Credit, money, limited participation and welfare*

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Abstract

Access to credit has greatly increased in the last decades. However, credit markets still feature limited participation. We present a model in which credit and outside money can be used as media of exchange in order to analyze how a heterogeneous access to credit affects welfare. Allowing more agents to use credit has an ambiguous effect on welfare because it may make consumption-risk sharing more inefficient. We calibrate the model using U.S. data and show that the increase in access to credit from 1990 to the near present has had a slightly negative impact on welfare.

Keywords : money, search, risk sharing, limited participation

JEL Classification : E41, E50, E51

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1 Introduction

In most economies, two types of money are widely used: fiat money, i.e., notes and coins issued by a central bank, and private money issued by commercial banks, such as credit cards. We usually call the former "outside money", which stands for "outside the private sector", and the latter "inside money". Over the last few decades, the relative importance of inside money as a means of payment has increased compared to outside money (Humphrey, Pulley and Vesala (1996), Ize, Kovanen and Henckel (1999)), which has given rise to a well-known debate about the eventual disappearance of outside money (Friedman (1999), King (1999)). However, outside money keeps on being used to a large extent: for instance, in 2003 in the United States, payments in cash accounted for 20.9% of the volume of consumer transactions and 41.3% of the number of consumer transactions.[1]

In this paper, we analyze the implications of the coexistence of different means of payment in terms of allocations and welfare in a model that explicitly describes the advantages of using credit versus fiat money. For this, we build a model *à la* Lagos and Wright (2005) in which agents can use both outside and inside money.[2] In order to allow for inside money, we adopt the way undertaken by Berentsen, Camera and Waller (2007) where bank credit is feasible, but money is still essential owing to anonymity and the absence of double coincidence in the goods market.[3] In particular, we intend to study the coexistence of inside and outside money in the presence of limited participation in the credit market. We accomplish this by assuming that recognizability of inside money requires a technology that can fail, a feature that makes inside money less liquid, whereas outside money is exposed to inflation. The resulting trade-off allows us to study an economy in which inside and outside money coexist.[4]

The effects of inflation on the consumption pattern are the following. When the economy is away from the Friedman rule (that is, when the nominal interest rate is higher than zero), equilibrium consumption quantities differ for agents who are able to use inside money and those who use outside money only. Agents who are able to borrow not only attain a higher consumption than those who cannot borrow, but they also attain a consumption higher than the socially efficient consumption quantity. Moreover, a rise in inflation has an asymmetric

[1] U.S. Census Bureau, Statistical Abstract of the United States, 2006.

[2] Throughout this paper, we use the terms "credit" and "inside money" indistinguishably because credit is the only type of inside money that we will allow for in our model.

[3] By essentiality we mean that money expands the set of allocations (see Kocherlakota (1998) and Wallace (2001)).

[4] Coexistence of inside and outside money is difficult to get as an equilibrium phenomenon, given that their rates of return are generally different. Hence, the explanations for this coexistence that we find in the literature are based on features that cause the liquidity of inside and outside money to differ, such as legal restrictions (Wallace (1983)), anonymity (Goodhart (2000)) and technology, with regard to recognizability (Powers (2005)) or information structure (Townsend (1989), Kocherlakota and Wallace (1998)).

effect on buyers, so that consumption-risk sharing becomes more inefficient. On the other hand, we analyze the impact on allocations of an increase in the proportion of borrowers. Interestingly, allowing more agents to use credit has an ambiguous effect on welfare, because it can make consumption-risk sharing more inefficient.

As stated by Green (2001), several studies, like Schreft (1992) and Aiyagari, Braun and Eckstein (1998), predict that greater innovation in the credit sector would reduce the welfare cost of inflation. Indeed, an increase in access to credit is expected to generate a welfare gain stemming from a lower exposure to inflation: if agents can rely more on credit, they can reduce their money holdings and hence suffer a lower impact from inflation.[5] However, we calibrate our model to U.S. data and show that the improvement in the credit sector that yielded a greater access to consumer credit from 1990 to the near present entailed a slightly negative welfare gain. The reason is that consumption-risk sharing across agents (borrowers and non-borrowers) became more inefficient.

In addition, our quantitative analysis allows us to calculate the welfare cost of inflation when credit is available and thereby advancing the literature that aims at introducing the banking sector into the computations of the welfare cost of inflation. The refinements to these calculations have consisted mainly in taking the interest-bearing assets (in particular, bank deposits) in agents' monetary holdings into consideration, which are shown to affect the estimates of the cost of inflation.[6] Instead, we calculate the welfare cost of inflation taking into account consumer credit. We find that reducing annual inflation from 10% to 0% is worth slightly more than 1% of steady-state output. This figure is close to the one reported by Lucas (2000) and Lagos and Wright.[7]

Coexistence of inside and outside money has been studied in a microfounded framework of monetary exchange by Shi (1996), He, Huang and Wright (2005, 2006), Williamson (1999, 2002) and Sun (2007), among others. However, they abstract from the heterogeneity in the use of inside money that interests us. In Cavalcanti, Erosa and Temzelides (1999) and Cavalcanti and Wallace (1999a, 1999b), all agents consume and produce, but only a subset of them, called banks, are able to issue inside money. These works, however, do not focus on welfare when different means of payment are used in equilibrium and access to them varies, but instead on the feasibility and optimality of private money systems compared to systems of outside money. Besides, money is assumed to be indivisible so that results strongly depend on the amount of outside money initially assumed and the effect of inflation

[5] Snellman, Vesala and Humphrey (2001) attribute the fall in the share of cash transactions in a sample of European countries to the more extensive use of debit and credit cards.

[6] See, for instance, Simonsen and Cisne (2001), Alvarez and Lippi (2007) and Attanasio, Guiso and Jappelli (2002).

[7] Lagos and Wright present several calculations of the welfare cost of inflation. We refer here to the one that assumes that pricing is competitive which is comparable to ours, as we will see below.

cannot be analyzed.[8] Telyukova and Wright (2008) consider a heterogeneity related to the use of inside money to explain why agents hold debt and money in their portfolios, but this concerns a particular subset of trades and, therefore, affects all agents equally.

Our work is very close to Reed and Waller (2006) who assume heterogeneity across agents arising from endowment shocks to study how money can help to overcome inefficient risk sharing. Aiyagari and Williamson (2000) also study the distribution of consumption and welfare in a different framework, with dynamic contracts offered by financial intermediaries, limited participation and private information. However, in these papers, the consequences of increased access to inside money are not analyzed. Other articles consider a heterogeneity regarding the use of inside money across agents for different purposes. For instance, Antinolfi, Azariadis and Bullard (2007) assume a fixed subset of agents who can borrow in their analysis of optimal inflation targets. Williamson (2008) assumes limited participation to show that the effect of monetary policy depends on the arrangements for clearing and settling credit instruments.

The rest of the paper proceeds as follows. In section 2 we describe the environment. In section 3 we develop the model, define the symmetric equilibrium and point out its main features. Section 4 is devoted to the quantitative analysis. Finally, section 5 concludes.

2 Environment

The original framework we build on is the divisible money model by Lagos and Wright. The main advantage of this framework is that it facilitates the introduction of heterogeneity in production and consumption preferences as well as the divisibility of money, keeping the distribution of money holdings degenerate and, thus, analytically tractable. More precisely, we base our model on the model developed by Berentsen et al. The difference is that, while in that model only outside money is considered, here we also allow agents to have access to inside money; i.e., they can borrow money issued by banks on their request. Besides, we do not allow agents to deposit money and earn interest on it as in Berentsen et al. because our focus is on the choice that agents make about which money to use in trade.

In addition to the accessibility to inside money, we introduce a probability of not being able to borrow it. This assumption is made in order to capture the feature that the money issued by banks potentially has a lower liquidity than cash. This seems quite reasonable as it reflects different situations involving the use of inside money in transactions: for instance, the credit card may not work or the technological device to recognize it may fail.

Time is discrete and goes for ever. There is a continuum of infinitely lived agents of

[8] More generally, results are shown to be affected when money is indivisible because of the way prices are determined (*cf.* Berentsen and Rocheteau (2002)).

unit mass and one perfectly divisible and non-storable good that all agents can potentially consume and produce. Agents discount across periods with factor $\beta \in (0, 1)$.

In each period, two competitive markets open sequentially (the second market opens only when the first market has closed). Before the first market opens, agents get a preference shock by which they either want to consume but cannot produce (with probability $(1 - n)$) or can produce but do not want to consume (with probability n). We call "buyers" the agents who get the first type of shock and "sellers" those who get the second type. In the first market, buyers get utility $u(q)$ when they consume a quantity q of the unique good, with $u'(q) > 0$, $u''(q) < 0$, $u'(0) = +\infty$ and $u'(\infty) = 0$. For sellers, producing a quantity q represents a disutility equal to $c(q)$ with $c'(q) > 0$ and $c''(q) > 0$.

In the second market all agents consume, produce and adjust their money holdings. Consuming x gives utility $v(x)$ with $v'(x) > 0$, $v''(x) \leqslant 0$, $v'(0) = \infty$ and $v'(+\infty) = 0$. Disutility cost from producing x is equal to h, where one unit of labor yields one unit of the consumption good.

In addition, there is an intrinsically useless object we refer to as outside money, which is issued by a central bank. Agents can also borrow inside money which is issued by competitive banks on agents' requests.[9] Inside money is then issued as a bilateral contract between an agent and a bank by which the bank gives an amount l (for loans) of inside money to the agent at the beginning of the period and the agent must pay it back at the end of the period. Inside money cannot, therefore, be taken from one period to another.[10] Besides, in our model banks have enforcement power. Thus default and, consequently, loans' size are not an issue.[11] For simplicity, we also assume that banks operate at zero cost.

Each period, agents face a probability $(1 - \delta)$ of not being able to use money borrowed to buy the consumption good. Hence, limited participation is idiosyncratic and random (as in Aiyagari and Williamson). Agents learn whether they will be able to use money borrowed or not simultaneously with (or immediately after) learning that they are buyers, before the first market opens.

We assume that agents who hold outside money at the beginning of each period exchange

[9] We could also assume that there is only one bank, the "central bank", which issues both outside money and inside money. In that case, the difference would be that only inside money would be issued on agents' requests. The assumption on competitive banks is also made in Berentsen et al. and in both papers by He et al. already cited.

[10] We attempt to capture one basic distinction between inside and outside money, which is that the former is cancelled out inside the private sector whereas the latter does not cancel out and so private agents may hold it across periods.

[11] Berentsen et al. actually propose two different settings to analyze money and credit. The first one assumes that banks have enforcement power, whereas the second rules out enforcement power but assumes that banks have a technology that allows them to exclude defaulters from the financial system. In our model we take the first of these two possibilities.

it for inside money before first market opens. This assumption is made only for simplicity, since it allows only one type of money to be dealt with in the first market, as will be seen in the next section. Agents will then have to repay the amount of loans, which will be equal to the difference between the amount of inside money they hold when entering the first market and the amount of outside money taken from the previous period.[12]

In order to motivate a role for money, we assume anonymity of traders so that, for trade to take place, sellers require compensation at the same time as they produce. This assumption rules out bilateral credit; however, it does not conflict with the existence of lending in this model because this only requires that agents are identified by banks (which is not the same as being identified by partners in trade).

Markets are competitive so that pricing is competitive. Competitive pricing was first analyzed in a Lagos-Wright framework by Rocheteau and Wright (2004). As they, and previously Temzelides and Yu (2004), point out, the existence of competitive markets does not make money inessential as long as the double coincidence problem and anonymity are still features of the environment studied.[13]

Supply of outside money is under central bank' decisions which we assume to be exogenous. We call M_t the per capita money stock in period t. Money stock grows at a rate γ where $\gamma > 0$. Agents receive lump-sum transfers equal to τM_{t-1} from the central bank at the beginning of the second market in period t, where the subscript -1 indicates the previous period (and $+1$ indicates the following period). Thus $M_t = (1+\tau) M_{t-1} = \gamma M_{t-1}$.

3 Symmetric equilibrium

We will consider symmetric and stationary equilibria in which strategies are the same across agents, real allocations are constant over time, γ is time-invariant and end-of-period real money balances are constant. This implies $\phi M = \phi_{+1} M_{+1}$, where ϕ is the price of money in the second market in period t, and

$$\gamma = \frac{M_{+1}}{M} = \frac{\phi}{\phi_{+1}} \tag{1}$$

We indicate by $U(m_I, l)$ the expected value of entering the second market with an amount

[12] We do not explicitly include an exchange rate between inside money and outside money, even if it would be more general to do so. The choice here is made by simplicity and because we will only consider stationary equilibria in which the real amount of loans and the real money balances are time-invariant. This allows us to consider an exchange rate also time-invariant, that for simplicity we assume equal to 1.

[13] Competitive pricing is also analyzed in Aruoba, Waller and Wright (2006), Berentsen, Camera and Waller (2005), Lagos and Rocheteau (2005) and the model in Berentsen et. al we mostly follow. We use here competitive pricing and leave the comparison with the determination of prices by bargaining for future research.

m_I of inside money (outside money taken from the previous period plus the money borrowed) and l, the amount of loans. $V(m_O)$ is the expected value of entering the first market with an amount m_O of outside money taken from the previous period. In this section, we solve the model backwards, the second market first and then the first market, for a representative period t.

3.1 The second market

In the second market, agents consume x, produce h, repay inside money borrowed at the beginning of the current period and choose the amount of outside money they will take into the following period.

The representative agent's program is

$$U(m_I, l) = \max_{x,h,m_{O_{+1}}} \left[v(x) - h + \beta V_{+1}\left(m_{O_{+1}}\right)\right] \tag{2}$$
$$\text{s.t. } x + \phi m_{O_{+1}} = h + \phi\left[m_I + \tau m_{O_{-1}} - (1+i)\, l\right]$$

where $l = m_I - m_O$ is the amount of inside money borrowed (loans); m_I is the amount of inside money brought into the second market and i is the interest rate. $m_{O_{-1}}$ and $m_{O_{+1}}$ are the amounts of outside money held during the previous period and taken to the following period, respectively.

If we rewrite this inserting the budget constraint into (2), we have

$$\begin{aligned} U(m_I, l) &= \phi\left[m_I + \tau m_{O_{-1}} - (1+i)\, l\right] \\ &\quad + \max_{x,m_{O_{+1}}} \left[v(x) - x + \beta V_{+1}\left(m_{O_{+1}}\right) - \phi m_{O_{+1}}\right] \end{aligned}$$

The first-order conditions are

$$v'(x) = 1 \tag{3}$$
$$\beta V'_{+1}\left(m_{O_{+1}}\right) = \phi \tag{4}$$

where $V'_{+1}\left(m_{O_{+1}}\right)$ is the marginal value of outside money taken into the following period.[14] As it is standard in the Lagos-Wright model, x is identical for all agents and $m_{O_{+1}}$ is independent of the amount of outside money brought into the second market m_O. This makes the distribution of money holdings degenerate: all agents carry the same amount of outside money from one period to the following one.

The envelope conditions are

$$U_{m_I} = \phi \tag{5}$$
$$U_l = -\phi(1+i) \tag{6}$$

[14] We show in the appendix that V is a concave function in m so that the solution to (4) is well-defined.

3.2 The first market

Agents deposit all their outside money in exchange for inside money at the beginning of the period.

The expected lifetime utility for an agent who holds an amount m_O of outside money before entering the first market is

$$\begin{aligned} V(m_O) &= (1-n)\,\delta\left[u\left(q_b^L\right)+U\left(m_I - pq_b^L, l\right)\right] \\ &+ (1-n)(1-\delta)\left[u\left(q_b^N\right)+U\left(m_I - pq_b^N, 0\right)\right] \\ &+ n\left[-c(q_s)+U(m_I + pq_s, 0)\right] \end{aligned} \tag{7}$$

where $(1-\delta)$ is the probability of not being able to borrow inside money in the current period. p is the price of the good in the first market. q_b^L is the quantity of good the buyer can consume by spending an amount of money equal to pq_b^L when he is able to borrow inside money (subscript L stands for "loans"), whereas q_b^N is the quantity of good he can consume if he is not able to borrow inside money (subscript N indicates that he is not able to borrow). q_s is the quantity the seller sells in exchange of an amount of money equal to pq_s.

3.2.1 Sellers' decisions

The problem for an agent that is a seller in the first market is

$$\max_{q_s}\left[-c(q_s)+U(m_I + pq_s, 0)\right]$$

The first-order condition is

$$-c'(q_s)+U_{m_I}p = 0$$

Using (5), it becomes

$$c'(q_s) = \phi p \tag{8}$$

As usual in Lagos-Wright models with two competitive markets, the seller's decision on how much to produce is such that relative marginal costs are equal to relative prices across markets. This decision is then independent of his money holdings.

3.2.2 Buyers who can borrow inside money

Buyers face a different problem depending on whether they can borrow or not. The decision's variables for a buyer who is able to use inside money are q_b^L and l. His problem is to solve

$$\max_{q_b^L, l}\left[u\left(q_b^L\right)+U\left(m_I - pq_b^L, l\right)\right]$$

$$\text{s.t. } pq_b^L \le m_I = m_O + l$$

Buyers maximize their utility subject to the cash constraint, which means that they cannot spend more money than the amount that they bring into the market. In the case of the buyer who can borrow inside money, this amount is given by the sum of m_O, the outside money taken from the previous period, and l, the amount of loans borrowed at the beginning of the current period.

The first-order condition on q_b^L is

$$u'\left(q_b^L\right) - pU_{m_I} - p\lambda_{m_I} = 0$$

where λ_{m_I} is the multiplier on the cash constraint.

Using (5) and (8) this condition reduces to

$$\frac{u'\left(q_b^L\right)}{c'\left(q_s\right)} = 1 + \frac{\lambda_{m_I}}{\phi} \tag{9}$$

If the inside-money constraint is binding ($\lambda_{m_I} > 0$), trades are inefficient for the buyer who can borrow inside money; if the inside-money constraint is not binding ($\lambda_{m_I} = 0$), then trades are efficient.

The first-order condition on l is

$$u'\left(q_b^L\right)\frac{dq_b^L}{dm_I}\frac{dm_I}{dl} + \left(U_{m_I} + \lambda_{m_I}\right)\left(\frac{dm_I}{dl} - p\frac{dq_b^L}{dm_I}\frac{dm_I}{dl}\right) + U_l = 0$$

Using the budget constraint of the buyer and (6), this condition becomes

$$\frac{u'\left(q_b^L\right)}{c'\left(q_s\right)} = 1 + i \tag{10}$$

If the interest rate i is zero then trades are efficient in the first market for the buyer able to borrow. Comparing this condition to the first-order condition for q_b^L, we verify that a non-binding borrowing constraint ($\lambda_{m_I} = 0$) is equivalent to $i = 0$.

3.2.3 Buyers who cannot borrow inside money

For a buyer who is not able to borrow inside money, the problem is to choose only q_b^N

$$\max_{q_b^N}\left[u\left(q_b^N\right) + U\left(m_I - pq_b^N, 0\right)\right]$$

$$\text{s.t. } pq_b^N \leq m_I = m_O$$

The maximal amount of money that the buyer can spend here is given by m_O only, because he is unable to borrow.

The first-order condition is

$$u'\left(q_b^N\right) - pU_{m_I} - p\lambda_{m_O} = 0$$

where λ_{m_O} is the multiplier on the cash constraint for the agent unable to borrow. Using (5) and (8) this reduces to

$$\frac{u'\left(q_b^N\right)}{c'\left(q_s\right)} = 1 + \frac{\lambda_{m_O}}{\phi}$$

If the cash constraint is binding ($\lambda_{m_O} > 0$), trades are inefficient for the buyer who cannot borrow.

Finally, for market clearing, the following condition must hold in equilibrium:

$$nq_s = (1-n)\left[\delta q_b^L + (1-\delta)\, q_b^N\right] \tag{11}$$

3.2.4 Marginal value of outside money

From (7), the marginal value of outside money is

$$V'\left(m_O\right) = \phi + (1-n)\left[\delta\lambda_{m_I} + (1-\delta)\,\lambda_{m_O}\right]$$

which can be rewritten as

$$V'\left(m_O\right) = \phi + (1-n)\left[\delta i\phi + (1-\delta)\left(\frac{u'\left(q_b^N\right)}{c'\left(q_s\right)} - 1\right)\phi\right] \tag{12}$$

In any competitive equilibrium, given that banks operate at no cost, the interest rate must be zero. In a stationary equilibrium, we can use (4) lagged one period and (1) to obtain

$$\frac{\gamma-\beta}{\beta} = (1-n)(1-\delta)\left[\frac{u'\left(q_b^N\right)}{c'\left(q_s\right)} - 1\right] \tag{13}$$

From (11), we replace q_s in (10) and (13) to get

$$\frac{u'\left(q_b^L\right)}{c'\left(\frac{(1-n)\left[\delta q_b^L + (1-\delta) q_b^N\right]}{n}\right)} = 1 \tag{14}$$

and

$$\frac{\gamma-\beta}{\beta} = (1-n)(1-\delta)\left[\frac{u'\left(q_b^N\right)}{c'\left(\frac{(1-n)\left[\delta q_b^L + (1-\delta) q_b^N\right]}{n}\right)} - 1\right] \tag{15}$$

We now state the following definition:

Definition 1 *Given γ and $\{\delta, \beta, n\} \in (0,1)$, a monetary equilibrium with both outside money and inside money is a quantity q_b^N and a quantity q_b^L satisfying (14) and (15).*

Before stating our first proposition, we derive the planner's solution; i.e., consumption and production quantities that maximize welfare. Since we assume that all agents are treated symmetrically, maximizing welfare implies maximizing the expected steady state lifetime utility of the representative agent, which is

$$(1-\beta) W = (1-n) \delta u \left(q_b^L\right) + (1-n)(1-\delta) u \left(q_b^N\right) - nc(q_s) + v(x) - x \tag{16}$$

while the feasibility constraint is

$$nq_s = (1-n)\left[\delta q_b^L + (1-\delta) q_b^N\right] \tag{17}$$

The planner maximizes (16) subject to (17) to get the first-best allocation. This satisfies

$$v'(x^*) = 1$$

as well as

$$u'\left(q_b^{L*}\right) = u'\left(q_b^{N*}\right) = c'(q_s^*)$$

Thus, welfare maximization implies $q_b^{N*} = q_b^{L*} = q^*$ where q^* is defined by

$$u'(q^*) = c'\left(\frac{1-n}{n} q^*\right) \tag{18}$$

Proposition 1 *a) If $\gamma > \beta$ and $\delta \in (0,1)$, a unique monetary equilibrium with both outside money and inside money exists. Moreover, equilibrium consumption quantities satisfy $q_b^N < q^* < q_b^L$.*

b) If $\delta = 1$, then in a competitive equilibrium outside money is driven out by inside money, unless the Friedman rule prevails ($\gamma = \beta$). Consumption quantity q_b^L satisfies $q_b^L = q^*$.

c) If $\gamma = \beta$ and $\delta \in (0,1)$, inside money is driven out by outside money. Consumption quantities satisfy $q_b^N = q_b^L = q^*$.

According to Proposition 1, a unique equilibrium exists in which both outside money and inside money are used, when economy is away from the Friedman rule and the event of not being able to borrow inside money in the following period occurs with some probability. The consumption quantity that the buyer who is not able to borrow attains is lower than the consumption quantity acquired by the buyer able to borrow. This is because the former is cash-constrained: with outside money only, the efficient consumption quantity cannot be attained because a positive inflation requires a higher marginal value of outside money for agents to accept it. A higher marginal value of outside money is equivalent to a higher marginal utility from consumption, and thus a lower consumption quantity.

Interestingly, buyers who borrow get a consumption quantity that is higher than the efficient quantity defined in (18). This is because these buyers benefit from the constraint on the buyers who cannot borrow which keeps sellers' marginal cost below the marginal cost at the efficient quantity. Thus, in this equilibrium there is inefficient consumption-risk sharing across buyers.

If $\delta = 1$ (i.e., agents can always borrow inside money), nobody is willing to hold outside money if the inflation rate is higher than the discount factor, as inside money becomes a costless alternative. Outside money need not play the insurance role anymore. Therefore, an equilibrium with outside money cannot be sustained. On the contrary, when $\gamma = \beta$ and $\delta \in (0, 1)$, inside money turns out to be useless because outside money becomes a costless way of acquiring consumption. Agents take the necessary amount of outside money across periods in order to get efficient trade and, each time a period starts, they do not need inside money (they are not cash-constrained). In both equilibria with either inside money or outside money, consumption-risk sharing is efficient. For the rest of the analysis in this section, we will focus on the case $\gamma > \beta$ and $\delta < 1$.

Proposition 2 *q_b^L is increasing in γ while q_b^N and q_s are decreasing in γ. An increase in γ is welfare-worsening.*

Proposition 2 states that an increase in inflation has an asymmetric effect on buyers. Consumption quantity decreases for buyers who cannot borrow and increases for buyers who can borrow, since the latter benefit from a higher constraint on buyers who use only outside money. Overall production decreases and welfare worsens because higher inflation makes consumption-risk sharing more inefficient; i.e., consumption decreases for buyers whose marginal utility is higher and increases for those whose marginal utility is lower.[15]

Proposition 3 *q_b^N is decreasing in δ while the effect of δ on q_b^L and q_s is ambiguous. An increase in δ has a negative effect on welfare along the intensive margin and a positive effect along the extensive margin. The overall effect on welfare is ambiguous.*

According to Proposition 3, when δ increases q_b^N certainly decreases, while this is not always the case for q_b^L. Buyers face a different situation when δ increases depending on whether they have access to credit or not. Given that q_s can decrease or increase when δ increases, buyers who can borrow could consume either more or less: they adjust their marginal utility to the marginal cost of sellers. However, an increase in δ has a direct effect that affects only buyers who do not borrow. As we can see in (13), increasing δ reduces the marginal value of money, which makes agents desire a lower level of money holdings to

[15] This result is similar to those in studies already cited by Reed and Waller and Aiyagari and Williamson.

be taken across periods. As a result, prices increase in the following-period first market; hence, buyers who cannot borrow are more cash-constrained and consume less. The key point is that production does not need to increase to make non-borrower buyers consume less provided that first-market prices increase, which is always the case when δ rises. On the contrary, borrowers consumption decreases only when overall production increases and increases when production decreases.

The ambiguous effect of an increase in δ on welfare is more intuitive, since it makes some high marginal utility buyers consume more and some consume less. We can interpret it as the combination of an extensive margin effect and an intensive margin effect. The extensive margin effect consists in an increase in consumption and production owing to a higher measure of agents who can borrow; i.e., $u\left(q_b^L\right) - u\left(q_b^N\right) - c'\left(q_s\right)\left(q_b^L - q_b^N\right)$, which is unambiguously positive. The intensive margin effect reflects the changes in quantities traded as a consequence of an increase in δ. This effect is always negative for non-borrowers. The negative effect on non-borrowers is sufficient for a negative intensive margin effect when computing welfare for the whole population. The reason is that, regardless of how q_b^L varies when δ does, borrowers get efficient trade in equilibrium, so that an increase (decrease) in their utility is exactly compensated for by an increase (decrease) in sellers' disutility. The overall intensive margin effect is then $(1-\delta)\left(dq_b^N/d\delta\right)\left[u'\left(q_b^N\right) - c'\left(q_s\right)\right] < 0$.[16]

Proposition 4 *If $u''' < 0$, then an increase in δ extends the difference between q_b^L and q_b^N.*

As stated by Proposition 4, a sufficient condition for risk sharing to become more inefficient among buyers when δ becomes higher is that the third derivative of the utility function is negative. The economic intuition for this can be better understood if we think of the opposite case; i.e., $u''' > 0$. In this case, the agent is said to be prudent (as defined by Kimball (1990)), in the sense that his demand for precautionary savings increases when he faces a greater risk. In our case, an increase in δ may imply that the agent faces a higher risk (that is, $\left(q_b^L - q_b^N\right)$ increases), which would lead him to demand higher money holdings at the end of each period. Hence, he will be less constrained if he is unable to borrow, so he will consume more. As overall prices will increase with respect to a situation in which δ is lower, q_b^L will increase less (decrease more). In contrast, if $u''' < 0$, then the opposite effect takes place; i.e., borrowers may profit from a lower demand owing to smaller precautionary savings.

[16] We have examined another version of this model in which one group of agents can borrow permanently while another group is permanently excluded from the credit market. Even though there are differences with the version that we present, the effect of δ on welfare is also ambiguous in that case.

4 Quantitative Analysis

Given that our formal analysis does not allow us to conclude on how δ affects welfare, we proceed to a calibration of the theoretical model. In addition, the calibration allows us to measure the welfare cost of inflation in the presence of limited participation in the credit market as well as the cost of inefficient risk sharing arising from limited participation. For this, we use postwar U.S. data generally reported in the literature, with the exception of data on credit cards transactions which have only become available in recent years. We choose the model period as a quarter and use the following functional forms:

$$u(q_b) = \frac{(q_b)^\alpha}{\alpha}, \quad v(x) = B_0 + B\ln(x), \quad c\left(q_s\right) = \left(q_s\right)^\theta$$

Therefore, the parameters to be identified are as follows: (i) preference parameters: $(\beta, \alpha, B, B_0, \theta)$; (ii) technology parameters: δ, n; and (iii) policy parameter: the money growth rate, γ.[17] This list contains eight parameters.

Table 1 lists the calibration parameters and the targets. Two parameters, (β, γ), are identified in "obvious" ways. The standard choice $\beta = 0.99$ gives an annual real interest rate of 4%. The quarterly average of inflation gives $\gamma - 1 = 1.2\%$.

Table 1. Calibrated parameters and targets

Parameters	Targets	Targets values
β	real interest rate	0.01
γ	average money growth	1.012
δ	ratio κ	0.83
n	ratio $\varkappa$	0.222
α	(Loans $+M1)/M1$	1.25
B	money demand	0.169
θ	elasticity of money demand	-0.5
B_0	normalization	1.00

There are six parameters still to be identified, $(\alpha, B, B_0, \theta, \delta, n)$. While B_0 is normalized to one, the other five parameters are identified jointly with the following restrictions.

First, we can determine the value of δ by using the ratio κ of the number of transactions carried out with money to the number of all transactions. In the model, this ratio is:

$$\kappa\left(\alpha, B, \theta, \delta, n\right) \equiv 1 - \delta + \delta\psi$$

[17] The parameter B_0 helps to better match our targets, but it does not change our results; in particular, it does not affect our computation of the welfare cost of inflation and inefficient risk sharing described below.

where ψ is the proportion of buyers able to use credit who choose to pay with outside money. We set $\psi = 0$.[18]

If we take only credit card payments as inside money payments, the sample average of κ is 0.83 (data being available for the years 1990 and 1999-2003).[19] Therefore, $\delta = 0.17$.[20]

Second, we call $\varkappa$ the ratio of the value of transactions carried out with inside money to the value of all transactions. In the model, $\varkappa$ is

$$\begin{aligned}\varkappa(\alpha, B, \theta, \delta, n) &\equiv \frac{2(1-n)\delta l}{(1-n)p\left[\delta q_b^L + (1-\delta) q_b^N\right] + (1-n)(\delta l + m_O)} \\ &= \frac{\delta\left(q_b^L - q_b^N\right)}{\delta q_b^L + (1-\delta) q_b^N}\end{aligned}$$

where m_O is the amount of outside money held by every agent at the beginning of each period and we used the fact that $l = pq_b^L - m_O$ and $m_O = pq_b^N$.

The numerator of $\varkappa$ shows that the amount of inside money $((1-n)\delta l)$ is used twice in each period: it is spent by buyers who borrow in the first market and then by sellers in the second market. In the denominator, the values of all transactions in both markets are added together.

To compute the value of transactions carried out with inside money, we can consider different possibilities. We choose to compute the volume of transactions paid by credit card, which is consistent with the data to compute ratio κ.[21]

We get the second equation to pin down $(\alpha, B, \theta, \delta, n)$ by equating $\varkappa(\alpha, B, \theta, \delta, n)$ to the sample average 22.2% of the credit card share of consumer payments in volume (years 1990 and 1999-2003). Given that we know δ, this equation allows us to get q_b^N as a proportion of

[18] In the real world, people use cash for some transactions and, say, credit cards for others. Our setup does not allow us to reflect this, since we assume a unique (decentralized) competitive market. In our model, agents able to use inside money could choose to use only inside money or use some outside money and some inside money, which implies that multiple equilibria exist. For our calibration procedure, we assume that agents able to use inside money do not spend outside money; i.e., $\psi = 0$. Of course, we could instead assume $\psi \in (0, 1)$.

[19] Survey of Consumer Finances, several years.

[20] Of course, we should not interpret $(1-\delta)$ here as "the proportion of agents not able to borrow; e.g., that cannot use credit card". There are many features in the real world we are not considering such as the costs of holding credit cards, even though, in general, it tends to be almost costless (provided that we repay immediately after the grace period), or the fact that there are some "cash-goods" that cannot be purchased with credit cards. Moreover, we should interpret the borrowing in our framework as the grace period granted by credit cards companies, as we do not allow for revolving debt.

[21] Alternatively, we could take the amount of consumer credit (which is very similar to the sum of credit card payments and consumer individual loans). Even though we are not able to compute consumption inside-money payments exactly, we know that the true value is somewhere in between both figures: the former figure underestimates it, since it only includes credit-card payments, whereas the latter includes not only new credit but also revolved debt and thus overestimates it.

q_b^L:

$$
\begin{aligned}
q_b^N &= \frac{(1-\varkappa)\,\delta}{\varkappa+(1-\varkappa)\,\delta} q_b^L \\
q_b^N &= 0.373342 q_b^L
\end{aligned}
$$

Third, in the model the ratio of the amount of loans plus money to the stock of money is as follows:

$$
\begin{aligned}
\frac{\text{Loans } + \ M}{M} &\equiv \frac{(1-n)\,\delta l + m_O}{m_O} \\
&= \frac{(1-n)\,\delta\left(q_b^L - q_b^N\right) + q_b^N}{q_b^N}
\end{aligned}
$$

If we take the ratio (annual credit card payments in trade/4 $+M1)/M1$, we get a sample average equal to 1.25 (for the period for which we report information on credit cards).[22] With this equation, we can pin down n. We get $n = 0.12387$.

Fourth, in the model, the steady state quantity of output traded in the decentralized market $q(i, \alpha, \theta, \delta, n)$ and the household's steady state money balance $g(i, \alpha, \theta, \delta, n)$ are functions of the nominal interest rate and the preference parameters α and θ. Denote $\overline{q}(\alpha, \theta, \delta, n)$ and $\overline{g}(\alpha, \theta, \delta, n)$, respectively, the model's steady state output and the household's steady state money balances in the decentralized market when $i = \bar{\imath}$. In the model, then, money demand satisfies

$$
\begin{aligned}
L(\alpha, B, \theta, \delta, n) &\equiv \frac{\overline{g}(\alpha, \theta, \delta, n)}{\overline{q}(\alpha, \theta, \delta, n) + B} \\
&= \frac{q_b^N}{(1-n)\left[\delta q_b^L + (1-\delta)\, q_b^N\right] + B}
\end{aligned}
$$

where $B = x^*$ is the output in the centralized market.

By equating the steady-state (annualized) money demand, $L(\alpha, B, \theta, \delta, n)/4$, to the sample average 0.169, we get the fourth equation to pin down $\alpha, B, \theta, \delta, n$.

Fifth, the interest elasticity of money demand is $\zeta \equiv (\partial L/\partial i)\, i/L$.

We can approximate $\overline{\xi}(\alpha, B, \theta, \delta, n)$ in the model by simply calculating $(\partial L/\partial i)\,\bar{\imath}/L$:

$$
\frac{\partial L}{\partial i}\frac{\bar{\imath}}{L} = \frac{(1-n)\,\delta\left(\frac{q_b^L}{q_b^N} - \frac{c''(q_s)(1-n)(1-\delta)}{u''\left(q_b^L\right)n - c''(q_s)(1-n)\delta}\right) + \frac{B}{q_b^N}}{(1-n)\left[\delta q_b^L + (1-\delta)\, q_b^N\right] + B}\left(\bar{\gamma} - \beta\right)\frac{\partial q_b^N}{\partial \gamma} < 0
$$

[22] We choose $M1$ as the monetary aggregate to measure money holdings so that our calibration results are comparable with previous studies.

By equating $\overline{\xi}(\alpha, B, \theta, \delta, n)$ to the sample average -0.5, we obtain the fifth equation to pin down $\alpha, B, \theta, \delta, n$. Table 2 reports calibrated parameters as well as consumption and production quantities:

Table 2. Calibration results

Calibrated parameters and allocations			
δ	0.17	θ	1.0032
n	0.1239	q_b^N	0.3023
α	0.9694	q_b^L	0.8098
B	0.1068	q_s	2.7476

Once we have determined the values of the calibrated parameters, it is possible to calculate the effect that an increase in δ would have on equilibrium allocations and welfare in the particular steady-state consistent with the data (see Table 3). Both quantities q_b^N and q_b^L shrink when δ becomes higher, even though consumption quantity for buyers who do not borrow falls much more than the quantity for buyers who do borrow (elasticities are -0.2074 and -0.0089, respectively). This explains that the change in δ is welfare worsening: in this case, the positive extensive effect does not compensate for the intensive effect that specially affects higher marginal utility buyers. In addition, as we could anticipate from the first-order conditions, a decrease in q_b^L is accompanied by an increase in q_s. We also report comparative statics on allocations and welfare given a change in the rate of inflation to corroborate our findings, since we know the signs of the derivatives from Proposition 2.

Table 3. Calibration results

Comparative statics: δ		Comparative statics: γ	
$(dW/d\delta)\,\delta/W$	$-7.3 * 10^{-3}$	$(dW/d\gamma)\,\gamma/W$	-0.4273
$(dq_s/d\delta)\,\delta/q_s$	$+0.0850$	$(dq_s/d\gamma)\,\gamma/q_s$	-26.3681
$\left(dq_b^L/d\delta\right)\delta/q_b^L$	-0.0089	$\left(dq_b^L/d\gamma\right)\gamma/q_b^L$	$+2.7574$
$\left(dq_b^N/d\delta\right)\delta/q_b^N$	-0.2074	$\left(dq_b^N/d\gamma\right)\gamma/q_b^N$	-42.3480

Table 4 illustrates the cost of inflation. We calculate it by computing how much consumption an agent would give up at a 0% inflation rate (i.e., $\gamma = 1$) to have the expected utility that corresponds to an annual rate of inflation of 10%, which is approximately equivalent to a quarterly inflation rate of 2.4%. Expected utility at $\gamma = 1$ is:

$$(1-\beta)\, V_{\gamma=1} = v(x^*) - x^* + (1-n)\left[\delta u\left(q^L_{b(\gamma=1)}\right) + (1-\delta)\, u\left(q^N_{b(\gamma=1)}\right)\right] - nc\left(q_{s(\gamma=1)}\right)$$

while the expected utility at $\gamma = 1.024$ is:

$$\begin{aligned}(1-\beta) V_{\gamma=1.024} &= v(x^*) - x^* + (1-n)\,\delta u\left(q^L_{b(\gamma=1.024)}\right) \\ &\quad + (1-n)(1-\delta)\, u\left(q^N_{b(\gamma=1.024)}\right) - nc\left(q_{s(\gamma=1.024)}\right)\end{aligned}$$

Hence, we calculate the cost of inflation by finding the value $\Delta_{\gamma=1.024}$ that solves the following equation:

$$\begin{aligned}(1-\beta) V_{\gamma=1.024} &= v(x^*\Delta_{\gamma=1.024}) - x^* + (1-n)\,\delta u\left(q^L_{b(\gamma=1)}\Delta_{\gamma=1.024}\right) \\ &\quad + (1-n)(1-\delta)\, u\left(q^N_{b(\gamma=1)}\Delta_{\gamma=1.024}\right) - nc\left(q_{s(\gamma=1)}\right)\end{aligned}$$

We also calculate $\Delta_{\gamma=1.012}$, the factor that would render an agent indifferent between 0% of inflation and the calibrated value of inflation, in a similar fashion. In our calibrated model, diminishing the quarterly inflation from 1.2% to 0% is worth 0.52% of steady-state consumption (or output), while diminishing the quarterly inflation from 2.4% to 0% is worth 1.08% of steady-state consumption. This estimation is in line with that presented by Lucas. It is also close to the estimates made by Lagos and Wright in the case where the buyer has all the bargaining power, the one that admits a comparison to our competitive pricing set-up.

Table 4. Welfare cost of inflation

$\gamma = 1$		$\gamma = 1.024$		$\gamma = 1.012$	
$q^N_{b(\gamma=1)}$	0.4983	$q^N_{b(\gamma=1.024)}$	0.1839	$q^N_{b(\gamma=1.012)}$	0.3023
$q^L_{b(\gamma=1)}$	0.7821	$q^L_{b(\gamma=1.024)}$	0.8344	$q^L_{b(\gamma=1.012)}$	0.8098
		$1-\Delta_{\gamma=1.024}$	1.08%	$1-\Delta_{\gamma=1.012}$	0.52%

Table 5 reports the cost of inefficient consumption-risk sharing owing to an asymmetric access to credit. We calculate how much of steady-state consumption would render agents indifferent between the value of δ in 1990 and the value of δ in both 2003 and 2005.[23] For this, we equate the expected utility for the value of δ in 2003; i.e., $\delta = 0.173$,

$$\begin{aligned}(1-\beta) V_{\delta_{2003}} &= v(x^*) - x^* + (1-n)\,\delta_{2003} * u\left(q^L_{b(\delta_{2003})}\right) \\ &\quad + (1-n)(1-\delta_{2003})\, u\left(q^N_{b(\delta_{2003})}\right) - nc\left(q_{s(\delta_{2003})}\right)\end{aligned}$$

to the expected utility that corresponds to the value of δ in 1990; i.e., $\delta = 0.141$, and consumption quantities are multiplied by a factor $\Delta_{\delta_{2003}}$,

$$\begin{aligned}(1-\beta) V_{\delta_{2003}} &= v(x^*\Delta_{\delta_{2003}}) - x^* + (1-n)\,\delta_{1990} * u\left(q^L_{b(\delta_{1990})}\Delta_{\delta_{2003}}\right) \\ &\quad + (1-n)(1-\delta_{1990})\, u\left(q^N_{b(\delta_{1990})}\Delta_{\delta_{2003}}\right) - nc\left(q_{s(\delta_{1990})}\right)\end{aligned}$$

[23] We report results for both 2003 and 2005 because only estimates are available for 2005.

We repeat the exercise for the value of δ in 2005; i.e., $\delta = 0.185$. We find that the increase of δ that took place from 1990 to 2003 and 2005 has entailed a welfare loss equivalent to 0.018% and 0.025% of steady-state consumption, respectively. We think that these figures are reasonable, since it would not seem sensible to argue that a higher proportion of agents having access to credit deteriorates welfare considerably. The point we want to make here is that improvements in the credit sector that allowed the number of borrowers to increase did not give rise to a welfare gain for the overall population, since the negative effect on consumption of agents unable to borrow has been sufficiently strong compared to the benefit of allowing more agents to borrow.

Table 5. Welfare cost of an increase in δ

$\delta = 0.141$ (1990)		$\delta = 0.173$ (2003)		$\delta = 0.185$ (2005)	
$q^N_{b(\delta_{1990})}$	0.3135	$q^N_{b(\delta_{2003})}$	0.3019	$q^N_{b(\delta_{2005})}$	0.2974
$q^L_{b(\delta_{1990})}$	0.8116	$q^L_{b(\delta_{2003})}$	0.8102	$q^L_{b(\delta_{2005})}$	0.8097
		$1 - \Delta_{\delta_{2003}}$	0.018%	$1 - \Delta_{\delta_{2005}}$	0.025%

To see that this result pertains to a particular combination of parameter values, in Figure 1 we depict the welfare loss (or gain) in terms of steady-state consumption for a range of values of δ. We compute the percentage of steady-state consumption that would render agents indifferent between each value of δ depicted and a value 1% lower $(1 - \Delta_{\delta_{0.01}})$. We acknowledge that welfare losses stemming from changes in risk sharing occur at relatively low values of δ, for which the intensive margin effect happens to be quantitatively more important than the extensive margin effect. In contrast, for higher values of δ we could expect that increasing the access to credit would actually be welfare improving. Increasing the proportion of borrowers appears to be costly in terms of welfare up to $\delta \approx 0.55$.

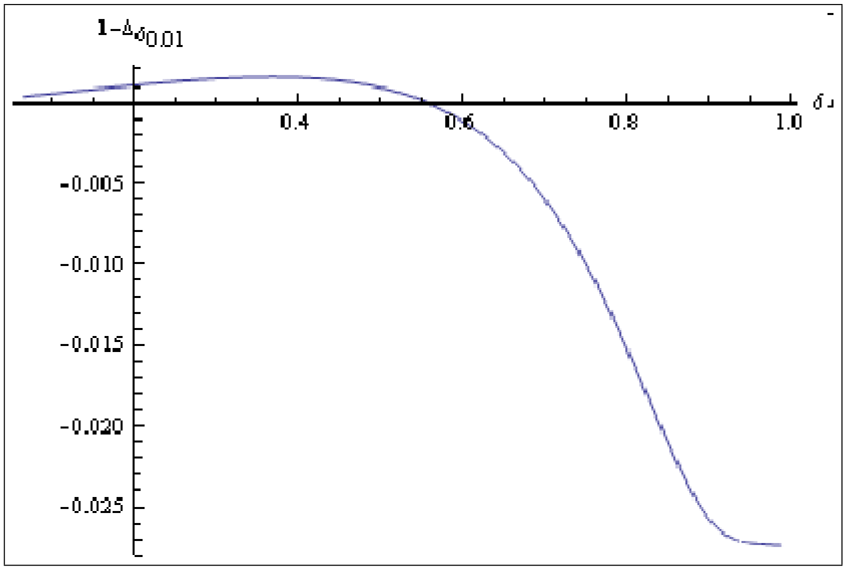

Figure 1: Welfare cost of increasing the proportion of borrowers as a function of δ

Finally, Figure 2 shows how the welfare cost of inflation changes when δ increases. The measure depicted is $(1-\Delta_{\gamma=1.012})$ (expressed in percentage), that is, the fraction of steady-state consumption that would provide the same expected utility for the calibrated value of γ and for $\gamma = 1$. The existence of a critical point is clear: the welfare cost of inflation is increasing in δ up to this point and decreases for higher values of δ.

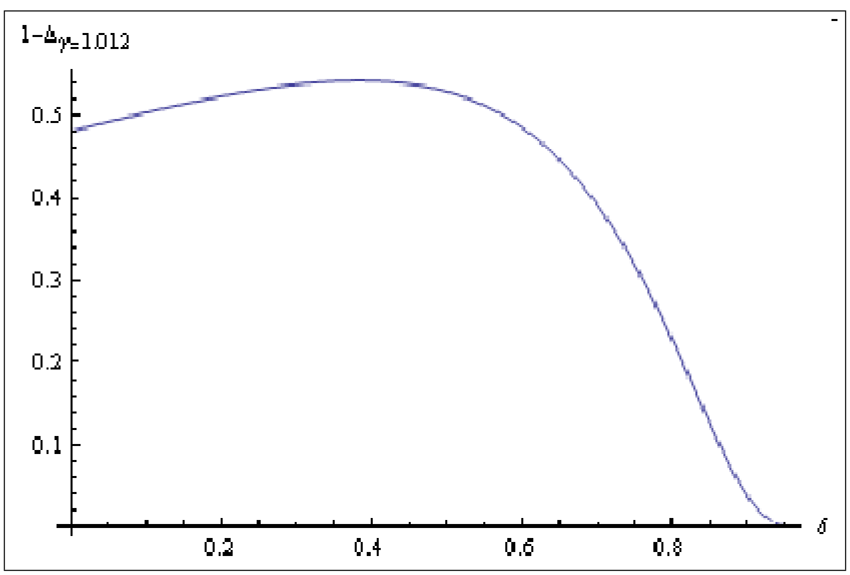

Figure 2: Welfare cost of inflation as a function of δ

5 Conclusion

In this paper we developed a model in which outside money and inside money are used as media of exchange in order to analyze how limited participation in the credit market impacts on welfare when the economy is away from the Friedman rule. Inflation is shown to be unambiguously welfare-worsening as it makes risk-sharing between borrowers and non-borrowers more inefficient. An increase in the proportion of agents able to borrow each period has an ambiguous impact on welfare as, on the one hand, it expands access to credit and, on the other, reduces the utility of non-borrowers. The quantitative analysis shows that the greater access to credit experienced in the United States since 1990 has entailed a slightly negative change in welfare. However, if the access to credit becomes sufficiently larger, the improvements that could increase the proportion of borrowers in the economy could be actually welfare improving.

Appendix

PROOF of PROPOSITION 1: Since by assumption $u(q)$ is strictly concave and $c(q)$ is convex, there are only one quantity q_b^N and one quantity q_b^L that solve (14) and (15) when $\gamma > \beta$ and $\delta \in (0,1)$. To see this, we need to compare the slope of the function $q_b^L\left(q_b^N\right)$ implicit in (14) with the slope of the function $q_b^L\left(q_b^N\right)$ implicit in (15). We indicate the former by $\partial q_b^L/\partial q_b^N$ and the latter by $\hat{\partial} q_b^L/\hat{\partial} q_b^N$.

From (14), we deduce by using the implicit function theorem:

$$\frac{\partial q_b^L}{\partial q_b^N} = \frac{c''(q_s)\, u'\left(q_b^L\right) \frac{(1-n)(1-\delta)}{n}}{u''\left(q_b^L\right) c'(q_s) - c''(q_s)\, u'\left(q_b^L\right) \frac{(1-n)\delta}{n}} < 0 \tag{19}$$

and from (15) we get:

$$\frac{\hat{\partial} q_b^L}{\hat{\partial} q_b^N} = \frac{u''\left(q_b^N\right) c'(q_s) - c''(q_s)\, u'\left(q_b^N\right) \frac{(1-n)(1-\delta)}{n}}{c''(q_s)\, u'\left(q_b^N\right) \frac{(1-n)\delta}{n}} < 0 \tag{20}$$

If we compare (19) to (20) it turns out that $\left|\partial q_b^L/\partial q_b^N\right| < \left|\hat{\partial} q_b^L/\hat{\partial} q_b^N\right|$ iff

$$\begin{aligned} & \frac{(1-n)}{n} c''(q_s) \left[u'\left(q_b^N\right) u''\left(q_b^L\right)(1-\delta) + u''\left(q_b^N\right) u'\left(q_b^L\right)\delta\right] \\ < \; & c'(q_s)\, u''\left(q_b^N\right) u''\left(q_b^L\right) \end{aligned} \tag{21}$$

The left-hand side in (21) is negative whereas the right-hand side is positive. This means that this inequality holds and so we can conclude that $\left|\partial q_b^L/\partial q_b^N\right| < \left|\hat{\partial} q_b^L/\hat{\partial} q_b^N\right|$. To prove that both curves intersect at only one point $\left(q_b^N, q_b^L\right)$ we also need to determine the points at which they intercept both axes q_b^N and q_b^L. From (14), it turns out that $q_b^N \to \infty$ when $q_b^L \to 0$ and $q_b^L = \breve{q}_b^L$ when $q_b^N \to 0$, where $\breve{q}_b^L$ is the quantity that satisfies $u'\left(\breve{q}_b^L\right) = c'\left(\delta \breve{q}_b^L (1-n)/n\right)$. From (15), it turns out that $q_b^L \to \infty$ when $q_b^N \to 0$ and $q_b^N = \breve{q}_b^N$ when $q_b^L \to 0$, where $\breve{q}_b^N$ is the quantity that satisfies $u'\left(\breve{q}_b^N\right) = \{\gamma + \beta[(1-n)(1-\delta)-1]\}/[\beta(1-n)(1-\delta)] * c'\left(\breve{q}_b^N (1-\delta)(1-n)/n\right)$ (it is then straightforward to see that $\breve{q}_b^N < \breve{q}_b^L$). Given that the slope of the second curve is steeper than the first one (for all q_b^N) and that the first curve intercepts only the axis q_b^L at a finite number while the second curve intercepts only the axis q_b^N, this implies that both curves intersect in the space (q_b^N, q_b^L) at only one point.

That in equilibrium $q_b^N < q_b^L$ when $\gamma > \beta$ and $\delta \in (0,1)$ can be deduced also from concavity of $u(q)$ and (14) and (15).

To see that $q_b^L > q^*$, compare (18) to (14). $\delta q_b^L + (1-\delta) q_b^N < q_b^L$ since $\delta < 1$ and $q_b^L > q_b^N$. Then $u'\left(q_b^L\right) < c'\left(q_b^L (1-n)/n\right)$ which implies $q^* < q_b^L$. In addition, since (18) implies $u'(q)$ and $\check{c}'(q) = c'(q(1-n)/n)$ intersect at q^* and $q_b^L > q^*$, it must be $q_s = (1-n)/n * \left(\delta q_b^L + (1-\delta) q_b^N\right) < (1-n)/n * q^*$ for (14) to hold.

If $\delta = 1$ and $\gamma > \beta$, it is straightforward to see that (15) cannot hold, which implies that $\beta V'(m_O) < \phi_{-1}$ for all $m_O > 0$. Thus $m_O^* = 0$ and outside money is driven out by inside money.

If $\gamma = \beta$ and $\delta \in (0,1)$, (14) and (15) are identical. Therefore, $q_b^N = q_b^L = q^*$. Then all traders choose $m^* = pq^*$ and $l = 0$.[24]

Finally, we have to verify that $V(m_O)$ is a concave function, so that the solution to (4) is well-defined.

Rewrite (12) as

$$V'(m_O) = (1-n)\left[\delta \frac{u'\left(q_b^L\right)}{p} + (1-\delta)\frac{u'\left(q_b^N\right)}{p}\right] + \phi n$$

Let $m^* = pq^*$. As long as $\delta < 1$, if $m < m^*$ then $q_b = \delta q_b^L + (1-\delta) q_b^N < q^*$ which means $dq_b/dm > 0$ so that $V''(m_O) < 0$. If $m = m^*$ then $q_b = q^*$ which means $dq_b/dm = 0$, so $V''(m_O) = 0$. This implies $V(m_O)$ is concave.

PROOF of PROPOSITION 2: Deriving (14) and (15) with respect to γ yields:

$$\frac{dq_b^N}{d\gamma} = \frac{\frac{\gamma}{(1-n)(1-\delta)} c'(q_s)\left[u''\left(q_b^L\right) - c''(q_s)\frac{1-n}{n}\delta\right]}{\beta u''\left(q_b^N\right)\left[u''\left(q_b^L\right) - c''(q_s)\frac{1-n}{n}\delta\right] - \frac{\gamma + [\beta(1-n)(1-\delta)-1]}{n} u''\left(q_b^L\right) c''(q_s)} < 0$$

[24] Actually, given that borrowing is costless for buyers able to borrow, we should consider the existence of multiple equilibria when $\gamma = \beta$. Buyers could borrow different amounts of inside money, even though they may not use it in trade.

and

$$\frac{dq_b^L}{d\gamma} = \frac{c''(q_s)(1-n)(1-\delta)}{u''(q_b^L)n - c''(q_s)(1-n)\delta}\frac{dq_b^N}{d\gamma} > 0$$

Deriving (17) with respect to γ yields:

$$\frac{dq_s}{d\gamma} = \frac{1-n}{n}\frac{u''\left(q_b^L\right)(1-\delta)}{u''(q_b^L) - c''(q_s)\frac{1-n}{n}\delta}\frac{dq_b^N}{d\gamma} < 0$$

Finally, the derivative of (16) with respect to γ gives the effect of γ on welfare, which is negative since $\left[u'\left(q_b^N\right) - c'(q_s)\right](1-\delta)\left(dq_b^N/d\gamma\right) < 0$.

PROOF of PROPOSITION 3: Using (14) and (15) we get:

$$\frac{dq_b^L}{d\delta} = \left[\frac{dq_b^N}{d\delta}(1-\delta) + q_b^L - q_b^N\right]\frac{c''(q_s)(1-n)}{u''(q_b^L)n - c''(q_s)\delta(1-n)} \tag{22}$$

and

$$\frac{dq_b^N}{d\delta} = \frac{1}{(1-\delta)}\frac{\{\gamma + [\beta(1-n)(1-\delta)-1]\}c''(q_s)\left(q_b^L - q_b^N + \delta\frac{dq_b^L}{d\delta}\right) + c'(q_s)\frac{(\gamma-\beta)n}{(1-n)(1-\delta)}}{\beta u''(q_b^N) - c''(q_s)\{\gamma + [\beta(1-n)(1-\delta)-1]\}} \tag{23}$$

Combining (22) and (23) yields:

$$\frac{dq_b^N}{d\delta} = \frac{u''\left(q_b^L\right)c''(q_s)\left(q_b^L - q_b^N\right) + \frac{\gamma-\beta}{(1-n)(1-\delta)\{\gamma+\beta[(1-n)(1-\delta)-1]\}}c'(q_s)\left[u''\left(q_b^L\right)n - \delta c''(q_s)(1-n)\right]}{\frac{(1-\delta)\beta u''\left(q_b^N\right)\left[u''\left(q_b^L\right)n - \delta c''(q_s)(1-n)\right]}{\gamma+\beta[(1-n)(1-\delta)-1]} - (1-\delta)u''(q_b^L)c''(q_s)} \tag{24}$$

which is negative.

Since the effect of δ on q_b^L, q_s and welfare is ambiguous, it is sufficient to consider different examples that exhibit an opposite relationship between δ and each of those variables.[25]

The intensive margin effect is negative because $\delta\left(dq_b^L/d\delta\right)\left[u'\left(q_b^L\right) - c'(q_s)\right]$
$+(1-\delta)\left(dq_b^N/d\delta\right)\left[u'\left(q_b^N\right) - c'(q_s)\right] < 0$.

The extensive margin effect is given by $u\left(q_b^L\right) - u\left(q_b^N\right) - u'\left(q_b^L\right)\left(q_b^L - q_b^N\right)$ since $c'(q_s) = u'\left(q_b^L\right)$. By the mean value theorem there is a $q_m \in \left(q_b^N, q_b^L\right)$ such that $u\left(q_b^L\right) - u\left(q_b^N\right) - u'(q_m)\left(q_b^L - q_b^N\right) = 0$. Hence, $u\left(q_b^L\right) - u\left(q_b^N\right) - u'\left(q_b^L\right)\left(q_b^L - q_b^N\right) > 0$ since u is strictly concave.

[25] See Section 4 for an example of q_b^L decreasing in δ, q_s increasing in δ and both negative and positive effects of δ on welfare. The function $u(q) = q^{0.01}\log(q+1)$ defined for $q \in (0.010152, 1)$ provides an example of q_b^L increasing in δ and q_s decreasing in δ for low values of δ, if the parameters' values presented in the Section 4 are used.

PROOF of PROPOSITION 4: From (22) and (24), $dq_b^L/d\delta > dq_b^N/d\delta$ is equivalent to

$$\left\{\beta(1-\delta)(1-n)\left[u''\left(q_b^L\right)-u''\left(q_b^N\right)\right]+(\gamma-\beta)u''\left(q_b^L\right)\right\}$$
$$< \frac{(\gamma-\beta)c'(q_s)}{(q_b^L-q_b^N)}\frac{(1-n)c''(q_s)-u''\left(q_b^L\right)n}{c''(q_s)(1-\delta)(1-n)}$$

If $u''' < 0$, then $u''\left(q_b^L\right)-u''\left(q_b^N\right) < 0$ and this condition always holds.

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