

# Money and Optimal Capital Taxation

S. Borağan Aruoba \*  
University of Maryland

Sanjay K. Chugh†  
University of Maryland

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## Abstract

In existing models of jointly-optimal fiscal and monetary policy, the monetary aspects of the economic environment have little to do with capital taxation prescriptions. Instead, the capital-taxation prescriptions of the underlying purely real economy in such models carry over unchanged, qualitatively and very nearly quantitatively, to the monetary economy. In this paper, we employ a micro-founded model of money in order to more meaningfully connect optimal fiscal *and* monetary policy, with a particular focus on optimal capital taxation. Our main result is that deep-rooted frictions underlying monetary trade in and of themselves provide a rationale for nonzero capital taxation — specifically, for capital subsidies. Optimal capital subsidies arise in versions of our model where monetary trades lead to capital holdup problems — in which case the prescription to subsidize capital follows readily — as well as versions of our model where holdup problems are absent. The latter result especially highlights the unique connection between fiscal *and* monetary policy our model articulates because the underlying purely real economy in our model features zero capital taxation. Connecting our results with some other recent advances in optimal capital taxation, we prove that for some versions of our environment, capital-income subsidies are consistent with zero intertemporal distortions. Our main conclusion is that capital-tax policy can fundamentally be driven by monetary issues.

**Keywords:** holdup problems, intertemporal distortions, micro-founded models of money, Friedman Rule

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\*E-mail address: aruoba@econ.umd.edu.

†E-mail address: chughs@econ.umd.edu

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# 1 Introduction

Ramsey models of jointly-optimal fiscal and monetary policy have only recently begun considering how the presence of capital accumulation affects policy prescriptions. Schmitt-Grohe and Uribe (2005) and Chugh (2007) both conclude that monopoly power in goods markets leads to capital subsidies as part of the Ramsey policy. The intuition is of course straightforward: monopoly power in goods markets leads to an inefficiently-low capital stock in the private economy, and a benevolent government attempts to correct this by subsidizing capital. In neither Schmitt-Grohe and Uribe (2005) nor Chugh (2007) is optimal capital taxation the main focus. However, in terms of their capital-taxation implications, their results are essentially just an extension of Judd’s (2002) demonstration — which does not rely at all on the existence of money or even a fully-specified Ramsey optimal-taxation problem — that monopoly power in goods markets calls for subsidies to capital. The monetary aspects of Schmitt-Grohe and Uribe’s (2005) and Chugh’s (2007) models therefore have nothing to do with their capital taxation results. They both use simple reduced-form models of money demand, which seems important for the lack of a deep connection between fiscal policy and monetary policy.<sup>1</sup> In particular, there is little sense in such models that monetary issues drive fiscal policy prescriptions.

In this paper, we instead employ a micro-founded model of money in order to more meaningfully connect optimal fiscal *and* monetary policy, with a particular focus on optimal capital taxation. Our main result is that deep-rooted frictions underlying monetary trade in and of themselves provide a rationale for subsidizing capital. Moreover, this result is specific to the fully-optimal joint fiscal *and* monetary policy: if monetary policy were conducted in a (particular) sub-optimal way, the capital-income tax rate would be zero. None of our optimal-taxation results is driven by incompleteness of the tax system, as can often happen especially with regards to the capital-income tax. To the contrary, our model features a complete set of tax instruments, which means there is (at least) one policy instrument for every independent margin in the decentralized economy. Our results thus illustrate that capital-tax policy can fundamentally be driven by monetary issues and can depend in particular on the primitive reasons for money demand.

Our work is a policy application of the model of Aruoba, Waller, and Wright (2007), who build on Lagos and Wright (2005) in order to integrate search-based monetary theory with a standard RBC dynamic general equilibrium model featuring capital accumulation. As Aruoba, Waller, and Wright (2007) — henceforth, AWW — show, if capital is used for production of goods that are exchanged in bilateral trades, then holdup problems in capital accumulation generically arise when the terms of trade in bilateral monetary transactions are determined via bargaining. These holdup

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<sup>1</sup>Schmitt-Grohe and Uribe (2005) use a transactions-based velocity model, and Chugh (2007) uses a cash-good/credit-good structure.

problems in capital investment — that is, the fact that some parties must make unilateral capital investment decisions but then share the fruits of the capital via production in bilateral trades — lead to inefficiently-low capital accumulation. In the baseline AWW bargaining environment, we find, perhaps not too surprisingly, that capital subsidies are optimal.

However, AWW also prove that if terms of trade in bilateral monetary transactions are determined by competitive price determination, then capital holdup problems disappear (as does another holdup problem related to money demand inherent in a micro-founded model of money). A natural conjecture then is that a zero capital tax would be optimal. We show that this is in fact not the case. Even though capital holdup problems disappear with competitive pricing, the optimal *monetary* policy, which entails a deviation from the Friedman Rule of a zero net nominal interest rate, distorts private-sector capital accumulation. Inflation thus acts as an indirect tax on capital accumulation, which a capital-income subsidy then either partially or wholly offsets. This latter result in particular is a nontrivial interaction between monetary and fiscal policy, one that can only be revealed by our full Ramsey framework — that is, one that can arise only in an environment with no lump-sum taxation. In this sense, optimal capital taxation is driven by fundamentally monetary issues, in contrast to the results of Schmitt-Grohe and Uribe (2005) and Chugh (2007), in which capital-taxation prescriptions are, qualitatively, independent of money demand. This is the essence of our conclusion that the primitive reasons underlying money demand may have important implications for how one thinks about optimal fiscal policy.

Our results can in a broad sense be viewed as a monetary counterpart to Acemoglu and Shimer (1999), who demonstrate, in a purely real environment, that search and bargaining frictions in labor markets lead to holdup problems in capital investment. Acemoglu and Shimer (1999) also consider environments in which holdup problems are absent.<sup>2</sup> While they do not explicitly draw optimal taxation implications, it seems clear from their analysis that capital subsidies would be optimal in their bargaining environment, but would be unnecessary when holdup problems are absent. Similar to Acemoglu and Shimer (1999), we find that capital subsidies are optimal in our bargaining environment, driven primarily by holdup problems. However, in contrast to Acemoglu and Shimer (1999) and as we noted above, we find that capital subsidies continue to be optimal even in the absence of holdup problems, driven by the unique connection between monetary policy and fiscal policy that operates through monetary trades.

Our result can also be related to Albanesi and Armenter (2007), who recently provided a unified framework in which to think about long-run capital taxation in both the Ramsey representative-agent framework and the Mirrleesian heterogenous-agent framework. One particularly important distinction they make is that between zero capital taxation and zero intertemporal distortions.

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<sup>2</sup>In their model this occurs when labor-market search is directed by ex-ante price (wage) posting.

They provide sufficient conditions under which a zero wedge between intertemporal marginal rates of substitution and marginal rates of transformation is optimal. Whether this zero wedge requires a zero capital tax or a nonzero capital tax then depends on the details of the economic environment. Albanesi and Armenter’s (2007) framework does not encompass (either reduced-form or micro-founded) monetary environments. However, we are able to prove for two important variants of our model that a zero intertemporal distortion is optimal and, moreover, that capital subsidies are in fact required to support a zero intertemporal distortion. For other versions of our model, in which holdup problems afflicting *money* demand are present, we find that intertemporal distortions are optimal. This latter result is yet another way in which fundamentally monetary issues in our model have implications for capital-income taxation.

The main focus in this paper is on the long-run optimal capital tax. However, our model also leads to other novel policy prescriptions. We have already mentioned that the Ramsey policy deviates from the classic Friedman Rule of a zero net nominal interest rate. We also conduct stochastic business-cycle simulations of our model; our central finding with regards to policy dynamics is that inflation is extremely stable over the business cycle. This latter result is novel because it arises in a flexible-price environment, which is counter to the conventional high-inflation-volatility result in the flexible-price Ramsey literature since Chari, Christiano, and Kehoe (1991). The deviation from the Friedman Rule and optimal inflation stability both arise in the simpler monetary environment that was studied in Aruoba and Chugh (2007), an environment that featured no capital accumulation. Thus, we document here that the results of Aruoba and Chugh (2007) readily extend to a monetary model with capital; although we briefly discuss these results in our model here, we refer interested readers to Aruoba and Chugh (2007) for more in-depth analysis on these two points.

As emphasized above, our results have nothing to do with incompleteness of the tax system. As Chari and Kehoe (1999, p. 1679-1680) explain, an incomplete tax system is in place if, for at least one pair of goods in the economy, the government has *no* policy instrument that drives a wedge between the marginal rate of substitution (MRS) and the corresponding marginal rate of transformation (MRT). As the examples of Correia (1996), Jones, Manuelli, and Rossi (1997), and Armenter (2008) illustrate, incompleteness of the tax system typically leads to non-zero capital-income taxation because the capital tax ends up substituting for the ability to create certain wedges. Incompleteness of tax instruments does not plague our model. In our analysis, neither the non-zero capital tax nor the deviation from the Friedman Rule arises due to any inability on the part of the government to create wedges between one or more MRS/MRT pairs. Indeed, in Section 5, we prove that our model features a complete set of policy instruments by demonstrating that the government has one unique policy tool for each wedge between MRS and MRT that it might want to create.

The rest of our work is organized as follows. Section 2 describes the environment and charac-

terizes the private sector equilibrium. Section 3 describes efficient allocations. Section 4 presents the Ramsey problem. Section 5 proves that the tax system is complete in the sense defined above. Section 6 presents analytical results, and Section 7 presents numerical results. Section 8 concludes. We provide the details of some of our analytical results in the Appendix.

## 2 Model

Our model follows closely the baseline model in AWW, which extends the framework in Lagos and Wright (2005) — henceforth, LW — to allow capital accumulation. The economy is populated by a measure one of infinitely-lived ex-ante identical households. In any period  $t$ , households first trade in a centralized market. During the centralized market (CM), a household rents its previously-accumulated capital and supplies its labor on spot factor markets; it chooses CM consumption in a spot goods market; and it adjusts its holdings of capital, a one-period nominal government bond, and money. Prices are determined competitively in all trades executed in the CM.

Upon exiting the CM, each household receives an idiosyncratic taste shock which governs its trading status in the second subperiod of period  $t$ . Trades in this second subperiod occur in a decentralized fashion, hence the label decentralized market (DM). A given household is a buyer in the DM with fixed probability  $\sigma$ , a seller with fixed probability  $\sigma$ , and neither a buyer nor a seller with probability  $1 - 2\sigma$ . That is, with probability  $1 - 2\sigma$ , a household does not trade at all in the DM.<sup>3</sup>

In the DM, buyers and sellers meet randomly, and trade is bilateral. In a given trade, a seller household produces goods for the buyer household using effort and capital, and receives money in return from the buyer. The terms of trade in a bilateral meeting are determined either through bargaining, which has become fairly standard in this class of models, or through Walrasian pricing (price-taking). We implement the latter pricing mechanism following Rocheteau and Wright (2005). Relative to LW, the critical innovation in AWW is that capital is accumulated in the CM and used in production in both the CM and the DM. As AWW show, with capital productive in both markets, the “dichotomy” result of both LW and Aruoba and Wright (2003), in which CM and DM allocations have nothing to do with each other, disappears.

Compared to AWW, we have reorganized the timing of markets by assuming that the CM meets before the DM in a given period. We make this change to mimic as closely as possible the timing assumption in standard monetary models — in particular, the cash-in-advance models of Lucas and Stokey (1983) and Chari, Christiano, and Kehoe (1991) — in which asset trade occurs before trade

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<sup>3</sup>This “taste shock” structure is simply a shortcut for the fully-specified environment in LW and the earlier monetary theory literature, in which the environment is specified explicitly in terms of search and double-coincidence problems. The two environments are formally identical.

in those goods markets in which money is the medium of exchange. This change in the timing of markets compared to AWW facilitates comparisons with these early seminal contributions to the macro-Ramsey literature, but it turns out to be inconsequential for all of our results.

We next describe economic events in the model in some more detail and then derive equilibrium conditions.

## 2.1 Production

In the CM, production takes place according to a constant-returns technology subject to TFP shocks,  $Y_t = Z_t F(K_t, H_t)$ . The notation is standard:  $K_t$  denotes aggregate capital,  $H_t$  denotes aggregate labor, and  $Z_t$  is the aggregate TFP shock. Profit-maximization by perfectly-competitive firms leads to standard factor-price conditions: the wage satisfies  $w_t = Z_t F_H(K_t, H_t)$ , and the rental price of capital satisfies  $r_t = Z_t F_K(K_t, H_t)$ .

In the period- $t$  DM, sellers produce using their capital carried out of the period- $t$  CM, which, according to our timing and notational conventions, is  $K_{t+1}$ .<sup>4</sup> Output in the DM is produced according to the technology  $q_t = Z_t f(K_{t+1}, e_t)$ , where  $e_t$  is the effort exerted by the seller, which creates a disutility given by  $v(e_t)$ .<sup>5</sup> Solving for the effort necessary for producing  $q_t$  units of DM output using capital  $K_{t+1}$  and aggregate technology state  $Z_t$  and computing its disutility leads to a cost function  $c(q_t, K_{t+1}, Z_t)$ , which describes the total (utility) cost of production. With  $f(\cdot)$  strictly increasing and strictly concave in each of its two arguments, and  $v(\cdot)$  strictly increasing and strictly convex, it readily follows that  $c_q > 0$ ,  $c_k < 0$ ,  $c_z < 0$ ,  $c_{qq} > 0$ ,  $c_{qk} < 0$  and  $c_{kk} > 0$ .

## 2.2 Households

A household enters the period- $t$  CM with money holdings  $m_{t-1}$ , nominal government bond holdings  $b_{t-1}$ , and capital holdings  $k_t$ . Before events unfold in the CM, the government spending state,  $G_t$ , and the TFP state  $Z_t$  are realized; we denote the exogenous aggregate state collectively as  $S_t = [G_t, Z_t]$ . Denoting the household's value function at the beginning of the period- $t$  CM by  $W_t(\cdot)$  and the household's value function at the beginning of the period- $t$  DM by  $V_t(\cdot)$ , the household's CM problem is

$$W_t(m_{t-1}, b_{t-1}, k_t, S_t) = \max_{x_t, h_t, m_t, b_t, k_{t+1}} \{U(x_t) - Ah_t + V_t(m_t, b_t, k_{t+1}, S_t)\}$$

subject to

$$P_t x_t + P_t \left[ k_{t+1} - (1 - \tau_t^k)(r_t - \delta)k_t \right] + m_t + b_t = P_t w_t (1 - \tau_t^h) h_t + m_{t-1} + R_{t-1} b_{t-1}.$$

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<sup>4</sup>Specifically, with the CM convening before the DM, households exit the period- $t$  CM with  $K_{t+1}$  units of capital, which is used in both period- $t$  DM production and period- $t + 1$  CM production.

<sup>5</sup>To preserve the tractability of the model, we do not link DM effort to labor supply in the CM.

$P_t$  is the nominal price of the (only) consumption good in the CM, which is  $x_t$ ;  $R_{t-1}$  is the gross nominal return on the one-period government bond purchased in period  $t-1$ ; and  $\tau_t^k$  and  $\tau_t^h$  are the tax rates on capital income (net of depreciation) and labor income, respectively.

The first-order conditions of this problem are

$$U'(x_t) = \frac{A}{w_t(1 - \tau_t^h)}, \quad (1)$$

$$\frac{A}{P_t w_t(1 - \tau_t^h)} = V_{m,t}(m_t, b_t, k_{t+1}, S_t), \quad (2)$$

$$\frac{A}{P_t w_t(1 - \tau_t^h)} = V_{b,t}(m_t, b_t, k_{t+1}, S_t), \quad (3)$$

and

$$\frac{A}{w_t(1 - \tau_t^h)} = V_{k,t}(m_t, b_t, k_{t+1}, S_t). \quad (4)$$

What makes the LW and AWW frameworks so tractable is the quasi-linearity of utility in the CM (and, as a prerequisite of course, the very existence of the CM) compared to similar search-based models in which households in equilibrium end up holding arbitrary non-negative quantities of money — for example of the latter, see Molico (2006). As LW and AWW show, quasi-linear CM utility implies that asset holdings — most importantly, money holdings — are identical across households in equilibrium. This degeneracy of the asset distribution follows from the first-order conditions (1)-(4), which together show that a given household's CM marginal utility of wealth is independent of its trading status in the previous DM.<sup>6</sup> Thus, all households in equilibrium make identical decisions in the CM. Another tractable implication of the CM problem is that  $W_t(\cdot)$  is linear in all its arguments, with marginal values given by

$$W_{m,t}(m_{t-1}, b_{t-1}, k_t, S_t) = \frac{A}{P_t w_t(1 - \tau_t^h)},$$

$$W_{b,t}(m_{t-1}, b_{t-1}, k_t, S_t) = \frac{AR_{t-1}}{P_t w_t(1 - \tau_t^h)},$$

and

$$W_{k,t}(m_{t-1}, b_{t-1}, k_t, S_t) = \frac{A [1 + (r_t - \delta) (1 - \tau_t^k)]}{w_t (1 - \tau_t^h)}.$$

Turning to the DM, we can write the problem of a household that enters the DM with portfolio  $(m_t, b_t, k_{t+1})$  as

$$\begin{aligned} V_t(m_t, b_t, k_{t+1}, S_t) &= \sigma \left\{ u \left( q_t^b \right) + \beta E_t W_{t+1} \left( m_t - d_t^b, b_t, k_{t+1}, S_{t+1} \right) \right\} \\ &+ \sigma \left\{ -c \left( q_t^s, k_{t+1}, Z_t \right) + \beta E_t W_{t+1} \left( m_t + d_t^s, b_t, S_{t+1} \right) \right\} \\ &+ (1 - 2\sigma) \beta E_t W_{t+1} \left( m_t, b_t, k_{t+1}, S_{t+1} \right), \end{aligned} \quad (5)$$

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<sup>6</sup>We refer interested readers to LW and AWW for more technical details.

where the first line describes the expected payoff if the household is a buyer, the second line describes the expected payoff if the household is a seller, and the last line describes the expected payoff if the household does not participate in the DM. We use  $(q_t^b, d_t^b)$  and  $(q_t^s, d_t^s)$  to represent the terms of trade from the viewpoints of the buyers and sellers, respectively.<sup>7</sup>

Exploiting the linearity of  $W_t(\cdot)$ , (5) simplifies to

$$V_t(m_t, b_t, k_{t+1}, S_t) = \sigma \left\{ u \left( q_t^b \right) - c \left( q_t^s, k_{t+1}, Z_t \right) - \beta \chi_t d_t^b + \beta \chi_t d_t^s \right\} + \beta E_t W_{t+1} \left( m_t, b_t, k_{t+1}, S_{t+1} \right).$$

Thus, all we have to do to characterize the solution to the household's problem is to compute the partial derivatives of  $V_t(\cdot)$ , which we do below for both pricing schemes we consider, bargaining and price-taking. As in AWW, capital's role in the DM is only as a productive input; it cannot be used as a medium of exchange. Those familiar with the AWW model may choose to skip the following exposition and proceed directly to the Ramsey problem in Section 4.

### 2.2.1 Household DM Problem - Bargaining

In this class of models, the most commonly-used pricing protocols for DM trades is bargaining — generalized Nash bargaining, in particular. Denote by  $\theta$  the (generalized Nash) bargaining power of the buyer, by  $(m_t, b_t, k_{t+1})$  the portfolio of the buyer, and by  $(\tilde{m}_t, \tilde{b}_t, \tilde{k}_{t+1})$  the portfolio of the seller. The generalized Nash bargaining problem is thus

$$\begin{aligned} & \max_{q_t, d_t} \left[ u(q_t) + \beta E_t W_{t+1} \left( m_t - d_t, b_t, k_{t+1}, S_{t+1} \right) - \beta E_t W_{t+1} \left( m_t, b_t, k_{t+1}, S_{t+1} \right) \right]^\theta \\ & \times \left[ -c(q_t, \tilde{k}_{t+1}, Z_t) + \beta E_t W_{t+1} \left( \tilde{m}_t + d_t, \tilde{b}_t, \tilde{k}_{t+1}, S_{t+1} \right) - \beta E_t W_{t+1} \left( \tilde{m}_t, \tilde{b}_t, \tilde{k}_{t+1}, S_{t+1} \right) \right]^{1-\theta} \end{aligned}$$

subject to

$$d_t \leq m_t. \tag{6}$$

The amount of cash that a buyer turns over to a seller in a DM trade is  $d_t$ ; the constraint (6) is thus simply a feasibility condition stating the buyer cannot spend more cash than he has on hand before meeting the seller. The threat points in the bargaining problem are the values of continuing on to the next CM, which occurs in period  $t + 1$ , without consummating a trade.

Once again using the linearity of  $W_t(\cdot)$  and defining  $\chi_t \equiv E_t \left[ A / \{ P_{t+1} w_{t+1} (1 - \tau_{t+1}^h) \} \right]$ , the bargaining problem simplifies to

$$\max_{q_t, d_t} \left\{ u(q_t) - \beta d_t \chi_t \right\}^\theta \left\{ -c(q_t, \tilde{k}_{t+1}, Z_t) + \beta d_t \chi_t \right\}^{1-\theta}$$

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<sup>7</sup>Strictly speaking, we need to integrate these payoffs over the distribution of capital and money holdings in the economy, but as LW and AWW show and as we summarized above, these distributions are degenerate.

subject to (6). In equilibrium, one can show, as AWW do, that (6) binds and that the quantity produced solves

$$\beta\chi_t m_t = g(q, \tilde{k}_{t+1}, Z_t),$$

where

$$g(q, k, Z) \equiv \frac{\theta c(q, k, Z) u'(q) + (1 - \theta) u(q) c_q(q, k, Z)}{\theta u'(q) + (1 - \theta) c_q(q, k, Z)}.$$

These last two conditions show that  $(q, d)$  depends only on the buyer's money holdings and the seller's capital holdings (along with, of course, the TFP realization), and on neither the seller's money holdings nor the buyer's capital holdings.

Using the results from the bargaining problem, and taking the appropriate partial derivatives of  $(q_t^b, q_t^s, d_t^b, d_t^s)$  with respect to the appropriate household's own money and capital holdings, we can compute the partial derivatives of  $V_t(\cdot)$  to finish solving the household's problem:

$$\begin{aligned} V_{m,t}(m_t, b_t, k_{t+1}, S_t) &= \beta\chi_t \left[ \sigma \frac{u'(q)}{g_q(q, k_{t+1}, Z)} + 1 - \sigma \right], \\ V_{b,t}(m_t, b_t, k_{t+1}, S_t) &= \beta R_t \chi_t, \end{aligned} \tag{7}$$

and

$$\begin{aligned} V_{k,t}(m_t, b_t, k_{t+1}, S_t) &= -\sigma \left[ -c_q(q_t, k_{t+1}, Z_t) \frac{g_k(q_t, k_{t+1}, Z_t)}{g_q(q_t, k_{t+1}, Z_t)} + c_k(q_t, k_{t+1}, Z_t) \right] \\ &\quad + \beta A E_t \left[ \frac{1 + (r_{t+1} - \delta)(1 - \tau_{t+1}^k)}{w_{t+1}(1 - \tau_{t+1}^h)} \right]. \end{aligned}$$

In these expressions, we used  $\partial q_t^b / \partial m_t = \beta\chi_t / g_q(q_t, Z_t)$  and  $\partial q_t^s / \partial k_t = -g_k(q_t, k_{t+1}, Z_t) / g_q(q_t, k_{t+1}, Z_t)$ .

Finally, defining

$$\gamma(q_t, k_{t+1}, Z_t) \equiv \frac{c_q(q_t, k_{t+1}, Z_t) g_k(q_t, k_{t+1}, Z_t) - c_k(q_t, k_{t+1}, Z_t) g_q(q_t, k_{t+1}, Z_t)}{g_q(q_t, k_{t+1}, Z_t)}, \tag{8}$$

we have

$$V_{k,t}(m_t, b_t, k_{t+1}, S_t) = \sigma \gamma(q_t, k_{t+1}, Z_t) + \beta E_t \left\{ \chi_t \left[ 1 + (r_{t+1} - \delta)(1 - \tau_{t+1}^k) \right] \right\}.$$

The definition of  $\gamma(\cdot)$  in (8) will be quite useful in understanding our bargaining model's holdup problem in capital investment.

### 2.2.2 Household DM Problem - Price-Taking

An alternative to bargaining is price taking, in which buyers and sellers each take the price of a unit of good in the DM,  $\tilde{p}$ , as given and solve their respective demand and supply problems. The buyer's problem is

$$V^b(m_t, b_t, k_{t+1}, S_t) = \max_q \{ u(q) + \beta E_t W_{t+1}(m_t - \tilde{p}q, b_t, k_{t+1}, S_{t+1}) \}$$

subject to  $\tilde{p}q \leq m_t$ . In equilibrium, this constraint binds, and we have  $q_t = M_t/\tilde{p}$ .

The seller's problem is

$$V^s(m_t, b_t, k_{t+1}, S_t) = \max_q \{-c(q, k_{t+1}, Z_t) + \beta E_t W_{t+1}(m_t + \tilde{p}q, b_t, k_{t+1}, S_{t+1})\},$$

with yields first-order condition  $c_q(q_t, k_{t+1}, Z_t) = \beta \tilde{p} \chi_t$ . Using these two expressions, the two envelope conditions we need to solve the problem of the household are

$$V_{m,t}(m_t, b_t, k_{t+1}, S_t) = (1 - \sigma) \beta \chi_t + \sigma \beta \chi_t \frac{u'(q_t)}{c_q(q_t, k_{t+1}, Z_t)}$$

and

$$V_{k,t}(m_t, b_t, k_{t+1}, S_t) = -\sigma c_k(q_t, k_{t+1}, Z_t) + \beta E_t \left\{ \chi_t \left[ 1 + (r_{t+1} - \delta) (1 - \tau_{t+1}^k) \right] \right\};$$

$V_{b,t}(\cdot)$  is still given by (7).

### 2.3 Government

Government consumption takes place in the CM and is financed by taxes on capital and labor income as well as money creation and debt issuance. Its CM flow budget constraint is thus

$$M_t + B_t + P_t w_t \tau_t^h H_t + P_t \tau_t^k (r_t - \delta) K_t = P_t G_t + M_{t-1} + R_{t-1} B_{t-1}.$$

We show below that permitting the government to have access to other policy instruments that affect either the CM or the DM has no bearing whatsoever on the Ramsey allocation.

### 2.4 Private Sector Equilibrium

Imposing equilibrium ( $m_t = M_t$ ,  $k_t = K_t$ , etc.), and combining the optimality conditions for firms and households, we now list the equilibrium conditions we use in writing the Ramsey problem.

#### 2.4.1 Bargaining

Given policy processes  $\{\tau_t^h, \tau_t^k, R_t\}_{t=0}^\infty$ , the technology process  $\{Z_t\}_{t=0}^\infty$ , the government spending process  $\{G_t\}_{t=0}^\infty$ , and initial conditions  $(M_0, B_0, K_0)$ , equilibrium is a collection of state-contingent processes  $\{q_t, B_t, M_t, K_t, X_t, H_t, P_t\}_{t=0}^\infty$  satisfying

$$U'(X_t) = \frac{A}{(1 - \tau_t^h) Z_t F_H(K_t, H_t)}, \quad (9)$$

$$\beta M_t E_t \left[ \frac{U'(X_{t+1})}{P_{t+1}} \right] = g(q_t, K_{t+1}, Z_t), \quad (10)$$

$$U'(X_t) = \beta E_t \left\{ U'(X_{t+1}) \left[ 1 + (1 - \tau_{t+1}^k) (Z_{t+1} F_K(K_{t+1}, H_{t+1}) - \delta) \right] \right\} + \sigma \gamma(q_t, K_{t+1}, Z_t), \quad (11)$$

$$R_t = \sigma \frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} + 1 - \sigma, \quad (12)$$

$$\frac{U'(X_t)}{P_t} = \beta R_t E_t \left[ \frac{U'(X_{t+1})}{P_{t+1}} \right], \quad (13)$$

$$X_t + G_t + K_{t+1} = Z_t F(K_t, H_t) + (1 - \delta)K_t, \quad (14)$$

and

$$M_t + B_t + P_t Z_t F_H(K_t, H_t) \tau_t^h H_t + P_t \tau_t^k [Z_t F_K(K_t, H_t) - \delta] K_t = P_t G_t + M_{t-1} + R_{t-1} B_{t-1}. \quad (15)$$

Some of these equilibrium conditions are identical to what one would get in a standard RBC model, such as the consumption-leisure optimality condition (9) and the intertemporal Euler equation for bonds in (13). Condition (10) follows from the solution to the DM bargaining problem, and (12) is a no-arbitrage condition that links the nominal return on bonds to the implied return of holding money.

Given our focus on capital income taxation, (11) deserves special attention. Except for the last term on the right-hand-side, (11) is a standard intertemporal Euler equation for capital investment. Because capital is used not only in the CM but also in the DM (with probability  $\sigma$ , if the household turns out to be a seller), optimal investment decisions take this into account. The additional term  $\sigma\gamma(\cdot)$  captures the return to capital in the DM, which reflects the fact that, all else equal, producing a given quantity of DM output is cheaper if a seller has more capital.

To construct the Ramsey problem, we need a compact expression for real money balances; combining (10), (12) and (13) we can express real money balances as

$$\frac{M_t}{P_t} = \frac{g(q_t, K_{t+1}, Z_t)}{U'(x_t)} \left[ \sigma \frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} + 1 - \sigma \right]. \quad (16)$$

We should also note that any monetary equilibrium must satisfy  $R_t \geq 1$ , which, when expressed in terms of allocations using (12), translates into what we call the zero lower bound (ZLB) constraint

$$\sigma \left( \frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} - 1 \right) \geq 0. \quad (17)$$

## 2.4.2 Price-Taking

Given policy processes  $\{\tau_t^h, \tau_t^k, R_t\}_{t=0}^\infty$ , the technology process  $\{Z_t\}_{t=0}^\infty$ , the government spending process  $\{G_t\}_{t=0}^\infty$ , and initial conditions  $(M_0, B_0, K_0)$ , equilibrium is a collection of state-contingent processes  $\{q_t, B_t, M_t, K_t, X_t, H_t, P_t\}_{t=0}^\infty$  satisfying (9), (13), (14), and (15), along with

$$\beta M_t E_t \left[ \frac{U'(X_{t+1})}{P_{t+1}} \right] = q_t c_q(q_t, K_{t+1}, Z_t), \quad (18)$$

$$U'(X_t) = \beta E_t \left\{ U'(X_{t+1}) \left[ 1 + (1 - \tau_{t+1}^k)(Z_{t+1} F_K(K_{t+1}, H_{t+1}) - \delta) \right] \right\} - \sigma c_k(q_t, K_{t+1}, Z_t), \quad (19)$$

and

$$R_t = \sigma \frac{u'(q_t)}{c_q(q_t, K_{t+1}, Z_t)} + 1 - \sigma. \quad (20)$$

In the price-taking version of our environment, (19) replaces (11) as the intertemporal Euler equation for capital. Instead of the  $\gamma(\cdot)$  that appears in (11), (19) features  $-c_k(\cdot)$  as the DM return to capital. From the definition in (8),  $\gamma(\cdot)$  contains  $-c_k(\cdot)$ , which describes the cost reduction for a marginal change in capital, holding fixed output  $q$ . However, in  $\gamma(\cdot)$ , the cost reduction captured by  $-c_k(\cdot)$  is at least partially offset by the increased cost due to the production of higher  $q$ , as captured by the  $c_q(\cdot) > 0$  term inside the  $\gamma(\cdot)$  function.<sup>8</sup>

The presence of  $c_q(\cdot) > 0$  inside  $\gamma(\cdot)$  is a manifestation of the capital-holdup problem identified by AWW. Because a household that turns out to be a seller does not enjoy the full return to its own capital accumulation decision, but rather must share some of the return via ex-post bargaining with a buyer, generically there will be under-accumulation of capital compared to the socially-optimal level of capital. In the price-taking environment, however, the competitive nature of DM trades circumvents this holdup problem — the terms of trade the seller faces,  $(q, d)$ , are not affected by his actions.

Next, real money balances can be expressed in the price-taking environment as

$$\frac{M_t}{P_t} = \frac{q_t c_q(q_t, K_{t+1}, Z_t)}{U'(X_t)} \left[ \sigma \frac{u'(q_t)}{c_q(q_t, K_{t+1}, Z_t)} + 1 - \sigma \right],$$

which follows from (13), (18) and (20). Finally using (20), the ZLB constraint ensuring a monetary equilibrium is

$$\sigma \left( \frac{u'(q_t)}{c_q(q_t, K_{t+1}, Z_t)} - 1 \right) \geq 0. \quad (21)$$

### 3 Efficient Allocations, Holdup Problems and Intertemporal Efficiency

Before turning to the Ramsey problem, we list the conditions that characterize the allocations  $\{q_t, K_{t+1}, X_t, H_t\}$  that a social planner would choose. Social efficiency is described by

$$u'(q_t) = c_q(q_t, K_{t+1}, Z_t), \quad (22)$$

$$U'(X_t) = \beta E_t \{ U'(X_{t+1}) [1 + (Z_{t+1} F_K(K_{t+1}, H_{t+1}) - \delta)] \} - \sigma c_k(q_t, K_{t+1}, Z_t), \quad (23)$$

$$U'(X_t) = \frac{A}{Z_t F_H(K_t, H_t)}, \quad (24)$$

and

$$X_t + G_t + K_{t+1} = Z_t F(K_t, H_t) + (1 - \delta) K_t, \quad (25)$$

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<sup>8</sup>This latter reflects the fact that the bargaining solution yields  $\partial q / \partial k > 0$  for the seller.

which do not require much comment because they follow readily from the definition of efficiency.<sup>9</sup> Comparing (22)-(25) with the decentralized equilibrium conditions, however, is instructive.

We make several observations. First, obviously, in the presence of proportional taxes, neither price-taking nor bargaining can achieve the social optimum. Second, shutting down proportional taxes, the equilibrium under price-taking achieves the social optimum if  $R_t = 1$ , i.e. if the Friedman Rule of a zero net nominal interest rate is in place. Third, even in the absence of proportional taxes and at the Friedman Rule, the equilibrium under Nash bargaining can never achieve the social optimum. This is due to the two holdup problems present in the bargaining environment, one that afflicts money demand and one that, as we mentioned above, afflicts investment in capital.

The holdup problem in capital investment disappears if  $\theta = 0$ , which leads to  $\gamma(\cdot) = -c_k(\cdot)$  and hence, as we discussed above, socially-optimal capital-accumulation incentives.<sup>10</sup> However, any  $\theta < 1$  means that households that turn out to be buyers have not accumulated the socially-optimal level of money balances. This follows because  $\theta < 1 \Rightarrow g_q(\cdot) \neq c_q(\cdot)$ , which in turn means that the social efficiency condition (22) will never coincide with the equilibrium condition (12). Intuitively, unless the buyer knows he will receive the entire surplus from a bilateral DM trade (which occurs only if  $\theta = 1$ ), ex-ante (i.e., in the immediately preceding CM), the buyer does not have the socially-optimal incentives to accumulate money. This is a holdup problem in money accumulation. Thus, while  $\theta = 0$  completely eliminates the capital holdup problem, the money holdup problem is present.

Alternatively, the money holdup problem disappears if  $\theta = 1$ . However, this also means  $\gamma(\cdot) \neq -c_k(\cdot)$ , which we know from our discussion above distorts capital-accumulation incentives. Thus, while  $\theta = 1$  completely eliminates the money holdup problem, the holdup problem in capital investment is present.

For the intermediate cases  $\theta \in (0, 1)$ , both the money holdup problem and the capital holdup problem are present because both  $g_q(\cdot) \neq c_q(\cdot)$  and  $\gamma(\cdot) \neq -c_k(\cdot)$ . But, interestingly, under price-taking, both of these holdup problems disappear because no households — neither buyers nor sellers — can affect the terms of DM trade through ex-ante accumulation choices. Thus, price-taking achieves social efficiency under the Friedman Rule (again, though, absent proportional taxation — we have not yet considered the Ramsey equilibrium).

Finally, for the subsequent analysis of the Ramsey problem and solution, it is useful to restate the conditions describing efficient allocations in terms of marginal rates of substitution (MRS) and corresponding marginal rates of transformation (MRT). To do this, note that our definition of the cost function of DM sellers mixes notions of preferences with notions of technology. As

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<sup>9</sup>They can be derived from maximizing social welfare given by (32) subject to the resource constraint (14).

<sup>10</sup>However, when  $\theta = 0$ , or when sellers make take-it-or-leave-it offers, a monetary equilibrium does not exist since the buyers have no incentive to carry money in to the DM.

we described in Section 2.1, the primitives behind this reduced-form cost function are that DM production occurs as sellers operate the technology  $q = Zf(k, e)$  while incurring the disutility of effort  $v(e)$ . It is easy to verify that the relationship between DM cost, utility, and production functions implies  $c_q(q, k, Z) = v'(e)/Zf_e(k, e)$  and  $c_k(q, k, Z) = -f_k(k, e)v'(e)/f_e(k, e)$ .

**Proposition 1.** *If the DM production function is  $q = Zf(k, e)$  and DM disutility of effort is  $v(e)$ , then the MRS and MRT for the pairs  $(e_t, q_t)$ ,  $(X_t, H_t)$  and  $(X_t, X_{t+1})$  are defined by*

$$\begin{aligned}
MRS_{e_t, q_t} &\equiv -\frac{u'(q_t)}{v'(e_t)} & MRT_{e_t, q_t} &\equiv -\frac{1}{Z_t f_e(K_{t+1}, e_t)} \\
MRS_{X_t, H_t} &\equiv -\frac{A}{U'(X_t)} & MRT_{X_t, H_t} &\equiv -Z_t F_H(K_t, H_t) \\
MRS_{X_t, X_{t+1}} &\equiv \frac{\beta U'(X_{t+1})}{U'(X_t)} & MRT_{X_t, X_{t+1}} &\equiv \frac{1 - \frac{\sigma v'(e_t) f_k(K_{t+1}, e_t) Z_t F_H(K_t, H_t)}{A f_e(K_{t+1}, e_t)}}{Z_{t+1} F_K(K_{t+1}, H_{t+1}) + 1 - \delta}
\end{aligned} \tag{26}$$

*Proof.* See Appendix B. □

Each MRS in Proposition 1 has the standard interpretation as a ratio of marginal utilities. Similarly, each MRT has the interpretation as a ratio of the marginal products of an appropriately-defined transformation frontier.<sup>11</sup> In the static MRS/MRT pairs in the first two lines above, negative signs appear because we have defined both CM and DM utility functions in terms of one good (consumption) and one bad (labor, or, in the DM, effort). The MRS between  $X_t$  and  $X_{t+1}$  is of course standard.

The intertemporal MRT (IMRT) in the third line above deserves further discussion. We formally derive the IMRT in Appendix B; an intuitive description suffices here. In the standard one-sector RBC model, in which there is only one type of produced good, it is straightforward to define the IMRT using the economy-wide intertemporal resource constraint. Due to our model's two-sector structure (DM and CM), however, defining the IMRT (in terms of CM consumption goods) is not as simple. By definition, the IMRT measures how many units of  $X_t$  the economy must forego in order to gain a given amount of  $X_{t+1}$ , holding output of all other goods in the economy constant. If  $X_t$  is reduced by one unit, the economy gains one additional unit of capital  $K_{t+1}$ , which increases  $X_{t+1}$  via period- $t+1$  CM production. Following period- $t+1$  production and subsequent depreciation, the one unit reduction in  $X_t$  leads to a gain of  $Z_{t+1} F_K(K_{t+1}, H_{t+1}) + 1 - \delta$ . This channel is standard in an RBC model, and it is present in our environment, as well. However, a second channel that affects the IMRT is also at work in our environment. The additional unit of  $K_{t+1}$  will also lead to increased period- $t$  DM production. Our definition of IMRT thus adjusts for this increase in  $q_t$ .

Reverting to using the cost function in the DM, the following corollary characterizes the efficient allocations in terms of MRSs and MRTs.

<sup>11</sup>We have in mind a very general notion of transformation frontier as in Mas-Colell et al (1995, p. 129).

**Corollary 1.** *The solution to the social planner's problem is characterized by the CM resource constraint (25) along with*

$$\frac{MRS_{e_t, q_t}}{MRT_{e_t, q_t}} \equiv \frac{u'(q_t)}{c_q(q_t, K_{t+1}, Z_t)} = 1, \quad (27)$$

$$E_t \left[ \frac{MRS_{X_t, X_{t+1}}}{MRT_{X_t, X_{t+1}}} \right] \equiv E_t \left\{ \frac{\beta U'(X_{t+1}) [Z_{t+1} F_K(K_{t+1}, H_{t+1}) + 1 - \delta]}{U'(X_t) \left[ 1 + \frac{\sigma c_k(q_t, K_{t+1}, Z_t) Z_t F_H(K_t, H_t)}{A} \right]} \right\} = 1, \quad (28)$$

and

$$\frac{MRS_{X_t, H_t}}{MRT_{X_t, H_t}} \equiv \frac{A}{U'(X_t) [Z_t F_H(K_t, H_t)]} = 1. \quad (29)$$

*Proof.* Obvious from the definition of the cost function  $c(\cdot)$ , Proposition 1, and (22)-(24)  $\square$

Corollary 1 shows that the economically-efficient allocations in our model can be described in terms of “zero-wedge conditions” between MRSs and MRTs. This way of understanding efficiency is of course standard, but given the novelty of our model, it is important to show how to precisely express efficiency in terms of the zero-wedge expressions (27), (28), and (29). This is especially important because, as Chari and Kehoe (1999, p. 1674) emphasize, optimal tax theory is really about the determination of optimal wedges between MRSs and MRTs. In what follows, we take expressions (27), (28), and (29) as the conditions that define zero wedges.

## 4 Ramsey Problem

As is common in the public finance approach to macroeconomic policy since Lucas and Stokey (1983), we use the primal approach and cast the Ramsey problem as the problem of a benevolent planner who chooses allocations subject to their implementation as a monetary equilibrium. We prove the following Proposition in Appendix A.1.

**Proposition 2.** *The allocations in a monetary equilibrium satisfy the resource constraint (14), the ZLB constraint ((17) for the bargaining model or (21) for the price-taking model), and the present-value implementability constraint (PVIC),*

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ U'(X_t) X_t - A H_t + \sigma g(q_t, K_{t+1}, Z_t) \left( \frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} - 1 \right) + \sigma \gamma(q_t, K_{t+1}, Z_t) K_{t+1} \right] = U'(X_0) A_0 \quad (30)$$

for the bargaining model or

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ U'(X_t) X_t - A H_t + \sigma q_t c_q(q_t, K_{t+1}, Z_t) \left( \frac{u'(q_t)}{c_q(q_t, K_{t+1}, Z_t)} - 1 \right) - \sigma c_k(q_t, K_{t+1}, Z_t) K_{t+1} \right] = U'(X_0) A_0 \quad (31)$$

for the price-taking model, where the constant  $A_0$  depends on  $M_0, B_0$ , and  $K_0$ ,

$$A_0 = \frac{M_{-1} + R_{-1}B_{-1}}{P_0} + \left[ 1 + (1 - \tau_0^k)(Z_0 F_K(K_0, H_0) - \delta) \right] K_0.$$

Comparing these PVICs with ones from standard flexible-price models, such as the ones in Chari and Kehoe (1999), the third and fourth terms on the left-hand-sides of (30) and (31) are novel. As Aruoba and Chugh (2007) argue, the third term is simply the marginal utility of money times the amount of real money balances, where the “marginal utility” stems from being able to consume more in the DM if the household happens to be a buyer. Similarly, the fourth term is nothing but the marginal DM benefit of capital times the capital holdings of the household. This extra benefit accrues when the household is a seller (with probability  $\sigma$ ), and the benefit terms  $\gamma(\cdot)$  (bargaining) or  $-c_k(\cdot)$  (price-taking) arise directly from the intertemporal Euler equations for capital in the respective environments.

Integrating over all households, the Ramsey problem is to choose  $\{X_t, H_t, K_{t+1}, q_t\}$  to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \{U(X_t) - AH_t + \sigma [u(q_t) - c(q_t, K_{t+1}, Z_t)]\} \quad (32)$$

subject to the resource constraint (14), the PVIC ((30) for the bargaining model or (31) for the price-taking model), and the ZLB constraint ((17) for the bargaining model or (21) for the price-taking model), taking as given  $\{G_t, Z_t\}$  and  $K_0$ . In Appendix A.2, we list the conditions that characterize the solution to this problem for both pricing schemes, along with the conditions that allow us to construct the policies and prices that support the Ramsey allocation.

## 5 Completeness of the Tax System

As we discussed in the introduction, an important issue in models of optimal taxation is whether or not the assumed tax instruments constitute a *complete* tax system.<sup>12</sup> In this section, we establish that the tax system is complete in our model. Establishing this is important for two reasons. First, at a technical level, proving completeness reaffirms that the Ramsey problem as formulated in Section 4, in which the only constraints are the sequence of CM resource constraints and the single PVIC, is indeed the correct Ramsey problem.<sup>13</sup> As shown by Chari and Kehoe (1999, p. 1680), Correia (1996), Armenter (2008), and many others, incompleteness of the tax system requires

<sup>12</sup>For convenience, we restate the definition of Chari and Kehoe (1999, 1679-1680) that an *incomplete* tax system is in place if, for at least one pair of goods in the economy, the government has *no* policy instrument that drives a wedge between the marginal rate of substitution (MRS) and the corresponding marginal rate of transformation (MRT). If this is not the case, then the tax system is instead said to be *complete*.

<sup>13</sup>For the purpose of establishing completeness, the ZLB constraint is irrelevant because it stems from the need to implement a monetary equilibrium and has nothing to do with completeness/incompleteness of the tax system.

imposing additional constraints that reflect the incompleteness. Incompleteness is not an issue in our model, therefore we do not need to impose additional constraints. Second, it is well-known in Ramsey theory that incomplete tax systems can lead to a wide range of “non-standard” policy prescriptions in which some instruments stand in for the ability to create certain wedges that *cannot*, by assumption of the available tax instruments, be created in a decentralized economy. Proving completeness therefore establishes that none of our results is due to any policy instrument serving as imperfect proxies for other, unavailable, instruments.

As we showed in Section 3, there are three independent MRS/MRT pairs in the environment, and expressions (27)-(29) state the efficiency conditions in terms of these MRS/MRT pairs. Completeness of the tax system requires that each of these margins is affected by (at least) one policy instrument. To establish completeness, we first express explicitly in terms of MRS/MRT pairs the private-sector equilibrium conditions that are the analogs of the efficiency conditions (27)-(29). For simplicity and brevity, we do this for the price-taking version of the model, but the ensuing arguments and logic hold for bargaining as well.

Using (9), (19), (20), and the definitions of MRSs and MRTs presented in Proposition 1, we have that in the decentralized economy

$$\frac{MRS_{e_t, q_t}}{MRT_{e_t, q_t}} = 1 + \frac{R_t - 1}{\sigma}, \quad (33)$$

$$E_t \left\{ \frac{MRS_{X_t, X_{t+1}}}{MRT_{X_t, X_{t+1}}} \right\} = \left[ 1 + \frac{\sigma c_k(q_t, K_{t+1}, Z_t) Z_t F_H(K_t, H_t)}{A} \right]^{-1} \times \quad (34)$$

$$\left\{ \left[ 1 + \frac{\sigma c_k(q_t, K_{t+1}, Z_t)}{U'(X_t)} \right] + \beta E_t \left[ \frac{\tau_{t+1}^k U'(X_{t+1}) [Z_{t+1} F_K(K_{t+1}, H_{t+1}) - \delta]}{U'(X_t)} \right] \right\}$$

and

$$\frac{MRS_{X_t, H_t}}{MRT_{X_t, H_t}} = 1 - \tau_t^h. \quad (35)$$

Next, we express in the same way the first-order conditions of the Ramsey planner (which are derived in Appendix A.2); doing so gives

$$\frac{MRS_{e_t, q_t}}{MRT_{e_t, q_t}} = 1 - \frac{\xi}{1 + \xi} \left[ \frac{q_t u''(q_t) - q_t c_{qq}(q_t, K_{t+1}, Z_t) - c_{qk}(q_t, K_{t+1}, Z_t) K_{t+1}}{c_q(q_t, K_{t+1}, Z_t)} \right] \quad (36)$$

$$- \frac{\iota_t}{1 + \xi} \left[ \frac{u''(q_t) c_q(q_t, K_{t+1}, Z_t) - u'(q_t) c_{qq}(q_t, K_{t+1}, Z_t)}{[c_q(q_t, K_{t+1}, Z_t)]^3} \right],$$

$$E_t \left\{ \frac{MRS_{X_t, X_{t+1}}}{MRT_{X_t, X_{t+1}}} \right\} = \left[ 1 + \frac{\sigma c_k(q_t, K_{t+1}, Z_t) Z_t F_H(K_t, H_t)}{A} \right]^{-1} \times \quad (37)$$

$$\left\{ 1 + \frac{\sigma c_k(q_t, K_{t+1}, Z_t) + \xi U''(X_t) X_t + \sigma \xi C_{1t} + \sigma \iota_t C_{2t} - \beta \xi E_t \{ U''(X_{t+1}) X_{t+1} [Z_{t+1} F_K(t+1) + 1 - \delta] \}}{(1 + \xi) U'(X_t)} \right\}$$

and

$$\frac{MRS_{X_t, H_t}}{MRT_{X_t, H_t}} = 1 + \left( \frac{\xi}{1 + \xi} \right) \frac{U''(X_t)X_t}{U'(X_t)}. \quad (38)$$

In (37),  $C_{1t}$  and  $C_{2t}$  are expressions defined in Appendix A.2, and  $\xi$  and  $\iota_t$  are the Lagrange multipliers of the Ramsey problem associated, respectively, with the PVIC and the sequence of ZLB constraints.

We are now ready to establish completeness of the tax system.

**Proposition 3. *Completeness of the Tax System.***

1. *Suppose that the Ramsey allocation converges to a deterministic steady state. In the steady state of a competitive equilibrium, the three policy instruments  $R$ ,  $\tau^h$  and  $\tau^k$  can uniquely create, in the margins defined by conditions (27), (28), and (29), the wedges implied by the steady-state Ramsey allocation.*
2. *Along the dynamic path of any competitive equilibrium, the instruments  $R_t$  and  $\tau_t^h$  can uniquely create, in the margins defined by conditions (27) and (29), the wedges implied by the dynamic path of the Ramsey allocation.*

*Proof.* For the first part of the statement, we compare the steady versions of (33)-(35) with those of (36)-(38). The second part of the statement follows directly because (33), (35), (36) and (38) are all static conditions. □

Note that, with reference to wedges between the IMRS and IMRT, Proposition 3 covers only the steady state. As is well-understood in Ramsey theory, outside of steady state, if both a state-contingent capital-income tax and state-contingent government debt returns are possible (the latter can be achieved in our environment via state-contingent movements in the inflation rate, which affect the realized real returns on nominal government debt), both cannot be simultaneously pinned down.<sup>14</sup> In fact, as in virtually all of the literature, our analytical results regarding capital-income taxes are only steady-state results (while our results regarding the optimality of the Friedman rule cover both in- and out-of-steady-state allocations).

Having proven completeness of the tax system, we next proceed to characterizing the optimal policy by solving the Ramsey problem as defined in Section 4 and decentralizing it using policy instruments. We emphasize that none of the policy prescriptions obtained is because one (or more) of the policy instruments that is assumed available is acting as an imperfect substitute for a policy instrument that is assumed unavailable.

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<sup>14</sup>See, for example, the discussion in Chari and Kehoe (1999, p. 1708) or Ljungqvist and Sargent (2004, p. 500-502).

## 6 Optimal Policy: Analytical Results

Now that we have proven completeness of the tax system, we are able to prove the following Propositions that characterize the optimal policy. We focus on two cases because analytical proofs can be obtained for them: bargaining with  $\theta = 1$  (buyer-take-all) and price-taking. We find numerically that most of the results we prove here also hold for the calibrated version of the model with  $\theta < 1$ ; for those results that do not hold under  $\theta < 1$ , there are clear economic explanations.<sup>15</sup>

### 6.1 Optimal Positive Nominal Interest Rate

**Proposition 4. (*Optimal Deviation from the Friedman Rule under Bargaining*)** *Under bargaining, if  $\theta = 1$ , the optimal policy features a strictly positive net nominal interest rate in every period  $t \geq 1$ . Furthermore, if  $u(\cdot)$  is CRRA (constant relative risk aversion), and the DM production function  $f(k, e)$  is constant-returns-to-scale, then the optimal nominal interest rate is constant over time.*

*Proof.* Consider the first-order condition of the Ramsey problem with respect to  $q_t$ , which is expression (51) in Appendix A.2. With  $\theta = 1$ , we have  $g(q, K, Z) = c(q, K, Z)$  and  $\gamma(q, K, Z) = 0$ . Dropping all arguments of functions, dropping time subscripts because everything is period- $t$ , and assuming the ZLB never binds (i.e., the multiplier  $\iota_t = 0$  in expression (51)), this first-order condition simplifies to

$$u' - c_q = - \left( \frac{\xi}{1 + \xi} \right) \frac{c}{c_q} \left[ u'' - \frac{u' c_{qq}}{c_q} \right]. \quad (39)$$

Because the multiplier on the PVIC  $\xi > 0$  under the Ramsey allocation,  $u$  is strictly concave,  $c > 0$ ,  $c_q > 0$ , and  $c_{qq} > 0$ , the right-hand-side of (39) is strictly positive. This in turn implies  $u' > c_q$ , and

$$\sigma \frac{u'(q_t)}{c_q(q_t, K_{t+1}, Z_t)} + 1 - \sigma > 1.$$

But this implies, by the equilibrium condition (17) with  $\theta = 1$  imposed, that  $R_t > 1$ . We have thus proven that the Friedman Rule is never optimal and that the ZLB never binds.

Now assume  $u(q) = q^{(1-\eta)}/(1-\eta)$  and  $f(k, e) = k^{1-\phi}e^\phi$ . These assumptions lead to

$$R_t = \sigma \left[ 1 - \left( \frac{\xi}{1 + \xi} \right) \eta \phi \left( 1 + \frac{1 - \phi}{\eta \phi} \right) \right]^{-1} + 1 - \sigma,$$

which clearly shows the optimal nominal interest rate is constant over time; further details appear in Appendix B. □

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<sup>15</sup>In the bargaining version with  $\theta < 1$ , some proofs depend on the signs of higher-order derivatives of already complicated functions such as  $g_{qk}$  which are indeterminate in general.

**Proposition 5. (Optimal Deviation from the Friedman Rule under Price-Taking)** *Under price-taking, if the DM production function  $f(k, e)$  is constant-returns-to-scale, the optimal policy features a strictly positive net nominal interest rate in every period  $t \geq 1$ . Furthermore, if  $u(\cdot)$  is CRRA (constant relative risk aversion), then the optimal nominal interest rate is constant over time.*

*Proof.* We start from the first-order condition of the Ramsey problem with respect to  $q_t$ , which is expression (56) in Appendix A.2. Once again dropping all arguments of functions, dropping time subscripts because everything is period- $t$ , (except, recall, that  $K$  chosen in period  $t$  is  $K_{t+1}$  by our timing and notational convention), and assuming the ZLB never binds (i.e., the multiplier  $\iota_t = 0$  in expression (56)), this first-order condition simplifies to

$$u' - c_q = - \left( \frac{\xi}{1 + \xi} \right) [qu'' - qc_{qq} - c_{qk}K].$$

As we demonstrate in Appendix B, if the DM production function is constant-returns-to-scale, then  $qc_{qq} + c_{qk}K = 0$ . In this case, we get

$$u' - c_q = - \left( \frac{\xi}{1 + \xi} \right) qu'' > 0, \tag{40}$$

which together with (21) imply that  $R_t > 1$ . The Friedman Rule is thus never optimal and the ZLB constraint never binds. Comparing (40) with (36) it is clear that supporting the Ramsey allocation requires creating a wedge in the  $(e_t, q_t)$  margin; condition (33) shows this is achieved by setting  $R_t > 1$ .

If we also assume CRRA utility in the DM, we get

$$R_t = \sigma \left[ 1 - \left( \frac{\xi}{1 + \xi} \right) \eta \right]^{-1} + 1 - \sigma, \tag{41}$$

which clearly shows the optimal nominal interest rate is constant over time; further details appear in Appendix B. □

The Ramsey allocation features a  $q_t$  smaller than the Pareto-optimal level. This is true in both the price-taking and buyer-take-all bargaining environments. In a Ramsey allocation, in general one should expect all final goods —  $q$  is, after all, a final good in the economy — to be below their Pareto-optimal levels, and so it is in our Ramsey allocation as well.<sup>16</sup> To decentralize this feature of the Ramsey allocation, what is needed is the ability to create a wedge between the MRS and MRT in condition (27). Expression (33) shows that there is a policy instrument — the nominal interest rate — that achieves precisely this. Thus, a strictly positive net nominal interest rate (i.e.,  $R_t > 1$ ) creates exactly the wedge between MRS and MRT that the Ramsey allocation

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<sup>16</sup>For example, see Diamond and Mirrlees (1971) or Atkinson and Stiglitz (1980).

requires. The deviation from the Friedman Rule does not arise as an imperfect substitute for some other instrument. In other words, as we proved in Section 5, this result has nothing to do with incompleteness of the tax system.

The policy prescription of deviating from the Friedman Rule is in contrast to the environments of Lucas and Stokey (1983), Chari, Christiano, Kehoe (1991), and others in which taxation of final activities that require cash ends up being achieved without having to resort to a positive nominal interest rate; in these basic Ramsey monetary models, other instruments (in particular, the labor income tax) end up indirectly taxing cash-intensive activities. This point is discussed extensively in Aruoba and Chugh (2007).

## 6.2 Optimal Capital Taxation

Having established results regarding the monetary aspects of optimal policy in some important versions of our model, we now turn to characterizing the associated fiscal aspects of optimal policy.

**Proposition 6. (*Optimal Tax for Labor*)** *Under both bargaining and price-taking, the optimal labor income tax is positive.*

*Proof.* See Appendix C. □

**Corollary 2.** *If the CM sub-utility function for consumption is CRRA, then the optimal labor tax is constant.*

*Proof.* See Appendix C. □

Proposition 6 follows simply by comparing (35) with (38), which shows that supporting the Ramsey allocation requires creating a wedge in the  $(X_t, H_t)$  margin, which the CM labor-income tax achieves.

We are now ready to prove our main result, which is that a steady-state capital subsidy (i.e.,  $\tau^k < 0$ ) is optimal in some important versions of our environment. We discuss how optimal monetary policy has important consequences for optimal fiscal policy in the price-taking version of our environment, in which capital-holdup problems are absent and hence, ostensibly, zero-capital taxation would seemingly be optimal. Finally, we show how our results connect with the synthesis of Albanesi and Armenter (2007) even though our model does not fit their canonical framework.

**Proposition 7. (*Optimal Subsidy for Capital*)** *Assuming CRS production in the DM, under (a) bargaining with  $\theta = 1$  or (b) price-taking, the optimal long-run policy will include a subsidy to capital income except when (1) the DM is shut down (i.e. no trades are carried out exclusively with money) or (2) capital is not used in the DM (i.e. the monetary and non-monetary sides of the economy are decoupled).*

*Proof.* **(a) (Bargaining with  $\theta = 1$ )** Imposing steady state on the Ramsey planner's first-order condition for capital, (53), dropping arguments of functions, and imposing both  $\iota = 0$  (since we showed ZLB doesn't bind) and  $\theta = 1$  (which implies  $\gamma(\cdot) = 0$  and  $g(\cdot) = c(\cdot)$ ), we have

$$-\sigma c_k - \rho + \beta [\rho (F_K + 1 - \delta)] + \sigma \xi \left[ c_k \left( \frac{u'}{c_q} - 1 \right) - \frac{cu'c_{qk}}{[c_q]^2} \right] = 0.$$

The last term in square brackets on the left-hand-side can be rearranged to

$$-c_k + \frac{u'}{c_q} \left[ \frac{c_k c_q - c c_{qk}}{c_q} \right] = -c_k;$$

the equality follows because  $c_k c_q - c c_{qk} = 0$  due to our CRS assumption (further details are provided in Appendix B). Collecting terms, we have

$$\beta [1 + (F_K - \delta)] - 1 = \frac{\sigma(1 + \xi)c_k}{\rho}.$$

Also, note that the multiplier on the resource constraint,  $\rho$  can be solved as

$$\rho = \frac{A(1 + \xi)}{F_H} \quad (42)$$

from (54), which is the Ramsey planner's first-order condition for CM labor. This leads to

$$\beta [1 + (F_K - \delta)] - 1 = \frac{\sigma c_k F_H}{A} \quad (43)$$

Similarly imposing steady state and  $\theta = 1$  on the private sector equilibrium condition (11) yields

$$\beta \left[ 1 + (1 - \tau^k)(F_K - \delta) \right] - 1 = 0. \quad (44)$$

Combining (43) and (44), we get the Ramsey-optimal capital tax rate,

$$\tau^k = \frac{\sigma F_H c_k}{A\beta (F_K - \delta)}.$$

Standard assumptions imply  $F_H > 0$  and  $(F_K - \delta) > 0$ , and of course  $\beta > 0$  and  $A > 0$  by assumption. The only way, therefore,  $\tau^k$  can equal zero is if  $\sigma = 0$  (which means DM trades never occur) or  $c_k = 0$  (which means capital is not used for DM production). So long as both the DM exists ( $\sigma > 0$ ) and capital is used for DM production, we must have  $\tau^k < 0$  because  $c_k < 0$ .

**(b) (Price-Taking)** Following similar algebra, the Ramsey planner's first-order condition for capital in the price-taking case can be simplified to exactly the expression in (43), using the CRS property of the DM production function (in particular, applying Euler's theorem to the marginals of the DM cost function, which yields  $c_{qk}q + c_{kk}k = 0$ ). The steady state multiplier  $\rho$  on the resource constraint is also again given by (42).

For price-taking, the steady-state version of the private sector equilibrium condition (19) is given by

$$\beta \left[ 1 + (1 - \tau^k)(F_K - \delta) \right] - 1 = \frac{\sigma c_k}{U'}; \quad (45)$$

solving (43), and (45) for the Ramsey-optimal capital tax rate,

$$\tau^k = \frac{\sigma c_k \left[ \frac{F_H}{A} - \frac{1}{U'} \right]}{\beta(F_K - \delta)}. \quad (46)$$

Note that we showed in Proposition 6 that  $\tau^h > 0$  and using (9) this implies  $F_H/A > 1/U'$ . As in case (a), standard assumptions on production and this result guarantee that the only way  $\tau_k$  can equal zero is if  $\sigma = 0$  or  $c_k = 0$ . Otherwise, we must have  $\tau_k < 0$ .  $\square$

The two parts of Proposition 7 allow us to disentangle two distinct motivations for capital subsidies. One obvious motivation is the capital holdup problem. As we discussed above, provided  $\theta > 0$  in the bargaining environment, a capital holdup problem is present, which leads to sub-optimally low private-sector capital accumulation. A subsidy to capital income naturally alleviates this problem. We think this result is quite interesting because existing optimal-taxation models that descend from Chamley (1986) and Judd (1985) are not suited to consider how holdup problems affect capital accumulation and hence optimal capital tax rates. And yet, as, say, Acemoglu and Shimer (1999) or Caballero (2007) argue, holdup problems are prevalent in the economy and are likely to be important for macroeconomic issues. Search-based environments featuring capital accumulation decisions made before bilateral trades naturally can give rise to holdup problems. Although optimal-capital taxation implications are not explicitly drawn by Acemoglu and Shimer (1999), it seems clear in their (non-monetary) environment that search and bargaining frictions associated with labor would also lead to optimal capital subsidies.

However, it is not just holdup problems that lead to capital subsidies in our model. Price-taking removes capital holdup (as well as money holdup) problems, as we discussed above and as AWW show. Nonetheless, the Ramsey policy features a capital subsidy in the price-taking environment, too. The capital subsidy arises in the price-taking environment because optimal monetary policy spills over into optimal fiscal policy. As we discussed above, the deviation from the Friedman Rule — a monetary tax — achieves taxation of the final good  $q$ , which is required by basic optimal-taxation principles, as also discussed above. Intuitively, by taxing DM goods, the monetary tax is also a tax on capital used for DM production. To correct this distortion, capital income must be subsidized.

At a more technical level, if, counterfactually, the Friedman Rule were part of the optimal policy in the price-taking environment, it would imply that a lump-sum tax/transfer exists in the economy.<sup>17</sup> But a lump-sum instrument would allow the Ramsey planner to achieve efficiency

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<sup>17</sup>This can be seen from condition (41), which shows that the Friedman Rule ( $R = 1$ ) necessarily requires  $\xi = 0$ . But  $\xi = 0$  means the PVIC is completely relaxed, which can only occur if there is a lump-sum tax in the environment. See also Ljungqvist and Sargent (2004, p. 494).

along the CM consumption-leisure margin and this would imply  $\tau^h = 0$  and  $1/U' = F_H/A$ . But  $1/U' = F_H/A$  immediately implies, from (46), that  $\tau^k = 0$ . Thus, if the Friedman Rule were somehow optimal, it would lead to a zero capital tax. Because the Friedman Rule is not optimal, there is a capital subsidy even though price-taking eliminates all holdup problems.

This result that the monetary aspects of optimal policy meaningfully affect the fiscal aspects of optimal policy is unlike Schmitt-Grohe and Uribe (2005) and Chugh (2007). As we mentioned in the introduction, the Schmitt-Grohe and Uribe (2005) and Chugh (2007) capital-taxation prescriptions are essentially just monetary translations of the Judd (2002) prescription that if monopoly problems afflict goods markets, capital accumulation should be subsidized. Judd's (2002) prescription has nothing to do with Ramsey taxation — one could allow a lump-sum instrument in Judd's (2002) environment and the prescription would be the same. In contrast, in our micro-founded monetary model, the causality in some sense runs from monetary issues (optimal monetary policy in the presence of anonymous goods trades that require the existence of money) to fiscal policy. Such a channel does not arise in Schmitt-Grohe and Uribe (2005) and Chugh (2007), in which the reduced-form models of money demand mean that the monetary aspects of policy and fiscal aspects of policy are more-or-less simply laid side-by-side rather than deeply integrated with each other.

Regardless of one's preferred stand on a model of money demand, we can also connect our results to Albanesi and Armenter (2007), who provide a unified framework with which to think about long-run capital taxation. One important distinction they make is that nonzero capital taxation may be consistent with zero intertemporal distortions. Specifically, their work highlights that the essence of the celebrated Chamley (1986) and Judd (1985) result is not that zero-capital-taxation *per se* is optimal, but rather that it is optimal because it supports a zero wedge between intertemporal marginal rates of substitution and marginal rates of transformation. Whether a zero intertemporal wedge requires a zero capital tax or a nonzero capital tax then depends on the details of the economic environment.

Many economic environments — including Chamley's (1986) and Judd's (1985) — have the feature that a zero intertemporal wedge requires a zero capital tax, but this need not always be. Although Albanesi and Armenter's (2007) framework does not encompass (either reduced-form or micro-founded) monetary environments, we can show, for the cases covered by Proposition 7, that capital *subsidies* are in fact needed for a zero intertemporal distortion, and, moreover, a zero intertemporal distortion is optimal.

**Proposition 8. (*Zero Intertemporal Distortions*)** *For the cases studied in Proposition 7, the Ramsey-optimal policy features a zero intertemporal distortion in the long run.*

*Proof.* In Proposition 1, we derived the IMRT and IMRS for our economy. Imposing steady state, the IMRS is simply  $\beta$ . To prove the result, it will be sufficient to show that in a deterministic

steady-state, the IMRT at the Ramsey-optimal allocation equals  $\beta$ .

We established above that (43) is the Ramsey planner's first-order condition for capital at the steady state for both of the cases considered in this Proposition. Rearranging (43), we have

$$\beta = \frac{1 + \frac{\sigma c_k F_H}{A}}{1 + F_K - \delta},$$

in which the right-hand-side is precisely the IMRT we derived in Proposition 1. (recall that  $c_k = -v'(e)f_k/f_e$ ). For the price-taking case, another way to prove this result would be to substitute the optimal tax rate we derived in (46) in to (34) to get IMRS equals IMRT.  $\square$

Thus, although our model cannot be cast in Albanesi and Armenter's (2007) canonical form, we can add micro-founded models of monetary trade to their list of environments as ones in which a long-run zero intertemporal distortion can be optimal.<sup>18</sup> Of course, in our environment, supporting this allocation requires a capital subsidy for the various reasons we have already discussed.

## 7 Optimal Policy: Numerical Results

Having established a number of analytical results for the steady state and some dynamic results for two important versions of the model, we resort to numerical methods to obtain results for the bargaining case with  $\theta < 1$  and for other dynamic results.

### 7.1 Solution Strategy and Parameterization

To the extent possible, we use the parameters and functional forms that AWW provide, whose model is calibrated to match some long-run features of the US economy. Specifically, the DM utility function is  $u(q) = \ln(q+b) - \ln(b)$  with  $b = 0.0001$ , which is a parameter that forces  $u(0) = 0$ , which can occur in the DM if a household does not meet another party with whom to trade. The DM production function is  $q = Ze^\phi k^{1-\phi}$ , which implies the cost function  $c(q, K, Z) = (1/Z^\psi)q^\psi K^{1-\psi}$ , with  $\psi \equiv 1/\phi$ . In the CM, instantaneous utility is  $B \ln(X) - AH$ , and the production function is  $Y = ZK^\alpha H^{1-\alpha}$ . We use the parameter values provided in AWW.<sup>19</sup> It should be noted that our functional forms conform to all of the restrictions specified in our analytical results, i.e. both utility

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<sup>18</sup>Technically, what prevents our model from being cast in Albanesi and Armenter's (2007) canonical form is the presence of the  $K$  terms in the implementability constraint, arising from the trading arrangements in the DM. Such a feature of the Ramsey problem does not arise in standard Chamley (1986) or Judd (1985)-type analyses.

<sup>19</sup>The calibration in AWW is annual. The following parameters are fixed across different version of the model :  $(\beta, \delta, \alpha, \eta) = (0.976, 0.07, 0.288, 1)$ . For the bargaining version with  $\theta = 1$ , we use  $(A, B, \psi, \sigma) = (3.702, 1.395, 1.738, 0.210)$ , for the bargaining version with  $\theta < 1$ , we use  $(A, B, \psi, \sigma, \theta) = (4.928, 1.852, 1.302, 0.227, 0.735)$ , and for the price-taking version we use  $(A, B, \psi, \sigma) = (6.453, 2.401, 1.15, 0.215)$ .

functions are CRRA and both production functions are CRS. As such, the nominal interest rate and the labor income tax will be constant over time.

The exogenous government spending and TFP processes each evolve as an AR(1) in logs,

$$\ln G_{t+1} = (1 - \rho_G) \ln \bar{G} + \rho_G \ln G_t + \epsilon_{t+1}^G,$$

$$\ln Z_{t+1} = \rho_Z \ln Z_t + \epsilon_{t+1}^Z,$$

with  $\epsilon^G \sim N(0, \sigma_{\epsilon^G}^2)$  and  $\epsilon^Z \sim N(0, \sigma_{\epsilon^Z}^2)$ . We calibrate  $\bar{G} = 0.1$ , so that government purchases constitute about 18 percent of total GDP in steady-state.<sup>20</sup> In line with Schmitt-Grohe and Uribe (2004) and the RBC literature, we set the parameters of the stochastic processes  $\sigma_{\epsilon^G} = 0.03$ ,  $\sigma_{\epsilon^Z} = 0.023$ ,  $\rho_G = 0.9$ , and  $\rho_Z = 0.82$ . Finally, we choose the level of steady-state real government debt, an object not pinned down by the model, so that it is 45 percent of steady-state output, consistent with the parameterizations of CCK and Schmitt-Grohe and Uribe (2004).

For Ramsey steady-states, we use standard numerical solvers for nonlinear systems of equations. To study Ramsey dynamics, we approximate time-invariant decision rules for  $q, K, X, H$  using the Ramsey first-order conditions.<sup>21</sup> We construct global nonlinear approximations because of the presence of the potentially occasionally-binding ZLB constraint.<sup>22</sup> Of interest to many practitioners, however, should be our (unreported) findings that, for the versions of the model in which we know for sure the ZLB constraint is always slack, first-order and second-order local approximations yielded results virtually identical to our global approximation.<sup>23</sup> With decision rules in hand, we conduct 1000 simulations of 500 periods each and discard the first 100 periods. As in Khan, King, and Wolman (2003) and others, we assume that the initial state of the economy is the asymptotic Ramsey steady state. For each simulation, we then compute first and second moments and report the averages of these moments over the 1000 simulations.

## 7.2 Steady-State

In Figure 1, we present the optimal steady-state nominal interest rate and the optimal steady-state capital income tax as  $\theta$  varies between 0 and 1. At  $\theta = 1$ , as we proved in Propositions 4 and 7,

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<sup>20</sup>Real GDP takes into account both CM and DM output:  $\sigma M/P + Y$ .

<sup>21</sup>Along with approximating decision rules for the allocation variables of direct interest, we also must approximate the decision rule for the multiplier on the ZLB,  $\iota$ , as well as the functions that define policies and prices.

<sup>22</sup>We approximate these functions using linear combinations of Chebyshev polynomials, following Judd (1992). Results from Aruoba, Fernandez-Villaverde and Rubio-Ramirez (2006) and Aruoba, Waller, and Wright (2007) indicate that this approximation method is very accurate. While our algorithm allows the ZLB to be an occasionally binding constraint, which means the multiplier  $\iota$  may have one or more kinks in it, our quantitative results indicate that for the parameterizations we use the ZLB either always binds or never binds.

<sup>23</sup>Of course, this statement only holds for sufficiently-small driving shocks; the business-cycle magnitude shocks that we assume are apparently small enough.

the optimal net nominal interest rate is positive and the optimal capital tax is negative — Figure 1 shows that quantitatively, these are, respectively, 9.8% and  $-80\%$ . The latter in particular may seem especially large, but it is not out of line with capital subsidy rates found in other Ramsey studies. For example, Schmitt-Grohe and Uribe (2005, Table 2) find an optimal capital subsidy rate of 44 percent in their benchmark model featuring a host of real and nominal rigidities; they also report that it can be as high as 85 percent.

As  $\theta$  falls, the optimal nominal interest rate also falls, eventually hitting the zero lower bound, just as in Aruoba and Chugh (2007). On the other hand, the capital-income subsidy falls (i.e.,  $\tau^k$  rises) as  $\theta$  falls below 1. For sufficiently-low  $\theta$ , the ZLB constraint binds and the Friedman rule becomes the constrained optimum.<sup>24</sup> Both of these results — that the nominal interest rate falls towards zero and the capital-tax rate rises towards zero as  $\theta$  decreases below unity — can be explained by the holdup problems in the environment.

First, as  $\theta$  falls, the holdup problem for money demand becomes more severe because a larger share of the surplus of a DM trade is grabbed by the seller. In addition, as shown by Aruoba, Rocheteau, and Waller (2007), a side-effect of generalized Nash bargaining is that the buyer’s surplus is not monotonic in the amount of money the buyer carries into the DM. The non-monotonicity of the Nash outcome and a positive net nominal interest rate each separately induce households to choose a suboptimally-low level of money balances, resulting in a suboptimally-low  $q$ . The Ramsey planner tries to remedy this by lowering the nominal interest rate so long as it is consistent with monetary equilibrium. The dotted lines in Figure 1 show the policies that would arise if somehow the ZLB was not a restriction on monetary equilibria; and they show that the Ramsey planner would then continue reducing the net nominal interest rate below 0% as  $\theta$  falls sufficiently far below unity.

To explain the fall in the capital income subsidy (the rise in  $\tau^k$ ), note that as  $\theta$  falls, the share of the surplus of a DM trade earned by the seller increases, which means the capital holdup problem becomes less severe.<sup>25</sup> This allows the Ramsey planner to ease the capital subsidy.

Table 1 reports the steady-state values of the three important policy tools ( $R, \tau^k, \tau^h$ ) under three different model environments: bargaining with  $\theta = 1$ , bargaining with our calibrated value of  $\theta < 1$ , and price-taking. For the calibrated value of  $\theta = 0.735$ , the ZLB constraint binds and the Friedman rule is optimal. For the price-taking version, the optimal net nominal interest rate is 7.7%. The capital income subsidy, at 80% when  $\theta = 1$ , falls to 25.1% in the calibrated version of the bargaining environment, and falls further to 2.7% in the price-taking environment. Finally, the labor income tax is in the range 25 – 30% for all versions of our model.

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<sup>24</sup>It is important to note that due to Proposition 4, there will always be an interior  $\bar{\theta}$  where the Friedman rule will not be optimal for all  $\theta > \bar{\theta}$ .

<sup>25</sup>Recall that in the limiting case  $\theta = 0$ , the capital holdup problem in principle completely disappears.

We proved in Proposition 8 that the Ramsey equilibrium features a zero steady-state intertemporal distortion in both the price-taking environment and the bargaining environment with  $\theta = 1$ . This result does not carry over to bargaining with  $\theta < 1$ . Numerically, we find for these cases that the Ramsey equilibrium does not equate IMRS and IMRT. As we discussed in Section 3, what is different about the  $\theta < 1$  environment compared to either the  $\theta = 1$  or the price-taking environment is that holdup problems in money accumulation are present. The presence of the money-holdup problem thus prevents the Ramsey planner from attaining a zero distortion along the capital-accumulation margin. We emphasize that although the capital holdup problem becomes less severe as  $\theta$  falls below 1, the discrete appearance of the money-holdup problem as we move from  $\theta = 1$  to any  $\theta < 1$  adds yet another private-sector distortion with which the Ramsey planner must deal. Our quantitative findings show that dealing with this problem requires deviating from efficiency along the capital-accumulation margin.

### 7.3 Dynamics

In Table 2, we report simulation-based moments for the Ramsey allocations and policy variables for the same three environments presented in Table 1. Because we proved for our assumed functional forms that the optimal nominal interest rate and the optimal labor income tax are constant, we do not report their dynamics. Also, as we mentioned above, when both a state-contingent capital-income tax and state-contingent government debt returns are possible (the latter can be achieved in our environment via state-contingent movements in the inflation rate, which changes the real return of nominal government debt), both cannot be simultaneously pinned down. We choose to fix capital-income-tax rate at its deterministic steady-state level and allow fluctuations in inflation to achieve the required state-contingency in government debt returns. This strategy allows the most scope to test whether or not the primitive frictions captured by our environment have anything quantitatively-important to say regarding the optimality of inflation stability, an issue of interest in its own right.

Turning to the results, Table 2 shows that state-contingent inflation has very low volatility in all three versions we study. At a standard deviation of about 60 basis points at an annual rate, we would characterize this as inflation stability. This inflation-stability result also arises in the micro-founded monetary environment absent capital studied by Aruoba and Chugh (2007), but it is strikingly different from the typical inflation *volatility* result that arises in standard flexible-price Ramsey environments either with or without capital.<sup>26</sup> That the endogeneity of capital

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<sup>26</sup>Quantitatively, baseline inflation volatility results in the flexible-price Ramsey literature are generally in the range of 5% to 10%, depending on the precise model and/or calibration. The result here of roughly 0.6% inflation volatility is thus an order of magnitude smaller. See Chari and Kehoe (1999), Schmitt-Grohe and Uribe (2005), and Chugh (2007) for a few examples.

accumulation does not change the optimality volatility of inflation is in line with the results of Chugh (2007) and Schmitt-Grohe and Uribe (2005), who both find that, so long as prices are flexible, capital accumulation in and of itself does little to change Ramsey inflation dynamics. As Aruoba and Chugh (2007) explain in detail, the intuition for the optimality of inflation stability is as follows: the relative price between CM and DM consumption depends on the inflation rate, and distorting this relative price imposes welfare costs so large that the Ramsey planner largely refrains from varying inflation despite its ability to absorb shocks to the government budget. We refer the interested reader to Aruoba and Chugh (2007) for more in-depth analysis of this result because it and its intuition carry over unchanged to our environment here.

## 8 Conclusion

The capital-taxation prescriptions that arise in existing Ramsey models of jointly-optimal fiscal and monetary policy have little to do with monetary aspects of the environment. Such results leave the impression that monetary issues are at best of minor importance for capital-income taxation. Our work shows that this impression changes once one adopts a more fundamental view of money demand than existing work adopts. In our model, capital-taxation prescriptions stem directly from primitive informational and coordination frictions that make monetary trade endogenously valuable. We find that capital-income subsidies are optimal in such an environment. A holdup problem in capital accumulation is one reason for this subsidy. This holdup problem can be traced to the bilateral nature of monetary exchange, exchanges in which capital is a required input to production. But even when holdup problems in capital accumulation do not exist, as occurs in our competitive-pricing environment, the optimal monetary policy, which entails a deviation from the Friedman Rule, spills over into and distorts capital accumulation, calling for a capital-income subsidy. Thus, monetary issues may indeed be important for capital-taxation prescriptions.

A point that we think is not as widely-appreciated as it should be is that non-zero capital income taxes may or may not be consistent with intertemporal efficiency, even in standard models used to consider capital taxation. We showed that whenever holdup problems afflicting *money* demand are absent, capital subsidies lead to intertemporal efficiency. However, if money demand is plagued by holdup problems, then capital subsidies do not bring about intertemporal efficiency. This is yet another channel by which fundamentally monetary issues can have implications for capital-income taxation, a channel about which reduced-form models of money demand are necessarily silent.

Finally, our model also inherits the general properties of optimal policy discovered by Aruoba and Chugh (2007) in a similar environment lacking capital accumulation. Primary among these results is that a deviation from the Friedman Rule is optimal and inflation exhibits very low volatility in the face of business-cycle magnitude shocks. As we mentioned above, we refer interested readers

to Aruoba and Chugh (2007) for more in-depth analysis of these results.

We see several promising avenues for future research. The AWW environment lacks an extensive dimension of decentralized activity. As Rocheteau and Wright (2005) show in a micro-founded monetary model without capital accumulation, the interplay between extensive and intensive margins of decentralized activity can be important for policy prescriptions. Reconsidering our capital-taxation results in an extended version of the AWW environment that allows for an extensive margin could prove quite interesting. More broadly, holdup problems are an understudied topic in the context of macroeconomics. Our results provide an example in which various holdup problems have important implications for policy prescriptions. As Caballero (2007) has also emphasized, it would be worthwhile to think about how holdup problems affect other questions in macroeconomics.

## A The Ramsey Problem

### A.1 Proof of Proposition 2

That allocations from a monetary equilibrium should satisfy the CM resource constraint (14), the ZLB constraint ((17) for the bargaining model or (21) for the price-taking model) is obvious. Here we derive the PVIC for the bargaining version of the model. The expression for the price-taking version follows the same steps.

We begin by summing the budget constraints of the three types of agents (buyer, sellers and nonparticipants in the previous DM) to get

$$P_t X_t + B_t + M_t + P_t K_{t+1} = P_t w_t (1 - \tau_t^h) H_t + M_{t-1} + R_{t-1} B_{t-1} + \left[ 1 + (1 - \tau_t^k)(F_K(K_t, H_t) - \delta) \right] K_t.$$

Multiplying by  $\beta^t U'(X_t)/P_t$ , summing from  $t = 0, 1, \dots, \infty$ , and dropping all  $E_t$  terms to keep notation simple, we get

$$\begin{aligned} & \sum_{t=0}^{\infty} \beta^t U'(X_t) X_t + \sum_{t=0}^{\infty} \beta^t U'(X_t) \frac{B_t}{P_t} + \sum_{t=0}^{\infty} \beta^t U'(X_t) \frac{M_t}{P_t} + \sum_{t=0}^{\infty} \beta^t U'(X_t) K_{t+1} = \\ & \sum_{t=0}^{\infty} \beta^t U'(X_t) (1 - \tau_t^h) w_t H_t + \sum_{t=0}^{\infty} \beta^t U'(X_t) \frac{M_{t-1}}{P_t} + \sum_{t=0}^{\infty} \beta^t U'(X_t) \frac{R_{t-1} B_{t-1}}{P_t} \\ & + \sum_{t=0}^{\infty} \beta^t U'(X_t) \left[ 1 + (1 - \tau_t^k)(F_K(K_t, H_t) - \delta) \right] K_t. \end{aligned} \quad (47)$$

Substitute into the second term on the left-hand-side of (47) using expression (13) to get

$$\sum_{t=0}^{\infty} \beta^{t+1} U'(x_{t+1}) \frac{R_t B_t}{P_{t+1}}.$$

This term cancels with the the last summation on the right-hand-side of (47) to leave only the initial bond position,

$$U'(x_0) \frac{R_{-1} B_{-1}}{P_0}.$$

Next, substitute into the third term on the left-hand-side of (47) using (12) and (13) to get

$$\sum_{t=0}^{\infty} \beta^{t+1} U'(x_{t+1}) \frac{M_t}{P_{t+1}} \left[ \sigma \frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} + 1 - \sigma \right];$$

expanding this summation, we have

$$\sum_{t=0}^{\infty} \beta^{t+1} U'(x_{t+1}) \frac{M_t}{P_{t+1}} + \sigma \sum_{t=0}^{\infty} \beta^{t+1} U'(x_{t+1}) \frac{M_t}{P_{t+1}} \left[ \frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} - 1 \right].$$

Canceling the first summation in this last expression with the second summation on the right-hand-side of (47) to leave only the initial money holdings,

$$U'(x_0) \left( \frac{M_{-1}}{P_0} \right),$$

and writing  $\frac{M_t}{P_{t+1}} = \frac{M_t}{P_t} \frac{P_t}{P_{t+1}}$ , we can express the second summation just above as

$$\sigma \sum_{t=0}^{\infty} \beta^{t+1} U'(x_{t+1}) \frac{M_t}{P_t} \frac{P_t}{P_{t+1}} \left[ \frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} - 1 \right].$$

Use (16) to substitute for  $M_t/P_t$ ,

$$\sigma \sum_{t=0}^{\infty} \beta^{t+1} \frac{U'(x_{t+1})}{P_{t+1}} \frac{P_t}{U'(X_t)} g(q_t, K_{t+1}, Z_t) \left[ \sigma \frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} + 1 - \sigma \right] \left[ \frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} - 1 \right].$$

Using (12) and (13), we can make the substitution  $\beta E_t \left[ \frac{U'(x_{t+1})}{P_{t+1}} \right] = \frac{U'(X_t)}{P_t} \left[ \sigma \frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} + 1 - \sigma \right]^{-1}$  which yields

$$\sigma \sum_{t=0}^{\infty} \beta^t g(q_t, K_{t+1}, Z_t) \left[ \frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} - 1 \right].$$

Next, using (9), we can substitute into the first term on the right-hand-side of (47) to get

$$\sum_{t=0}^{\infty} \beta^t A H_t;$$

and using (11), we can express the fourth term on the left-hand-side of (47) as

$$\sum_{t=0}^{\infty} \beta^{t+1} U'(X_{t+1}) \left[ 1 + (1 - \tau_{t+1}^k)(F_K(K_{t+1}, H_{t+1}) - \delta) \right] K_{t+1} + \sum_{t=0}^{\infty} \beta^t \sigma \gamma(q_t, K_{t+1}) K_{t+1}.$$

Canceling the first summation with the last term on the right-hand-side of (47) yields

$$U'(X_0) \left[ 1 + (1 - \tau_0^k)(F_K(K_0, H_0) - \delta) \right] K_0.$$

Defining  $A_0$  as

$$A_0 \equiv \frac{M_{-1} + R_{-1}B_{-1}}{P_0} + \left[ 1 + (1 - \tau_0^k)(F_K(K_0, H_0) - \delta) \right] K_0,$$

re-introducing the expectation operator  $E_0$ , and collecting all remaining terms, we arrive at

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ U'(X_t) X_t - A H_t + \sigma g(q_t, K_{t+1}, Z_t) \left( \frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} - 1 \right) + \sigma \gamma(q_t, K_{t+1}, Z_t) K_{t+1} \right] = U'(X_0) A_0,$$

which is expression (30) in the text.

## A.2 The Solution to the Ramsey Problem

The Ramsey problem described in Section 4 is to choose  $\{X_t, H_t, K_{t+1}, q_t\}$  to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \{U(X_t) - AH_t + \sigma [u(q_t) - c(q_t, K_{t+1}, Z_t)]\}$$

subject to the resource constraint

$$Z_t F(K_t, H_t) + (1 - \delta)K_t - X_t - G_t - K_{t+1} \geq 0, \quad (48)$$

the PVIC, which is

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ U'(X_t)X_t - AH_t + \sigma g(q_t, K_{t+1}, Z_t) \left( \frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} - 1 \right) + \sigma \gamma(q_t, K_{t+1}, Z_t) K_{t+1} \right] = U'(X_0)A_0. \quad (49)$$

for the bargaining model or

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ U'(X_t)X_t - AH_t + \sigma q_t c_q(q_t, K_{t+1}, Z_t) \left( \frac{u'(q_t)}{c_q(q_t, K_{t+1}, Z_t)} - 1 \right) - \sigma c_k(q_t, K_{t+1}, Z_t) K_{t+1} \right] = U'(X_0)A_0 \quad (50)$$

for the price-taking model,<sup>27</sup> and the ZLB constraint, which is

$$\sigma \left( \frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} - 1 \right) \geq 0$$

for the bargaining model or

$$\sigma \left( \frac{u'(q_t)}{c_q(q_t, K_{t+1}, Z_t)} - 1 \right) \geq 0$$

for the price-taking model, taking as given  $\{G_t, Z_t\}$  and  $K_0$ . We associate multipliers  $\beta^t \rho_t$  with the time- $t$  resource constraint,  $\xi$  with the PVIC, and  $\beta^t \iota_t$  with the time- $t$  ZLB constraint.

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<sup>27</sup>Note the simplification in the third term which leads to  $\sigma [q_t u'(q_t) - q_t c_q(q_t, K_{t+1}, Z_t)]$ .

### A.2.1 Bargaining

The Kuhn-Tucker conditions for the problem above are the first-order conditions

$$\begin{aligned} \mathbf{q}_t : & \sigma [u'(q_t) - c_q(q_t, K_{t+1}, Z_t)] + \sigma \xi \left[ g_q(q_t, K_{t+1}, Z_t) \left( \frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} - 1 \right) \right] \\ & + \sigma \xi \left[ g(q_t, K_{t+1}, Z_t) \left( \frac{u''(q_t)g_q(q_t, K_{t+1}, Z_t) - u'(q_t)g_{qq}(q_t, K_{t+1}, Z_t)}{[g_q(q_t, K_{t+1}, Z_t)]^2} \right) + \gamma_q(q_t, K_{t+1}, Z_t)K_{t+1} \right] \\ & + \sigma \iota_t \left[ \frac{u''(q_t)g_q(q_t, K_{t+1}, Z_t) - u'(q_t)g_{qq}(q_t, K_{t+1}, Z_t)}{[g_q(q_t, K_{t+1}, Z_t)]^2} \right] = 0, \end{aligned} \quad (51)$$

$$\mathbf{X}_t : U'(X_t) - \rho_t + \xi [U''(X_t)X_t + U'(X_t)] = 0, \quad (52)$$

$$\begin{aligned} \mathbf{K}_{t+1} : & -\sigma c_k(q_t, K_{t+1}, Z_t) - \rho_t + \beta E_t \{ \rho_{t+1} [Z_{t+1}F_K(K_{t+1}, H_{t+1}) + 1 - \delta] \} \\ & + \sigma \xi \left[ g_k(q_t, K_{t+1}, Z_t) \left( \frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} - 1 \right) - \frac{g(q_t, K_{t+1}, Z_t)u'(q_t)g_{qk}(q_t, K_{t+1}, Z_t)}{[g_q(q_t, K_{t+1}, Z_t)]^2} \right] \\ & + \sigma \xi \gamma_k(q_t, K_{t+1}, Z_t)K_{t+1} + \sigma \xi \gamma(q_t, K_{t+1}, Z_t) - \sigma \iota_t \frac{u'(q_t)g_{qk}(q_t, K_{t+1}, Z_t)}{[g_q(q_t, K_{t+1}, Z_t)]^2} = 0, \end{aligned} \quad (53)$$

$$\mathbf{H}_t : -A + \rho_t Z_t F_H(K_t, H_t) - \xi A = 0, \quad (54)$$

along with (48), (49), and the complementary slackness condition

$$\iota_t \sigma \left[ \frac{u'(q_t)}{g_q(q_t, K_{t+1}, Z_t)} - 1 \right] = 0, \text{ and } \iota_t \geq 0. \quad (55)$$

We can represent the right-hand side of the PVIC in terms of allocations as

$$U'(X_0)A_0 = U'(X_0) \left[ \frac{g(q_{-1}, K_0, Z_{-1})}{\beta U'(X_0)} + \frac{B_{-1}/P_{-1}}{\beta} \right] + \frac{U'(X_0)}{\beta} K_0 \left[ 1 - \sigma \frac{\gamma(q_{-1}, K_0, Z_{-1})}{U'(X_0)} \right],$$

in which the initial real bond position  $B_{-1}/P_{-1}$  is a parameter that we set as described in Section 7.1.

With these Ramsey FOCs in hand, we proceed as follows. Imposing steady state on (51)-(54), and taking the timeless perspective, i.e. setting time-zero allocations equal to their steady state value, we solve for the steady state values of allocations and the multiplier  $\xi$ . Next, given  $\xi$  and  $\{Z_t, G_t\}$ , (51)-(55) characterize  $\{q_t, X_t, K_t, H_t, \iota_t\}$ . We back out policies  $\{\tau_t^h, R_t\}$  from (9) and (12) statically. Finally ex-post inflation can be solved dynamically from (13).

### A.2.2 Price-Taking

The Kuhn-Tucker conditions for the problem above are the first-order conditions

$$\mathbf{q}_t : \sigma [u'(q_t) - c_q(q_t, K_{t+1}, Z_t)] + \sigma \xi [u'(q_t) + q_t u''(q_t) - c_q(q_t, K_{t+1}, Z_t) - q_t c_{qq}(q_t, K_{t+1}, Z_t)] - \sigma \xi c_{qk}(q_t, K_{t+1}, Z_t) K_{t+1} + \sigma \iota_t \left[ \frac{u''(q_t) c_q(q_t, K_{t+1}, Z_t) - u'(q_t) c_{qq}(q_t, K_{t+1}, Z_t)}{[c_q(q_t, K_{t+1}, Z_t)]^2} \right] = 0, \quad (56)$$

$$\mathbf{X}_t : U'(X_t) - \rho_t + \xi [U''(X_t) X_t + U'(X_t)] = 0, \quad (57)$$

$$\mathbf{K}_{t+1} : -\sigma c_k(q_t, K_{t+1}, Z_t) - \rho_t + \beta E_t \{ \rho_{t+1} [Z_{t+1} F_K(K_{t+1}, H_{t+1}) + 1 - \delta] \} - \sigma \xi [q_t c_{qk}(q_t, K_{t+1}, Z_t) + c_{kk}(q_t, K_{t+1}, Z_t) K_{t+1} + c_k(q_t, K_{t+1}, Z_t)] - \sigma \iota_t \frac{u'(q_t) c_{qk}(q_t, K_{t+1}, Z_t)}{[c_q(q_t, K_{t+1}, Z_t)]^2} = 0, \quad (58)$$

$$\mathbf{H}_t : -A + \rho_t Z_t F_H(K_t, H_t) - \xi A = 0, \quad (59)$$

along with (48), (50), and the complementary slackness condition

$$\iota_t \sigma \left[ \frac{u'(q_t)}{c_q(q_t, K_{t+1}, Z_t)} - 1 \right] = 0, \text{ and } \iota_t \geq 0. \quad (60)$$

We can represent the right-hand side of the PVIC in terms of allocations as

$$U'(X_0) A_0 = U'(X_0) \left[ \frac{q_{-1} c_q(q_{-1}, K_0, Z_{-1})}{\beta U'(X_0)} + \frac{B_{-1}/P_{-1}}{\beta} \right] + \frac{U'(X_0)}{\beta} K_0 \left[ 1 + \sigma \frac{c_k(q_{-1}, K_0, Z_{-1})}{U'(X_0)} \right],$$

in which, once again, the initial real bond position  $B_{-1}/P_{-1}$  is a parameter that we set as described in Section 7.1.

With these FOCs in hand, we proceed as follows. Imposing steady state on (56)-(59), and taking the timeless perspective, i.e. setting time-zero allocations equal to their steady state value, we solve for the steady state values of allocations and the multiplier  $\xi$ . Next, given  $\xi$  and  $\{Z_t, G_t\}$ , (56)-(60) characterize  $\{q_t, X_t, K_t, H_t, \iota_t\}$ . We back out policies  $\{\tau_t^h, R_t\}$  from (9) and (20) statically. Finally inflation can be solved dynamically from (13).

For reference, we define

$$C_{1t} = [q_t c_{qk}(q_t, K_{t+1}, Z_t) + c_{kk}(q_t, K_{t+1}, Z_t) K_{t+1} + c_k(q_t, K_{t+1}, Z_t)]$$

$$C_{2t} = \frac{u'(q_t) c_{qk}(q_t, K_{t+1}, Z_t)}{[c_q(q_t, K_{t+1}, Z_t)]^2}$$

## B Details of the Proofs of Propositions 1, 4, 5 and 7

### B.1 Proposition 1

The expression for IMRS,  $\beta U'(X_{t+1})/U'(X_t)$ , simply follows from the social welfare function. To see where the expression for IMRT comes from, consider a decrease in  $X_t$  by one unit. This will increase  $K_{t+1}$  by one unit. This marginal increase in capital will have two effects. First, as occurs in a standard RBC model with capital, output in period  $t+1$  will increase by  $Z_{t+1}F_K(K_{t+1}, H_{t+1}) + 1 - \delta$  units, which in period  $t+1$  can be converted one-for-one into  $X_{t+1}$ . Because of the existence of the DM, though, a second effect arises:  $q_t$  will rise by  $\sigma Z_t f_k(K_{t+1}, e_t)$  units. In order to properly define IMRT in our environment, then, this increase in  $q_t$  needs to be taken into account.

To properly account for this second effect, consider the following thought experiment. In order to hold production of  $q_t$  fixed following an initial reduction of  $X_t$  by one unit, DM effort must be reduced by  $\sigma f_k(K_{t+1}, e_t)/f_e(K_{t+1}, e_t)$ . This reduction in DM effort will increase utility  $\sigma v'(e_t) f_k(K_{t+1}, e_t)/f_e(K_{t+1}, e_t)$ . This increase in utility is equivalent to a decrease in CM labor by  $\sigma v'(e_t) f_k(K_{t+1}, e_t)/f_e(K_{t+1}, e_t)A$ , under the maintained assumption of linear disutility of CM labor. Next, this reduction in CM labor would lead to a reduction of period- $t$  CM output by the amount  $\sigma Z_t F_H(K_t, H_t) v'(e_t) f_k(K_{t+1}, e_t)/f_e(K_{t+1}, e_t)A$ , which, because there is of course a unit rate of transformation between CM output and CM consumption, means a decrease in  $X_t$  by the same amount.

Thus, we have demonstrated that a *reduction* in  $X_t$  in the amount  $1 - \sigma Z_t F_H(K_t, H_t) v'(e_t) f_k(K_{t+1}, e_t)/f_e(K_{t+1}, e_t)A$  leads to an *increase* in  $X_{t+1}$  by  $Z_{t+1}F_K(K_{t+1}, H_{t+1}) + 1 - \delta$  units. Clearly, if  $\sigma = 0$  as in the standard RBC model, we have that  $MRT_{X_t, X_{t+1}} = Z_{t+1}F_K(K_{t+1}, H_{t+1}) + 1 - \delta$ , which has the usual interpretation that a one unit decrease in  $X_t$  leads to increased period- $t+1$  CM consumption by the amount  $Z_{t+1}F_K(K_{t+1}, H_{t+1}) + 1 - \delta$ . With  $\sigma > 0$ , in order to achieve the same increase in period- $t+1$  CM consumption, the required decrease in  $X_t$  is *less* than one unit. This is due to the fact that DM output also increases when one unit of  $X_t$  is foregone.

In the text we claimed that  $c_k$  corresponds to  $-v'(e)f_k/f_e$ . To see this, remember that  $c_k$  refers to the marginal change in utility for having more capital, holding output constant. Using the production function  $q = Zf(k, e)$ , we can write  $dq = Zf_k dk + Zf_e de$ . If we consider a change in  $k$  with no change in  $q$ , this corresponds to a change in  $e$  in the amount  $-f_k dk/f_e$ . The change in utility due to this change in  $k$  will therefore be  $-v'(e)f_k/f_e$ . A similar argument shows that  $c_q$  corresponds to  $v'(e)/Zf_e$ .

## B.2 Proposition 4

We start by

$$u' - c_q = - \left( \frac{\xi}{1+\xi} \right) \frac{c}{c_q} \left[ u'' - \frac{u' c_{qq}}{c_q} \right]. \quad (61)$$

which along with (17) show that  $R_t$  is constant if and only if  $u'/c_q$  is constant. We assume  $u(q) = q^{(1-\eta)}/(1-\eta)$ , which has the property

$$q_t u''(q_t) = -\eta u'(q_t). \quad (62)$$

We also assume  $q = Zk^{1-\phi} e^\phi$  which yield the following cost function and related derivatives

$$c(q, K, Z) = \frac{1}{Z^\psi} q^\psi K^{1-\psi} > 0 \quad (63)$$

$$c_q(q, K, Z) = \frac{1}{Z^\psi} \psi q^{\psi-1} K^{1-\psi} > 0 \quad (64)$$

$$c_k(q, K, Z) = \frac{1}{Z^\psi} (1-\psi) q^\psi K^{-\psi} < 0 \quad (65)$$

$$c_{qq}(q, K, Z) = \frac{1}{Z^\psi} \psi(\psi-1) q^{\psi-2} K^{1-\psi} > 0 \quad (66)$$

$$c_{qk}(q, K, Z) = \frac{1}{Z^\psi} \psi(1-\psi) q^{\psi-1} K^{-\psi} < 0 \quad (67)$$

where  $\psi \equiv 1/\phi$ . These imply the properties

$$\frac{c_{qq}q}{c_q} = \frac{\psi(\psi-1)q^{\psi-2}K^{1-\psi}q}{\psi q^{\psi-1}K^{1-\psi}} = \psi - 1 \quad (68)$$

$$\frac{c}{qc_q} = \frac{q^\psi K^{1-\psi}}{\psi q^{\psi-1} K^{1-\psi} q} = \frac{1}{\psi} \quad (69)$$

where we dropped arguments for notational simplicity. Rewriting (61)

$$\begin{aligned} \frac{u'}{c_q} &= 1 - \left( \frac{\xi}{1+\xi} \right) \frac{c}{c_q^2} \left[ u'' - u' \frac{c_{qq}}{c_q} \right] \\ \frac{u'}{c_q} &= 1 - \left( \frac{\xi}{1+\xi} \right) \frac{cu''}{c_q^2} \left[ 1 + \frac{c_{qq}q}{\eta c_q} \right] \\ \frac{u'}{c_q} &= 1 - \left( \frac{\xi}{1+\xi} \right) \frac{cu''}{c_q^2} \left[ 1 + \frac{\psi-1}{\eta} \right] \\ \frac{u'}{c_q} &= 1 - \left( \frac{\xi}{1+\xi} \right) \frac{-c\eta u'}{qc_q^2} \left[ 1 + \frac{\psi-1}{\eta} \right] \\ \frac{u'}{c_q} &= 1 - \left( \frac{\xi}{1+\xi} \right) \frac{-\eta u'}{\psi c_q} \left[ 1 + \frac{\psi-1}{\eta} \right] \end{aligned}$$

where in the second and fourth lines we use (62), in the third line we use (68) and in the fifth line we use (69). Collecting terms we get

$$\frac{u'}{c_q} = \left[ 1 - \left( \frac{\xi}{1+\xi} \right) \frac{\eta}{\psi} \left( 1 + \frac{\psi-1}{\eta} \right) \right]^{-1}$$

which show that  $u'/c_q$  is indeed constant.

### B.3 Proposition 5

Using (66) and (67) we get

$$c_q q q + c_{qk} k = \frac{1}{Z^\psi} \left[ \psi(\psi - 1) q^{\psi-2} K^{1-\psi} q + \psi(1 - \psi) q^{\psi-1} K^{-\psi} K \right] = 0$$

which is in fact an application of the Euler's Theorem.

Using (62), we can rewrite (40) as

$$\frac{u'}{c_q} = 1 - \left( \frac{\xi}{1 + \xi} \right) \eta \frac{u'}{c_q}$$

Collecting terms we get

$$\frac{u'}{c_q} = \left[ 1 - \left( \frac{\xi}{1 + \xi} \right) \eta \right]^{-1}$$

which show that  $u'/c_q$  is indeed constant.

### B.4 Proposition 7

Using (63), (64), (65) and (67) we get

$$c_q c_k - c c_{qk} = \frac{1}{Z^{2\psi}} \left[ \psi q^{\psi-1} K^{1-\psi} (1 - \psi) q^\psi K^{-\psi} - q^\psi K^{1-\psi} \psi (1 - \psi) q^{\psi-1} K^{-\psi} \right] = 0$$

## C Proof of Proposition 6 and Corollary 2

Combining (35) and (38) we get

$$1 - \tau_t^h = 1 + \left( \frac{\xi}{1 + \xi} \right) \frac{U''(X_t) X_t}{U'(X_t)}$$

which shows that  $\tau_t^h > 0$  as  $\xi > 0$ ,  $U''(X) < 0$  and  $U'(X) > 0$ , all of which hold under our assumptions.

Moreover, if  $U(x) = X^{1-\gamma}/(1 - \gamma)$  then we have

$$\tau_t^h = \gamma \left( \frac{\xi}{1 + \xi} \right)$$

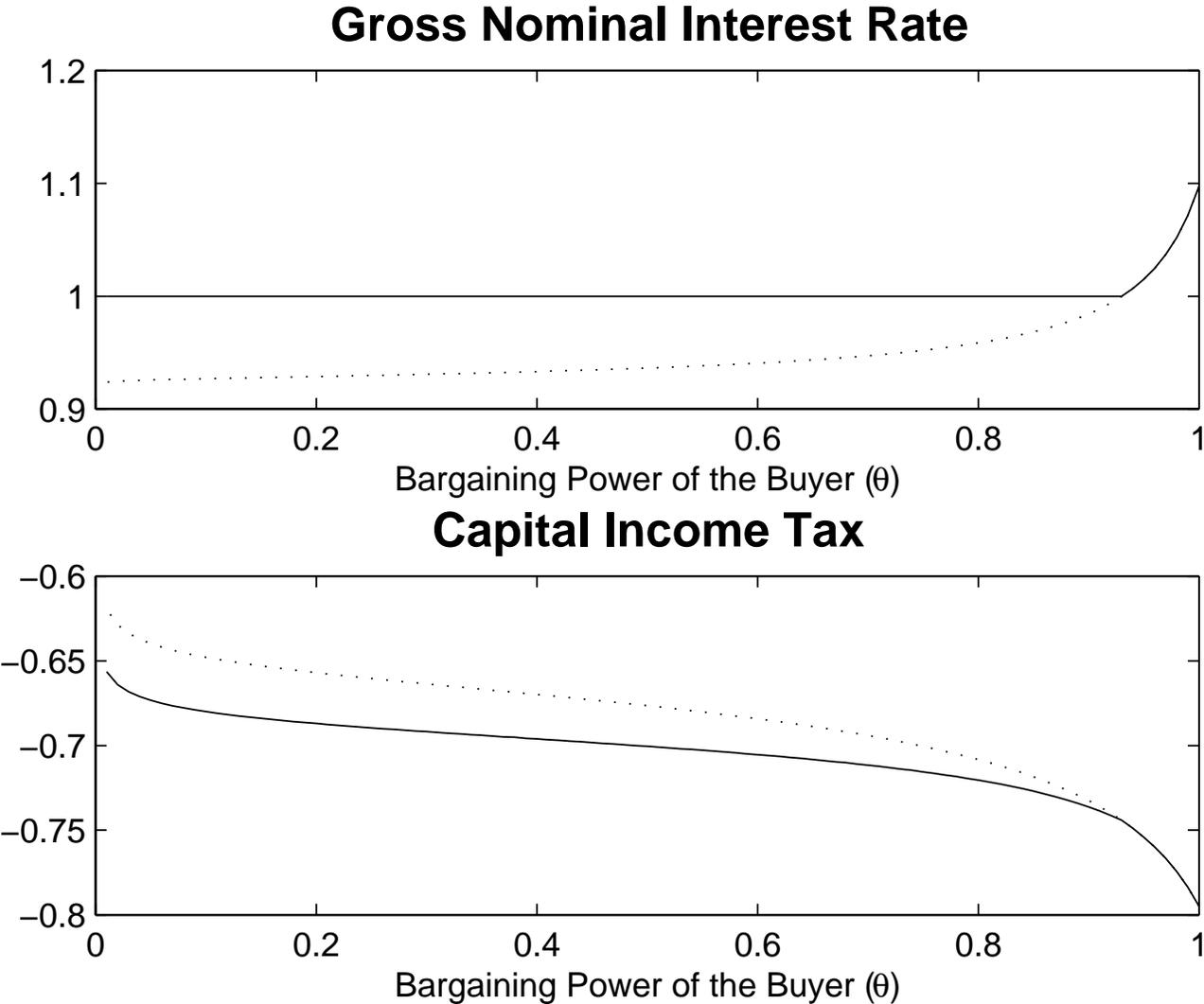
since  $U''(X)X = -\gamma U'(X)$ .

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Figure 1 - Ramsey Steady-State Under Bargaining



Notes : Ramsey steady-state policy as a function of the bargaining power of the buyer ( $\theta$ ) with the ZLB constraint (solid line) and without the ZLB constraint (dotted line).

## Table 1 - Steady State Policies

Variable	Bargaining ( $\theta = 1$ )	Bargaining ( $\theta < 1$ )	Price-Taking
$\tau^h$	0.317	0.312	0.263
$\tau^k$	-0.795	-0.251	-0.027
$R - 1$	9.761	0	7.673

**Notes:** Nominal interest rate reported in percentage points.

## Table 2 - Dynamics

(a) *Bargaining* ( $\theta = 1$ )

Variable	Mean	Std. Dev.	Auto corr.	Corr( $x, GDP$ )	Corr( $x, Z$ )	Corr( $x, G$ )
$\pi - 1$	7.151	0.609	0.705	-0.474	-0.398	-0.115
$q$	0.757	0.049	0.903	0.923	0.956	0.033
$K$	1.836	0.126	0.979	0.553	0.609	0.061
$X$	0.303	0.015	0.951	0.811	0.874	-0.067
$H$	0.322	0.014	0.692	0.700	0.592	0.391
$GDP$	0.551	0.039	0.791	1	0.981	0.180
$P_{DM}/P$	0.124	0.002	0.735	-0.966	-0.912	-0.294

(b) *Bargaining* ( $\theta < 1$ )

Variable	Mean	Std. Dev.	Auto corr.	Corr( $x, GDP$ )	Corr( $x, Z$ )	Corr( $x, G$ )
$\pi - 1$	-2.378	0.568	0.703	-0.452	-0.387	-0.123
$q$	0.294	0.014	0.874	0.956	0.981	0.027
$K$	1.634	0.113	0.979	0.571	0.614	0.069
$X$	0.296	0.015	0.950	0.824	0.879	-0.064
$H$	0.314	0.013	0.691	0.674	0.572	0.407
$GDP$	0.534	0.037	0.796	1	0.981	0.183
$P_{DM}/P$	0.418	0.007	0.765	-0.287	-0.186	-0.293

(c) *Price-Taking*

Variable	Mean	Std. Dev.	Auto corr.	Corr( $x, GDP$ )	Corr( $x, Z$ )	Corr( $x, G$ )
$\pi - 1$	5.113	0.617	0.701	-0.451	-0.384	-0.122
$q$	0.725	0.034	0.852	0.970	0.992	0.019
$K$	1.633	0.114	0.978	0.572	0.616	0.071
$X$	0.311	0.016	0.949	0.825	0.881	-0.060
$H$	0.327	0.013	0.689	0.675	0.572	0.404
$GDP$	0.540	0.037	0.795	1	0.981	0.183
$P_{DM}/P$	0.123	0.002	0.797	-0.163	-0.070	-0.212

**Notes:** Simulation-based moments. Inflation reported in percentage points.