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Abstract: This paper analyzes the cyclical properties of the optimal labor wedges in a Mirrleesean business cycle model. Agents value consumption and leisure and receive persistent idiosyncratic shocks to their value of leisure, which are private information. The production structure is identical to a standard real business cycle model with aggregate productivity shocks. Solving the mechanism design problem for a version of this economy calibrated under logarithmic preferences, I find that the optimal labor wedges are approximately constant over the business cycle.

***JEL* classification:** D39; D82; E32; H21

Keywords: Optimal taxes; Business cycles; Private information; Social insurance

1 Introduction

A fundamental question in public finance and macroeconomics is how should taxes be optimally set over the business cycle. Should the incentives to work and save vary over booms and busts? Should the amount of income redistribution and social insurance insurance vary over the business cycle? Or should taxes be optimally set using balanced growth path considerations but left unchanged in the presence of aggregate shocks? Instead of exogenously restricting the tax instruments available to the government, I will answer these questions following a Mechanism Design approach. In particular, I will solve the optimal allocation of a Mirrleesean economy with aggregate shocks

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in which agents are subject to time-varying idiosyncratic shocks and these shocks are private information. As in the Dynamic Public Finance literature, the optimal provision of incentives and insurance in the presence of private information endogenously determines optimal savings and labor wedges that can be interpreted as non-linear taxes. It is the cyclical behavior of these wedges that I am interested in characterizing.

The model considered is a standard perpetual youth real business cycle model in which agents are subject to idiosyncratic shocks to their value of leisure. These shocks are persistent over time and are private information. The production technology is standard. Output, which can be consumed or invested, is produced using capital and labor. The aggregate production function is subject to aggregate productivity shocks that follow a standard AR(1) process.

A social planner seeks to maximize the present lifetime value of the current and future generations of agents subject to individual incentive compatibility constraints and aggregate resource feasibility constraints. In the absence of aggregate shocks I show that the optimal allocation must satisfy two necessary conditions every period: an intratemporal cross-sectional inverse Euler condition, and an intertemporal cross-sectional inverse Euler equation. Interestingly, when the utility functions with respect to consumption and leisure both take logarithmic forms, these cross sectional conditions deliver equations in the aggregate variables of the economy that coincide with the optimal intratemporal and intertemporal Euler equations of the representative economy social planner's problem with full information. This not only provides an important irrelevance result of private information for aggregate dynamic in a deterministic benchmark case, but will be extremely valuable for computing optimal allocations under aggregate shocks.

In order to be able to handle the case of aggregate shocks in a parsimonious way, I restrict the idiosyncratic values of leisure to take only two values: high and low. I also turn to a recursive formulation similar to Fernandes and Phelan (2000) in which the social planner designs dynamic contracts for young and old agents.[2] The current state of a dynamic contract is given by three variables: the type (high or low) that the agent reported in the previous period, the promised value (in expected discounted terms) that the agent will receive at the beginning of the current period if the previous-period report was truthful, and a threat value (in expected discounted terms)

[2]The young agents are those born in the beginning of the current period while the old are those born in any other previous period.

that the agent will receive at the beginning of the current period if the previous-period report was untruthful.[3] Given the current state, the contract specifies current consumption, current hours worked, and next-period state-contingent promised and threat values as a function of the value of leisure reported by the agent. It is important to note that the next-period promised and threat values that the dynamic contract determines are contingent on the realization of the aggregate shock at the beginning of the following period (and, hence, represent highly dimensional objects). Since the model has a large number of agents and the shocks to the value of leisure are idiosyncratic, the social planner needs to keep track of the whole distribution of individuals across previous-period reports, promised values and threat values. Given this distribution, the aggregate stock of capital, and the aggregate productivity level, the social planner seeks to maximize the present discounted utility of agents subject to incentive compatibility, promise-keeping, threat-keeping and aggregate resource feasibility constraints.

This economy-wide social planner's problem can be decomposed into a series of sub-planning problems, in which each sub-planner takes the economy-wide shadow price processes as given. These shadow prices (and associated allocations) correspond to the economy-wide social planning solution if the aggregate resource feasibility constraints are satisfied. There are sub-planning problems for insuring old agents, for insuring young agents and for making production decisions.

The advantage of decomposing the economy-wide planner's problem into sub-planning problems is that it makes computing a solution more tractable. However, this is still a daunting task given the high dimensionality of the recursive contracts. In order to make progress I focus on the case of logarithmic preferences. The reason is that, given the previous results in the paper, the shadow prices are known for this case: They are the shadow prices of the stochastic representative agent social planner's problem. These shadow prices, which are easily obtained, can then be plugged into the sub-planning problems in order to obtain the optimal dynamic contracts. Solving these sub-planning problems still poses considerable difficulties, given that next-period promised and threat values are contingent on the realization of the aggregate productivity shock in the next period. However, I explain a simple way of constructing these state-contingent promised and threat values using the solutions of the linearized deterministic sub-planning problems, which are

[3]The social planner is fully committed to delivering these promised and threat values to the individuals, and agents have no outside options.

relatively easy to obtain.

Calibrating the model to standard observations in the literature, I find that the intertemporal wedges are negligible. However, the labor wedges for agents with the high value of leisure are significant (the labor wedges of workers with the low value of leisure are always equal to zero). The main finding of the paper regards the behavior of these labor wedges in response to an aggregate productivity shock. I find that these wedges increase in response to the shock, but that the increase is negligible: Optimal labor wedges are roughly constant over the business cycle. That is, labor wedges vary with the idiosyncratic shocks that the agents receive, but are roughly invariant to aggregate shocks.

In its goals, this paper is closely related to Chari et al. (1994): both papers are interested in characterizing optimal labor and capital taxes over the business cycle. However, both papers follow very different approaches. While Chari et al. (1994) restricts the set of tax instruments available to linear taxes and considers Ramsey taxation, in this paper non-linear tax distortions are endogenously determined in an attempt to optimally insure agents in the presence of private information. Despite these differences, both papers obtain similar results: They both find that capital taxes are virtually zero and labor taxes are roughly constant over the business cycle.

Within the Mechanism Design camp, the paper is related to Farhi and Werning (2013) since both characterize optimal labor wedges in a life-cycle model. Important differences are that Farhi and Werning (2013) consider agents with finite lifetimes instead of a perpetual youth model, and that they evaluate preferences with a constant Frisch elasticity of labor supply instead of the logarithmic preferences considered in this paper. The most fundamental difference, however, is that Farhi and Werning (2013) consider a deterministic environment while this paper analyzes optimal labor wedges over the business cycle.

The irrelevance of private information for aggregate dynamics presented here for the case of logarithmic preferences is closely related to Farhi and Werning (2012) and Veracierto (2021). Farhi and Werning (2012) present this result in a similar economy, except that there are no aggregate shocks and the social planner is only allowed to optimize with respect to the consumption allocations (labor allocations are taken to be beyond the planner's control). This paper, in contrast, considers an economy with aggregate shocks and allows the social planner to optimize with respect to labor as well as consumption. Similar to this paper, Veracierto (2021) also shows the irrelevance result under logarithmic preferences in an economy with aggregate productivity shocks and endogenous labor supply decisions. However, the results are presented only for the case of i.i.d. idiosyncratic shocks. This paper, extends that result to the case of persistent idiosyncratic shocks.

This paper is also closely related to Kapicka (2024), which studies optimal Mirrlees taxes over the business cycle for two set of agents that permanently differ in their incomes and preferences. Similarly to this paper, Kapicka (2024) finds that if preferences of the two groups were identical that labor taxes would be constant over the business cycles. However, under preference heterogeneity labor taxes would vary. An important difference, though, is that in Kapicka (2024) the private information is about a permanent type that gets revealed at the beginning of time, while in this paper the private information is about idiosyncratic shocks that vary over time.

The paper is organized as follows. Section 2 describes the economic environment. Section 3 describes the mechanism design problem for the deterministic version of the economy and provides the irrelevance result of private information for the logarithmic preferences case. Section 4 describes the mechanism design problem for the stochastic version of the economy in recursive form. Section 5 describes how to compute the stochastic optimal allocation when preferences are logarithmic. Section 6 provides the quantitative results. Finally, Section 7 concludes the paper.

2 The economy

The economy is populated by a unit measure of agents who value consumption and leisure, and who are subject to idiosyncratic shocks to their value of leisure. With probability $1-\sigma$ an agent dies from one period to the next and is immediately replaced by a newborn.[4] Agents do not care about the utility levels of their descendants. The idiosyncratic shocks take values in a finite set. The realized value at date $t \geq a$ of an agent born at date a (who is therefore of age $t-a$) will be denoted by θ_{t-a} and the history of shocks of length $t-a$ that the agent received since they were born will be denoted by $\theta^{t-a} = (\theta_0, \theta_1, ..., \theta_{t-a})$. Of major interest in this paper is the case in which the realizations of the value of leisure θ_{t-a} are private information of the individuals. The

[4] As in Phelan (1994), the stochastic lifetime guarantees that there will be a stationary distribution of agents across individual states, which will play a crucial role in the recursive formulation of the following sections.

preferences of an agent born at date a are given by

$$\sum_{t=a}^{\infty}\sum_{\theta^{t-a}}(\sigma\beta)^{t-a}\left[u\left(c_t^a\left(\theta^{t-a}\right)\right)+\theta_{t-a}n\left(1-h_t^a\left(\theta^{t-a}\right)\right)\right]\phi_{t-a}\left(\theta^{t-a}\right),$$

where c_t^a and h_t^a are consumption and hours worked, respectively; and u and n are continuously differentiable, strictly increasing, and strictly concave utility functions. The initial distribution over θ_0 is denoted by ϕ_0 and realizations of θ_{t-a}, conditional on θ_{t-a-1}, are determined by a transition matrix Q. Thus, $\phi_{t-a}\left(\theta^{t-a}\right)=\phi_0\left(\theta_0\right)Q\left(\theta_0,\theta_1\right)...Q(\theta_{t-a-1},\theta_{t-a})$.

Output, which can be consumed or invested, is produced with the following production function:

$$Y_t=e^{z_t}F(K_{t-1},H_t),$$

where Y_t is output, z_t is aggregate productivity, K_{t-1} is capital, H_t is hours worked, and F is a neoclassical production function. The aggregate productivity level z_t follows a standard AR(1) process given by:

$$z_{t+1}=\rho z_t+\varepsilon_{t+1}, \tag{2.1}$$

where $0<\rho<1$ and ε_{t+1} is normally distributed with mean zero and standard deviation σ_ε.

Capital is accumulated using a standard linear technology given by

$$K_t=\left(1-\delta\right)K_{t-1}+I_t,$$

where I_t is gross investment and $0<\delta<1$. The initial values of z_0 and K_{-1} are given.

3 Deterministic optimal allocation

This section characterizes the optimal allocation when the aggregate productivity level z_t is set to zero. The stochastic process in equation (2.1) will be reintroduced later on.

In what follows it will be convenient to follow the literature in specifying the social planner's problem in terms of directly choosing utilities of consumption and leisure. By the Revelation Principle it suffices to specify u_t^a and leisure n_t^a as a function of the history of reports θ^{t-a} made by the individuals. The social planner's problem is then given by

$$\max\left\{\sum_{t=0}^{\infty}\sum_{\theta^t}(\sigma\beta)^t\left[u_t^0\left(\theta^t\right)+\theta_t n_t^0\left(\theta^t\right)\right]\phi_t\left(\theta^t\right)\right.$$

$$+\sum_{a=1}^{\infty}\pi^{a}(1-\sigma)\sum_{t=a}^{\infty}\sum_{\theta^{t-a}}(\sigma\beta)^{t-a}\left[u_t^a\left(\theta^{t-a}\right)+\theta_{t-a}n_t^a\left(\theta^{t-a}\right)\right]\phi_{t-a}\left(\theta^{t-a}\right)\Bigg\} \tag{3.1}$$

subject to

$$C_t+K_{t+1}-(1-\delta)K_t\leq F\left(K_t,H_t\right), \tag{3.2}$$

$$C_t=\sigma^t\sum_{\theta^t}c\left(u_t^0\left(\theta^t\right)\right)\phi_t\left(\theta^t\right)+\sum_{a=1}^{t}(1-\sigma)\sigma^{t-a}\sum_{\theta^{t-a}}c\left(u_t^a\left(\theta^{t-a}\right)\right)\phi_{t-a}\left(\theta^{t-a}\right) \tag{3.3}$$

$$H_t=\sigma^t\sum_{\theta^t}h\left(n_t^0\left(\theta^t\right)\right)\phi_t\left(\theta^t\right)+\sum_{a=1}^{t}(1-\sigma)\sigma^{t-a}\sum_{\theta^{t-a}}h\left(n_t^a\left(\theta^{t-a}\right)\right)\phi_{t-a}\left(\theta^{t-a}\right) \tag{3.4}$$

for every $t\geq 0$, and

$$\begin{gathered}u_t^a\left(\theta^{t-a-1},\theta_{t-a}\right)+\theta_{t-a}n_t^a\left(\theta^{t-a-1},\theta_{t-a}\right)\\
+\sum_{j=1}^{\infty}\sum_{\theta_{t-a+1}^{t-a+j}}(\sigma\beta)^j\left[u_{t+j}^a\left(\theta^{t-a-1},\theta_{t-a},\theta_{t-a+1}^{t-a+j}\right)+\theta_{t-a+j}n_{t+j}^a\left(\theta^{t-a-1},\theta_{t-a},\theta_{t-a+1}^{t-a+j}\right)\right]\left(\prod_{k=1}^{j}Q\left(\theta_{t-a+k-1},\theta_{t-a+k}\right)\right)\\
\geq u_t^a\left(\theta^{t-a-1},\theta_{t-a}^c\right)+\theta_{t-a}n_t^a\left(\theta^{t-a-1},\theta_{t-a}^c\right)\\
+\sum_{j=1}^{\infty}\sum_{\theta_{t-a+1}^{t-a+j}}(\sigma\beta)^j\left[u_{t+j}^a\left(\theta^{t-a-1},\theta_{t-a}^c,\theta_{t-a+1}^{t-a+j}\right)+\theta_{t-a+j}n_{t+j}^a\left(\theta^{t-a-1},\theta_{t-a}^c,\theta_{t-a+1}^{t-a+j}\right)\right]\left(\prod_{k=1}^{j}Q\left(\theta_{t-a+k-1},\theta_{t-a+k}\right)\right),\end{gathered} \tag{3.5}$$

for every $a\geq 0$, $t\geq a$, θ^{t-a-1}, θ_{t-a} and θ_{t-a}^c , with K_0 given.[5] The functions $c(u_t)=u^{-1}(u_t)$ and $h(n_t)=1-n^{-1}(n_t)$ in equations (3.3) and (3.4) are the consumption and hours worked implied by the utility levels u_t and n_t, respectively. Observe that the objective function (3.1) is the weighted lifetime utility of all agents (current and future), where agents of generation a receive a Pareto weight π^a, with $0<\pi<1$. Equation (3.2) is the aggregate feasibility constraint at date t. Equation (3.3) defines aggregate consumption at date t. The first term is the total consumption of the measure one of agents initially alive at date 0, of which a fraction σ^t survives to date t. The second term sums the total consumption of the measure $1-\sigma$ of agents born at each other date $1\leq a\leq t$, of which a fraction σ^{t-a} survives to date t. Equation (3.4) is similar to equation (3.3) but for aggregate hours worked. Equation (3.5) is the incentive compatibility constraint at date t for agents of generation a, after the history of reports θ^{t-a-1}.[6] It states that truthfully

[5] θ_{t-a}^c refers to an element of the set $\{\theta_{t-a}\}^c$

[6] Observe that $\theta_{t-a+1}^{t-a+j}=(\theta_{t-a+1},\theta_{t-a+2},...,\theta_{t-a+j})$ denotes a history of shocks between ages $t-a+1$ and $t-a+j$ for an agent born at date $a\leq t$.

reporting θ_{t-a} weakly dominates making any other report θ^c_{t-a}. The true value of leisure θ_{t-a} is used to evaluate deviations from truth-telling, the conditional expectations are always taken with respect to the true transition probabilities, and only one-time deviations from truth-telling need to be considered.[7] Also observe that the social planner's problem under full information reduces to (3.1)-(3.4) (that is, the incentive compatibility constraints in equation (3.5) are dropped from the problem).

In order to simplify the arguments that follow, I hereon assume that $\pi = \beta$. That is, that the social planner discounts future utilities at the same rate as individual agents.

3.1 The cross-sectional inverse Euler conditions

In this subsection I derive necessary conditions to the social planner problem that will play a central role in determining the importance of private information for aggregate dynamics.

3.1.1 The intratemporal condition

I first show that if $\left\{\{u^a_t, n^a_t\}^t_{a=0}, K_{t+1}\right\}^\infty_{t=0}$ is a solution to the social planner's problem (3.1)-(3.5), then it satisfies the following condition:

$$F_H(K_t,H_t)\left(\sigma^t\sum_{\theta^t}\frac{1}{n'\left(1-h\left(n^0_t\left(\theta^t\right)\right)\right)}\phi_t\left(\theta^t\right)+\sum_{a=1}^{t}(1-\sigma)\,\sigma^{t-a}\sum_{\theta^{t-a}}\frac{1}{n'\left(1-h\left(n^a_t\left(\theta^{t-a}\right)\right)\right)}\phi_{t-a}\left(\theta^{t-a}\right)\right)$$

$$=\bar{\theta}_t\left(\sigma^t\sum_{\theta^t}\frac{1}{u'\left(c\left(u^0_t\left(\theta^t\right)\right)\right)}\phi_t\left(\theta^t\right)+\sum_{a=1}^{t}(1-\sigma)\,\sigma^{t-a}\sum_{\theta^{t-a}}\frac{1}{u'\left(c\left(u^a_t\left(\theta^{t-a}\right)\right)\right)}\phi_{t-a}\left(\theta^{t-a}\right)\right), \tag{3.6}$$

for every t, where

$$\bar{\theta}_t=\sigma^t\sum_{\theta^t}\theta_t\phi_t\left(\theta^t\right)+\sum_{a=1}^{t}(1-\sigma)\,\sigma^{t-a}\sum_{\theta^{t-a}}\theta_{t-a}\phi_{t-a}\left(\theta^{t-a}\right) \tag{3.7}$$

is the average value of θ at date t.

To see this, suppose that the left-hand side of equation (3.6) is strictly larger than the right-hand side at some $t = t^*$. Consider an alternative allocation $\left\{\{\hat{u}^a_t, \hat{n}^a_t\}^t_{a=0}, K_{t+1}\right\}^\infty_{t=0}$, where

[7] The incentive compatibility constraints only consider one time deviations because it can be shown that if an agent has lied in the past it always dominates to tell the truth in the future (see Fernandes and Phelan (2000)).

$\left\{\{\hat{u}_t^a, \hat{n}_t^a\}_{a=0}^t\right\}_{t=0}^\infty$ is identical to $\left\{\{u_t^a, n_t^a\}_{a=0}^t\right\}_{t=0}^\infty$ except at t^*, which is given by

$$\begin{aligned} \hat{u}_{t^*}^a\left(\theta^{t^*-a}\right) &= u_{t^*}^a\left(\theta^{t^*-a}\right) + \bar{\theta}_{t^*}\varepsilon, \\ \hat{n}_t^a\left(\theta^{t^*-a}\right) &= n_{t^*}^a\left(\theta^{t^*-a}\right) - \varepsilon, \end{aligned}$$

for every $0 \leq a \leq t^*$ and θ^{t^*-a}, where ε is a small positive number. Observe that in going from $\left\{\{u_t^a, n_t^a\}_{a=0}^t, K_{t+1}\right\}_{t=0}^\infty$ to $\left\{\{\hat{u}_t^a, \hat{n}_t^a\}_{a=0}^t, K_{t+1}\right\}_{t=0}^\infty$, the objective function (3.1) changes by

$$\begin{aligned} & \sum_{\theta^{t^*}} (\sigma\beta)^{t^*}\left[\bar{\theta}_{t^*}\varepsilon - \theta_{t^*}\varepsilon\right]\phi_{t^*}\left(\theta^{t^*}\right) + \sum_{a=1}^{t^*}(1-\sigma)\sum_{\theta^{t^*-a}}\sigma^{t^*-a}\beta^{t^*}\left[\bar{\theta}_{t^*}\varepsilon - \theta_{t^*-a}\varepsilon\right]\phi_{t^*-a}\left(\theta^{t^*-a}\right) \\ = {} & \bar{\theta}_{t^*}\varepsilon\beta^{t^*}\left[\sum_{\theta^{t^*}}\sigma^{t^*}\phi_{t^*}\left(\theta^{t^*}\right) + \sum_{a=1}^{t^*}(1-\sigma)\sum_{\theta^{t^*-a}}\sigma^{t^*-a}\phi_{t^*-a}\left(\theta^{t^*-a}\right)\right] \\ & -\varepsilon\beta^{t^*}\left[\sum_{\theta^{t^*}}\sigma^{t^*}\theta_{t^*}\phi_{t^*}\left(\theta^{t^*}\right) + \sum_{a=1}^{t^*}(1-\sigma)\sum_{\theta^{t^*-a}}\sigma^{t^*-a}\theta_{t^*-a}\phi_{t^*-a}\left(\theta^{t^*-a}\right)\right] \\ = {} & \bar{\theta}_{t^*}\varepsilon\beta^{t^*} - \varepsilon\beta^{t^*}\bar{\theta}_{t^*}. \end{aligned} \tag{3.8}$$

That is, the objective function remains unchanged.[8] Observe that the incentive compatibility constraint (3.5) can be written as

$$\begin{aligned} 0 \leq {} & \left(u_t^a\left(\theta^{t-a-1}, \theta_{t-a}\right) - u_t^a\left(\theta^{t-a-1}, \theta_{t-a}^c\right)\right) + \theta_{t-a}\left(n_t^a\left(\theta^{t-a-1}, \theta_{t-a}\right) - n_t^a\left(\theta^{t-a-1}, \theta_{t-a}^c\right)\right) \\ & + \sum_{j=1}^\infty \sum_{\theta_{t-a+1}^{t-a+j}} (\sigma\beta)^j\left(u_{t+j}^a\left(\theta^{t-a-1}, \theta_{t-a}, \theta_{t-a+1}^{t-a+j}\right) - u_{t+j}^a\left(\theta^{t-a-1}, \theta_{t-a}^c, \theta_{t-a+1}^{t-a+j}\right)\right)\left(\prod_{k=1}^j Q\left(\theta_{t-a+k-1}, \theta_{t-a+k}\right)\right) \\ & + \sum_{j=1}^\infty \sum_{\theta_{t-a+1}^{t-a+j}} (\sigma\beta)^j\theta_{t-a+j}\left(n_{t+j}^a\left(\theta^{t-a-1}, \theta_{t-a}, \theta_{t-a+1}^{t-a+j}\right) - n_{t+j}^a\left(\theta^{t-a-1}, \theta_{t-a}^c, \theta_{t-a+1}^{t-a+j}\right)\right)\left(\prod_{k=1}^j Q\left(\theta_{t-a+k-1}, \theta_{t-a+k}\right)\right) \end{aligned} \tag{3.9}$$

for every $a \geq 0$, $t \geq a$, θ^{t-a-1}, θ_{t-a} and θ_{t-a}^c. Since $\left\{\{\hat{u}_t^a, \hat{n}_t^a\}_{a=0}^t, K_{t+1}\right\}_{t=0}^\infty$, preserves all the same differences in equation (3.9) as $\left\{\{u_t^a, n_t^a\}_{a=0}^t, K_{t+1}\right\}_{t=0}^\infty$, it is incentive compatible.

Next, I verify that $\left\{\{\hat{u}_t^a, \hat{n}_t^a\}_{a=0}^t, K_{t+1}\right\}_{t=0}^\infty$ satisfies the aggregate feasibility condition (3.2) at $t = t^*$ (which is the only date at which the allocations are affected by the ε deviations). Define $\hat{C}_t$ and $\hat{H}_t$ as the aggregate consumption and hours worked that satisfy equations (3.3) and (3.4) under $\{\hat{u}_t^a, \hat{n}_t^a\}_{a=0}^t$, respectively. Observe that

$$F\left(K_{t^*}, \hat{H}_{t^*}\right) - \hat{C}_{t^*} - K_{t^*+1} + (1-\delta)K_{t^*}$$

[8]Observe that the last line of equation (3.8) uses the facts that $\sigma^t + \sum_{a=1}^t (1-\sigma)\sigma^{t-a} = 1$, that ϕ_{t-a} is a probability distribution, and that $\bar{\theta}_{t^*}$ satisfies equation (3.7).

$$\simeq F\left(K_{t^*},H_{t^*}\right)+F_H\left(K_{t^*},H_{t^*}\right)\times$$
$$\left(\sigma^{t^*}\sum_{\theta^{t^*}}\frac{1}{n'\left(1-h\left(n_{t^*}^0\left(\theta^{t^*}\right)\right)\right)}\phi_{t^*}\left(\theta^{t^*}\right)+\sum_{a=1}^{t^*}\left(1-\sigma\right)\sigma^{t^*-a}\sum_{\theta^{t^*-a}}\frac{1}{n'\left(1-h\left(n_{t^*}^a\left(\theta^{t^*-a}\right)\right)\right)}\phi_{t^*-a}\left(\theta^{t^*-a}\right)\right)\varepsilon$$
$$-\left(\sigma^{t^*}\sum_{\theta^{t^*}}\frac{1}{u'\left(c\left(u_{t^*}^0\left(\theta^{t^*}\right)\right)\right)}\phi_{t^*}\left(\theta^{t^*}\right)+\sum_{a=1}^{t^*}\left(1-\sigma\right)\sigma^{t^*-a}\sum_{\theta^{t^*-a}}\frac{1}{u'\left(c\left(u_{t^*}^a\left(\theta^{t^*-a}\right)\right)\right)}\phi_{t^*-a}\left(\theta^{t^*-a}\right)\right)\bar{\theta}_{t^*}\varepsilon$$
$$-C_{t^*}-K_{t^*+1}+\left(1-\delta\right)K_{t^*}$$
$$>F\left(K_{t^*},H_{t^*}\right)-C_t-K_{t^*+1}+\left(1-\delta\right)K_{t^*}$$

where the approximate equality in the first line becomes arbitrarily close to an equality when ε is sufficiently small, and the strict inequality follows from the assumption that the left-hand side of equation (3.6) is strictly larger than the right-hand side. Since $\left\{\left\{u_t^a,n_t^a\right\}_{a=0}^t,K_{t+1}\right\}_{t=0}^\infty$ satisfies equation (3.2) at t^*, it follows that $\left\{\left\{\hat{u}_t^a,\hat{n}_t^a\right\}_{a=0}^t,K_{t+1}\right\}_{t=0}^\infty$ satisfies it with strict inequality. This surplus of consumption goods at t^* can then be used to increase $\hat{u}_{t^*}^a\left(\theta^{t^*-a}\right)$ by an additional η (sufficiently small amount) for every $0\leq a\leq t^*$ and θ^{t^*-a}. The resulting allocation satisfies the aggregate feasibility constraint (3.2) at t^* (provided that η is sufficiently small) and the incentive compatibility constraint (3.9) (since $\hat{u}_{t^*}^a\left(\theta^{t^*-a}\right)$ increases a uniform additional amount across all θ^{t^*-a}). Moreover, the objective function (3.1) evaluates at a strictly larger number (since η is positive) than $\left\{\left\{u_t^a,n_t^a\right\}_{a=0}^t,K_{t+1}\right\}_{t=0}^\infty$. This contradicts the assumption that $\left\{\left\{u_t^a,n_t^a\right\}_{a=0}^t,K_{t+1}\right\}_{t=0}^\infty$ solves the social planner's problem (3.1)-(3.5).

3.1.2 The intertemporal condition

I now show that if $\left\{\left\{u_t^a,n_t^a\right\}_{a=0}^t,K_{t+1}\right\}_{t=0}^\infty$ is a solution to the social planner's problem (3.1)-(3.5), then it satisfies the following cross-sectional intertemporal inverse Euler condition:

$$\sigma^{t+1}\sum_{\theta^{t+1}}\phi_t\left(\theta^{t+1}\right)\frac{1}{u'\left(c_{t+1}^0\left(\theta^{t+1}\right)\right)}+\sum_{a=1}^{t+1}\left(1-\sigma\right)\sigma^{t+1-a}\sum_{\theta^{t+1-a}}\frac{1}{u'\left(c_{t+1}^a\left(\theta^{t+1-a}\right)\right)}\phi_{t+1-a}\left(\theta^{t+1-a}\right)$$
$$=\beta\left(F_K\left(K_{t+1},H_{t+1}\right)+1-\delta\right)\times$$
$$\left(\sigma^t\sum_{\theta^t}\phi_t\left(\theta^t\right)\frac{1}{u'\left(c_t^0\left(\theta^t\right)\right)}+\sum_{a=1}^{t}\left(1-\sigma\right)\sigma^{t-a}\sum_{\theta^{t-a}}\frac{1}{u'\left(c_t^a\left(\theta^{t-a}\right)\right)}\phi_{t-a}\left(\theta^{t-a}\right)\right)\qquad(3.10)$$

for every $t\geq0$.

To see this, suppose that the right-hand side of equation (3.10) is strictly larger than the left-hand side at some t^*. Consider an alternative allocation $\left\{\left\{\hat{u}_t^a,\hat{n}_t^a\right\}_{a=0}^t,\hat{K}_{t+1}\right\}_{t=0}^\infty$, given by $\hat{n}_t^a=n_t^a$

for every t and $0 \leq a \leq t$, $\hat{u}_t^a = u_t^a$ for every t and $0 \leq a \leq t$, except at t^* and t^*+1, which is given by

$$\hat{u}_{t^*}^a \left(\theta^{t^*-a}\right) = u_{t^*}^a \left(\theta^{t^*-a}\right) - \beta\varepsilon,$$

for every $0 \leq a \leq t^*$ and θ^{t^*-a}, and

$$\hat{u}_{t^*+1}^a \left(\theta^{t^*+1-a}\right) = u_{t^*+1}^a \left(\theta^{t^*+1-a}\right) + \varepsilon,$$

for every $0 \leq a \leq t^*+1$ and every θ^{t^*+1-a}, where ε is a small positive number, and $\hat{K}_t$ identical to K_t except at t^*+1, which is given by

$$\hat{K}_{t^*+1} = K_{t^*+1} + C_t - \hat{C}_t \tag{3.11}$$

Observe that in going from $\left\{\{u_t^a, n_t^a\}_{a=0}^t, K_{t+1}\right\}_{t=0}^\infty$ to $\left\{\{\hat{u}_t^a, \hat{n}_t^a\}_{a=0}^t, \hat{K}_{t+1}\right\}_{t=0}^\infty$, the Pareto-weighted welfare of the agents alive at date 0 changes by

$$\sum_{\theta^{t^*}} (\sigma\beta)^{t^*} (-\beta\varepsilon)\, \phi_t \left(\theta^t\right) + \sum_{\theta^{t^*+1}} (\sigma\beta)^{t^*+1} \varepsilon\phi_{t^*+1} \left(\theta^{t^*+1}\right), \tag{3.12}$$

the sum of Pareto-weighted welfare of the agents born between $1 \leq a \leq t^*$ changes by

$$\sum_{a=1}^{t^*} \pi^a (1-\sigma) \left(\sum_{\theta^{t^*-a}} (\sigma\beta)^{t^*-a} (-\beta\varepsilon)\, \phi_{t^*-a} \left(\theta^{t^*-a}\right) + \sum_{\theta^{t^*+1-a}} (\sigma\beta)^{t^*+1-a} \varepsilon\phi_{t^*+1-a} \left(\theta^{t^*+1-a}\right) \right) \tag{3.13}$$

and the Pareto-weighted welfare of the agents born at date t^*+1 changes by

$$\pi^{t^*+1} (1-\sigma) \sum_{\theta^0} \varepsilon\phi_0 \left(\theta^0\right) \tag{3.14}$$

Adding the changes in equations (3.12)-(3.14), using the assumption that $\pi = \beta$, that the ϕ's are probability distributions, and the fact that

$$\sigma^{t^*} + \sum_{a=1}^{t^*} (1-\sigma)\, \sigma^{t^*-a} = 1,$$

we get that the objective function (3.1) remains unchanged.

Also, since $\left\{\{\hat{u}_t^a, \hat{n}_t^a\}_{a=0}^t, \hat{K}_{t+1}\right\}_{t=0}^\infty$, preserves all the same differences in equation (3.9) as $\left\{\{u_t^a, n_t^a\}_{a=0}^t, K_{t+1}\right\}_{t=0}^\infty$, it is incentive compatible.

Next, I verify that $\left\{\{\hat{u}_t^a, \hat{n}_t^a\}_{a=0}^t, \hat{K}_{t+1}\right\}_{t=0}^\infty$ satisfies the aggregate feasibility condition (3.2) at $t = t^*$ and $t = t^*+1$ (which are the only two dates at which the allocations are affected by the

ε deviations). Using that $\hat{n}^a_{t^*} = n^a_{t^*}$ for every $0 \leq a \leq t^*$ and equation (3.11), which implies that aggregate investment plus aggregate consumption does not change at $t = t^*$, we have that $\left\{\{\hat{u}^a_t, \hat{n}^a_t\}^t_{a=0}, \hat{K}_{t+1}\right\}^\infty_{t=0}$ satisfies the aggregate feasibility condition (3.2) at $t = t^*$ simply from the fact that $\left\{\{u^a_t, n^a_t\}^t_{a=0}, K_{t+1}\right\}^\infty_{t=0}$ does.

Observe that

$$C_t - \hat{C}_t \simeq \left(\sigma^{t^*} \sum_{\theta^{t^*}} \frac{1}{u'\left(c^0_{t^*}\left(\theta^{t^*}\right)\right)} \phi_{t^*}\left(\theta^{t^*}\right) + \sum_{a=1}^{t^*} (1-\sigma)\,\sigma^{t^*-a} \sum_{\theta^{t^*-a}} \frac{1}{u'\left(c^a_{t^*}\left(\theta^{t^*-a}\right)\right)} \phi_{t^*-a}\left(\theta^{t^*-a}\right)\right) \beta\varepsilon.$$

Then,

$$\begin{aligned}
&F\left(\hat{K}_{t^*+1}, H_{t^*+1}\right) - \hat{C}_{t^*+1} - \hat{K}_{t^*+2} + (1-\delta)\,\hat{K}_{t^*+1} \\
&\simeq F\left(K_{t^*+1}, H_{t^*+1}\right) + F_K\left(K_{t^*+1}, H_{t^*+1}\right) \times \\
&\left(\sigma^t \sum_{\theta^t} \frac{1}{u'\left(c^0_{t^*}\left(\theta^{t^*}\right)\right)} \phi_{t^*}\left(\theta^{t^*}\right) + \sum_{a=1}^{t^*} (1-\sigma)\,\sigma^{t^*-a} \sum_{\theta^{t^*-a}} \frac{1}{u'\left(c^a_{t^*}\left(\theta^{t^*-a}\right)\right)} \phi_{t^*-a}\left(\theta^{t^*-a}\right)\right) \beta\varepsilon \\
&-\sigma^{t^*+1} \sum_{\theta^{t^*+1}} \frac{1}{u'\left(c^0_{t^*+1}\left(\theta^{t^*+1}\right)\right)} \phi_{t^*}\left(\theta^{t^*+1}\right) \varepsilon \\
&-\sum_{a=1}^{t^*+1} (1-\sigma)\,\sigma^{t^*+1-a} \sum_{\theta^{t^*+1-a}} \frac{1}{u'\left(c^a_{t^*+1}\left(\theta^{t^*+1-a}\right)\right)} \phi_{t^*+1-a}\left(\theta^{t^*+1-a}\right) \varepsilon \\
&-C_{t^*+1} - K_{t^*+2} + (1-\delta)\,K_{t^*+1} \\
&+(1-\delta)\left(\sigma^t \sum_{\theta^t} \frac{1}{u'\left(c^0_{t^*}\left(\theta^{t^*}\right)\right)} \phi_{t^*}\left(\theta^{t^*}\right) + \sum_{a=1}^{t^*} (1-\sigma)\,\sigma^{t^*-a} \sum_{\theta^{t^*-a}} \frac{1}{u'\left(c^a_{t^*}\left(\theta^{t^*-a}\right)\right)} \phi_{t^*-a}\left(\theta^{t^*-a}\right)\right) \beta\varepsilon \\
&> F\left(K_{t^*+1}, H_{t^*+1}\right) - C_{t^*+1} - K_{t^*+2} + (1-\delta)\,K_{t^*+1}
\end{aligned}$$

where the approximate equality becomes arbitrarily close to an equality when ε is sufficiently small, and the strict inequality follows from the assumption that the right-hand side of equation (3.10) is smaller than the left-hand side. Since $\left\{\{u^a_t, n^a_t\}^t_{a=0}, K_{t+1}\right\}^\infty_{t=0}$ satisfies equation (3.2) at $t = t^* + 1$, it follows that $\left\{\{\hat{u}^a_t, \hat{n}^a_t\}^t_{a=0}, \hat{K}_{t+1}\right\}^\infty_{t=0}$ satisfies it with strict inequality. This surplus of consumption goods at $t = t^* + 1$ can then be used to increase $\hat{u}^a_{t^*+1}\left(\theta^{t^*+1-a}\right)$ by an additional η (sufficiently small amount) at every $0 \leq a \leq t^* + 1$ and every θ^{t^*+1-a}. The resulting allocation satisfies the aggregate feasibility constraint (3.2) (provided that η is sufficiently small) and the incentive compatibility constraints (3.9) (since $\hat{u}^a_{t^*+1}\left(\theta^{t^*+1-a}\right)$ increases by a uniform additional amount across all θ^{t^*+1-a}). Moreover, the objective function (3.1) evaluates at a strictly larger number (since η is positive) than $\left\{\{u^a_t, n^a_t\}^t_{a=0}, K_{t+1}\right\}^\infty_{t=0}$. This contradicts the assumption that $\left\{\{u^a_t, n^a_t\}^t_{a=0}, K_{t+1}\right\}^\infty_{t=0}$ solves the social planner's problem (3.1)-(3.5).

3.2 An irrelevance result

Observe that if $\left\{\left\{u_t^a, n_t^a\right\}_{a=0}^t, K_{t+1}\right\}_{t=0}^{\infty}$ is a solution to the social planner's problem (3.1)-(3.5) under full information, then it must also satisfy the cross-sectional intratemporal and intertemporal inverse Euler conditions (3.6) and (3.10). Otherwise, we could consider exactly the same deviations as above to reach exactly the same contradictions. The only difference is that now there would be no need to verify that the incentive compatibility constraints (3.5) hold under the proposed deviations, since there are no such constraints under full information.

An important question to consider is: How do the frictions to risk sharing that arise from the presence of private information affect the evolution of the macroeconomic variables $\{C_t, H_t, K_{t+1}\}_{t=0}^{\infty}$? Are these macroeconomic variables the same under private information as under full information? In this subsection I provide an important benchmark case in which these macroeconomic variables are exactly the same in the private and full information economies. This is the case in which the utility functions u and n both take logarithmic forms.

To see this observe that when u and n are both logarithmic, equation (3.6) becomes

$$(1 - H_t)\, F_H\left(K_t, H_t\right) = \bar{\theta}_t C_t, \tag{3.15}$$

for every t.

Also, when u is logarithmic equation (3.10) becomes

$$C_{t+1} = \beta\left(F_K\left(K_{t+1}, H_{t+1}\right) + 1 - \delta\right) C_t, \tag{3.16}$$

for every t.

Since the aggregate feasibility constraint (3.2) as well as equations (3.3)-(3.4) must be satisfied both in the private and full information economies, we conclude that $\{C_t, H_t, K_{t+1}\}_{t=0}^{\infty}$ are exactly the same in both economies. That is, under logarithmic preferences the information frictions are completely irrelevant for macroeconomic dynamics.

Observe that the solution $\{C_t, H_t, K_{t+1}\}_{t=0}^{\infty}$ in both economies coincides with the solution to the following representative agent social planner's problem:

$$max \sum_{t=0}^{\infty} \beta^t \left[\ln C_t + \bar{\theta}_t \ln\left(1 - H_t\right)\right], \tag{3.17}$$

subject to

$$C_t + K_{t+1} - (1 - \delta)\, K_t \leq F\left(K_t, H_t\right), \tag{3.18}$$

where $\bar{\theta}_t$ satisfies equation (3.7). Moreover, the shadow values of labor q_t and consumption λ_t also coincide.[9]

4 Stochastic mechanism design problem in recursive form

This section reintroduces the original stochastic process for aggregate productivity in equation (2.1) and provides a recursive formulation to the mechanism design problem. In order to simplify the recursive formulation and with an eye already on computations, hereon I will assume that the idiosyncratic shock to leisure can take only two values: either θ_L or θ_H, with $\theta_L < \theta_H$.

4.1 Economy-wide mechanism design problem

In order to provide a recursive representation of the problem it will be important to distinguish between two types of agents: young and old. A young agent is one that has been born at the beginning of the current period. An old agent is one that has been born in some previous period. The social planner must decide recursive plans for both types of agents.[10] The state of a recursive plan is described by three variables: 1) the type (either L or H) that the agent reported in the previous period, 2) the promised value (i.e., discounted expected utility) that the agent receives at the beginning of the current period if the agent made a truthful report in the previous period, and 3) the threat value that the agent receives at the beginning of the current period if the agent made a false report in the previous period. Given the current state, the recursive plan specifies the current utility of consumption, the current utility of leisure, the next-period state-contingent promised value, and the next-period state-contingent threat value, as functions of the value of leisure currently reported by the agent. The social planner is fully committed to the recursive plans they choose and agents have no outside opportunities available.

A key difference between the young and the old is in terms of promised and threat values.

[9] To see this, add all the first order conditions with respect to $u_0^0(\theta^0)$ in problem (3.1)-(3.5) to get that $\lambda_0 = 1/C_0$ (both in the private and public information economies), differentiate with respect to H_t to get that $q_t = F(K_t, H_t)$, and differentiate with respect to K_{t+1} to get that $\lambda_t = \beta\lambda_{t+1}(F_K(K_{t+1}, H_{t+1}) + 1 - \delta)$. The result then follows from equation (3.16) and the fact that all aggregate allocations are identical.

[10] Hereon, I will follow the representation of recursive contracts introduced by Fernandes and Phelan (2000)

Because during the previous period the social planner has already decided on some recursive plan for a currently old agent, the planner is restricted to delivering the corresponding promised and threat values during the current period. In contrast, the social planner is free to deliver any value to a currently young agent since this is the first period they are alive. Reflecting this difference, I will specify the individual state of an old agent to be the type r that they reported in the previous period, their promised value v, their threat value $\hat{v}$, and their current value of leisure s (heron, I will refer to the value of leisure θ_s by its subindex s). At date t, their current utility of consumption, utility of leisure, next-period promised value and next-period threat value are denoted by $u_{ost}(r, v, \hat{v})$, $n_{ost}(r, v, \hat{v})$, $w_{os,t+1}(r, v, \hat{v})$ and $\hat{w}_{os,t+1}(r, v, \hat{v})$, respectively, where $w_{os,t+1}(r, v, \hat{v})$ and $\hat{w}_{os,t+1}(r, v, \hat{v})$ are random variables contingent on the realization of z_{t+1}.[11] In turn, the individual state of a young agent is solely given by their current value of leisure s. At date t, the agent's current utility of consumption, utility of leisure, next-period promised value, and next-period threat value are denoted by u_{yst}, n_{yst}, $w_{ys,t+1}$ and $\hat{w}_{ys,t+1}$, respectively, where $w_{ys,t+1}$ and $\hat{w}_{ys,t+1}$ are also contingent on the realization of z_{t+1}.

The aggregate state of the economy is given by the triplet (z_t, K_{t-1}, μ_t), where z_t is the aggregate productivity level, K_{t-1} is the stock of capital, and μ_t is a measure describing the number of old agents across individual states $(r, v, \hat{v})$. The social planner seeks to maximize the weighted sum of welfare levels of current and future generations of young agents (the welfare levels of old agents are predetermined by their promised and threat values at the beginning of the period). In recursive form, the planner's problem at $t \geq 1$ is given by[12]

$$V_t\left(z_t, K_{t-1}, \mu_t\right) = \max \left\{ (1-\sigma) \sum_s \left[u_{yst} + \theta_s n_{yst} + \beta\sigma E_t\left(w_{ys,t+1}\right)\right] \psi_s + \pi E_t\left[V_{t+1}\left(z_{t+1}, K_t, \mu_{t+1}\right)\right] \right\} \tag{4.1}$$

subject to:

$$\begin{aligned} & (1-\sigma) \sum_s c\left(u_{yst}\right) \psi_s + \sum_r \int \sum_s c\left(u_{ost}\left(r, v, \hat{v}\right)\right) Q(r,s) \mu_t(r, dv \times d\hat{v}) + K_t - (1-\delta) K_{t-1} \\ \leq \;& e^{z_t} F(K_{t-1}, H_t), \end{aligned} \tag{4.2}$$

[11] I follow the convention that a variable is dated t if it becomes known at date t.

[12] Actually, the planning problem described here has a recursive structure, and therefore, the time subscripts of all variables could be removed. The advantages of leaving them explicit will become apparent below.

$$H_t \leq (1-\sigma)\sum_s h\left(n_{yst}\right)\psi_s + \sum_r \int \sum_s h\left(n_{ost}\left(r,v,\hat{v}\right)\right) Q(r,s)\mu_t(r, dv \times d\hat{v}), \tag{4.3}$$

$$u_{yst} + \theta_s n_{yst} + \beta\sigma E_t\left[w_{ys,t+1}\right] \geq u_{ys^ct} + \theta_s n_{ys^ct} + \beta\sigma E_t\left[\hat{w}_{ys^c,t+1}\right], \tag{4.4}$$

$$\begin{aligned} &u_{ost}\left(r,v,\hat{v}\right) + \theta_s n_{ost}\left(r,v,\hat{v}\right) + \beta\sigma E_t\left[w_{os,t+1}\left(r,v,\hat{v}\right)\right] \\ &\geq u_{os^ct}\left(r,v,\hat{v}\right) + \theta_s n_{os^ct}\left(r,v,\hat{v}\right) + \beta\sigma E_t\left[\hat{w}_{os^c,t+1}\left(r,v,\hat{v}\right)\right], \end{aligned} \tag{4.5}$$

$$v = \sum_s \left\{u_{ost}\left(r,v,\hat{v}\right) + \theta_s n_{ost}\left(r,v,\hat{v}\right) + \beta\sigma E_t\left[w_{os,t+1}\left(r,v,\hat{v}\right)\right]\right\} Q(r,s), \tag{4.6}$$

$$\hat{v} = \sum_s \left\{u_{ost}\left(r,v,\hat{v}\right) + \theta_s n_{ost}\left(r,v,\hat{v}\right) + \beta\sigma E_t\left[w_{os,t+1}\left(r,v,\hat{v}\right)\right]\right\} Q(r^c,s), \tag{4.7}$$

$$\begin{aligned} \mu_{t+1}\left(s, B \times \hat{B}\right) &= \sigma \sum_r \int_{\left\{(v,\hat{v}):\ w_{os,t+1}(r,v,\hat{v}) \in B \text{ and } \hat{w}_{os,t+1}(r,v,\hat{v}) \in \hat{B}\right\}} Q(r,s)\mu_t(r, dv \times d\hat{v}) \\ &+ (1-\sigma)\sigma\psi_s \chi\left(w_{ys,t+1} \in B \text{ and } \hat{w}_{ys,t+1} \in \hat{B}\right), \end{aligned} \tag{4.8}$$

where E_t denotes expectation conditional on z_t, $\beta\sigma < \pi < 1$ is the welfare weight of the next-period generation relative to the current-period generation, $c(u) = u^{-1}(u)$, $h(n) = 1 - n^{-1}(n)$, and ψ denotes the initial distribution of idiosyncratic shocks when agents are born.[13] Equation (4.2) describes the aggregate feasibility constraint for the consumption good. It states that the total consumption of young and old agents plus aggregate investment cannot exceed aggregate output.[14] Equation (4.3) is the aggregate labor feasibility constraint. It states that the input of hours into the production function cannot exceed the total hours worked by young and old agents. Equations (4.4) and (4.5) are the incentive-compatibility constraints of young and old agents, respectively, and must hold for every s. Observe that in these equations (and hereafter) s^c denotes the value opposite to s (i.e. $\{s^c\} = \{s\}^c$). Also observe that in the right hand sides of equations (4.4) and (4.5), which correspond to the payoffs from reporting type s^c falsely, the expected present value that the agent will obtain at the beginning of the following period is the threat value $\hat{w}$ corresponding to the false report s^c. Equation (4.6) is the promise-keeping constraint. It states that the expected value that the agent receives when r was reported truthfully in the previous

[13] ψ corresponds to ϕ_0 in the sequential notation of Section 2.

[14] Observe that, given the constant probability of dying $1-\sigma$ and the immediate replacement with newborns, the number of young agents in the economy is always equal to $1-\sigma$.

period is equal to the promised value v. Observe that the probability distribution used in this equation is the true distribution $Q(r, \cdot)$. Equation (4.7) is the threat-keeping constraint. It states that the expected value that the agent receives when r was reported falsely in the previous period is equal to the threat value $\hat{v}$. Observe that the probability distribution used in this equation is the true distribution $Q(r^c, \cdot)$. Finally, equation (4.8) is the law of motion for the measure of old agents across individual states. It states that the number of old agents that at the beginning of the following period will have a previous-period report s, a promised value in the Borel set B, and as threat-value in the Borel set $\hat{B}$ is given by the sum of two terms. The first term sums all currently old agents that receive a current type s, a next-period promised value in the set B, a next-period threat-value in the set $\hat{B}$, and do not die. The second term does the same for all currently young agents, where $\chi()$ is an indicator function that evaluates to one if the argument is true and evaluates to zero otherwise. Observe that because next-period promised values $w_{os,t+1}(v)$ and $w_{ys,t+1}$ and because next-period threat-values $\hat{w}_{os,t+1}(v)$ and $\hat{w}_{ys,t+1}$ are all contingent on the realization of next-period aggregate productivity z_{t+1}, that the same is true for the measure μ_{t+1}.

4.2 Sub-planning problems

In what follows it will be useful to decompose the economy-wide planning problem of the previous section into a series of sub-planning problems. In these sub-planning problems the stochastic process for shadow prices are taken as given. These shadow prices (and associated allocations) correspond to the economy-wide social planning solution if certain side-conditions are satisfied. There are sub-planning problems for insuring old agents, for insuring young agents, and for making production decisions.

The sub-planning problem for old agents is the following:

$$P_t\left(r, v, \hat{v}\right) = max \sum_s Q\left(r, s\right) \left\{ q_t h(n_{ost}) - c(u_{ost}) + \pi\sigma E_t \left[\frac{\lambda_{t+1}}{\lambda_t} P_{t+1}\left[s, w_{os,t+1}, \hat{w}_{os,t+1}\right] \right] \right\}, \quad (4.9)$$

subject to

$$u_{ost} + \theta_s n_{ost} + \beta\sigma E_t\left[w_{os,t+1}\right] \geq u_{os^c,t} + \theta_s n_{os^c,t} + \beta\sigma E_t\left[\hat{w}_{os^c,t+1}\right] \quad (4.10)$$

$$v = \sum_s \left\{ u_{ost} + \theta_s n_{ost} + \beta\sigma E_t\left[w_{os,t+1}\right] \right\} Q\left(r, s\right) \quad (4.11)$$

$$\hat{v} = \sum_s \left\{ u_{ost} + \theta_s n_{ost} + \beta\sigma E_t\left[w_{os,t+1}\right] \right\} Q\left(r^c, s\right) \quad (4.12)$$

where $(r, v, \hat{v})$ is the state of the recursive contract of the old agent, q_t is the shadow price of labor (in terms of the consumption good), and λ_t is the shadow price of the consumption good (in utiles). Observe that the "social profits" in equation (4.9) are given by the social value of the hours worked by the old agent, net of the consumption transferred to them. The reason for the opposite signs is that the hours worked by the old agent relaxes the aggregate feasibility constraint (4.3) while the consumption transferred to them tightens the aggregate feasibility constraint (4.2).

The sub-planning problem for young agents is the following:

$$\max \sum_s \psi_s \left\{ \frac{u_{yst} + \theta_s \ n_{yst} + \beta\sigma E_t \left[w_{ys,t+1}\right]}{\lambda_t} + q_t h(n_{yst}) - c\left(u_{yst}\right) \right.$$
$$\left. + \pi\sigma E_t \left[\frac{\lambda_{t+1}}{\lambda_t} P_{t+1}\left[s, w_{ys,t+1}, \hat{w}_{ys,t+1}\right] \right] \right\} \tag{4.13}$$

subject to

$$u_{yst} + \theta_s n_{yst} + \beta\sigma E_t \left[w_{ys,t+1}\right] \geq u_{ys^c t} + \theta_s n_{ys^c t} + \beta\sigma E_t \left[\hat{w}_{ys^c,t+1}\right]. \tag{4.14}$$

Observe that the "social profits" of young agents include their lifetime welfare since it directly contributes to the economy-wide objective function (4.1).[15]

Finally, the sub-planning problem for production decisions is

$$J_t\left(K_{t-1}\right) = \max_{\{H_t, I_t\}} \left\{ e^{z_t} F(K_{t-1}, H_t) - q_t H_t - I_t + \pi E_t \left[\frac{\lambda_{t+1}}{\lambda_t} J_{t+1}\left(\left(1-\delta\right) K_{t-1} + I_t\right) \right] \right\}$$

Observe that the social surplus in this planning problem is given by output net of the value of the labor input and the value of investment.

The stochastic process for the shadow prices q_t and λ_t that are taken as given in these sub-planning problems are the true economy-wide shadow prices if, given the stochastic process for μ_t generated by equation (4.8), the solutions to the sub-planning problems satisfy the aggregate feasibility constraints (4.2) and (4.3) almost surely.

5 Computations

In Veracierto (2026) I described a computational method that can handle a large class of economies with private information and aggregate shocks. However, that method is extremely costly to

[15] This term is divided by λ_t to convert it into current consumption units.

implement for the economy in this paper.[16] In order to make progress, hereon I will have to focus on a very special case that makes computations feasible. This is the case in which the functions u and n are both logarithmic. In Section 3 we saw that starting from any initial K_0, the optimal path $\{C_t, K_{t+1}, H_t, \lambda_t, q_t\}_{t=0}^{\infty}$ that solves the deterministic private information social planner's problem (3.1)-(3.5) coincides with the deterministic public information solution and with that of the deterministic representative agent social planner's problem (3.17)-(3.18). If we were to introduce the stochastic process (2.1) and linearize the model around the deterministic optimal path, the solution would then be fluctuating around that path and all aggregate variables would converge (in probability) to the stationary solution of the following representative agent social planner's problem:

$$max E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\ln C_t + \bar{\theta}_t \ln (1 - H_t) \right] \right\}, \tag{5.1}$$

subject to

$$C_t + K_{t+1} - (1 - \delta) K_t \leq e^{z_t} F(K_t, H_t), \tag{5.2}$$

$$z_{t+1} = \rho z_t + \varepsilon_{t+1}, \tag{5.3}$$

where $\bar{\theta}_t$ is the cross-sectional average value of θ at date t.

My strategy will then be to linearize problem (5.1)-(5.3) at the deterministic steady state and find the associated linear shadow price functions and capital accumulation decision rule

$$\lambda_t = \bar{\lambda} + \lambda^K K_t + \lambda^z z_t, \tag{5.4}$$

$$q_t = \bar{q} + q^K K_t + q^z z_t, \tag{5.5}$$

$$K_{t+1} = \bar{K} + \kappa^K K_t + \kappa^z z_t. \tag{5.6}$$

These functions can then be fed into the sub-planning problems for old agents (4.9)-(4.12) and the sub-planning problem for young agents (4.13)-(4.14) in order to obtain optimal allocations conditional on those functions. In what follows I describe how the sub-planning problem for old agents can be solved.[17]

[16] The reason is that since decision rules are two dimensional (they are functions of v and $\hat{v}$), many grid points are needed to describe them accurately. As a consequence, the history of past decision rules evaluated at these grid points (which the computational method uses to describe the aggregate state of the economy) becomes too large.

[17] The sub-planning problem for young agents can be similarly handled.

In recursive form, all individual allocations rules for old agents are functions of the triplet $(r, v, \hat{v})$. It turns out that when $r = L$ that the set of admissible pairs $(v, \hat{v})$ are all those in which $v \geq \hat{v}$, and when $r = H$ the set of admissible pairs $(v, \hat{v})$ are all those in which $v \leq \hat{v}$ (see Figure 1). In what follows I find it convenient to rotate the coordinate system so that the x-axis coincides with the 45-degree line and the y-axis is perpendicular to it. When $r = L$ I then restrict the domain for $(v, \hat{v})$ to be a rectangle bordering the 45-degree line right below it, and when $r = H$ I restrict the domain to be the reflection of that previous rectangle on the 45-degree line (see Figure 1). Having rectangular domains is not only crucial for approximating functions as splines, but my choice of rectangular domains is quite natural. The reason is that, for any state $(r, v, \hat{v})$ in the sub-planning problem (4.9)-(4.12), if $[u_{os,t}, n_{os,t}, w_{os,t+1}, \hat{w}_{os,t+1}]_s$ is feasible (i.e. satisfies equations 4.10-4.12) then

$$\beta\sigma(\hat{w}_{oH,t+1}) - w_{oH,t+1}) - \frac{1}{\pi(r^c, L) - \pi(r, L)}(\hat{v} - v) = (\theta_H - \theta_L)\, n_{oHt}.$$

That is, conditional on any value for n_{oHt}, only the differences between threat values and promised values matter for feasibility (i.e. only the location of the lines parallel to the 45 degree line matter for feasibility).

Having rotated the coordinate system and defined the rectangular domains, the relation between the new (x, y) pairs and the original $(v, \hat{v})$ pairs are given by

$$v = v_{\min} + \frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y \tag{5.7}$$

$$\hat{v} = v_{\min} + \frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y \tag{5.8}$$

where $v_{\min}$ is the lowest value of v (and $\hat{v}$) defined by the rectangles along the 45 degree line.

The sub-planning problem for old agents (4.9)-(4.12) can then redefined in terms of the new variables x and y as follows:

$$P_t\,(r, x, y) =$$
$$max \sum_s Q\,(r, s) \left\{ q_t(z_t, K_t)h(n_{ost}) - c(u_{ost}) + \pi\sigma E_t \left[\frac{\lambda_{t+1}(z_{t+1}, K_{t+1})}{\lambda_t(z_t, K_t)} P_{t+1}\,[s, a_{os,t+1}, b_{os,t+1}] \right] \right\}, \tag{5.9}$$

subject to

$$u_{ost} + \theta_s n_{ost} + \beta\sigma E_t\,[v(a_{os,t+1}, b_{os,t+1})] \geq u_{os^c,t} + \theta_s n_{os^c,t} + \beta\sigma E_t\,[\hat{v}(a_{os,t+1}, b_{os,t+1})] \tag{5.10}$$

$$v(x,y) = \sum_s \{u_{ost} + \theta_s n_{ost} + \beta\sigma E_t [v(a_{os,t+1}, b_{os,t+1})]\} Q(r,s) \tag{5.11}$$

$$\hat{v}(x,y) = \sum_s \{u_{ost} + \theta_s n_{ost} + \beta\sigma E_t [v(a_{os,t+1}, b_{os,t+1})]\} Q(r^c,s) \tag{5.12}$$

$$K_{t+1} = g(z_t, K_t), \tag{5.13}$$

$$z_{t+1} = \rho z_t + \varepsilon_{t+1}, \tag{5.14}$$

where the functions $\lambda(z_t, K_t)$, $q(z_t, K_t)$, $g(z_t, K_t)$, $v(x,y)$ and $\hat{v}(x,y)$ are defined by equations (5.4)-(5.8), respectively, and $(a_{os,t+1}, b_{os,t+1})$ denotes next-period state-contingent values for (x,y).

Before solving the stochastic solutions to this problem I first need to obtain the deterministic steady state solution, in which the aggregate productivity z is set to zero and the stock of capital K is set to its deterministic steady state value. This problem is solved defining grid points $\{x_i\}_{i=1}^I$ and $\{y_j\}_{j=1}^J$ for x and y, respectively, and using cubic splines to evaluate the first order conditions for $a_{os,t+1}$ and $b_{os,t+1}$ outside their grid points.

In order to solve the stochastic solution I group variables into the following two vectors

$$m_{t+1}^1 = \left([\triangle a_{os,t+1}(r, x_i, y_j)]_{s,r,i,j}, [\triangle b_{os,t+1}(r, x_i, y_j)]_{s,r,i,j} \right) \tag{5.15}$$

$$\begin{aligned} m_t^2 = \Big(&[\triangle \xi_{ot}(r, x_i, y_j)]_{r,i,j}, [\triangle \eta_t(r, x_i, y_j)]_{r,i,j}, [\triangle \hat{\eta}_t(r, x_i, y_j)]_{r,i,j}, \\ &[\triangle u_{os,t}(r, x_i, y_j)]_{s,r,i,j}, [\triangle n_{os,t}(r, x_i, y_j)]_{s,r,i,j} \Big), \end{aligned} \tag{5.16}$$

where $\xi_{ot}(r, x_i, y_j)$, $\eta_{ot}(r, x_i, y_j)$ and $\hat{\eta}_{ot}(r, x_i, y_j)$ are the Lagrange multipliers to equations (5.10)-(5.12), respectively, and $\triangle$ denotes differences with respect to deterministic steady state values.[18] Observe that the vector m_{t+1}^1 includes all variables that are contingent on the realization of z_{t+1} while the vector m_t^2 includes all non-contingent variables.

Having grouped variables in that way, the first-order conditions to the sub-planning problem for old agents can be linearized around the deterministic steady state to obtain a linear model of

[18]The grid points $\{x_i\}_{i=1}^I$ and $\{y_j\}_{j=1}^J$ are exactly the same as in the deterministic steady state solution.

the following form:

$$0 = J_1 m^1_{t+1} + L_1 m^2_t + M_1 m^2_{t+1} + z_t A_1 + k_t B_1 + z_{t+1} C_1 + k_{t+1} D_1 \tag{5.17}$$

$$0 = L_2 m^2_t + z_t A_2 + k_t B_2 \tag{5.18}$$

$$0 = J_3 E_t \left\{ m^1_{t+1} \right\} + L_3 m^2_t \tag{5.19}$$

$$0 = -z_{t+1} + \rho z_t + \varepsilon_{t+1} \tag{5.20}$$

$$0 = -k_{t+1} + a k_t + b z_t \tag{5.21}$$

where $k = \triangle K$. Equation (5.17) describes the linearized first-order conditions for $[\triangle a_{os,t+1}(r, x_i, y_j)]_{s,r,i,j}$ and $[\triangle b_{os,t+1}(r, x_i, y_j)]_{s,r,i,j}$, equation (5.18) describes the linearized first-order conditions for $[\triangle u_{os,t}(r, x_i, y_j)]_{s,r,i,j}$ and $[\triangle n_{os,t}(r, x_i, y_j)]_{s,r,i,j}$, equation (5.19) describes equations (5.10)-(5.12), equation (5.20) describes equation (5.14) and equation (5.21) describes equation (5.13).[19].

I seek a recursive solution of the following form:

$$m^1_{t+1} = k_t \Phi_1 + z_t \Gamma_1 + z_{t+1} \Lambda_1, \tag{5.22}$$

$$m^2_t = k_t \Phi_2 + z_t \Gamma_2. \tag{5.23}$$

The state-contingent nature of the recursive decision rule (5.22) is a non-standard feature. My strategy will be to construct it from the recursive solution to the deterministic version of equations (5.17)-(5.21), in which ε_{t+1} is set to zero and the expectations operator is dropped from equation (5.19). The solution to this deterministic model is standard and has the following form:

$$m^1_{t+1} = k_t R_1 + z_t S_1, \tag{5.24}$$

$$m^2_t = k_t R_2 + z_t S_2. \tag{5.25}$$

The way that the state-contingent solution can be obtained from the deterministic solution is provided in the following Proposition.

Proposition 1 *Let (5.24)-(5.25) be the solution to the deterministic version of equations (5.17)-*

[19]The reason why m^2_{t+1} appears in equation (5.17) is that in order to evaluate the marginal values of $[\triangle a_{os,t+1}(r, x_i, y_j)]_{s,r,i,j}$ and $[\triangle b_{os,t+1}(r, x_i, y_j)]_{s,r,i,j}$ in the next period, we need to use the spline coefficients described by $[\triangle \eta_t(r, x_i, y_j)]_{r,i,j}$ and $[\triangle \hat{\eta}_t(r, x_i, y_j)]_{r,i,j}$.

(5.21). Define

$$\Phi_1 = R_1 \tag{5.26}$$

$$\Phi_2 = R_2 \tag{5.27}$$

$$\Lambda_1 = -J_1^{-1} M_1 S_2 - J_1^{-1} C_1 \tag{5.28}$$

$$\Gamma_1 = S_1 - \rho \Lambda_1 \tag{5.29}$$

$$\Gamma_2 = S_2 \tag{5.30}$$

Then, (5.22)-(5.23) solves the stochastic system (5.17)-(5.21).

Proof: It follows from guessing and verifying.

6 Quantitative results

Before turning to the quantitative results the model must by calibrated. In what follows I describe how parameter values are selected using a model time period equal to one year (which simplifies computations quite considerably).

The survival probability σ is chosen to be 0.975 so that the expected lifetime of a young agent is equal to 40 years. As is standard in the macro literature, the discount factor β is set to 0.96. The relative Pareto weight π is chosen to be equal to β.

The production function is chosen to have the following Cobb-Douglas form:

$$F(K, H) = K^{1-\alpha} H^{\alpha}.$$

Following standard calibrations of the neoclassical growth model the labor share α is selected to be 0.64. Similarly, the depreciation rate of capital δ is set to 0.10. In turn, the stochastic process chosen for aggregate productivity is standard in the Real Business Cycle literature: I select $\rho = 0.9$ and $\sigma_{\varepsilon} = 0.014$.

Since there are only two values for the idiosyncratic shocks, I choose a symmetric transition matrix Q as well as an initial distribution ψ that puts equal mass on both values. I then set $Q(L, L) = Q(H, H) = 0.90$ to make the idiosyncratic shocks highly persistent and choose θ_L and θ_H to match two targets: 1) that aggregate hours worked equal 0.33 (a standard value in the Real

Business Cycle literature), and 2) that the variance of idiosyncratic labor income growth rates has a variance of 0.086 (consistent with the persistent component in Guvenen et al. (2023)).

Having parametrized the model I now turn to the quantitative results. I am particularly interested in analyzing the behavior of the optimal intratemporal and intertemporal wedges $\tau_{ost}(r, v, \hat{v})$ and $\kappa_{ost}(r, v, \hat{v})$, which satisfy

$$\frac{1}{c(u_{ost}(r, v, \hat{v}))} q_t (1 - \tau_{ost}(r, v, \hat{v})) = \theta_s \frac{1}{1 - h(n_{ost}(r, v, \hat{v}))} \tag{6.1}$$

and

$$\frac{1}{c(u_{ost}(r, v, \hat{v}))} = (1 - \kappa_{ost}(r, v, \hat{v})) \beta E_t \left[\sum_{s'} \frac{e^{z_{t+1}} F(K_{t+1}, H_{t+1}) + 1 - \delta}{c(u_{os',t+1}(s, w_{os,t+1}(r, v, \hat{v}), \hat{w}_{os,t+1}(r, v, \hat{v})))} Q(s, s') \right], \tag{6.2}$$

respectively. That is, $\tau_{ost}(r, v, \hat{v})$ and $\kappa_{ost}(s, r, v, \hat{v})$ are the wedges that preclude the intratemporal and intertemporal individual Euler conditions under perfect risk-sharing to hold, and are interpretable as labor and savings taxes.

It turns out that in the calibrated economy the intertemporal wedges are negligible. As a consequence, I will hereon ignore them. Also, it can be shown that the labor supply of agents with the low value of leisure is always undistorted. That is, $\tau_{ost}(r, v, \hat{v})$ is always equal to zero for $s = L$. I will therefore focus on the behavior of $\tau_{ost}(r, v, \hat{v})$ for $s = H$. Still, there is so much heterogeneity in this economy that it is hard to report the cross-sectional behavior of these wedges over the business cycle in a parsimonious way. In order to streamline the presentation I will therefore focus on the wedges of individuals that have always enjoyed the low value of leisure since birth and transit to the high value of leisure for the first time.[20]

Figure 2 shows the labor wedges of this group of agents at the deterministic steady state of the economy (i.e. in the absence of aggregate shocks). We see that the labor wedge increases monotonically with the age at which the agent receives the high value of leisure. However the increase is relatively small: the labor wedge increases from 1.97% at the beginning of life to only 2.20% after 30 years of continuously receiving the low value of leisure before transiting to the high value for the first time.

[20]Considering other groups of agents, such as those that have always received the high value of leisure since birth, gives similar results.

Figure 3 shows the impulse responses of the labor wedges for the same group of agents across a selection of ages when they transit to the high value of leisure. These impulse responses correspond to an innovation in aggregate productivity equal to one standard deviation. We see that the labor wedges for this group of agents all increase with the aggregate productivity shock and peak one period after the shock takes place. The largest response (in raw units) is for agents that have an age of 30 when they transit to the high value of leisure for the first time: their labor wedge increase from a steady-state value of 2.20% to 2.23%. Since this is a negligible difference, this indicates that optimal labor wedges are roughly constant over the business cycle.

7 Conclusions

This paper considered a perpetual youth real business cycle economy in which agents receive persistent idiosyncratic shocks to their value of leisure and these shocks are private information. In the absence of aggregate productivity shocks the paper showed that aggregate allocations and economy-wide shadow prices are identical to the case of public information when the utility functions for consumption and leisure are both logarithmic. Under these preferences, the paper then introduced a simple computational approach that allows to obtain optimal allocations when aggregate productivity shocks are present. In particular, it allows to compute state-contingent next-period promised values in a parsimonious way. Calibrating the economy to the U.S. economy, the paper found that optimal labor wedges are roughly constant over the business cycle.

For computational reasons, the paper only evaluated the cyclical behavior of optimal labor wedges for the case of logarithmic preferences. Computing an economy-wide solution for other preferences is prohibitively expensive. However, it would be possible to estimate empirical processes for the stochastic discount rate and shadow value of leisure that could then be plugged into the sub-planning problems, similarly to how I plugged the solution to the representative agent social planner's problem. Characterizing the optimal behavior of labor wedges conditional on those empirical shadow price processes under preferences other than logarithmic would be extremely interesting but left for future research.

References

Chari, V. V., L. J. Christiano and P. J. Kehoe, "Optimal Fiscal Policy in a Business Cycle Model," *Journal of Political Economy* 102 (1994), 617–652.

Farhi, E. and I. Werning, "Capital Taxation: Quantitative Explorations of the Inverse Euler Equation," *Journal of Political Economy* 120 (2012), 398 – 445.

———, "Insurance and Taxation over the Life Cycle," *The Review of Economic Studies* 80 (April 2013), 596–635.

Fernandes, A. and C. Phelan, "A Recursive Formulation for Repeated Agency with History Dependence," *Journal of Economic Theory* 91 (April 2000), 223–247.

Guvenen, F., A. McKay and C. B. Ryan, "A Tractable Income Process for Business Cycle Analysis," Working Paper 30959, National Bureau of Economic Research, February 2023.

Kapicka, M., "Optimal Fiscal Policy with Preference Heterogeneity," mimeo, CERGE-EI, 2024.

Phelan, C., "Incentives and Aggregate Shocks," *Review of Economic Studies* 61 (1994), 681–700.

Veracierto, M., "Business cycle fluctuations in Mirrlees economies: The case of i.i.d. shocks," *Journal of Economic Theory* 196 (2021).

———, "Computing Aggregate Fluctuations of Economies with Private Information," *Computational Economics* (2026), https://doi.org/10.1007/s10614–025–11176–9.

Figure 1: Value function domains for old agents

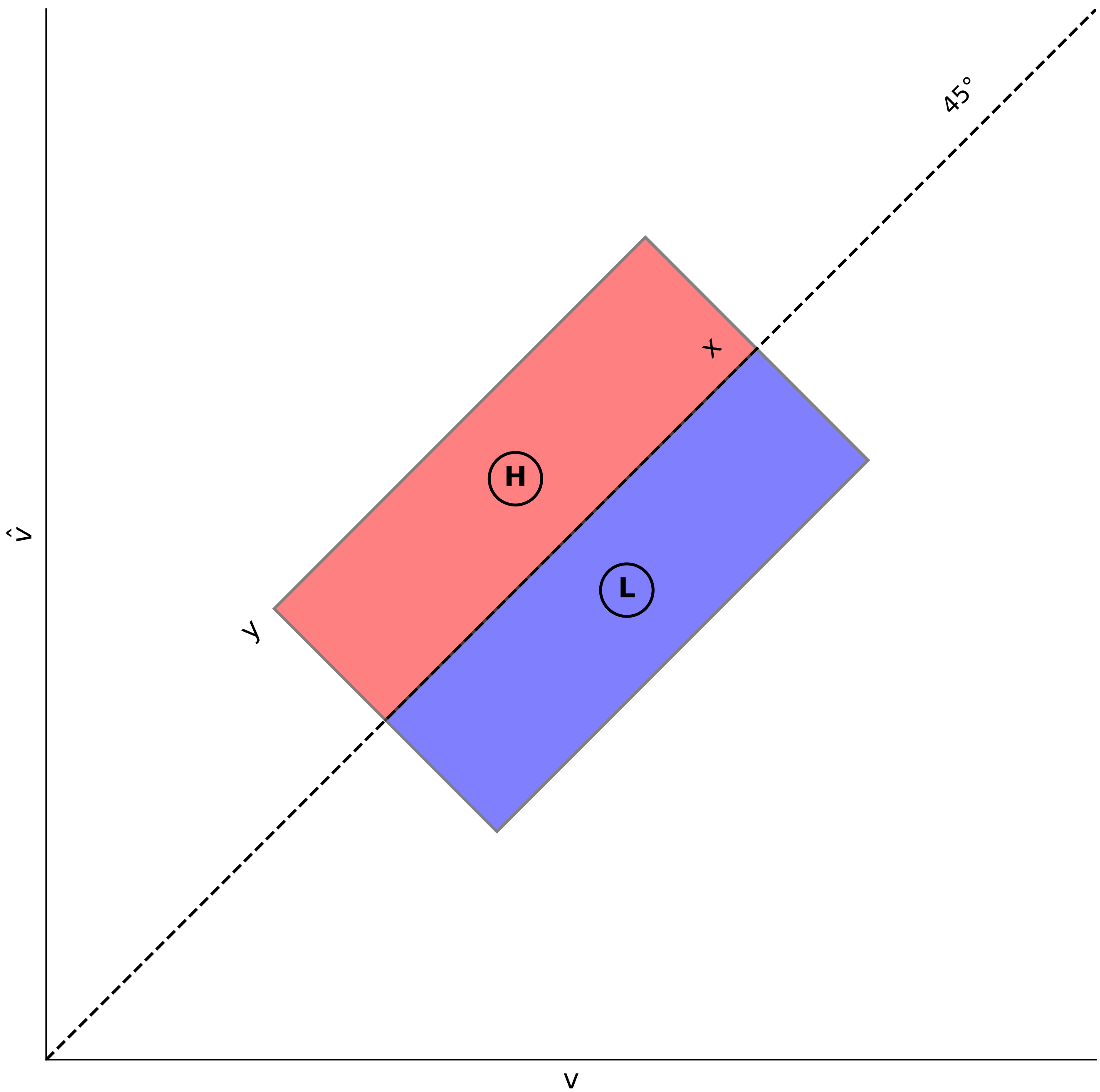

Figure 2: Steady state labor wedges across ages

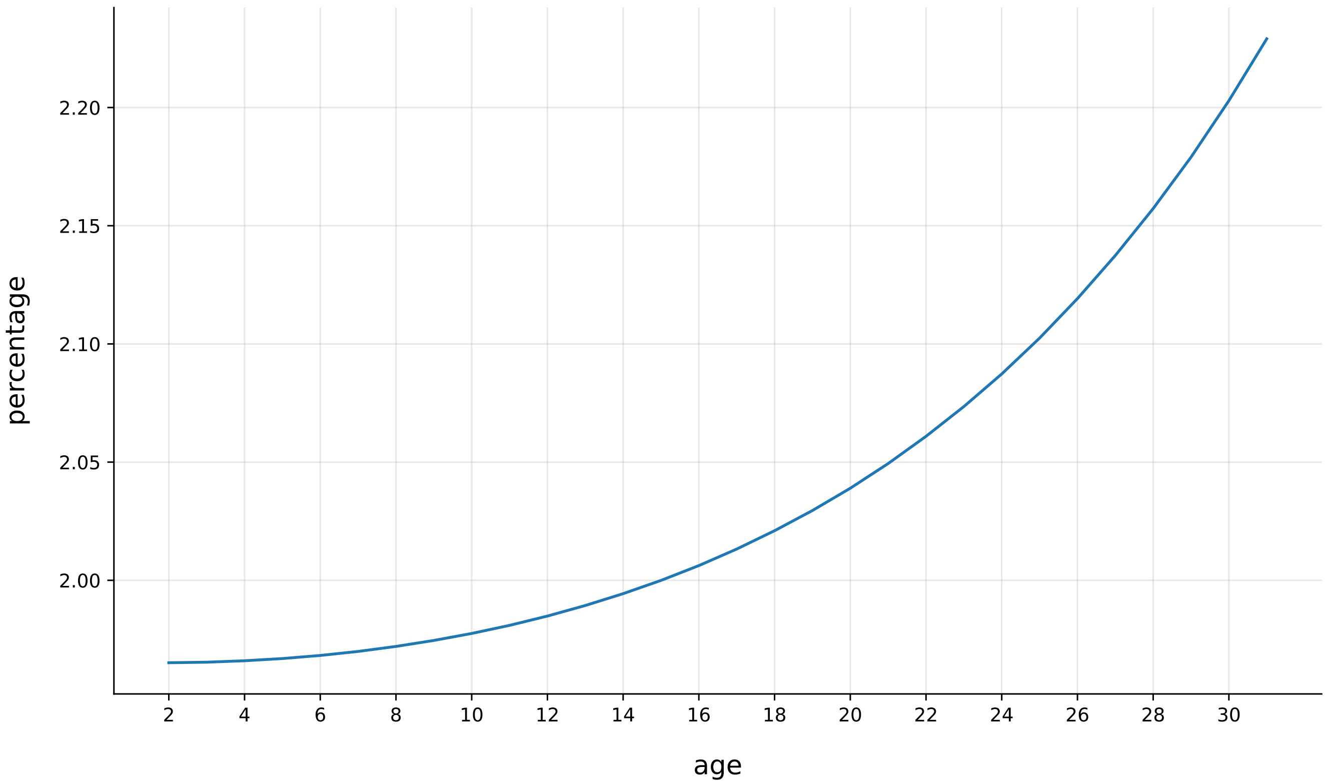

Figure 3: Impulse responses of labor wedges across ages