Skewness and Time-Varying Second Moments in a Nonlinear Production Network:

Theory and Evidence

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Abstract

This paper studies asymmetry in economic activity in a multisector model with shocks to productivity and labor wedges. Complementarity across inputs – creating nonlinear intersectoral interactions – creates negative skewness. The analysis generates additional predictions: skewness is smaller at the sector than aggregate level, sector-specific shocks are unskewed, and sector centrality rises following negative shocks – and finds empirical support for them. Skewness arising out of intersector interactions helps reconcile differences in skewness at the micro and macro level. Finally, we show the model's ability to match the data comes from the wedge shocks, rather than variation in productivity.

1 Introduction

A defining feature of the business cycle is the existence of recessions as distinct episodes. Rather than simply experiencing symmetric random fluctuations around a trend, output, employment, and other aggregate measures of the state of the economy display sharp declines and relatively smooth expansions. In other words, levels and growth in real activity are skewed left – they have negative third moments, and hence relatively long left tails.[1] At the same time, many measures of volatility are countercyclical. As a mathematical matter,

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[1] See Sichel (1993), McKay and Reis (2008), Morley and Piger (2012), Berger, Dew-Becker and Giglio (2020), and Dupraz, Nakamura and Steinsson (2021), among others.

there is a mechanical link between skewness and countercyclical volatility—high volatility in bad times leads to a long left tail of outcomes—but only a handful of models capture that feature of the economy.

In general, negative skewness endogenously arises in models when they feature a form of concavity, mapping symmetrical shocks into asymmetrical outcomes. That can happen, for example, when there are occasionally binding constraints or capacity limits.[2] In past work the concavity typically arises either because the mapping from shocks to output is concave for individual firms (e.g. Ilut et al. (2018) and Straub and Ulbricht (2019)), or because aggregate shocks have a nonlinear effect on output (e.g. Kozeniauskas et al. (2018)).

This paper studies a multisector production network in the tradition of Long and Plosser (1983). Whereas past work on production networks has primarily studied the effects only of productivity shocks, we also examine shocks to frictions giving rise to an aggregate labor wedge – a gap between the marginal product of labor and the household's marginal rate of substitution (which are equated in an efficient economy) – building on La'O and Bigio (2020). A key question then becomes, in this paper's setting, can we say anything about which type of shock is empirically relevant for understanding skewness?

When intermediate inputs are gross complements, the paper shows that aggregate output is a concave function of the micro shocks – both to productivity and wedges – even though shocks linearly affect micro units. That is, concavity arises *endogenously* due the interaction of shocks through the production network. Intuitively, this occurs because the local sensitivity of GDP to a shock to a given sector – whether to productivity or the wedge – is proportional to the sector's size. The elasticity of substitution determines how sector sales shares respond to shocks – in the presence of complementarity (an elasticity below 1), sales shares rise following negative shocks and fall following positive shocks, creating asymmetry and thus left skewness.

Baqaee and Farhi (2019), primarily numerically, show that productivity shocks in a model like the one we study generates skewness in aggregate output. This paper goes beyond that work in three dimensions – by formalizing the argument for a model that can be solved in closed form, by adding frictional shocks as in La'O and Bigio (2020) and Baqaee and Rubbo (2023), and by generating a range of additional predictions for the joint behavior of aggregate and sector output and employment. Finally, we evaluate whether an economy driven relatively more by productivity or wedge shocks is better able to match the array of empirical findings.

[2]See, among many others, Kiyotaki and Moore (1997), Gilchrist and Williams (2000), Kocherlakota (2000), Hansen and Prescott (2005), Bianchi (2011), and Bianchi et al. (2018). See also Acemoglu and Scott (1997) and Dupraz et al. (2021) for alternative mechanisms.

Mathematically, the equilibrium of the economy has a simple representation. For a set of symmetrical sector-level shocks $\{\varepsilon_i\}$ (for $i = 1$ to N):

$$\log GDP = f(\{\varepsilon_i\}) \quad (1)$$

$$\log y_i = a_0 \log GDP + a_1 \varepsilon_i \quad (2)$$

where y_i is the output of sector i and a_0 and a_1 are coefficients. The function f is concave in each ε_i. As an extreme example, when production is Leontief in inputs,[3] then $f(\{\varepsilon_i\}) = \min_i \varepsilon_i$. A very similar result holds with frictional shocks.

The paper argues that the concave aggregation arising in a production network model fits a wide range of features of the economy. In particular, it develops three sets of theoretical predictions.

1. *Unconditional skewness:* Aggregate and sector activity are skewed left, but the effect is stronger at the aggregate level. The sector-specific component of activity is unskewed.

2. *Countercyclical dispersion:* The cross-sectional variance of output and employment is countercyclical.

3. *Conditional covariances:* When a sector receives a negative shock, it subsequently covaries more strongly with other sectors and with aggregate activity.

The key mechanism driving all three predictions is that the (endogenous) function f is concave, immediately generating negative skewness and countercyclical volatility.[4] The fact that sector output has an independent symmetrically distributed component explains why it is less negatively skewed than aggregate output. Empirically, we confirm past results on the negative skewness of aggregate time series, but further show that for industrial production, employment, and stock returns, skewness is significantly more negative at high than low levels of aggregation, by factors of two to five.[5]

On the other hand, when we examine sector-specific shocks in the data—estimates of the ε_i—skewness is near zero with tight confidence intervals: all the skewness observed at the sector level is explained by exposure to an aggregate factor. These observations imply that skewness is an aggregate phenomenon, rather than being due to, for example, skewed sectoral

[3] Along with other restrictions on the model.

[4] On countercyclical aggregate volatility, see, recently, Jurado, Ludvigson and Ng (2015), among many others.

[5] Albuquerque (2012) discusses that fact for stock returns.

shocks.[6] Being able to simultaneously match aggregate, sectoral, and residual skewness is a novel feature of the paper's model and distinguishes it from mechanisms based on concave micro responses to shocks, which, under the usual assumption of linear aggregation (e.g. in Ilut et al. (2018)), imply skewness is greater at the micro than the macro level.

Countercyclical dispersion arises directly from the concavity of GDP as a function of the shocks. When the shocks are more dispersed, a Jensen's inequality effect causes output to be on average lower.

The third set of results on conditional covariances directly addresses the key mechanism in the model. In the presence of complementarities, when the supply of one input shrinks, downstream output becomes relatively more sensitive to it and less sensitive to other inputs. Empirically, following a positive statistical innovation in output in a given sector and month (which, in the model, identifies ε_i), that sector's industrial production, employment, stock returns, and TFP covary less strongly with other sectors and with aggregate activity for the next three to 12 months.

When we say that the concavity arises from interactions among sectors, what that means more specifically is that aggregate TFP and the labor wedge are concave in the sector productivity and wedge shocks, respectively. That gives us a way to examine which of the shocks gives the model the ability to match the data. Empirically, there is little or no skewness in aggregate TFP, and, quantitatively, it contributes little or nothing to the skewness of GDP growth. Conversely, the labor wedge is very strongly skewed – more than any of the other macro aggregates, in fact – and contributes a significant fraction of overall skewness in GDP. Similarly, it is the labor wedge that is correlated with cross-sectional dispersion, not TFP. Finally, in simulations, we find that the model fits the data best when sector productivity shocks are simply shut off, or are set to be much smaller than frictional shocks.

Those findings are important because the majority of the literature on production networks has focused on productivity shocks. Theoretically, we show that the second-order effects of frictional shocks are very similar to those of productivity shocks in our setting, but when searching for micro shocks in the data, these results suggest that trying to measure sector productivity may not be the relevant concern.

Through the analysis, the paper also discusses the model's distinctions from some other potential explanations for skewness. For example, one might naturally assume that there

[6] There is some evidence that firm-level shocks may be skewed left. The paper discusses how substitutability across firms within sectors can cause negatively skewed firm-level shocks to map into symmetrically distributed shocks when measured at the sector level.

are skewed aggregate shocks, such as rare disasters,[7] or perhaps skewness in a universal input (e.g. Brunnermeier and Sannikov (2014)). But such a model does not necessarily generate cyclical cross-sectional moments. Conversely, models of micro uncertainty shocks, such as Bloom (2009) and Christiano, Motto and Rostagno (2014), imply that cross-sectional moments are cyclical, but they do not have any implications for aggregate skewness.

Finally, one might also consider a model in which skewness arises due to concave decision rules at the *micro* level, but without any nonlinearity in interactions across economic units, as in Ilut et al. (2018) or Straub and Ulbricht (2019). Those models can match many of our empirical results, but micro asymmetry counterfactually predicts that the magnitude of skewness is greater at the micro than the macro level. We show, though, that micro asymmetry can be reconciled with the apparent symmetry in sectoral behavior if firms within sectors are substitutes, while sector outputs are complements.

It is important to note that these alternative models depend on a variety of shocks. This paper shows that both sector-level productivity and frictional shocks can lead to skewness via intersectoral interactions, but that obviously need not be true of all shocks. A simple aggregate labor supply shock, for example, would have no nonlinear effects here.

Related work As already mentioned, the paper is related to the literature on time-variation in time-series and cross-sectional moments of output.[8] It also belongs to the growing literature studying the role of production networks in propagating and amplifying shocks (Long and Plosser, 1983; Acemoglu et al., 2012).[9] Particularly relevant is the body of work that emphasizes complementarities in production, such as Horvath (2000), Jones (2011), Atalay (2017), Baqaee and Farhi (2019), and Dew-Becker (2023).

Empirically, Atalay (2017) and Atalay et al. (2018) estimate the elasticity of substitution between intermediate inputs using sectoral data and find evidence for strong complementarity, as do Barrot and Sauvagnat (2016), and Boehm, Flaaen and Pandalai-Nayar (2019).[10] Peter and Ruane (2023) study long-term substitution and obtain mixed results, with some elasticities below 1 and others well above. That suggests this paper's mechanism is more

[7] E.g. Barro (2006), Gourio (2012, 2013), and Wachter (2013).

[8] For work on unconditional skewness, see Sichel (1993), McKay and Reis (2008), Morley and Piger (2012), and Berger, Dew-Becker and Giglio (2020). For recent work on time-varying volatility in the real economy, see Justiniano and Primiceri (2008), Clark and Ravazzolo (2015), and Schorfheide, Song and Yaron (2018). Work on time-varying cross-sectional moments includes Guvenen, Ozkan and Song (2014), Salgado, Guvenen and Bloom (2020), and Dew-Becker and Giglio (2023).

[9] For other recent related work, see Grassi and Sauvagnat (2019), Frohm and Gunnella (2021), and Liu and Tsyvinski (2024).

[10] On the other hand, Carvalho et al. (2021) estimate elasticities of substitution between primary factors and intermediates that are well below 1, but elasticities among material inputs that are slightly above 1 (1.1 to 1.3).

relevant in the short-term – i.e. over the business cycle, instead of the seven-year span that Peter and Ruane (2023) study. More generally, those papers focus on testing the micro implications of production network models, while this paper's contribution is to test the model's predictions for the joint distribution of output, employment, and stock returns at the sector and aggregate levels, and also to distinguish it from other theories. It thus builds on the literature studying the joint behavior of aggregate and sectoral output.[11]

More generally, this paper is related to the broader literature that studies the macroeconomic impacts of microeconomic shocks, such as Gabaix's (2011) work on granularity (among many others). More recently, Gourieroux et al. (2021) examine how shocks to systemically important firms may affect aggregate consumption and asset prices (see also Seo and Wachter (2018)).

2 Motivating evidence

This section briefly presents evidence for two simple and fairly well-known facts: aggregate measures of activity are skewed left and the cross-sectional variance of sector-level measures of activity is countercyclical.[12] These moments will also be used below in evaluating a quantitative version of the paper's model.

2.1 Time-series skewness

The first column in Table 1 reports skewness for growth rates (Panel A) and levels (Panel B) of ten measures of aggregate activity: industrial production, employment, stock market returns, the unemployment rate, and the Chicago Fed National Activity Index (CFNAI) at the monthly frequency, and at the quarterly frequency GDP, real consumption, real investment, TFP (from John Fernald; see Fernald (2014)), and the aggregate labor wedge (the log marginal product of labor minus the log marginal rate of substitution from Loukas Karabarbounis; see Karabarbounis (2014)). In all cases, here and below, levels are detrended by the AR(4)-based filter suggested by Hamilton (2018) (results are very similar with either

[11]E.g. Horvath (2000), Foerster et al. (2011), Carvalho and Gabaix (2013), Atalay et al. (2018), and Caliendo et al. (2018).

[12]Berger, Dew-Becker and Giglio (2020) show that growth rates of employment, capacity utilization, industrial production, GDP, durable and non-durable consumption, and residential and nonresidential investment are all skewed left. Furthermore, returns on the S&P 500 are skewed left, as is their option-implied distribution. Morley and Piger (2012) provide a much more thorough analysis of asymmetry in the output gap—that is, on skewness in levels, rather than growth rates—and finding similar results— while Sichel (1993) provides an earlier analysis distinguishing asymmetry in levels from growth rates. See also references therein for the literature on business cycle asymmetry.

an exponentially weighted moving average or Hodrick–Prescott filter). Note that in the data a high labor wedge is associated with low employment, so negative time-series skewness in the economy will be associated with positive skewness in unemployment and the labor wedge.

The table includes two measures of skewness: the moment-based skewness coefficient (i.e. the scaled third moment, $\mu_3/\mu_2^{3/2}$, where μ_i is the i'th central moment) and the Kelly skewness ($\frac{p95-2\times p50+p5}{p95-p5}$, where pX is the Xth percentile of a given data series).[13] 90-percent confidence intervals from a block bootstrap are reported in brackets.

Across the ten series, in both levels and growth rates, the skewness coefficients are negative in all nearly all cases. Industrial production and the labor wedge are most skewed (at 1.51 and –1.23). The results hold in both levels and growth rates, for both monthly and quarterly series, and across both skewness measures. The magnitudes are economically large – the Kelly skewness coefficients show that the left tail of the distribution is on average 20 percent longer than the right for growth rates, and 40 percent longer for levels.[14] While the confidence bands are fairly wide, due to the well known difficulty of estimating higher-order moments, they mostly rule out zero or positive skewness.

The bottom two rows of the table report results for TFP and the labor wedge, which we will use to evaluate the model. TFP shows no significant skewness in either direction, while the labor wedge is very strongly positively skewed.

2.2 Cross-sectional dispersion

Countercyclical volatility is well known to also appear in the cross-section of industries. In each month t, we calculate the cross-sectional variance of monthly growth rates of industrial production and employment at the four-digit NAICS level, along with the average daily cross-sectional variance of sector-level stock returns.[15] That is done for both total sector activity and also residuals from a regression of sector activity on aggregate activity, to purge effects due to differential exposure to a common factor (results are similar using residuals from a principal components analysis).

[13] Kelly skewness is more often measured at the 10th and 90th percentiles. We use the 5th and 95th percentiles here because they better represent recessions, given that the US economy in our sample is in recession about 12% of the time, so the 5th percentile captures roughly the middle of those periods. Table A.1 in the appendix shows that the results are qualitatively similar though somewhat weaker when Kelly skewness is calculated at the 90th and 10th percentiles.

[14] Manipulating the Kelly skew, and denoting by k, we have $(p95 - p50) / (p50 - p5) = (1 + k) / (1 - k)$.

[15] That is, using daily returns in sector i, $r_{i,d}$, the monthly cross-sectional variance is $\frac{1}{\#(days \in t)} \sum_{d\in t} \text{var}_d\left(r_{i,d}\right)$ where var_d is the cross-sectional standard deviation on day d. We use 140 three-digit SIC sectors for stock returns for data availability reasons—there are not enough firms in the CRSP dataset to calculate sector returns with too much detail.

Panel A of Table 2 reports results from separate univariate regressions of the cross-sectional variances on an NBER recession indicator and aggregate employment growth as two different measures of the state of the business cycle along with TFP growth and growth in the labor wedge. All of the variables, except the recession indicator, are normalized to have unit variance so that the regression coefficients can be interpreted as correlations.[16]

Consistent with past work (Davis and Haltiwanger (1992), Bloom et al. (2018), Ilut et al. (2018), and Salgado et al. (2020)), there are statistically and economically significant increases in cross-sectional dispersion when the economy is weak. Across the various estimates, in both levels and growth rates, variance is on average higher by 0.80 standard deviations during recessions and its correlation with aggregate employment growth is -0.30. Additionally, the cyclicality of cross-sectional variance is very similar for the sector-specific residuals, so it is not due just to differential exposures to a common factor.

Finally, as with skewness, the driving force here appears to be the labor wedge, rather than aggregate TFP.

3 Network model

This section presents the baseline model. The goals are to give conditions under which aggregate output is a concave function of sector-level shocks and then develop empirical predictions that describe the broader behavior of the economy. To do so, the model is meant to be as simple as possible – and, critically, remain analytically tractable – while still having meaningful intersectoral interactions.

3.1 Model structure

This section first lays out the technological structure of the economy and then describes the sources of the labor wedge.

3.1.1 Tastes and technology

The economy consists of N sectors. Each sector produces a distinct output according to the technology

$$Y_{i,t} \equiv Y\left(L_{i,t}, X_{i,t}; Z_{i,t}\right) \equiv \zeta Z_{i,t} L_{i,t}^{1-\alpha} X_{i,t}^{\alpha} \tag{3}$$

[16]Similar results hold looking at the cross-sectional variance of sector investment growth rates. Bachmann and Bayer (2014) find different results at the firm level, but that can be explained by aggregation within sectors, which smooths out much of the firm-level lumpiness in investment.

where $Z_{i,t}$ is sector i's productivity on date t, $L_{i,t}$ is its use of labor, $X_{i,t}$ is its use of material inputs, and $\zeta \equiv \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}$ is a normalizing constant. $0 < \alpha < 1$ determines the relative importance of labor and material inputs.

A composite good is produced contemporaneously from the sector outputs via the production function

$$X_t \equiv \left(\sum_{i=1}^{N} A_i^{1/\sigma} Y_{i,t}^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)} \tag{4}$$

where σ is the elasticity of substitution and A_i determines the importance of good i in production. The good X can be used either as an intermediate or directed to final consumption, with the resource constraint

$$X_t = \sum_i X_{i,t} + C_t \tag{5}$$

Since there is no investment and the economy is closed, real GDP is equal to real consumption, C_t. Note that the specification here is identical to one in which each sector uses inputs from other sectors (as in more general setups like those in Baqaee and Farhi (2019) and section 6), with the restriction that the weights on the different goods in each sector and the elasticities of substitution are identical both across sectors and in final consumption.[17] The production network here is one of the simplest possible — all sectors are connected to all others, with all being both upstream and downstream of each other and also directly upstream of final consumption. The same structure is also studied in Jones (2011).

Finally, household utility is

$$U(C_t, L_t) \equiv \frac{C_t^{1-\rho}}{1-\rho} - \chi \frac{L_t^{1+\eta}}{1+\eta} \tag{6}$$

where $L_t \equiv \sum_i L_{i,t}$ is aggregate employment. In the efficient allocation, a social planner would maximize $U(C, L)$ subject to the production functions and resource constraint (3–5).

[17] That is, the production function can equivalently be written as $Y(L_{i,t}, X_{i,t}; Z_{i,t}) = \zeta Z_{i,t} L_{i,t}^{1-\alpha} (\sum_{j=1}^{N} A_{i,j}^{1/\sigma_i} X_{i,j,t}^{(\sigma_i-1)/\sigma_i})^{\sigma_i/(\sigma_i-1)}$ where $X_{i,j,t}$ is sector i's use of good j on date t. The required restriction is that σ_i and $A_{i,j}$ are both identical across sectors (i.e. $\sigma_i = \sigma$ and $A_{i,j} = A_j$ for all i). The equivalence holds because the cost-minimization problem is then identical regardless of the scale of a sector and they all use the same mix of inputs (as do households in selecting final consumption).

3.1.2 Wedges

In order to study frictions and inefficiencies, we allow for the presence of a wedge at the sector level. Specifically, firms optimize

$$\max_{L_{i,t} X_{i,t}} P_{i,t} Y\left(L_{i,t}, \left(1-\phi_{i,t}\right) X_{i,t}; Z_{i,t}\right) - L_{i,t} - P_{X,t} X_{i,t} \tag{7}$$

where $P_{i,t}$ is the price of good i, $P_{X,t}$ is the price of the composite good, and the nominal wage is normalized to 1. $\phi_{i,t}$ represents the wedge in sector i on date t. A fraction of the material inputs that firms purchase "leak" out, getting paid back to households in lump sum. This sort of wedge can be motivated in various ways – it might be due to the government collecting taxes on purchased inputs (denominated by the consumption good), it might represent a friction in the market for inputs, or it could represent the effects of bankruptcy or random firm death – a fraction $\phi_{i,t}$ of the firms in sector i declare bankruptcy prior to producing and liquidate, returning their purchased inputs to households. The core economic effect of the sector wedges is to create a gap between the private and social marginal products of inputs at the sector level.

With the wedge refunded to the household, the household's optimization problem becomes

$$\max_{C_t, L_t} U\left(C_t, L_t\right) \text{ such that } P_{X,t} C_t = L_t + P_{X,t} \sum_i \phi_{i,t} X_{i,t} \tag{8}$$

The wedge shock is on material inputs because it has the economic interpretation of returning some inputs to the household to consume. But it can also affect labor input instead and the results go through identically.[18] In the case where $\phi_i X_{i,t}$ is not refunded to the household at all, it is isomorphic to a TFP shock, while when it is partially refundable it affects both the labor wedge and TFP. See appendix A.2.1.[19]

Definition 1. *A competitive equilibrium is a set of prices, $\{P_{i,t}\} \cup P_{X,t}$, and quantities, $\{L_{i,t}, X_{i,t}\}$, such that households solve (8), firms solve (7), both taking prices as given, and the goods market clears (equations (4) and (5)).*

[18]In such settings, a labor wedge can, in addition to the reasons above, arise from labor taxes, price markups, or pledgability constraints.

[19]Note also that the presence of wedges and other inefficiencies can contaminate measurement of TFP at the firm or sector level (e.g. see the discussion in Baqaee and Farhi (2020)). The empirical analysis here, however, primarily focuses on outcomes like output and employment whose measurement is not subject to such contamination.

3.2 Solution

The model can be solved by hand in terms of aggregate TFP and an aggregate labor wedge:

Theorem 2. *Given a set of productivities and input taxes, aggregate consumption (representing GDP) and employment in the competitive equilibrium are*

$$C_t = \chi^{-1/(\eta+\rho)} \Lambda_t^{-1/(\eta+\rho)} \mathrm{TFP}_t^{(\eta+1)/(\eta+\rho)} \tag{9}$$

$$L_t = C_t \mathrm{TFP}_t^{-1} \tag{10}$$

where

$$\Lambda_t \equiv \frac{1}{1-\alpha}\left(1-\alpha\frac{\sum_i A_i Z_{i,t}^{\sigma-1}\left(1-\phi_{i,t}\right)^{1+\alpha(\sigma-1)}}{\sum_i A_i Z_{i,t}^{\sigma-1}\left(1-\phi_{i,t}\right)^{\alpha(\sigma-1)}}\right) \tag{11}$$

$$\mathrm{TFP}_t = \Lambda_t \left(\sum_i A_i \left(1-\phi_{i,t}\right)^{\alpha(\sigma-1)} Z_{i,t}^{\sigma-1}\right)^{-1/(1-\alpha)(1-\sigma)} \tag{12}$$

Aggregate output and employment are log-linear functions of the aggregate labor wedge, Λ_t, and TFP, each of which is itself a *nonlinear* function of the underlying shocks. The key task is to understand that nonlinearity.

Remark 3. TFP_t *measures aggregate total factor productivity (an efficiency wedge):*

$$\mathrm{TFP}_t \equiv \partial C_t / \partial L_t, \tag{13}$$

while Λ_t *is the labor wedge:*

$$\Lambda_t = \frac{\partial C_t / \partial L_t}{-U_{L,t}/U_{C,t}} \tag{14}$$

Remark 4. *GDP is always increasing in TFP and decreasing in the labor wedge. Employment always decreases with the labor wedge, and increases with* TFP *if* $\rho < 1$.

Where necessary we assume that $\rho < 1$, ensuring that employment is procyclical when the model is driven by productivity shocks.

The labor wedge here is defined as in Karabarbounis (2014). That paper argues that the labor wedge is driven by household frictions based on the fact that labor's share of income is relatively stable empirically. Note that in this model, labor's share of income is constant at 1, consistent with Karabarbounis's (2014) findings.

Remark 5. *As a special case, if* $\phi_i = 0 \ \forall \ i$*, then* $\Lambda_t = 1$ *and the competitive equilibrium coincides with the social optimum.*

3.2.1 First-order responses to shocks

Consistent with La'O and Bigio (2020), we have the following:

Proposition 6. *Starting from an efficient steady-state ($\phi_{i,t} = \log Z_{i,t} = 0 \; \forall \; i$), the derivatives of TFP and the labor wedge are*

$$\frac{\mathrm{d}\log \mathrm{TFP}_t}{\mathrm{d}\log Z_{i,t}} = \frac{1}{1-\alpha}s_i \qquad \frac{\mathrm{d}\log \Lambda_t}{\mathrm{d}\log Z_{i,t}} = 0$$
$$\frac{\mathrm{d}\log \mathrm{TFP}_t}{\mathrm{d}\phi_{i,t}} = 0 \qquad \frac{\mathrm{d}\log \Lambda_t}{\mathrm{d}\phi_{i,t}} = \frac{\alpha}{1-\alpha}s_i$$

where $s_i \equiv P_{i,t}Y_{i,t}/\sum_j P_{j,t}Y_{j,t}$ is the sales share of sector i.

To first order, the sector productivity shocks affect only TFP and not the labor wedge, while the frictions affect only the wedge and not TFP. Intuitively, productivity shocks directly reduce the productive capacity of the economy, while the wedge shocks just reallocate resources, and as a result have no first-order impact on TFP around the efficient steady state.

It is also notable – and will be important for understanding the mechanism – that the derivatives are proportional to the sales shares (again, from La'O and Bigio (2020), and in line with Hulten's (1979) theorem for TFP).[20]

The effects of the shocks on GDP and employment ultimately depend on the household's preference parameters (ρ and η), which determine aggregate labor supply.

3.2.2 Necessary and sufficient conditions for concavity

The next section shows that all of the qualitative predictions of the model come from concavity of aggregates with respect to the sector shocks. Since GDP and employment are decreasing in Λ and increasing in TFP, concavity in the model with respect to a given shock arises when $\mathrm{d}^2 \log \Lambda_t/\mathrm{dshock}_{i,t}^2 > 0$ and $\mathrm{d}^2 \log \mathrm{TFP}_t /\mathrm{dshock}_{i,t}^2 < 0$.[21] The conditions for that are as follows:

Proposition 7. *The second derivatives of Λ_t and TFP_t evaluated at the efficient steady-state satisfy*

[20] In Hulten's theorem, the denominator is nominal GDP, not the sum of sector sales. The ratio of the two is $1/(1-\alpha)$, hence the difference.

[21] It would be preferable to give results on negative definiteness for the Hessian. That can be done when there are only productivity shocks, but is infeasible for the wedge shocks in this model. The individual second derivatives capture the core mechanism, though, and the numerical results below confirm they hold in a much more general setting.

$$\frac{d^2 \log \Lambda_t}{d\left(\log Z_{i,t}\right)^2} = 0 \qquad \frac{d^2 \log \Lambda_t}{d\phi_{i,t}^2} > 0 \iff \sigma < 1 - \frac{A_i}{2\left(1-A_i\right)\left(1-\alpha\right)} \approx 1$$

$$\frac{d^2 \log \mathrm{TFP}_t}{d\phi_{i,t}^2} < 0 \qquad \frac{d^2 \log \mathrm{TFP}_t}{d\left(\log Z_{i,t}\right)^2} < 0 \iff \sigma < 1$$

We denote the condition for $d^2 \log \Lambda_t / d\phi_{i,t}^2 > 0$ by $\sigma \lesssim 1$.

First, consider convexity in the labor wedge (which, recall, yields concavity in GDP and employment with respect to the sector wedge shocks). Sector TFP shocks have no second-order effect on the labor wedge, Λ_t, since in an efficient economy, productivity changes create no wedges. In order for the labor wedge to be convex in the wedge shocks, the elasticity parameter σ must be less than a number slightly below 1 (depending on the exact value of A_i). The reason for this comes from Proposition 6: the change in the first derivative depends on the change in the sales share in response to the shock. A 1% increase in $\phi_{i,t}$ makes sector i's output α% more expensive and also, in the absence of any reallocation in inputs, reduces $Y_{i,t}$ by α%. The degree to which demand for good i declines, though, depends on σ. When $\sigma < 1$, so that demand is inelastic, demand falls by less than α%, so that the sales share, $s_{i,t}$, rises on net. Production then shifts *towards* the sector receiving the negative shock. When $\sigma > 1$, sales shares fall when prices rise, and production moves *away* from the shocked sector.

That is the core economic mechanism in the paper, and it applies to both the labor wedge and TFP. Concavity arises when production shifts towards sectors hit by negative shocks, while convexity arises when production can shift towards sectors with positive shocks. The curvature thus comes from interactions *across* sectors and goods, rather than from behavior *within* sectors. Each sector responds log-linearly to its own shocks. But downstream demand (whether from households for consumption goods or firms for inputs) is nonlinear. The analysis here is thus critically about inter-sectoral linkages. While the model is obviously simplistic, it is now possible to see that it is just complex enough to capture the key mechanism.

One might naturally ask whether this isn't just completely about substitution by households, instead of linkages among firms. In theory, it can certainly be both – the result is driven by elasticities of demand, which here are identical for all purchasers. In the numerical simulations below, though, we show that a version of the model in which households have an elasticity of substitution that is different from those of firms, the model mechanism can work entirely via low elasticities of substitution for firms.

Proposition 7 shows that shocks to sector-level wedges additionally have second-order effects on TFP via a misallocation channel, since they cause resources to be allocated differently than is socially optimal.

Finally, note that it is possible to see from Theorem 2 that not all shocks will have concave effects. A shock to labor supply via the parameter χ, for example, would just have log-linear effects. Nonlinearity arises with respect to sector-level shocks here. Common aggregate shocks will in general not generate any skewness.[22]

4 Empirical predictions

This section uses the closed-form solution to the model to develop a series of empirical predictions. In Section 6, we show that the predictions derived for the simple solvable version of the model also all hold in a simulation of a more realistic but not analytically tractable setting.

4.1 Time-series skewness

4.1.1 Aggregate skewness

If log GDP, or employment, is a function of some shock x with volatility v, then to first order in v the moment skewness coefficient satisfies,

$$skewness\left(gdp_t\right) = \frac{3\kappa}{2} \frac{d^2\left[gdp_t\right]/dx_t^2}{\left|d\left[gdp_t\right]/dx_t\right|} v + o\left(v\right) \tag{15}$$

where $gdp_t \equiv \log GDP_t$ and κ is a positive constant increasing in the kurtosis of x.[23] Intuitively, skewness depends on the relative curvature of gdp_t with respect to x, and is negative when gdp_t is concave in the shocks. We then immediately have the following:

Proposition 8. *Taking one shock at a time, and to first order in the volatility of the shocks, v, the logs of GDP, employment, and TFP are negatively skewed and the log labor wedge is positively skewed when $\sigma \lesssim 1$.*

4.1.2 Sector skewness

In addition to the solution for aggregates, it is also possible to solve for sector output and employment:

[22] Note that this is not a statement about all demand shocks – in Acemoglu, Akcigit and Kerr (2015), for example, demand shocks are industry-specific.

[23] Specifically, the approximation here is holding the shape of the distribution of shocks—including the kurtosis, defined as $\mathbb{E}\left[x^4\right]/\mathbb{E}^2\left[x^2\right]$—constant while varying the scale via v.

Proposition 9. *In the competitive equilibrium*

$$L_{i,t} \propto Z_{i,t}^{\sigma-1} (1-\phi_{i,t})^{\alpha(\sigma-1)} \Lambda_t^{a_L} TFP_t^{b_L} \tag{16}$$

$$Y_t \propto Z_{i,t}^{\sigma} (1-\phi_{i,t})^{\alpha\sigma} \Lambda_t^{a_Y} TFP_t^{b_Y} \tag{17}$$

for coefficients a_Y, a_L, b_Y and b_L (which do not depend on the identity of the sector i).

Sector output and employment therefore both satisfy a factor structure in logs. They are exposed to the two composite aggregate shocks that drive GDP and employment, and also are log-linear functions of their own shocks. That log-linearity is a key distinction of the mechanism here from other work generating concavity in output – Ilut, Kehrig, and Schneider (2018; IKS) and Straub and Ulbricht (2019; SU) in particular, where sectors have concave responses to their own shocks. There two immediate implications of that result:

Proposition 10. *If sector-level shocks are symmetrically distributed in logs, then:*
1. residuals from regressions of sector output and employment on their common components are unskewed
2. sector output and employment are less skewed in absolute value than their common components.

That is, if the sector shocks are symmetrically distributed and $\sigma \lesssim 1$, skewness should be most negative at the aggregate level, less negative at the sector level, and smallest or nonexistent when looking at residuals from a factor model for sector output or employment, since in the model that will isolate the sector-specific shocks.

Proposition 10 distinguishes the model's mechanism from models based on concave responses at the micro level, e.g. due to adjustment frictions or fixed factors, as in IKS and SU. If skewness is due to concave sector-level responses, then sector-level activity if anything would be *more* negatively skewed than the aggregate (this can be seen in the simulations reported in IKS). In addition, such a model would predict that the sector-specific residuals would be negatively skewed, since it is exactly the sector's own behavior that creates skewness. In the production network model, on the other hand, it is not the sector's own optimization problem that creates concavity, but rather how the sectors interact.

That is not to say that there are not or cannot be concave responses at the firm level. Rather, that is simply a different mechanism than what this paper focuses on. A simple way to reconcile the two mechanisms simultaneously is if there are concave responses at the firm level, but if firms within a sector are substitutes. Appendix A.2.2 then shows that the substitutability across firms can cause sector output to appear as though there are unskewed sector-level shocks, which is what section 5.2.2 finds in the data.

4.2 The cross-sectional distribution of activity

To capture the time-varying cross-sectional dispersion documented in section 2.2, suppose the volatility of shocks drawn on date t is equal to some v_t, which varies over time. Using a second-order approximation, if for some mean-zero shock x, $\text{var}(x_t) = v_t^2$, then

$$E[gdp_t \mid v_t] = gdp(0) + \frac{1}{2}\left(\frac{d^2}{dx^2} gdp(x)\right) v_t^2 + o\left(v_t^2\right) \tag{18}$$

If f represents GDP or employment, then exactly the same concavity that leads to negative skewness also leads to a negative relationship between the level of output and the cross-sectional variance of shocks.

Proposition 11. *Taking one shock at a time, to first order in the variance of the shocks, v_t^2, the logs of GDP, employment, and TFP covary negatively and the log labor wedge covaries positively with the volatility of the sector shocks when $\sigma \lesssim 1$.*

Note also that the volatility of the shocks, v_t, drives cross-sectional dispersion, via proposition 9, linking the cross-sectional dispersion of output and employment to the level of GDP and employment.

The observation that there is a link between concavity, countercyclical dispersion, and time-series skewness is also in IKS and SU, but again via a different mechanism, and with a different interpretation of causation. Here, dispersion is exogenous and drives aggregate output. In IKS and SU, on the other hand, dispersion endogenously rises when aggregate output is low. That by itself does not lead to any observable differences between the two models, but it highlights the different economic mechanisms underlying the effects.

Remark 12. *A similar result to Proposition 11 holds even with v_t constant. Due to concavity, in periods when the cross-sectional dispersion of realized shocks randomly happens to be higher – even if they are drawn from a fixed distribution – GDP tends to be lower.*

4.3 Conditional covariances

The final aspect of the model that we examine is its predictions for how covariances across sectors change in response to shocks.

Suppose some shock x_t follows an AR(1) process, with

$$x_{t+1} = \Phi x_t + \varepsilon_{t+1} \tag{19}$$

where Φ as a coefficient. Then to first order

$$\operatorname{cov}_t\left(gdp_{t+1}, x_{t+1}\right) \approx \frac{d}{dx} gdp\left(\gamma x_t\right) \times \operatorname{var}\left(\varepsilon_{t+1}\right) \tag{20}$$

There is then an obvious prediction, nearly a comparative static: if $\frac{d^2}{dx^2} gdp < 0$, then when x_t rises, $\operatorname{cov}_t\left(gdp_{t+1}, x_{t+1}\right)$ should fall.

Proposition 13. *Suppose $\sigma \lesssim 1$ and the sector shocks are positively serially correlated. To first order and taking one shock at a time, for any sector i, a decline in sector i's output or employment is associated with an increase in the covariance of sector i with the aggregate. (For employment this result requires A_i to be sufficiently small).*

In a formal sense, a sector effectively becomes more central – in terms of first derivatives – after being hit by a negative shock. This prediction directly tests the idea that aggregate output is a concave function of the sector shocks. When the inter-sector interactions are concave, it is exactly the sectors that receive negative shocks that should rise most in importance.

The results in this and the previous section are also useful for distinguishing the predictions of the production network model from a model with skewed aggregate shocks. Skewed aggregate shocks obviously generate negative aggregate skewness and it would not be surprising if sector output were less skewed than the aggregate (e.g. if it is just assumed to involved a sector-specific shock that averages out). However, there is no immediate reason why the presence of a skewed aggregate shock would cause cross-sectional dispersion (of the sector residuals, especially) to be countercyclical, nor would it naturally create these time-varying conditional covariances.

4.4 Extensions

Appendix A.2 reports results for two variations of the baseline model that relax some of the strict assumptions above. Appendix A.2.3 shows that the characterization in theorem 2 remain valid with minor modifications in the presence of sector-specific factors of production (e.g. capital or inflexible labor). Those results show that the empirical predictions are robust to the specific assumptions made on factor elasticities and mobility. It also shows that sectoral and aggregate payments to any fixed factor share the same characteristics as sectoral and aggregate output. To the extent that stock returns move with payments to capital, then, the predictions in Section 4 also apply to stock returns, which we study in the empirical analysis. That said, the analysis here is certainly far from a fully realistic model of equity markets.

Appendix A.2.2 extends the model to more explicitly allow multiple layers of aggregation, as in the data studied below. Section A.2.2 models firms within each sector and derives results for skewness at the firm, sector, and aggregate levels when the outputs of firms within each sector are substitutes while those across sectors remain complements. With complementarity skewness becomes progressively more negative across levels of aggregation. In addition, if there is concavity at the firm level within sectors, but the firms are substitutes, then *sector* residuals can appear to be unskewed even when there is negative skewness at the *firm* level. More generally, this paper's analysis is taking the sector production function as a reduced-form representation of whatever goes on inside sectors, which might involve both concavity within firms and substitutability across them.

The model here is totally static, but obviously many economic decisions are fundamentally dynamic. One simple way to think about the effect of dynamics here is that adjustments, like reallocating inputs, are easier in the long-run than the short-run. While this model's mechanism is driven by concavity, reallocation pushes in the direction of convexity. A dynamic version of this paper's model would therefore likely have stronger concavity in the short-run, since here we are assuming that inputs can be frictionlessly reallocated in response to shocks. Dew-Becker (2023) analyzes that mechanism in more detail. Consistent with that intuition, we provide evidence below that the paper's mechanisms are significantly attenuated at longer horizons.

5 Empirical analysis

5.1 Data

The analysis focuses on measures of activity that have data at the monthly frequency or higher and are measured at a high level of sectoral detail. The two primary monthly series we focus on are industrial production (IP), available from the Federal Reserve, which is measured at up to the five-digit NAICS level of detail in manufacturing industries; and employment, available from the Current Employment Survey of the U.S. Bureau of Labor Statistics (BLS), which is measured up to the six-digit NAICS level and covers the entire economy. For industrial production, we follow Foerster et al. (2011) and study data since 1972. For employment, the sample with detailed NAICS coverage begins in 1990, while data on two-digit BLS-defined supersectors is available since 1972. The sample period ends in 2019 so as to avoid the large shocks associated with covid.

In addition to the macroeconomic data, we also examine the behavior of stock returns. They have the drawback that they do not directly measure activity, being driven not just by current conditions but also by expectations for the future. Being measured at much higher frequencies, though – we use up to daily data – makes them useful for estimating time-variation in moments. For sector-level measures of stock returns, we construct value-weighted portfolios according to SIC sectors, requiring at least five firms in a given sector/month pair to include it in the analysis.

5.2 Time-series skewness

Beyond the basic negative skewness in macro aggregates discussed in section 2.1, the model also predicts that TFP and the labor wedge should be skewed. Their skewness coefficients are reported in the bottom two rows of table 1. Skewness for the labor wedge is highly positive, consistent with the model, while TFP is nearly unskewed. The model's predictions thus appear to run through the wedge, rather than TFP.

Tables A.2 and A.3 in the appendix report time-series skewness at horizons of one, three, and five years. It shows that skewness converges towards zero at longer horizons, showing that the paper's mechanisms are primarily relevant at higher frequencies. Note also, though, that under weak assumptions, the central limit theorem applies to longer-term growth rates, pulling in the direction of normality and hence zero skewness.

5.2.1 Skewness declines with the level of aggregation

The model's more novel predictions have to do with skewness not just at the aggregate but also at the sectoral level. Figure 1 reports skewness across different levels of aggregation for industrial production, employment, and stock returns, in both growth rates and levels. At a given level of aggregation, we calculate the moment skewness coefficient (see Figure A.1 for a replication with Kelly skewness) in each sector's time-series, and then report the average of those skewness coefficients at each level of aggregation. Squares are point estimates, and circles measure the difference from the aggregate skewness. The vertical lines are bootstrapped 90-percent confidence bands.

The top panels report results for industrial production. The skewness of total IP growth is -1.22. At the two-digit level—just three sectors: durable and nondurable manufacturing and mining—average skewness is -0.96. At the three- and four-digit levels, where there are 43 and 81 total sectors, respectively, skewness declines in magnitude to -0.55 then -0.45. Finally, at the five-digit level skewness is only -0.41.

At the point estimates, the skewness of IP growth is *three times* greater at the aggregate level than for the most disaggregated sectors. The red series show confidence bands for those differences, and three of the four are statistically significant.

The second and third rows report results for employment growth and stock returns. For aggregate employment, skewness is -1.72, compared to -0.55 at the five-digit level.[24] The pattern is similar for stock returns, where aggregate skewness is -0.92, while average skewness at the five-digit level is only -0.35.[25]

If there is measurement error in sector-level data, and it is greater at lower levels of aggregation, then that could potentially explain the skewness patterns in figure 1. However, to do so would require so much noise that the monthly data would in fact have more noise than signal (see appendix A.3). Since we are using the fully revised data (rather than preliminary estimates) for industrial production and employment, errors of that magnitude are unlikely. And for stock returns, the most likely source of "measurement" error would be bid/ask spreads, which are de minimis in comparison to the monthly returns used for Figure 1. In addition, Figure A.2 shows that the same results hold in annual data, where measurement error is minimal, especially for employment since it is based on complete population data from the Census.

Finally, as with tables A.2 and A.3, figure A.3 shows that the results in this section essentially disappear at the five-year horizon, again emphasizing that the behavior highlighted here is again a shorter-run phenomenon, consistent both with the fact that factors become much more flexible at longer horizon, and potentially also that firms may otherwise be able to substitute across inputs more easily at longer horizons (i.e. σ is effectively higher).

5.2.2 Sector residuals are unskewed

In the model, sector shocks can be identified from a regression of sectoral activity on a common component. To that end, we estimate principal components models

$$y_{i,t} = a_i + \sum_{j=1}^{P} b_i pc_{j,t} + \nu_{i,t}, \tag{21}$$

$$\Delta y_{i,t} = a_{\Delta,i} + \sum_{j=1}^{P} b_{\Delta,i} pc^{\Delta}_{j,t} + \nu_{\Delta,i,t}, \tag{22}$$

where $y_{i,t}$ denotes some measure of activity in sector i and Δ is the first-difference operator. $pc_{j,t}$ is the jth principal component ($pc^{\Delta}_{j,t}$ is the same for the growth rates). In the model, the

[24] Note this differs from table 1 because here we use the detailed employment data only available since 1990, whereas table 1 uses the aggregate time series since 1972.

[25] Moment and Kelly skewness also decline with aggregation for sector investment levels and growth rates. The data is less detailed, and not available at the monthly frequency, but, similar to what reported in the figures, moment skewness for investment growth goes from -0.19 for sector growth rates to -0.72 in the aggregate.

number of factors P is naturally equal to 2 (due to proposition 9), but we set $P = 5$ here to capture any additional common factors.[26] The third and fourth columns of panels in figure 1 plot skewness for the residuals $\nu_{i,t}$ and $\nu_{\Delta,i,t}$ as well as the difference between skewness for the residuals and skewness for the original data, $y_{i,t}$, at the same level of aggregation (the blue diamonds). Recall that the assumption of the theoretical model is that $\nu_{i,t}$ and $\nu_{\Delta,i,t}$ are unskewed, so that skewness in sector activity in the model is due to exposure to the aggregate component.

Figure 1 shows that the skewness of the residuals is, across all variables, economically close to zero. For stock returns and employment, skewness for residuals is indistinguishable from zero, and for industrial production it is less than half the magnitude that is observed in the original data, and those differences are statistically meaningful.

So while there is strong evidence for negative skewness in raw growth rates and levels, the distribution of residuals displays little to no skewness, consistent with the model. This finding further strengthens the notion that skewness is an endogenous outcome: sector-specific shocks themselves add little to asymmetry across sectors. Rather, it is how those shocks combine into aggregate outcomes that matters.

As above, figures A.2 and A.1 show that the results are very similar in annual data and using Kelly skewness.

Consistent with the findings here, Atalay et al. (2018) also estimate sectoral productivity shocks and find, for a low elasticity of substitution as studied here, that average skewness is close to zero. Similarly, TFP growth rates in the NBER-CES manufacturing dataset have skewness close to zero.

5.3 Countercyclical dispersion

As with skewness, the main evidence on countercyclical dispersion is not novel here and is reviewed in section 2.2. More specific to this paper, though, the bottom section of table 2 shows, similar to table 1, that cross-sectional dispersion is positively correlated with the aggregate labor wedge and uncorrelated with aggregate TFP. Consistent with the evidence on aggregate skewness, the model's mechanism appears to run through the labor wedge rather than TFP. Another way to put it is that this suggests – and we confirm more formally below – that a version of this paper's production network model with only productivity shocks would fail to match the data, in that it would imply that all skewness and countercyclical

[26]Setting $P = 1$, or just replacing the principal components with the value for the aggregate series (e.g. using aggregate employment growth as the common factor for sector employment growth) gives qualitatively and quantitatively similar results.

dispersion runs through TFP.

5.4 Conditional moments

This final section tests a core prediction of the network model: since GDP is a concave function of the sector shocks, when a sector receives a negative shock it should become more central.

We examine variation in centrality based on conditional covariances for three datasets: (1) employment and industrial production, directly measuring activity at the monthly level; (2) stock returns, indirectly measuring activity, but at the daily level; and (3) total factor productivity, which is most tightly linked to the model primitives, but available only at the annual level.

5.4.1 Industrial production and employment

Define $\Sigma_{t,i}$ to be the (unobservable) average of the date-t conditional covariances of sector i with all other sectors. When we say that sector i covaries more strongly with other sectors, we mean $\Sigma_{t,i}$ rises. Also define $\beta_{i,t}$ to be the conditional covariance of activity in sector i with aggregate activity.

The goal is to estimate relationships of the form

$$\tilde{x}_{i,t} = \tilde{a}_i + \sum_{j=0}^{J-1} \tilde{b}_j \varepsilon_{i,t-j} + \tilde{c}_t + \tilde{\eta}_{i,t}, \tag{23}$$

for $\tilde{x}_{i,t}$ equal to $\Sigma_{t,i}$ or $\beta_{i,t}$ and where $\varepsilon_{i,t}$ measures the innovation to the level of activity in sector i on date t (included up to lag $J-1$). $\tilde{a}_i$, $\tilde{b}_j$ and $\tilde{c}_t$ are coefficients and $\tilde{\eta}_{i,t}$ is a residual. Since $\Sigma_{t,i}$ and $\beta_{i,t}$ are not directly observable, we proxy for them with date-t products, similar to the literature on heteroskedasticity and feasible generalized least squares.

More specifically, define $\varepsilon_{i,t}$ to be the statistical innovation in activity in sector i (i.e. from a forecasting regression).[27] Note that $\varepsilon_{i,t}$ here is not meant to capture the sector-specific TFP or wedge shock. Rather, it is just the innovation in sector activity conditional on date-$(t-1)$ information, which will contain both sector and aggregate components.

We then proxy for $\Sigma_{t,i}$ with $\sum_{j\neq i} \varepsilon_{i,t+1}\varepsilon_{j,t+1}$ and $\beta_{i,t}$ with $\varepsilon_{i,t+1}\varepsilon_{agg,t+1}$ (where $\varepsilon_{agg,t}$ is the statistical innovation in aggregate activity). Those products are single-observation sample

[27] Specifically, we forecast activity in each sector using four lags of sector activity and the lagged value of the first three principal components of activity across all sectors.

moments when the conditional expectation of $\varepsilon_{i,t}$ is zero, with the property that

$$\mathbb{E}_t\left[\sum_{j\neq i}\varepsilon_{i,t+1}\varepsilon_{j,t+1}\right] = \Sigma_{t,i}, \tag{24}$$

$$\mathbb{E}_t\left[\varepsilon_{i,t+1}\varepsilon_{agg,t+1}\right] = \beta_{i,t}. \tag{25}$$

This leads to the following regression

$$x_{i,t} = a_i + \sum_{j=0}^{J-1} b_j\varepsilon_{i,t-j} + c_t + \eta_{i,t}, \tag{26}$$

$$\text{where } x_{i,t} = \sum_{j\neq i}\varepsilon_{i,t+1}\varepsilon_{j,t+1} \text{ or } \varepsilon_{i,t+1}\varepsilon_{agg,t+1}, \tag{27}$$

and $\eta_{i,t}$ captures both the true residual, $\tilde{\eta}_{i,t}$, and also the measurement error in the dependent variable, $x_{i,t}-\tilde{x}_{i,t}$ (e.g., $\varepsilon_{i,t+1}\varepsilon_{agg,t+1}-\beta_{i,t}$). Because there may be common components across sectors in the innovations, $\varepsilon_{i,t}$, we include time fixed effects in the estimation (c_t) and cluster the standard errors by date. Similarly, some sectors will covary with others more strongly on average, so we also include sector fixed effects, a_i. The inclusion of time fixed effects means that changes in the conditional moments are all interpreted as changes relative to those in other sectors. For example, since the date-t mean of $\Sigma_{t,i}$ is equal to the mean of all pairwise covariances, positive values for the b_j coefficients mean that a positive shock to sector i raises its covariances *relative to those between other sectors*. Because the $x_{i,t}$ variables are functions of date-$t+1$ observations, the regressions all represent forecasts and hence conditional moments. That is, the fitted value of the right-hand side is a date-t conditional expectation.

The top three rows of Table 3 report results of the forecasting regressions for IP and employment. In each case, we use the level of aggregation that yields the largest number of sectors. For IP it is the 4-digit level. For employment, we use 2-digit data that extends to 1972 and 5-digit when using data since 1990 in separate regressions. In all cases, we use three monthly lags of activity on the right-hand side ($J = 3$) and report the sum of the coefficients in the table. Standard errors clustered by date are reported in brackets.

For the regressions forecasting $\Sigma_{t,i}$ and $\beta_{i,t}$, the estimated coefficients are negative for both IP and employment. For the first two rows, the coefficients are of similar magnitude, about -0.05, while they are close to zero for the short employment sample. Since the variables are all standardized, a value of -0.05 implies that when a sector's activity rises by one standard deviation, the product on the left-hand side variable falls by 0.05 standard deviations. The regressions thus give consistent support to the model's prediction that following a negative shock, a sector becomes more central and more correlated with aggregate activity.

5.4.2 Stock returns

Since IP and employment are only available at the monthly frequency, forcing us to use a single observation to proxy for a covariance, one might naturally worry that the proxies for the moments in (24)–(25) would have a substantial amount of measurement error.[28] Stock returns have the advantage that they are available at the daily frequency and thus allow us to measure the covariance matrix for each month more accurately. We denote the sample covariance in month t by $\hat{\Sigma}_t$, and then $\hat{\Sigma}_{t,i}$ is again the sum of the i'th row, excluding the diagonal element. Similarly, the covariance of each sector's returns with returns on the overall market can be estimated using daily data within each month, $\hat{\beta}_{i,t}$, and the sector's variance can be estimated from the monthly sample variance. The fourth row of Table 3 reports results for regressions of the form

$$x_{i,t} = a_i + \sum_{j=0}^{J-1} b_j r_{i,t-j} + c_t + \eta_{i,t}, \tag{28}$$

for $x_{i,t} = \hat{\Sigma}_{t,i}$ or $\hat{\beta}_{i,t}$, where $r_{i,t}$ is the return in sector i in month t. That is, we use the same specification as for IP and employment, just replacing $\sum_{j\neq i} \varepsilon_{i,t}\varepsilon_{j,t}$ with $\hat{\Sigma}_{t,i}$, etc.

The results are highly similar to those for IP and employment, with coefficients again close to -0.05. While there is less measurement error in this regression, we also have fewer observations since we use a higher level of aggregation (due to the number of firms available), so the magnitude of the standard errors is similar to that for our proxies of economic activity.

5.4.3 TFP shocks

To get a more direct measure of productivity shocks in each sector, this section uses the NBER-CES manufacturing database, which measures productivity at the 6-digit level among manufacturing industries, but is only available at the annual frequency (Acemoglu, Akcigit and Kerr, 2015).

We estimate regressions using the same specification as for IP and employment above (equations (26) and (27)), but with two modifications. First, we use data on real gross output and hours of production workers instead of IP and employment on the left-hand

[28] Notice, however, that such measurement error appears in the residual in the regression, and therefore is accounted for in the standard errors.

side. Second, the independent variable, instead of being the lagged statistical innovation in activity, is the lagged statistical innovation in total factor productivity.

The bottom two rows of Table 3 report the results from this exercise. For shipments, as with IP, the coefficients are close to -0.05. For hours, similar to employment in the shorter sample, the coefficients are indistinguishable from zero.

This is the one place where we find some direct evidence in favor of the paper's mechanism running through TFP, though even here the results are somewhat ambiguous.

6 Calibrations

This section develops a more general and quantitatively realistic version of the benchmark model that relaxes its various symmetry assumptions. A full-blown structural estimation is well beyond the scope of this paper. Intsead, the key question this section asks is the extent to which the model can match the data moments from the previous section quantitatively, and whether that fit comes via productivity or wedge shocks.

6.1 Model and calibration

The specification closely follows that of Baqaee and Farhi (2019), with three key generalizations from our baseline model: heterogeneity in the intermediate input mix used by each industry, differential elasticities of substitution for production inputs versus final consumption, and differential elasticities for labor versus material inputs.

The model is now described by

$$X_{i,t} = \left(\sum_j A_{i,j}^{1/\sigma} X_{i,j,t}^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)} \tag{29}$$

$$Y_{i,t} = \zeta Z_{i,t} \left((1-\alpha)^{1/\beta} L_{i,t}^{(\beta-1)/\beta} + \alpha^{1/\beta} \left((1-\phi_{i,t}) X_{i,t}\right)^{(\beta-1)/\beta}\right)^{\beta/(\beta-1)} \tag{30}$$

$$Y_{i,t} = \sum_j X_{j,i,t} + C_{i,t} \tag{31}$$

$$C_t = \left(\sum_{j=1}^n A_{C,j}^{1/\xi} C_{j,t}^{(\xi-1)/\xi}\right)^{\xi/(\xi-1)}. \tag{32}$$

With household income and utility unchanged.

The calibration is similar to Baqaee and Farhi (2019). The production weights are chosen to match the 2012 detailed input-output table from the BEA. The elasticities of substitution are set to $\sigma = 0.1$, $\xi = 0.9$, and $\beta = 0.5$, implying that goods are less substitutable in firms' production technology than in the consumption good bundle, which is close to a Cobb-Douglas specification. The strong complementarity in sectoral production technologies—in line with the estimates of Atalay (2017)—means that the mix of material inputs is not amenable to adjustment.

The inverse Frisch elasticity η is set to 1/2 and the inverse elasticity of intertemporal substitution, ρ, to 3/4.

We use the same specification for shocks to log productivity and the wedge. For either, denoting it by $x_{i,t}$, we assume

$$x_{i,t} = 0.85 x_{i,t-1} + v_x \sigma_t \varepsilon_{i,t} \quad (33)$$

$$\sigma_t \sim \min \{Exp[1], 3\} \quad (34)$$

$$\varepsilon_{i,t} \sim N(0,1) \quad (35)$$

The distribution for σ_t is chosen to match the variation in the cross-sectional variance of output and employment growth studied above and capped at 3 (its mean is 1) to control the kurtosis of the shocks. The volatility of the shocks, $v_x = 0.026$, is set to match the volatility of TFP shocks in the NBER–CES manufacturing database (with wedge shocks set to be the same size for comparability). The persistence is chosen to generate reasonable business-cycle dynamics.[29]

We study three separate simulations: one with only productivity shocks, one with only wedge shocks, and one with a combination of the two (where the volatility of the productivity shocks is reduced by half (to 0.013) while the volatility of wedge shocks remains the same; this choice fits the data reasonably well). The version with only productivity shocks corresponds to the majority of past work on production networks. The version with only wedge shocks takes the polar opposite view and asks how well the model can match the data when there are only frictional shocks. The third is a combination of the two that fits the data better.

None of the calibrations is meant to capture all features of the economy. Rather, the goal is simply to understand whether the quantitative magnitudes are broadly in line with the data and whether we can say anything about which of the shocks does a better job of jointly explaining the empirical findings.

[29] The unconditional moments of GDP and employment growth are broadly similar to what is observed in the data. The standard deviation of employment in levels (again detrended using the Hamilton (2018) filter) is 1.97 percent, compared to 2.7 percent in the data. The standard deviation of GDP is 2.27 percent, versus 3.0 percent in the data. The kurtosis of employment and GDP are, respectively, 4.07 and 4.08, compared to 2.78 and 3.69 in the data. In differences, kurtosis for employment and GDP are 8.52 and 4.47 in the model and 6.37 and 5.55 in the data.

6.2 Results

Tables 4 and A.4 report the simulation results. Table 4 reports results using the moment skewness coefficient, and table A.4 the Kelly skewness.

Looking at the first two sections, all three calibrations of the model are able to generate quantitatively realistic amounts of skewness, with generally more coming from the wedge shocks. The second section shows that the model matches the fact that sector skewness is smaller than aggregate, but the quantitative magnitude of the difference is smaller than observed in the data. The results are qualitatively similar for Kelly skewness.

In all three versions of the model, TFP is significantly negatively skewed, unlike in the data (recall that the wedges have a negative second-order effect on TFP via a misallocation channel). However, since the skewness measures are scaled by volatility, they are not ideal measures of the contribution of TFP to skewness in GDP. In the model with only wedge shocks, for example, aggregate TFP growth has a standard deviation of only 0.04 percent (i.e. 0.0004 in logs), so the skewness is not really economically meaningful, in that the unscaled third moment is actually very small.

To measure the contribution of the two aggregate shocks to GDP skewness, we construct the following measure. Considering a linear regression of log GDP on log TFP and the log labor wedge (which, recall, holds with equality in the baseline model), contributions to skewness are

$$gdp_t = b_0 + b_{tfp} tfp_t + b_{wedge} wedge_t + \varepsilon_t \tag{36}$$

$$\text{skew contribution}\,(tfp) = \frac{b_{tfp}^3 E\left[(tfp - E[tfp])^3\right]}{E\left[(gdp - E[gdp])^2\right]^{3/2}} \tag{37}$$

$$\text{skew contribution}\,(wedge) = \frac{b_{wedge}^3 E\left[(wedge - E[wedge])^3\right]}{E\left[(gdp - E[gdp])^2\right]^{3/2}} \tag{38}$$

Each variable's contribution to skewness is measured here as the (univariate) contribution to the third moment from that variable scaled by the standard deviation of GDP.[30] TFP or the labor wedge will only have a large contribution to skewness if their third moment is large or they have a large impact on GDP. Obviously there are other methods of decomposing skewness, and one might be interested in nonlinear interactions. These can always simply be viewed as moments that can be calculated in both the model and the data and then compared.

[30] If tfp_t, $wedge_t$, and the residual ε_t are jointly independent, then the contributions sum to the total skewness in x_t.

The third section of table 4 reports results for the skewness contributions in the data and the three simulations. In the data, only the labor wedge has a large negative contribution to GDP skewness. For TFP, the contribution is zero or positive (both due to its empirical lack of skewness and since its coefficient in the *gdp* regression is small). These moments are the first place we see a strong difference between the two calibrations. In the productivity-based model, TFP contributes significantly to GDP skewness, while the labor wedge does nothing. In the wedge-based model and the model with both shocks (but where the wedges are still dominant) the results are reversed

The bottom two sections of table 4 present the model's results for the findings in tables 2 and 3. All three versions of the model generate significant countercyclicality of cross-sectional dispersion. Again, the model with both shocks, where the wedge shocks are larger, works quantitatively best, in that the correlation is significantly larger for the labor wedge than TFP.

Finally, for the conditional covariances, all three versions of the model generate negative coefficients as observed in the data, but their magnitude is smaller than we observe empirically.

Overall, we draw the following conclusions from the three simulations. First, at a high level, the model is quantitatively reasonable. The fit is far from perfect, but it lines up with the data well in terms of orders of magnitudes.

Second, the model with only TFP shocks, not surprisingly, implies that TFP has too much explanatory power for the skewness of GDP compared to what is observed empirically. The model with only wedge shocks fixed that problem – all skewness in GDP is from the labor wedge. On the other hand, though, that specification generates TFP that is both very highly skewed and strongly negatively correlated with cross-sectional dispersion, again counterfactually. That is solved in the third simulation by adding a little extra variation in TFP, which reduces its skewness quantitatively and also its correlation with dispersion.

As a final point, we note that solving this model numerically is not hard, taking only a few thousandths of a second at most. So it could in principle easily be combined with a richer model of the business cycle (though adding sector-specific forward-looking optimizations would significantly complicate the solution).

7 Conclusion

The goal of this paper is to understand the sources of asymmetries in aggregate output. One theoretical way to generate that asymmetry is for aggregate output to be a concave function of micro shocks. A production network model in which inputs are complements is a particular example of that mechanism. The paper shows that it naturally generates time-series skewness in aggregate and sector output. It then goes on to test a range of additional empirical predictions, all of which have support in the data. The idea of complementarity is powerful in understanding both the aggregate and cross-sectional behavior of the economy.

The paper's driving force is changes in the centrality of different sectors over time. In some models, recessions have common causes, e.g. aggregate technology shocks. Here, however, every episode is different. When a sector receives a negative shock, it becomes relatively more important. For example, in a period where oil stocks are low, shocks to the oil sector become a major driving force (e.g., Hamilton (2003), Kilian (2008)), whereas in periods when the financial sector is highly constrained, financial shocks become more relevant (e.g., Brunnermeier and Sannikov (2014)). This paper explores that mechanism and its implications for a range of empirical features of the economy.

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Table 1: **Measures of Aggregate Time-Series Skewness**

	Growth rates		Levels	
	Skewness coef.	Kelly skew	Skewness coef.	Kelly skew
Monthly data				
IP	-1.23	-0.01	-0.96	-0.16
	[-2.01,-0.45]	[-0.1,0.08]	[-1.57,-0.35]	[-0.37,0.05]
Employment	-0.52	-0.17	-0.68	-0.21
	[-1.19,0.15]	[-0.33,-0.01]	[-1.24,-0.12]	[-0.47,0.05]
Stock Returns	-0.58	-0.15	-1.14	-0.29
	[-0.96,-0.2]	[-0.2,-0.1]	[-1.62,-0.66]	[-0.45,-0.13]
Unempl. rate	0.56	-0.01	1.25	0.54
	[0.08,1.04]	[-0.2,0.18]	[0.62,1.88]	[0.37,0.71]
CFNAI	-1.15	-0.12	-0.82	-0.18
	[-1.7,-0.6]	[-0.27,0.03]	[-1.46,-0.18]	[-0.46,0.1]
Quarterly data				
GDP	-0.37	-0.1	-0.46	-0.19
	[-1.24,0.5]	[-0.26,0.06]	[-1.01,0.09]	[-0.43,0.05]
Consumption	-0.79	-0.04	-0.45	-0.19
	[-1.65,0.07]	[-0.16,0.08]	[-0.94,0.04]	[-0.4,0.02]
Investment	-0.86	-0.03	-1.2	-0.23
	[-1.87,0.15]	[-0.16,0.1]	[-2.27,-0.13]	[-0.46,0]
TFP	0.28	0.02	0.12	0.12
	[-0.08,0.64]	[-0.11,0.15]	[-0.2,0.44]	[-0.06,0.28]
Labor wedge	1.51	0.33	1.17	0.50
	[0.27,2.75]	[0.07,0.59]	[0.32,2.02]	[0.11,0.89]

Notes: The table presents estimated skewness coefficients (the scaled third moment) and Kelly skewness ((p95-2p50+p5)/(p95-p5)). 90-percent confidence intervals from a block bootstrap are reported in brackets. The data sample runs from January 1972 to December 2019. CFNAI is the Chicago Fed National Activity Index. TFP is from John Fernald and the labor wedge from Loukas Karabarbounis. Data in levels is detrended with an exponentially weighted moving average with weights decaying 5% per month.

Table 2: **Cross-Sectional Moments and Cyclicality**

	IP	IP resid	Employ	Employ resid	Returns resid	Returns
			Panel A: Growth Rates			
NBER	0.47***	0.41***	0.92***	0.56***	1.00**	1.00**
	[0.16]	[0.14]	[0.23]	[0.18]	[0.41]	[0.40]
Employment growth	-0.14	-0.11	-0.37***	-0.27***	-0.36**	-0.35**
	[0.09]	[0.08]	[0.06]	[0.07]	[0.15]	[0.14]
Labor wedge growth	0.07	0.05	0.36***	0.27***	0.43***	0.42***
	[0.07]	[0.07]	[0.06]	[0.06]	[0.11]	[0.10]
TFP growth	-0.01	-0.02	0.07	0.03	0.14**	0.14**
	[0.07]	[0.07]	[0.07]	[0.05]	[0.07]	[0.06]
			Panel B: Levels			
NBER	0.76***	1.01***	0.68	1.20		
	[0.20]	[0.27]	[0.41]	[0.74]		
Employment growth	-0.22***	-0.30***	-0.32***	-0.56***		
	[0.07]	[0.10]	[0.11]	[0.20]		
Labor wedge growth	0.15***	0.21***	0.30***	0.53***		
	[0.06]	[0.08]	[0.11]	[0.20]		
TFP growth	-0.10	-0.14	0.07	0.13		
	[0.07]	[0.09]	[0.09]	[0.15]		
# Obs	566	566	352	352		

Notes: This table reports univariate regressions of cross-sectional variance of growth rates (Panel A) or levels (Panel B) on various measures economic activity. NBER is a dummy variable equal to one in recessions and zero otherwise. Growth in employment, TFP, and the labor wedge are all standardized to have unit variance. Newey–West (1987) standard errors with 12 monthly lags are reported in brackets. The columns labeled residuals use the cross-sectional variance of residuals from regressions of sector growth rates on aggregate growth. * indicates significance at the 10- ** 5-, and *** 1-percent level, respectively.

Table 3: **Conditional covariance regressions**

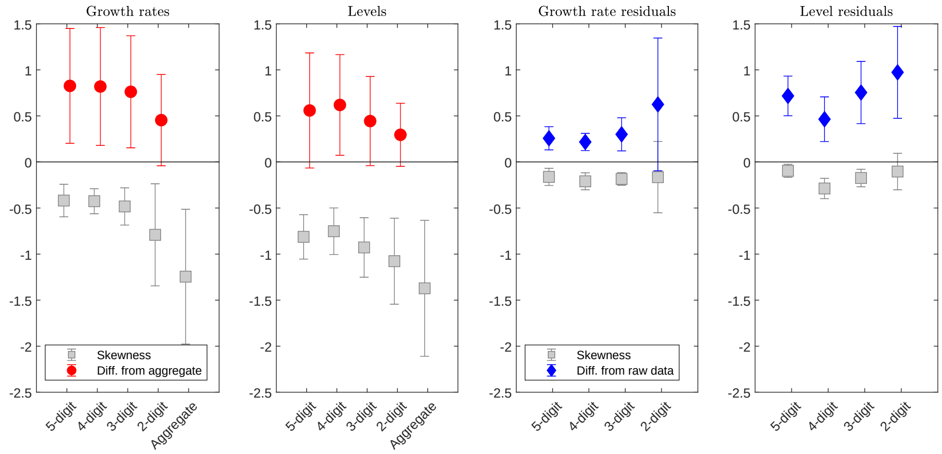
	$\Sigma_{t,i}$		$\beta_{t,i}$	
IP	-0.050***	[0.019]	-0.043***	[0.017]
Employment (1972–2019)	-0.062*	[0.037]	-0.121***	[0.043]
Employment (1990–2019)	-0.012	[0.015]	0.017	[0.017]
Stock returns	-0.045***	[0.011]	-0.069***	[0.019]
Shipments	-0.026*	[0.015]	-0.024	[0.016]
Hours	-0.012	[0.015]	-0.013	[0.011]

Notes: Each entry is an estimate of the sum of the coefficients in equation (26). Standard errors clustered by date are reported in brackets.

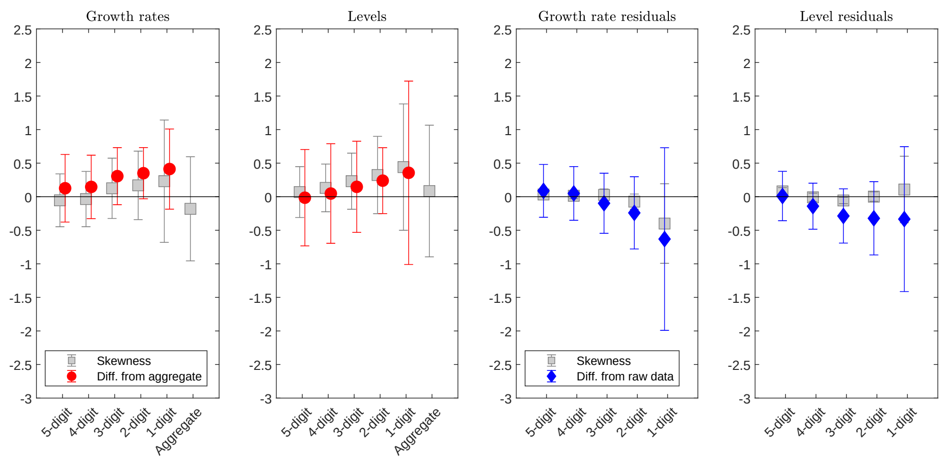
Table 4: **Model simulation results**

	Data		$v_{tfp} = 0.026$ $v_{wedge} = 0$		$v_{tfp} = 0$ $v_{wedge} = 0.026$		$v_{tfp} = 0.013$ $v_{wedge} = 0.026$	
Table 1: Time-series skewness								
	Growth rates	Levels	Growth rates	Levels	Growth rates	Levels	Growth rates	Levels
GDP	-0.37	-0.46	-0.47	-0.40	-1.36	-0.64	-1.31	-0.58
Employment	-0.52	-0.68	-0.47	-0.32	-1.26	-0.56	-1.27	-0.54
TFP	0.28	0.11	-0.08	-0.40	-0.58	-1.80	-0.12	-0.41
Wedge	1.51	1.17	-0.04	0.01	0.39	0.53	0.39	0.51
Figure 1: Aggregate vs. average sector skewness								
	IP	Empl.	IP	Empl.	IP	Empl.	IP	Empl.
Aggregatge	-1.31	-1.72	-0.47	-0.47	-1.36	-1.26	-1.31	-1.27
Avg. sector	-0.46	-0.56	-0.32	-0.17	-1.26	-0.98	-1.08	-0.89
Skewness contributions								
	Growth rates	Levels	Growth rates	Levels	Growth rates	Levels	Growth rates	Levels
TFP	0.01	0.00	-0.08	-0.40	-0.00	-0.00	-0.01	-0.05
Labor wedge	-0.07	-0.47	N/A	N/A	-0.36	-0.48	-0.24	-0.31
Table 2: Cyclicality of cross-sectional dispersion								
	IP	Empl.	IP	Empl.	IP	Empl.	IP	Empl.
Employment	-0.14	-0.37	-0.11	-0.10	-0.40	-0.37	-0.37	-0.36
TFP	-0.01	0.07	-0.11	-0.10	-0.82	-0.81	-0.14	-0.14
Labor wedge	0.07	0.36	N/A	N/A	0.39	0.35	0.35	0.34
Sum of regression coefficients								
	Sigma	*beta*	*Sigma*	*beta*	*Sigma*	*beta*	*Sigma*	*beta*
IP	-0.05	-0.04	0.00	-0.01	-0.01	-0.03	-0.01	-0.02
Employment	-0.06	-0.12	-0.01	-0.01	-0.01	-0.03	-0.01	-0.02

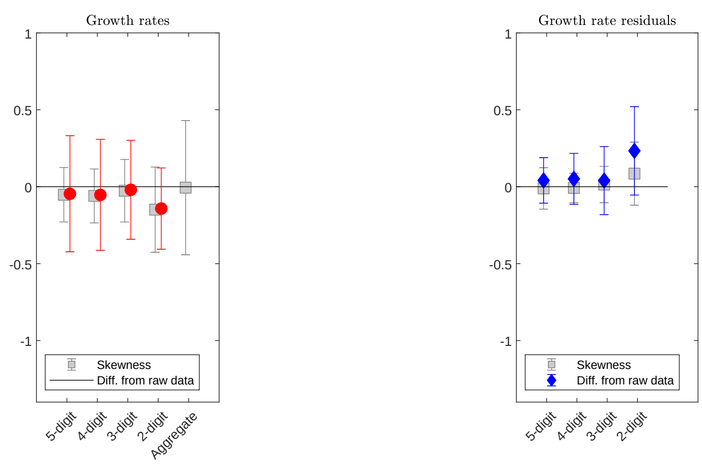
Notes: Results from simulations of the model. Calibrations for the shock volatilities vary across columns and are reported in the top row. The first column reports analogous moments from the empirical analysis in previous tables and figures.



(a) Industrial Production



(b) Employment



(c) Stock Returns

Figure 1: **Time-Series Skewness**

Notes: This figure plots skewness for different levels of aggregation for industrial production, employment, and stock returns for growth rates (left two panels) or in levels (right two panels) together with 90-percent confidence intervals. Data is monthly and starts in January 1972 and ends in December 2019 for industrial production and stock returns and starts in January 1990 and ends in December 2019 for employment.

A.1 Proofs

The proofs drop the time subscripts anywhere that they are not relevant (which is through most of the analysis).

A.1.1 Equilibrium conditions

The equilibrium conditions underlying the results are collected in the following lemma.

Lemma 14. *The equilibrium is characterized by the following set of conditions:*
Cost minimization for inputs:

$$Y_j = \left(P_j^{-1} P_X\right)^{\sigma} X A_j \tag{A.1}$$

Price of composite good:

$$P_X = \left(\sum_j A_j \left(1 - \phi_i\right)^{\alpha(\sigma-1)} Z_j^{\sigma-1} \right)^{1/(1-\alpha)(1-\sigma)} \tag{A.2}$$

Marginal cost minimization:

$$L_i = P_i \left(1 - \alpha\right) Y_i \tag{A.3}$$

$$P_X X_i = P_i \alpha Y_i \tag{A.4}$$

Prices:

$$P_i = Z_i^{-1} \left(1 - \phi_i\right)^{-\alpha} P_X^{\alpha} \tag{A.5}$$

Household budget:

$$\sum_i L_i + \sum_i \phi_i X_i = P_X C \tag{A.6}$$

Household labor optimization:

$$\chi L^{\eta} = P_X^{-1} C^{-\rho} \tag{A.7}$$

Resource:

$$X = \sum_i X_i + C \tag{A.8}$$

Labor adding up:

$$\sum_i L_i = L \tag{A.9}$$

Proof. Equations (A.6), (A.8), and (A.9) are just assumptions (from equations (8), (5), and immediately following (6), respectively). Equation (A.1) is a standard cost minimization for a CES aggregator (i.e. finding the minimum cost for the bundle X_t). Equation (A.7) is just the standard household optimality condition given the utility function (6) and the budget constraint.

Writing the cost minimization problem for sector i in its Lagrangian form (with P_i being the Lagrange multiplier due to competitive markets), we have

$$\min L_i + P_X X_i - P_i \left(\zeta Z_i (1-\phi_i)^\alpha L_i^{1-\alpha} X_i^\alpha - Y_i\right) \tag{A.10}$$

$$L_i = P_i (1-\alpha) Y_i \tag{A.11}$$

$$P_X X_i = P_i \alpha Y_i \tag{A.12}$$

yielding equations (A.3) and (A.4). In addition,

$$L_i / (P_X X_i) = \frac{1-\alpha}{\alpha} \tag{A.13}$$

Inserting the production function into (A.11) and using (A.13) and the definition of ζ,

$$L_i = P_i (1-\alpha) \zeta Z_i L_i^{1-\alpha} X_i^\alpha (1-\phi_i)^\alpha \tag{A.14}$$

$$1 = P_i (1-\alpha) \zeta Z_i p_X^{-\alpha} (L_i / (P_X X_i))^{-\alpha} (1-\phi_i)^\alpha \tag{A.15}$$

$$P_i = Z_i^{-1} (1-\phi_i)^{-\alpha} P_X^\alpha \tag{A.16}$$

where

$$\zeta = (1-\alpha)^{-(1-\alpha)} \alpha^{-\alpha} \tag{A.17}$$

yielding (A.5). Finally, using cost minimization for X yields

$$P_X = \left(\sum_j A_j P_j^{1-\sigma}\right)^{1/(1-\sigma)} \tag{A.18}$$

$$= \left(\sum_j A_j \left(Z_i^{-1} (1-\phi_i)^{-\alpha}\right)^{1-\sigma}\right)^{1/(1-\alpha)(1-\sigma)} \tag{A.19}$$

which is equation (A.2). □

A.1.2 Proof of theorem 2

Taking first (A.3) then (A.1), the production function, and (A.4) yields

$$
\begin{aligned}
L_i &\propto P_i Y_i && \text{(A.20)}\\
&\propto A_i^{1/\sigma} Y_i^{(\sigma-1)/\sigma} && \text{(A.21)}\\
&\propto A_i^{1/\sigma} \left(Z_i \left(1-\phi_i\right)^{\alpha} L_i\right)^{(\sigma-1)/\sigma} && \text{(A.22)}\\
&\propto A_i Z_i^{(\sigma-1)} \left(1-\phi_i\right)^{\alpha(\sigma-1)} && \text{(A.23)}
\end{aligned}
$$

which then means due to the adding-up constraint (A.9),

$$
L_i = L \frac{A_i Z_i^{\sigma-1} \left(1-\phi_i\right)^{\alpha(\sigma-1)}}{\sum_i A_i Z_i^{\sigma-1} \left(1-\phi_i\right)^{\alpha(\sigma-1)}} \tag{A.24}
$$

Aggregate consumption is then

$$
\begin{aligned}
C &= P_X^{-1} \left(L + \sum_i \phi_i P_X X_i\right) && \text{(A.25)}\\
&= P_X^{-1} \left(L + \sum_i \phi_i \frac{\alpha}{1-\alpha} L_i\right) && \text{(A.26)}\\
&= P_X^{-1} \frac{1}{1-\alpha} \left(\sum_i L_i \left(1-\alpha\left(1-\phi_i\right)\right)\right) && \text{(A.27)}\\
&= P_X^{-1} \frac{1}{1-\alpha} L \left(1 - \alpha \sum_i \frac{A_i Z_i^{\sigma-1} \left(1-\phi_i\right)^{\alpha(\sigma-1)+1}}{\sum_i A_i Z_i^{\sigma-1} \left(1-\phi_i\right)^{\alpha(\sigma-1)}}\right) && \text{(A.28)}
\end{aligned}
$$

We can then write

$$
\begin{aligned}
L\Lambda &= P_X C && \text{(A.29)}\\
\text{where } \Lambda &\equiv \frac{1}{1-\alpha} \left(1 - \alpha \frac{\sum_i A_i Z_i^{\sigma-1} \left(1-\phi_i\right)^{\alpha(\sigma-1)+1}}{\sum_i A_i Z_i^{\sigma-1} \left(1-\phi_i\right)^{\alpha(\sigma-1)}}\right) && \text{(A.30)}
\end{aligned}
$$

Then the model is solved by using, from (A.7),

$$
\begin{aligned}
\chi L^{\eta} &= P_X^{-1} C^{-\rho} && \text{(A.31)}\\
\chi \left(P_X C \Lambda^{-1}\right)^{\eta} &= P_X^{-1} C^{-\rho} && \text{(A.32)}\\
\chi^{-1/(\eta+\rho)} \Lambda^{-1/(\eta+\rho)} \left(\Lambda / P_X\right)^{(\eta+1)/(\eta+\rho)} &= C && \text{(A.33)}
\end{aligned}
$$

$$\frac{d\Lambda_t}{d\phi_i} = \frac{-\alpha}{1-\alpha} A_i \tag{A.34}$$

A.1.2.1 Remark 3

The definition of the labor wedge is

$$\frac{\partial C / \partial L}{-U_L / U_C} \tag{A.35}$$

From equation (A.29), we have $\partial C/\partial L = \Lambda P_X^{-1}$, while we have from the household's first-order condition that $V'(L)/U'(C) = P_X^{-1}$ (since the nominal wage is normalized to 1). That immediately implies that the labor wedge is equal to Λ.

The efficiency wedge, or TFP, is defined by

$$TFP = \partial C / \partial L \tag{A.36}$$

so that equation (A.29) gives

$$TFP = \Lambda P_X^{-1} \tag{A.37}$$

A.1.2.2 Propositions 6 and 7

We give derivations for the first derivatives here. The second derivatives were calculated using Mathematica and code is included with the replication files.

First, define

$$g \equiv \frac{-1}{(1-\alpha)(1-\sigma)} \log \left(\sum_i A_i \left(1-\phi_{i,t}\right)^{\alpha(\sigma-1)} \exp\left((\sigma-1)\log Z_{i,t}\right) \right) \tag{A.38}$$

We then have $\log TFP = g + \log \Lambda_t$

Directly taking derivatives, we have

$$\frac{dg}{d\log Z_{i,t}} = \frac{-(\sigma-1)}{(1-\alpha)(1-\sigma)} \frac{A_i \left(1-\phi_{i,t}\right)^{\alpha(\sigma-1)} \exp\left((\sigma-1)\log Z_{i,t}\right)}{\sum_i A_i \left(1-\phi_{i,t}\right)^{\alpha(\sigma-1)} Z_{i,t}^{\sigma-1}} \tag{A.39}$$

$$\rightarrow \frac{1}{1-\alpha} A_i \tag{A.40}$$

where the notation "$\rightarrow$" in this section indicates evaluating at $Z_i = 1$ and $\phi_i = 0$ for all i.

Similarly,

$$\frac{d\log\Lambda_t}{d\log Z_{i,t}} = \left(1-\alpha\frac{\sum_i A_i Z_{i,t}^{\sigma-1}\left(1-\phi_{i,t}\right)^{1+\alpha(\sigma-1)}}{\sum_i A_i Z_{i,t}^{\sigma-1}\left(1-\phi_{i,t}\right)^{\alpha(\sigma-1)}}\right)^{-1} \tag{A.41}$$

$$\times\left(-\alpha\frac{\begin{array}{c}\left(\sum_i A_i Z_{i,t}^{\sigma-1}\left(1-\phi_{i,t}\right)^{\alpha(\sigma-1)}\right)A_i\left(\sigma-1\right)Z_{i,t}^{\sigma-1}\left(1-\phi_{i,t}\right)^{1+\alpha(\sigma-1)}\\ -\left(\sum_i A_i Z_{i,t}^{\sigma-1}\left(1-\phi_{i,t}\right)^{1+\alpha(\sigma-1)}\right)\left(\sigma-1\right)A_i Z_{i,t}^{\sigma-1}\left(1-\phi_{i,t}\right)^{\alpha(\sigma-1)}\end{array}}{\left(\sum_i A_i Z_{i,t}^{\sigma-1}\left(1-\phi_{i,t}\right)^{\alpha(\sigma-1)}\right)^2}\right) \tag{A.42}$$

$$\to 0 \tag{A.43}$$

We then have

$$\frac{d\log TFP}{d\log Z_{i,t}} \to \frac{1}{1-\alpha}A_i \tag{A.44}$$

For ϕ_i, the analogous derivatives are

$$\frac{dg}{d\phi_{i,t}} = \frac{\alpha\left(\sigma-1\right)}{\left(1-\alpha\right)\left(1-\sigma\right)}\frac{A_i\left(1-\phi_{i,t}\right)^{\alpha(\sigma-1)}\exp\left(\left(\sigma-1\right)\log Z_{i,t}\right)}{\sum_i A_i\left(1-\phi_{i,t}\right)^{\alpha(\sigma-1)}Z_{i,t}^{\sigma-1}} \tag{A.45}$$

$$\to \frac{-\alpha}{1-\alpha}A_i \tag{A.46}$$

$$\frac{d\log\Lambda_t}{d\phi_{i,t}} = \left(1-\alpha\frac{\sum_i A_i Z_{i,t}^{\sigma-1}\left(1-\phi_{i,t}\right)^{1+\alpha(\sigma-1)}}{\sum_i A_i Z_{i,t}^{\sigma-1}\left(1-\phi_{i,t}\right)^{\alpha(\sigma-1)}}\right)^{-1} \tag{A.47}$$

$$\times\left(\alpha\frac{\begin{array}{c}\left(\sum_i A_i Z_{i,t}^{\sigma-1}\left(1-\phi_{i,t}\right)^{\alpha(\sigma-1)}\right)A_i Z_{i,t}^{\sigma-1}\left(1-\phi_{i,t}\right)^{\alpha(\sigma-1)}\left(1+\alpha\left(\sigma-1\right)\right)\\ -\left(\sum_i A_i Z_{i,t}^{\sigma-1}\left(1-\phi_{i,t}\right)^{1+\alpha(\sigma-1)}\right)A_i Z_{i,t}^{\sigma-1}\left(1-\phi_{i,t}\right)^{\alpha(\sigma-1)-1}\alpha\left(\sigma-1\right)\end{array}}{\left(\sum_i A_i Z_{i,t}^{\sigma-1}\left(1-\phi_{i,t}\right)^{\alpha(\sigma-1)}\right)^2}\right) \tag{A.48}$$

$$\to \frac{\alpha}{1-\alpha}A_i \tag{A.49}$$

$$\frac{d\log TFP}{d\phi_i} \to \frac{-\alpha}{1-\alpha}A_i + \frac{\alpha}{1-\alpha}A_i = 0 \tag{A.50}$$

The fact that the sales shares s_i are equal to A_i at the steady-state is shown in section A.1.2.5.

A.1.2.3 Proposition 8

This section first derives equation (15) for a general function and then shows that the required condition on the derivatives holds for the variables listed in proposition 8.

Consider a variable $y = f(\varepsilon)$ for some shock ε. Assume f has a power series representation,

$$y = \sum_{j=0}^{\infty} \frac{1}{j!} f^{(j)} \varepsilon^j \tag{A.51}$$

where $f^{(j)}$ is the jth derivative of f evaluated at $\varepsilon = 0$ (and $f^{(0)} \equiv f(0)$). The variance is then

$$E\left[(y - E[y])^2\right] = (f')^2 \sigma^2 + o\left(\sigma^2\right) \tag{A.52}$$

where, as usual, the term $o(x)$ means that for a residual r, $\lim_{x \to 0} r/x = 0$ (and where the equation uses the more typical notation of f' for the derivative, again evaluated at $\varepsilon = 0$). A similar analysis holds for the third moment. Specifically,

$$\begin{aligned}
(y - E[y])^3 &= \left(\sum_{j=1}^{\infty} \frac{1}{j!} f^{(j)} \left(\varepsilon^j - E\left[\varepsilon^j\right]\right) \right)^3 && \text{(A.53)} \\
&= \sum_{j=1}^{\infty} \left(\frac{1}{j!}\right)^3 \left(f^{(j)}\right)^3 \left(\varepsilon^j - E\left[\varepsilon^j\right]\right)^3 && \text{(A.54)} \\
&\quad +3 \sum_{j \equiv k} \left(\frac{1}{j!}\right)^2 \left(f^{(j)}\right)^2 \frac{1}{k!} f^{(k)} \left(\varepsilon^j - E\left[\varepsilon^j\right]\right)^2 \left(\varepsilon^k - E\left[\varepsilon^k\right]\right) && \text{(A.55)} \\
&\quad +6 \sum_{j \neq k \neq m} \frac{1}{j!k!m!} f^{(j)} f^{(k)} f^{(m)} \left(\varepsilon^j - E\left[\varepsilon^j\right]\right) \left(\varepsilon^k - E\left[\varepsilon^k\right]\right) \left(\varepsilon^m - E\left[\varepsilon^m\right]\right) && \text{(A.56)}
\end{aligned}$$

Taking expectations,

$$\begin{aligned}
E\left[(y - E[y])^3\right] &= (f')^3 E\left[(\varepsilon - E[\varepsilon])^3\right] + 3 \sum_{2j+k \leq 4} \left(\frac{1}{j!}\right)^2 \left(f^{(j)}\right)^2 \frac{1}{k!} f^{(k)} E\left[\left(\varepsilon^j - E\left[\varepsilon^j\right]\right)^2 \left(\varepsilon^k - E\left[\varepsilon^k\right]\right)\right] && \text{(A.57)} \\
&\quad + o\left(\sigma^4\right) && \text{(A.58)}
\end{aligned}$$

We then have

$$
\begin{aligned}
E\left[(y-E[y])^3\right] &= E\left[\frac{3}{2}(f')^2 f''\varepsilon^2\left(\varepsilon^2-E\left[\varepsilon^2\right]\right)\right]+o\left(\sigma^4\right) && \text{(A.59)}\\
&= \frac{3}{2}(f')^2 f''\left(E\left[\varepsilon^4\right]-\sigma^4\right)+o\left(\sigma^4\right) && \text{(A.60)}\\
&= \frac{3}{2}(f')^2 f''(\kappa-1)\sigma^4+o\left(\sigma^4\right) && \text{(A.61)}
\end{aligned}
$$

where $\kappa \equiv E[\varepsilon^4]/E[\varepsilon^2]^2$ is the kurtosis of ε.

When $f' \neq 0$, the skewness coefficient is

$$
\begin{aligned}
skew(y) &= \frac{E\left[(y-E[y])^3\right]}{E\left[(y-E[y])^2\right]^{3/2}} && \text{(A.62)}\\
&= \frac{\frac{3}{2}(f')^2 f''(\kappa-1)\sigma^4+o(\sigma^4)}{\left(f'^2\sigma^2+o(\sigma^2)\right)^{3/2}} && \text{(A.63)}\\
&= \frac{\frac{3}{2}(f')^2 f''(\kappa-1)\sigma^4+o(\sigma^4)}{\left|f'^6\sigma^6+o(\sigma^6)\right|^{1/2}} && \text{(A.64)}\\
&= \frac{3}{2}\frac{f''}{|f'|}(\kappa-1)\sigma+o(\sigma) && \text{(A.65)}
\end{aligned}
$$

When $f' = 0$ the skewness diverges as $\sigma \to 0$, with sign equal to that of f'' (it is straightforward to show that by taking the next order term in the expansion of the variance).

To show that the logs of GDP, employment, and TFP are negatively skewed with respect to the individual shocks, we need to show that the ratio of their second derivatives to the first derivatives with respect to each shock is negative, while we need to show the same ratio is positive for the log labor wedge. For TFP and the labor wedge, those results follow directly from propositions 6 and 7. For GDP and employment, they follow from the facts that both are linearly increasing in log TFP and linearly decreasing in the log labor wedge (theorem 2 and remark 4).

A.1.2.4 Proposition 13

For a given shock x_i, we have

$$
\begin{aligned}
\operatorname{cov}(y_i, c) &= \frac{dy_i}{dx_i}\frac{dc}{dx_i}\operatorname{var}(x_i) && \text{(A.66)}\\
\frac{d}{dx_i}\operatorname{cov}(y_i, c) &= \left(\frac{d^2y_i}{dx_i^2}\frac{dc}{dx_i}+\frac{dy_i}{dx_i}\frac{d^2c}{dx_i^2}\right)\operatorname{var}(x_i) && \text{(A.67)}
\end{aligned}
$$

where $y_i \equiv \log Y_i$ and $c \equiv \log C$. Furthermore,

$$\frac{d\operatorname{cov}(y_i, c)}{dy_i} = \frac{d\operatorname{cov}(y_i, c)}{dx_i}\left(\frac{dy_i}{dx_i}\right)^{-1} \tag{A.68}$$

The same expressions obviously hold replacing log sector output with log employment. We therefore consider four cases – output and labor each with the two types of shocks.

Output and shocks to z_i. We show that

$$\frac{d^2 y_i}{dz_i^2}\frac{dc}{dz_i} + \frac{dy_i}{dz_i}\frac{d^2 c}{dz_i^2} < 0 \tag{A.69}$$

From theorem 2 and propositions 6 and 7, we have that $\frac{dc}{dz_i} > 0$ and $\frac{d^2 c}{dz_i^2} < 0$. So we need to show $\frac{d^2 y_i}{dz_i^2} < 0$ and $\frac{dy_i}{dz_i} > 0$.

As to $\frac{d^2 y_i}{dz_i^2}$, that follows from the fact (from proposition 9) that $\frac{d^2 y_i}{dz_i^2} = \frac{dy_i}{d\log TFP}\frac{d^2 \log TFP}{dz_i}$ and that the latter two terms are positive and negative, respectively. Note that the proof of proposition 9 gives the formulas for the coefficients in the text.

Similarly, $\frac{dy_i}{dz_i} > 0$ follows from propositions 6 and 9. We thus have that $\frac{d\operatorname{cov}(y_i,c)}{dz_i} < 0$, and since $\frac{dy_i}{dx_i} > 0$, $\frac{d\operatorname{cov}(y_i,c)}{dy_i} < 0$.

Output and shocks to ϕ_i. We now show that $\frac{d\operatorname{cov}(y_i,c)}{d\phi_i} > 0$. Again, proposition 6 gives $dc/d\phi_i < 0$. Next, from proposition 9,

$$\frac{d^2 y_i}{d\phi_i^2} = -\alpha\sigma + \left(-1/(\eta+\rho) - \alpha - (1-\alpha)(1-\sigma)\right)\frac{d^2 \log \Lambda}{d\phi_i^2} \tag{A.70}$$

$$+\left((1-\rho)/(\eta+\rho) + \alpha + (1-\alpha)(1-\sigma)\right)\frac{d^2 \log TFP}{d\phi_i} \tag{A.71}$$

The first term is obviously negative. $\frac{d^2 \log \Lambda}{d\phi^2} > 0$ by the assumption of the proposition that $\sigma \lesssim 1$. Similarly, the coefficient in the second line is positive by assumption, and $\frac{d^2 \log TFP}{d\phi_i} > 0$ from proposition 7. This implies that $\frac{d^2 y_i}{d\phi_i^2}\frac{dc}{d\phi_i} > 0$. $\frac{d^2 c}{d\phi_i^2} < 0$ and $\frac{dy_i}{d\phi_i} < 0$ follow by the same type of argument.

Finally, since $\frac{dy_i}{d\phi_i} < 0$ and $\frac{d\operatorname{cov}(y_i,c)}{d\phi_i} > 0$, $\frac{d\operatorname{cov}(y_i,c)}{dy_i} < 0$.

Employment and shocks to z_i. We show here first that

$$\frac{d}{dz_i}\operatorname{cov}(\log L_i, \log L) > 0 \tag{A.72}$$

From equation (A.102) and theorem 2 (and using equation (A.29)),

$$L_i = (1-\alpha) A_i Z_i^{\sigma-1} (1-\phi_i)^{\alpha(\sigma-1)} \Lambda_t^{-1/(\eta+\rho)} TFP^{(1-\rho)/(\eta+\rho)} (\Lambda/TFP)^{-(1-\alpha)(1-\sigma)} \tag{A.73}$$

We then have,

$$\frac{d\log L_i}{dz_i} = \underbrace{(\sigma-1)}_{<0} + \underbrace{\left[-\left(\frac{1}{\eta+\rho} + (1-\alpha)(1-\sigma)\right)\right]}_{<0} \underbrace{\frac{d\log\Lambda}{dz_i}}_{=0} + \underbrace{\left[\frac{1-\rho}{\eta+\rho} + (1-\alpha)(1-\sigma)\right]}_{>0} \underbrace{\frac{d\log TFP}{dz_i}}_{>0} \tag{A.74}$$

$$\frac{d^2\log L_i}{dz_i^2} = \underbrace{\left[-\left(\frac{1}{\eta+\rho} + (1-\alpha)(1-\sigma)\right)\right]}_{<0} \underbrace{\frac{d^2\log\Lambda}{dz_i^2}}_{=0} + \underbrace{\left[\frac{1-\rho}{\eta+\rho} + (1-\alpha)(1-\sigma)\right]}_{>0} \underbrace{\frac{d^2\log TFP}{dz_i^2}}_{<0} \tag{A.75}$$

For aggregate employment,

$$L_t = C_t TFP_t^{-1} = \chi^{-1/(\eta+\rho)} \Lambda_t^{-1/(\eta+\rho)} TFP^{(1-\rho)/(\eta+\rho)} \tag{A.76}$$

$$\frac{d\log L}{dz_i} = \frac{-1}{\eta+\rho} \underbrace{\frac{d\log\Lambda}{dz_i}}_{=0} + \underbrace{\frac{1-\rho}{\eta+\rho}}_{>0} \underbrace{\frac{d\log TFP}{dz_i}}_{>0} \tag{A.77}$$

$$\frac{d^2\log L}{dz_i^2} = \frac{-1}{\eta+\rho} \underbrace{\frac{d^2\log\Lambda}{dz_i^2}}_{=0} + \underbrace{\frac{1-\rho}{\eta+\rho}}_{>0} \underbrace{\frac{d^2\log TFP}{dz_i^2}}_{<0} \tag{A.78}$$

Combining,

$$\frac{d^2\log L_i}{dz_i^2}\frac{d\log L}{dz_i} + \frac{d\log L_i}{dz_i}\frac{d^2\log L}{dz_i^2} \tag{A.79}$$

$$= \left(\left[\frac{1-\rho}{\eta+\rho} + (1-\alpha)(1-\sigma)\right] \frac{d^2\log TFP}{dz_i^2}\right) \frac{1-\rho}{\eta+\rho}\frac{d\log TFP}{dz_i} \tag{A.80}$$

$$+ \left[(\sigma-1) + \left[\frac{1-\rho}{\eta+\rho} + (1-\alpha)(1-\sigma)\right] \frac{d\log TFP}{dz_i}\right] \frac{1-\rho}{\eta+\rho}\frac{d^2\log TFP}{dz_i^2} \tag{A.81}$$

$$= \left[2\frac{1-\rho}{\eta+\rho}\frac{A_i}{1-\alpha} + \underbrace{(\sigma-1)(1-A_i)}_{<0}\right] \underbrace{\frac{1-\rho}{\eta+\rho}}_{>0} \underbrace{\frac{d^2\log TFP}{dz_i^2}}_{<0} \tag{A.82}$$

So for A_i sufficiently small,

$$\frac{d}{dz_i}\operatorname{cov}(\log L_i, \log L) > 0 \tag{A.83}$$

Since $\frac{d\log L_i}{dz_i} < 0$ for A_i sufficiently small, $\frac{d}{d\log L_i}\text{cov}\left(\log L_i, \log L\right) < 0$

Employment and shocks to ϕ_i

We first show that

$$\frac{d}{d\phi_i}\text{cov}\left(\log L_i, \log L\right) < 0 \tag{A.84}$$

As above,

$$\frac{d\log L_i}{d\phi_i} = \underbrace{-\alpha\left(\sigma-1\right)}_{>0} + \underbrace{\left[-\left(\frac{1}{\eta+\rho} + \left(1-\alpha\right)\left(1-\sigma\right)\right)\right]}_{>0}\underbrace{\frac{d\log\Lambda}{d\phi_i}}_{>0} + \underbrace{\left[\frac{1-\rho}{\eta+\rho} + \left(1-\alpha\right)\left(1-\sigma\right)\right]}_{>0}\underbrace{\frac{d\log TFP}{d\phi_i}}_{=0} \tag{A.85}$$

$$\frac{d^2\log L_i}{d\phi_i^2} = \underbrace{-\alpha\left(\sigma-1\right)}_{>0} + \underbrace{\left[-\left(\frac{1}{\eta+\rho} + \left(1-\alpha\right)\left(1-\sigma\right)\right)\right]}_{<0}\underbrace{\frac{d^2\log\Lambda}{d\phi_i^2}}_{>0} + \underbrace{\left[\frac{1-\rho}{\eta+\rho} + \left(1-\alpha\right)\left(1-\sigma\right)\right]}_{>0}\underbrace{\frac{d^2\log TFP}{d\phi_i^2}}_{<0} \tag{A.86}$$

$$\frac{d\log L}{d\phi_i} = \frac{-1}{\eta+\rho}\underbrace{\frac{d\log\Lambda}{d\phi_i}}_{>0} + \underbrace{\frac{1-\rho}{\eta+\rho}}_{>0}\underbrace{\frac{d\log TFP}{d\phi_i}}_{=0} \tag{A.87}$$

$$\frac{d^2\log L}{d\phi_i^2} = \frac{-1}{\eta+\rho}\underbrace{\frac{d^2\log\Lambda}{d\phi_i^2}}_{>0} + \underbrace{\frac{1-\rho}{\eta+\rho}}_{>0}\underbrace{\frac{d^2\log TFP}{d\phi_i^2}}_{<0} \tag{A.88}$$

Again combining,

$$\frac{d^2\log L_i}{d\phi_i^2}\frac{d\log L}{d\phi_i} + \frac{d\log L_i}{d\phi_i}\frac{d^2\log L}{d\phi_i^2} \tag{A.89}$$

$$= \left(\frac{\partial L_i}{\partial\phi_i} + \frac{d\log L_i}{d\log\Lambda}\frac{d^2\log\Lambda}{d\phi_i} + \frac{d\log L_i}{d\log TFP}\frac{d^2\log TFP}{d\phi_i}\right)\frac{d\log L}{d\log\Lambda}\frac{d\log\Lambda}{d\phi_i} \tag{A.90}$$

$$+\left(\frac{\partial\log L_i}{\partial\phi_i} + \frac{d\log L_i}{d\log\Lambda}\frac{d\log\Lambda}{d\phi_i}\right)\left(\frac{d\log L}{d\log\Lambda}\frac{d^2\log\Lambda}{d\phi_i^2} + \frac{d\log L}{d\log TFP}\frac{d^2\log TFP}{d\phi_i^2}\right) \tag{A.91}$$

$$
\begin{aligned}
&\frac{d^2 \log L_i}{d\phi_i^2}\frac{d \log L}{d\phi_i} + \frac{d \log L_i}{d\phi_i}\frac{d^2 \log L}{d\phi_i^2} \\
&= \left[\begin{array}{c} \underbrace{\alpha(1-\sigma)}_{>0} + \underbrace{\left[-\left(\frac{1}{\eta+\rho} + (1-\alpha)(1-\sigma)\right)\right]}_{<0}\underbrace{\frac{d^2 \log \Lambda}{d\phi_i^2}}_{>0} \\ + \underbrace{\left[\frac{1-\rho}{\eta+\rho} + (1-\alpha)(1-\sigma)\right]}_{>0}\underbrace{\frac{d^2 \log TFP}{d\phi_i^2}}_{<0} \end{array}\right] \frac{-1}{\eta+\rho}\underbrace{\frac{d \log \Lambda}{d\phi_i}}_{>0} \\
&\quad + \left[\underbrace{-\alpha(\sigma-1)}_{>0} + \underbrace{\left[-\left(\frac{1}{\eta+\rho} + (1-\alpha)(1-\sigma)\right)\right]}_{>0}\underbrace{\frac{d \log \Lambda}{d\phi_i}}_{>0}\right]\left(\frac{-1}{\eta+\rho}\underbrace{\frac{d^2 \log \Lambda}{d\phi_i^2}}_{>0} + \underbrace{\frac{1-\rho}{\eta+\rho}}_{>0}\underbrace{\frac{d^2 \log TFP}{d\phi_i^2}}_{<0}\right)
\end{aligned}
$$

As in the previous section, the terms in brackets on the second two lines are both positive for A_i sufficiently small, implying that the entire derivative is negative. Since $d \log L_i / d\phi_i > 0$, we again have that as employment in sector i rises, its covariance with aggregate employment falls.

A.1.2.5 Proposition 9

Combining equation (A.1) with $\tilde{W}$, equation (A.2), and (A.29)

$$
P_i = Z_i^{-1}(1-\phi_i)^{-\alpha} P_X^{\alpha} \tag{A.92}
$$

$$
\begin{aligned}
Y_i &= \zeta Z_i L_i (L_i/X_i)^{-\alpha}(1-\phi_i)^{\alpha} && \text{(A.93)} \\
&= (1-\alpha)^{-1} Z_i L_i P_X^{-\alpha}(1-\phi_i)^{\alpha} && \text{(A.94)} \\
&= \frac{A_i Z_i^{\sigma}(1-\phi_i)^{\alpha\sigma}}{\sum_i A_i Z_i^{\sigma-1}(1-\phi_i)^{\alpha(\sigma-1)}} L P_X^{-\alpha} && \text{(A.95)} \\
&= A_i Z_i^{\sigma}(1-\phi_i)^{\alpha\sigma} L P_X^{-\alpha-(1-\alpha)(1-\sigma)} && \text{(A.96)}
\end{aligned}
$$

It is then immediate that at $Z_i = 1$ and $\phi_i = 0 \; \forall \; i$, the sales share of sector i is equal to A_i.

To map this into the proposition,

$$
\begin{aligned}
C_t &= \chi^{-1/(\eta+\rho)} \Lambda_t^{-1/(\eta+\rho)} TFP^{(\eta+1)/(\eta+\rho)} && \text{(A.97)} \\
L_t &= C_t TFP_t^{-1} = \chi^{-1/(\eta+\rho)} \Lambda_t^{-1/(\eta+\rho)} TFP^{(1-\rho)/(\eta+\rho)} && \text{(A.98)}
\end{aligned}
$$

$$Y_i = A_i Z_i^{\sigma} (1-\phi_i)^{\alpha\sigma} \chi^{-1/(\eta+\rho)} \Lambda_t^{-1/(\eta+\rho)} TFP^{(1-\rho)/(\eta+\rho)} \left(\Lambda TFP^{-1}\right)^{-\alpha-(1-\alpha)(1-\sigma)} \quad (A.99)$$

$$Y_i = A_i Z_i^{\sigma} (1-\phi_i)^{\alpha\sigma} \chi^{-1/(\eta+\rho)} \Lambda_t^{-1/(\eta+\rho)-\alpha-(1-\alpha)(1-\sigma)} TFP^{(1-\rho)/(\eta+\rho)+\alpha+(1-\alpha)(1-\sigma)} \quad (A.100)$$

For sector employment, we have

$$L_i = P_i (1-\alpha) Y_i \quad (A.101)$$

$$= (1-\alpha) A_i Z_i^{\sigma-1} (1-\phi_i)^{\alpha(\sigma-1)} LP_X^{-(1-\alpha)(1-\sigma)} \quad (A.102)$$

A.2 Extensions and Variations

A.2.1 Partially refundable wedge shocks

In the case where the wedges are partially refunded, the only equation in the model that changes is the household's budget constraint,

$$\sum_i L_i + \gamma \sum_i \phi_i X_i = P_X C \quad (A.103)$$

where the parameter $0 \leq \gamma \leq 1$ determines the fraction of the wedges that is refunded.

It is then straightforward to solve the model, following the proof of theorem 2, and show that the only difference is that Λ is now

$$\Lambda \equiv \frac{1}{1-\alpha} \left(1 - \gamma\alpha \frac{\sum_i A_i Z_i^{\sigma-1} (1-\phi_i)^{\alpha(\sigma-1)+1}}{\sum_i A_i Z_i^{\sigma-1} (1-\phi_i)^{\alpha(\sigma-1)}} \right) \quad (A.104)$$

All other equations remain unaffected. We then have the following modifications to propositions 6 and 7:

Proposition 15. *Starting from an efficient steady-state ($\phi_{i,t} = \log Z_{i,t} = 0 \ \forall \ i$), the derivatives of TFP and the labor wedge are*

$$\frac{d \log TFP_t}{d \log Z_{i,t}} = \frac{1}{1-\alpha} s_i \qquad \frac{d \log \Lambda_t}{d \log Z_{i,t}} = 0$$

$$\frac{d \log TFP_t}{d\phi_{i,t}} = \frac{\alpha(\gamma-1)}{(1-\alpha)(1-\alpha\gamma)} s_i \leq 0 \qquad \frac{d \log \Lambda_t}{d\phi_{i,t}} = \frac{\alpha\gamma}{1-\alpha\gamma} s_i$$

where $s_i \equiv P_{i,t} Y_{i,t} / \sum_i P_{i,t} Y_{i,t}$ is the nominal sales share of sector i.

Proposition 16. *The second derivatives of Λ and TFP evaluated at the efficient steady-state satisfy*

$$\frac{d^2 \log \Lambda_t}{d\left(\log Z_{i,t}\right)^2} = 0 \qquad \frac{d^2 \log \Lambda_t}{d\phi_{i,t}^2} > 0 \iff \sigma < 1 - \frac{A_i \gamma}{2\left(1 - A_i\right)\left(1 - \alpha\gamma\right)} \approx 1$$

$$\frac{d^2 \log TFP_t}{d\phi_{i,t}^2} < 0 \qquad \frac{d^2 \log TFP_t}{d\left(\log Z_{i,t}\right)^2} < 0 \iff \sigma < 1$$

A.2.2 Model with multiple levels of aggregation

This section examines a version of the model with multiple subunits within each sector – they could be either firms or subsectors, but this section will just refer to them as firms (the following section interprets them as subsectors). As in the model from the main text, there is a set of goods indexed by j, which we refer to as the sector outputs. Those goods are combined into the composite good according to the same function,

$$X = \left(\sum_{j=1}^{n} A_j^{1/\sigma} Y_j^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)}. \tag{A.105}$$

Now, though, instead of production occurring at the sector level, we assume that there is a set of firms (or, again, subsectors) within each sector that each produce a differentiated output, denoted $Y_{j,i}$, according to the production function

$$Y_{j,i} = \zeta Z_{j,i} L_{j,i}^{\alpha} X_{j,i}^{1-\alpha} \tag{A.106}$$

where $Z_{j,i}$ is the productivity of firm i in sector j, $L_{j,i}$ is its use of labor, and $X_{j,i}$ is its use of the intermediate. ζ is again a normalization factor with $\zeta = (1-\alpha)^{-(1-\alpha)} \alpha^{-\alpha}$. The total output of sector j is a CES aggregate of the outputs of the firms in that sector,

$$Y_j = \left(\sum_{i} N^{-1/\gamma} Y_{j,i}^{(\gamma-1)/\gamma}\right)^{\gamma/(\gamma-1)}, \tag{A.107}$$

where γ is the elasticity of substitution across firms and here we are normalizing the weight on each firm in the sector to N^{-1}.

Proposition 17. *In the model of this section with multiple levels of aggregation,*

$$C = \chi^{-1/(\eta+\rho)} \Lambda^{-1/(\eta+\rho)} \left(\Lambda / P_X\right)^{(\eta+1)/(\eta+\rho)} \tag{A.108}$$

$$\chi L^{\eta} = P_X^{-1} C^{-\rho} \tag{A.109}$$

as in the benchmark model except now

$$\Lambda = \frac{1}{1-\alpha}\left(1-\alpha\sum_i \frac{\tilde{Z}_i^{\sigma-1}A_i}{\sum_i \tilde{Z}_i^{\sigma-1}A_i}\frac{\sum_j Z_{i,j}^{\gamma-1}\left(1-\phi_{i,j}\right)^{\alpha(\gamma-1)+1}}{\tilde{Z}_i^{\gamma-1}}\right) \quad \text{(A.110)}$$

$$\textit{where } \tilde{Z}_i \equiv \left(\sum_i N^{-1/\gamma} Z_{i,j}^{\gamma-1}\left(1-\phi_{i,j}\right)^{\alpha(\gamma-1)}\right)^{1/(\gamma-1)} \quad \text{(A.111)}$$

Sector and firm output satisfy

$$Y_i = L\frac{\tilde{Z}_i^{\sigma}A_i}{P_X^{(1-\alpha)(1-\sigma)}}\frac{\alpha}{1-\alpha}P_X^{-\alpha} \quad \text{(A.112)}$$

$$Y_{i,j} = Z_{i,j}\left(1-\phi_{i,j}\right)^{\alpha} L\frac{\tilde{Z}_i^{\sigma-1}A_i}{\sum_i \tilde{Z}_i^{\sigma-1}A_i}\frac{Z_{i,j}^{\gamma-1}\left(1-\phi_{i,j}\right)^{\alpha(\gamma-1)}}{\tilde{Z}_i^{\gamma-1}}\frac{\alpha}{1-\alpha}P_X^{-\alpha} \quad \text{(A.113)}$$

Sector and firm employment satisfy

$$L_i = L\frac{\tilde{Z}_i^{\sigma-1}A_i}{\sum_i \tilde{Z}_i^{\sigma-1}A_i} \quad \text{(A.114)}$$

$$L_{i,j} = L\frac{\tilde{Z}_i^{\sigma-1}A_i}{\sum_i \tilde{Z}_i^{\sigma-1}A_i}\frac{Z_{i,j}^{\gamma-1}\left(1-\phi_{i,j}\right)^{\alpha(\gamma-1)}}{\tilde{Z}_i^{\gamma-1}} \quad \text{(A.115)}$$

And the price of the final good is

$$P_X = \left(\sum_j A_j \tilde{Z}_i^{\sigma-1}\right)^{1/(1-\alpha)(1-\sigma)} \quad \text{(A.116)}$$

The derivation of this result is a straightforward generalization of the baseline case.

A.2.2.1 Discussion

It is now possible to see how the two elasticities – σ across sectors and γ within sectors – affect aggregation. $\tilde{Z}$ is an aggregate of the firm-level shocks. If $\gamma > 1$, it is a convex aggregate, and will tend to be positively skewed. Overall skewness in the economy depends on the relative values of γ and σ.

Note, though, that if $\gamma > 1$, then the sector specific shocks, $\tilde{Z}_i$ would be positively skewed, which we do not see (specifically in the analysis of the residuals for output and employment). We also do not observe any skewness in sector-level TFP in the NBER-CES manufacturing database. That is all consistent with a value of γ near 1.

In addition, the empirical evidence at the firm level from Barrot and Sauvagnat (2016) and Carvalho et al. (2021) implies that in the short-run, substitution across firms is in fact limited, implying that γ is finite, and potentially even smaller than 1.

A.2.3 Elastic and fixed factors

This section presents a version of the model in which certain factors are fixed in each sector. Specifically, the sector production function is

$$Y_i = \zeta Z_i K_i^{1-\beta-\alpha} L_i^{\beta} \left(1-\phi_i\right)^{\alpha} X_i^{\alpha} \tag{A.117}$$

and we normalize $K_i = 1 \; \forall \; i$.

This section derives the following solution to this generalized model

Proposition 18. *Taking the baseline model with the only modification that some inputs cannot be adjusted (i.e. equation (A.117)), we have*

$$C = \chi \left(\tilde{W} \bar{P}_X^{-1}\right)^{\frac{\beta+\eta(1-\alpha)-(1-\alpha-\beta)}{\rho\beta+\eta(1-\alpha)-(1-\alpha-\beta)}} \tilde{W}^{-\beta/(\rho\beta+\eta(1-\alpha)-(1-\alpha-\beta))} \tag{A.118}$$

$$L = \left(C\tilde{W}^{-1}\bar{P}_X\right)^{(1-\alpha)/\beta} \tag{A.119}$$

where

$$\tilde{W} \equiv \beta^{-1}\left(1-\alpha\sum_i \left(1-\phi_i\right)\frac{\left(A_i Z_i^{(\sigma-1)}\left(1-\phi_i\right)^{\alpha(\sigma-1)}\right)^{1/(\sigma-(\beta+\alpha)(\sigma-1))}}{\sum_i \left(A_i Z_i^{(\sigma-1)}\left(1-\phi_i\right)^{\alpha(\sigma-1)}\right)^{1/(\sigma-(\beta+\alpha)(\sigma-1))}}\right) \tag{A.120}$$

$$\bar{P}_X = \left(\sum_j A_j \left(Z_i^{-1}\left(1-\phi_i\right)^{-\alpha}\left(\frac{\left(A_i Z_i^{(\sigma-1)}\left(1-\phi_i\right)^{\alpha(\sigma-1)}\right)^{1/(\sigma-(\beta+\alpha)(\sigma-1))}}{\sum_i \left(A_i Z_i^{(\sigma-1)}\left(1-\phi_i\right)^{\alpha(\sigma-1)}\right)^{1/(\sigma-(\beta+\alpha)(\sigma-1))}}\right)^{1-\alpha-\beta}\right)^{1-\sigma}\right)^{1/(1-\alpha)(1-\sigma)} \tag{A.121}$$

$$P_X = L^{(1-\alpha-\beta)/(1-\alpha)}\bar{P}_X \tag{A.122}$$

Sector output and employment are

$$Y_i = Z_i\left(1-\phi_i\right)^{\alpha} L_i^{\beta+\alpha}\alpha P_X^{-1}\beta^{-1} \tag{A.123}$$

$$L_i = L\frac{\left(A_i Z_i^{(\sigma-1)}\left(1-\phi_i\right)^{\alpha(\sigma-1)}\right)^{1/(\sigma-(\beta+\alpha)(\sigma-1))}}{\sum_i \left(A_i Z_i^{(\sigma-1)}\left(1-\phi_i\right)^{\alpha(\sigma-1)}\right)^{1/(\sigma-(\beta+\alpha)(\sigma-1))}} \tag{A.124}$$

Profits are

$$\Pi_t = \frac{(1-\beta-\alpha)}{\beta} L_i \tag{A.125}$$

A.2.3.1 Equilibrium conditions

Lemma 19. *The equilibrium conditions are the same as in the benchmark case except for Price of input:*

$$P_X = \left(\sum_j A_j \left(Z_i^{-1} \left(1-\phi_i\right)^{-\alpha} L_i^{1-\alpha-\beta} \right)^{1-\sigma} \right)^{1/(1-\alpha)(1-\sigma)} \tag{A.126}$$

Marginal cost minimization:

$$L_i = P_i \beta Y_i \tag{A.127}$$
$$P_X X_i = P_i \alpha Y_i \tag{A.128}$$

Prices:

$$P_i = Z_i^{-1} \left(1-\phi_i\right)^{-\alpha} L_i^{1-\alpha-\beta} P_X^{\alpha} \tag{A.129}$$

HH budget:

$$\sum_i L_i + \sum_i \phi_i X_i + \sum_i \Pi_i = p_X C \tag{A.130}$$

Writing the cost minimization problem for sector i in its Lagrangian form (with P_i being the Lagrange multiplier due to competitive markets), we have

$$\min L_i + P_X X_i - P_i \left(\zeta Z_i \left(1-\phi_i\right)^{\alpha} L_i^{\beta} X_i^{\alpha} - Y_i \right) \tag{A.131}$$
$$L_i = P_i \beta Y_i \tag{A.132}$$
$$P_X X_i = P_i \alpha Y_i \tag{A.133}$$

$$X_i = \alpha P_X^{-1} \beta^{-1} L_i \tag{A.134}$$

where $\zeta = \alpha^{-\alpha}\beta^{\alpha-1}$. Plugging that back into the production function to eliminate Y_i,

$$Y_i = \zeta Z_i \left(1-\phi_i\right)^{\alpha} L_i^{\beta} X_i^{\alpha} \tag{A.135}$$
$$\left(P_i\beta\right)^{-1} L_i = \zeta Z_i \left(1-\phi_i\right)^{\alpha} L_i^{\beta} \left(\alpha P_X^{-1} \beta^{-1} L_i \right)^{\alpha} \tag{A.136}$$
$$P_i = Z_i^{-1} \left(1-\phi_i\right)^{-\alpha} L_i^{1-\beta-\alpha} P_X^{\alpha} \tag{A.137}$$

$$P_X = \left(\sum_j A_j \left(Z_i^{-1} (1-\phi_i)^{-\alpha} L_i^{1-\beta-\alpha} \right)^{1-\sigma} \right)^{1/(1-\alpha)(1-\sigma)} \tag{A.138}$$

We also have

$$\begin{aligned} Y_i &= Z_i (1-\phi_i)^{\alpha} L_i^{\beta} X_i^{\alpha} & \text{(A.139)} \\ &= Z_i (1-\phi_i)^{\alpha} L_i^{\beta+\alpha} \alpha P_X^{-1} \beta^{-1} & \text{(A.140)} \end{aligned}$$

A.2.3.2 Sector employment

Taking first (A.3) then (A.1), the production function, and (A.4) (and dropping time subscripts for concision) yields

$$\begin{aligned} L_i &\propto P_i Y_i & \text{(A.141)} \\ &\propto A_i^{1/\sigma} Y_i^{(\sigma-1)/\sigma} & \text{(A.142)} \\ &\propto A_i^{1/\sigma} \left(Z_i (1-\phi_i)^{\alpha} L_i^{\beta+\alpha} \right)^{(\sigma-1)/\sigma} & \text{(A.143)} \\ L_i &\propto \left(A_i Z_i^{(\sigma-1)} (1-\phi_i)^{\alpha(\sigma-1)} \right)^{1/(\sigma-(\beta+\alpha)(\sigma-1))} & \text{(A.144)} \end{aligned}$$

which then means due to the adding-up constraint,

$$L_i = L \frac{\left(A_i Z_i^{(\sigma-1)} (1-\phi_i)^{\alpha(\sigma-1)} \right)^{1/(\sigma-(\beta+\alpha)(\sigma-1))}}{\sum_i \left(A_i Z_i^{(\sigma-1)} (1-\phi_i)^{\alpha(\sigma-1)} \right)^{1/(\sigma-(\beta+\alpha)(\sigma-1))}} \tag{A.145}$$

The pricing formula gives

$$P_X = L^{\frac{(1-\alpha-\beta)}{(1-\alpha)}} \left(\sum_j A_j \left(Z_i^{-1} (1-\phi_i)^{-\alpha} \left(\frac{\left(A_i \left[Z_i (1-\phi_i)^{\alpha} \right]^{(\sigma-1)} \right)^{\frac{1}{(\sigma-(\beta+\alpha)(\sigma-1))}}}{\sum_i \left(A_i \left[Z_i (1-\phi_i)^{\alpha} \right]^{(\sigma-1)} \right)^{\frac{1}{(\sigma-(\beta+\alpha)(\sigma-1))}}} \right)^{1-\alpha-\beta} \right)^{1-\sigma} \right)^{\frac{1}{(1-\alpha)(1-\sigma)}} \tag{A.146}$$

If $1-\alpha-\beta=0$, then this returns the price from the benchmark specification.

A.2.3.3 Profits

Profits are

$$\begin{aligned} \Pi_i &= P_i Y_i - L_i - P_X X_i && \text{(A.147)} \\ &= (1-\alpha-\beta) P_i Y_i && \text{(A.148)} \\ &= (1-\alpha-\beta) \beta^{-1} L_i && \text{(A.149)} \end{aligned}$$

Aggregate consumption is then

$$\begin{aligned} C &= P_X^{-1} \left(L + \sum_i (\phi_i P_X X_i + \Pi_i) \right) && \text{(A.150)} \\ &= P_X^{-1} \left(L + \beta^{-1} \sum_i (\phi_i \alpha + (1-\alpha-\beta)) L_i \right) && \text{(A.151)} \\ &= P_X^{-1} \left(\beta^{-1} \sum_i (1 - \alpha (1-\phi_i)) L_i \right) && \text{(A.152)} \\ &= P_X^{-1} \beta^{-1} L \left(1 - \alpha \sum_i (1-\phi_i) \frac{\left(A_i Z_i^{(\sigma-1)} (1-\phi_i)^{\alpha(\sigma-1)} \right)^{1/(\sigma-(\beta+\alpha)(\sigma-1))}}{\sum_i \left(A_i Z_i^{(\sigma-1)} (1-\phi_i)^{\alpha(\sigma-1)} \right)^{1/(\sigma-(\beta+\alpha)(\sigma-1))}} \right) && \text{(A.153)} \end{aligned}$$

We can then write

$$\begin{aligned} L\Lambda &= P_X C && \text{(A.154)} \\ \text{where } \Lambda &\equiv \beta^{-1} \left(1 - \alpha \sum_i (1-\phi_i) \frac{\left(A_i Z_i^{(\sigma-1)} (1-\phi_i)^{\alpha(\sigma-1)} \right)^{1/(\sigma-(\beta+\alpha)(\sigma-1))}}{\sum_i \left(A_i Z_i^{(\sigma-1)} (1-\phi_i)^{\alpha(\sigma-1)} \right)^{1/(\sigma-(\beta+\alpha)(\sigma-1))}} \right) && \text{(A.155)} \end{aligned}$$

Note also that

$$P_X = L^{(1-\alpha-\beta)/(1-\alpha)} \bar{P}_X \quad \text{(A.156)}$$

$$\begin{aligned} L\Lambda &= L^{(1-\alpha-\beta)/(1-\alpha)} \bar{P}_X C && \text{(A.157)} \\ C &= L^{\beta/(1-\alpha)} \Lambda \bar{P}_X^{-1} && \text{(A.158)} \end{aligned}$$

TFP, measured as the Solow residual, is

$$\log C - \frac{\beta}{1-\alpha} \log L = \log \left(\tilde{W} / \bar{P}_X \right) \quad \text{(A.159)}$$

where $\beta/(1-\alpha)$ is labor's share of income, and capital here is constant so can be ignored (up to a level shift).

Then the model is solved by using, from (A.7),

$$L^{\beta/(1-\alpha)} = \Lambda^{-1}\bar{P}_X C \tag{A.160}$$

$$\begin{aligned}
\chi L^{\eta} &= P_X^{-1} C^{-\rho} && \text{(A.161)}\\
\chi L^{\eta-(1-\alpha-\beta)/(1-\alpha)} &= \bar{P}_X^{-1} C^{-\rho} && \text{(A.162)}\\
\chi\left(\tilde{W}^{-1}\bar{P}_X\right)^{\frac{\eta(1-\alpha)-(1-\alpha-\beta)+\beta}{\beta}}\Lambda &= C^{-\rho-\frac{\eta(1-\alpha)-(1-\alpha-\beta)}{\beta}} && \text{(A.163)}
\end{aligned}$$

$$C = \chi\left(\tilde{W}\bar{P}_X^{-1}\right)^{\frac{\beta+\eta(1-\alpha)-(1-\alpha-\beta)}{\rho\beta+\eta(1-\alpha)-(1-\alpha-\beta)}}\Lambda^{-\beta/(\rho\beta+\eta(1-\alpha)-(1-\alpha-\beta))} \tag{A.164}$$

Again, when $1-\alpha-\beta=0$, this reduces to the result from the benchmark model.

A.2.3.4 Analog to proposition 3

The definition of the labor wedge, Λ, is

$$\Lambda \equiv \left(\frac{V'(L)}{U'(C)}\right) / \left(\frac{dC}{dL}\right) \tag{A.165}$$

From equation (A.29), we have

$$\begin{aligned}
C &= L^{\beta/(1-\alpha)}\Lambda\bar{P}_X^{-1} && \text{(A.166)}\\
\partial C/\partial L &= \frac{\beta}{1-\alpha}L^{\frac{\beta}{1-\alpha}-1}\Lambda\bar{P}_X^{-1} && \text{(A.167)}
\end{aligned}$$

while we have from the household's first-order condition that $V'(L)/U'(C) = P_X^{-1}$ (since the nominal wage is normalized to 1). So then

$$\begin{aligned}
\frac{\partial C/\partial L}{-U_L/U_C} &\equiv \frac{\frac{\beta}{1-\alpha}L^{\frac{\beta}{1-\alpha}-1}\Lambda\bar{P}_X^{-1}}{P_X^{-1}} && \text{(A.168)}\\
&= \frac{\beta}{1-\alpha}L^{\frac{\beta}{1-\alpha}-1}\Lambda && \text{(A.169)}
\end{aligned}$$

A.2.3.5 Analog to proposition 9

$$
\begin{aligned}
Y_i &= Z_i \left(1-\phi_i\right)^{\alpha} L_i^{\beta+\alpha} \alpha P_X^{-1} \beta^{-1} && \text{(A.170)} \\
&= Z_i \left(1-\phi_i\right)^{\alpha} L_i^{\beta+\alpha} \alpha \left(L^{(1-\alpha-\beta)/(1-\alpha)} \bar{P}_X\right)^{-1} \beta^{-1} && \text{(A.171)}
\end{aligned}
$$

$$
L_i = L \frac{\left(A_i Z_i^{(\sigma-1)} \left(1-\phi_i\right)^{\alpha(\sigma-1)}\right)^{1/(\sigma-(\beta+\alpha)(\sigma-1))}}{\sum_i \left(A_i Z_i^{(\sigma-1)} \left(1-\phi_i\right)^{\alpha(\sigma-1)}\right)^{1/(\sigma-(\beta+\alpha)(\sigma-1))}} \quad \text{(A.172)}
$$

A.2.3.6 Payments to capital

$$
\begin{aligned}
L_i &= P_i \beta Y_i && \text{(A.173)} \\
\Pi_i &= \left(1-\beta-\alpha\right) P_i Y_i && \text{(A.174)} \\
&= \frac{\left(1-\beta-\alpha\right)}{\beta} L_i && \text{(A.175)}
\end{aligned}
$$

A.2.3.7 Discussion

Here we continue to have the result that log sector output and employment are equal to aggregate factors plus log-linear functions of the sector shocks. In addition, the aggregates remain concave functions of the shocks for $\sigma < 1$.

Profits in this model behave identically to sector employment, inheriting whatever its skewness is and also any countercyclical dispersion.

A.2.4 Skewness of growth rates

This section gives conditions under which the growth rates of output in the model are skewed left. We study a continuous time specification. In particular, a case where productivity follows a mean-reverting version of a finite-activity Lévy process (finite activity means that in any given time period, the number of jumps is almost surely finite). A Lévy process is a general class in which the increments are independent and stationary. There are well known results on representations for such processes. We impose one restriction, which is that the jump component of the process (from the Lévy–Itô representation) has finite activity (i.e. the Lévy measure is finite). This is a restriction to allow only certain types of jumps, which we impose because it allows the jump process to be expressed as a simple compound Poisson process, keeping the analysis relatively simple (see Cont and Tankov (2004) sections 3.4 and 4.1.1). Jump diffusions are widely studied in economics, particularly within finance.

Formally, we assume that for all i, $\varepsilon_{i,t}$ follows

$$d\varepsilon_{i,t} = -m(\varepsilon_{i,t})\,dt + \sigma dW_{i,t} + \Delta\varepsilon_{i,t} \tag{A.176}$$

where $W_{i,t}$ is a standard Wiener process and $\Delta\varepsilon_{i,t}$ is a compound Poisson process, equal to a random number $k_{i,t}$ with probability λdt and zero otherwise (for a more formal definition, see Cont and Tankov (2004) section 8.3). $k_{i,t}$ is a symmetrically distributed random variable. We assume that the function m is such that ε has a well-defined unconditional distribution with finite moments. Intuitively, that requires that $m(\cdot)$ induces mean reversion in ε, for example as in an Ornstein–Uhlenbeck process.

The solution of the model is such that aggregate output is a function f of the sector productivities with the characteristics that $f_i > 0$ and $f_{ii} < 0\ \forall i$ and $f_{ij} > 0\ \forall i \neq j$. The question here is under what circumstances df_t is skewed left. That is, when do our results on skewness in levels also apply to growth rates?

Itô's lemma in the case of a jump diffusion (Cont and Tankov, 2004, section 8.3.2) yields

$$\begin{aligned} df_t &= \sum_i \left(-m(\varepsilon_{i,t}) f_{i,t} + \frac{\sigma^2}{2} f_{ii,t}\right) dt \\ &\quad + \sum_i f_i \sigma dW_{i,t} + \sum_i f(\ldots, \varepsilon_{i,t-} + \Delta\varepsilon_i, \ldots) - f(\ldots, \varepsilon_i, \ldots) \end{aligned} \tag{A.177}$$

Now first assume that there are no jumps, so that the final summation above is equal to zero. Then we have

$$\mathbb{E}\left[df_t^3\right] = O\left(dt^3\right) \tag{A.178}$$

$$\mathbb{E}\left[df_t^2\right] = O(dt) \tag{A.179}$$

and hence skewness is $O\left(dt^{3/2}\right) \to 0$.

Alternatively, suppose there are jumps. Then

$$\mathbb{E}\left[df_t^3\right] = \sum_i \lambda dt \mathbb{E}\left[\left(f(\ldots, \varepsilon_{i,t-} + k_t, \ldots) - f(\ldots, \varepsilon_i, \ldots)\right)^3\right] + o(dt) \tag{A.180}$$

The fact that f is globally concave implies that the expectation on the right-hand side is negative. Furthermore, $\mathbb{E}\left[df_t^2\right] = O(dt)$, so that skewness is negative and $O\left(dt^{-1/2}\right)$.

A.3 Measurement error

This section examines the effect of measurement error on the results for skewness across levels of aggregation. It first provides a simple calculation quantifying how much measurement error would be needed in order to generate the observed differences, and second it reports results for annual instead of monthly growth rates and shows that they are highly similar.

First, denote some aggregate variable by x_{agg} and a sector-level variable x_{sect}. The x's are the true variables, and assume they have identical skewness. Now suppose that while the aggregate variable is observed directly, we only see $y_{sect} = x_{sect} + \varepsilon$, where ε is symmetrically distributed measurement error. For skewness, we assume

$$\frac{skew\left(x_{sect}\right)}{skew\left(x_{agg}\right)} = 1 \tag{A.181}$$

It is then straightforward to show that

$$\frac{skew\left(y_{sect}\right)}{skew\left(x_{agg}\right)} = \left(\frac{\text{var}\left(x_{sect}\right) + \text{var}\left(\varepsilon\right)}{\text{var}\left(x_{sect}\right)}\right)^{-3/2} \tag{A.182}$$

In the data, for both employment and industrial production, the ratio of skewness at the most disaggregated level to skewness at the most aggregated level is 0.35. That is, $\frac{skew(y_{sect})}{skew(x_{agg})} = 0.35$. In order for that to be due to measurement error, we must have

$$0.35 = \left(\frac{\text{var}\left(x_{sect}\right) + \text{var}\left(\varepsilon\right)}{\text{var}\left(x_{sect}\right)}\right)^{-3/2} \tag{A.183}$$

$$\Rightarrow \frac{\text{var}\left(\varepsilon\right)}{\text{var}\left(x_{sect}\right)} = 1.01 \tag{A.184}$$

In other words, as discussed in the main text, for measurement error to explain the magnitude of the decline in skewness observed with disaggregation, it would need to be the case that more than half of the variation in the observed sector-level growth rates comes from measurement error.

To the extent that there is measurement error, though, it is more likely to appear in monthly than annual data. For employment, in particular, annual data is based on the full universe of firms (as it comes from the full set of unemployment insurance filings). Furthermore, aggregation over time increases the strength of the signal relative to noise, since true changes in the underlying series become larger. Figure A.2 therefore reproduces the analysis of skewness of growth rates using annual rather than monthly changes. The results are highly similar to those in Figure 1, providing further evidence that the results are not driven by measurement error.

Table A.1: **Time-Series Skewness with 5- vs. 10-percent asymmetry**

	Growth rates		Levels	
	5%/95% Kelley	5%/95% Kelley	10%/90% Kelley	10%/90% Kelley
Monthly data				
IP	-0.01	-0.07	-0.16	0.01
	[-0.1,0.08]	[-0.14,0]	[-0.37,0.05]	[-0.19,0.21]
Employment	-0.17	-0.1	-0.21	-0.19
	[-0.33,-0.01]	[-0.28,0.08]	[-0.47,0.05]	[-0.47,0.09]
Stock Returns	-0.15	-0.12	-0.29	-0.17
	[-0.2,-0.1]	[-0.18,-0.06]	[-0.45,-0.13]	[-0.35,0.01]
Unempl. rate	-0.01	0	0.54	0.49
	[-0.21,0.19]	[-0.15,0.15]	[0.37,0.71]	[0.32,0.66]
CFNAI	-0.12	-0.09	-0.18	-0.19
	[-0.26,0.02]	[-0.24,0.06]	[-0.46,0.1]	[-0.51,0.13]
Quarterly data				
GDP	-0.1	-0.02	-0.19	-0.06
	[-0.26,0.06]	[-0.21,0.17]	[-0.43,0.05]	[-0.3,0.18]
Consumption	-0.04	0.04	-0.19	-0.22
	[-0.16,0.08]	[-0.1,0.18]	[-0.4,0.02]	[-0.45,0.01]
Investment	-0.03	0.13	-0.23	-0.2
	[-0.16,0.1]	[0,0.26]	[-0.46,0]	[-0.41,0.01]
TFP	0.02	0.04	0.11	0.12
	[-0.11,0.15]	[-0.08,0.16]	[-0.06,0.28]	[-0.06,0.3]
Labor wedge	0.31	0.2	0.56	0.5
	[0.07,0.55]	[0.01,0.39]	[0.18,0.94]	[0.14,0.86]

Notes: This table shows the effect of calculating Kelly skewness in table 1 using the 5th and 95th percentiles, as in the main text, versus the 10th and 90th percentiles, as is more common.

Table A.2: **Time-Series skewness coefficient across horizons**

	1-period	1 year	3 years	5 years
Monthly data				
IP	-1.23	-1.31	-0.38	0.03
	[-2.03,-0.43]	[-1.87,-0.75]	[-0.91,0.15]	[-0.4,0.46]
Employment	-0.52	-0.86	-0.51	-0.44
	[-1.2,0.16]	[-1.46,-0.26]	[-1.13,0.11]	[-0.98,0.1]
Stock Returns	-0.58	-0.92	-0.76	-0.09
	[-0.96,-0.2]	[-1.26,-0.58]	[-1.14,-0.38]	[-0.52,0.34]
Unempl. rate	0.56	1.34	0.73	0.1
	[0.07,1.05]	[0.64,2.04]	[0.27,1.19]	[-0.32,0.52]
CFNAI	-1.15	-1.15	-0.71	-0.65
	[-1.72,-0.58]	[-1.79,-0.51]	[-1.46,0.04]	[-1.24,-0.06]
Quarterly data				
GDP	-0.37	-0.53	-0.23	-0.34
	[-1.26,0.52]	[-1.16,0.1]	[-0.76,0.3]	[-0.89,0.21]
Consumption	-0.79	-0.43	-0.3	-0.4
	[-1.65,0.07]	[-0.9,0.04]	[-0.82,0.22]	[-0.99,0.19]
Investment	-0.86	-0.66	-0.48	-0.51
	[-1.86,0.14]	[-1.64,0.32]	[-1.06,0.1]	[-1.19,0.17]
TFP	0.28	-0.08	0.2	0.81
	[-0.08,0.64]	[-0.49,0.33]	[-0.22,0.62]	[0.25,1.37]
Labor wedge	1.38	1.53	0.88	0.45
	[0.23,2.53]	[0.28,2.78]	[0.21,1.55]	[0,0.9]

Notes: This table replicates results from table 1, but measuring skewness in growth rates across horizons. The first column reports results for one-month and one-quarter changes, while the other columns are for 1-, 3- and 5-year changes. All results in this table are for the moment-based skewness coefficient.

Table A.3: **Time-Series Kelly skewness across horizons**

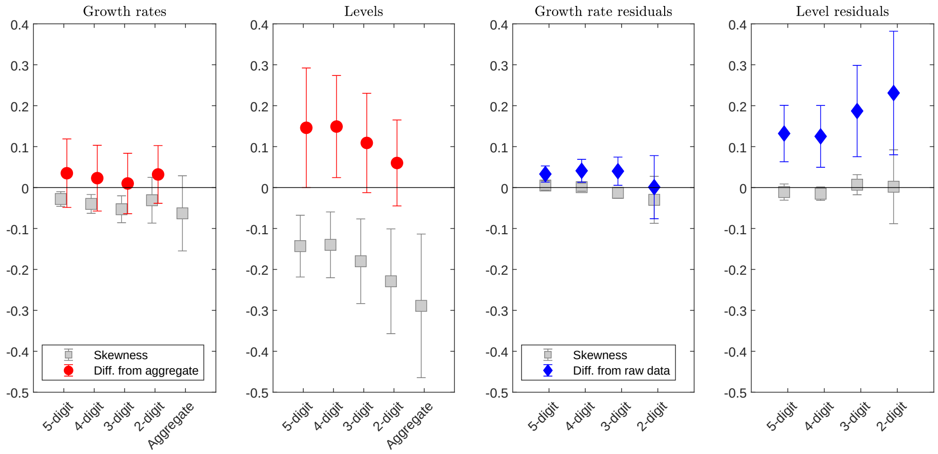
	1-period	1 year	3 years	5 years
Monthly data				
IP	-0.01	-0.26	-0.22	0.02
	[-0.1,0.08]	[-0.42,-0.1]	[-0.47,0.03]	[-0.21,0.25]
Employment	-0.17	-0.16	-0.21	-0.24
	[-0.33,-0.01]	[-0.35,0.03]	[-0.52,0.1]	[-0.53,0.05]
Stock Returns	-0.15	-0.32	-0.29	-0.04
	[-0.21,-0.09]	[-0.42,-0.22]	[-0.46,-0.12]	[-0.24,0.16]
Unempl. rate	-0.01	0.48	0.44	0.06
	[-0.2,0.18]	[0.31,0.65]	[0.18,0.7]	[-0.2,0.32]
CFNAI	-0.12	-0.26	-0.24	-0.25
	[-0.26,0.02]	[-0.45,-0.07]	[-0.62,0.14]	[-0.58,0.08]
Quarterly data				
GDP	-0.1	-0.16	-0.09	-0.13
	[-0.26,0.06]	[-0.38,0.06]	[-0.39,0.21]	[-0.39,0.13]
Consumption	-0.04	-0.15	-0.16	-0.17
	[-0.16,0.08]	[-0.35,0.05]	[-0.4,0.08]	[-0.44,0.1]
Investment	-0.03	-0.22	-0.15	-0.21
	[-0.16,0.1]	[-0.48,0.04]	[-0.39,0.09]	[-0.49,0.07]
TFP	0.02	-0.02	0.13	0.41
	[-0.11,0.15]	[-0.21,0.17]	[-0.08,0.34]	[0.13,0.69]
Labor wedge	0.31	0.5	0.54	0.19
	[0.06,0.56]	[0.14,0.86]	[0.1,0.98]	[-0.04,0.42]

Notes: This table is identical to table A.2, but using the Kelly skewness measure (at the 5th and 95th percentiles) instead of the moment skewness.

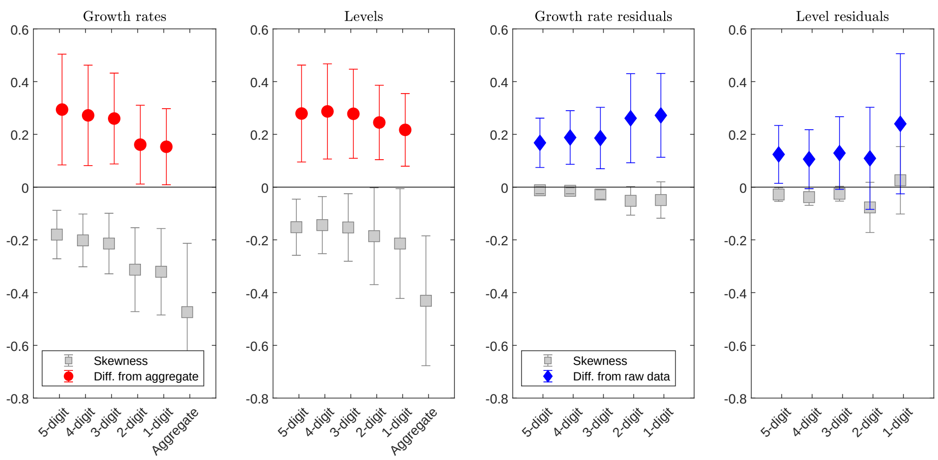
Table A.4: **Simulation results, Kelly skewness**

	Data		$v_{tfp} = 0.026$ $v_{wedge} = 0$		$v_{tfp} = 0$ $v_{wedge} = 0.026$		$v_{tfp} = 0.013$ $v_{wedge} = 0.026$	
Table 1: Time-series skewness								
	Growth rates	Levels	Growth rates	Levels	Growth rates	Levels	Growth rates	Levels
GDP	-0.01	-0.16	-0.07	-0.04	-0.26	-0.16	-0.23	-0.17
Employment	-0.17	-0.21	-0.07	-0.05	-0.24	-0.15	-0.25	-0.14
TFP	0.02	0.12	-0.03	-0.04	-0.16	-0.47	-0.03	-0.06
Wedge	0.33	0.50	-0.00	0.01	0.09	0.14	0.09	0.13
Figure 1: Sector vs. aggregate skewness								
	IP	Empl.	IP	Empl.	IP	Empl.	IP	Empl.
Aggregate	-0.05	-0.46	-0.07	-0.07	-0.26	-0.24	-0.23	-0.25
Sector	-0.02	-0.18	-0.06	-0.04	-0.23	-0.20	-0.20	-0.18

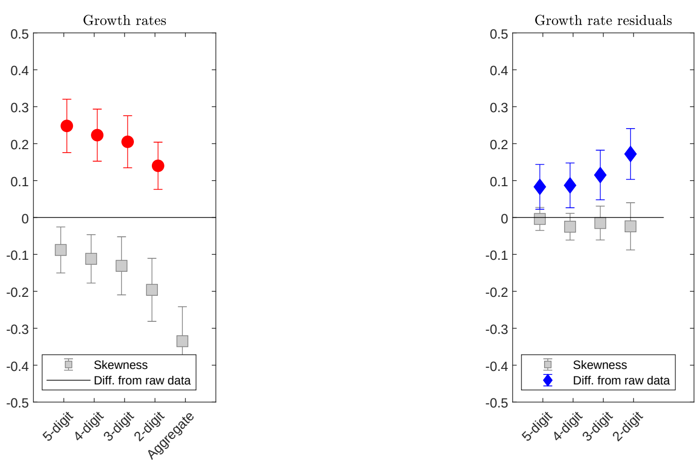
Notes: Replicates 4, but with Kelly instead of moment skewness.



(a) Industrial Production



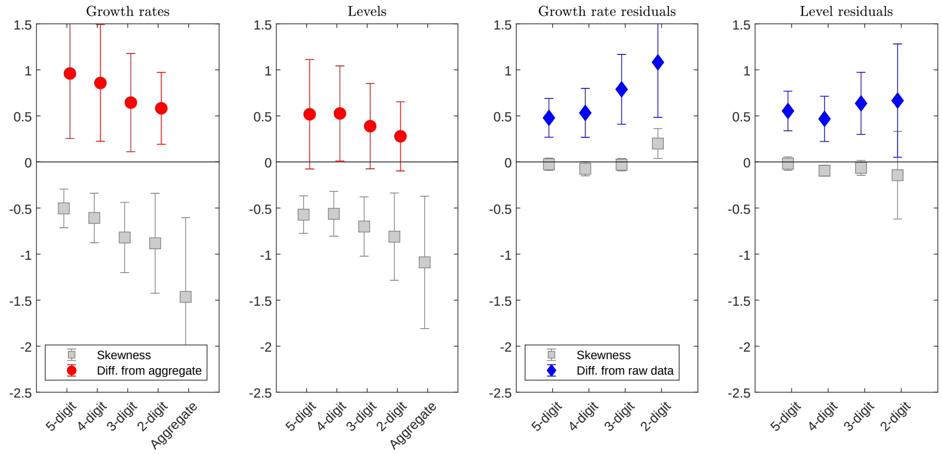
(b) Employment



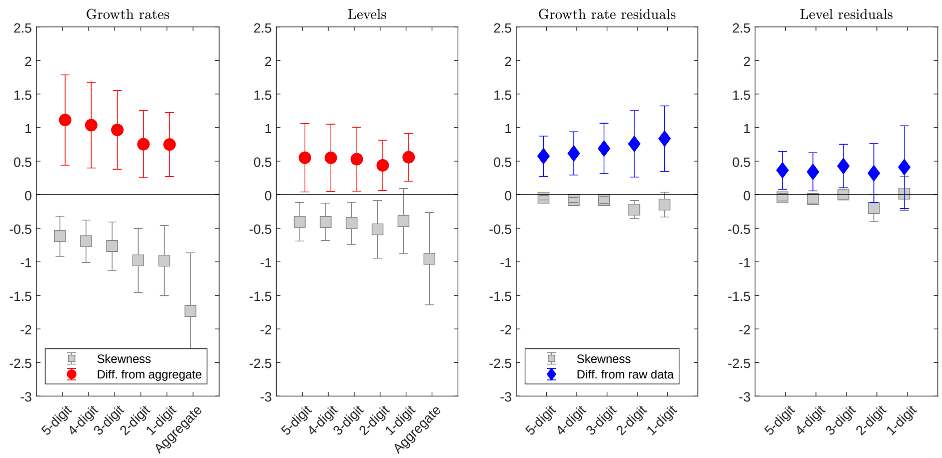
(c) Stock Returns

Figure A.1: **Time-Series Kelly Skewness**

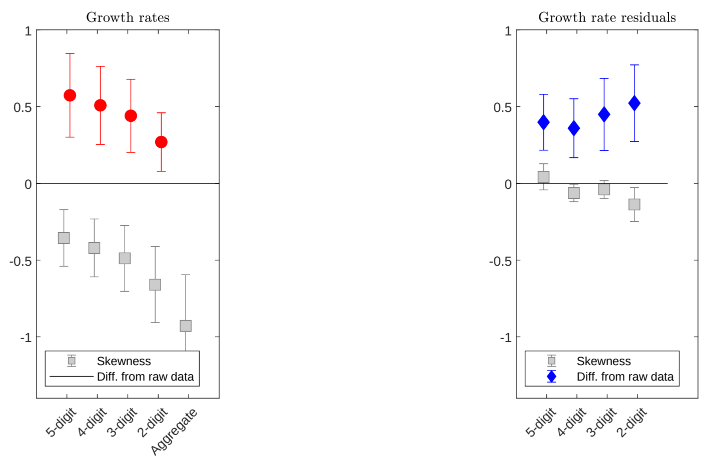
Notes: This figure replicates figure 1, but with Kelly skewness instead of the moment skewness coefficient.



(a) Industrial Production



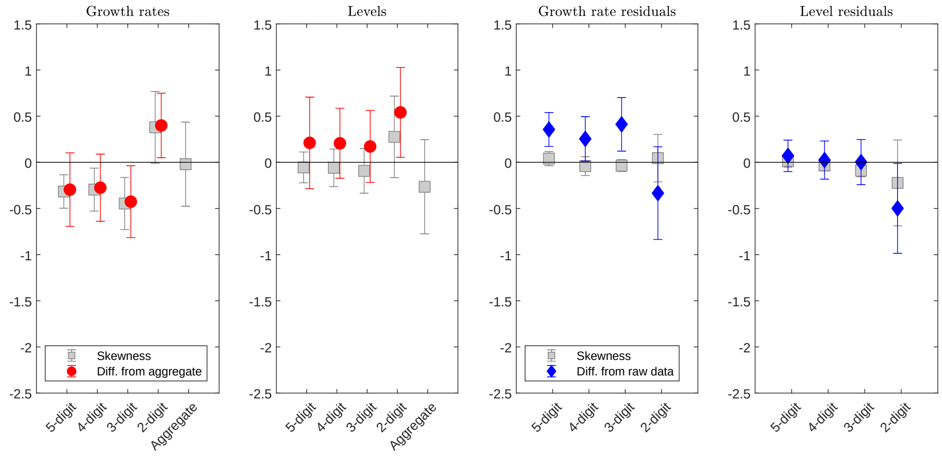
(b) Employment



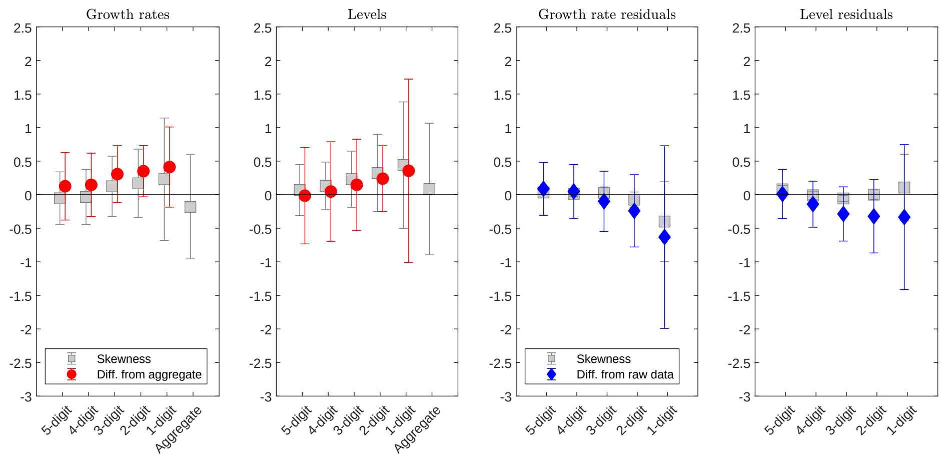
(c) Stock Returns

Figure A.2: **Time-Series Skewness, 12-month growth rates**

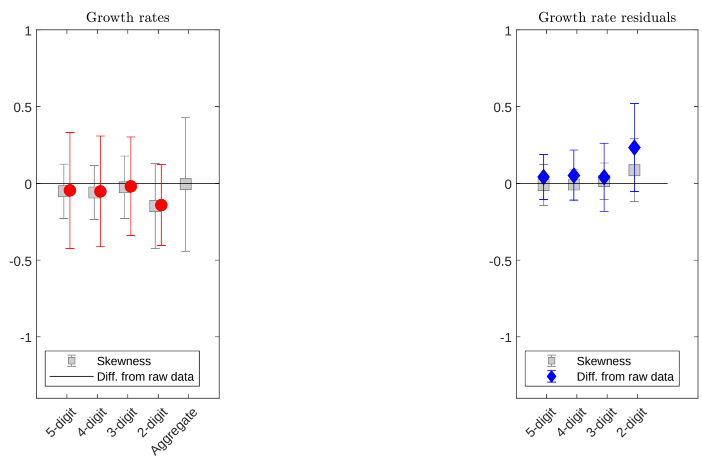
Notes: This figure replicates figure 1, but with annual instead of monthly growth rates.



(a) Industrial Production



(b) Employment



(c) Stock Returns

Figure A.3: **Time-Series Skewness, 12-month growth rates**

Notes: This figure replicates figure 1, but with five-year instead of monthly growth rates.