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Insights

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LOSS GIVEN DEFAULT AND ECONOMIC CAPITAL

Lenders know that any loan might default. They also know that the default rate can rise and produce additional loss. To absorb this possible loss, lenders must hold capital.

A second effect can compound the loss. This is a rise in the loss given default rate, or LGD rate. LGD is the fraction of exposure lost when a loan defaults. When the default rate rises, the LGD rate tends to rise as well. So when lenders think about possible loss, the variation of the default rate is only half the picture. The variation of the LGD rate can be just as important.

This article outlines a framework for understanding the coordinated rise of the default rate and the LGD rate.[1] It views both rates as connected to a single underlying factor. A single factor model provides insights into what happens to lending portfolios when the economy has a bad year. The loss expected in an extremely bad year—quantified at a point on a probability scale—equals economic capital.

Loss Given Default and ELGD

Default occurs when an obligor fails to meet a financial obligation. A bank's working definition of default might depend on the product, but all we really need to know is whether a given loan has defaulted or not.

For a defaulted loan, loss given default (LGD) is the proportion of exposure that is lost. This is an economic concept that does not necessarily appear in conventional financial accounting records. Since LGD is a fraction, it takes values between zero and one.

For a loan that has not (yet) defaulted, LGD is a random variable. Much of what follows is an exploration of the distribution of random LGD, but an important observation arrives at the outset: LGD is independent of default. That is, the mere fact that a default occurs tells us nothing about the loss given default that results from that default. That is because LGD already presupposes the default event has occurred—consistent with its name, it is loss given default.[2]

The independence of the default of a loan and its LGD allows the following insight. The expected loss on a loan equals its expected default rate (usually denoted "PD" for "probability of default") times its expected LGD rate (ELGD). Stated symbolically,

$$EL = PD \bullet ELGD$$

For example, if a loan has probability of default equal to 5% and expected LGD equal to 40%, its expected loss is 2%. Naturally, this set-up applies to non-defaulted firms and loans. When a default actually occurs, we deal with realizations, not with expectations.

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It is important to note the difference between the symbols. LGD is a random variable that has some distribution, and ELGD is the mean of that random variable. In the same way, PD is the mean of the distribution of default; a default either occurs or it does not, but its mean occurrence equals PD.

Even though a realized LGD is independent of the default that brings it into being, there is a correlation between *different* LGDs, and this is where things get interesting.
If recent LGDs have been elevated, the economy is probably in a depressed condition. In depressed conditions, the next LGD is apt to be elevated as well. This imparts a correlation between the LGDs that implies a double misfortune for lenders—in a bad year both the default rate and the LGD rate tend to rise. To prepare for the losses that can come about, bankers use the idea of economic credit capital.

Economic Credit Capital

The concept of economic capital quantifies the risk faced by a financial institution over a defined period, usually one year. All types of loss, not just credit loss, are included in the economic capital concept. While keeping this in mind, we look only at economic credit capital.

Economic credit capital is sometimes called "credit value-at-risk", or "credit VaR". That is because, like market VaR, economic capital is defined as a high percentile of the loss distribution.[3] The 99.9th percentile loss would mean the loss occurring once in a millennium—*if* the model reflected all the elements of reality. But every model simplifies and stylizes reality. We use the 99.9th percentile without really believing that the chance of losing economic capital exactly equals one in one thousand.

Economic capital models tend to estimate loss at very high percentiles. Against that backdrop, a most important fact is that good data on default rates and LGD rates are available for only about twenty years. Therefore, economic capital models depend considerably on statistical theory. By contrast, market VaR depends less on theory, both because a lower percentile (perhaps the 99th) is used, and because there are more data (perhaps 1000 market days) available to characterize the loss distribution.

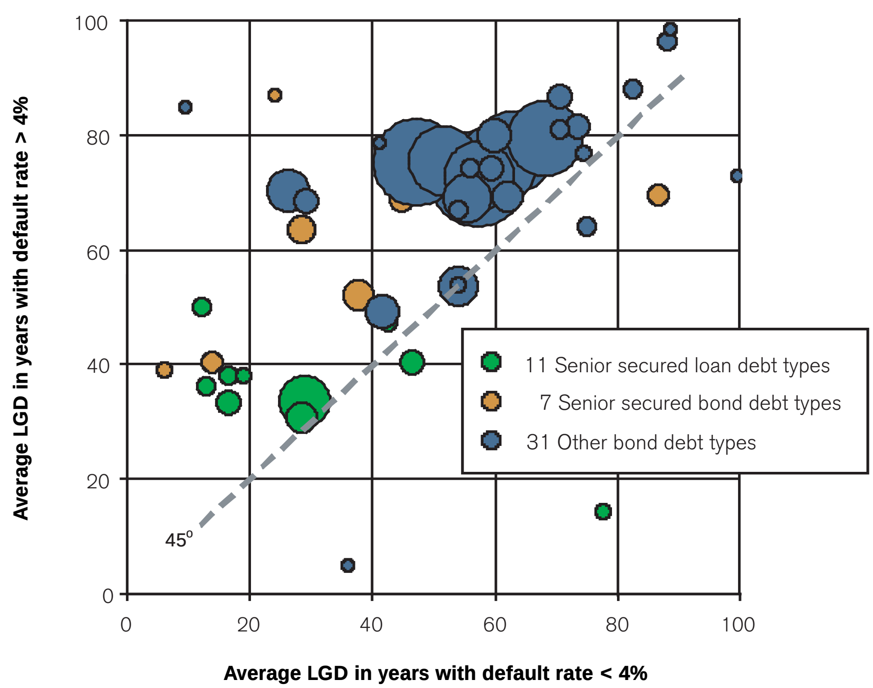
A loss distribution depends on so-called "risk factors". In a market VaR model, the factors are the prices of securities or derivatives that closely resemble portfolio holdings. In an economic capital model, the risk factors are more abstract. They are simply the driving forces behind variations in default and LGD—whatever makes the rates rise and fall. There is no need to associate credit risk factors to economic data such as GDP, stock prices, or interest rates, though it can be illuminating to do so.

Single Risk Factor Models

The economy, for all its diversity, can be viewed as a single entity. Though different localities or industries might be affected more or less, a "recession" is a recession for the whole economy. Therefore, it is reasonable to think of a single economic factor as driving the ups and downs of economic activity. Similarly, we think of a single risk factor driving the ups and downs of default rate and the LGD rate.

At a given bank, purely random influences can cause its default rate and its LGD rate to deviate from the rates experienced system-wide. But if a bank portfolio is large enough and well diversified enough, the deviation is very small. This is an example of the law of large numbers at work. A given firm might or might not default, but there are enough

Figure 1:
LGD in good years and bad years



Bubble size indicates number of LGDs in a debt type
Source: Moody's Default Risk Service

firms that the main concern is the (conditionally) expected default rate. A given defaulted loan might have any LGD, but there are enough defaulted loans that the main concern is the (conditionally) expected LGD rate.

These two features—a single risk factor and a large, well-diversified portfolio of loans—are key features of the model described here. In this model, default and LGD vary from year to year because of variation in a risk factor called Z. The annual rates of default and LGD are said to be conditioned on Z. A greater value of Z for a year brings about a greater conditionally expected default rate and a greater conditionally expected LGD rate. Since Z has an effect throughout the portfolio, it is called the "systematic" risk factor.

If Z reflects "stress" conditions—if it is drawn at the 99.9th percentile of its distribution—both the LGD rate and the default rate are at the 99.9th percentiles of their respective distributions. The product of the two is the loss rate at the 99.9th percentile. We can summarize this by saying that in a single factor model, economic capital for a loan equals its stress default rate times its stress LGD rate.

This context shows the potential importance of LGD for economic capital. The stronger the effect of Z on the LGD rate, the greater is stress LGD, and the greater in turn is economic capital.

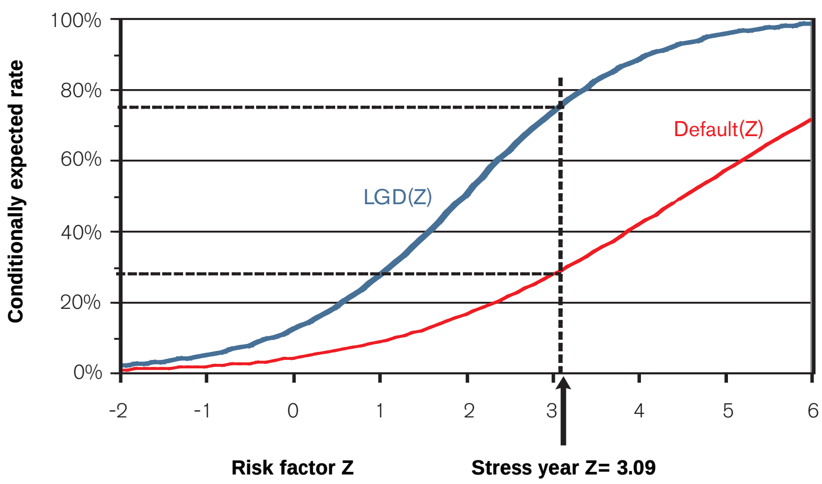
Evidence of Coordinated Variation

This section examines evidence that the LGD rate and the default rate are linked. Moody's Default Risk Service provides for rated firms both the dates of any defaults and the prices of defaulted loans and bonds a few weeks later. The percentage difference between par and a post-default price measures an LGD.

The data sample comprises nineteen years, 1983-2001. The years separate into "good" (low default) years and "bad" (high default) years. The purpose of separating is to see what happens to LGD when the default rate is high. We take a bad year to mean any year where the default rate is greater than 4%. This identifies four bad years: 1990, 1991, 2000, and 2001. The four bad years have about as many defaults as the fifteen good years.

Moody's observes a "debt type" for each defaulted loan or bond. A debt type designation is rather detailed, and there are thousands of debt types in all. Of them, only forty-nine debt types let us see LGD in both good years and bad years, because they are the only debt types that have experienced at least one default in each kind of year.

Figure 2:
LGD and default as functions of risk factor z



The LGDs of the forty-nine debt types are depicted as forty-nine "bubbles" in Figure 1.[4] The horizontal position of a bubble reflects average LGD in good years, and the vertical position reflects average LGD in bad years. The size of a bubble represents the total number of LGDs observed within the debt type. The smallest bubbles represent only two or three LGDs. The largest bubble ("Senior Subordinated Notes," which is mostly covered up by other bubbles) represents 111 LGDs. There are 859 LGDs in total.

If the default rate and the LGD rate were unconnected, the debt type bubbles in Figure 1 would scatter equally above and below the 45-degree line. That is because the 45-degree line indicates the LGD rate is the same whether the default rate is high or low. But the appearance of Figure 1 is far different. Many more than half of debt types appear above the 45-degree line, and this result is statistically significant.

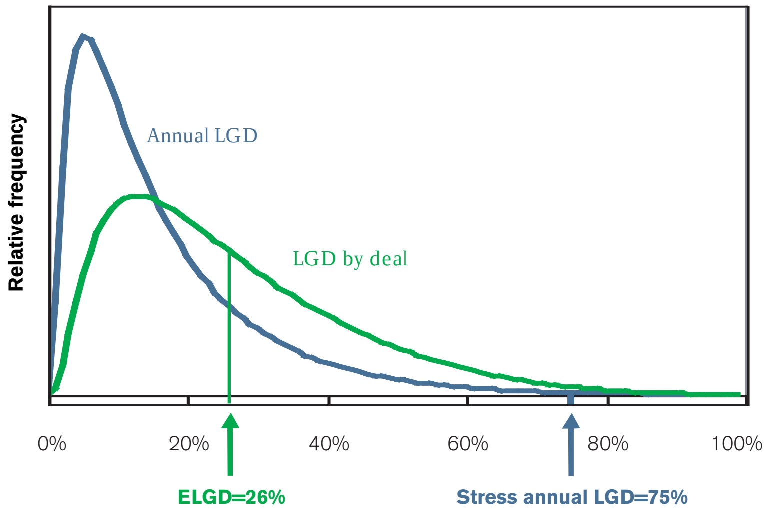
The main message of Figure 1 is that the default rate and the LGD rate tend to rise together. In the context of our model, both rates respond a common systematic risk factor.

Modeling LGD and Default Rates

Returning to the simple model introduced earlier, we tie the default rate and the LGD rate to Z. Worse conditions produce a greater default rate and a greater LGD rate. Figure 2 displays examples of LGD and default functions. Though they have not been calibrated to data, they are useful to show the basics of working with conditionally expected default and LGD rates.

To convert the mathematical functions of Figure 2 into probability distributions, we need to know the probability distribution of Z. The most common

Figure 3:
Two distributions of LGD



assumption is that Z follows a standard normal distribution. Thus, each year we imagine a randomly drawn normal variable, Z. The realized level of Z implies the default rate and the LGD rate for the year according to the functions shown in Figure 2.

The most common realizations of Z are near zero. Figure 2 shows that those years have an expected default rate near 4%. Away from Z = 0 there is a basic asymmetry: If Z is positive, the default rate can become quite elevated; but if Z is negative, the default rate can fall only to 0%. When all the possible default rates are weighted by their probabilities, the "unconditionally" expected default rate of the example function—in other words, PD—equals 5%. Figure 2 also shows that when Z takes its 99.9th percentile value (3.09), it produces the stress default rate of 28%.

LGD distributions

Given the LGD function shown in Figure 2 and the distribution of Z, one can use standard mathematical techniques to derive the distribution of LGD. The distribution appears in Figure 3. As in any probability distribution, the horizontal axis shows values of the random variable (LGD) and the vertical axis shows the relative frequency of the values.

In fact, Figure 3 shows two distributions of LGD. The one labeled "Annual LGD" is the one that we just derived. It shows the distribution of LGD that comes about when each year a randomly drawn Z produces a year's LGD rate.

Figure 3 also shows the distribution of *LGD by deal*, and it is quite different. The distribution of LGD by deal depends on both the distribution of annual LGD and the default function. The idea is that when the annual LGD rate is elevated, there are more defaults than usual. (This is because Z is elevated.) So compared to the distribution of annual LGD, the distribution of LGD by deal has more weight on the right and less weight on the left.

It is usually easier to think about data than distributions. Imagine there are three years of default and loss data as shown in Table 1.

Table 1:
Annual Average LGD

Year	Number of defaults	Average LGD
1	20	20%
2	50	50%
3	30	20%

The table shows three annual averages for LGD. Conceptually, these are drawn from the distribution of annual LGD. The average of these averages is 30%. This is the data representation of the mean of the annual LGD distribution, though this quantity usually plays no role in modeling credit risk.

The table shows 100 defaults in total. Conceptually, the associated LGDs are drawn from the distribution of LGD by deal. Average LGD is 35% (half the LGDs are from years averaging 20% and half the LGDs are from years averaging 50%). This is the data representation of the mean LGD by deal, or ELGD.

Using some math that mimics these procedures, finding the distribution of *LGD by deal* depends on bringing together the *distribution of LGD by year* and the *default rate by year*. If the default rate is greater in years where LGD is elevated, ELGD exceeds the mean annual LGD. It is not exaggerated to say that a bank lives by the distribution of LGD by deal, but it dies by the distribution of LGD by year. A bank lives by making profitable loans. The pricing for each should include expected loss. As we have seen, the expected loss (EL) for a loan equals PD for the loan times expected LGD of the loan, which is the mean of the distribution of LGD by deal. Figure 3 shows its mean equals 26%.

By contrast, a bank dies in an adverse year. In a year when Z is at its 99.9th percentile, LGD can be found from Figure 2. It is also noted in Figure 3 on the distribution of annual LGD. Stress LGD—the 99.9th percentile of annual LGD—equals 75%.

An economic capital function expresses the risk of a loan as a function of its characteristics, principally, its PD and its ELGD. To find ELGD we must dig a bit deeper than the conditionally expected LGD rates, as we have seen. Once this subtlety is understood, it presents no impediment to converting a single factor model into an economic capital function.

This provides a framework for understanding the systematic variation of default and LGD, and for creating a model that can be calibrated to default and loss data. That calibration, and the economic capital function that results, is the subject of a subsequent article in *Capital and Market Insights.*

Conclusion

Investigation has shown that when the default rate is elevated, the LGD rate also tends to be elevated. Tying both rates to a single systematic risk factor, we derive the distribution of the default rate and the distribution of annual LGD. The 99.9th percentiles of these distributions provide the respective stress values. The product of the two stress values equals economic capital.

An economic capital function states economic capital as a function of the characteristics of a loan—its PD and its expected LGD (ELGD). ELGD comes from another distribution, the distribution of LGD by deal. The distribution of LGD by deal depends on the distribution of annual LGD and the default rate function. Since more defaults happen in years when LGD is elevated, the distribution of LGD by deal is shifted right compared to the distribution of annual LGD.

The framework presented here keeps a clear focus on the quantities that matter: PD, stress LGD, and ELGD. In doing so, it avoids many of the complications encountered by previous models of systematic LGD variation. It also allows for direct calibration of the model, using the technique of maximum likelihood estimation. The estimated model, and the resulting economic capital function, will be presented in a subsequent issue of *Capital and Market Insights.*

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Footnotes

1 It draws heavily from Frye, "Loss Given Default and Economic Capital," forthcoming as a chapter in Economic Capital edited by Ashish Dev.

2 LGD might depend on anything but the default event itself. For example, the reason for default (say, fraud by senior management) may tell something about LGD.

3 Some analysts prefer to work with "unexpected credit loss," which is loss minus EL.

4 Figure 1 originally appeared in Frye, A False Sense of Security, Risk Magazine, August 2003, pp 63-67.

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