Involuntary Unemployment and the Business Cycle*

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Abstract

We propose a monetary model in which the unemployed satisfy the official US definition of unemployment: they are people without jobs who are (i) currently making concrete efforts to find work and (ii) willing and able to work. In addition, our model has the property that people searching for jobs are better off if they find a job than if they do not (i.e., unemployment is 'involuntary'). We integrate our model of involuntary unemployment into the simple New Keynesian framework with no capital and use the resulting model to discuss the concept of the 'non-accelerating inflation rate of unemployment'. We then integrate the model into a medium sized DSGE model with capital and show that the resulting model does as well as existing models at accounting for the response of standard macroeconomic variables to monetary policy shocks and two technology shocks. In addition, the model does well at accounting for the response of the labor force and unemployment rate to the three shocks.

Keywords: DSGE, unemployment, business cycles, monetary policy, Bayesian estimation.

JEL codes: E2, E3, E5, J2, J6

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1. Introduction

The unemployment rate is a key variable of interest to policy makers. A shortcoming of standard monetary dynamic stochastic general equilibrium (DSGE) models is that they are silent about this important variable. Work has begun recently on the task of introducing unemployment into DSGE models. However, the approaches taken to date assume the existence of perfect consumption insurance against labor market outcomes, so that consumption is the same for employed and non-employed households. With this kind of insurance, a household is delighted to be unemployed because it is an opportunity to enjoy leisure without a drop in consumption.[1] In contrast, the theory of unemployment developed here has the implication that the unemployed are worse off than the employed. Our approach follows the work of Hopenhayn and Nicolini (1997) and others, in which finding a job requires exerting a privately observed effort.[2] In this type of environment, the higher utility enjoyed by employed households is necessary for people to have the incentive to search for and keep jobs.[3]

We define unemployment the way it is defined by the agencies that collect the data. To be officially 'unemployed' a person must assert that she (i) has recently taken concrete steps to secure employment and (ii) is currently available for work.[4] To capture (i) we assume that people who wish to be employed must undertake a costly effort. Our model has the implication that a person who asserts (i) and (ii) enjoys more utility if she finds a job than if she does not, i.e., unemployment is 'involuntary'. Empirical evidence appears to be consistent with the notion that unemployment is in practice more of a burden than a blessing. For example, Chetty and Looney (2006) and Gruber (1997) find that US households suffer roughly a 10 percent drop in consumption when they lose their job. Also, there is a substantial literature which purports to find evidence that insurance against labor market outcomes is imperfect. An early example is Cochrane (1991). These observations motivate

[1] The drop in utility reflects that models typically assume preferences that are additively separable in consumption and labor or that have the King, Plosser, Rebelo (1988) form. Examples include Blanchard and Gali (2009), Christiano, Ilut, Motto and Rostagno (2008), Christiano, Trabandt and Walentin (2009,2009a), Christoffel, Costain, de Walque, Kuester, Linzert, Millard, and Pierrard (2009), Christoffel, and Kuester (2008), Christoffel, Kuester and Linzert (2009), den Haan, Ramey and Watson (2000), Gali (2009), Gali, Smets and Wouters (2010), Gertler, Sala and Trigari (2009), Groshenny (2009), Krause, Lopez-Salido and Lubik (2008), Lechthaler Merkl and Snower (2009), Sala, Soderstrom and Trigari (2008), Sveen and Weinke (2008, 2009), Thomas (2008), Trigari (2009) and Walsh (2005).

[2] An early paper that considers unobserved effort is Shavell and Weiss (1979). Our approach is also closely related to the efficiency wage literature, as in Alexopoulos (2004).

[3] Lack of perfect insurance in practice probably reflects other factors too, such as adverse selection. Alternatively, Kocherlakota (1996) explores lack of commitment as a rationale for incomplete insurance. Lack of perfect insurance is not necessary for the unemployed to be worse off than the employed (see Rogerson and Wright, 1988).

[4] See the Bureau of Labor Statistics website, http://www.bls.gov/cps/cps_htgm.htm#unemployed, for an extended discussion of the definition of unemployment, including the survey questions used to determine a household's employment status.

our third defining characteristic of unemployment: (iii) a person looking for work is worse off if they fail to find a job than if they find one.[5]

To highlight the mechanisms in our model, we introduce it into the simplest possible DSGE framework, the model presented by Clarida, Gali and Gertler (1999) (CGG). The CGG model has frictions in the setting of prices, but it has no capital accumulation and no wage-setting frictions. In our model, individual households gather into families for the purpose of insuring themselves against idiosyncratic shocks. We view the family as a stand-in for the various market and non-market arrangements that actual households have for dealing with idiosyncratic labor market outcomes. Households experience a privately observed shock that determines their aversion to work. In addition, they exert a costly, privately observed effort that increases the likelihood that job search will be successful. Although consumption insurance is desirable in our environment, perfect insurance is not feasible because it destroys the incentive to look for work.

In principle, in an environment like ours the wage would be set through a bargaining mechanism. Instead, for simplicity we suppose the wage rate is determined competitively so that firms and families take the wage rate as given.[6] Firms face no search frictions and hire workers up to the point where marginal costs and benefits are equated. Although individual households face uncertainty as to who will work and who will not, families are sufficiently large that there is no uncertainty at the family level. Once the family sets incentives by allocating more consumption to employed households than to non-employed households, it knows exactly how many households will find work. The family takes the wage rate as given and adjusts employment incentives until the marginal cost (in terms of foregone leisure and reduced consumption insurance) of additional market work equals the marginal benefit. The firm and family first order necessary conditions of optimization are sufficient to determine the equilibrium wage rate.

Our environment has a simple representative agent formulation, in which the representative agent has an indirect utility function that is a function only of market consumption and

[5] Although all the monetary DSGE models that we know of fail (iii), they do not fail (ii). In these models there are workers who are not employed and who would say 'yes' in response to the question, 'are you currently available for work?'. Although such people in effect declare their willingness to take an action that reduces utility, they would in fact do so. This is because they are members of a large family insurance pool. They obey the family's instruction that they value a job according to the value assigned by the family, not themselves. In these models everything about the individual household is observable to the family, and it is implicitly assumed that the family has the technology necessary to enforce verifiable behavior. In our environment - and we suspect this is true in practice - the presence of private information makes it impossible to enforce a labor market allocation that does not completely reflect the preferences of the individual household. (For further discussion, see Christiano, Trabandt and Walentin, 2009, 2009a).

[6] One interpretation of our environment is that job markets occur on Lucas-Phelps-Prescott type 'islands'. Effort is required to reach those islands, but a person who finds the island finds a perfectly competitive labor market. For recent work that uses a metaphor of this type, see Veracierto (2007).

labor. As a result, our model is observationaly equivalent to the CGG model when only the data addressed by CGG are considered. In particular, our model implies the three equilibrium conditions of the New Keynesian model: an IS curve, a Phillips curve and a monetary policy rule. The conditions can be written in the usual way, in terms of the 'output gap'. The output gap is the difference between actual output and output in the 'efficient equilibrium': the equilibrium in which there are no price setting frictions and distortions from monopoly power are extinguished. In our model there is a simple relation between the output gap and the 'unemployment gap': the difference between actual and efficient unemployment.[7] The presence of this gap in our model allows us to discuss the microeconomic foundations of the non-accelerating inflation rate of unemployment (NAIRU). The NAIRU plays a prominent role in public discussions about the inflation outlook, as well as in discussions of monetary and labor market policies. In practice, these discussions leave the formal economic foundations of the NAIRU unspecified. This paper (as well as others), in effect takes a step towards integrating the NAIRU into the formal quantitative apparatus of monetary DSGE models.

We investigate the welfare cost of business cycles in our model with unemployment. We consider parameterizations of our model and the CGG model that make the two observationally equivalent from the perspective of an econometrician using standard macroeconomic data that do not include observations on unemployment and the labor force. We obtain the numerical result that the implications of the two models for the welfare cost of business cycle are the same to all available significant digits. This result stands as a challenge to the widespread view that a representative agent model like CGG implies a low cost of business cycles only because it does not factor in limitations on households' ability to insure themselves against labor market risk.

Next, we introduce our model of unemployment into a standard DSGE model that has been fit to actual data. In particular, we work with a version of the model proposed in Christiano, Eichenbaum and Evans (2005) (CEE). In this model there is monopoly power in the setting of wages, there are wage setting frictions, capital accumulation and other features.[8] As in CGG, our unemployment model aggregates into a representative agent formulation, and so our model is observationally equivalent to the standard DSGE model when only the data that are common to the two models are considered. However, our model also has implications for the dynamics of the labor force and unemployment. We estimate our model using the Bayesian version of the impulse response matching procedure proposed in Christiano, Trabandt and Walentin (2009a) (CTW). The three shocks we consider are the ones considered in Altig, Christiano, Eichenbaum and Linde (2004) (ACEL). In particular,

[7] This relationship is a formalization of the widely discussed 'Okun's law'.

[8] The model of wage setting in the standard DSGE model is the one proposed in Erceg, Henderson and Levin (2000).

we consider VAR-based estimates of the impulse responses of macroeconomic variables to a monetary policy shock, a neutral technology shock and an investment-specific technology shock. Not surprisingly, in view of the observational equivalence result, our model can match the impulse responses of standard variables as well as the standard model. However, our model also does a good job matching the responses of the labor force and unemployment to the three shocks.

Our paper emphasizes labor supply in its explanation of the dynamics of unemployment and the labor force. Another recent paper that adopts this perspective is Gali (2009). To better explain our model, it is useful to compare its properties with those of Gali's model. Gali demonstrates that with a modest reinterpretation of variables, the standard DSGE model already contains a theory of unemployment. In particular, one can define the unemployed as the difference between the number of people actually working and the number of people that would be working if the marginal cost of work were equated to the wage rate. This difference is positive and fluctuating in the standard DSGE model because of the presence of wage-setting frictions and monopoly power. In effect, unemployment is a symptom of social inefficiency. People inflict unemployment upon themselves in the quest for monopoly profits. By contrast, in our model unemployment reflects frictions that are necessary for people to find jobs. The existence of unemployment does not require monopoly power. This point is dramatized by the fact that we introduce our model in the CGG framework, in which wages are set in competitive labor markets. At the same time, the logic of our model does create a positive relationship between monopoly power and unemployment. In our model, the employment contraction resulting from an increase in the monopoly power of unions produces a reduction in the incentives for households to work. Households' response to the reduced incentives is to allocate less effort to search, implying higher unemployment. So, our model shares the prediction of Gali's model that unemployment should be higher in economies with more union monopoly power. However, our model has additional implications that could differentiate it from Gali's. Ours implies that in economies with more union power both the labor force and the disparity in consumption between employed and non-employed households are reduced. Gali's model predicts that with more union monopoly power, the labor force will be larger. The exact amount by which the labor force increases depends on the strength of wealth effects on leisure.

Other important differences between our model of unemployment and Gali's is that the latter fails to satisfy characteristics (i) and (iii) above. The model assumes that the available jobs can be found without effort. Because the model does not satisfy (i), unemployment does not meet the official US definition of unemployment. In addition, the presence of perfect insurance in Gali's model implies that the employed have lower utility than the non-employed.

There are more differences between ours and Gali's theory unemployment. In standard

DSGE models, labor supply plays little role in the dynamics of standard macro variables like consumption, output, investment, inflation and the interest rate. The reason is that the presence of wage setting frictions reduces the importance of labor supply. This is why the New Keynesian literature has been relatively unconcerned about all the old puzzles about income effects on labor and labor supply elasticities that were a central concern in the real business cycle literature. However, CTW show that these problems are back in full force if one adopts Gali's theory of unemployment. This is because labor supply corresponds to the labor force in that theory. To see how this brings back the old problems, we study the standard DSGE model's predictions for unemployment and the labor force in the wake of an expansionary monetary policy shock. Because that model predicts a rise in consumption, the model also predicts a decline in labor supply, as the income effect associated with increased consumption produces a fall in the value of work. The drop in labor supply is counterfactual, according to our VAR-based evidence. In addition, the large drop in the labor force leads to an counterfactually large drop in unemployment in the wake of an expansionary monetary policy shock.

Gali, Smets and Wouters (2010) and CTW show, in different ways, that changes to the household utility function that offset income effects reduce the counterfactual implications of the standard model for the labor force. In effect, our paper proposes a different strategy. We preserve the additively separable utility function that is standard in monetary DSGE models, and our model nevertheless does not display the labor force problems in the standard DSGE model. This is because in our model the labor force and employment have a strong tendency to comove. In our model, the rise in employment in the wake of an expansionary monetary policy shock is accomplished by increasing people's incentives to work. The additional incentives not only encourage households to intensify their job search, but also to shift into the labor force. More generally, the analysis highlights the fact that modeling unemployment requires thinking carefully about the determinants of the labor force.[9]

The next section lays out our model in the context of CGG. The section after that places the model in a more elaborate DSGE model, suitable to be fit to data. After that, we estimate the model and report our results. The paper ends with some concluding remarks. In those remarks we draw attention to some microeconomic implications of our model. We describe evidence that provides tentative support for the model.

[9] Our argument complements the argument in Krusell, Mukoyama, Rogerson, and Sahin (2009), who also stress the importance of understanding employment, unemployment and the labor force.

2. An Unemployment-based Phillips Curve

To highlight the mechanisms in our model of unemployment, we embed it into the framework with price setting frictions, flexible wages and no capital analyzed in CGG. Our model has heterogeneous agents, since there are some households that are in the labor force and some that are out. Moreover, of those who are in the labor force, some are employed and some are unemployed. Despite this heterogeneity, the model has a representative agent representation. As a result, the linearized equilibrium conditions of the model can be written in the same form as those in CGG. Indeed, relative to a standard macroeconomic data set that includes consumption, employment, inflation and the interest rate, but not unemployment and the labor force, our model and CGG are observationally equivalent.

In our environment, the output gap is proportional to what we call the unemployment gap, the deviation between the actual and efficient rates of unemployment. As a result, the Phillips curve can also be expressed in terms of the unemployment gap. The discuss the implications of the theory developed here for the NAIRU and for the problem of forecasting inflation. Finally, the last subsection below investigates the impact on the welfare cost of business cycles of the assumption that insurance against labor market outcomes is imperfect.

2.1. Families, Households and the Labor Market

The economy is populated by a large number of identical families. The representative family's optimization problem is:

$$\max_{\{C_t, h_t, B_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, h_t\right), \ \beta \in (0,1), \tag{2.1}$$

subject to

$$P_t C_t + B_{t+1} \leq B_t R_{t-1} + W_t h_t + Transfers\ and\ profits_t. \tag{2.2}$$

Here, C_t, h_t denote family consumption and market work, respectively. In addition, B_{t+1} denotes the quantity of a nominal bond purchased by the family in period t. Also, R_t denotes the one-period gross nominal rate of interest on a bond purchased in period t. Finally, W_t denotes the competitively determined nominal wage rate. The family takes W_t as given and makes arrangements to set h_t so that the relevant marginal conditions are satisfied.

The representative family is composed of a large number of ex ante identical households. The households band together into families for the purpose of insuring themselves as best they can against idiosyncratic labor market outcomes. Individual households have no access to credit or insurance markets other than through their arrangements with the family. In part, we view the family construct as a stand-in for the market and non-market arrangements that actual households use to insure against idiosyncratic labor market experiences. In part,

we are following Andolfatto (1996) and Merz (1995), in using the family construct as a technical device to prevent the appearance of difficult-to-model wealth dispersion among households. We emphasize that, although there is no dispersion in household wealth in our model, there is dispersion in consumption.

The family utility function, $u(\cdot,\cdot)$ in (2.1), is the utility attained by the solution to an efficient risk sharing problem subject to incentive constraints, for given values of C_t and h_t. Our simplifying assumptions guarantee that $u(\cdot,\cdot)$ has a simple analytic representation. An important simplifying assumption is that consumption allocations across households within the family are contingent only upon a household's current employment status, and not on its employment history.[10]

The representative family is composed of a unit measure of households. We follow Hansen (1985) and Rogerson (1988) in supposing that household employment is indivisible. A household can either supply one unit of labor, or none at all.[11] This assumption is consistent with the fact that most variation in total hours worked over the business cycle reflects variations in numbers of people employed, rather than in hours per person.

At the start of the period, each household in the family draws a privately observed idiosyncratic shock, l, from a uniform distribution with support, $[0,1]$.[12] The random variable, l, determines the household's utility cost of working:

$$F + \varsigma_t (1+\sigma_L) l^{\sigma_L}. \tag{2.3}$$

The parameters, $\varsigma_t, \sigma_L \geq 0$ and F are common to all households. The object, ς_t, is potentially stochastic. We include it in the analysis in order to document the observations we made about what happens when the NAIRU is stochastic. After drawing l, a household decides whether or not to participate in the labor market. A household that chooses to participate must choose a privately observed job search effort, e. The larger is e, the greater is the household's chance of finding a job.

Consider a household which has drawn an idiosyncratic work aversion shock, l, and

[10] The analysis of Atkeson and Lucas (1995) and Hopenhayn and Nicolini (1997) suggests that ex ante utility would be greater if consumption allocations could be made contingent on past labor market outcomes.

[11] The indivisible labor assumption has attracted substantial attention recently. See, for example, Mulligan (2001), and Krusell, Mukoyama, Rogerson, and Sahin (2008, 2009).

[12] A recent paper which emphasizes a richer pattern of idiocyncracies at the individual firm and household level is Brown, Merkl and Snower (2009).

chooses to participate in the labor market. This household has utility given by:[13]

$$p\left(e_t\right) \overbrace{\left[\log\left(c_t^w\right) - F - \varsigma_t\left(1+\sigma_L\right) l^{\sigma_L} - \frac{1}{2}e_t^2\right]}^{\text{ex post utility of household that joins labor force and finds a job}} \tag{2.4}$$

$$+\left(1-p\left(e_t\right)\right) \overbrace{\left[\log\left(c_t^{nw}\right) - \frac{1}{2}e_t^2\right]}^{\text{ex post utility of household that joins labor force and fails to find a job}} .$$

Here, c_t^w and c_t^{nw} denote the consumption of employed and non-employed households, respectively. An individual household's consumption can only be dependent on its employment status and labor type because these are the only household characteristics that are publicly observed. In (2.4), $p\left(e_t\right)$ denotes the probability that a household which participates in the labor market and exerts effort, e_t, finds a job. This probability is the following linear function of $e_t \geq 0$:

$$p\left(e_t\right) = \eta + ae_t, \ \ \eta, a \geq 0. \tag{2.5}$$

The only admissible model parameterizations are those that imply $0 \leq p\left(e_t\right) \leq 1$ in equilibrium.[14] The object, $e_t^2/2$ is the utility cost associated with effort. In (2.4) we have structured the utility cost of employment so that σ_L affects its variance in the cross section and not its mean.[15]

A household which participates in the labor force and has idiosyncratic work aversion, l, selects search effort $e_{l,t} \geq 0$ to maximize (2.4). This leads to the following necessary and sufficient condition:

$$e_{l,t} = \max\left\{a\left(\log\left[\frac{c_t^w}{c_t^{nw}}\right] - F - \varsigma_t\left(1+\sigma_L\right) l^{\sigma_L}\right), 0\right\}.$$

The corresponding probability of finding a job is:

$$p\left(e_{l,t}\right) = \eta + a^2 \max\left\{\log\left[\frac{c_t^w}{c_t^{nw}}\right] - F - \varsigma_t\left(1+\sigma_L\right) l^{\sigma_L}, 0\right\}. \tag{2.6}$$

Collect the terms in $p\left(e_t\right)$ in (2.4) and then substitute out for $p\left(e_t\right)$ using $p\left(e_{l,t}\right)$ in (2.6). We then find that the utility of a household that draws work aversion index, l, and chooses

[13] The utility function of the household is assumed to be additively separable, as is the case in most of the DSGE literature. In the appendix, we show how to implement the anlaysis when the utility function is non-separable.

[14] The specification of $p\left(e\right)$ in (2.5) allows for probabilities greater than unity. We could alternative specify the probability function to be $min\left\{\eta + ae_t, 1\right\}$. This would complicate some of the notation and the corner would have to be ignored anyway given the solution strategy that we pursue.

[15] To see this, note:

$$\int_0^1 \left(1+\sigma_L\right) l^{\sigma_L} dl = 1, \ \ \int_0^1 \left[\left(1+\sigma_L\right) l^{\sigma_L} - 1\right]^2 dl = \frac{\sigma_L^2}{1+2\sigma_L}.$$

to participate in the labor force is:

$$\left[\eta + a^2 \max\left\{\log\left[\frac{c_t^w}{c_t^{nw}}\right] - F - \varsigma_t\left(1+\sigma_L\right) l^{\sigma_L}, 0\right\}\right] \tag{2.7}$$
$$\times\left[\log\left(\frac{c_t^w}{c_t^{nw}}\right) - F - \varsigma_t\left(1+\sigma_L\right) l^{\sigma_L}\right] + \log\left(c_t^{nw}\right)$$
$$-\frac{1}{2}\left[\max\left\{a\left(\log\left[\frac{c_t^w}{c_t^{nw}}\right] - F - \varsigma_t\left(1+\sigma_L\right) l^{\sigma_L}\right), 0\right\}\right]^2.$$

The utility of household members which do not participate in the labor force is simply:

$$\log\left(c_t^{nw}\right). \tag{2.8}$$

Let m_t denote the smallest value of l for which a household is just indifferent between participating and not participating in the labor force (i.e., (2.7) is equal to (2.8)):

$$\log\left[\frac{c_t^w}{c_t^{nw}}\right] = F + \varsigma_t\left(1+\sigma_L\right) m_t^{\sigma_L}. \tag{2.9}$$

For households with $1 \geq l \geq m_t$, (2.7) is smaller than (2.8). They choose to be out of the labor force. For households with $0 \leq l < m_t$ (2.7) is greater than (2.8), and they strictly prefer to be in the labor force. By setting c_t^w and c_t^{nw} according to (2.9) the family incentivizes the m_t households with the least work aversion to participate in the labor force. Imposing (2.9) on (2.7), we find that the ex ante utility of households which draw $l \leq m_t$ is:

$$\eta\varsigma_t\left(1+\sigma_L\right)\left(m_t^{\sigma_L} - l^{\sigma_L}\right) + \frac{1}{2}a^2\varsigma_t^2\left(1+\sigma_L\right)^2\left(m_t^{\sigma_L} - l^{\sigma_L}\right)^2 + \log\left(c_t^{nw}\right). \tag{2.10}$$

If household members with work aversion index $l \in [0, m_t]$ participate in the labor force, then the number of employed household members, h_t, is:

$$h_t = \int_0^{m_t} p\left(e_{l,t}\right) dl, \tag{2.11}$$

or, after making use of (2.6) and (2.9) and rearranging,

$$h_t = m_t\eta + a^2\varsigma_t\sigma_L m_t^{\sigma_L+1}. \tag{2.12}$$

Note that the right side is equal to zero for $m_t = 0$. In addition, the right side of (2.12) is unbounded above and monotonically increasing in m_t. As a result, for any value of $h_t \geq 0$ there exists a unique value of $m_t \geq 0$ that satisfies (2.12), which we express as follows:

$$m_t = f\left(h_t, \varsigma_t\right), \tag{2.13}$$

where f is monotonically increasing in h_t.

Let $\bar{p}_t$ denote the largest value of $p(e_{t,l})$. Evidently, $\bar{p}_t$ is the probability associated with the household having the least aversion to work, $l = 0$. Setting $l = 0$ in (2.6) and imposing (2.9):

$$\bar{p}_t = \eta + \varsigma_t a^2 (1 + \sigma_L) m_t^{\sigma_L}. \tag{2.14}$$

We require

$$\bar{p}_t \leq 1, \tag{2.15}$$

for all t. We assume that model parameters have been chosen to guarantee this condition holds.

From (2.11) and the fact that $p(e_{l,t})$ is strictly decreasing in l, we see that

$$h_t < m_t \bar{p}_t.$$

It then follows from (2.15) that $h_t < m_t$, so that the unemployment rate, u_t,

$$u_t \equiv \frac{m_t - h_t}{m_t}, \tag{2.16}$$

is strictly positive. We gain insight into the determinants of the unemployment rate in the model, by substituting out h_t in (2.16) using (2.12):

$$u_t = 1 - \eta - a^2 \varsigma_t \sigma_L m_t^{\sigma_L}. \tag{2.17}$$

According to (2.17), a rise in the labor force is associated with a proportionately greater rise in employment, so that the unemployment rate falls. This greater rise in employment reflects that an increase in the labor force requires raising employment incentives, and this simultaneously generates an increase in search intensity. From (2.11) we see that h_t is linear in m_t if search intensity is held constant, but that h_t/m_t increases with m_t if search intensity increases with m_t. That search intensity indeed does increase in m_t can be seen by substituting (2.9) into (2.6). It is important to note that the theory developed here does not imply that the empirical scatter plot of the unemployment rate against the labor force lies rigidly on a negatively sloped line. Equation (2.17) shows that disturbances in ς_t (or in the parameters of the search technology, (2.5)) would make the scatter of u_t versus m_t resemble a shotgun blast rather than a line. A similar observation can be made about the relationship between h_t and m_t in the context of (2.12).

Consider a household with aversion to work, l, which participates in the labor force. For such a household the ex post utility of finding work minus the ex post utility of not finding work is:

$$\Delta(l) = \log\left[\frac{c_t^w}{c_t^{nw}}\right] - F - \varsigma_t (1 + \sigma_L) l^{\sigma_L}.$$

Condition (2.9) guarantees that, with one exception, $\Delta(l) > 0$. That is, among households that participate in the labor force, those that find work are strictly better off than those that do not. The exceptional case is the marginal household with $m = l$, which sets search effort to zero and finds a job with probability η. The ex post utility enjoyed by the marginal household is the same, whether its job search is successful or not.

In addition to the incentive constraint, the allocation of consumption across employed and non-employed households must also satisfy the following resource constraint:

$$h_t c_t^w + (1 - h_t) c_t^{nw} = C_t. \tag{2.18}$$

Here, C_t is the aggregate consumption of the family and h_t is the fraction of households that is employed. Solving (2.18) and (2.9), for c_t^{nw}:

$$c_t^{nw} = \frac{C_t}{h_t \left(e^{F + \varsigma_t (1+\sigma_L) m_t^{\sigma_L}} - 1 \right) + 1}. \tag{2.19}$$

Integrating the utility, (2.10), of the m_t households in the labor force and the utility, (2.8), of the $1 - m_t$ households not in the labor force, we obtain:

$$\int_0^{m_t} \left[\eta \varsigma_t (1 + \sigma_L) (m_t^{\sigma_L} - l^{\sigma_L}) + \frac{1}{2} a^2 \varsigma_t^2 (1 + \sigma_L)^2 (m_t^{\sigma_L} - l^{\sigma_L})^2 \right] dl + \log(c_t^{nw}). \tag{2.20}$$

Evaluating the integral, and making use of (2.13) and (2.19), we obtain

$$u(C_t, h_t,) = \log(C_t) - z(h_t, \varsigma_t), \tag{2.21}$$

where

$$\begin{aligned} z(h_t, \varsigma_t) &= \log \left[h_t \left(e^{F + \varsigma_t (1+\sigma_L) f(h_t, \varsigma_t)^{\sigma_L}} - 1 \right) + 1 \right] \\ &\quad - \frac{a^2 \varsigma_t^2 (1 + \sigma_L) \sigma_L^2}{2\sigma_L + 1} f(h_t, \varsigma_t)^{2\sigma_L + 1} - \eta \varsigma_t \sigma_L f(h_t, \varsigma_t)^{\sigma_L + 1}. \end{aligned} \tag{2.22}$$

In (2.22) the function, f, is defined in (2.13).

We now briefly discuss expression (2.21). First, note that the derivation of the utility function, (2.21), involves no maximization problem by the family. This is because the family incentive and resource constraints, (2.9) and (2.18), are sufficient to determine c_t^w and c_t^{nw} conditional on h_t and C_t. In general, the constraints would not be sufficient to determine the household consumption allocations, and the family problem would involve non-trivial optimization. Second, we can see from (2.21) that our model is likely to be characterized by a particular observational equivalence property. To see this, note that although the agents in our model are in fact heterogeneous, C_t and h_t are chosen as if the economy were populated by a representative agent with the utility function specified in (2.21). A model such as CGG,

which specifies the representative agent utility as the sum of the log of consumption and a constant elasticity disutility of labor will be indistinguishable from our model, as long as data on the labor force and unemployment are not used. This is particularly obvious if, as is the case here, we only study the linearized dynamics of the model about steady state. In this case, the only properties of a model's utility function that are used are its second order derivative properties in nonstochastic steady state. This observational equivalence result reflects our simplifying assumptions. These assumptions are primarily driven by the desire for analytic tractability, so that the economics of the environment are as transparent as possible. Presumably, a careful analysis of microeconomic data would lead to different functional forms and the resulting model would then not be observationally equivalent to the standard model.

Our model and the standard CGG model are distinguished by two features. First, our model addresses a larger set of time series than the standard model does. Second, in our model the representative agent's utility function is a reduced form object. Its properties are determined by details of the technology of job search, and by cross-sectional variation in preferences with regard to attitudes about market work. As a result, the basic structure of the utility function in our model can in principle be informed by time use surveys and studies of job search.[16]

With the representative family's utility function in hand, we are in a position to state the necessary conditions for optimization by the representative family:

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\pi_{t+1}} \tag{2.23}$$

$$C_t z_h(h_t, \varsigma_t) = \frac{W_t}{P_t}. \tag{2.24}$$

Here, π_{t+1} is the gross, realized rate of inflation from t to $t+1$. The expression to the left of the equality in (2.24) is the family's marginal cost in consumption units of providing an extra unit of market employment. This marginal cost takes into account the need for the family to provide appropriate incentives to increase employment. A cost of the incentives, which involves increasing the consumption differential between employed and non-employed households, is that consumption insurance to family members is reduced.

[16] A similar point was made by Benhabib, Rogerson and Wright (1991). They argue that a representative agent utility function of consumption and labor should be interpreted as a reduced form object, after non-market consumption and labor activities have been maximized out. From this perspective, construction of the representative agent's utility function can in principle be guided by surveys of how time in the home is used.

2.2. Goods Production and Price Setting

Production is standard in our model. Accordingly, we suppose that a final good, Y_t, is produced using a continuum of inputs as follows:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{1}{\lambda_f}} di \right]^{\lambda_f}, \ 1 \leq \lambda_f < \infty. \tag{2.25}$$

The good is produced by a competitive, representative firm which takes the price of output, P_t, and the price of inputs, $P_{i,t}$, as given. The first order necessary condition associated with optimization is:

$$\left(\frac{P_t}{P_{i,t}} \right)^{\frac{\lambda_f}{\lambda_f - 1}} Y_t = Y_{i,t}. \tag{2.26}$$

A useful result is obtained by substituting out for Y_{it} in (2.25) from (2.26):

$$P_t = \left[\int_0^1 (P_{i,t})^{\frac{-1}{\lambda_f - 1}} di \right]^{-(\lambda_f - 1)}. \tag{2.27}$$

Each intermediate good is produced by a monopolist using the following production function:

$$Y_{i,t} = A_t h_{i,t},$$

where A_t is an exogenous stochastic process that will be discussed below. The marginal cost of the ith firm is, after dividing by P_t:

$$s_t = (1 - \nu) \frac{W_t}{A_t P_t} = (1 - \nu) \frac{C_t z_h (h_t, \varsigma_t)}{A_t}, \tag{2.28}$$

after using (2.24) to substitute out for W_t / P_t. Here, ν is a subsidy designed to remove the effects, in steady state, of monopoly power. To this end, we set

$$1 - \nu = \frac{1}{\lambda_f}. \tag{2.29}$$

Monopolists are subject to Calvo price frictions. In particular, a fraction ξ_p of intermediate good firms cannot change price:

$$P_{i,t} = P_{i,t-1}, \tag{2.30}$$

and the complementary fraction, $1 - \xi_p$, set their price optimally:

$$P_{i,t} = \tilde{P}_t.$$

The ith monopolist that has the opportunity to reoptimize its price in the current period is only concerned about future histories in which it cannot reoptimize its price. This leads to the following problem:

$$\max_{\tilde{P}_t} E_t \sum_{j=0}^{\infty} \left(\xi_p \beta\right)^j \upsilon_{t+j} \left[\tilde{P}_t Y_{i,t+j} - P_{t+j} s_{t+j} Y_{i,t+j}\right], \tag{2.31}$$

subject to (2.26). In (2.31), υ_t is the multiplier on the representative family's time t flow budget constraint, (2.2), in the Lagrangian representation of its problem. Intermediate good firms take υ_{t+j} as given. The nature of the family's preferences, (2.21), implies:

$$\upsilon_{t+j} = \frac{1}{P_{t+j} C_{t+j}}.$$

2.3. Market Clearing, Aggregate Resources and Equilibrium

Clearing in the loan market requires $B_{t+1} = 0$. Clearing in the market for final goods requires:

$$C_t = Y_t. \tag{2.32}$$

The relationship between aggregate output of the final good, Y_t, and aggregate employment, h_t, is given by (see Tak Yun, 1996):

$$Y_t = p_t^* A_t h_t, \tag{2.33}$$

where

$$p_t^* \equiv \left(\frac{P_t^*}{P_t}\right)^{\frac{\lambda_f}{\lambda_f - 1}}, \; P_t^* = \left[\int_0^1 P_{i,t}^{\frac{\lambda_f}{1-\lambda_f}} di\right]^{\frac{1-\lambda_f}{\lambda_f}}. \tag{2.34}$$

The model is closed once we specify how monetary policy is conducted and time series representations for the shocks. A sequence of markets equilibrium is a stochastic process for prices and quantities which satisfies market clearing and optimality conditions for the agents in the model.

2.4. Log-Linearizing the Private Sector Equilibrium Conditions

It is convenient to express the equilibrium conditions in linearized form relative to the 'efficient' equilibrium. We define the efficient equilibrium as the one in which $\pi_t = 1$ for all t, monopoly power does not distort the level of employment, and there are no price frictions. We refer to the equilibrium in our market economy with sticky prices as simply the 'equilibrium', or the 'actual equilibrium' when clarity requires special emphasis.

2.4.1. The Efficient Equilibrium

In the efficient equilibrium, the marginal cost of labor and the marginal product of labor are equated:

$$C_t z_h\left(h_t, \varsigma_t\right) = A_t.$$

The resource constraint in the efficient equilibrium is $C_t = A_t h_t$, which, when substituted into the previous expression implies:

$$h_t^* z_h\left(h_t^*, \varsigma_t\right) = 1, \tag{2.35}$$

where the '$*$' indicates an endogenous variable in the efficient equilibrium. Evidently, the efficient level of employment, h_t^*, fluctuates only in response to disturbances in ς_t.[17] The level of work in the nonstochastic steady state of the efficient equilibrium coincides with the level of work in the nonstochastic steady state of the actual equilibrium. This object is denoted by h in both cases. Because of the specification of our monetary policy rule (see below) the values of all variables in nonstochastic steady state coincide across actual and efficient equilibria.

Linearizing (2.35) about steady state,

$$\hat{h}_t^* = -\frac{\sigma_\varsigma}{1+\sigma_z}\hat{\varsigma}_t, \tag{2.36}$$

where

$$\sigma_z \equiv \frac{z_{hh} h}{z_h},\ \sigma_\varsigma \equiv \frac{z_{h\varsigma}\varsigma}{z_h}. \tag{2.37}$$

Here, z_{ij} denotes the cross derivative of z with respect to i and j $(i, j = h, \zeta)$, evaluated in steady state and z_h denotes the derivative of z with respect to h, evaluated in steady state. We follow the standard convention in that a hat over a variable denotes percent deviation from its steady state value.

The object, σ_z, is a measure of the curvature of the function, z, in the neighborhood of steady state. Also, $1/\sigma_z$ is a consumption-compensated elasticity of labor supply in the steady state of both the efficient and actual equilibria. It is related to the notion of a Frisch elasticity, except that fluctuation in an individual household's labor occurs exclusively on the extensive margin.

The efficient rate of interest, R_t^*, is derived from (2.23) with consumption and the inflation rate set at their efficient rates:

$$R_t^* = \frac{1}{\beta E_t\left(\frac{h_t^*}{g_{A,t+1} h_{t+1}^*}\right)}$$

[17] This result reflects the balanced growth property of the model, so that income and substitution effects associated with a movement in A_t cancel.

where we have used that consumption in the efficient equilibrium is $A_t h_t^*$ and

$$g_{A,t} \equiv \frac{A_t}{A_{t-1}}.$$

Linearizing the efficient rate of interest expression about steady state, we obtain:

$$\begin{aligned} \hat{R}_t^* &= E_t \hat{g}_{A,t+1} + E_t \hat{h}_{t+1}^* - \hat{h}_t^* \\ &= E_t \hat{g}_{A,t+1} - \frac{\sigma_\varsigma}{1+\sigma_z} \left(E_t \hat{\varsigma}_{t+1} - \hat{\varsigma}_t \right), \end{aligned} \tag{2.38}$$

using (2.36). That $\hat{R}_t^*$ jumps in response to a rise in expected $\hat{g}_{A,t+1}$ reflects that a higher $\hat{g}_{A,t+1}$ signals a higher level of future consumption in the efficient equilibrium. Given the desire to smooth consumption, this encourages families to borrow. Since there is no family that wishes to lend, clearing in the market for loans requires a higher rate of interest. Similar logic explains why the efficient rate of interest drops in response to a rise in $E_t \hat{\varsigma}_{t+1}$.

2.4.2. The Actual Equilibrium

We turn now to the linearized equilibrium conditions in the actual equilibrium. The monetary policy rule (displayed below) ensures that inflation and, hence, price dispersion, is zero in the steady state. Yun (1996) showed that under these circumstances, p_t^* in (2.33) is unity to first order. Linearizing (2.32) and (2.33), and taking into account Yun (1996)'s result,

$$\hat{C}_t = \hat{A}_t + \hat{h}_t. \tag{2.39}$$

Linearizing (2.28) about the non-stochastic steady state equilibrium and using (2.39), we obtain:

$$\hat{s}_t = (1+\sigma_z)\, \hat{h}_t + \sigma_\varsigma \hat{\varsigma}_t.$$

Subtracting (2.36) from this, we obtain:

$$\hat{s}_t = (1+\sigma_z)\, \hat{x}_t, \tag{2.40}$$

where $\hat{x}_t$ denotes the 'output gap', the percent deviation of actual output from its value in the efficient equilibrium:

$$\hat{x}_t \equiv \hat{h}_t - \hat{h}_t^*. \tag{2.41}$$

Condition (2.27), together with the necessary conditions associated with (2.31) leads (after linearization about a zero inflation steady state) to:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{\left(1-\beta\xi_p\right)\left(1-\xi_p\right)}{\xi_p} (1+\sigma_z)\, \hat{x}_t. \tag{2.42}$$

The derivation of (2.42) is standard, but is included in appendix A for completeness.

Log linearizing the household's intertemporal Euler equation after replacing C_t by $A_t h_t$:

$$\hat{h}_t = E_t \left[\hat{g}_{A,t+1} + \hat{h}_{t+1} - \left(\hat{R}_t - \hat{\pi}_{t+1} \right) \right],$$

or, substituting out for $\hat{g}_{A,t+1}$ from (2.38):

$$\hat{x}_t = E_t \hat{x}_{t+1} - \left(\hat{R}_t - \hat{\pi}_{t+1} - \hat{R}_t^* \right). \tag{2.43}$$

Expression (2.43) is the standard representation of the 'New Keynesian IS' curve, expressed in terms of the output gap, $\hat{x}_t$, and the efficient rate of interest, $\hat{R}_t^*$.

The equations that summarize the linearized private sector equilibrium conditions are (2.38), (2.42) and (2.43). Note that the reduced form parameters of these equations are σ_z, $\sigma_\varsigma \hat{\varsigma}_t$, ξ_p, and β. Consistent with the earlier discussion after (2.21), the only way that the parameters underlying the period utility function of the representative agent impact the reduced form of the model is through σ_z and $\sigma_\varsigma \hat{\varsigma}_t$.

The model is closed with the assumption that monetary policy follows a Taylor rule of the following form:

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) \left[r_\pi \hat{\pi}_t + r_y \hat{x}_t \right] + \varepsilon_t, \tag{2.44}$$

where ε_t is a monetary policy shock. The equilibrium conditions of the log-linearized system are (2.38), (2.42), (2.43) and (2.44). These equations are indistinguishable from the equilibrium conditions in standard version of the CGG model. They determine the equilibrium stochastic processes for $\hat{R}_t^*$, $\hat{R}_t$, $\hat{\pi}_t$ and $\hat{x}_t$ as a function of the exogenous stochastic process, $\hat{g}_{A,t}$, $\hat{\varsigma}_t$ and ε_t. The first two stochastic processes enter the system via the efficient rate of interest as indicated in (2.38) and the monetary policy shock enters via (2.44). The variables, $\hat{h}_t$ and $\hat{h}_t^*$ can be solved using (2.36) and (2.41). Relative to time series on the following 6 variables, $\hat{R}_t^*$, $\hat{R}_t$, $\hat{\pi}_t$, $\hat{x}_t$, $\hat{h}_t$, and $\hat{h}_t^*$, our model and the standard CGG model are observationally equivalent.

2.5. The NAIRU

We can solve for the labor force and unemployment from (2.16) and (2.12). Linearizing (2.12) about steady state, we obtain

$$\hat{m}_t = \frac{1-u}{1-u+a^2 \varsigma \sigma_L^2 m^{\sigma_L}} \hat{h}_t - \delta_\varsigma \hat{\varsigma}_t, \tag{2.45}$$

where[18]

$$\delta_\varsigma \equiv \frac{\eta}{1-u+a^2 \varsigma \sigma_L^2 m^{\sigma_L}} > 0. \tag{2.46}$$

[18] To see this, note from (2.12):

$$\begin{aligned} h\hat{h}_t &= \eta m \hat{m}_t + a^2 \varsigma \sigma_L m^{\sigma_L + 1} \left[(\sigma_L + 1) \hat{m}_t + \hat{\varsigma}_t \right] \\ &= \left[\eta m + (h - m\eta)(\sigma_L + 1) \right] \hat{m}_t + (h - m\eta) \hat{\varsigma}_t. \end{aligned}$$

Linearizing (2.17):

$$du_t = -a^2 \varsigma \sigma_L m^{\sigma_L} \left[\sigma_L \hat{m}_t + \hat{\varsigma}_t \right],$$

where

$$du_t \equiv u_t - u,$$

and u_t is a small deviation from steady state unemployment, u. Substituting from (2.45),

$$u_t = u - \kappa^{okun} \hat{h}_t - a^2 \varsigma \sigma_L m^{\sigma_L} \left(1 - \sigma_L \delta_\varsigma\right) \hat{\varsigma}_t, \tag{2.47}$$

where

$$\kappa^{okun} = \frac{a^2 \varsigma \sigma_L^2 m^{\sigma_L} (1-u)}{1 - u + a^2 \varsigma \sigma_L^2 m^{\sigma_L}} > 0.$$

The analogous equation holds in the efficient equilibrium, with $\hat{h}_t$ replaced by $\hat{h}_t^*$:

$$u_t^* = u - \kappa^{okun} \hat{h}_t^* - a^2 \varsigma \sigma_L m^{\sigma_L} \left(1 - \sigma_L \delta_\varsigma\right) \hat{\varsigma}_t. \tag{2.48}$$

Here, the notation reflects that the steady states in the actual and efficient equilibria coincide. In (2.48), u_t^* denotes the model's 'efficient rate of unemployment'. The coefficients on $\hat{\varsigma}_t$ in (2.47) and (2.48) are positive, because $\sigma_L \delta_\varsigma < 1$.[19]

Let u_t^g denote the 'unemployment gap', $u_t - u_t^*$. Subtracting (2.48) from (2.47), we obtain:

$$u_t^g = -\kappa^{okun} \hat{x}_t. \tag{2.49}$$

Note that the unemployment gap is the level deviation of the unemployment rate in the actual equilibrium from the efficient rate. The notation is chosen to emphasize that (2.49) represents the model's implication for Okun's law. In particular, a one percentage point rise in the unemployment rate above the efficient rate is associated with a $1/\kappa^{okun}$ percent fall in output relative to its efficient level. The general view is that $1/\kappa^{okun}$ is somewhere in the range, 2 to 3.

The model can be rewritten in terms of the unemployment gap instead of the output gap. Substituting (2.49) into (2.42), (2.43) and (2.44), respectively, we obtain:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} - \kappa u_t^g \tag{2.50}$$

$$u_t^g = \kappa^{okun} E_t u_{t+1}^g + \kappa^{okun} \left(\hat{R}_t - \hat{\pi}_{t+1} - \hat{R}_t^* \right) \tag{2.51}$$

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) \left[r_\pi \hat{\pi}_t - \frac{r_y}{\kappa^{okun}} u_t^g \right] + \varepsilon_t \tag{2.52}$$

Then, divide by h and rearrange using the identity, $u = 1 - h/m$. Finally, replace $1 - \eta - u$ in this expression with $a^2 \varsigma \sigma_L m^{\sigma_L}$ using the steady state version of (2.17) in the text.

[19] To see this, note

$$\frac{\eta \sigma_L}{1 - u + a^2 \varsigma \sigma_L^2 m^{\sigma_L}} = \frac{\left(1 - u - a^2 \varsigma_t \sigma_L m_t^{\sigma_L}\right) \sigma_L}{1 - u + a^2 \varsigma \sigma_L^2 m^{\sigma_L}} < 1$$

where

$$\kappa \equiv \frac{\left(1-\beta\xi_p\right)\left(1-\xi_p\right)}{\xi_p}\frac{1+\sigma_z}{\kappa^{okun}}$$

This is the expression Stock and Watson (1999) refer to as the unemployment rate Phillips curve.

We can relate the theory derived here to the idea of a non-accelerating inflation rate of unemployment (NAIRU). One interpretation of the NAIRU focuses on the first difference of inflation. Under this interpretation, the NAIRU is a level of unemployment such that whenever the actual unemployment rate lies below it, inflation is predicted to accelerate and whenever the actual unemployment rate is above it, inflation is predicted to decelerate. The efficient level of unemployment, u_t^*, does not in general satisfy this definition of the NAIRU. From (2.50) it is evident that a negative value of u_t^g does not predict an acceleration of inflation in the sense of predicting a positive value for

$$\beta E_t \hat{\pi}_{t+1} - \hat{\pi}_t. \tag{2.53}$$

On the contrary, according to the unemployment rate Phillips curve, (2.50), a negative value of u_t^g creates an anticipated *deceleration* in inflation.[20] Testing this implication of the data empirically is difficult, because u_t^* is not an observed variable. However, some insight can be gained if one places upper and lower bounds on u_t^*. For example, suppose $u_t^* \in (4, 8)$. That is, the efficient unemployment rate in the postwar US was never below 4 percent or above 8 percent. In the 593 months between February 1960 and July 2009, the unemployment rate was below 4 percent in 52 months and above 8 percent in 42 months. Of the months in which unemployment was above its upper threshold, the change in inflation from that month to three months later was positive 79 percent of the time. Of the months in which unemployment was below the 4 percent lower threshold, the corresponding change in inflation was negative 67 percent of the time. If one accepts our assumption about the bounds on u_t^*, these results lend empirical support to the proposition that there exists a NAIRU in the first difference sense. They also represent evidence against the model developed here.[21]

[20] In their discussion of the NAIRU, Ball and Mankiw (2002) implicitly reject (2.50) as a foundation for the notion that u_t^* is a NAIRU. Their discussion begins under a slightly different version of (2.50), with $\beta E_t \hat{\pi}_{t+1}$ replaced by $E_{t-1}\hat{\pi}_t$. They take the position that u_t^* in this framework is a NAIRU only when monetary policy generates the random walk outcome, $E_{t-1}\hat{\pi}_t = \hat{\pi}_{t-1}$. In this case, a negative value of u_t^g is associated with a deceleration of current inflation relative to what it was in the previous period. Ball and Mankiw argue that the random walk case is actually the relevant one for the US in recent decades.

[21] The bounds test of the model just discussed is proposed in Stiglitz (1997). It was implemented as follows. Monthly observations on the unemployment and the consumer price index were taken from the Federal Reserve Bank of St. Louis' online data base, FRED. We worked with the raw unemployment rate. The consumer price index was logged, and we computed a year-over-year rate of inflation rate, π_t. The percentages reported in the text represent the fraction of times that $u_t < 4$ and $\pi_{t+3} - \pi_t < 0$, and the fraction of times that $u_t > 8$ and $\pi_{t+3} - \pi_t > 0$.

An alternative interpretation of the NAIRU focuses on the level of inflation, rather than its change. Under this interpretation, u_t^* in the theory developed here is a NAIRU.[22] To see this, one must take into account that the theory (sensibly) implies inflation returns to steady state after a shock that causes u_t^g to drop has disappeared. That is, the eventual effect on inflation of a negative shock to u_t^g must be zero. That a negative shock to u_t^g also creates the expectation of a deceleration in inflation then implies that inflation converges back to steady state from above after a negative shock to u_t^g. Thus, a shock that drives u_t below u_t^* is expected to be followed by a higher level of inflation and a shock that drives u_t above u_t^* is expected to be followed by a lower level of inflation.[23]

Thus, u_t^* in the theory derived here is a NAIRU if one adopts the level interpretation of the NAIRU and not if one adopts the first difference interpretation. Interestingly, the object, u_t^*, is a NAIRU under the first difference interpretation if one adopts the price indexation scheme proposed in CEE, in which (2.30) is replaced by

$$P_{i,t} = \pi_{t-1} P_{i,t-1}.$$

In this case, $\hat{\pi}_t$ and $\hat{\pi}_{t+1}$ in (2.50) are replaced by their first differences. Retracing the logic of the previous two paragraphs establishes that with price indexation, u_t^* is a NAIRU in the first difference sense. Under our assumptions about the bounds on u_t^*, price indexation also improves the empirical performance of the model on the dimensions emphasized here.

It is instructive to consider the implications of the theory for the regression of the period $t+1$ inflation rate on the period t unemployment and inflation rates. If we assume that u_t^* is constant (in our example, this means ς_t is a constant) then the regression coefficient on u_t would be κ. In addition, the theory implies that if other variables beside unemployment are added to the regression, then they will not be significant. However, these predictions depend crucially on the assumption that u_t^* is constant. If it is stochastic, then u_t^* is part of

[22] In his discussion of the NAIRU, Stiglitz (1997) appears to be open to either the first difference or level interpretation of the NAIRU.

[23] A quick way to formally verify the convergence properties just described is to consider the following example. Suppose the monetary policy shock, ε_t, is an iid stochastic process. Let the response of the endogenous variables to ε_t be given by

$$u_t^g = u_\varepsilon \varepsilon_t,\ \hat{R}_t = R_\varepsilon \varepsilon_t,\ \hat{\pi}_t = \pi_\varepsilon \varepsilon_t,$$

where u_ε, R_ε and π_ε are undetermined coefficients to be solved for. Substituting these into the equations that characterize equilibrium and imposing that the equations must be satisfied for every realization of ε_t, we find:

$$u_\varepsilon = \frac{\kappa^{okun}}{1 + \kappa^{okun}\kappa r_\pi + r_y},\ \pi_\varepsilon = -\kappa u_\varepsilon,\ R_\varepsilon = \frac{1}{\kappa^{okun}} u_\varepsilon.$$

According to these expressions, a monetary policy shock drives u_t^g and R_t in the same direction. Thus, a monetary policy shock that drives the interest rate down also drives the unemployment gap down. The same shock drives current inflation up.

the error term. Since u_t^* is expected to be correlated with all other variables in the model, then adding these variables to the forecast equation should improve fit.

2.6. Welfare Cost of Business Cycles

There is a widespread view that standard assessments understate the welfare cost of business cycles because they do not take into account that insurance against unemployment is incomplete. We can evaluate that view in our model. Let

$$W\left(g_{A,t}, p_{t-1}^*, \varsigma_t, \sigma\right)$$

denote the welfare of the representative agent in our model, conditional on the period t state, $g_{A,t}$, p_{t-1}^*, ς_t, σ. Here, $g_{A,t}$ denotes the period t realization of the growth rate of technology, where

$$\begin{aligned} g_{A,t} &\equiv \frac{A_t}{A_{t-1}} \\ \log g_{A,t} &= (1-\rho_A)\log g_A + \rho_A \log g_{A,t-1} + \varepsilon_t^A, \end{aligned}$$

and g_A denotes the steady state value of $g_{A,t}$ when the innovation in $\log g_{A,t}$, ε_t^A, is zero. In addition, the previous period's measure of price distortions is p^* and the current realization of the aggregate utility shock is ς. The parameter, σ, is a scalar that multiplies the 2×1 vector composed of the innovations to g_A and ς. Thus, $\sigma = 0$ corresponds to the nonstochastic version of the model and $\sigma = 1$ corresponds to the stochastic version. We think of σ as a continuous variable, $0 \leq \sigma \leq 1$. In the numerical examples considered, we allow only the technology shock to be stochastic.

We define the welfare cost of business cycles as

$$\Delta = W\left(g_{A,t}, p_{t-1}^*, \varsigma_t, 0\right) - W\left(g_{A,t}, p_{t-1}^*, \varsigma_t, 1\right).$$

Let Δ corresponding to the CGG model be denoted Δ^{CGG} and let Δ corresponding to our model with involuntary unemployment be denoted Δ^u. We measure the impact on the welfare cost of business cycles of the assumption of imperfect insurance markets by $\Delta^{CGG} - \Delta^u$. Of course, this impact depends on how the models are parameterized. We do so as follows. We adopt a baseline parameterization for the CGG model. We then consider the parameterization of the model with involuntary unemployment that is observationally equivalent when data on unemployment and the labor force are not included.

We computed Δ^{CGG} and Δ^u by computing the second order approximation of W about $g_{A,t} = g_A$, $p_{t-1}^* = 1$ and $\sigma = 0$. Doing so, we found that

$$\Delta^{CGG} - \Delta^u = 0$$

so all available significant digits. We presume this must reflect the existence of a theorem that implies the above measure is mathematically zero, though we have not yet found the proof. The actual experiments are reported in the technical appendix to this paper.

3. Integrating Unemployment into a Standard DSGE Model with Capital

The following five subsections describe how we insert our model of unemployment into a version of CEE or Smets and Wouters (2003, 2007). The last section describes our representation of the 'standard DSGE model'. In our description of the standard model we incorporate Gali (2009)'s insight that that model can be interpreted as alrealy incorportating a model of unemployment. As Gali showed, incorporating this interpretation requires no change to the equilibrium conditions of the standard DSGE model. It simply requires the addition of an extra equation to define the labor force and the unemployment rate.

3.1. Final and Intermediate Goods

A final good is produced by competitive firms using (2.25). The ith intermediate good is produced by a monopolist with the following production function:

$$Y_{i,t} = (z_t H_{i,t})^{1-\alpha} K_{i,t}^{\alpha} - z_t^{+}\phi, \tag{3.1}$$

where $K_{i,t}$ denotes capital services used for production by the ith intermediate good producer. Also, $\log(z_t)$ is a technology shock whose first difference has a positive mean and ϕ denotes a fixed production cost. The economy has two sources of growth: the positive drift in $\log(z_t)$ and a positive drift in $\log(\Psi_t)$, where Ψ_t is the state of an investment-specific technology shock discussed below. The object, z_t^{+}, in (3.1) is defined as follows:

$$z_t^{+} = \Psi_t^{\frac{\alpha}{1-\alpha}} z_t.$$

Along a non-stochastic steady state growth path, Y_t/z_t^{+} and $Y_{i,t}/z_t^{+}$ converge to constants. The two shocks, z_t and Ψ_t, are specified to be unit root processes in order to be consistent with the assumptions we use in our VAR analysis to identify the dynamic response of the economy to neutral and capital-embodied technology shocks. The two shocks have the following time series representations:

$$\Delta \log z_t = \mu_z + \rho_n \Delta \log z_{t-1} + \varepsilon_t^n,\ E(\varepsilon_t^n)^2 = (\sigma_n)^2 \tag{3.2}$$

$$\Delta \log \Psi_t = \mu_\Psi + \rho_\Psi \Delta \log \Psi_{t-1} + \varepsilon_t^\Psi,\ E\left(\varepsilon_t^\Psi\right)^2 = (\sigma_\Psi)^2. \tag{3.3}$$

In (3.1), $H_{i,t}$ denotes homogeneous labor services hired by the ith intermediate good producer. Intermediate good firms must borrow a fraction of the wage bill, so that one unit

of labor costs is given by

$$W_t R_t^f,$$

where

$$R_t^f = \nu^f R_t + 1 - \nu^f. \tag{3.4}$$

Here, W_t denotes the aggregate wage rate, R_t denotes the gross nominal interest rate on working capital loans, and ν^f denotes the fraction of the wage bill that must be financed in advance.

Intermediate good firms are subject to Calvo price-setting frictions. With probability ξ_p the intermediate good firm cannot reoptimize its price, in which case it is assumed to set its price according to the following rule:

$$P_{i,t} = \tilde{\pi}_{f,t} P_{i,t-1},\ \tilde{\pi}_{f,t} \equiv (\pi_{t-1})^{\kappa_f} (\bar{\pi})^{1-\kappa_f}, \tag{3.5}$$

where $\kappa_f \in (0,1)$ is a parameter, π_{t-1} is lagged (gross) inflation rate and $\bar{\pi}$ is the steady state inflation rate. With probability $1-\xi_p$ the intermediate good firm can reoptimize its price. Apart from the fixed cost, the ith intermediate good producer's profits are:

$$E_t \sum_{j=0}^{\infty} \beta^j \upsilon_{t+j} \{P_{i,t+j} Y_{i,t+j} - s_{t+j} P_{t+j} Y_{i,t+j}\},$$

where s_t denotes the marginal cost of production, denominated in units of the homogeneous good. The object, s_t, is a function only of the costs of capital and labor, and is described in Appendix C. In the firm's discounted profits, $\beta^j \upsilon_{t+j}$ is the multiplier on the household's nominal period $t+j$ budget constraint. The equilibrium conditions associated with this optimization problem are reported in Appendix C.

We suppose that the homogeneous labor hired by intermediate good producers is itself 'produced' by competitive labor contractors. Labor contractors produce homogeneous labor by aggregating different types of specialized labor, $j \in (0,1)$, as follows:

$$H_t = \left[\int_0^1 (h_{t,j})^{\frac{1}{\lambda_w}} dj \right]^{\lambda_w},\ 1 \leq \lambda_w < \infty. \tag{3.6}$$

Labor contractors take the wage rate of H_t and $h_{t,j}$ as given and equal to W_t and $W_{t,j}$, respectively. Profit maximization by labor contractors leads to the following first order necessary condition:

$$W_{j,t} = W_t \left(\frac{H_t}{h_{t,j}} \right)^{\frac{\lambda_w - 1}{\lambda_w}}. \tag{3.7}$$

Equation (3.7) is the demand curve for the jth type of labor.

3.2. Family and Household Preferences

We integrate the model of unemployment in the previous section into the EHL model of sticky wages used in the standard DSGE model. Each type, $j \in [0,1]$, of labor is assumed to be supplied by a particular family of households. The jth family resembles the single representative family in the previous section, with one exception. The exception is that the unit measure of households in the jth family is only able to supply the jth type of labor service. Each household in the jth family has the utility cost of working, (2.3), and the technology for job search, (2.5). The five parameters of these functions are

$$F, \varsigma_t, \sigma_L, a, \eta,$$

where the first three pertain to the cost of working and the last two pertain to job search. In the analysis of the empirical model, the preference shock, ς_t, is constant. We assume that these parameters (including the stochastic process, ς_t) are identical across families. In order that the representative family in the current section have habit persistence in consumption, we change the way consumption enters the additive utility function of the household. In particular, we replace $\log(c_t^{nw})$ and $\log(c_t^{w})$ everywhere in the previous section with

$$\log\left(c_{j,t}^{nw} - bC_{t-1}\right), \ \ \log\left(c_{j,t}^{w} - bC_{t-1}\right),$$

respectively. Here, C_{t-1} denotes the family's previous period's level of consumption. When the parameter, b, is positive, then each household in the family has habit in consumption. Also, $c_{j,t}^{nw}$ and $c_{j,t}^{w}$ denote the consumption levels allocated by the jth family to non-employed and employed households within the family. Although families all enjoy the same level of consumption, C_t, for reasons described momentarily each family experiences a different level of employment, $h_{j,t}$. Because employment across families is different, each type j family chooses a different way to balance the trade-off between consumption insurance and the need to provide work incentives. The jth type of family with high $h_{j,t}$ provides a high level of consumption, $c_{j,t}^{w}$, in relation to level of consumption, $c_{j,t}^{nw}$, provided to the non-employed. It is easy to verify that the incentive constraint in the version of the model considered here is the analog of (2.9):

$$\log\left[\frac{c_{j,t}^{w} - bC_{t-1}}{c_{j,t}^{nw} - bC_{t-1}}\right] = F + \varsigma\left(1+\sigma_L\right) m_{j,t}^{\sigma_L},$$

where $m_{j,t}$ solves the analog of (2.12):

$$h_{j,t} = m_{j,t}\eta + a^2\varsigma\sigma_L m_{j,t}^{\sigma_L+1}. \tag{3.8}$$

Consider the jth family that enjoys a level of family consumption and employment, C_t and $h_{j,t}$, respectively. It is readily verified that the utility of this family, after it efficiently allocates

consumption across its member households subject to the private information constraints, is given by:

$$u\left(C_t - bC_{t-1}, h_{j,t}\right) = \log\left(C_t - bC_{t-1}\right) - z\left(h_{j,t}\right). \tag{3.9}$$

The z function in (3.9) is defined in (2.22) with ς_t replaced by ς. The jth family's discounted utility is:

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t - bC_{t-1}, h_{j,t}\right). \tag{3.10}$$

Note that this utility function is additively separable, like the utility functions assumed for the households. Additive separability is convenient because perfect consumption insurance at the level of families implies that consumption is not indexed by labor type, j. As we show later, this simplification appears not to have come at a cost in terms of accounting for aggregate data. Still, it would be interesting to explore the implications of non-separable utility. The appendix derives (3.9) for two non-separable specifications of utility for households. Moreover, Guerron-Quintana (2008) shows how to handle the fact that family consumption is now indexed by j.

3.3. The Family Problem

The jth family is the monopoly supplier of the jth type of labor service. The family understands that when it arranges work incentives for its households so that employment is $h_{j,t}$, then $W_{j,t}$ takes on the value implied by the demand for its type of labor, (3.7). The family therefore faces the standard monopoly problem of selecting $W_{j,t}$ to optimize the welfare, (3.10), of its member households. It does so, subject to the requirement that it satisfy the demand for labor, (3.7), in each period. We follow EHL in supposing that the family experiences Calvo-style frictions in its choice of $W_{j,t}$. In particular, with probability $1 - \xi_w$ the jth family has the opportunity to reoptimize its wage rate. With the complementary probability, the family must set its wage rate according to the following rule:

$$W_{j,t} = \tilde{\pi}_{w,t} W_{j,t-1} \tag{3.11}$$

$$\tilde{\pi}_{w,t} = \left(\pi_{t-1}\right)^{\kappa_w} \left(\bar{\pi}\right)^{(1-\kappa_w)} \mu_{z^+}, \tag{3.12}$$

where $\kappa_w \in (0,1)$. Note that in a non-stochastic steady state, non-optimizing families raise their real wage at the rate of growth of the economy. Because optimizing families also do this in steady state, it follows that in the steady state, the wage of each type of household is the same.

In principle, the presence of wage setting frictions implies that families have idiosyncratic levels of wealth and, hence, consumption. However, we follow EHL in supposing that each family has access to perfect consumption insurance. At the level of the family, there is

no private information about consumption or employment. The private information and associated incentive problems all exist among the households inside a family. Because of the additive separability of the family utility function, perfect consumption insurance implies equal consumption across families. We have used this property of the equilibrium to simplify our notation and not include a subscript, j, on the jth family's consumption.

The jth family's period t budget constraint is as follows:

$$P_t\left(C_t+\frac{1}{\Psi_t}I_t\right)+B_{t+1}\leq W_{t,j}h_{t,j}+X_t^k\bar{K}_t+R_{t-1}B_t+a_{jt}. \tag{3.13}$$

Here, B_{t+1} denotes the quantity of risk-free bonds purchased by the household, R_t denotes the gross nominal interest rate on bonds purchased in period $t-1$ which pay off in period t, and a_{jt} denotes the payments and receipts associated with the insurance on the timing of wage reoptimization. Also, P_t denotes the aggregate price level and I_t denotes the quantity of investment goods purchased for augmenting the beginning-of-period $t+1$ stock of physical capital, $\bar{K}_{t+1}$. The price of investment goods is P_t/Ψ_t, where Ψ_t is a unit root process with positive drift. This is our way of capturing the trend decline in the relative price of investment goods.[24]

The household owns the economy's physical stock of capital, $\bar{K}_t$, sets the utilization rate of capital and rents the services of capital in a competitive market. The household accumulates capital using the following technology:

$$\bar{K}_{t+1}=(1-\delta)\bar{K}_t+\left(1-S\left(\frac{I_t}{I_{t-1}}\right)\right)I_t. \tag{3.14}$$

Here, S is a convex function, with S and S' equal to zero on a steady state growth path. The function, S, is defined in Appendix E.

For each unit of $\bar{K}_{t+1}$ acquired in period t, the household receives X_{t+1}^k in net cash payments in period $t+1$,

$$X_{t+1}^k=u_{t+1}^kP_{t+1}r_{t+1}^k-\frac{P_{t+1}}{\Psi_{t+1}}a(u_{t+1}^k). \tag{3.15}$$

where u_t^k denotes the rate of utilization of capital. The first term is the gross nominal period $t+1$ rental income from a unit of $\bar{K}_{t+1}$. The family supply of capital services in period $t+1$ is:

$$K_{t+1}=u_{t+1}^k\bar{K}_{t+1}.$$

[24] We suppose that there is an underlying technology for converting final goods, Y_t, one-to-one into C_t and one to Ψ_t into investment goods. These technologies are operated by competitive firms which equate price to marginal cost. The marginal cost of C_t with this technology is P_t and the marginal cost of I_t is P_t/Ψ_t. We avoid a full description of this environment to avoid cluttering the presentation, and simply impose these properties of equilibrium on the family budget constraint.

It is the services of capital that intermediate good producers rent and use in their production functions, (3.1). The second term to the right of the equality in (3.14) represents the cost of capital utilization, $a(u_{t+1}^k)P_{t+1}/\Psi_{t+1}$. See Appendix E for the functional form of the capital utilization cost cost function.

The family's problem is to select sequences, $\left\{C_t, I_t, u_t^k, W_{j,t}, B_{t+1}, \bar{K}_{t+1}\right\}$, to maximize (3.10) subject to (3.7), (3.11), (3.12), (3.13), (3.14), (3.15) and the mechanism determining when wages can be reoptimized. The equilibrium conditions associated with this maximization problem are standard, and so appear in the appendix, in section C.

3.4. Aggregate Resource Constraint, Monetary Policy and Equilibrium

Goods market clearing dictates that the homogeneous output good is allocated among alternative uses as follows:

$$Y_t = G_t + C_t + \tilde{I}_t. \tag{3.16}$$

Here, C_t denotes household consumption, G_t denotes exogenous government consumption and $\tilde{I}_t$ is a homogenous investment good which is defined as follows:

$$\tilde{I}_t = \frac{1}{\Psi_t}\left(I_t + a\left(u_t^k\right)\bar{K}_t\right). \tag{3.17}$$

As discussed above, the investment goods, I_t, are used by the families to add to the physical stock of capital, $\bar{K}_t$, according to (3.14). The remaining investment goods are used to cover maintenance costs, $a\left(u_t^k\right)\bar{K}_t$, arising from capital utilization, u_t^k. The cost function, $a(\cdot)$, is increasing and convex, and has the property that in steady state, $u_t^k = 1$ and $a(1) = 0$. Finally, Ψ_t in (3.17) denotes a unit root investment specific technology shock with positive drift.

We suppose that monetary policy follows a Taylor rule of the following form:

$$\log\left(\frac{R_t}{R}\right) = \rho_R \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_R)\left[r_\pi \log\left(\frac{\pi_{t+1}}{\pi}\right) + r_y \log\left(\frac{gdp_t}{gdp}\right)\right] + \frac{\varepsilon_{R,t}}{4R}, \tag{3.18}$$

where gdp_t denotes scaled real GDP defined as:

$$gdp_t = \frac{G_t + C_t + I_t/\Psi_t}{z_t^+}, \tag{3.19}$$

and gdp denotes the nonstochastic steady state value of gdp_t. We adopt the model of government spending suggested in Christiano and Eichenbaum (1992), in which

$$G_t = gz_t^+.$$

In principle, g could be a random variable, though our focus in this paper is just on monetary policy and technology shocks. So, we set g to a constant. Lump-sum transfers are assumed to balance the government budget.

An equilibrium is a stochastic process for the prices and quantities which has the property that the family and firm problems are satisfied, and goods and labor markets clear.

3.5. Aggregate Labor Force and Unemployment in Our Model

We now derive our model's implications for unemployment and the labor market. At the level of the jth family, unemployment and the labor force are defined in the same way as in the previous section, except that the endogenous variables now have a j subscript (the parameters and shocks are the same across families). Thus, the jth family's labor force, $m_{j,t}$, and total employment, $h_{j,t}$, are related by (2.12) (or, (3.8)). We linearize the latter expression as in (2.45):

$$\hat{m}_{j,t} = \frac{1-u}{1-u+a^2\varsigma\sigma_L^2 m^{\sigma_L}}\hat{h}_{j,t}, \tag{3.20}$$

where $\delta_\varsigma > 0$ is defined in (2.46). Also, u and m denote the steady state values of unemployment and the labor force in the jth family. Because we have made assumptions that guarantee each family is identical in steady state, we drop the j subscripts from all steady state labor market variables (see the discussion after (3.11)).

Aggregate household hours and the labor force are defined as follows:

$$h_t \equiv \int_0^1 h_{j,t}dj,\ m_t \equiv \int_0^1 m_{j,t}dj.$$

Totally differentiating,

$$\hat{h}_t = \int_0^1 \hat{h}_{j,t}dj,\ \hat{m}_t \equiv \int_0^1 \hat{m}_{j,t}dj.$$

Using the fact that, to first order, type j wage deviations from the aggregate wage cancel, we obtain:

$$\hat{h}_t = \hat{H}_t. \tag{3.21}$$

See appendix D for a derivation. That is, to a first order approximation, the percent deviation of aggregate household hours from steady state coincides with the percent deviation of aggregate homogeneous hours from steady state. Integrating (3.20) over all j :

$$\hat{m}_t = \int_0^1 \hat{m}_{j,t}dj = \frac{1-u}{1-u+a^2\varsigma\sigma_L^2 m^{\sigma_L}}\hat{H}_t.$$

Aggregate unemployment is defined as follows:

$$u_t \equiv \frac{m_t - h_t}{m_t},$$

so that

$$du_t = \frac{h}{m}\left(\hat{m}_t - \hat{h}_t\right).$$

Here, du_t denotes he deviation of unemployment from its steady state value, not the percent deviation.

3.6. The Standard Model

We follow Gali (2009) in deriving the type j family's utility as the indirect utility function associated with an efficient allocation problem. The indirect utility function coincides exactly with the utility function used in standard DSGE models. The efficient allocation problem is a special case of the one in section 2.1. Gali's approach does not introduce any new free parameters. The equilibrium conditions that determine all model variables apart from unemployment and the labor force are identical to what they are in the standard DSGE model. Gali's insight in effect adds a block recursive system of two equations to the standard DSGE model which determine the size of the labor force and unemployment. Although the model does not satisfy all the criteria for unemployment that we described in the introduction (i.e., conditions (i) and (ii)), it nevertheless provides a natural benchmark for comparison with our model.

As in the previous subsections, we suppose that corresponding to each type j of labor, there is a unit measure of households which gather together into a family. As in the model described above, labor is indivisible: households are either employed or not. At the beginning of each period, each household draws a random variable, l, from a uniform distribution with support, $[0,1]$. The random variable, l, determines a household's aversion to work according to (2.3), with $F = 0$. Here, we suppose that finding a job requires no effort, so that $m_{t,j} = h_{t,j}$, and that a household's aversion to work is publicly observed. In this case, it is efficient for households with $l \leq h_{t,j}$ to work and for households with $h_{t,j} \leq l \leq 1$ to take leisure. With these changes, the type j family allocation problem is to maximize the utility of its member households with respect to $c_{t,j}^{nw}$ and $c_{t,j}^{w}$, subject to (2.18), and the given values of $h_{t,j}$ and C_t. In Lagrangian form, the problem is:

$$u\left(C_t - bC_{t-1}, h_{j,t}\right) = \max_{c_{t,j}^{w}, c_{t,j}^{nw}} \int_0^{h_{t,j}} \left[\log\left(c_{t,j}^{w} - bC_{t-1}\right) - \varsigma\left(1+\sigma_L\right) l^{\sigma_L}\right] dl$$
$$+ \int_{h_{t,j}}^{1} \log\left(c_{t,j}^{nw} - bC_{t-1}\right) dl + \lambda_{j,t}\left[C_t - h_{t,j}c_{t,j}^{w} - \left(1-h_{t,j}\right)c_{t,j}^{nw}\right].$$

Here, $\lambda_{j,t} > 0$ denotes the multiplier on the resource constraint. The first order conditions imply $c_{t,j}^{w} = c_{t,j}^{nw} = C_t$. In this environment there is nothing to stand in the way of perfect consumption insurance for the households in the family. Imposing the first order conditions

and evaluating the integral, we find:

$$u\left(C_t - bC_{t-1}, h_{j,t}\right) = \log\left(C_t - bC_{t-1}\right) - \varsigma h_{t,j}^{1+\sigma_L}. \tag{3.22}$$

The problem of the family is identical to what it is in section 3.3, with the sole exception that the utility function, (3.9), is replaced by (3.22).

We now deduce the implications of the standard model for unemployment and the labor force, following the suggestion of Gali (2009). A type j household that draws work aversion index l is defined to be unemployed if the following two conditions are satisfied:

$$(a)\ l > h_{j,t},\ (b)\ \upsilon_t W_{j,t} > \varsigma l^{\sigma_L}. \tag{3.23}$$

Here, υ_t denotes the multiplier on the budget constraint, (3.13), in the Lagrangian representation of the family optimization problem. Expression (a) in (3.23) simply says that to be unemployed, the household must not be employed. Expression (b) in (3.23) determines whether a non-employed household is unemployed or not in the labor force. The object on the left of the inequality in (b) is the value assigned by the family to the wage, $W_{j,t}$. The object on the right of (b) is the cost of going to work for the lth household. Gali (2009)'s proposal is to define households with l satisfying (3.23) as unemployed.

We use (3.23) to define the labor force, l_t^*, in the standard model. With l_t^* and aggregate employment, h_t, we obtain unemployment as follows

$$u_t = \frac{l_t^* - h_t}{l_t^*},$$

or, after linearization about steady state:

$$du_t = \frac{h}{l^*}\left(\hat{l}_t^* - \hat{h}_t\right).$$

Here, $h < l^*$ because of the presence of monopoly power. The object, $\hat{h}_t$ may be obtained from (3.21) and the solution to the standard model. We now discuss the computation of the aggregate labor force, l_t^*. We have

$$l_t^* \equiv \int_0^1 l_{j,t}^* dj,$$

where $l_{j,t}^*$ is the labor force associated with the jth type of labor and is defined by enforcing (b) in (3.23) at equality. After linearization,

$$\hat{l}_t^* \equiv \int_0^1 \hat{l}_{j,t}^* dj.$$

We compute $\hat{l}_{j,t}^*$ by linearizing the equation that defines $l_{j,t}^*$. After scaling that equation, we obtain

$$\psi_{z^+,t}\bar{w}_t\mathring{w}_{j,t} = \varsigma\left(l^*_{j,t}\right)^{\sigma_L}, \tag{3.24}$$

where

$$\psi_{z^+,t} \equiv \upsilon_t P_t z_t^+,\ \bar{w}_t \equiv \frac{W_t}{z_t^+ P_t},\ \mathring{w}_{j,t} \equiv \frac{W_{j,t}}{W_t}.$$

Linearizing (3.24) about steady state and integrating the result over all $j \in (0,1)$:

$$\hat{\psi}_{z^+,t} + \widehat{\bar{w}}_t + \int_0^1 \widehat{\mathring{w}}_{j,t}dj = \sigma_L \hat{l}^*_t.$$

From the result in appendix D, the integral in the above expression is zero, so that:

$$\hat{l}^*_t = \frac{\hat{\psi}_{z^+,t} + \widehat{\bar{w}}_t}{\sigma_L}.$$

4. Estimation Strategy

We estimate the parameters of the model in the previous section using the impulse response matching approach applied by Rotemberg and Woodford (1997), CEE, ACEL and other papers. We apply the Bayesian version of that method proposed in Christiano, Trabandt and Walentin (2009) (CTW). To promote comparability of results across the two papers and to simplify the discussion here, we use the impulse response functions and associated probability intervals estimated using the 13 variable, 2 lag vector autoregression (VAR) estimated in CTW. Here, we consider the response of 11 variables to three shocks: the monetary policy shock, $\varepsilon_{R,t}$ in equation (3.18), the neutral technology shock, ε_t in equation (3.2), and the investment specific shock, ε_t^{Ψ} in equation (3.3).[25] Nine of the eleven variables whose responses we consider are the standard macroeconomic variables displayed in Figures 2-4 in the appendix. The other two variables are the unemployment rate and the labor force. The VAR is estimated using quarterly, seasonally adjusted data covering the period 1952Q1 to 2008Q4.

The assumptions that allow us to identify the effects of our three shocks are the ones implemented in ACEL. To identify the monetary policy shock we suppose all variables aside from the nominal rate of interest are unaffected contemporaneously by the policy shock. We make two assumptions to identify the dynamic response to the technology shocks: (i) the only shocks that affect labor productivity in the long run are the two technology shocks and (ii) the only shock that affects the price of investment relative to consumption is the innovation to the investment specific shock. All these identification assumptions are satisfied

[25] The VAR in CTW also includes data on vacancies and separations, but these variables do not appear in the models in this paper and so we do not include their impulse responses in the analysis.

in our model. Details of our strategy for computing impulse response functions imposing the shock identification are discussed in ACEL.

Let $\hat{\psi}$ denote the vector of impulse responses used in the analysis here. Since we consider 15 lags in the impulses, there are in principle 3 (i.e., the number of shocks) times 11 (number of variables) times 15 (number of lags) = 495 elements in $\hat{\psi}$. However, we do not include in $\hat{\psi}$ the 10 contemporaneous responses to the monetary policy shock that are required to be zero by our monetary policy identifying assumption. Taking the latter into account, the vector $\hat{\psi}$ has 485 elements. To conduct a Bayesian analysis, we require a likelihood function for our 'data', $\hat{\psi}$. For this, we use an approximation based on asymptotic sampling theory. In particular, when the number of observations, T, is large, we have

$$\sqrt{T}\left(\hat{\psi}-\psi\left(\theta_0\right)\right) \overset{a}{\sim} N\left(0, W\left(\theta_0,\zeta_0\right)\right). \tag{4.1}$$

Here, θ_0 and ζ_0 are the parameters of the model that generated the data, evaluated at their true values. The parameter vector, θ_0, is the set of parameters used explicitly in our analysis while ζ_0 contains the parameters of stochastic processes not included in the analysis. In (4.1), $W\left(\theta_0,\zeta_0\right)$ is the asymptotic sampling variance of $\hat{\psi}$, which of course is a function of all model parameters. We find it convenient to express (4.1) in the following form:

$$\hat{\psi} \overset{a}{\sim} N\left(\psi\left(\theta_0\right), V\left(\theta_0,\zeta_0,T\right)\right), \tag{4.2}$$

where

$$V\left(\theta_0,\zeta_0,T\right) \equiv \frac{W\left(\theta_0,\zeta_0\right)}{T}.$$

We treat $V\left(\theta_0,\zeta_0,T\right)$ as though it were known. In practice, we work with a consistent estimator of $V\left(\theta_0,\zeta_0,T\right)$ in our analysis (for details, see CTW). That estimator is a diagonal matrix with only the variances along the diagonal. An advantage of this diagonality property is that our estimator has a simple graphical representation.

We treat the following object as the likelihood of the data, $\hat{\psi}$, conditional on the model parameters, θ :

$$f\left(\hat{\psi}|\theta, V\left(\theta_0,\zeta_0,T\right)\right) = \left(\frac{1}{2\pi}\right)^{\frac{N}{2}} \left|V\left(\theta_0,\zeta_0,T\right)\right|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}\left(\hat{\psi}-\psi\left(\theta\right)\right)' V\left(\theta_0,\zeta_0,T\right)^{-1}\left(\hat{\psi}-\psi\left(\theta\right)\right)\right].$$

The Bayesian posterior of θ conditional on $\hat{\psi}$ and $V\left(\theta_0,\zeta_0,T\right)$ is:

$$f\left(\theta|\hat{\psi}, V\left(\theta_0,\zeta_0,T\right)\right) = \frac{f\left(\hat{\psi}|\theta, V\left(\theta_0,\zeta_0,T\right)\right) p\left(\theta\right)}{f\left(\hat{\psi}|V\left(\theta_0,\zeta_0,T\right)\right)}, \tag{4.3}$$

where $p\left(\theta\right)$ denotes the priors on θ and $f\left(\hat{\psi}|V\left(\theta_0,\zeta_0,T\right)\right)$ denotes the marginal density of $\hat{\psi}$:

$$f\left(\hat{\psi}|V\left(\theta_0,\zeta_0,T\right)\right) = \int f\left(\hat{\psi}|\theta, V\left(\theta_0,\zeta_0,T\right)\right) p\left(\theta\right) d\theta.$$

As usual, the mode of the posterior distribution of θ can be computed by simply maximizing the value of the numerator in (4.3), since the denominator is not a function of θ. The marginal density of $\hat{\psi}$ is required when we want an overall measure of the fit of our model and when we want to report the shape of the posterior marginal distribution of individual elements in θ. This can be done using the MCMC algorithm, or by adopting a Laplace approximation. In this paper we do the latter. For details, see CTW.

5. Results

5.1. Parameters

Parameters whose values are set a priori are listed in Table 1. We found that when we included the parameters, κ_w and λ_w, among those to be estimated, the estimator drove them to their boundaries. This is why we then set them a priori to values near the boundary. The setting for λ_w implies a high level of competition among the different labor inputs and this in turn serves to dampen the response of wages to shocks. Having this parameter close to unity also serves to emphasize the fact that the theory of unemployment here has nothing to do with the presence of monopoly power. Setting κ_w near its boundary also serves to reduce the volatility of the wage. The steady state value of inflation (a parameter in the monetary policy rule), the steady state government consumption to output ratio, and the growth rates of neutral and investment-specific technology were chosen to coincide with their corresponding sample means in our data set.

The parameters whose values are estimated are listed in Table 2, which reports the associated priors and posteriors. We report the support of the prior distribution, the mean and the 95 percent probability interval. We report results for two estimation exercises for our model with involuntary unemployment. In the first exercise we did not include the responses of the unemployment rate or the labor force to shocks. Because this model is observationally equivalent to the standard DSGE model, the results based on this exercise have the heading, 'baseline model'. We refer to the results based on including the unemployment rate and labor force in the estimation with the heading, 'baseline model with involuntary unemployment'. We make several observations about the parameters listed in Table 2. First, the estimated parameters in the two columns take on very similar values. This reflects the observational equivalence result and the fact that no great adjustments to the parameters are required to fit the unemployment and labor force data. Second, in the case of the baseline model, the objects, σ_L and σ_z coincide. This is evident from (3.22), which shows that the steady state curvature of utility with respect to h_t, denoted by σ_z, is precisely equal to σ_L. In the case of the baseline model with involuntary unemployment, σ_z and σ_L are distinct. In both models, σ_L determines the cross-sectional variance of households' aversion to work. The two

parameters are individually identified only in the presence of the unemployment and labor force impulse responses.

The third observation concerns the idea of σ_z as a 'parameter'. In fact, σ_z is a parameter of our model's reduced form and so it is in fact an endogenous feature of the model. We find it convenient to estimate this reduced form as a primitive parameter, as well as the object, $\bar{p}$, which is the maximal value of the steady state function, $p(e_l)$ for $l \in [0,1]$. In addition, we fix the steady state unemployment rate, u, at its sample average, 0.056, and the steady state labor force participation rate, m, at a value of 2/3. For given values of the four objects, σ_z, $\bar{p}$, u and m, we can uniquely compute values for[26]:

$$F, \varsigma, a, \eta. \tag{5.1}$$

This explains why $\bar{p}$ and σ_z appear in the list of estimated parameters for our model, while the four parameters in (5.1) do not. In the case of the baseline model, u was not fixed and m and $\bar{p}$ do not appear.

Turning to the parameter values themselves, note first that the degree of price stickiness, ξ_p, is modest. The implied time between price reoptimizations averages 1.5 quarters. The posterior standard deviation for ξ_p is fairly large, so that the posterior probability interval includes values of ξ_p that are close to zero. The standard deviation of the posterior on ξ_p is roughly equal to the standard deviation of the associated prior, suggesting there is little information in the impulse response functions about ξ_p. The results for ξ_w are different. The results suggest that the average duration between wage reoptimization is three quarters and the associated posterior standard deviation is quite small. The fact that the standard deviation of the posterior is substantially smaller than the standard deviation of the prior suggests that there is substantial information in the impulse response functions about ξ_w. These findings complement those in CEE, where it was found that wage stickiness is more important for model fit than price stickiness. In the case of the other parameters, there appears to be a reasonably substantial amount of information about their value in the impulse response functions.

Table 3 reports steady state properties of the two models, evaluated at the posterior mode of the parameters. According to the results, the capital labor ratio is a little lower than its empirical value closer to 12 that is typically reported in the real business cycle literature. The replacement ratio, c^{nw}/c^{w}, is an interesting new model feature that does not appear in standard models of unemployment in monetary DSGE models. It is estimated to lie in the neighborhood of 80 percent. This is a somewhat lower replacement ratio than the 90 percent number reported in the introduction. It is higher than the number reported for developed countries in OECD (2006). However, those replacement ratios pertain to income, rather

[26] See section C.4.2.

than consumption.[27] So, they are likely to underestimate the consumption concept relevant for us. Not surprisingly, our model's implications for the consumption replacement ratio is very sensitive to the habit persistence parameter, b. If we set the value of that parameter to zero, then our model's steady state replacement ratio drops to 20 percent.

5.2. Impulse Response Functions of Unemployment and the Labor Force

Figure 1 displays the response of unemployment and the labor force to the monetary policy shock, the neutral technology shock, and the investment specific shock, respectively. The solid line in the figure depicts the point estimates of the VAR impulse response functions, while the grey area depicts the 95 percent probability intervals around the point estimates.[28] The posterior mode of the model parameters was in effect chosen so that the model produces impulse response functions as close to the middle of the grey areas as possible, subject to a penalty for deviating too far from the mode of the priors on model parameters. The key thing to note in Figure 1 is that the model has no difficulty accounting for the pattern of responses. These results differ sharply from what happens when we add unemployment and the labor force to the standard model in the way suggested by Gali (2009). These results are displayed in the appendix and they reflect the presence of strong income effects on the labor force and, hence, unemployment.

The response of standard macroeconomic aggregates to our three shocks is similar for the baseline model and the baseline model with involuntary unemployment. This is not surprising, in view of the similarity of the parameter values. In addition, these impulses are similar to what is reported in CEE or ACEL. For this reason, these impulse responses are discussed in the appendix.

6. Concluding Remarks

We constructed a model in which households must make an effort to find work. Because the effort is privately observed, perfect insurance against labor market outcomes is not feasible. To ensure that people have an incentive to find work, it must be that workers are better off if they find a job than if they do not. With additively separable utility, this translates into the proposition that employed workers have higher consumption than the non-employed.

[27] The income replacement ratio for the US is reported to be 54 percent in Table 3.2, which can be found at http://www.oecd.org/dataoecd/28/9/36965805.pdf.

[28] We compute the probability interval as follows. We simulate 2,500 sets of impulse response functions by generating an equal number of artificial data sets, each of length T, using the VAR estimated from the data. Here, T denotes the number of observations in our actual data set. We compute the standard deviations of the artificial impulse response functions. The grey areas in Figures 1-5 are the estimated impulse response functions plus and minus 1.96 times the corresponding standard deviation.

Our model of unemployment has several interesting implications that deserve closer attention. The model implies that the consumption premium of employed workers over the non-employed, c_t^w/c_t^{nw}, is procyclical. Although Chetty and Looney (2006) and Gruber (1997) report that there is a premium on average, we cannot infer anything about the cyclicality of the premium from the evidence they present. Studies of the cross section variance of log household consumption are a potential source of evidence on the cyclical behavior of the premium. To see this, let V_t denote the variance of log household consumption in the period t cross section in our model. We have that[29]

$$V_t = (1 - h_t)\, h_t \left(\log \left(\frac{c_t^w}{c_t^{nw}} \right) \right)^2 .$$

According to this expression, the model posits two countervailing forces on consumption dispersion, V_t, in a recession. First, for a given distribution of the population across employed and non-employed households (i.e., fixed h_t), a decrease in the consumption premium leads to a decrease in consumption dispersion in a recession. Second, holding the consumption premium fixed, consumption dispersion increases as people move from employment to non-employment with the fall in h_t.[30] These observations suggest that (i) if V_t is observed to drop in recessions, this is evidence in favor of the model's prediction that the consumption premium is procyclical and (ii) if V_t is observed to stay constant or rise in recessions then we cannot conclude anything about the cyclicality of the consumption premium. Evidence in Heathcote, Perri and Violante (2010) suggests that the US was in case (i) in three of the previous five recessions. In particular, they show that the dispersion in log household non-durable consumption decreased in the 1980, 2001 and 2007 recessions.[31] We conclude that the cross sectional dispersion of consumption across households lends support to our model's implication that the consumption premium is procyclical.

Another interesting implication of the model is its prediction that high unemployment in recessions reflects the procyclicality of effort in job search. There is some evidence that

[29] Strictly speaking, this formula is correct only for the model in the second section of this paper. The relevant formula is more complicated for the model with capital because that requires a non-trivial aggregation across households that supply different types of labor services. To see how we derived the formula in the text, note that the cross sectional mean of log household consumption is:

$$E_t = h_t \log\left(c_t^w\right) + (1 - h_t) \log\left(c_t^u\right),$$

so that

$$\begin{aligned} V_t &= h_t \left(\log c_t^w - E_t\right)^2 + (1 - h_t)\left(\log c_t^u - E_t\right)^2 \\ &= h_t (1 - h_t) \left(\log c_t^w - \log c_t^u\right)^2 . \end{aligned}$$

[30] This statement assumes that the empirically relevant case, $h_t > 1/2$.

[31] A similar observation was made about the 2007 recession in Parker and Vissing-Jorgensen (2009).

supports this implication of the model. The Bureau of Labor Statistics (2009) constructs a measure of the number of 'discouraged workers'. These are people who are available to work and have looked for work in the past 12 months, but are not currently looking because they believe no jobs are available. This statistic has only been gathered since 1994, and so it covers just two recessions. However, in both the recessions for which we have data, the number of discouraged workers increased substantially. For example, the number of discouraged workers jumped 70 percent from 2008Q1 to 2009Q1. In fact, the number of discouraged workers is only a tiny fraction of the labor force. However, to the extent that their sentiments are shared by workers more generally, a jump in the number of discouraged workers could be a signal of a general decline in job search intensity in recessions. But, this is an issue that demands a more careful investigation.

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Table 1: Non-Estimated Parameters

Parameter	Value	Description
α	0.25	Capital share
δ	0.025	Depreciation rate (quarterly)
β	0.999	Discount factor (quarterly)
π	1.0083	Gross inflation rate (quarterly)
η_g	0.2	Gov. cons. to GDP ratio
κ_w	1	Wage indexation to π_{t-1}
λ_w	1.01	Wage markup
μ_n	1.0041	Gross neutral tech. growth (quarterly)
μ_ψ	1.0018	Gross invest. tech. growth (quarterly)

Table 3: Model Steady State at Posterior Mode

Variable	Baseline Model	Baseline+Invol. Unemp. Model	Description
$p_{k'}k/y$	7.73	7.73	Capital to GDP ratio (quarterly)
c/y	0.56	0.56	Consumption to GDP ratio
i/y	0.24	0.24	Investment to GDP ratio
$H = h$	0.63	0.63	Steady state labor input
c^u/c^w	1.0	0.81	Replacement ratio
R	1.014	1.014	Gross nom. int. rate (quarterly)
R^{real}	1.006	1.006	Gross real int. rate (quarterly)
r^k	0.033	0.033	Capital rental rate (quarterly)
u	0.025	0.056	Unemployment rate
m	-	0.665	Participation rate
l^*	0.644	-	Labor force
ς	1.83	1.64	Disutility of labor shifter
a	-	0.47	Slope of $p(e)$
η	-	0.72	Intercept of $p(e)$
F	-	0.17	Disutility of labor parameter

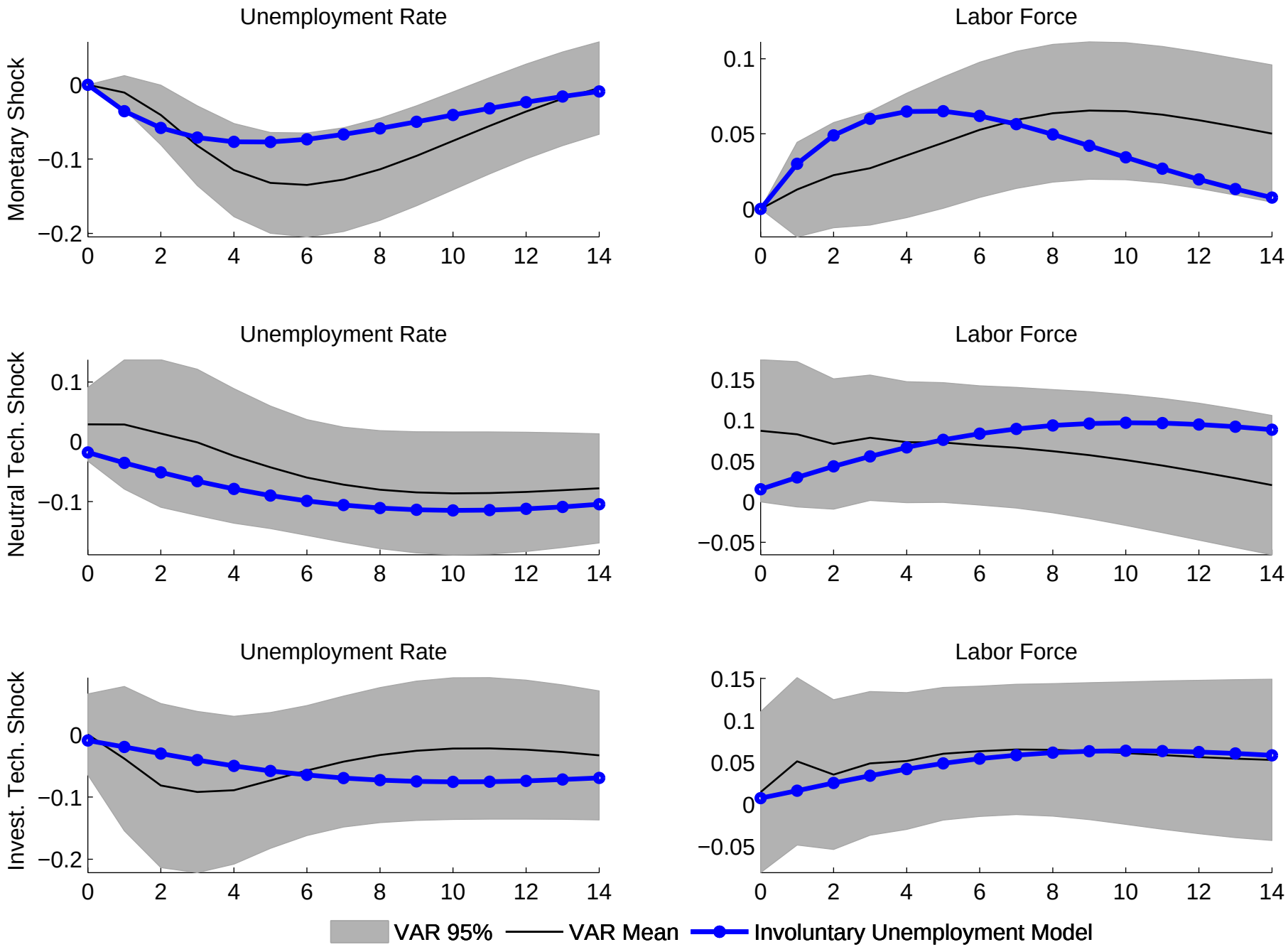
Table 2: Priors and Posteriors of Model Parameters

Parameter		Prior		Posterior Mode (Std.Dev.)[b]	
		Distribution [bounds]	Mean, Std.Dev. [5% and 95%]	**Baseline**	**Baseline + Involuntary Unemployment**
Price and wage setting parameters					
Price Stickiness	ξ_p	Beta	0.50, 0.15	0.33	0.38
		[0, 0.8]	[0.23, 0.72]	(0.13)	(0.13)
Wage Stickiness	ξ_w	Beta	0.50, 0.15	0.69	0.66
		[0, 0.8]	[0.23, 0.72]	(0.05)	(0.05)
Price Markup	λ_f	Gamma	1.20, 0.15	1.18	1.41
		[1.01, ∞]	[1.04, 1.50]	(0.09)	(0.1)
Price Indexation	κ_f	Beta	0.50, 0.20	0.50	0.44
		[0, 1]	[0.17, 0.83]	(0.27)	(0.26)
Monetary authority parameters					
Taylor Rule: Int. Smoothing	ρ_R	Beta	0.80, 0.8	0.88	0.89
		[0, 1]	[0.62, 0.94]	(0.01)	(0.01)
Taylor Rule: Inflation Coef.	r_π	Gamma	1.60, 0.15	1.45	1.42
		[1.01, 4]	[1.38, 1.87]	(0.11)	(0.11)
Taylor Rule: GDP Coef.	r_y	Gamma	0.20, 0.15	0.04	0.04
		[0, 2]	[0.03, 0.49]	(0.02)	(0.02)
Household parameters					
Consumption Habit	b	Beta	0.75, 0.15	0.78	0.80
		[0, 1]	[0.47, 0.95]	(0.02)	(0.02)
Curvature, labor disutility[a]	σ_L	Gamma	1.00, 0.5	0.39	5.23
		[0, ∞]	[0.34, 1.94]	(0.21)	(0.34)
Inverse labor supply elast.[a]	σ_z	Gamma	1.00, 0.5	0.39	0.49
		[0, ∞]	[0.34, 1.94]	(0.21)	(0.26)
Capacity Adj. Costs Curv.	σ_a	Gamma	1.00, 0.75	0.36	0.31
		[0, ∞]	[0.15, 2.46]	(0.09)	(0.09)
Inv. Adj. Costs Curv.	S''	Gamma	12.00, 8.00	17.44	21.38
		[0, ∞]	[2.45, 27.43]	(3.89)	(4.35)
Working Capital Fraction	ν^f	Beta	0.50, 0.20	0.35	0.35
		[0, 1]	[0.17, 0.83]	(0.10)	(0.10)
max $p(e)$	$\overline{p}$	Beta	0.95, 0.03	-	0.987
		[0, 0.99]	[0.89, 0.984]	-	(0.002)
Shocks					
Autocorr. Neutral Tech.	ρ_n	Beta	0.75, 0.15	0.80	0.81
		[0, 1]	[0.47, 0.95]	(0.03)	(0.04)
Autocorr. Invest. Tech.	ρ_ψ	Beta	0.75, 0.15	0.71	0.70
		[0, 1]	[0.47, 0.95]	(0.05)	(0.06)
Std.Dev. Neutral Tech. Shock	σ_n	Inv. Gamma	0.1, 0.05	0.06	0.05
		[0, ∞]	[0.05, 0.19]	(0.01)	(0.01)
Std.Dev. Invest. Tech. Shock	σ_ψ	Inv. Gamma	0.1, 0.05	0.12	0.12
		[0, ∞]	[0.05, 0.19]	(0.02)	(0.02)
Std.Dev. Monetary Shock	σ_R	Inv. Gamma	0.4, 0.2	0.48	0.47
		[0, ∞]	[0.21, 0.74]	(0.03)	(0.03)

[a] In the case of the baseline model, σ_z and σ_L coincide. In the case of the involuntary unemployment model these two parameters are different.

[b] Laplace approximation.

Figure 1: Dynamic Responses of Labor Market Variables to Three Shocks



7. Appendix: Response of Standard Macroeconomic Variables to Our Three Shocks

Figures 2-4 display the results of the indicated macroeconomic variables to our three shocks. Note how the model captures the slow response of inflation to a monetary policy shock. Indeed, the model even captures the 'price puzzle' phenomenon, aaccording to which inflation moves in the 'wrong' direction initially, in response to the expansionary monetary policy shock. In addition, the inflation response to a technology shock is initially slower relative to what it is in the data. These observations are discussed extensively in CEE, ACEL and CTW.

Figure 5 displays the response of unemployment and the labor force to our three shocks in the standard model modified in the way suggested in Gali (2009). Note how poorly the responses match the corresponding empirical estimates. In addition, the responses differ sharply from those in the standard model with involuntary unemployment. As disccused in the in the introduction, the poor performance of the Gali model reflects the strong impact of income effects on labor supply (e.g., the 'labor force' in the model).

Figure 2: Dynamic Responses of Non−Labor Market Variables to a Monetary Policy Shock

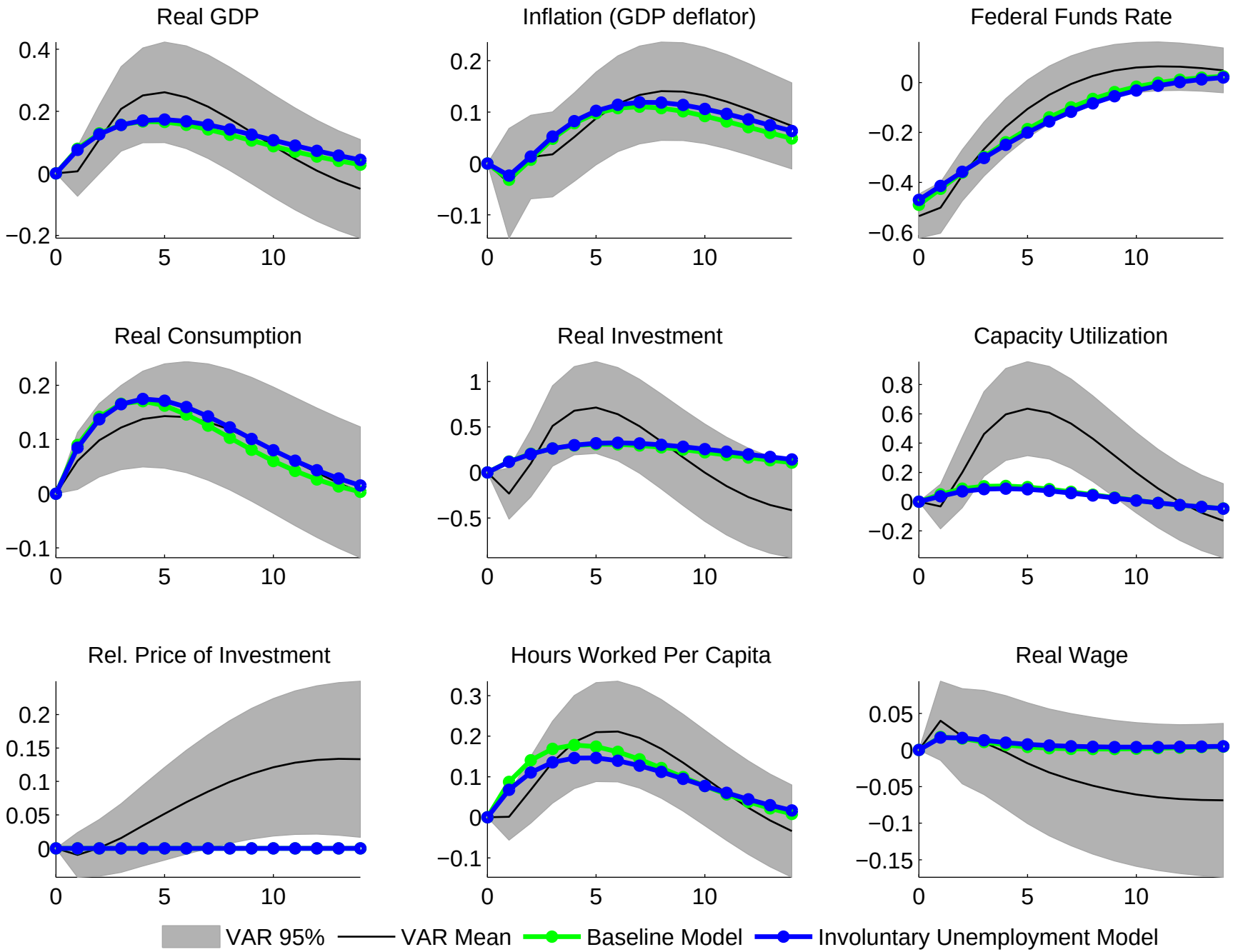


Figure 3: Dynamic Responses of Non−Labor Market Variables to a Neutral Technology Shock

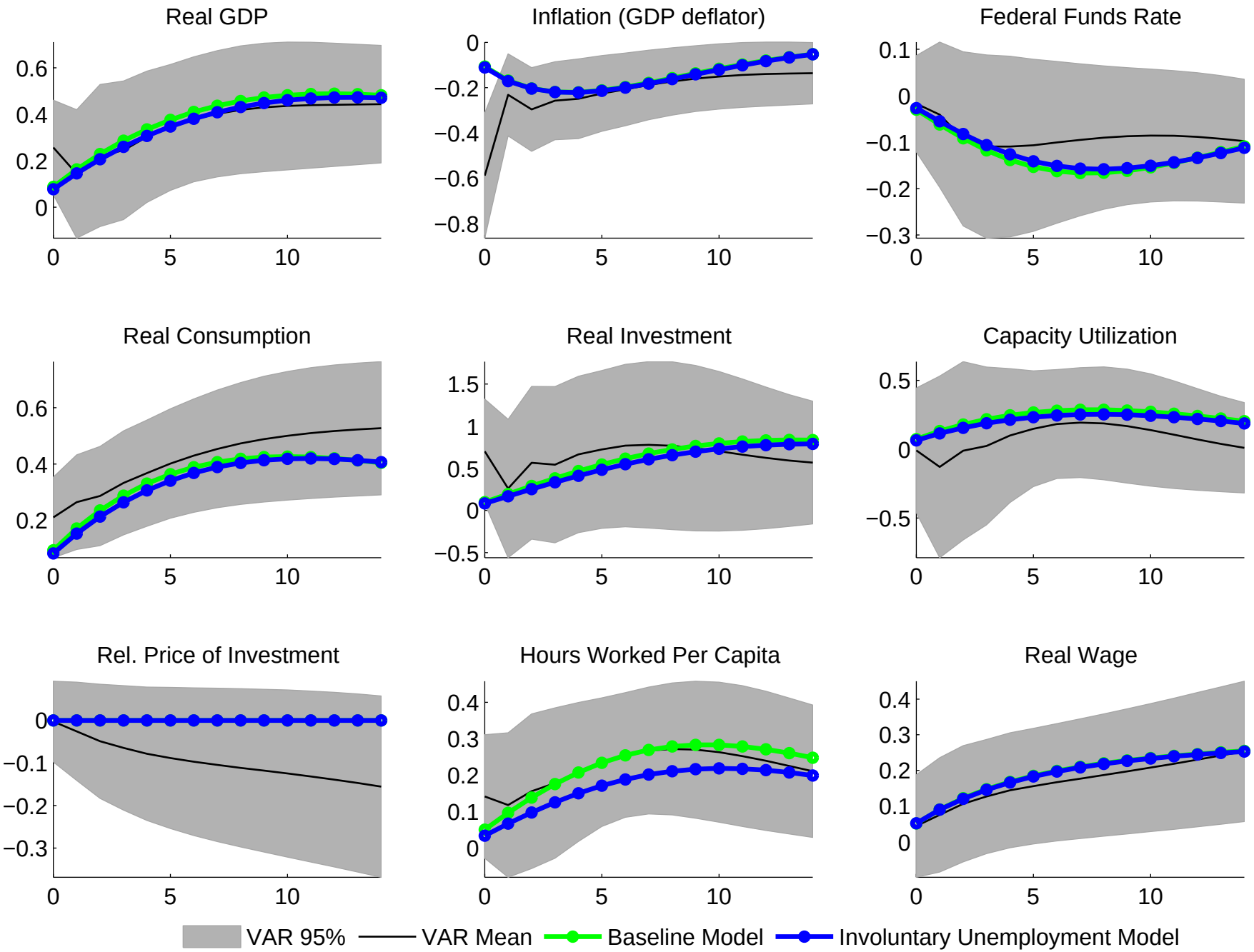


Figure 4: Dynamic Responses of Non−Labor Market Variables to an Investment Specific Technology Shock

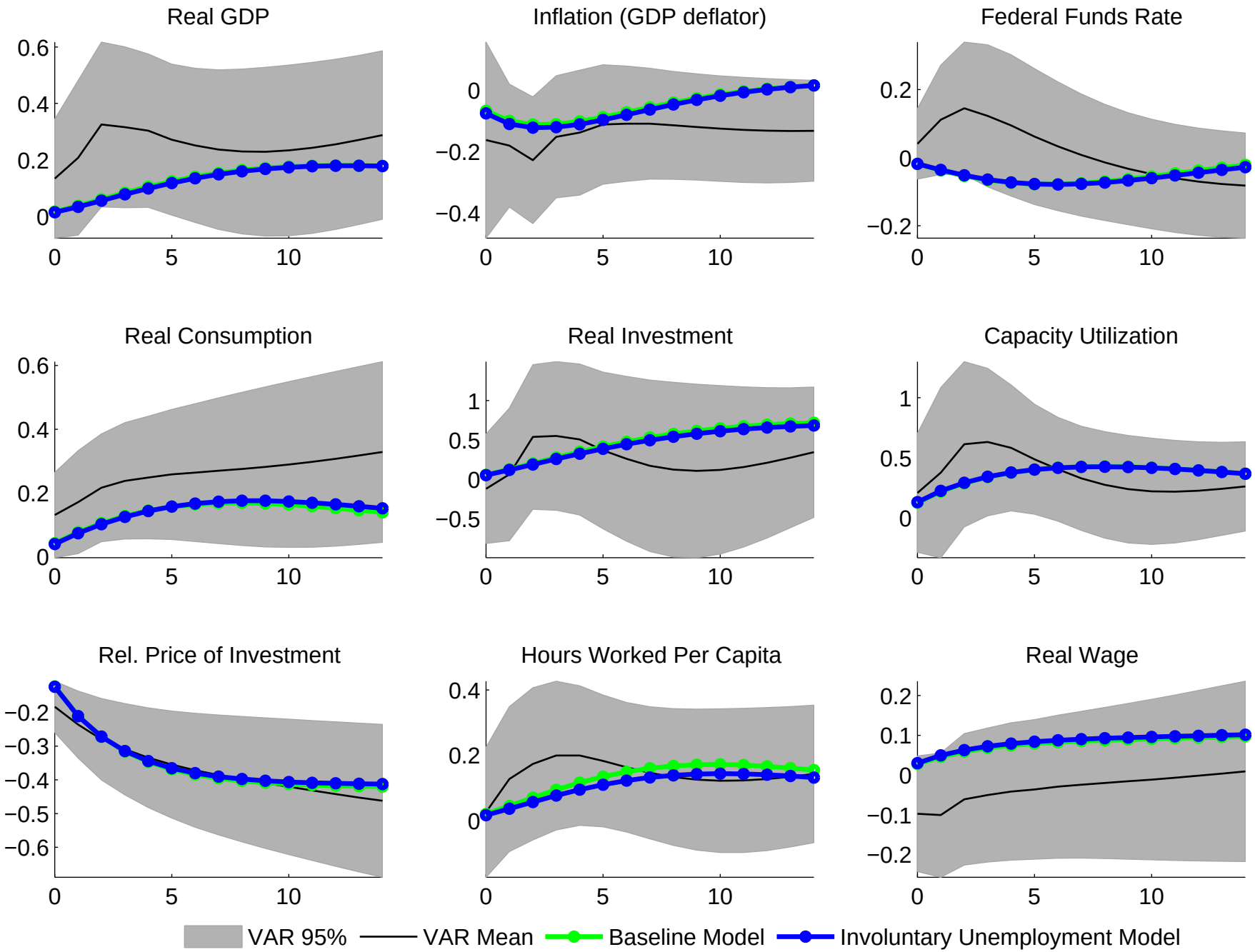
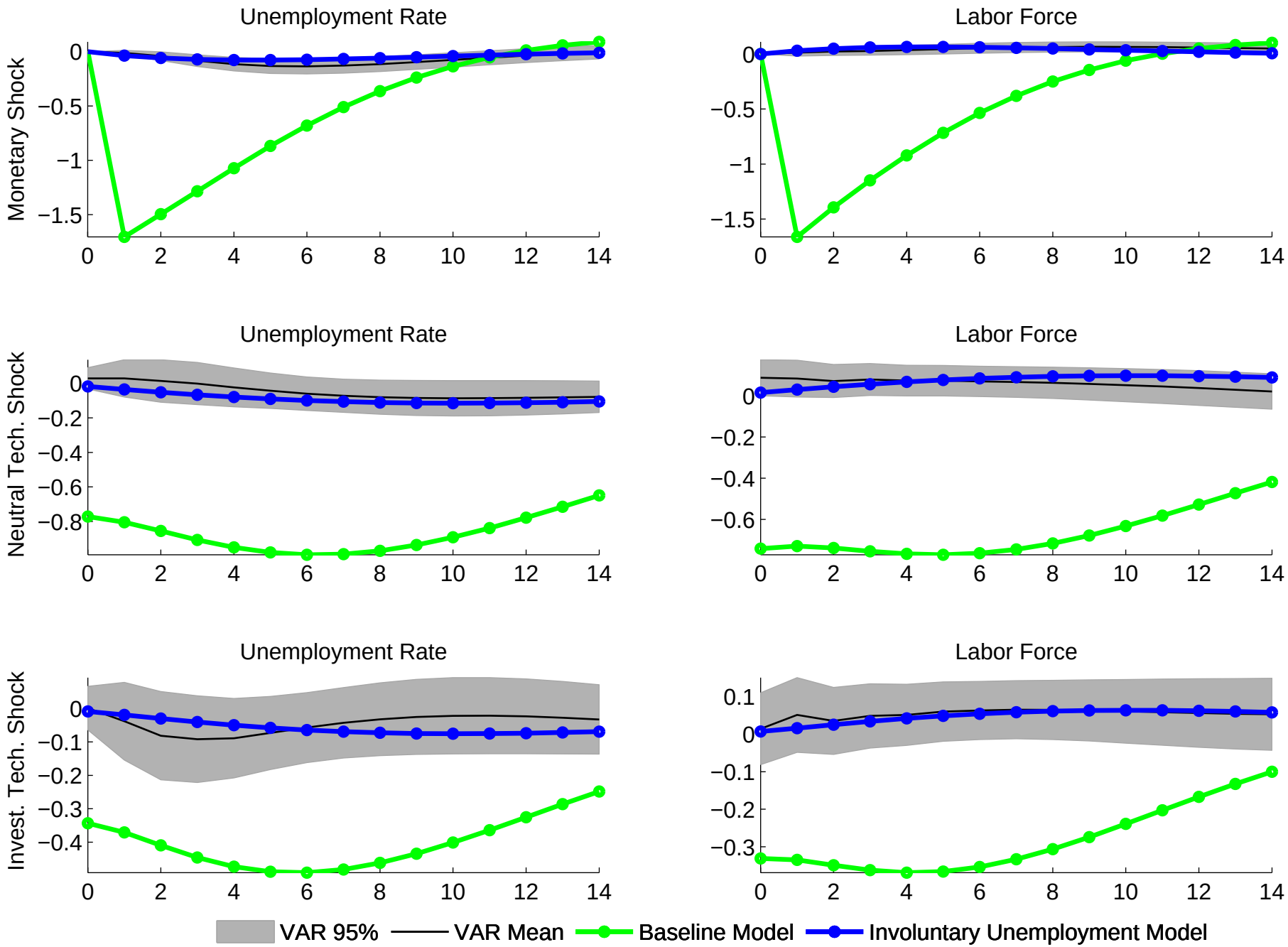



Figure 5: Dynamic Responses of Labor Market Variables to Three Shocks