A MODEL OF MORAL-HAZARD CREDIT CYCLES

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Abstract: This paper considers a simple model of credit cycles driven by moral hazard in financial intermediation. Investment advisers or bankers must earn moral-hazard rents, but the cost of these rents can be efficiently spread over a banker's entire career, by promising large back-loaded rewards if the banker achieves a record of consistently successful investments. The dynamic interactions among different generations of bankers can create equilibrium credit cycles with repeated booms and recessions. We find conditions when taxing workers to subsidize bankers can increase investment and employment enough to make the workers better off.

I. Introduction

This paper analyzes a simple model to show how boom-bust credit cycles can be sustained in economies with moral hazard in financial intermediation. Problems of moral hazard in banks and other financial institutions were evident at many stages of the recent financial crisis, but the role of moral hazard has been less clear in traditional macroeconomic theory. As Freixas and Rochet (1997) have noted, modern microeconomic models of banking depend on advances in information economics and agency theory which were not available when the traditional Keynesian and monetarist theories were first developed. So now, as economists confront the need for deeper insights into the forces that can drive macroeconomic instability, we should consider new models that can apply the microeconomic theory of banking to the macroeconomic theory of business cycles.

In particular, we should recognize that moral hazard in financial intermediation has an essential fundamental role at the heart of any capitalist economy. A successful economy requires industrial concentrations of capital that are vastly larger than any typical individual's wealth, and the mass of small investors must rely on specialists to do the work of identifying good investment opportunities. So the flow of capital to industrial investments must depend on a relatively small group of financial intermediaries, in banks and other financial institutions, who decide how to invest great sums of other people's wealth. But individuals who hold such financial power may be tempted to abuse it for their own personal profit. To solve this problem of financial moral hazard, a successful capitalist economy needs a system of incentives for bankers and other financial intermediaries that can deter such abuse of power.

Since Becker and Stigler (1974) and Shapiro and Stiglitz (1984), it has been well understood in agency theory that dynamic moral-hazard problems with limited liability are efficiently solved by promising large end-of-career rewards for agents who maintain good performance records. So an efficient solution to moral hazard in banking must involve long-term promises of large late-career rewards for individual bankers. Such back-loading of moral-hazard agency rents requires that bankers must anticipate some kind of long-term relationship with investors. So agency considerations can compel investors to accept limits on the liquidity of their investments, even in a world where physical investments may be short-term in nature. As the prospect of long-term career rewards is essential for motivating bankers to identify appropriate investments, investors' ability to trust their bankers must depend on expectations about long-term future profits in banking. At any point in time, the value of mid-career bankers' positions depends on the recent history of the economy and so becomes a state variable that can affect the level of current investment. When trusted bankers become scarce, aggregate investment must decline. Thus, long-term solutions to financial moral hazard can create dynamic forces that drive aggregate economic fluctuations. This basic insight underlies all the analysis in this paper.

The model in this paper is designed to probe these effects of financial moral hazard on dynamic economic equilibria in the simplest possible context. The analysis here shows how, even in an environment that is stationary and nonstochastic, boom-and-bust credit cycles can be driven purely by concerns of financial moral hazard. In such cycles, when investment is weak, a bailout or stimulus that uses poor workers' taxes to subsidize rich bankers may actually make the workers better off.

To highlight the effects of financial moral hazard, the model here simplifies away most other dynamic economic factors. We consider a simple economic environment with one commodity and labor, with no money or on other long-term assets that could become illiquid investments, and so questions of long-term asset pricing are absent from the analysis here. Bankers' contracts with investors are the only long-term assets that have nontrivial price dynamics in this model. The analytical focus here is on how expectations of future profits in banking can affect the cost of financial intermediation for current investment.

These simplifying assumptions make this paper complementary to other important contributions to this literature on agency effects in macroeconomic dynamics. The model here is closely related to the classic model of Bernanke and Gertler (1989) and to the moral-hazard model of Suarez and Sussman (1997), which also consider simple dynamic economies without long-term assets. In the Bernanke-Gertler and Suarez-Sussman models, the dynamic state variable is the aggregate wealth of entrepreneurs, who are subject to moral hazard in the second period of their two-period careers, but our model here shifts the focus to moral hazard of financial intermediaries whose careers can span any given number of periods. As in other standard models of financial intermediation (Diamond, 1984; Holmstrom and Tirole, 1997; Philippon, 2008), the problem of moral hazard in financial intermediation is derived here from a more basic problem of moral hazard in entrepreneurship, but financial intermediaries here are distinguished by their long-term relationships with investors.

Several other recent papers have also offered theoretical models to show how macroeconomic instability can be derived from incentive constraints in microeconomic transactions. Like this paper, Sussman and Suarez (1997, 2007) and Li and Wang (2010) have dynamic models in which macroeconomic fluctuations are driven by purely moral hazard in one sector, with no exogenous shocks. Closely related models that involve adverse selection have been developed by Azariadis and Smith (1998), Reichlin and Siconolfi (2004), Martin (2008), and Figueroa and Leukhina (2010). Other important credit-cycle models of Kiyotaki and Moore (1997), He and Krishnamurthy (2008), and Brunnermeier and Sannikov (2009) have analyzed how the prices of long-term assets can dynamically depend on the aggregate wealth of investors or intermediaries who are subject to moral-hazard constraints, but the investors in these models cannot solve moral-hazard problems by using long-term career incentives in agency contracts.

This paper may be distinguished from the previous literature by our consideration of long-term incentive contracts over more than two periods. One new result that we get from long-term contracting in this model is that the rate of growth must be gradual but contractions can be steep (see condition [5] below). This fundamental asymmetry between bounded growth rates and unbounded contraction rates could not be derived in a two-period model.

The most important goal of our model is to show how basic standard assumptions about long-term moral-hazard contracting in financial intermediation imply the general possibility of macroeconomic fluctuations. The long-term contracts that we analyze here are quite standard in the literature on dynamic moral hazard (see for example Tirole, 2006, p. 184), and the formulation here has been particularly influenced by Biais, Marrioti, Rochet, and Villeneuve (2007). (See also Myerson, 2009, for applications of such dynamic agency models to fundamental problems of government and politics.)

We assume that investors can freely recruit any number of new young bankers every period. So at any point in time there will be different cohorts of bankers of different ages who will have accumulated contractual promises and assets in long-term relationships with their investors. The aggregate values of the contractual positions of these different cohorts of mid-career bankers will form the dynamic state of our economic model, which can change cyclically over time.

One might compare the relational assets of bankers of various ages to the accumulated investments in physical capital of various ages in a standard growth model. But there is a crucial difference between investments in physical capital and investments in long-term relationships with financial agents. The standard economic assumption about physical capital is that investors incur the cost of a unit of physical capital at the beginning of its life, and then the productive value of the capital investment depreciates over time. In contrast, the standard economic assumption about dynamic moral-hazard relationships is that the cost of moral-hazard rents is largely incurred by investors at the end of the agent's career, and so the value of the relationship actually increases over time, as end-of-career rewards draw closer, until the agent retires. This crucial distinction implies that moral-hazard rents of different cohorts of financial agents cannot be simply aggregated like investments in capital of different vintages. Thus, a simple one-sector model with long-term moral-hazard rents can generate complex dynamics that are fundamentally different from a simple one-sector model with long-term capital investment.

II. Basic parameters of the model

Consider a simple economy that has just one commodity, which we may call grain. Grain can be consumed or invested at any time, but lasts only one time period. Individuals live n+1 periods, for some positive integer n. Each individual has risk-neutral utility for consumed grain with some time-discount rate ρ. We assume that agents can borrow grain globally for future repayment at this interest rate ρ.

An investment at any time period t will, if successful, return at the next time period t+1 an amount of grain that is proportional to the amount invested. Harvesting the fruits of successful investments requires scarce labor, however. So after subtracting labor-harvesting costs, the net returns to successful investments will depend on wages in the harvest period t+1, which in turn will depend on the aggregate amount of investment I_t at time t. Thus, we assume that some decreasing continuous investment-demand function R specifies the rate of return

$$r_{t+1} = R(I_t)$$

that any successful investment will yield, per unit invested, at any time t+1 when the aggregate investment in the previous period was I_t.

Investment projects differ in size and quality. A project's quality may be good or bad. A good investment project has some probability α of succeeding in the next period, and a bad project has some lower probability β of succeeding, where

$$\alpha > \beta.$$

These probabilities of success are independent across projects. The size of a project is the amount invested. There are always plenty of good investment projects, but they can only be found by special entrepreneurs, who are experienced individuals in their next-to-last period of life. The investment project must be managed by the entrepreneur who found it. For any investment project of size h at time t, the managing entrepreneur could undetectably divert some fraction γ of the investment, and then the entrepreneur would get a private consumption amount γh at time t+1. Such diversion of γh would convert a good project into a bad project, however, and so would reduce its probability of successfully yielding $r_{t+1}h$ from α to β.

We assume that good projects are available in any size that is greater than or equal to some given minimal size, which we may denote by 1 unit of grain, but this minimal investment size is assumed to be very large relative to the amounts that ordinary individuals can earn from their labor. So investment requires many individuals to pool their wealth and rely on an agent, whom we shall call a banker, to identify good projects.

The central moral-hazard problem is that, instead of finding a good entrepreneur with a good project, the banker could substitute a bad entrepreneur with a bad project. Assume that everybody can find bad projects of any size, and so the banker could use any trusted friend or stooge as the managing entrepreneur for a bad project. Since diverting a fraction γ from a bad project does not make it any worse, the banker could demand the diverted fraction γ as a kickback. Furthermore, by not bothering to verify that an investment is good, the banker would also save some verification costs (which are normally included in the overall investment cost) that are proportional to the investment size. We may suppose that, for any investment of size h at time t, the banker's private benefits from skipping the verification process would be worth an additional ηh in private consumption for the banker at the harvest time t+1. Even with the diversions of γh and ηh from a bad project of size h, it would still have a β probability of yielding a successful harvest worth $r_{t+1}h$.

As individuals are assumed to live n+1 periods, a banker can supervise an investment in each of the n periods before her last period. We are not assuming that banking requires any special talent or skill. Bankers only need to be trusted by investors. New bankers and entrepreneurs are not assumed to have any personal wealth comparable to the amount that is needed for investment, and their consumption in any period is bounded below by 0 (limited liability).

To guarantee that bad investment projects are unprofitable, we will assume that, at any time t, the rate of return on successful investments r_{t+1} satisfies

[1] $$\gamma + \eta + \beta r_{t+1} < 1+\rho,$$

This inequality says that, even when the banker's private benefits of γ and η are taken into account, the expected total return from a bad project is less than what investors could get by lending their grain at the global risk-free interest rate ρ. This inequality will be satisfied by all equilibria in our model if our parameters satisfy the following parametric inequality, which essentially says that γ and η are not too large relative to ρ:

[2] $$\gamma/(1-\beta/\alpha)^3 + \eta/(1-\beta/\alpha)^2 < 1+\rho.$$

III. One-period analysis of moral hazard for entrepreneurs and bankers

Let us begin with a simple one-period formulation of the moral hazard problems of entrepreneurs and bankers. Consider a good investment project of size h at time t, and consider a contract in which the entrepreneur and banker respectively will get payoffs e and b at time t+1 if the project is a success. We assume that the entrepreneur and banker have limited liability and so cannot get less than 0 payoff if the project fails.

The entrepreneur's expected payoff here is αe if the project is managed appropriately. But the entrepreneur could instead divert $h\gamma$ and make the project bad, reducing the probability of success from α to β, and then the entrepreneur would get the expected payoff $h\gamma+\beta e$. Thus the entrepreneur's moral-hazard incentive constraint is

$$\alpha e \geq h\gamma+\beta e.$$

This is equivalent to $e \geq h\gamma/(\alpha-\beta)$. Let us define the entrepreneur's moral-hazard coefficient

$$E = \gamma/(\alpha-\beta).$$

So for an investment of size h, the entrepreneurs rewards for success must be at least $e = hE$. This amount hE may be called the entrepreneur's moral-hazard rent.

The banker's expected payoff from a good project managed appropriately is αb. But if the banker instead put the investment h into a bad project, then next period she could take ηh in investment funds that were budgeted for quality verification, take an additional γh in diverted funds from the stooge-entrepreneur, and with probability β she could get a success-payment of b and demand a further kickback of e from the stooge-entrepreneur. So the banker's moral-hazard incentive constraint is

$$\alpha b \geq (\eta+\gamma)h+\beta(b+e).$$

Let us define the banker's moral-hazard coefficient

$$B = (\eta+\gamma+\beta E)/(\alpha-\beta) = \eta/(\alpha-\beta) + \gamma\alpha/(\alpha-\beta)^2.$$

These moral-hazard incentive constraints imply that minimal pay for the banker after success is

$$b = hB = h(\eta+\gamma+\beta E)/(\alpha-\beta) = h[\eta/(\alpha-\beta) + \gamma\alpha/(\alpha-\beta)^2].$$

This amount hB may be called the banker's moral-hazard rent.

We are assuming that minimal investment sizes are very large compared to the resources of a typical individual, and so the large expected moral-hazard rent αhB makes the position of banker here very attractive. But the assumption of limited resources also means that an individual cannot be asked to pay ex ante for her expected benefits of becoming a powerful banker. The moral-hazard constraint would be violated if a prospective banker raised funds for such an entry fee by borrowing against her future moral-hazard rents, because with limited liability the debt would have to be excused if her project failed, and so her net benefit from success would be reduced by the amount borrowed. So the banker's moral-hazard rent is an essential cost of investing in this economy, but investors can spread this cost over a sequence of investments when bankers serve for more than one period.

IV. Efficient financial intermediation by bankers hired with long-term contracts

At any time t, consider a consortium of investors that hires a young individual to be their financial intermediary or banker for an extended period of service. We are assuming that individuals can work for n periods. So suppose that the banker is hired at the start of her n-period career, when her career-age is 0, and she will retire at age n in period t+n. The consortium can ask the banker to choose investments for them in periods t+s, for all s in $\{0,...,n-1\}$. Suppose that the banker is to start at age 0 with the minimal investment $h_0=1$.

To simplify the analysis, let us here consider only contracts that have maximal back-loading of rewards and have maximal punishment for failure. That is, the banker's rewards are all concentrated in one big retirement payment that depends on good performance throughout her career, but any failure will cause a termination of the banker's contract without pay, which is the worst possible punishment under limited liability. For each s in $\{0,1...,n-1\}$, the contract must specify some amount h_s that the consortium will ask the banker to invest at time t+s if her previous s investments were all successful. The contract must also specify some final payment b_n that the banker will get on retirement at time t+n if all her n investments were successful. In the Appendix we show that the optimum among such contracts is also optimal more generally in the complete class of feasible n-period contracts subject to moral-hazard incentive constraints.

At any age s in $\{0,...,n-1\}$, the banker will invest h_s at time t+s if her first s projects succeed, which with good projects has probability α^s, and so the expected time-t discounted cost of this investment at time t+s is $\alpha^s h_s/(1+\rho)^s$. The probability that the banker will make a successful investment at time t+s is α^{s+1}, and so the investors' earnings in time t+s+1, after deducting the current entrepreneur's moral-hazard rent $h_s E$, have the time-t expected discounted value $\alpha^{s+1}h_s(r_{t+s+1}-E)/(1+\rho)^{s+1}$. The final payment to the banker at time t+n has the expected time-t discounted cost $\alpha^n b_n/(1+\rho)^n$. Thus, at time t, the consortium's expected discounted profit (above what it could earn by lending at the global interest rate ρ) from its contractual relationship with the banker is

$$\sum_{s\in\{0,...,n-1\}} \alpha^{s+1}h_s[r_{t+s+1} - E - (1+\rho)/\alpha]/(1+\rho)^{s+1} - \alpha^n b_n/(1+\rho)^n.$$

For good investments to be worthwhile, the rates of return r_{t+s+1} must satisfy

[3] $$r_{t+s+1} \geq E + (1+\rho)/\alpha, \ \forall s.$$

If the rate of returns for successful investments were less than this, then the investors' expected returns, after deducting the moral-hazard rents for entrepreneurs, would be less than they could get by lending at the global risk-free interest rate ρ. Thus, investments must yield a nonnegative surplus for banking, where the banking surplus is

$$\sigma_{t+s+1} = r_{t+s+1} - E - (1+\rho)/\alpha.$$

We know (from the previous section) that if the investment of h_s is made successfully at time t+s, then at time t+s+1 the banker's current expected value of her reward for success must be equal to her required moral-hazard rent $h_s B$. (As in the previous section, the banker would get 0 from failure at time t+s+1, as her contract would then be terminated without pay.) Given a successful outcome at time t+s+1, the contract offers the banker a chance of getting b_n in n−(s+1) periods if she gets n–(s+1) more successes, and this prospect has the current expected discounted value $b_n[\alpha/(1+\rho)]^{n-(s+1)}$. To satisfy the banker's moral-hazard incentive constraint at every period t+s, b_n and h_s must satisfy

$$b_n[\alpha/(1+\rho)]^{n-(s+1)} \geq h_s B.$$

Given b_n, with inequality [3], the optimal investment at each t+s is

$$h_s = b_n[\alpha/(1+\rho)]^{n-(s+1)}/B.$$

With $h_0=1$, we get

$$b_n = B[(1+\rho)/\alpha]^{n-1}, \text{ and so } h_s = [(1+\rho)/\alpha]^s, \forall t.$$

Thus, under the optimal contract, the amount that the banker invests will be multiplied by the factor $(1+\rho)/\alpha$ each period she succeeds. As the probability of success is α, the (unconditional) expected value of this investment is increased by the multiplicative factor $(1+\rho)$ each period during the banker's career. If the banker has success in all n periods of investment, then her consumption in retirement will equal B times the actual amount of her last investment. During a successful career, the banker's expected discounted value of this final payment grows, as the final payment becomes closer in time and more likely to be realized; and so the banker's investment responsibilities h_s grow during her career in proportion to this conditionally expected discounted value.

With this optimal plan of investments $(h_0,h_1,...h_{n-1})$ and final reward b_n, the investors' expected discounted value of profits at time t is

$$\sum_{s\in\{0,\ldots,n-1\}} [r_{t+s+1} - E - (1+\rho)/\alpha]\alpha/(1+\rho) - B\alpha/(1+\rho).$$

We are assuming that there is a global pool of risk-neutral investors who can freely hire any number of new young bankers at any time t. So if investors could earn a strictly positive expected discounted value from such a contract, then aggregate investment in this economy would go to infinity. On the other hand, investment in this economy would vanish if such optimal contracts had a negative expected discounted value for investors. So in equilibrium with finite positive investment, the investors' optimal expected discounted value of profits must equal 0. Thus, in equilibrium, we must have

$$\sum_{s\in\{0,\ldots,n-1\}} [r_{t+s+1} - E - (1+\rho)/\alpha] = B. \tag{4}$$

This equation tells us that banking surpluses over the n periods of a banker's career must cover the cost of the banker's moral-hazard rents. A consortium that hired an older banker would have to distribute the same moral-hazard rents over a shorter career and so would not be profitable.

At any time t+s+1, for $s\in\{0,1,...,n-1\}$, if all investments so far have been successful, then latest successful investment h_s can pay the investors the dividend

$$\sigma_{t+s+1}h_s = [r_{t+s+1} - E - (1+\rho)/\alpha]h_s$$

after the entrepreneur has been paid Eh_s and the amount $h_{s+1} = h_s(1+\rho)/\alpha$ has been reinvested. If the banker always succeeds then at time t+n the banker must be paid $h_{n-1}B$ for her retirement, and so the investors' final dividend is

$$[r_{t+n} - E - B]h_{n-1}.$$

The parametric condition [2] and the inequality $\alpha>\beta$ imply

$$B < B\alpha/(\alpha-\beta) < (1+\rho)/\alpha.$$

Thus, with the banking-surplus inequality [3], the dividends are all nonnegative. That is, the consortium does not need any external funding after the initial investment h_0.

At time t+s (with $s\in\{0,1,...,n-1\}$), the consortium's future dividends have expected value

$$h_s[(\sigma_{t+s+1} + ... + \sigma_{t+n}) + (1+\rho)/\alpha - B]\alpha/(1+\rho)$$

$$= h_s - h_s[B - (\sigma_{t+s+1} + ... + \sigma_{t+n})]\alpha/(1+\rho).$$

At the initial time t, this expected value just equals h_0, by the banking-rents equation [4]. At later times t+s, this expected value becomes strictly less than h_s, reflecting the fact that the investors have already amortized part of their investment. The investors would actually prefer to break the contract and pay the current investment funds h_s as a dividend to themselves if they had no contractual obligations to the banker. Thus, although the productive investments in this economy are all short term (spanning just one period), moral hazard in banking requires investors to make a long-term (n-period) commitment to their banker. In this sense, moral hazard in banking induces investors here to accept a kind of illiquidity in their investments.

There is one special case when an alternative optimum can be considered for the optimal long-term investment contract: when the rate of return satisfies [3] with equality. Let r^* be the lowest rate of return for successful investments that satisfies the banking-surplus inequality [3]:

$$r^* = (1+\rho)/\alpha + E.$$

At any time t+s when $r_{t+s+1} = r^*$, good investment projects yield no expected banking surplus over the risk-free bonds paying interest ρ. In this case, an efficient banking contract could also allow the banker to invest in risk-free bonds; but then, if $s+1<n$, the investors would have to re-invest all the risk-free returns $(1+\rho)h_s$, so as to provide the expected final payoff $\alpha Bh_0(1+\rho)^{n-1}$ that is required to motivate the banker's good behavior in all regular investment periods when [3] is a

strict inequality. Notice that, even with this modification, the expected value of the banker's investments increase by the multiplicative factor $(1+\rho)$ each period over her career.

We can now verify that, with the parametric assumption [2], the equilibrium conditions [3] and [4] imply that bad projects are unprofitable in any equilibrium. Bad projects are unprofitable at any time t when the rate of returns satisfies [1] $r_{t+1} < (1+\rho-\gamma-\eta)/\beta$. Given the definitions of B and E and $\alpha>\beta$, the inequality $(1+\rho)/\alpha+E+B < (1+\rho-\gamma-\eta)/\beta$ is equivalent to the parametric inequality [2] $\gamma/(1-\beta/\alpha)^3 + \eta/(1-\beta/\alpha)^2 < 1+\rho$.

Proposition 1. Equilibrium rates of return that satisfy conditions [3] and [4] will always satisfy the bounds $(1+\rho)/\alpha + E \leq r_{t+1} \leq (1+\rho)/\alpha + E + B$ and thus will satisfy the condition [1] that makes bad projects unprofitable when the parameters satisfy condition [2].

V. The full characterization of equilibria with investment-demand constraints

Applying the banking-rents equation [4] at any two successive times t−1 and t, we get

$$\sigma_t = B - (\sigma_{t+1}+...+\sigma_{t+n-1}) = \sigma_{t+n} \text{ where } \sigma_s = r_s - E - (1+\rho)/\alpha, \ \forall s.$$

Thus, for equation [4] to hold at every time t in this deterministic model, the returns r_{t+1} must form a cycle $(r_1,...,r_n)$ that repeats every n time periods. To check that such a cycle satisfies conditions [3] and [4], it is sufficient just to check them for the first n periods:

$$r_{t+1} \geq E + (1+\rho)/\alpha, \ \forall t\in\{0,1,...,n-1\},$$

$$\textstyle\sum_{s\in\{0,...,n-1\}} [r_{s+1} - E - (1+\rho)/\alpha] = B.$$

A steady-state equilibrium, in which returns are constant over time, must have

$$r_{t+1} = (1+\rho)/\alpha + E + B/n, \ \forall t.$$

This steady-state rate of return is a decreasing function of n, the length of bankers' careers. So bankers' ability to have longer relationships with investors decreases the cost of investment and thus increases aggregate investment through the investment-demand function.

Away from the steady state, however, equilibria in this economy must satisfy one additional dynamic condition that depends on the investment-demand function.

Let J_s denote the total investment that is handled at time s by young age-0 bankers. In each period of their careers, a fraction α of this cohort will succeed and have their investments multiplied by $(1+\rho)/\alpha$, and so the total investment of the cohort will be multiplied each period by $(1+\rho)$. So for any time t in $\{s,...,s+n-1\}$, the total investment that is handled by this t-cohort at time t will be $J_s(1+\rho)^{t-s}$. Thus, the total investment at any time t must be

$$I_t = \sum_{s\in\{t-(n-1),\ldots,t\}} J_s(1+\rho)^{t-s}, \ \forall t.$$

With these equations, $(1+\rho)I_{t-1}$ and I_t can be written as sums of terms that match for each cohort except that $(1+\rho)I_{t-1}$ includes a term $J_{t-n}(1+\rho)^n$ and I_t includes a term J_t. But in a cyclical solution, we must have $J_t = J_{t-n}$. So these equations have a cyclical solution that repeats $(J_0,\ldots,J_{n-1})$ iff

$$J_t = [(1+\rho)I_{t-1} - I_t]/[(1+\rho)^n - 1], \ \forall t.$$

The total investment of young bankers in any time period must be nonnegative. Such nonnegative J_t can be found iff the aggregate investments satisfy the inequalities

[5] $$I_t \leq (1+\rho)I_{t-1}, \ \forall t.$$

Condition [5] imposes no bound on how steeply aggregate investment can crash from one period to the next, but it tells us that aggregate investment cannot ever grow at a rate faster than ρ. Thus, our model yields a fundamental asymmetry between growth, which must be gradual, and contraction, which can be steep.

In this economy, aggregate investment I in good projects determines the rate of return for successful projects in the next period by a given investment-demand function R(I). But when $r_{t+1} = r^* = (1+\rho)/\alpha+E$, risk-free bonds with interest ρ can replace good investment projects. So we should apply an adjusted investment-demand function that does not go below r^*:

[6] $$r_{t+1} = R^*(I_t) = \max\{R(I_t), r^*\}, \forall t.$$

We can now formalize the main solution concept of this paper.

Definition. An *n-period equilibrium credit cycle* is any returns sequence $(r_1,\ldots,r_n)$ that satisfies the banking-surplus inequality [3] and the banking-rents equation [4], together with an aggregate investment sequence $(I_0,\ldots,I_{n-1})$ that cyclically satisfies the growth bounds [5] and the adjusted investment-demand equations [6].

The distinction between the investment-demand function R and the adjusted investment-demand function R^* is actually not essential to our concept of equilibrium. To see why, let I^* denote the aggregate investment such that $R(I^*)=r^*$. So I^* is the maximal investment in good projects that the economy can sustain. In any period when the bankers' contracts specify an aggregate investment I_t that exceeds I^*, the excess I_t-I^* must be invested in risk-free bonds (which investors are willing to allow, as $r_{t+1}=r^*$). But any equilibrium can be supported by an investment sequence where such bond investments do not occur. Given any equilibrium returns $(r_1,...,r_n)$ where conditions [5] and [6] are cyclically satisfied by an investment sequence $(I_0,...,I_{n-1})$, conditions [5] and [6] are also cyclically satisfied by $(\check{I}_0,...,\check{I}_{n-1})$ where

$$\check{I}_t = \min\{I_t, I^*\}, \ \forall t.$$

In effect, the transformation from I_t to $\check{I}_t$ shifts the recruitment of some young bankers earlier in time across periods when the banking surplus rates σ_{t+1} are 0.

Assuming that bankers are hired with efficient long-term contracts, as described in the previous section, the dynamic state of the economy at any point in time will depend on its history through the re-investments that successful bankers of various ages are entitled to make under the terms of their contracts. Let $\theta_s(0)$ denote the total investments that are handled by bankers of age s at time 0. The dynamic state of the economy at time 0 could be characterized by this vector $(\theta_1(0),...,\theta_{n-1}(0))$. Given these investment amounts at time 0, we can compute what the initial investment had to be for each cohort when it started in the previous n−1 periods

$$J_{-s} = \theta_s(0)/(1+\rho)^s \quad \text{for } s = 1,...,n-1.$$

It will be more convenient to characterize the initial conditions at time 0 by this vector

$$(J_{-(n-1)},...J_{-1}).$$

To characterize the dynamic equilibrium that follows from these initial conditions, we need to find the amount J_0 that is invested by new young bankers at time 0. Given any guess for J_0, each cohort's investments will grow by the multiplicative factor $(1+\rho)$ from each period to the next until it retires. Then, if we are in an equilibrium cycle, the cohort that retires at time n−s will be replaced by a new cohort with the same initial size as its predecessor of n periods before; that is $J_{n-s} = J_{-s}$ for all $s \in \{1,...,n-1\}$. In an equilibrium credit cycle, the resulting investments must yield banking surpluses that just cover the bankers' moral-hazard rents B over the next n periods:

$$[7] \qquad \sum_{t\in\{0,...,n-1\}} [R(\sum_{s\in\{0,...,n-1\}} J_{t-s}(1+\rho)^s) - E - (1+\rho)/\alpha] = B,$$

$$\text{with } J_0 \geq 0 \text{ and } J_t = J_{t-n} \text{ for all } t\geq 1.$$

The sum of banking surpluses in [7] is monotone decreasing in J_0.

But there might be a transient interval from time 0 to some time T>0 during which the continuing contractual investments at time 0 are too high to admit any profitable investment by new bankers. On such a transient path to an equilibrium credit cycle, we must have:

[8] $$\sum_{t\in\{0,\ldots,n-1\}} [R(\sum_{s\in\{0,\ldots,n-1\}} J_{\tau+t-s}(1+\rho)^s) - E - (1+\rho)/\alpha] < B \text{ and } J_\tau = 0$$

$$\text{for all } \tau \text{ such that } 0 \le \tau \le T-1,$$

$$\sum_{t\in\{0,\ldots,n-1\}} [R(\sum_{s\in\{0,\ldots,n-1\}} J_{T+t-s}(1+\rho)^s) - E - (1+\rho)/\alpha] = B,$$

$$J_T \ge 0, \text{ and } J_t = J_{t-n} \text{ for all } t \ge T+1.$$

If T becomes n−1 then the new cohort at T gets no competition from any other cohorts, and then the B-equation here can be satisfied by some $J_T>0$, provided that R satisfies the boundary conditions

$$R(0) > (1+\rho)/\alpha+E+B \text{ and } \lim_{I\to\infty} R(I) \le (1+\rho)/\alpha+E.$$

Thus, a transient path must reach an equilibrium credit cycle by some time $T \le n-1$.

Proposition 2. Suppose that the investment-demand function R is decreasing, continuous, and satisfies the boundary conditions above. Given any nonnegative initial conditions $(J_{-(n-1)},\ldots J_{-1})$, either there exists some $J_0 \ge 0$ that yields an equilibrium credit cycle satisfying [7], or else there exists a transient path to an equilibrium credit cycle satisfying [8] for some T in {1,...,n−1} and some $J_T \ge 0$.

VI. An alternative financial system with regulated independent bankers

In the equilibria of our economy, an investors' consortium would suffer expected losses if it recruited a banker without capital who would serve strictly less than n periods, because the surplus returns of banking over a shorter career would not cover the cost of the banker's moral-hazard rents. So we have found a kind of illiquidity here: although our economy has only short-term 1-period investments, investors need a long n-period relationship with their bankers.

With regulation, however, these equilibria may also be implemented by a system where bankers accumulate capital and invest under age-dependent leverage constraints. In this system, the bankers accumulate capital during their careers, which is invested by older bankers to cover part of the cost of their own moral-hazard rents.

Consider again a banker whose career starts at time t, when her age is 0. We saw that a consortium of investors would have such a banker handling investments $h_s = h_0[(1+\rho)/\alpha]^s$ at each time t+s in her career, as long as her previous investments have not failed. Now let us see how such investments could be handled with the banker repaying her investors each period but maintaining a regulated capital account. Let k_s denote the value of the banker's capital account at any time t+s in the banker's career. We assume that the young banker starts her investment career at time t with no capital, $k_0 = 0$. To invest h_s at time t+s, a banker with capital k_s must borrow $h_s - k_s$, and the banker must promise to repay her risk-neutral ρ-discounting investors the amount $(h_s - k_s)(1+\rho)/\alpha$ at time t+s+1, in the α-probability event of success. Thus, success at time t+s+1 will yield banker's capital

$$k_{s+1} = [r_{t+s+1} - E - (1+\rho)/\alpha]h_s + k_s(1+\rho)/\alpha = \sigma_{t+s+1}h_s + k_s(1+\rho)/\alpha.$$

With $k_0 = 0$ and $h_s = h_0[(1+\rho)/\alpha]^s$, induction yields the equations

$$k_s = h_{s-1}(\sigma_{t+1} + ... + \sigma_{t+s}) = h_s(\sigma_{t+1} + ... + \sigma_{t+s})\alpha/(1+\rho), \ \forall s \in \{1,...,n-1\}.$$

So the banker can invest h_s in each period t+s with the age-dependent leverage ratio

$$h_s/k_s = (1+\rho)/(\alpha\sigma_{t+1} + ... + \alpha\sigma_{t+s}), \ \forall s \in \{1,...,n-1\}.$$

At any time t+s, let v_s denote the expected discounted value of the ultimate payoff (k_n consumed at t+n) that the banker can earn, per unit of capital at time t+s, by investments that satisfy this leverage ratio. These value multipliers can be derived from the recursive equations

$$v_n = 1, \ v_s = \alpha[\sigma_{t+s+1}h_s/k_s + (1+\rho)/\alpha]v_{s+1}/(1+\rho), \ \forall s \in \{1,...,n-1\}.$$

With the equilibrium condition [4] and leverage ratios h_s/k_s as above, these multipliers are

$$v_s = B/(\sigma_{t+1} + ... + \sigma_{t+s}), \ \forall s \in \{1,...,n\}.$$

With these value multipliers, the banker's incentive constraint at time t+s can be written:

$$\alpha[\sigma_{t+s+1}h_s + k_s(1+\rho)/\alpha]v_{s+1}$$
$$\geq (\eta + \gamma + \beta E)h_s + \beta[\sigma_{t+s+1}h_s + k_s(1+\rho)/\alpha]v_{s+1}.$$

This incentive constraint is satisfied iff k_s and h_s satisfy

$$h_s/k_s \leq [(\alpha-\beta)v_{s+1}(1+\rho)/\alpha]/[\eta + \gamma + \beta E - (\alpha-\beta)\sigma_{t+s+1}v_{s+1}]$$
$$= [(1+\rho)/\alpha]/[B/v_{s+1} - \sigma_{t+s+1}] = (1+\rho)/(\alpha\sigma_{t+1} + ... + \alpha\sigma_{t+s}).$$

Thus, the banker's moral-hazard incentive constraint is equivalent to requiring that the banker's investments should not exceed this age-dependent leverage ratio.

This analysis, however, depends critically on an assumption that, in the banker's incentive constraint, the value of illegitimate earnings from diverted investments and kickbacks $(\eta+\gamma+\beta E)h_s$ is not increased by the multiplier v_{s+1}. This assumption means the banker's leveraged capital can only include the banker's legitimate earnings. Young bankers who have no capital can satisfy their moral-hazard incentive constraint here only because their legitimate earnings from banking can be leveraged in their future careers in a way that is not available for their illegitimate earnings. Thus, for this system of capital requirements to solve the problems of moral hazard in banking, financial regulation may be needed for two reasons: to assure investors that a banker's capital must be legitimately earned, and to assure investors that a banker's investments do not exceed the appropriate age-dependent multiple of her legitimate capital.

Under this alternative system of financial governance, the dynamic state of the economy would be defined in terms of the accumulated capital of each cohort of bankers. The total investment that can be made by a cohort with a given stock of accumulated capital would depend on the anticipated returns in future periods. A version of Proposition 2 for a financial system with independent regulated bankers would be somewhat more complicated and is omitted here.

In the rest of this paper, our analysis assumes a financial system where bankers are hired by consortiums of investors with long-term contracts, as in Section IV.

VII. An example with linear investment demand and labor supply: steady state

To construct simple examples, let us consider a simple linear investment-demand function R, that is characterized by two parameters ψ and π. Suppose that any investment of size h will, if it succeeds, return ψh units of grain in the next period, but then harvesting this grain will require h units of labor. So when I_t is aggregate investment in the economy at time t (per unit of population), then aggregate labor demand at time t+1 is αI_t. Suppose that the wage rate w is determined by a linear labor-supply curve with slope π, so that $w = \pi\alpha I_t$. This labor supply curve can be justified by assuming that workers who supply h units of labor at a wage rate w get utility $wh-0.5\pi h^2$, so that their optimal labor supply is $h = w/\pi$. Then the wage rate at time t+1 is $w_{t+1} = \pi\alpha I_t$, and the investment-demand function is

$$R(I_t) = \psi - w_{t+1} = \psi - \pi\alpha I_t.$$

Aggregate investment is strictly positive in all equilibria with this investment-demand function if

$$\psi > (1+\rho)/\alpha + E + B.$$

The workers' total wage income at t+1 is

$$W_{t+1} = \alpha I_t w_{t+1} = \pi(\alpha I_t)^2,$$

and the workers' net utility is $0.5W_{t+1}$.

For a specific numerical example, let us use the parameters

[9] $$\rho=0.1,\ \alpha=0.95,\ \beta=0.6,\ \gamma=0.05,\ \eta=0,\ \psi=1.74,\ \pi=0.233.$$

Then the entrepreneurs' moral-hazard coefficient is $E = 0.143$, and the bankers' moral-hazard coefficient is $B = 0.388$. With these parameters, the rate of return for successful investments in equilibria can range between the minimum of $r^* = (1+\rho)/\alpha+E = 1.301$ and the maximum possible rate of $(1+\rho)/\alpha+E+B = 1.689$.

It may be helpful to see how the steady-state equilibrium depends on the bankers' career length n. In a steady-state n-period equilibrium cycle, the banking surplus is always $\sigma = B/n$, and so, with the parameters in [9], and the constant rate of return on successful investments is

$$r = (1+\rho)/\alpha + E + B/n = 1.301 + 0.388/n.$$

With this rate of return, aggregate investment is

$$I = (\psi-r)/(\pi\alpha) = 1.983 - 1.751/n,$$

and expected total net product (less the cost of invested inputs with interest), is

$$Y = [\alpha\psi-(1+\rho)]I = 1.097 - 0.968/n.$$

In this net product, the total wage income for workers is

$$W = \pi(\alpha I)^2 = 0.828 - 1.461/n + 0.645/n^2,$$

the total income for entrepreneurs is

$$E\alpha I_t = 0.269 - 0.238/n,$$

and the total profit for bankers is

$$[r_{t+1}-E-(1+\rho)/\alpha]\alpha I_t = (B/n)\alpha I_t = 0.731/n - 0.645/n^2.$$

In the steady state with n=1, rate of return on successful investments is $r = 1.689$, aggregate investment is $I = 0.233$, and total net product is $Y = 0.129$, of which the profit for bankers is 0.086 (66% of the total net product), and total wage income for workers is only 0.011.

When the bankers' careers span n=10 periods, however, the steady-state equilibrium has rate of return $r = 1.340$, aggregate investment $I=1.808$, and total net product $Y = 1$, of which the profit for bankers is less than 7%, and the total wage income for workers is W=0.688. So in our model, an increase of the parameter n, which represents the length of time over which investors can maintain relationships of trust with financial intermediaries, can cause changes in output and distribution that resemble the great transformations of economic development.

In this steady-state equilibrium with n=10, new bankers each period start investing J=0.113. At any point in time, the total investment handled by bankers of age s is

$$\Theta_s = J(1+\rho)^s,$$

In the steady-state equilibrium, the vector of investments of these different age cohorts is

[10] $(\Theta_1,...,\Theta_9) = (0.125, 0.137, 0.151, 0.166, 0.183, 0.201, 0.221, 0.243, 0.268).$

The steady-state investment I=1.808 in each period is equal to the sum of these 9 continuing-cohorts' investments plus the new-bankers' investment J.

VIII. An example of recession dynamics

Now let us analyze some dynamic equilibria, assuming that bankers are hired with efficient long-term contracts, as described in Section IV. Consider the example with parameters as above in [9] with n=10, but suppose that the continuing bankers' investments at time 0 are 80% of the steady state amounts in line [10] above. Such a situation could occur if the economy was previously in steady state, but then an unanticipated technical change at time 0 increased investment demand by a permanent 20% reduction of the parameter π (to 0.233). So at time 0, the total contractually-mandated investments $\theta_s(0)$ for continuing bankers of each age s are

$$(\theta_1(0)...,\theta_9(0)) = (0.100, 0.110, 0.121, 0.133, 0.146, 0.161, 0.177, 0.195, 0.214).$$

Each of these current-investment amounts corresponds to an initial investment of $J_{-s} = \theta_s(0)/(1+\rho)^s = 0.091$ for $s = 1,...,n-1$.

To compute the equilibrium that evolves from these initial conditions, we only need to find J_0, the total investments that new bankers make at time 0. The contractual investments of each cohort grow by the multiplicative factor $(1+\rho)$ each period until the cohort retires at age n, and then it must be replaced by new cohort whose new investments will equal the final investment of the old retiring cohort divided by $(1+\rho)^{n-1}$, so that the new cohort will repeat the retiring cohort's investments n periods later. Any increase of J_0 would increase all future investments I_t and so would decrease all future returns $r_{t+1}=R^*(I_t)$. In equilibrium, we must have $\sigma_1+...+\sigma_{10} = B$, as in equation [7], this equation here has the solution $J_0 = 0.318$. The resulting 10-period equilibrium credit cycle is shown in Figures 1 and 2.

[Insert Figures 1 and 2 about here]

In this equilibrium, the shortage of bankers at time 0 causes a large cohort of new age-0 bankers to enter and handle investment $J_0 = 0.318$, which is almost three times larger than the steady-state J=0.113 that we found in the previous section. At time 1, the rate of returns on successful investments is $r_1= 1.369$, the banking surplus rate is $\sigma_1=0.069$, and total output at time 1 is 7.5% below steady state. Thereafter, in the shadow of the large J_0, subsequent cohorts of young bankers are smaller, with $J_t=0.091$ for t=1,2,...,9. The economy gradually grows, and just reaches steady-state output at time 6. Thereafter, the growing investments of the large cohort of bankers that entered at time 0 put the economy into a boom with investment and output greater than in the steady state, reaching a peak at time 10, with output 9.6% above steady state and returns $r_{10} = 1.301$.

At time 10, the generation-0 bankers retire and consume their accumulated profits, thus creating a new scarcity of investment intermediaries. Then investment at time 10 drops in a recession to the same level as at time 0, and the cycle repeats itself.

The workers' incomes W_t over this 10-period cycle are

$$(W_1,...,W_{10}) = (0.589, 0.605, 0.623, 0.643, 0.666, 0.691, 0.719, 0.751, 0.786, 0.826)$$

The workers' income and welfare are initially 14% below the steady-state W=0.688, although they later rise to 20% above steady state at the peak of the boom.

IX. Evaluating the benefits of subsidies for financial stabilization or stimulus

In the context of the above example, let us consider the consequences of a financial intervention by the government to stabilize the economy at the steady state. To achieve steady-state stability here at time 0, new investment consortia must hire enough older bankers to restore the steady-state profile of age-cohort investments Θ_s shown in line [10]. That is, for each s in $\{1,...,9\}$, bankers of age s must be given new investments equal to $\Theta_s - \theta_s(0)$. But in the steady state equilibrium, these new investments with age-s bankers would suffer expected losses worth $[\Theta_s - \theta_s(0)](sB/n)\alpha/(1+\rho)$. For stabilization, then, new investors who hire old bankers must get a subsidy worth

$$\sum_{s\in\{1,...,9\}} [\Theta_s - \theta_s(0)](sB/n)\alpha/(1+\rho) = 0.064.$$

This subsidy can be financed by selling bonds to be repaid with interest ρ by 0.070 from lump-sum taxes on the workers at time 1. The cost of this subsidy is less than the increase in wage income 0.688−0.589 = 0.099 that the workers get from the stabilization at time 1. The workers' utility gains here are only half of their wage gains (because of their quadratic disutility of working), but the wage gains continue for 5 time periods. At time 1, the discounted values of future utility gains from stabilization for workers who have 1 to 10 periods of employment remaining in their careers are respectively:

$$(0.049, 0.087, 0.114, 0.130, 0.138, 0.137, 0.128, 0.112, 0.089, 0.060).$$

Middle-aged workers gain the most here. Old workers have less future time to gain, and stabilization eliminates benefits of a future boom for young workers. Aggregating, we find that the time-1 workers' average long-run utility gains from stabilization (0.105) exceed its cost here.

Other examples can be found where stabilization subsidies are not worth the expense for tax-paying workers, however, and it seems difficult to characterize the cases where it is worthwhile. But we can make broader statements about another kind of financial stimulus that is simpler to analyze.

Suppose that we are given an n-period equilibrium credit cycle where, at time 0, continuing bankers of any age $s \in \{1,...,n-1\}$ have contracts to re-invest $\theta_s(0)$, and new bankers would invest J_0 in this equilibrium, for a total investment of

$$I_0 = J_0 + \sum_{s\in\{1,...,n-1\}} \theta_s(0).$$

As above, let us assume the linear investment-demand model, $R(I_t) = \psi - w_{t+1} = \psi - \pi\alpha I_t$.

Now suppose that the government is considering an unanticipated one-period stimulus of the following form: Some amount δ of new short-term investment will be handled by old bankers who will retire next period, and government subsidies will be offered as needed to finance these δ short-term investments and to maintain the given equilibrium quantity J_0 of new long-term investments by young (age-0) bankers at time 0. This plan may be called a *short-term balanced stimulus* because, if people do not expect it to be repeated in the future, then this stimulus at time 0 will have no effect on the equilibrium investment after time 0.

Let us assume that the new investment δ is small enough that

$$R(I_0+\delta) \geq r^* = (1+\rho)/\alpha+E.$$

The δ investment will decrease the rate of return on investments next period from $R(I_0)$ to $R(I_0+\delta)$. The δ investment by bankers who serve only one period requires a time-1 subsidy

$$\delta\alpha[B+E+(1+\rho)/\alpha - R(I_0+\delta)].$$

The J_0 investment by young bankers would have broken even for their investors in the long run if the time-1 rate of return on investments was $R(I_0)$, and so the time-1 subsidy that is needed to maintain this J_0 investment by young bankers is

$$J_0\alpha[R(I_0)-R(I_0+\delta)].$$

The workers' utility from their wage income at time 1 is $0.5\pi\alpha^2(I_0+\delta)^2$. So assuming that the subsidies are paid by lump-sum taxes on workers, the workers' net benefit is

$$0.5\alpha^2\pi(I_0+\delta)^2 - \delta\alpha[B+E+(1+\rho)/\alpha+\alpha\pi(I_0+\delta)-\psi] - J_0\alpha^2\pi\delta.$$

This quadratic in δ is maximized when

$$J_0+\delta = [\psi-(1+\rho)/\alpha-E-B]/(\pi\alpha).$$

The right-hand side here is what total investment would be if all bankers could serve for only one period (that is, it is the steady-state investment for n=1 with all other parameters unchanged). Thus, we find that the workers could benefit from a small short-term balanced stimulus when the investment of young bankers is not too large.

Proposition 3. In the model with linear investment-demand and labor-supply functions, a small short-term balanced stimulus at time 0 which is financed by workers' lump-sum taxes could benefit the workers if the rate of return $R(I_0)$ is greater than the minimal rate r^* and the investment of new bankers J_0 is less than what total investment would be in the economy with n=1 (one-period banking) but with all other parameters the same.

In our numerical example, the steady state with n=1 had total investment I=0.233, but the steady state with n=10 has young bankers investing J=0.113. So with n=10 here, a short-term balanced stimulus would actually increase workers' welfare when the economy is not too far from the steady state.

Such a stimulus reduces the profits of established contracts with older continuing bankers, but we are assuming that investors cannot alter the re-investment that these contracts stipulate. So this analysis relies critically on an assumption that the stimulus would not be anticipated by investors, and would not induce expectations of other such interventions in the future.

X. Other equilibrium scenarios

In Section VIII we considered an example with bankers starting 20% below the steady-state levels. Now let us consider the same example when, at time 0, all continuing cohorts of bankers are investing 20% more than their steady-state levels. So we have n=10 and all other parameters as in [9], but the re-invested funds $\theta_s(0)$ for bankers of each age s at time 0 are now

$$(\theta_1(0)...,\theta_9(0)) = (0.150, 0.165, 0.181, 0.199, 0.219, 0.241, 0.265, 0.292, 0.321).$$

Given these investment amounts at time 0, we can compute what the initial investment had to be for each cohort when it started in the previous n−1 periods

$$J_{-s} = \theta_s(0)/(1+\rho)^s = 0.136 \quad \text{for } s = 1,...,n-1.$$

To characterize the dynamic equilibrium that follows from this initial condition, we need to find the amount J_0 that is invested by new young bankers at time 0. Given any guess for J_0, each cohort's investments will grow by the multiplicative factor $(1+\rho)$ from each period to the next until it retires. Then, if we are in a cyclical equilibrium, the cohort that retires at time n−s will be replaced by a new cohort with the same initial size as its predecessor of n periods before; that is

$J_{n-s} = J_{-s}$ for s = 1,...,n−1. In a cyclical equilibrium, the resulting investments must yield return rates with banking surpluses that just cover the bankers' moral-hazard rent B over the next n periods, as in condition [7] above. But condition [7] cannot be satisfied for this example, because any nonnegative J_0 will yield investment returns in times 1 through n that are too low:

$$\sum_{t\in\{0,\dots,n-1\}} [R(\sum_{s\in\{0,\dots,n-1\}} J_{t-s}(1+\rho)^s) - E-(1+\rho)/\alpha] < B, \ \ \forall J_0 \geq 0.$$

That is, the continuing contractual investments at time 0 are too high to admit any profitable investment by new bankers at time 0. Thus we must have $J_0 = 0$ in a dynamic equilibrium here.

The failure to satisfy the banking-rents equation [4] at time 0 means that initial conditions cannot be part of a cyclical equilibrium, and so the cohort that retires at time 1 can be replaced by a new cohort that is smaller. That is, J_1 here may different from J_{-9}, and instead we can choose J_1 to start a new cyclical equilibrium with $(J_{-8},J_{-7},\dots,J_0)$, by satisfying the equation

$$\sum_{t\in\{1,\dots,n\}} [R(\sum_{s\in\{0,\dots,n-1\}} J_{t-s}(1+\rho)^s) - E-(1+\rho)/\alpha] = B$$

with $J_t = J_{t-n}$ for all $t\geq 2$. This is solved by $J_1 = 0.045$, satisfying condition [8] with T=1.

[Insert Figure 3 about here]

Thus, the vector $(\theta_1(0)\dots,\theta_9(0))$ here yields a dynamic equilibrium which begins with one transient period at time 0, when returns are too high for any new bankers to enter, and thereafter it evolves as an n-period equilibrium credit cycle repeating the sequence of investments and returns from times 1 to 10. Figure 3 shows the distribution of investments across age cohorts and time in this equilibrium. In the bar at time 0, the shaded parts indicate the pattern of investments that is repeated at time 10 and every 10th period thereafter, and the white rectangle at the top of the bar indicates an additional investment (0.214) by old bankers at time 0 that is not repeated by old bankers at time 10 or thereafter. Aggregate investment declines slowly from time 0 to time 9. Thereafter, in each subsequent pass through the 10-period cycle, the economy grows strongly in the first two periods, as the small cohorts retire, but then the economy drops into another long slow recession, as the large cohorts retire. Investment is 6.7% above the steady state at the top of the cycle (at times 11, 21, etc), but it is 8.5% below the steady state at the bottom of the cycle (at times 9, 19, etc.).

This example represents an economy that has inherited a banking system that is too large to be sustainable. The large cohorts of old bankers can keep investment above the steady state for several periods, but only as the start of a long economic decline. This model of "zombie" bankers, continuing beyond their natural economic lives, might be interpreted as a simple model of Japan's lost decade after the collapse of the 1980s boom.

Finally, let us consider what would happen in the worst-case scenario when the economy starts at time 0 with no bankers at all, so that $\theta_t(0)=0$ for all t. From this initial condition, with the parameters above in [9], the equilibrium credit cycle has initial investment $J_0 = I_0 = 1.338$. Investment then grows at the maximal rate ρ for 4 periods and thereafter levels off at the peak $I_t = 1.983$ from time t=5 onwards, with no new entry of bankers until the generation-0 bankers retire at period n. Thus, output at the trough in this worst-case scenario is 33% less than output at the peak, which takes 5 periods to reach from the trough. Remarkably, this result does not depend on the parameter n, as long as n>5. That is, the potential depth and duration of recessions in our model do not depend on the length of the bankers' careers.

XI. Conclusions

Financial crises and recessions are vast complex phenomena, but their inexorable momentum must be derived from factors that are fundamental in economic systems. Theoretical models are tools that can help us see what these driving factors might be. In this paper, we analyzed a theoretical model to show how moral hazard in financial intermediation can cause aggregate economic fluctuations, even in a stationary economic environment without money or long-term assets.

The key to our analysis is that, to efficiently solve financial moral-hazard problems, bankers must form some kind of long-term relationship with communities of investors. The state of these relationships can create complex dynamics, even in an economy that is otherwise completely stationary. These dynamics are driven by the basic fact that, at any point in time, investors' ability to trust their bankers depends critically on expectations of future profits in banking. Cyclically changing expectations can rationally sustain an equilibrium cycle of booms and recessions.

In the recessions of our model, aggregate production declines as productive investment is reduced by a scarcity of trusted financial intermediaries. Competitive recruitment of new bankers cannot fully remedy such an undersupply of financial intermediaries, because moral-hazard constraints imply that bankers can be hired efficiently only as part of a long-term career plan in which the bankers' expected responsibilities tend to grow during their careers. Because of this expected growth of bankers' responsibilities, a large adjustment to reach steady-state financial capacity in one period would create excess financial capacity in future periods. Thus, a financial recovery must drive gradually uphill into the next boom, when the economy will have an excess of bankers relative to what can be sustained in the steady state, and this boom can in turn contain the seeds of a future recession.

A stabilization that shifts the economy from such a recession to the steady state would require some new investments to be handled by older bankers who are more expensive, because their moral-hazard rents cannot be distributed over as many periods of future investment. Investors would be unwilling to use these costly shorter-term intermediaries without a subsidy. But we found that, in some cases, the workers' benefits from such macroeconomic stabilization may be greater than the cost of the required subsidies. In this sense, a tax on poor workers to subsidize rich bankers may actually benefit the workers, as the increase of investment and employment can raise their wages by more than the cost of the tax. Some of these wage increases, however, would come at the expense of other investors who must re-invest past earnings under previously negotiated financial contracts.

This paper is part of a growing theoretical literature on the important question of how macroeconomic instability may be derived from incentive constraints in microeconomic transactions, and more models are needed. The model here has made many simplifying assumptions which should be relaxed in future research.

Appendix: Recursive formulation of investors' optimal contracts with bankers

Consider a contractual relationship, at time t, between a consortium of investors and a banker who started at time 0. Let y_t denote the value at time t of rewards that were previously promised to the banker by the consortium. Let m_{t+1} denote the expected marginal cost to the consortium at time t+1 of increasing the banker's expected future rewards by one unit of value at time t+1. Rewards cannot be deferred at time n, so $m_n = 1$. At time 0, the investors have made no prior promise to the young banker, and so $y_0 = 0$.

In the contract, let h_t denote the size of their investment at time t. For the cases of success and failure, respectively, let e and f denote payments to the entrepreneur, and let b and c denote the value of rewards to the banker at time t+1. Then the consortium's optimization problem at time t is:

choose $h_t \geq 0$, $b \geq 0$, $c \geq 0$, $e \geq 0$, and $f \geq 0$ so as to

maximize $\alpha(r_{t+1}h_t - e - m_{t+1}b) - (1-\alpha)(f + m_{t+1}c) - (1+\rho)h_t$

subject to $\qquad$ [*Lagrange multipliers*]

$$\alpha e + (1-\alpha)f \geq \gamma h_t + \beta e + (1-\beta)f, \qquad [\lambda_e]$$

$$\alpha b + (1-\alpha)c \geq (\gamma+\eta)h_t + \beta(b + e) + (1-\beta)(c + f), \qquad [\lambda_b]$$

$$\alpha b + (1-\alpha)c \geq (1+\rho)y_t. \qquad [\mu_t]$$

If $\sigma_{t+1} > Bm_{t+1}$ then infinite solutions would be feasible with $c=0$, $f=0$, $e=Eh_t$, $b=Bh_t$, taking $h_t \to +\infty$. So we must have $m_{t+1} \geq \sigma_{t+1}/B$. Then we can show that the optimal solution is

$$c = 0,\ f = 0,\ e = h_tE,\ b = h_tB,\ \text{and}\ h_t = y_t(1+\rho)/(\alpha B).$$

This solution satisfies the three constraints with equality, and it maximizes the Lagrangean with multipliers:

$$\lambda_e = (\alpha+\lambda_b\beta)/(\alpha-\beta),\ \lambda_b = \alpha\sigma_{t+1}/[(\alpha-\beta)B],\ \mu_t = m_{t+1} - \sigma_{t+1}/B.$$

These make e, h_t, and b drop out of the Lagrangean, which becomes

$$-\mu_t(1+\rho)y_t - c\sigma_{t+1}/B - (1+\lambda_b)f.$$

Thus, in the optimal solution (with c=f=0) the consortium at time t+1 has an expected net cost

$$\mu_t(1+\rho)y_t = (m_{t+1} - \sigma_{t+1}/B)(1+\rho)y_t.$$

This expected cost is linear in the previously promised rewards y_t, and this linearity can recursively justify our assumption of linear costs for future promised rewards b and c. A unit increase in y_t would increase the consortium's expected cost at time t+1 by $(1+\rho)\mu_t$, and so it would increase the consortium's expected cost at time t by μ_t. So the Lagrange multiplier μ_t that we get from the above problem is equal to the parameter m_t that is the marginal cost of rewards promised to the banker at time t, for the consortium's analogous investment problem at time t–1:

$$m_t = \mu_t = m_{t+1} - \sigma_{t+1}/B.$$

Thus, with $m_n = 1$, we get by induction:

$$m_t = \mu_t = 1 - (\sigma_{t+1}+...+\sigma_n)/B,\ \forall t \in \{1,...,n-1\}.$$

If successful at time t+1, the banker will be promised $y_{t+1} = b = h_tB$, and her next investment will be $h_{t+1} = h_tB(1+\rho)/(\alpha B) = h_t(1+\rho)/\alpha$. Thus, success multiplies her investment by $(1+\rho)/\alpha$ each period.

At time t=0, we have $y_0=0$, but then a solution $e = h_0E$, $b = h_0B$, and $h_0 > 0$ can be optimal for the consortium, with slack in the promise-keeping constraint, if and only if this constraint has multiplier $\mu_0 = 0$, which is equivalent to the banking-rents equation [4].

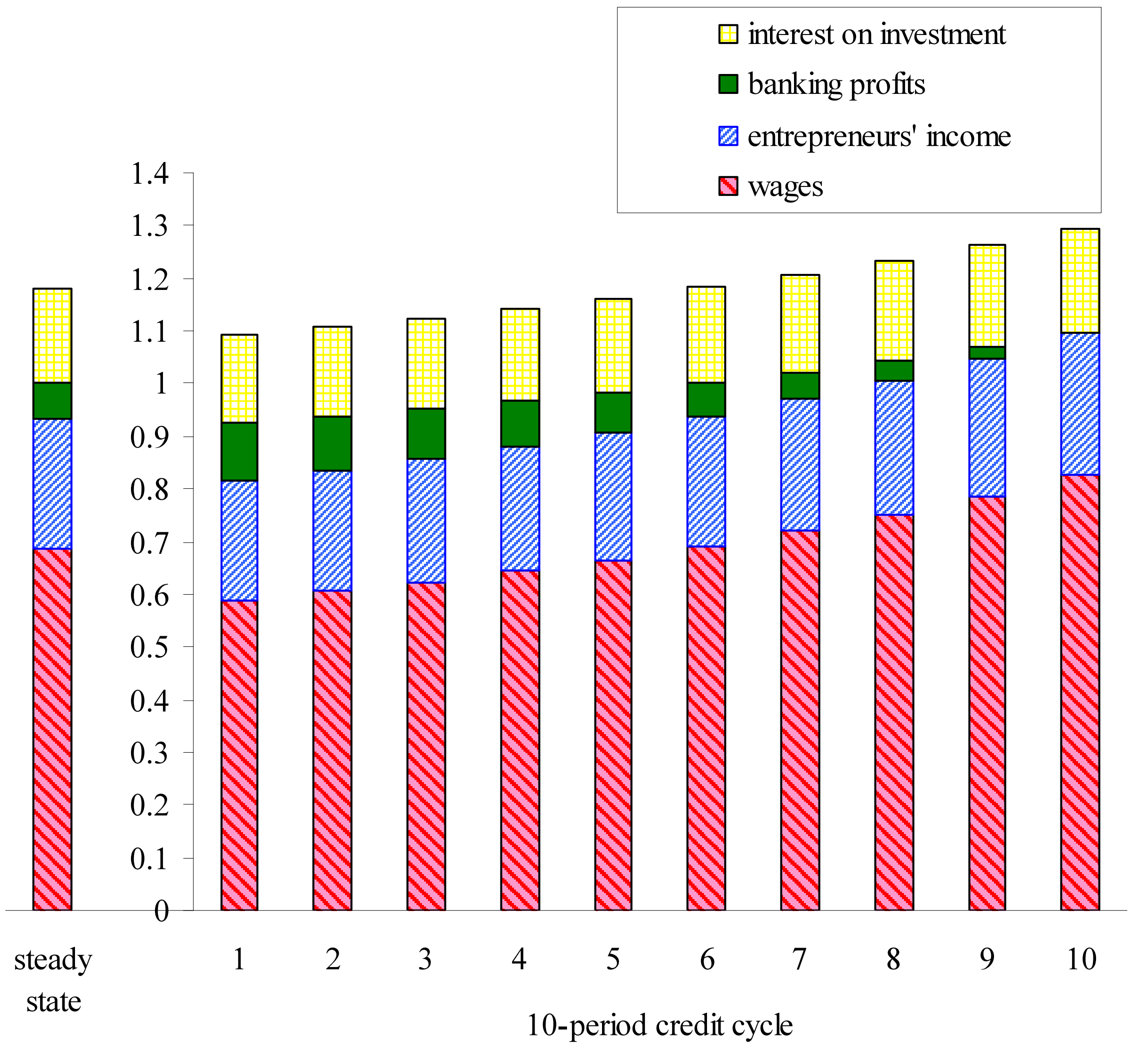

Figure 1. Net product in a 10-period credit cycle, with continuing bankers' investments at time 0 being 80% of steady state. Parameters: $\rho=0.1$, $n=10$, $\alpha=0.95$, $\beta=0.6$, $\gamma=0.05$, $\eta=0$, $\psi=1.74$, $\pi = 0.233$.

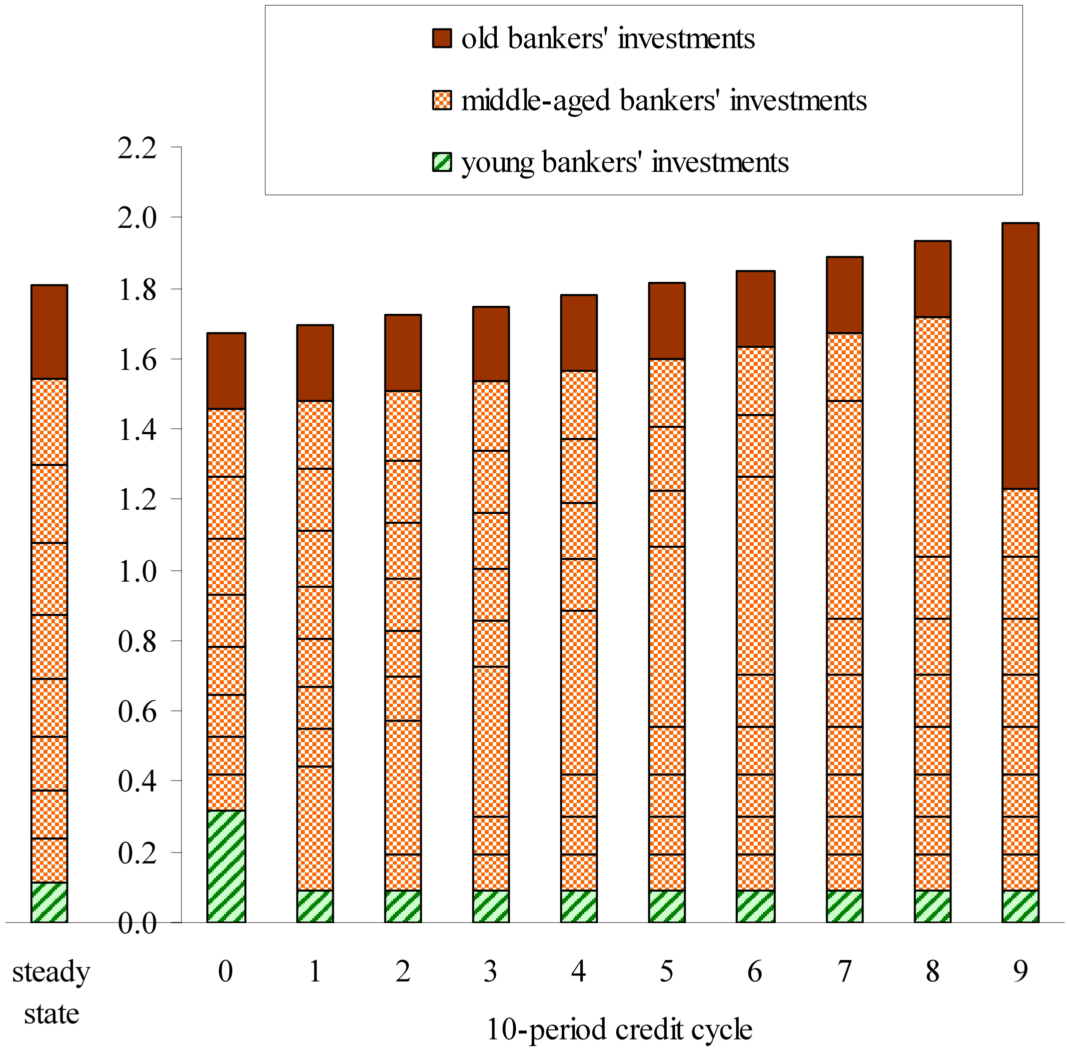

Figure 2. Investment amounts handled by different generations of bankers over a 10-period credit cycle, with continuing bankers' investments at time 0 being 80% of steady state. Same parameters as in Figure 1

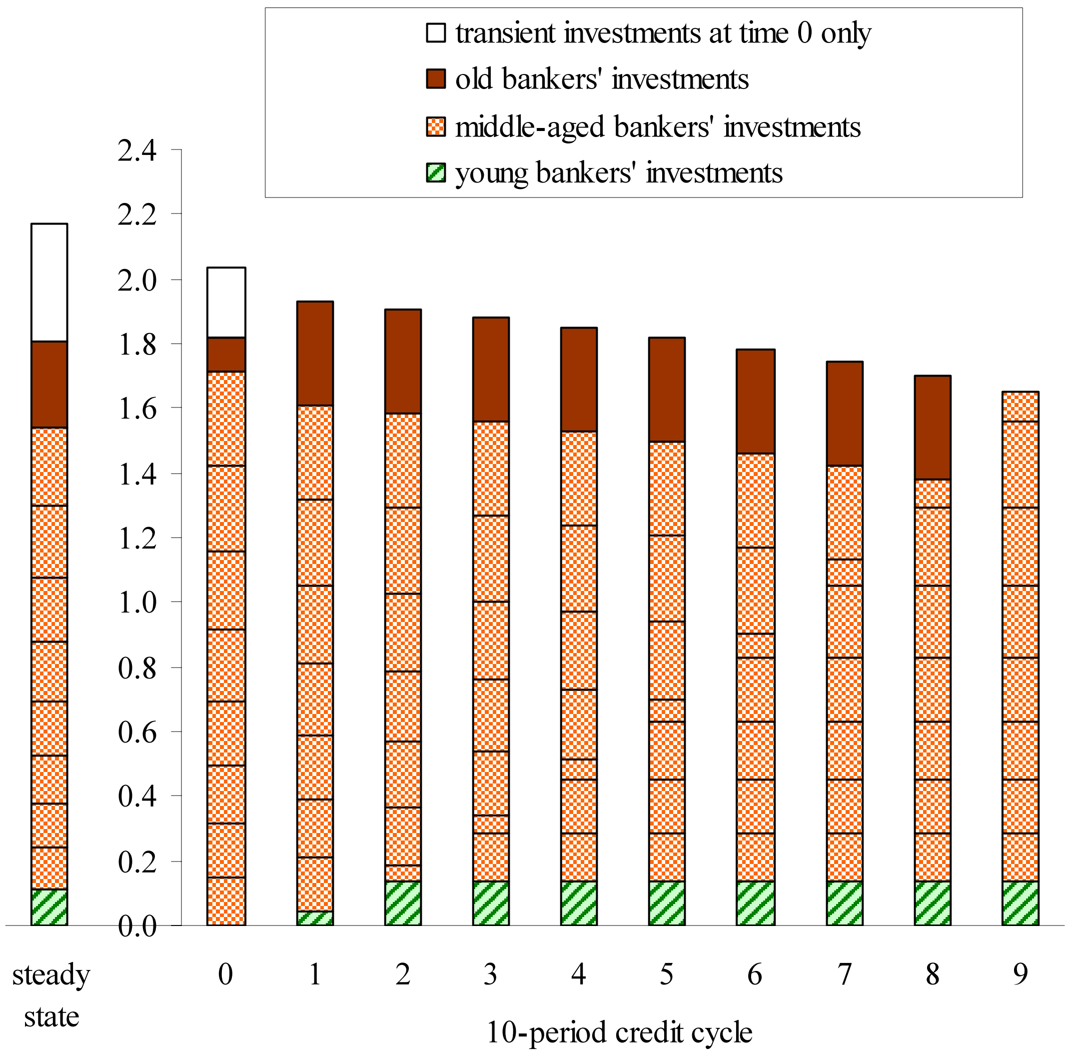

Figure 3. Investment amounts handled by different cohorts of bankers over a 10-period credit cycle, with continuing bankers' investments at time 0 being 120% of steady state (zombie banks). Same parameters as in Figure 1.

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