



Federal Reserve Bank of Chicago

**The Pitfalls in Inferring Risk from Financial
Market Data**

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Abstract

This paper examines two qualitative rules of thumb, frequently invoked in discussions of bank regulatory policy. The first, that equity holders prefer more risk to less, derives from a result in option pricing theory, that an option's value increases monotonically with the riskiness of the underlying asset. This result is shown to depend on very restrictive assumptions regarding the underlying assets return distribution and the type of option being considered. These restrictive assumptions do not generally obtain in practice. The second rule of thumb is that bondholders' and deposit insurers' interests are aligned. The paper shows that, in fact, their interests can diverge in the sense that bondholders and deposit insurers will not necessarily agree on the relative riskiness of different banks or bank portfolios. The conclusion of this paper is that rules of thumb can be misleading. Furthermore, the concept of risk is shown to be model and agent specific.

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The analysis and conclusions expressed herein represent the author's personal opinion, which do not necessarily coincide with those of the Federal Reserve System or the Federal Reserve Bank of Chicago.

“..., standard option pricing theory suggests that, all else being equal, the value of equity increases with the risk of a banking organization’s assets.” (Kwast et al., 1999)

“...the incentives of the subordinated debt holders and the deposit insurance agency are aligned.” (Kaufman et al., 2000).

1. Introduction

Financial models are useful for understanding the relation among economic factors, for instance risk and return. Models are, however, based on simplifying assumptions, and the qualitative relations that we infer from these models do not always hold when the assumptions are violated. Similarly, intuitions based on simple conceptual frameworks may not be valid under more complex circumstances.

This paper will examine two commonly rules of thumb that arise from such simple models. The first is the hypothesis that option value increases in the risk of the underlying asset. The second is that fixed claimants of a firm, sharing common attitudes towards risk, will agree in their assessments.

Empirical option pricing models, used to fit option prices, are susceptible to testing. However, the hypothesis that option value increases monotonically with risk is a qualitative assertion that is not generally tested. When the option-value/risk relation is combined with the idea that equity holders of a firm have an option-like claim, it leads to the conclusion the equity holders prefer more risk to less. It may be argued that, given the chance, equity holders will expropriate other creditors by undertaking more risky projects once the return on debt has been locked in. This asset substitution moral hazard problem is considered to be particularly severe when the cost of debt is insensitive to risk or is underpriced as happens with deposit insurance.

The second section of this paper examines the option-value/risk monotonicity hypothesis from a number of perspectives. It is shown that, in general, the hypothesis that option value increases in risk requires that the underlying asset values have a simple two-parameter distribution—for example, lognormal. When comparing options on two different assets both underlying distributions must come from the same two-parameter family for the hypothesis to be true. Two-parameter distributions work in this context because variance can be unambiguously equated with risk, and higher moments are fixed

functions of the first and second moments of the distribution. For arbitrary distributions, however, the concept of risk is ambiguous. A simple numerical example shows that variance does not necessarily provide a reliable ordering of option values. Indeed, in cases where the ordering of the values of two options varies with the strike price, then no measure of risk can possibly exist that satisfies the monotonicity hypothesis. Even under simple two-parameter distributions the option-value/risk relation depends on the type of option involved. Several cases applicable to the equity holder incentive question are discussed in the same section.

The hypothesis that all fixed claimants share common attitudes towards risk does not appear to arise from a specific model of required returns for providers of capital. Rather, it is based on intuition. *Ceteris paribus*, an increase in asset risk seems likely to increase the risk for all fixed claimants. This statement is, however, ambiguous: asset risk is undefined, as are the risks are claimants concerned with. Claimants may be concerned with the probability of default, if default *per se* triggers non-economic consequences. Or they may be interested in purely economic losses, in which case expected loss rather than default is the relevant measure of risk. Asset risk might be measured with variance, but again if all underlying distributions are not from the same two-parameter family similar-variance portfolios of assets can present different risks to different creditors.

The third section of this develops a rational model of bond pricing under the assumption that bondholders are risk-neutral, deposits have a senior claim on the assets of the bank, and that deposit insurers bear any losses if the bank cannot repay depositors in full.¹ The method of analysis utilizes comparisons of pairs of asset distributions, representing alternative investments a bank might make, holding capital structure fixed. The model shows that bondholders do not always price bonds so that higher coupons correspond to higher bond- or deposit-default risk. While higher coupons correspond (by construction) to higher expected bondholder losses, it is not true that they necessarily correspond to higher deposit insurer losses.

¹ Deposit insurance pricing is not considered. This is consistent with current U.S. practice where deposit insurance is minimally risk sensitive and is arguably under-priced. However, the qualitative results of this analysis are not dependent on this assumption.

The conclusion of this paper is that qualitative results from simple models and intuitions are apt to break down in more complex and realistic situations. In particular, the idea of risk turns out to be complex, and agent and model specific. The usual practice of identifying risk with variance works only in restrictive situations. In general, a single measure of risk consistent across models and agents will not exist.

2. The Monotonicity of the Option-Value/Risk Relation

Every introductory Investments text teaches that the value of an option increases monotonically as the risk of the underlying asset increases.² This relation is widely assumed to be true among academics and practitioners. The hypothesis appears in the real options approach to capital budgeting.³ In the literature on bank capital regulation, the monotonicity hypothesis leads to a moral hazard argument that since riskier investments benefit a bank's equity holders, banks have incentives for excessive risk taking at the expense of the bondholders and deposit insurers.

The monotonicity hypothesis is based on the models of Black-Scholes (1972) and Merton (1973). Within the context of the models and options studied in these papers the relation does indeed hold true. Outside this context, the hypothesis is not necessarily or even generally true.

Literature Review of Sufficient Conditions

A number of papers have established sufficient conditions under which the monotonicity hypothesis will hold. All of these papers considered only fixed-expiry, plain-vanilla European put and call options. Black-Scholes (1972) and Merton (1973) assume that the underlying asset follows a log-normal diffusion process:

$$\frac{dS}{S} = \alpha dt + \sigma(S, t) dz$$

and pays no dividends. In the Black-Scholes model, the volatility term is constant,

² It is common practice to identify the underlying asset's "risk" with the volatility or variance of the underlying stochastic process or the variance of the distribution of asset values at the time of option expiration.

³ See for example Dixit and Pyndyck (1994), p. 153.

$\sigma(S,t) = \sigma$. In the Merton model the volatility term can vary as a deterministic function of time, $\sigma(S,t) = \sigma(t)$. Bergman, Grundy and Weiner (1996) show that the monotonicity hypothesis holds for the general class of univariate diffusion processes even if volatility depends on the level of the underlying asset as long as $\sigma^*(s,t) \geq \sigma(s,t) \quad \forall s,t$ and $\sigma^*(s,t) > \sigma(s,t)$ for some s and t . Changing the distributional assumptions, the payout assumptions, or the types options considered can change the relation between risk and option value.

Jagannathan (1984) does not rely on the Black-Scholes-Merton framework based on assumptions of the underlying stochastic process. Jagannathan instead uses the Harrison and Kreps (1979) risk-neutral pricing framework. He shows in a two-period context that if terminal-value distributions of two underlying assets differ in risk in the Rothschild-Stiglitz (1970) sense of mean-preserving spreads, then otherwise similar options on the riskier asset will be more valuable. Jagannathan thus proves that Rothschild-Stiglitz risk ordering is a sufficient condition for the monotonicity hypothesis to obtain, but does not address necessary conditions. However, the Rothschild-Stiglitz definition of risk is very restrictive. In general, two distributions will not differ by mean-preserving spreads, and thus cannot be ranked on the basis of their “risk” in the Rothschild-Stiglitz sense.

Each of these papers establishes sufficient conditions under which the monotonicity hypothesis will be true. That is not to say however that these conditions obtain in general. No paper of which I am aware has proposed necessary conditions that must obtain if the monotonicity hypothesis is to be true. In the following analysis an alternative approach is taken to the problem. I answer the question “If the monotonicity hypothesis is true, what must be true of asset distributions?”

Distribution-based analysis with European options

The conclusion that the higher the underlying asset’s risk the higher the resulting (European) option value can lead to a potentially untenable conclusion. First we need to be specific about the claim. When comparing options on two underlying assets, the

moneynesses, times to expiry, and types of option must be equivalent.⁴ With this proviso, let us state the monotonicity hypothesis as:

$$\text{Risk}(S_T) \leq \text{Risk}(S_T^*) \Rightarrow C(S_T; K, S_t) \leq C(S_T^*; K, S_t^*) \quad \forall K \quad (1)$$

where K is the common strike price, t and T are the current time and option expiry respectively, and S_t and S_t^* are the current values of the two underlying assets, $S_t = S_t^*$, and S_T and S_T^* are the (random) values of the two underlying assets at the expiration of the option. “Risk” is, for the moment, undefined.

Let us assume for sake of argument that the monotonicity hypothesis is true. It then follows that the reverse of equation (1) also holds

$$\text{Risk}(S_T) \geq \text{Risk}(S_T^*) \Rightarrow C(S_T; K, S_t) \geq C(S_T^*; K, S_t^*) \quad \forall K$$

which in turn implies that

$$\text{Risk}(S_T) = \text{Risk}(S_T^*) \Leftrightarrow C(S_T; K, S_t) = C(S_T^*; K, S_t^*) \quad \forall K. \quad (2)$$

Under the monotonicity assumption, equation (1), equation (2) must hold for all strikes, K .⁵ Using the Cox-Ross (1976) option-pricing framework: the value of an option is the discounted expected payoff under the risk-neutral distribution of the value of the underlying asset at option expiration. For a call option this means

$$C(S_t; K, T) = e^{-r(T-t)} \int_K^{\infty} (x - K) f(x) dx$$

and similarly

$$C^*(S_t^*; K, T) = e^{-r(T-t)} \int_K^{\infty} (x - K) f^*(x) dx$$

where $f(\cdot)$ and $f^*(\cdot)$ are risk neutral density functions for the terminal, time- T , value of S_T . To guarantee that $f(\cdot)$ and $f^*(\cdot)$ are proper distributions

⁴ Obviously a deep in-the-money option will be worth more than a deep out-of-the-money option on a slightly riskier underlying asset. Equivalence is most easily achieved by considering similar-strike call (or put) options on underlying assets that currently have the same value (achievable in practice by scaling—i.e. adjusting the contract quantity).

⁵ An important implication of the monotonicity hypothesis is that

$$C(S_t; K, S_t) > C(S_t^*; K, S_t^*) \text{ for some } K \Rightarrow C(S_t; K, S_t) \geq C(S_t^*; K, S_t^*) \quad \forall K.$$

In other words, option value ordering cannot be strike-dependent.

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{\infty} f^*(x)dx = 1$$

and

$$f(x) \geq 0, f^*(x) \geq 0, \quad \forall x \in (-\infty, \infty).$$

and under risk-neutrality the discounted expected value of the underlying asset must equal its current price:

$$E_{f(\cdot)}(S_T; S_t) = \int_{-\infty}^{\infty} x f(x)dx = S_t e^{r(T-t)}$$

$$E_{f^*(\cdot)}(S_T^*; S_t^*) = \int_{-\infty}^{\infty} x f^*(x)dx = S_t^* e^{r(T-t)}$$

to ensure that the drifts under the risk-neutral distribution equal the risk-free rate, r .

Suppose $\text{Risk}(S_T) = \text{Risk}(S_T^*)$, for some candidate measure of risk, for example variance. Under the monotonicity hypothesis this would imply that $C(S_T; K, S_t) = C(S_T^*; K, S_t^*) \quad \forall K$. Thus

$$\int_K^{\infty} (x - K) f(x)dx = \int_K^{\infty} (x - K) f^*(x)dx \quad \forall K. \quad (3)$$

Following Breeden and Litzenberger (1978) and differentiating equation (3) twice with respect to K results in

$$f(K) = f^*(K) \quad \forall K.$$

In other words, if greater “risk” is invariably associated with greater option value (subject to the above caveats), this implies that equal risk implies equal option value, which in turn implies that the two underlying-asset-value risk-neutral distributions must be identical whenever two similar-strike options have the same value (on two underlying assets that have the same current value). Necessary conditions therefore for the monotonicity hypothesis to be true are that all distributions can be ordered in terms of an appropriate measure of risk (for valuing options) and that distributions of equal risk be exactly identical.

This condition is naturally met whenever both underlying asset distributions come from the same two-parameter parametric distribution, as for instance is assumed in the

Black-Scholes-Merton option-pricing framework. Within a given two-parameter family of distributions, the higher moments are fixed functions of the mean and variance so that “risk” may be unambiguously measured by variance. If, however,

1. The two underlying assets come from different two-parameter parametric distributions, say one is normally distributed and the other is log-normally distributed, it will not necessarily be the case that equal risk will result in equal option prices for all strikes, even though both parameters are tied down.
2. If higher moments are priced variance is no longer a sufficient statistic for risk.
3. If the two underlying asset distributions come from the same parametric family having more than two parameters, then it is possible for drift (expected value of the underlying) and variance to be the same while allowing the distributions to differ in their higher moments resulting in different option prices for the same variance.
4. If the underlying asset distributions are non-parametric, equal variance combined with equal means are insufficient to guarantee equal option values. Thus, defining risk as variance or volatility works only in limited cases, and in those cases where variance is not an unambiguous measure of risk, variance does not provide the hypothesized ordering of option values.

None of these conditions are impossible. Indeed they are quite likely to occur in the real world. It is thus almost certain that the necessary conditions for the monotonicity hypothesis to obtain do not in general obtain. This is not to say that there do not exist some special classes of underlying assets for which the monotonicity hypothesis is true. However, that is an empirical question.

It is possible to construct counter-examples to the monotonicity hypothesis in a Harrison-Kreps no-arbitrage pricing framework—examples where the relative ordering of option values depends on the strike price.⁶ Consider the following example:

Let the risk-free rate be 5 percent. Two assets, A and B, both have current values of 1. Each asset has three equally likely end-of-period values as follows:

⁶ Jaganathan (1984) also presents a counter-example using discrete asset payoffs. His example relies on state-contingent utility of payoffs. The example given here does not.

Example 1

State	Payoffs			σ^2	C(K=1)	C(K=2)
	High	Medium	Low			
Probability	1/3	1/3	1/3			
Asset A	2.041	0.559	0.500	0.491	0.330	0.013
Asset B	2.020	1.130	0.000	0.683	0.365	0.006

The expected payoff of each asset is 1.05, consistent with risk-neutral pricing.⁷ The variance of A is 0.491 and that of B is 0.683. Under the monotonicity hypothesis an option on A should always be worth less than a similar-strike option on B. A call option on A with a strike price of 1 has a value of 0.330, while a similar call option on B has a (higher) value of 0.365, as expected. However, a call option with a strike price of 2 on A has a value of 0.013, while a similar call option on B has a (lower) value of 0.006. Thus, for this distribution of payoffs, the monotonicity hypothesis does not hold.

If one attempts to risk as the property that results in ordering of option values, it will not be possible to assign “risk” ordering to most arbitrary pairs of distributions. As the above example shows it is easy to find option pairs where order changes with the strike price leading to the conclusions that no monotonicity hypothesis-consistent measure of risk exists in these cases. Unlike the Rothschild-Stiglitz mean-preserve spread definition of risk, variance can always be used to order underlying distributions (those with finite variance) in terms of that measure of risk, but cannot always order the values of options on those distributions of payoffs. Variance as a measure of risk is relevant for the monotonicity hypothesis only in very restrictive situations.

Unless one is willing to assume that all underlying asset distributions come from the same two-parameter parametric family or that all underlying asset distributions differ by mean-preserving spreads, both clearly counter-factual assumptions, one cannot conclude that higher “risk,” variance or volatility invariably implies higher European

⁷ A risk-free bond that pays off 1.05 in each state has the required present value of 1.

option value. Thus, the monotonicity hypothesis is not invariably true and is most probably not even generally true.

Different kinds of options and underlying payoffs

The monotonicity hypothesis does not invariably hold, even under the Black-Scholes-Merton log-normal diffusion assumption, for all types of options or underlying payoff streams. When applied to pricing down-and-out barrier options the Black-Scholes-Merton analysis produces a complex relation between option value and volatility. For barrier options close to the barrier, an increase in volatility decreases the value of the option. For a barrier option deeper into the money, option value will increase as volatility increases from zero, but only up to a point, beyond which further increases in volatility again lead to a decline in the value of the option. Merton (1978) found a similar effect modeling the value of equity as a perpetual call option on the value of the firm with the firm subject to random audits. If an audit finds that the value of the firm had declined below the value of debt, the firm is liquidated on the spot. This is an intermediate case between the European option and the barrier option. Merton found that as the value of the firm declined towards the value of debt, equity holders became less risk tolerant (value of equity varied inversely with risk). However, for firms with plenty of economic equity the positive equity-value/asset-risk relation obtained, but only up to a point. If perchance the value of the firm declined below the value of debt before an audit, equity holders become extremely risk seeking in an effort to regain positive equity prior to the next audit.

Ritchken, Thomson, DeGennaro and Li (1993) modeling bank equity as a call option added charter value that would be lost in the event insolvency resulting in a negative payoff to the equity holder rather than the equity call option simply expiring worthless. Ritchken et al also permitted the equity holder to dynamically manage firm risk. Their finding was that as the firm approaches insolvency equity holders will decrease risk. This is consistent with the non-monotone relation between risk and option value found in the barrier option framework. Thus, even under restrictive distributional assumptions the monotonicity hypothesis is not true except for a subset of types of options.

Geske and Shastri (1981) examined American options with uncertain (suspendable) discrete dividends. They show that that in this case the relation between option value and underlying asset risk is not always monotonically increasing.

Summary

The monotonicity hypothesis is a qualitative result that arose from early investigations into the determinants of options prices. These early investigations were made in the context of a specific model (log-normal diffusion) and option type (European puts and calls) where the hypothesis happened to be true. In addition to the original Black-Scholes (1972) and Merton (1973) studies, two previous studies have examined examples of sufficient distributional assumptions under which the hypothesis, applied to European puts and calls, obtains. This note has derived distributional implications of the monotonicity hypothesis that constitute necessary conditions for the hypothesis to obtain. I have shown that in general the hypothesis leads to implausible restrictions on the possible distribution of stochastic generating process and asset terminal value distributions.

An alternative indictment of the monotonicity hypothesis can and has been made by considering alternative options types rather than distributional assumptions. Numerous papers have noted the complexities of risk/option value relations, even in the context of log-normal diffusion processes. Unfortunately these papers are frequently forgotten when the monotonicity hypothesis is invoked, for instance in valuing real options or assessing equity holder moral hazard incentives.⁸

The cases where the monotonicity hypothesis obtains unambiguously are really rather specialized. Few contingent claims, other than some traded options, are simple European puts and calls on non-dividend paying underlying assets. As Merton himself admits, log-normality is a crude approximation of stock return distributions (Merton, 1990, p. 59). For other assets, for instance loan portfolios with their bounded returns, the

⁸ Geske and Shastri (1981) and Bergman, Grundy and Weiner (1996) study other commonly held beliefs: 1) that option values increase monotonically with the value of the underlying asset; 2) that option values increase monotonically with the time to expiry; 3) that option values are convex functions of the value of the underlying. All of these hypotheses can be violated when one moves away from the Black-Scholes-Merton framework.

assumption is clearly untenable. Most corporate budgeting, regulatory and risk management problems are multi-period with boundary conditions, uncertain intermediate payouts and the potential for adaptive adjustments of portfolio characteristics.

The monotonicity hypothesis cannot be taken for granted. Whether it applies in a particular situation is an empirical question that must be established on a case by case basis before the hypothesis can be invoked. As a general qualitative result that can be used in thinking about incentive and real option problems in a general sense, the monotonicity hypothesis is almost certainly false.

3. Bondholders, Deposit Insurers and Their Risk Alignment

If bond investors price their investments rationally, will we necessarily observe that bond coupon rates (or yields) are monotonically increasing in 1) bond default risk or 2) the measures of risk of interest to bank supervisors and deposit insurers? In both cases the answer is “No.”

In the first case bond investors care both about default and losses in the event of default. A higher default risk can be more than compensated by a higher recovery in the event of default resulting in a lower coupon rate. In the second case, bond investors and deposit insurers are sensitive to different possible payoffs of the projects their funds are invested in. Depositors, and hence deposit insurers are paid first. Once their obligations are covered they are (theoretically) indifferent as to how much more the project pays off. On the other hand, if the project cannot cover the depositors' claims the bond investors recover nothing and are therefore indifferent as to the amount of the shortfall.

The remainder of this note develops this intuition mathematically and gives simple numerical examples. The note concludes with a brief discussion of the policy implications of these results.

The Basic Argument

Rational bond investors demand coupon rates that set the expected return on the bond equal to some risk-adjusted benchmark.⁹ For example if bond investors are risk neutral the expected returns on bonds will equal the risk free rate.¹⁰

For simplicity, assume a single period bond that promises to repay principal (\$1) plus the coupon, c (to be determined). The borrower invests in a project which repays x with a probability density function of $f(x)$.

The expected repayment to the bond investors consists of the principal and coupon if the borrower's project pays off at least that amount:

$$(1+c)\int_{(1+c)}^{\infty} f(x)dx,$$

and the expected project value itself if the project payoff is insufficient to cover the principal and coupon, that is if the loan defaults (we will address the issue of bankruptcy costs later), down to a minimum bondholder payoff of zero due to limited bondholder liability:

$$\int_0^{(1+c)} x f(x)dx.$$

By assumption of risk-neutrality the expected payoff of the bond will be equal to 1 plus the risk free rate in equilibrium:

$$1+r_f = (1+c)\int_{(1+c)}^{\infty} f(x)dx + \int_0^{(1+c)} x f(x)dx.$$

The bond default probability, which we denote π_B is

$$\pi_B = \int_{-\infty}^{(1+c)} f(x)dx = 1 - \int_{(1+c)}^{\infty} f(x)dx$$

which allows us to rewrite the equilibrium coupon rate equation as

$$1+r_f = (1+c)(1-\pi_B) + \int_0^{(1+c)} x f(x)dx. \quad (4)$$

⁹ Covitz, Hancock and Kwast, 2000, "Mandatory Subordinated Debt: Would Banks Face More Market Discipline?" Federal Reserve Board working paper, made the point that bondholders care about expected return and not default probability *per se* and thus motivated this analysis.

¹⁰ The analysis that follows does not depend on risk-neutral bond investors, however it does simplify the math.

Now consider two different, but equal-sized, projects with payoff densities $f(x)$ and $f^*(x)$. The resulting coupons, c and c^* will in equilibrium satisfy:

$$1 + r_f = (1 + c)(1 - \pi_B) + \int_0^{(1+c)} x f(x) dx = (1 + c^*)(1 - \pi_B^*) + \int_0^{(1+c^*)} x f^*(x) dx.$$

Can we conclude from this bond pricing relation that $c > c^* \Leftrightarrow \pi_B > \pi_B^*$? We cannot. The reason is that knowing both $\pi_B > \pi_B^*$ and eq. (1.1) is still insufficient to determine

$$\int_0^{(1+c)} x f(x) dx < ? > \int_0^{(1+c^*)} x f^*(x) dx.$$

A higher default probability can be offset by a higher payoff in the event of default, resulting in a lower expected return and hence a lower coupon rate (even though the default threshold is itself a function of the coupon rate).

Example 2 illustrates this assertion. Each of the two projects to be compared has two possible outcomes, “good” and “bad.” The projects are calibrated to have the same expected payoffs. In this example Project A has a higher payoff variance, 7.29 vs. 4.36, and a higher probability that the borrower will default on the bond, 10% vs. 5%. Nonetheless, bond investors will require a lower coupon rate to lend to Project A than to Project B, 5.55% vs. 10.53%.

Bankruptcy Costs

Fixed bankruptcy costs have the effect of lowering the payoff in the event of default. If bankruptcy costs are $b < 1 + c$ then the equilibrium coupon rate must satisfy

$$1 + r_f = (1 + c)(1 - \pi) + \int_b^{(1+c)} (x - b) f(x) dx. \quad (5)$$

Again, knowing $\pi > \pi^*$ and equation (5) are insufficient to determine

$$\int_b^{(1+c)} (x - b) f(x) dx < ? > \int_b^{(1+c^*)} (x - b) f^*(x) dx.$$

Example 2

Outcome s	Project A		Project B	
	Probability π_s	Payoff x_s	Probability π_s^*	Payoff x_s^*
Up: u	0.90	10.00	0.95	9.58
Down: d	0.10	1.00	0.05	0.00
Project Statistics				
Expected payoff	9.10		9.10	
Payoff variance	7.29		4.36	
Bond Investor's Solution				
Bond default probability	0.10		0.05	
Equilibrium coupon rate	5.55%		10.53%	

- Each project is financed with a bond that promises to repay principle, 1, plus the coupon, c , at the end of the project.
- Bond investors are risk neutral.
- The equilibrium coupon rate is set so that expected bond payoff equals 1 plus the risk free rate, 5%.

$$1 + r_f = \pi_u \min(1 + c, x_u) + \pi_d \min(1 + c, x_d)$$

If bankruptcy costs are proportional, $b < 1$, to the value of assets, x , then the equilibrium coupon rate must satisfy

$$1 + r_f = (1 + c)(1 - \pi) + \int_0^{(1+c)} x(1-b)f(x)dx,$$

and again we cannot determine

$$\int_0^{(1+c)} x(1-b)f(x)dx < ? > \int_0^{(1+c^*)} x(1-b)f^*(x)dx.$$

Thus, the existence of bankruptcy costs do not generally effect our conclusion that there does not exist an unambiguously monotonic relation between the coupon rate that bond investors demand in equilibrium and the probability that the bond will default.

Insurers' Losses

Suppose bank supervisor/insurers care primarily about defaults to bondholders, perhaps because they will incur financial or political costs from recognizing and resolving insolvency even when the project payoff exceeds the insured-deposit obligations.¹¹ Then the results in the previous section will continue to apply and coupon rates will not increase monotonically in expected harm to supervisor/insurers.

Next consider bank supervisor/insurers who are free of non-economic considerations such as a tendency to equate closure with supervisory failure. Let the quantity of insured deposit obligations at the end of the period be D . Dead-weight losses due to reorganization are ignored (following the above arguments they do not materially change the analysis). Thus, the probability that losses will occur to the deposit fund is

$$\pi_D = \int_{-\infty}^D f(x)dx,$$

and the expected loss to the deposit fund is

$$EL_D = \int_0^D (D-x)f(x)dx.$$

¹¹ Insolvency with attendant costs is assumed to result when the bank cannot payoff all fixed claimants including bondholders. In some cases this may not be true and regulators may prevent bondholders from legally enforcing their claims. In such cases bonds are *de facto* non-voting, tax-advantage preferred stock and not debt in the usual sense.

The bondholders stand behind the insured depositors for pay off from the project, so in the presence of insured depositors the equilibrium bond pricing equation (4) becomes

$$\begin{aligned} 1 + r_f &= (1 + c) \int_{(D+1+c)}^{\infty} f(x) dx + \int_D^{(D+1+c)} (x - D) f(x) dx \\ &= (1 + c)(1 - \pi_B) + \int_D^{(D+1+c)} (x - D) f(x) dx. \end{aligned}$$

It is immediately apparent that bondholders and deposit insurers are concerned about different portions of the project-payoff distribution. Bondholders care only about payoffs above D , while deposit insurers care about payoffs between zero and D . It is thus possible for equilibrium bond coupons for different projects to be ordered differently than probability of deposit defaults or expected losses to deposit insurers. Bondholders are willing to trade of a greater risk of being paid nothing, for a greater expected payoff when depositors are fully covered. Example 3 illustrates this result.

Again the competing projects have been calibrated to have the same expected payoffs. In this example the projects have the same variance as well. Even though the projects also have the same probability of defaulting on the bonds, 10%, they nonetheless have different equilibrium coupon rates: 9.74% for Project A vs. 10.53% for Project B. The low-coupon-rate project A has the lower deposit-default probability, 2% vs. 3%, but also has the higher expected loss to the deposit insurance fund, 0.020 vs. 0.015.¹² Example 4 illustrates a case where the low-bond-coupon project has the higher depositor default rate and the higher expected losses to the deposit insurance fund.

Summary

The analysis has shown that rationally-set equilibrium bond coupon rates, and by extension secondary-market bond yields, are not proxies for

- Probability of bond default,
- Probability that depositors will not be repaid in full by a bank, or
- Expected financial losses to the deposit insurance fund,

¹² This illustrates a potential inconsistency between a deposit insurer's goals of minimizing probability of default (on deposit repayment) and minimizing losses to the deposit insurance fund.

Example 3

Outcome s	Project A		Project B	
	Probability π_s	Payoff x_s	Probability π_s^*	Payoff x_s^*
Good: g	0.95	4.00	0.95	4.00
Bad: b	0.03	1.25	0.02	1.00
Worse: w	0.02	0.00	0.03	0.50
Project Statistics				
Expected payoff	3.84		3.84	
Payoff variance	0.52		0.52	
Bond Investor's Solution				
Bond default probability	0.05		0.05	
Equilibrium coupon rate	9.74%		10.53%	
Deposit Insurer's Solution				
Default probability	0.02		0.03	
Expected losses	0.020		0.015	

- Each project is financed with a bond that promises to repay principle, 1, plus the coupon, c , at the end of the project, and insured deposits of $D = 1$.
- Cost of deposits and interest rate on deposits are both assumed to be zero.
- Deposit repayment takes priority over bond repayment.
- Bond investors are risk neutral.
- Expected deposit insurer losses are

$$EL(D) = \sum_{s=g,b,w} \pi_s \max\{D - x_s, 0\}$$

- The equilibrium coupon rate is set so that expected loan payoff equals 1 plus the risk free rate, 5%

$$1 + r_f = \sum_{s=g,b,w} \pi_s \max\{\min(1 + c, x_s - D), 0\}$$

in the sense that when comparing two or more bonds, a higher coupon rate does not reliably indicate the relative magnitudes that obtain for any of these other measures of risk.

The numerical examples were, of course, contrived to illustrate the possibilities for inconsistency in ordering of projects depending on which risk measure or proxy is used. However, the distributions used in these examples are by no means bizarre.¹³ If one is willing to make sufficient assumptions about project returns (e.g. normally distributed with equal means) and capital structure (identical), then under such specific assumptions one might be able to assert that all risk measures and proxies should be identically ordered.¹⁴ But the situations that guarantee consistency may well be the exception rather than the rule. The restrictions for obtaining consistency are likely to be severe and we usually do not know enough about real-world return distributions to be sure that the required conditions obtain, even approximately.¹⁵

Policy Implications

Economic arguments suggest that bank supervisor/insurers should price deposit insurance so to cap at a socially optimal level the probability that bank-default-related insurance-fund payments to depositors in any given period do not exceed the funds in the deposit insurance pool. However, we observe that deposit insurance is under-priced by any reasonable measure of bank risk and simultaneously that supervisors are extremely reluctant to close banks that appear to be economically insolvent (forbearance). This suggests that there are regulatory costs to bank failure that are independent of the actual financial losses that closing the bank would occur.

¹³ Preliminary results of ongoing work for this paper show that the same ambiguity obtains with normally distributed project returns.

¹⁴ The development of necessary and sufficient conditions for consistent risk ordering is beyond the scope of this note.

¹⁵ Note in Example2 that equality of payoff means and variances together are still insufficient to guarantee consistency. Normally distributed payoffs, without restrictions on means, are also insufficient.

Example 4

Outcome s	Project A		Project B	
	Probability π_s	Payoff x_s	Probability π_s^*	Payoff x_s^*
Good: g	0.920	4.00	0.900	4.05
Bad: b	0.060	1.00	0.035	1.90
Worse: w	0.020	0.00	0.065	0.40
Project Statistics				
Expected payoff	3.74		3.74	
Payoff variance	0.79		0.93	
Bond Investor's Solution				
Bond default probability	0.08		0.10	
Equilibrium coupon rate	14.13%		13.17%	
Deposit Insurer's Solution				
Default probability	0.020		0.065	
Expected losses	0.020		0.039	

- Each project is financed with a bond that promises to repay principle, 1, plus the coupon, c , at the end of the project, and insured deposits of $D = 1$.
- Cost of deposits and interest rate on deposits are both assumed to be zero.
- Deposit repayment takes priority over bond repayment.
- Bond investors are risk neutral.
- Expected deposit insurer losses are

$$EL(D) = \sum_{s=g,b,w} \pi_s \max\{D - x_s, 0\}$$

- The equilibrium coupon rate is set so that expected loan payoff equals 1 plus the risk free rate, 5%

$$1 + r_f = \sum_{s=g,b,w} \pi_s \max\{\min(1 + c, x_s - D), 0\}$$

It has been widely suggested that reliance on the yields of mandated sub-ordinated bonds issued by banks can circumvent non-economic incentives of supervisors by proxying for economically-rational supervisory goals. The analysis in this note suggests that this is not necessarily true. The trade-off of default-probability and loss-given-default facing the bondholder is not the same as that facing the supervisor/insurer. This because depositor-priority forces bondholders and deposit insurers to be concerned with different parts of the project-return distribution. Thus, it is unclear that subordinated debt yields will achieved the desired result of classifying banks in order of economically-rational regulatory concern. Nor could a maximum-allowable bond-yield threshold reliably discriminate with any precision between banks that pose expected deposit-insurance losses or insolvency risk above a socially optimal threshold, and those that do not.

The basic intuition that higher bond yields signal higher risk is, of course, generally true, even if it is not invariably true, so long as “risk” is used as an intuitive concept rather than a measurable one and yield differences are large. However, any attempt to define the idea of “risk” precisely, and to then link bondholder risk to deposit insurer risk will inevitably run into the problem that bondholders and deposit insurers are not the same and hence do not share the same risks.

4. Summary and Conclusion

This paper has shown that two commonly held conceptions regarding risk are subject to error. That option value increases in risk is true only for limited types of options and for restrictive types of distributions, situations that do not generally obtain. The related idea that equity holders present a moral hazard risk because the value of their call option on the assets of the firm may be increased by increase the riskiness of investments, at the expense of bondholders, is flawed for related reasons.

The coincidence of fixed claimants’ assessments of risk is similarly dependent on restrictive, and generally unverifiable, assumptions about the distributions of returns of bank (or firm) investments. This is due to the priority of payoffs different claimants enjoy, and due to the effects that return distributions have on the expectations of those payoffs being realized. Not only is it quite possible for different fixed claimants to

disagree on the relative risks they face, but the relative risks for a given claimant may depend on whether they are interested in economic losses or the event of default itself.

Rules of thumb such as those discussed in this paper are only useful so long as they do not mislead. That is where they are generally, if perhaps not absolutely, reliable. The examples given here are sensitive to effects about which we know very little, for example the true distribution of bank portfolio expected returns. That leaves the empirical question of how important the questions raised here are in practice.

There is little evidence on the degree to which increases in bond yields parallel increases in risk (default or economic losses) to deposit insurers. What evidence there is is indirect at best.¹⁶ Nonetheless, an empirical investigation of this question is, in theory, possible. The incentives of equity holders is a more complex, and probably illusive, question. Incentives are not easily measurable. The potential effects on firm performance of equity holder incentives are difficult to measure, and the ability of equity holders to influence managers is uncertain.¹⁷ Nonetheless, theory strongly suggests that equity holder moral hazard risk is easily overstated. Equity holders' incentives to induce firm risk taking are clearly offset by incentives to ensure firm survival—to avoid losing future profits. Models that consider this issue suggest that equity holders will wish to reduce firm risk as the firm approaches insolvency. Most models also suggest that equity holder incentives for risk taking, betting the bank, increases sharply only when the firm becomes economically insolvent. This likely discontinuity presents an interesting, and as yet unresolved, problem for those interested in corporate governance or regulation.

Finally, these two examples show that while we may have an intuitive understanding of what risk means, any attempt to pin down the concept can lead to ambiguity. Clearly defining which (and whose) risks are being discussed is a necessary preliminary to any meaningful analysis.

¹⁶ See Bliss(2001) for a discussion.

¹⁷ See Bliss and Flannery (2001).

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