxes and debt. Defense capital stock serves a dual purpose. It drives down the disaster risk, which we denotes as the *deterrence* role. Higher existing defense capital stock also helps to meet the additional spending needs in the war state. We denote this as the *insurance* role. We show that *deterrence* motives are quantitatively more important. Indeed, it is optimal to finance defense spending by borrowing and giving up tax smoothing across states to favor tax smoothing over time. The model where disasters are endogenous not only features higher levels of debt but also more responsive debt issuance in response to both government expenditure and uncertainty shocks. More broadly, we view this paper as relevant and applicable to questions such as climate change or public equity investment. Looking further, it would be interesting to explore settings where the planner cannot commit to future policies and to solve for the optimal time-consistent policy, following Klein, Krusell, and Rios-Rull (2008). In this latter case, we speculate that the negative pressure of defense investment on the bond's price could counteract the current planner's temptation to postpone tax distortions, but we leave this for future work.

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A Mathematical Appendix

The mathematical appendix is organized as follows. First, we reports all derivations and proofs. Second, we report the full derivation of the optimal policy under Full Commitment and details about the solution algorithm.

A.1 Derivations and Proofs

This subsection contains the derivation of the Optimal Tax Rate and the 5 propositions reported in the main body of the paper.

A.1.1 Optimal Tax Rate

Start by laying out the optimality conditions for consumption and leisure:

$$\begin{aligned} &u_{c,t} + \mu_t \Omega_{c,t} - b_t u_{cc,t}(\mu_t - \mu_{t-1}) = \lambda_t, \\ &- v_{l_t} + \mu_t \Omega_{h,t} = -\lambda_t z_t. \end{aligned}$$

Divide through to eliminate the multiplier λ_t to get an expression for z_t that reads

$$z_t = \frac{v_{l,t}(1 - \mu_t \Omega_{h,t}/v_{l,t})}{u_{c,t}(1 + \mu_t \Omega_{c,t}/u_{c,t} - u_{cc,t} b_t (\mu_t - \mu_{t-1})/u_{c,t})}.$$

Note that $\tau_t = 1 - \frac{v_{l,t}}{z_t u_{c,t}}$. Also, define $\epsilon_{cc} \equiv -\frac{u_{cc} c}{u_c}$, $\epsilon_{hh} \equiv -\frac{v_{ll} h}{v_l}$, and $\epsilon_{ch} \equiv \frac{v_{cl} h}{u_c}$. This allows to express primary surpluses in terms of elasticities:

$$\frac{\Omega_{c,t}}{u_{c,t}} = \frac{u_{cc,t}}{u_{c,t}} + u_{c,t}/u_{c,t} - \frac{v_{lc,t}}{u_{c,t}} = 1 - \epsilon_{cc} - \epsilon_{ch},$$

$$\frac{\Omega_h}{v_l} = -\frac{u_{cl} c}{v_l} - \frac{v_l}{v_l} + \frac{v_{ll} h}{v_l} = -\epsilon_{hc} - 1 - \epsilon_{hh}.$$

Substitute in taxes and elasticities to get the following expression for z_t

$$z_t = (1 - \tau_t) z_t \frac{1 + \mu_t (1 + \epsilon_{hh})}{1 + \mu_t (1 - \epsilon_{cc}) + \epsilon_{cc} b_t / c_t (\mu_t - \mu_{t-1})},$$

and rearrange the equation above for τ_t to finally get

$$\tau = \frac{\mu_t(\epsilon_{hh} + \epsilon_{cc}) - \epsilon_{cc} b_t / c_t (\mu_t - \mu_{t-1})}{1 + \mu_t (1 + \epsilon_{hh})}.$$

A.1.2 Proof of Proposition 2

Proof.

Start by substituting out labor supply in terms of c, D, and g using the three aggregate resource constraints:

$$\begin{aligned} &c_0 + g_0 + D_1 = h_0 \rightarrow h_0 = h(c_0, g_0, D_1), \\ &c^N + g^N = h^N \rightarrow h^N = h(c^N, g^N), \\ &c^W + g^W - \mathcal{S}((1-\delta) D_1, \phi(g^W - g^N)) = h^W \rightarrow h^W = h(c^W, g^W, g^N, D_1). \end{aligned}$$

Hence, substitute out labor supply from the household's intra-temporal optimality condition $\tau_t = 1 - \frac{v_l(1-h_t)}{c_t}$ and express consumption as a function of D, τ, and g:

$$\tau_0 = 1 - \frac{v_l(1-h(c_0, g_0, D_0))}{u_c(c_0)} \rightarrow c_0 = c_0(\tau_0, g_0, D_1),$$
$$\tau^N = 1 - \frac{v_l(1-h(c^N, g^N))}{u_c(c^N)} \rightarrow c^N = c^N(\tau^N, g^N),$$
$$\tau^W = 1 - \frac{v_l(1-h(c^W, g^W, D_1))}{u_c(c^W)} \rightarrow c^W = c^W(\tau^W, g^W, g^N, D_1).$$

The bond's price $Q_0 = P(D_1)\frac{u_c(c^W)}{u_c(c_0)} + (1-P(D_1))\frac{u_c(c^N)}{u_c(c_0)}$ is then a function of $(g_0, g^N, g^W, D_1, \tau_0, \tau^N, \tau^W)$. These substitutions also allow to express the government's revenue in marginal utility terms $\Omega \equiv h\tau u_c(c)$ as a function of $(g_0, g^N, g^W, D_1, \tau_0, \tau^N, \tau^W)$. Finally, the planner's implementability constraints define a system that relates $(\tau_0, \tau^N, \tau^W, b_1)$ to $(D_1, g_0, g^N, g^W, b_0)$ as follows:

$$(g_0 + b_0)u_c(F(\tau_0, g_0, D_1)) = \Omega_0 + b_1 P(D_1)u_c(c^W) + (1-P(D_1))u_c(c^N), \tag{20}$$
$$(g^N + b_1)u_c(F(\tau^N, g^N)) = \Omega^N, \tag{21}$$
$$(g^W + b_1 - D_1(1-\delta))u_c(F(\tau^W, g^W, D_1)) = \Omega^W. \tag{22}$$

This can be simplified further by substituting out b_1 using the period 0's implementability constraint $b_1 = \frac{(g_0+b_0)u_c(F(\tau_0,g_0,D_1))-\Omega_0}{Q_0}$, which yields $b_1 = b_1(g_0, b_0, \tau_0, D_1, g^N, \tau^N, g^W, \tau^W)$. This gives the following system of two equations in two endogenous variables τ^N and τ^W:

$$(g^N + b_1)u_c(F(\tau^N, g^N)) = \Omega^N,$$
$$(g^W + b_1 - \mathcal{S}((1-\delta)D_1, \phi(g^W - g^N)))u_c(F(\tau^W, g^W, D_1)) = \Omega^W.$$

Implicitly defining $f(\tau^W, \tau^N, D_1, g_0)$ and assuming that τ_0 is constant, one can use the implicit function theorem to calculate $\left(\frac{\partial \tau^W}{\partial D_1}, \frac{\partial \tau^N}{\partial D_1}\right)$ and $\left(\frac{\partial \tau^W}{\partial g_0}, \frac{\partial \tau^N}{\partial g_0}\right)$. These objects then allow to get $\frac{\partial Q_0}{\partial D_1}$ and $\frac{\partial Q_0}{\partial g_0}$. It remains to be shown that $\beta P'(D_1)\frac{u_c(c^W)-u_c(c^N)}{u_c(c_0)} < 0$ and $\beta P(D_1)u_{cc}(c^W)\frac{\partial c^W}{\partial D_1} < 0$.

For what regards $\beta P'(D_1)\frac{u_c(c^W)-u_c(c^N)}{u_c(c_0)}$, $P'(D_1) < 0$ by assumption and $c^W \le c^N$ as long as $g^W \ge g^N$. For what regards $\beta P(D_1)u_{cc}(c^W)\frac{\partial c^W}{\partial D_1}$, $u_{cc}(c^W) < 0$, we need to show the sign of $\frac{\partial c^W}{\partial D_1}$ using the household's intra-temporal condition:

$$u_c(c^W)(\tau^W - 1) + v_l(1 - c^W - g^W + \mathcal{S}((1-\delta)D_1, \phi(g^W - g^N))) = 0.$$

Finally, the implicit function theorem yields:

$$\frac{\partial c^W}{\partial D_1} = -\frac{(1-\delta)v_{ll}(1 - c^W - g^W + \mathcal{S}((1-\delta)D_1\phi, \phi(g^W - g^N)))\frac{\partial \mathcal{S}}{\partial D_1}}{u_{cc}(c^W)(\tau^W - 1) - v_{ll}(1 - c^W - g^W + \mathcal{S}((1-\delta)D_1, \phi(g^W - g^N)))} < 0.$$

■

A.1.3 Proof of Proposition 3

Proof. Using the optimal tax formula and assuming quasilinear preferences, so that $\epsilon_{cc} = 0$ one can express the multiplier in terms of tax rates:

$$\mu^W = \frac{\tau^W}{\epsilon_{hh} - \tau^W(1 + \epsilon_{hh})}. \tag{23}$$

Hence, the model can be characterized by the following system of equations:

$$\begin{aligned}
&\frac{\tau_0}{\epsilon_{hh} - \tau_0(1 + \epsilon_{hh})} = P(D_0)\frac{\tau^W}{\epsilon_{hh} - \tau^W(1 + \epsilon_{hh})} + (1 - P(D_0))\frac{\tau^N}{\epsilon_{hh} - \tau^N(1 + \epsilon_{hh})},\\
&g^W + b_1 = h(\tau^W)\tau^W,\\
&g^N + b_1 = h(\tau^N)\tau^N,\\
&g_0 + b_0 + D_1 = h(\tau_0)\tau_0 + \beta b_1,
\end{aligned}$$

where we have used the household's optimality condition $\tau_t = 1 - v_l(1 - h_t)$ to express h as functions of the tax rate.

To simplify the system further, first express τ in terms of the multiplier: $\tau^x = \frac{\mu^x \epsilon_{hh}}{1+\mu^x(1+\epsilon_{hh})}$. Substitute out μ_0 using $\mu_0 = P(D_1)\mu^W + (1 - P(D_1))\mu^N$. Also substitute b_1 using $b_1 = 1/\beta(g_0 + b_0 + D_1 - h_0\tau_0)$, where it is understood that h^x is a function of τ^x, and τ^x is a function of μ^x. μ_0 is a function of μ^W, μ^N and D_1. Then the model can be summarized by the following system:

$$\begin{aligned}
h^W\tau^W - g^W - 1/\beta(g_0 + b_0 + D_1 - h_0\tau_0) = 0,\\
h^N\tau^N - g^N - 1/\beta(g_0 + b_0 + D_1 - h_0\tau_0) = 0.
\end{aligned}$$

This system can be thought as an implicit function $f(\mu^W, \mu^N, D_1) = 0$. Apply the implicit function theorem to compute:

$$\frac{\partial \boldsymbol{\mu}}{\partial D_1} = -f_\mu^{-1} f_D, \tag{24}$$

where, after defining $H^x \equiv \frac{\partial h^x \tau^x}{\partial \mu^x}$ for ease of notation,

$$f_\mu = \begin{pmatrix} H^W + 1/\beta H_0 P(D_1) & 1/\beta H_0(1 - P(D_1)) \\ 1/\beta H_0 P(D_1) & H^N + 1/\beta H_0(1 - P(D_1)) \end{pmatrix},$$

and

$$f_D = \begin{pmatrix} -1/\beta + 1/\beta H^0 P'(D_1)(\mu^W - \mu^N) \\ -1/\beta + 1/\beta H^0 P'(D_1)(\mu^W - \mu^N) \end{pmatrix}.$$

In order to compute f_u^{-1} in equation (24), we need to compute the determinant $det(f_\mu)$ and the adjugate

$adj(f_\mu)$. Respectively, these are:

$$det(f_\mu) = [H^W + 1/\beta H_0 P(D_1)][H^N + 1/\beta H_0(1-P(D_1))] - 1/\beta H_0 P(D_1) 1/\beta H_0 (1-P(D_1)) = \\ = H^W H^N + 1/\beta H^W H_0(1-P(D_1)) + 1/\beta H^N H_0 P(D_1),$$

and

$$adj(f_\mu) = \begin{pmatrix} H^N + 1/\beta H_0(1-P(D_1)) & -H_0 1/\beta(1-P(D_1)) \\ -H_0 1/\beta P(D_1) & H^W + H_0 1/\beta P(D_1) \end{pmatrix}.$$

Finally, multiply $f_\mu^{-1} = adj(f_\mu)/det(f_\mu)$ by f_D to rewrite (24) as:

$$\frac{\partial \mu}{\partial D_1} = -\frac{-1/\beta + 1/\beta H^0 P'(D_1)(\mu^W - \mu^N)}{det(f_\mu)} \begin{pmatrix} H^N + 1/\beta H_0(1-P(D_1)) - 1/\beta H_0(1-P(D_1)) \\ -H_0 1/\beta P(D_1) + H^W + H_0 1/\beta P(D_1) \end{pmatrix},$$

which further simplifies to

$$\frac{\partial \boldsymbol{\mu}}{\partial D_1} = -\underbrace{\frac{-1/\beta + 1/\beta H^0 P'(D_1)(\mu^W - \mu^N)}{det(f_\mu)}}_{\equiv \mathcal{Z}} \begin{pmatrix} H^N \\ H^W \end{pmatrix}, \tag{25}$$

and where H^x can be written as:

$$H^x = \frac{\partial h^x \tau^x}{\mu^x} = \frac{\partial h^x \tau^x}{\partial \tau^x}\frac{\partial \tau^x}{\partial \mu^x} = \underbrace{\frac{\partial h^x \tau^x}{\partial \tau^x}}_{\text{Laffer slope}} \frac{\epsilon_{hh}}{1 + \mu^x(1+\epsilon_{hh})}.$$

Assuming the economy is on the left-hand-side of the Laffer curve, then $H^W > 0$, $H^N > 0$, and $H_0 > 0$. Hence, $det(f_\mu) > 0$. On the left-hand-side of the Laffer curve $\tau^W \geq \tau^N$. According to equation (23), this implies that $\mu^W > \mu^N$, hence $-1/\beta + 1/\beta H^0 P'(D_1)(\mu^W - \mu^N) < 0$ and $\frac{\epsilon_{hh}}{1+\mu^W(1+\epsilon_{hh})} \leq \frac{\epsilon_{hh}}{1+\mu^N(1+\epsilon_{hh})}$. The fact that $det(f_u) > 0$ and that $-1/\beta + 1/\beta H^0 P'(D_1)(\mu^W - \mu^N) < 0$ imply that $\mathcal{Z} > 0$. Hence, in order to establish whether $\frac{\partial \mu^W}{\partial D_1} > \frac{\partial \mu^N}{\partial D_1}$, we need to investigate whether or not $H^N > H^W$. For this purpose, we are left with the task to study the terms $\frac{\partial h^N \tau^N}{\partial \tau^N}$ and $\frac{\partial h^W \tau^W}{\partial \tau^W}$. Given that we assumed that the economy is on the left-hand-side of the Laffer curve and that the Laffer curve is single peaked and, given differentiability, this also means that the Laffer curve is concave in τ. When the Laffer curve is concave in τ, we have $\frac{\partial h^N \tau^N}{\partial \tau^N} > \frac{\partial h^W \tau^W}{\partial \tau^W}$. Hence, $\frac{\partial \mu^W}{\partial D_1} > \frac{\partial \mu^N}{\partial D_1}$. ■

A.1.4 Proof of Proposition 4

Proof. Using the period 0's budget constraint, the response of debt to g_0 and D_1 is given by

$$\frac{\partial b_1}{\partial g_0} = 1/\beta\left(1 - \frac{\partial h_0 \tau_0}{\partial g_0}\right),$$
$$\frac{\partial b_1}{\partial D_1} = 1/\beta\left(1 - \frac{\partial h_0 \tau_0}{\partial D_1}\right).$$

Debt is more responsive to D_1 when $\frac{\partial h_0 \tau_0}{\partial D_1} < \frac{\partial h_0 \tau_0}{\partial g_0}$. Following the notation from the proof above

$$\frac{\partial h_0 \tau_0}{\partial g_0} = \frac{\partial h_0 \tau_0}{\partial \mu_0} \frac{\partial \mu_0}{\partial g_0} = H^0 \frac{\partial \mu_0}{\partial g_0},$$
$$\frac{\partial h_0 \tau_0}{\partial D_1} = \frac{\partial h_0 \tau_0}{\partial \mu_0} \frac{\partial \mu_0}{\partial D_1} = H^0 \frac{\partial \mu_0}{\partial D_1}.$$

One needs to compare $\frac{\partial \mu_0}{\partial g_0}$ and $\frac{\partial \mu_0}{\partial D_1}$

$$\frac{\partial \mu_0}{\partial g_0} = P(D_1) \frac{\partial \mu^W}{\partial g_0} + (1 - P(D_1)) \frac{\partial \mu^N}{\partial g_0},$$
$$\frac{\partial \mu_0}{\partial D_1} = P(D_1) \frac{\partial \mu^W}{\partial D_1} + (1 - P(D_1)) \frac{\partial \mu^N}{\partial D_1} + P'(D_1)(\mu^W - \mu^N),$$

where, through the bond's optimality condition, μ_0 is a function of D_1, μ^W, and μ^N. Use the implicit function theorem to get the effect on μ^W and μ^N. In the proof above we have shown that

$$\frac{\partial \boldsymbol{\mu}}{\partial D_1} = -\frac{-1/\beta + 1/\beta H^0 P'(D_1)(\mu^W - \mu^N)}{det(f_\mu)} \begin{pmatrix} H^N \\ H^W \end{pmatrix},$$

which was equation (25). Similarly, the marginal effect of g_0 is

$$\frac{\partial \boldsymbol{\mu}}{\partial g_0} = -\frac{-1/\beta}{det(f_\mu)} \begin{pmatrix} H^N \\ H^W \end{pmatrix}. \tag{26}$$

Assuming the economy is on the left-hand side of the Laffer curve, $\frac{\partial \mu^x}{\partial D_1} > \frac{\partial \mu^x}{\partial g_0}$ for $x \in \{N, W\}$. The debt choice b_1 responds more to D_1 than to g_0, iff $\frac{\partial \mu_0}{\partial D_1} < \frac{\partial \mu_0}{\partial g_0}$. Equivalently, using the bond optimality condition (15), $\frac{\partial \mathbb{E}_0(\mu_1)}{\partial D_1} < \frac{\partial \mathbb{E}_0(\mu_1)}{\partial g_0}$

Expanding and rearranging terms gives

$$P(D_1) \left(\frac{\partial \mu^W}{\partial D_1} - \frac{\partial \mu^W}{\partial g_0} \right) + (1 - P(D_1)) \left(\frac{\partial \mu^N}{\partial D_1} - \frac{\partial \mu^N}{\partial g_0} \right) < P'(D_1)(\mu^N - \mu^W). \tag{27}$$

Using (25) and (26) it is easy to show that

$$\frac{\partial \mu^W}{\partial D_1} - \frac{\partial \mu^W}{\partial g_0} = \frac{1}{det(f_\mu)} \frac{1}{\beta} H^N H_0 P'(D_1)(\mu^N - \mu^W),$$
$$\frac{\partial \mu^N}{\partial D_1} - \frac{\partial \mu^N}{\partial g_0} = \frac{1}{det(f_\mu)} \frac{1}{\beta} H^W H_0 P'(D_1)(\mu^N - \mu^W).$$

Using these expressions, the left-hand side of (27) is

$$P(D_1) \frac{1}{det(f_\mu)} \frac{1}{\beta} H^N H_0 P'(D_1)(\mu^N - \mu^W) + (1 - P(D_1)) \frac{1}{det(f_\mu)} \frac{1}{\beta} H^W H_0 P'(D_1)(\mu^N - \mu^W) =$$
$$\underbrace{\left(P(D_1) \frac{1}{det(f_\mu)} \frac{1}{\beta} H^N H_0 + (1 - P(D_1)) \frac{1}{det(f_\mu)} \frac{1}{\beta} H^W H_0 \right)}_{\mathcal{K}} P'(D_1)(\mu^N - \mu^W).$$

Given this, $\frac{\partial\mu_0}{\partial D_1} < \frac{\partial\mu_0}{\partial g_0}$ is equivalent to

$$\underbrace{\left(P(D_1)\frac{1}{det(f_\mu)}\frac{1}{\beta}H^N H_0 + (1-P(D_1))\frac{1}{det(f_\mu)}\frac{1}{\beta}H^W H_0\right)}_{\mathcal{K}} P'(D_1)(\mu^N - \mu^W) < \frac{\partial P^W(D_1)}{\partial D_1}(\mu^N - \mu^W)$$

It remains to show that the $\mathcal{K}$ is less than one:

$$\mathcal{K} = \frac{1/\beta H_0 H^N P(D_1) + (1-P(D_1))H_0 H^W 1/\beta}{1/\beta H_0 H^N P(D_1) + (1-P(D_1))H_0 H^W 1/\beta + H^W H^N} < 1,$$

since both the numerator and the denominator are positive and $H^W H^N > 0$. Hence, $\frac{\partial b_1}{\partial D_1} > \frac{\partial b_1}{\partial g_0}$. ■

A.1.5 Proof of Proposition 5

Proof.
As shown in A.1.2, one can express hours and consumption as functions of policy variables, such as

$$c_0 = c_0(\tau_0, D_1, g_0); \quad c^N = c^N(\tau^N, g^N); \quad c^W = c^W(\tau^W, D_1, g^W, g^N),$$

and

$$h_0 = h_0(\tau_0, D_1, g_0); \quad h^N = h^N(\tau^N, g^N); \quad h^W = h^W(\tau^W, D_1, g^W, g^N).$$

Using the notation from A.1.2, the Ramsey problem reads as:

$$\max_{b_1, D_1, \tau_0, \tau^N, \tau^W} U(c_0, h_0) + \beta(P(D_1)U(c^W, h^W) + (1-P(D_1))U(c^N, h^N))$$

s.t.

$$(g_0 + b_0)u_c(F(\tau_0, g_0, D_1)) = \Omega_0 + b_1 P(D_1)u_c(c^W) + (1-P(D_1))u_c(c^N),$$
$$(g^N + b_1)u_c(F(\tau^N, g^N)) = \Omega^N,$$
$$(g^W + b_1 - D_1(1-\delta))u_c(F(\tau^W, g^W, D_1)) = \Omega^W.$$

The optimality condition for D_1 would read as:

$$\frac{\partial U}{\partial c_0}\frac{\partial c_0}{\partial D_1} + \frac{\partial U}{\partial h_0}\frac{\partial h_0}{\partial D_1} + \mu_0\left(\frac{\partial \Omega_0}{\partial D_1} - (g_0 + b_0)u_{cc}\frac{\partial c_0}{\partial D_1}\right) + \beta\frac{\partial P}{\partial D_1}\left(U(c^W, h^W) - U(c^N, h^N)\right) + \beta\mu^W\left(\frac{\partial \Omega^W}{\partial D_1}\right).$$

One can solve the model for an arbitrary sequence of b_1 by choosing $D_1, \tau_0, \tau^N, \tau^W$ as well as μ_0, μ^N, μ^W. This would give a system of four optimality conditions and three implementability constraints and seven variables, namely $D_1, \tau_0, \tau^N, \tau^W$, μ_0, μ^N, μ^W, for any given value of b_1. This would define an implicit function between $D_1, \tau_0, \tau^N, \tau^W$, μ_0, μ^N, μ^W and b_1. As a last step, one could choose b_1 and let other variables adjust optimally. More formally, the Planner's objective at this last step would be written as

$$\max_{b_1} \quad U_0(\tau_0(b_1), D_1(b_1)) + \beta(P(D_1)U^W(\tau^W(b_1), D_1(b_1)) + (1-P(D_1))U^N(\tau^N(b_1))).$$

At the optimum the following must be true:

$$\frac{\partial U_0}{\partial \tau_0}\frac{\partial \tau_0}{\partial b_1}+\beta P(D_1)\frac{\partial U^W}{\partial \tau^W}\frac{\partial \tau^W}{\partial b_1}+\beta(1-P(D_1))\frac{\partial U^N}{\partial \tau^N}\frac{\partial \tau^N}{\partial b_1}+\left(\frac{\partial U_0}{\partial D_1}+\underbrace{\beta\frac{\partial P}{\partial D_1}(U^W-U^N)}_{\text{Risk Management}}+\underbrace{\beta\frac{\partial U^W}{\partial D_1}}_{\text{Insurance}}\right)\frac{\partial D_1}{\partial b_1}=0,$$

where derivatives $\frac{\tau_0}{\partial b_1}, \frac{\tau^W}{\partial b_1}, \frac{\tau^N}{\partial b_1}, \frac{D_1}{\partial b_1}$ capture the optimal responses consistent with the Ramsey plan.

The model with endogenous disasters will have higher b_1 than the standard model iff

$$\left(\frac{\partial U_0}{\partial D_1}+\beta\frac{\partial P}{\partial D_1}(U^W-U^N)+\beta\frac{\partial U^W}{\partial D_1}\right)\frac{\partial D_1}{\partial b_1}>0.$$

This can be shown by showing that both terms are positive.

1. To show that $\frac{\partial D_1}{\partial b_1}>0$, use Topkis (1978). Define $x=\{-\tau_0,\tau^W,\tau^N,D_1\}$. Denote $x\wedge x'=\{min(x_1,x_1'),...,min(x_n,x_n')\}$ and $x\vee x'=\{max(x_1,x_1'),...,max(x_n,x_n')\}$. And say that $x'\geq x$ if $x_i'\geq x_i\forall i$.

   Define planners objective function as:

$$\begin{aligned}L=&U(c_0,h_0)+\beta P(D_1)U(c^W,h^W)+\beta(1-P(D_1))U(c^N,h^N)+\mu_0(h_0\tau_0+\beta b_1-D_1-b_0-g_0)+\\&\beta P(D_1)\mu^W(h_W\tau^W-b_1-g^W)+\beta(1-P(D_1))\mu^N(h^N\tau^N-b_1-g^N),\end{aligned}$$

   where it is understood that c and h are functions of taxes and D_1.

   If $L(x,b_1)$ is supermodular in x and has increasing differences in x and b_1, then x is nondecreasing in b_1. Or if $L(x,b_1)$ is differentiable, $\frac{\partial^2 L(x,b_1)}{\partial x\partial b_1}\geq 0$ (Topkis, 1978).

   One can show that $\frac{\partial D_1}{\partial b_1}>0$ by showing that the planner's lagrangian satisfies increasing differences in a sense that $L(x',b_1')-L(x,b_1')>L(x',b_1)-L(x,b_1)$ (Topkis, 1978), where $b_1'>b_1$ and $x'>x$, and $x\in\{\tau_0,\tau^W,\tau^N,D_1\}$. $x'>x$ then means that every element of x' is at least as high as the respective element of x.

   $L(x,b_1)$ exhibits increasing differences in x and b_1 iff $L(x',b_1')-L(x,b_1')-(L(x',b_1)-L(x,b_1))>0$. Calculating it gives

$$\beta(\mu^W-\mu^N)(b_1'-b_1)(P(D_1)-P(D_1'))>0.$$

   Hence, $L(x,b_1)$ has increasing differences and, therefore, $\frac{\partial D_1}{\partial b_1}>0$.

   $L(x,b_1)$ is supermodular in x iff $L(x\vee x',b_1)+L(x\wedge x',b_1)\geq L(x',b_1)+L(x,b_1)\geq$. When $x'>x$, this holds with equality and $L(x,b_1)$ is supermodular in x.

2. To show that $\frac{\partial U_0}{\partial D_1}+\beta\frac{\partial P}{\partial D_1}(U^W-U^N)>0$, take the first order condition of L with respect to D_1 to

get:

$$\frac{\partial U_0}{\partial D_1} + \beta \frac{\partial P}{\partial D_1}(U^W - U^N) + \mu_0 \left(\tau_0 \frac{\partial h_0}{\partial D_0} - 1 \right) + \lambda^D = 0.$$

Assuming $\lambda^D = 0$, $\tau_0 \frac{\partial h_0}{\partial D_0} = \frac{\partial c_0}{\partial D_1} - 1$, therefore $\tau_0 \frac{\partial h_0}{\partial D_0} - 1 < 0$ and $\frac{\partial U_0}{\partial D_1} + \beta \frac{\partial P}{\partial D_1}(U^W - U^N) > 0$.

■

A.2 Optimal Policy under Full Commitment

We consider a full commitment approach to optimal debt and disaster management with incomplete bond markets.

Incomplete Markets

In this subsection, we solve for the time-inconsistent Ramsey plan under incomplete debt markets. The Ramsey planner seeks to maximize the household's utility (1) subject to a series of implementability constraints

$$b_t = \mathbb{E}_0 \left[\sum_{j=t}^{\infty} \beta^j \frac{u_c(c_{t+j})}{u_c(c_t)} \cdot s_{t+j} \right],$$

with multiplier μ and the law of motion for the defense stock

$$DS_t = DS_{t-1}(1-\delta) + D_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e), \tag{28}$$

with multiplier μ_t^D. The Ramsey planner also needs to take into account that defense stock affects the disaster probability, i.e. $P(DS, \xi)$, and needs to take into account the $D_t > 0$ constraint, to which we assign multiplier λ_t^D. Additionally, the planner needs to respect the aggregate resource constraint

$$c_t + D_t + g_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e) = z_t h_t = z_t(1 - l_t).$$

From the resource constraint we obtain an expression for l_t and substitute it out. Hence, the recursive Lagrangian of the planner reads:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ u(c_t) + v(l_t) + \mu_t(\Omega_t + \beta \mathbb{E}_t u_c(c_{t+1}) b_{t+1} - u_c(c_t) b_t) \\ + \mu_t^D (DS_{t-1}(1-\delta) + D_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e) - DS_t) + \lambda_t^D D_t \Big\},$$

where $\Omega_t \equiv s_t u_c(c_t) = u_c(c_t) c_t - v_l(l_t) h_t$.

The first-order condition with respect to c_t is:

$$0 = u_c(c_t) + v_l(l_t) \frac{\partial l_t}{\partial c_t} + \mu_t \left(\frac{\partial (s_t u_c(c_t))}{\partial c_t} \right) - u_{cc}(c_t) b_t (\mu_t - \mu_{t-1}). \tag{29}$$

The first-order condition with respect to b_{t+1} is:

$$\mu_t = \frac{\mathbb{E}_t (u_c(c_{t+1}) \mu_{t+1})}{\mathbb{E}_t (u_c(c_{t+1}))}. \tag{30}$$

The first-order condition with respect to D_t is:

$$0 = \lambda_t^D + v_l(l_t)\frac{\partial l_t}{\partial D_t} + \mu_t\left(\frac{\partial s_t u_c(c_t)}{\partial D_t}\right) + \mu_t^D\left(1 - \mathcal{I}_t\frac{\partial \mathcal{S}(DS_{t-1}(1-\delta)+D_t,\phi g^e)}{\partial D_t}\right). \tag{31}$$

The first-order condition with respect to DS_t is:

$$\begin{aligned}\mu_t^D =& \beta\frac{\partial P(DS_t,\xi_t)}{\partial DS_t}\mathbb{E}_t^x\left(u(c_{t+1}^W) + v(l_{t+1}^W) - u(c_{t+1}^N) - v(l_{t+1}^N)\right) + \beta\mathbb{E}_t\left(\mu_{t+1}\frac{\partial s_{t+1}u_c(c_{t+1})}{\partial DS_t} + v'(l_{t+1})\frac{\partial l_{t+1}}{\partial DS_t}\right)\\ &+ \beta\mathbb{E}_t\left(\mu_{t+1}^D(1-\delta) - \mu_{t+1}^D\frac{\mathcal{I}_{t+1}\partial\mathcal{S}(DS_t(1-\delta)+D_{t+1},\phi g^e))}{\partial DS_t}\right).\end{aligned} \tag{32}$$

The implicit derivatives contained in these first-order conditions are:

$$\begin{aligned}\frac{\partial s_t u'(c_t)}{\partial D_t} &= -\frac{v'(l_t)}{z_t} - v''(l_t)\frac{\partial l_t}{\partial D_t}h_t - v'(l_t)\frac{\partial h_t}{\partial D_t}\\ \frac{\partial s_t u'(c_t)}{\partial c_t} &= u''(c_t)c_t + u'(c_t) - \frac{v'(l_t)}{z_t} - v''(l_t)\frac{\partial l_t}{\partial c_t}\frac{c_t + g_t + D_t - \mathcal{I}_t\mathcal{S}(DS_{t-1},\phi g^e))}{z_t}\\ \frac{\partial s_t u'(c_t)}{\partial DS_{t-1}} &= -\frac{v'(l_t)}{z_t}\frac{\mathcal{I}_t\partial\mathcal{S}(DS_{t-1},\phi g^e))}{\partial DS_{t-1}} - \frac{c_t + D_t + g_t - \mathcal{I}_t\mathcal{S}(DS_{t-1},g^e\phi)}{z_t}v''(l_t)\frac{\partial l_t}{\partial DS_{t-1}}\end{aligned}$$

$$\begin{aligned}\frac{\partial l_t}{\partial D_t} &= -\frac{1}{z_t}\left(1 - \mathcal{I}_t\frac{\partial\mathcal{S}(DS_{t-1}(1-\delta)+D_t)}{\partial D_t}\right)\\ \frac{\partial h_t}{\partial D_t} &= \frac{1}{z_t}\left(1 - \mathcal{I}_t\frac{\partial\mathcal{S}(DS_{t-1}(1-\delta)+D_t)}{\partial D_t}\right)\\ \frac{\partial l_t}{\partial c_t} &= -\frac{1}{z_t}\\ \frac{\partial l_t}{\partial DS_{t-1}} &= \frac{1}{z_t}\mathcal{I}_t\frac{\partial\mathcal{S}(DS_{t-1}(1-\delta)+D_t,g^e\phi))}{\partial DS_{t-1}}.\end{aligned}$$

Note that we did not explicitly take the optimality condition with respect to leisure. Instead, we used the aggregate resource constraint to substitute out leisure in terms of consumption. Also note that $\mathbb{E}^x$ denotes the expectation operator over g_{t+1} and ξ_{t+1} after integrating out uncertainty over the disaster state. These four optimality conditions together with the implementability constraints

$$\Omega_t + \beta\mathbb{E}_t u_c(c_{t+1})b_{t+1} - u_c(c_t)b_t = 0, \tag{33}$$

and the law of motion for DS_t equation (28) characterize the model equilibrium dynamics.

A.3 First-best Optimal Policy

We consider a social planner who directly chooses optimal allocations, optimal tax, debt, and disaster management. The law of motion for the defense stock is the same as equation (28), with associated multiplier μ_t^D. The social planner still needs to take into account that defense stock affects the disaster probability, i.e. $P(DS, \xi)$, and needs to take into account the $D_t > 0$ constraint, to which we assign multiplier λ_t^D. Additionally, the planner needs to respect the aggregate resource constraint

$$c_t + D_t + g_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e) = z_t(1 - l_t),$$

with associated multiplier λ_t. More formally, the Lagrangian of the social planner reads:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ u(c_t) + v(l_t) + \mu_t^D (DS_{t-1}(1-\delta) + D_t - \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e) - DS_t) +$$
$$+ \lambda_t (z_t(1 - l_t) - c_t - D_t - g_t + \mathcal{I}_t \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e)) + \lambda_t^D D_t \Big\}.$$

The first-order condition with respect to c_t is:

$$0 = u_c(c_t) - \lambda_t. \tag{34}$$

The first-order condition with respect to l_t is:

$$0 = v_l(l_t) - \lambda_t z_t. \tag{35}$$

The first-order condition with respect to D_t is:

$$0 = \mu_t^D + (\lambda_t^D - \lambda_t) \left(1 - \mathcal{I}_t \frac{\partial \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e)}{\partial D_t} \right). \tag{36}$$

The first-order condition with respect to DS_t is:

$$\mu_t^D = \beta \frac{\partial P(DS_t, \xi_t)}{\partial DS_t} \mathbb{E}_t^x \left(u(c_{t+1}^W) + v(l_{t+1}^W) - u(c_{t+1}^N) - v(l_{t+1}^N) \right) + \beta \mathbb{E}_t \left(\lambda_{t+1} \mathcal{I}_{t+1} \frac{\mathcal{S}(DS_t(1-\delta) + D_{t+1}, \phi g^e)}{\partial DS_t} \right)$$
$$\beta \mathbb{E}_t \left(\mu_{t+1}^D (1-\delta) - \mu_{t+1}^D \frac{\mathcal{I}_{t+1} \partial \mathcal{S}(DS_t(1-\delta) + D_{t+1}, \phi g^e))}{\partial DS_t} \right). \tag{37}$$

Substitute out terms using $\lambda_t = \frac{v'(l_t)}{z_t}$ to get:

$$\frac{\partial l_t}{\partial D_t} = -\frac{1}{z_t} \left(1 - \mathcal{I}_t \frac{\partial \mathcal{S}(DS_{t-1}(1-\delta) + D_t)}{\partial D_t} \right)$$

$$\frac{\partial l_t}{\partial DS_{t-1}} = \frac{1}{z_t} \mathcal{I}_t \frac{\partial \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e))}{\partial DS_{t-1}}.$$

This substitution gives expressions that can be directly compared to the analogous optimality conditions of the Ramsey problem

$$0 = \lambda_t^D + \mu_t^D \left(1 - \mathcal{I}_t \frac{\partial \mathcal{S}(DS_{t-1}(1-\delta) + D_t, \phi g^e)}{\partial D_t}\right) + v'(l_t)\frac{\partial l_t}{\partial D_t} \tag{38}$$

$$\begin{aligned}\mu_t^D =& \beta \frac{\partial P(DS_t, \xi_t)}{\partial DS_t} \mathbb{E}_t^x \left(u(c_{t+1}^W) + v(l_{t+1}^W) - u(c_{t+1}^N) - v(l_{t+1}^N)\right) + \beta \mathbb{E}_t \left(v'(l_{t+1}) \frac{\partial l_{t+1}}{\partial DS_t}\right) \\ & \beta \mathbb{E}_t \left(\mu_{t+1}^D (1-\delta) - \mu_{t+1}^D \frac{\mathcal{I}_{t+1} \partial \mathcal{S}(DS_t(1-\delta) + D_{t+1}, \phi g^e))}{\partial DS_t}\right). \end{aligned} \tag{39}$$

Comparing the First-best and Ramsey planner's optimality conditions for D_t and DS_t, we see that the optimal values of D_t and DS_t coincide when

$$\mu_t \frac{\partial \Omega_t}{\partial D_t} = \beta \mathbb{E}_t \left(\mu_{t+1} \frac{\partial \Omega_{t+1}}{\partial DS_t}\right).$$

A.4 Solution Algorithm

Here we provide a brief summary of the algorithm. More implementation details along with the sample code can be found in Valaitis and Villa (2024). PEA algorithm requires making a projection of expected value terms on the state variables. We do this by projecting the integrands in the expected value terms in the system of equations (29),(30), (31), (32), (28), (33) onto the state variables using an artificial neural network. The we use Gaussian quadrature to approximate the expected value terms. Solution algorithm:

1. Generate a sequence of shocks $\{g_t, \xi_t\}_{t=1}^T$. Given an educated guess, initialize the neural network $\mathcal{ANN}(g_t, \xi_t, \mu_{t-1}, b_{t-1}, DS_{t-1}, \mathcal{I}_t)$, where $\mathcal{I}_t$ indicates whether economy is n the disaster state.
2. Given this guess, simulate the model by solving the system of equations (29),(30), (31), (32), (28), (33) at every t to obtain sequences of endogenous variables.
3. Given the simulated sequence train the neural network and update network weights.
4. Check if the $\mathcal{ANN}$ predictions are consistent with the simulated data and the network weights do not change across iterations. If not, go back to step 2 and simulate the model again using the updated neural network.

B Data

In this section we discuss our data sources and variables construction, used to calculate moments and the empirical impulse responses. We also provide a detailed description of the data used in the calibration.

B.1 Data used in the Calibration and in Section 2.1

Government Spending. Our measure of exogenous government spending is the total US government spending net of federal defense spending. Specifically, we use:

- GSP: government spending and investment (NIPA Table 3.9.5, line 1)
- ND: Federal defense spending (NIPA Table 3.9.5, line 17)
- $GSPF$: Government deflator (NIPA Table 3.9.4 line 1)
- NDF: Federal defense spending (NIPA Table 3.9.5, line 17)
- GDP: nominal GDP (NIPA Table 1.1.5, line 1)

In order to estimate ρ_g and σ_g, we construct the real government expenditure series as

$$g_t = GSP/GSPF - ND/NDF.$$

We then filter the linear trend before estimating ρ_g and σ_g. Our estimate of μ_g is:

$$\mu_g = \frac{GSP - ND}{GDP}.$$

B.2 Estimation of $P(DS, \xi)$ Parameters

Throughout the paper we use the historical series (GPRH, GPRHA, GPRHT). We measure ξ using the threats index (GPRHT) and we measure DSY_t by interpreting it as the US military capital stock (NIPA FA Table 7.1 line 22) as a fraction of GDP (NIPA Table 3.9.5, line 1). These data sources contain annual data for the period $1929-2023$. We then convert the acts index (GPRHA) into a probability using the following transformation:

$$P_t = \frac{GPRHA_t - min(GPRHA)}{max(GPRHA) - min(GPRHA)}.$$

In order to harmonize our data into annual frequency, we take a 12-month moving average of P_t and ξ_t and pick the midyear value as the measure of P_t and ξ_t in period t. Our normalized series P_t rises above 0.5 twice: 1) during WWII and 2) during the September 11 episode. We label these as disaster episodes. Our objective is to estimate the role of defense investment on geopolitical risk in peace time. Hence, we add dummy variables to capture defense investment dynamics during our identified disaster episodes, which we denote as $D1_t$ and $D2_t$. We then estimate coefficients by minimizing the least squares of the nonlinear equation below:

$$P_t = \frac{1}{1 + e^{-\beta_1 - \beta_2 log(DSY_{t-1}) - \beta_3 log(\xi_{t-1}) - \beta_4 D1_{t-1} - \beta_5 D2_{t-1}}}.$$

Consistently with the model, our data for ξ_t is measured in the middle of period t, while data for DSY_t records the end of period quantity. For robustness, we consider the following four possibilities: 1) We use a single dummy variable to capture disaster episodes. 2) We fit absolute values instead of least squares. 3) We do not use disaster dummy variables. 4) We run a linear OLS in logs. All these cases always yield a negative coefficient for β_2 and a positive coefficient for β_3.

In our numerical work we then remain consistent with our estimation and use the defense stock DS scaled by the model GDP as an input in the $P(DS, \xi)$ function.

B.3 Construction of Tax Variables

To measure capital and labor taxes in the data we follow the approach of measuring average tax rates using national accounts data. This is a common approach initially introduced in Mendoza, Razin, and Tesar (1994). Since then, it has been repeatedly used in empirical work on taxation, such as Burnside, Eichenbaum, and Fisher (2004), Mertens and Ravn (2013), Leeper, Plante, and Traum (2010), Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015) and Clymo, Lanteri, and Villa (2023). An alternative approach would be to account for non-linear tax schedules, with varying marginal tax rates Bhandari and McGrattan (2020). They construct a full marginal tax schedule for personal income taxes from US data, and model how business profits are taxed either as personal or corporate taxes. While this approach has the advantage of better capturing the specifics of a given country's tax system, it maps less directly into our representative agent model with linear tax schedule. In practice, however, Jones (2002) finds that the two approaches yield taxes of a similar magnitude and with a high correlation between the series.

Data for constructing tax variables For consistency with our model, we use annual data. We follow Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015) and include both federal and state taxes when constructing our data, so that our taxes capture all taxes paid domestically by households in the US. The data come from the National Accounts (NIPA) tables provided by the Bureau of Economic Analysis (BEA). We name variables for use in the formulas below:

- Output and spending
  - DEF: output deflator (NIPA Table 1.1.4, line 1)
  - $NGDP$: nominal GDP (NIPA Table 1.1.5, line 1)
  - PCE: personal consumption expenditures (NIPA Table 1.1.5, line 2)
  - GSP: government spending and investment (NIPA Table 1.1.5, line 22)
- Incomes
  - CEM: compensation of employees (NIPA Table 1.12, line 2)
  - WSA: wage and salary accruals (NIPA Table 1.12, line 3)
  - PRI: proprietor's income (NIPA Table 1.12, line 9)

- RI: rental income (NIPA Table 1.12, line 12),
- CP: corporate profits (NIPA Table 1.12, line 13)
- NI: interest income (NIPA Table 1.12, line 18)

- Taxes
  - TPI: taxes on production and imports (NIPA Table 3.1, line 4)
  - CT: taxes on corporate income (NIPA Table 3.1, line 5)
  - CSI: contributions to Social Security (NIPA Table 3.1, line 7)
  - PIT: federal, state, and local taxes on personal income (NIPA Table 3.2, line 3 plus NIPA Table 3.3, line 4)
  - PRT: state and local property taxes (NIPA Table 3.3, line 9)

Data for, and construction of, other basic variables. Real GDP is nominal GDP over the price deflator (NGDP / DEF). Government spending to GDP is the GSP divided by $NGDP$ series. For government debt we start with data on nominal "Debt held by the public", which excludes debt held by other government departments. This is available from FRED (series FYGFDPUB) from 1939 onwards, and we extend back to earlier years using data from the CBO.[10] The data is annual year-end debt. We convert this to real debt by dividing by the GDP deflator.

For the local projection estimation, we use defense news shocks from Ramey and Zubairy (2018), which we convert from quarterly to annual by dividing the sum of news within the year by the total potential GDP of that year. The time spans of our raw data series are as follows. Our NIPA data run from 1929 to 2024, as does our debt data once extended with the CBO data. The news shock data runs from 1890 until 2016. We thus have complete coverage for 1929 to 2016 that we use to estimate the impulse-responses. Next, we turn to describing the construction of our tax series. The procedure closely follows Clymo, Lanteri, and Villa (2023). Nevertheless, we still explain it here in detail.

First step: Personal Income Tax. The personal income tax is a key tax in the US, which applies to income which will be classified as either labor or capital income in our model. Hence, a first step is to measure the average personal income tax in the data. In the data we directly measure before-tax personal income, PI_t, and personal income taxes paid, PIT_t, so measuring the personal income tax rate is simply done as $\tau_{p,t} = PIT_t/PI_t$. The personal income tax in the data is

$$\tau_{p,t} = \frac{PIT_t}{LI_t + CI_t} \tag{40}$$

where total taxable personal income, PI_t, is split into personal labor income, PLI_t, and personal capital income, PCI_t. These are defined as

$$PLI_t = WSA_t + PRI_t/2 \tag{41}$$

[10] Specifically, there is historical CBO data which we choose not to use as our main data since it is measured as debt to GDP and only given to the first decimal place, and only up to the year 2000. The two data series are almost identical in their overlapping years, and we splice them together. The CBO data is as the Economic and Budget Issue Brief "Historical Data on Federal Debt Held by the Public", available at https://www.cbo.gov/publication/21728.

$$PCI_t = PRI_t/2 + RI_t + CP_t + NI_t \tag{42}$$

Labor income is wages and salaries plus half of proprietors income. The half split is arbitrary and from Jones (2002), who finds results are robust to how proprietors income is split. Capital income is made up of four components: half of proprietors income, and then rental income, corporate profits, and interest income.

Capital Tax. In the data we measure capital taxes paid, KIT_t, and total taxable capital income, TCI_t. So measuring the tax is simply done as $\tau_{k,t} = KIT_t/TCI_t$:

$$\tau_{k,t} = \frac{\tau_{p,t}PCI_t + CT_t + PRT_t}{PCI_t + PRT_t} \tag{43}$$

The numerator is capital taxes paid. This is capital taxes paid out of personal income, plus taxes paid on corporate income and property taxes. The denominator measures total capital income which adds property taxes, PRT_t, back to personal capital income ($TCI_t = PCI_t + PRT_t$). Property taxes are subtracted from profits and hence missing from personal capital income, and so are added back to the denominator to properly measure total capital income.

Labor Tax. In the data we measure labor taxes paid, LIT_t, and total taxable labor income, TLI_t. So measuring the tax is simply done as $\tau_{l,t} = LIT_t/TLI_t$:

$$\tau_{l,t} = \frac{\tau_{p,t}PLI_t + CSI_t}{CEM_t + PRI_t/2} \tag{44}$$

The numerator is total income taxes. These come from two sources. Firstly, personal labor income is taxed at rate $\tau_{p,t}$. Secondly, there are additional contributions to social security, CSI_t, which are not taxed as personal income. The denominator is total labor income, which is total labor compensation, CEM_t, plus half of proprietor income ($TLI_t = CEM_t + PRI_t/2$).

Consumption Taxes. We do not use consumption taxes in our model or empirical exercises, but include details on how to construct consumption taxes using the Jones (2002) method here for reference. In a model, we would think of total consumption expenditure as $CS_t = (1 + \tau_{c,t})C_t$, where C_t is the amount of real good that is bought and CS_t is the total spending. The data gives CS_t and the tax bill, $CTAX_t = \tau_{c,t}C_t$, giving $\tau_{c,t} = CTAX_t/C_t = CTAX_t/(CS_t - CTAX_t)$:

$$\tau_{c,t} = \frac{TPI_t - PRT_t}{PCE_t - (TPI_t - PRT_t)} \tag{45}$$

The numerator is taxes on production and imports (TPI_t) less state and local property taxes (PRT_t). Production taxes are equivalent to consumption taxes in the standard model. Property taxes are included in production taxes in the data, but are better thought of as capital taxes, so are subtracted and counted in the capital tax instead. The denominator is consumption spending before taxes: PCE_t is personal consumption expenditure, which includes taxes, so the tax is subtracted.

B.4 Impulse Responses to Military Spending Shocks

In this section we provide details of the impulse responses presented in Section 5.1. We are interested in the dynamics of taxes and debt to an exogenous increase in government spending, which we identify as the defense news shocks of Ramey and Zubairy (2018). They use a narrative approach to measure announced planned changes in government defense spending, as a fraction of potential GDP. In their work, they study the response of GDP to this shock to measure government spending multipliers, using local projections (Òscar Jordà, 2005). We adapt their approach to instead measure the response of fiscal instruments. Specifically, we use the actual values of the defense news shock as an instrument and apply local projection to estimate the impact on taxes and debt.

Our baseline specification, adapted from Ramey and Zubairy (2018), is as follows:

$$x_{t+h} - x_{t-1} = \alpha_h + A_h \cdot z_t + \beta_h \cdot Z_t + \phi \cdot \text{trend}_t + \epsilon_{t+h}, \qquad \text{for } h = 0, 1, 2, ..., H. \tag{46}$$

For any left-hand-side variable x, we are regressing the forward difference h periods ahead, $x_{t+h} - x_{t-1}$, on the the military spending shock, z_t, and a set of controls, Z_t. Our left-hand-side variables include i) government spending to GDP, ii) federal debt to GDP iii) the level of labor taxes, iv) level of capital taxes, v) level of consumption taxes. Each is regressed separately, giving $5 \times H$ regressions with associated coefficients. We control for a trend which is a fourth-order polynomial of time. Z_t are the controls used in a typical local projection set up. In particular, Z_t consists of lags of all of the three left hand side variables and the shock z_t. Since we are using yearly data, we use two lags. We use robust standard errors.[11] Our impulse responses plot the coefficients A_h for each variable, multiplied by a scaling factor that is common to each variable. The scale is chosen to create a an increase in government spending that is equivalent to an increase following a two standard deviations shock in the g_t AR(1) process estimated in section 5.1.

Notice that the regression above is a simple OLS regression, but has an instrumental variables flavor since the independent variable of interest, z_t, is considered as an exogenous shock due to the narrative identification. Since government spending itself is one of the variables regressed on this variable we are automatically performing an effective "first stage" regression to check that total government spending does rise in response to the military spending shock. The remaining variables then investigate how the rest of the economy responds to this military-spending-induced rise in government spending.

Our results use all available data to maximize power. Accordingly, we use the full dataset from 1929 to 2016 for our regressions. The estimates using the full dataset are heavily influences by the early military buildups in the twentieth century, as pointed out in Ramey and Zubairy (2018). Nevertheless, the use of full sample allows us to obtain precise estimates to an exogenous increase in government spending.

[11]Specifically, the estimations are run in Stata using the ivreg2 package with options robust and bw(auto).