Credit Default Swap Spreads and Systemic Financial Risk

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May 3, 2011
Introduction

What is the joint probability of default of large financial institutions?

Systemic default risk:

\[ \Pr\{ \text{at least } r \text{ LFIs default} \} \]

- Banks are interconnected and exposed to common shocks
- Defaults not independent even at short horizons
- Severe consequences of multiple defaults
Introduction: this paper

- Difficult to measure
  - Rare events
  - Prices that reflect *individual* defaults (bonds, CDSs) but not multiple defaults

- In this paper
  1. Exploit counterparty risk to learn about $P(A_i \cap A_j)$: enrich information set
  2. Derive tightest bounds on multiple default risk
CDS and counterparty risk

- Credit Default Swap is an OTC contract designed to transfer the credit risk of the *reference entity*.

- Counterparty risk in CDSs: if seller defaults, contract terminates.

- “Double default” relevant for pricing: discount relative to the corresponding bond.
CDS and counterparty risk

- Collateral
  - static: very costly \(\rightarrow\) dynamic
  - Not widely used with dealers (66% of contracts in 2008)
  - When margin set to current exposure, subject to jumps
  - Collateral can be less than current exposure (Goldman)
  - Buyers aware of counterparty risk (CDS against seller)
Theory

- Assume we observe

\[ p_i : P(A_i) \]

\[ z_{ji} : P(A_i), P(A_i \cap A_j) \]

- Look for \( P_r : P\{\text{at least } r \text{ default}\} \) (information of order \( N \) - systemic)

\[ P_1 = P(A_1 \cup A_2 \cup A_3) \]

\[ P_2 = P((A_1 \cap A_2) \cup (A_2 \cap A_3) \cup (A_1 \cap A_3)) \]

\[ P_3 = P(A_1 \cap A_2 \cap A_3) \]
Theory

- Becomes

\[
\max P\{\text{at least } r \text{ default}\} \quad \max p \ c'_r p \\
\]

\[
P(A_i) = a_i \quad Ap = b \\
P(A_i \cap A_j) = a_{ij} \\
Consistent \ probability \ system \quad p \geq 0 \\
i'p = 1
\]
Implementation

- Assume a simple, discretized pricing model for bonds and CDSs
- Constant hazard rates
- Assume recovery rates $R=30\%$, $S=30\%$
- Impose a *lower bound* for the liquidity process $\gamma$ of bonds
  - Nonnegative
  - Calibrated to 2004
  - Calibrated to nonfinancial firms
Implementation

- Observe average CDS spreads:
  - $z_{ji}$ linear function of $P(A_i)$ and $P(A_i \cap A_j)$
  - $\bar{z}_i$ linear function of $P(A_i)$ and $\frac{1}{N-1} \sum_{j \neq i} P(A_i \cap A_j)$
  - One constraint for each $i$
Systemic risk

Introduction Step 1: learn about $P(A_i \cap A_j)$

Step 2: construct bounds

Results

Extra Slides

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Systemic risk - $\gamma^i_t \geq 0$
Systemic risk - $\gamma_t^i \geq 0$
Systemic risk measures: assumptions on liquidity
Picture of network 8/4/2008
Marginal and pairwise probabilities

- Marginal probability
- Joint probability

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Contribution: $Pr\{\text{at least } 4 \cap j\}$
Explore: Contrib vs. MES

Citigroup

- Contrib
- MES

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Explore: Contrib vs. MES

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Explore: Contrib vs. MES

- Contrib
- CDS

Lehman

2008

0

0.1

0

5

$5 \times 10^{-3}$

0

0

May 3, 2011
Conclusion

- We learn that systemic risk really started to increase in late 2008:
  - *If systemic risk was so high in January-March 2008, why did the average CDS spread go up so much?*
  - *Why were people so keen to buy insurance from unreliable counterparties?*

- Things to explore
  - Correlation of contribution to systemic risk with other measures (MES, CoVar, stress test)
  - Pairwise default risk and correlation of equity returns
  - Systemic risk and puts
Extra Slides

- Additional details
  - Simple Example of Bounds
  - Linear Programming Algorithm
  - Implementation
  - Pricing Formulas
  - Data
  - Symmetry
Extra Slides

- Robustness to assumptions on recovery rates:
  - Robustness to R and S
  - Stochastic and Time Varying Recovery Rates

- All other robustness tests:
  - Robustness Results
  - Derivations

- References:
  - Arora et al.
Theory: simple example
Theory: simple example

- Suppose we observe, from bonds:

  \[ P(A_1) = P(A_2) = P(A_3) = 0.2 \]

- From CDSs:

  \[ P(A_1 \cap A_2) = P(A_2 \cap A_3) = 0.07 \]
  \[ P(A_1 \cap A_3) = 0.01 \]

- Tightest bounds

  \[ 0.45 \leq P_1 \leq 0.46 \]
  \[ 0.13 \leq P_2 \leq 0.15 \]
  \[ 0 \leq P_3 \leq 0.01 \]
Theory: simple example

- **Heterogeneity.** Suppose still

\[ P(A_1) = P(A_2) = P(A_3) = 0.2 \]

but now we only know

\[ \frac{P(A_1 \cap A_2) + P(A_2 \cap A_3) + P(A_1 \cap A_3)}{3} = 0.05 \]

- Then

\[
\begin{align*}
\text{Full information} & \quad \text{Only average} \\
0.45 \leq P_1 \leq 0.46 & \quad 0.45 \leq P_1 \leq 0.50 \\
0.13 \leq P_2 \leq 0.15 & \quad 0.05 \leq P_2 \leq 0.15 \\
0 \leq P_3 \leq 0.01 & \quad 0 \leq P_3 \leq 0.05
\end{align*}
\]
Linear Programming Algorithm

- Start from problem:

\[
\begin{align*}
\max P_r \\
\text{s.t.} \\
P(A_i) &= a_i \\
\ldots \\
P(A_i \cap A_j) &= a_{ij}
\end{align*}
\]
Linear Programming Algorithm

- Obtain:

\[
\begin{align*}
\max_p & \ c_r'p \\
\text{s.t.} & \ p \geq 0 \\
& \ i'p = 1 \\
& \ Ap = b
\end{align*}
\]
**Linear Programming Algorithm**

- Start with matrix $B (2^N, N)$
- Rows are binary representation of $0...2^N - 1$
  \[
  B = \begin{bmatrix}
  0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 1 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 1 & 1 \\
  \vdots \\
  1 & 1 & 1 & 1
  \end{bmatrix}
  \]
- Each row is event of type
  \[A_1^* \cap A_2^* \cap ... \cap A_N^*\]
- where $A_j^* = A_j$ if element $j$ of the row is 1, and $A_j^* = \overline{A_j}$ otherwise
Linear Programming Algorithm

- $p$ contains probabilities of these events

\[ p \geq 0 \]

\[ p' i = 1 \]

- $P(A_i)$:

\[ P(A_i) = \sum_{j: B(j,i) = 1} p_j \]

or

\[ P(A_i) = a^i p \]

\[ a_j = B(j, i) \]
Linear Programming Algorithm

- \( P(A_i \cap A_k) \):

\[
P(A_i \cap A_k) = \sum_{j: B(j,i)=1 \text{ and } B(j,k)=1} p_j
\]

or:

\[
P(A_i \cap A_k) = b^{ik'} p
\]

for a vector \( b_{ik} \) of size \( (2^N, 1) \) s.t.:

\[
b_{jk}^{ik} = B(j, i)B(j, k)
\]
Linear Programming Algorithm

\[ P_r: \]
\[ P_r = \sum_{j: (\sum_{h=1:N} B(j, h)) \geq r} p_j \]

or
\[ P_r = c^{r'} p \]

for a vector \( c^r \) of size \((2^N, 1)\) s.t.:
\[ c^r_j = I \left[ \sum_{h=1:N} B(j, h) \geq r \right] \]
Implementation: 1 - Pricing

- Contracts span long horizons
- Contracts priced again every time $t$ looking forward, assuming
  - Constant hazard rates $h_i^t$
  - Constant bond liquidity premium $\gamma_i^t$
- Discretize by month
- Joint default in a month $\iff$ double default
- Seller default and reference survives until next month $\implies$ small change in reference risk
- Recovery $R=30\%, \ S=30\%$
Implementation: 1 - Pricing (bonds)

\[ B^{ij}(t, T^{ij}) = c^{ij} \left( \sum_{s=t+1}^{T^{ij}} \delta(t,s)(1 - h_t^i)^{s-t}(1 - \gamma_t^i)^{s-t} \right) + \]

\[ + \delta(t, T^{ij})(1 - h_t^i)^{T^{ij}-t}(1 - \gamma_t^i)^{T^{ij}-t} \]

\[ + R \left( \sum_{s=t+1}^{T^{ij}} \delta(t,s)(1 - h_t^i)^{s-t-1}(1 - \gamma_t^i)^{s-t-1} h_t^i \right) \]
Implementation: 1 - Pricing (bonds)

- Bond liquidity: constant conveniency yield $\gamma_t^i$

$$E_t[R_{t+1}^{ij} - R_{t+1}^f] = -\frac{\text{Cov}_t(M_{t+1}, R_{t+1}^{ij} - R_{t+1}^f)}{E_t[M_{t+1}]} + m_t^i x_t \psi_t$$

- $m_t^i$ margin for senior unsecured bonds of firm $i$
- $x_t$ proportion of agents constrained
- $\psi_t$ shadow cost of capital

$$\gamma_t^i \approx m_t^i x_t \psi_t = \alpha^i \lambda_t$$
Implementation: 1 - Pricing (CDSs)

\[
\sum_{s=t}^{T-1} \delta(t, s)(1 - P(A_i \cup A_j))^{s-t} z_{ji} = \\
= \sum_{s=t+1}^{T} \delta(t, s)(1 - P(A_i \cup A_j))^{s-t-1}
\]

\[
\left\{\left[ P(A_i) - P(A_i \cap A_j) \right] (1 - R) + S \left[ P(A_i \cap A_j) \right] (1 - R) \right\}
\]
Implementation: 2 - Calibration of liquidity $\gamma_t^i$

- Bond liquidity: constant conveniency yield $\gamma_t^i = \alpha^i \lambda_t$
  - $\lambda_t$: common variations in margins, cost of capital, constrained agents
- Calibrating $\gamma_t^i \geq \gamma_t^i$, obtain

$$P(A_i) \leq h_i(\gamma_t^i)$$

1. $\gamma_t^i \geq 0$
2. $\gamma_t^i \geq \alpha^i$: liquidity at least as of 2004
Implementation: 2 - Calibration of liquidity $\gamma_t^i$

3. For a group $K$ of A-rated (or better) nonfinancial firms

   ▶ double default risk is low

   ▶ calibrate matching the bond-CDS basis

$$\gamma_t^k = \alpha^k \lambda^*_t$$

   ▶ and assume that for financials

$$\gamma_t^i \geq \alpha^i \lambda^*_t$$
Implementation: 3 - Availability of CDS spreads

- Observe average CDS spreads:
  - $z_{ji}$ linear function of $P(A_i)$ and $P(A_i \cap A_j)$
  - $\bar{z}_i$ linear function of $P(A_i)$ and $\frac{1}{N-1} \sum_{j \neq i} P(A_i \cap A_j)$
  - One constraint for each $i$

- Do not observe contributors of Markit quotes
  - Pick 15 dealers covering 90% of CDS market
Pricing formulas: bonds

Bonds:

\[ B^{ij}(t, T^{ij}) = c^{ij} \left( \sum_{s=t+1}^{T^{ij}} \delta(t, s)(1 - h^i_t)^{s-t}(1 - \gamma^i_t)^{s-t} \right) + \]

\[ + \delta(t, T^{ij})(1 - h^i_t)^{T^{ij}-t}(1 - \gamma^i_t)^{T^{ij}-t} \]

\[ + R \left( \sum_{s=t+1}^{T^{ij}} \delta(t, s)(1 - h^i_t)^{s-t-1}(1 - \gamma^i_t)^{s-t-1} h^i_t \right) \]
Pricing formulas: CDSs

\[ T-1 \sum_{s=t}^{T} \delta(t, s)(1 - P(A_i \cup A_j))^{s-t} z_{ji} = \]

\[ = \sum_{s=t+1}^{T} \delta(t, s)(1 - P(A_i \cup A_j))^{s-t-1} \]

\[ \left\{ [P(A_i) - P(A_i \cap A_j)] (1 - R) + S [P(A_i \cap A_j)] (1 - R) \right\} \]
Pricing formulas: CDSs

Linearize to use as a constraint:

\[ z_{ji,t} = (P(A_i) - (1 - S)P(A_i \cap A_j)) \frac{\sum_{s=t+1}^{T} \delta(t, s)}{\sum_{s=t}^{T-1} \delta(t, s)} \text{(1 - R)} \]
Data

- Bonds
  - Look on Bloomberg and Markit for all bonds that are issued by institution $i$
  - Restrict to senior unsecured fixed or zero coupon: no callable, putable, sinkable, structured
  - TRACE-eligible bonds: use TRACE closing price
  - Other bonds: generic closing price
Data

- Risk-free rate: zero-coupon government bonds
- CDS: Markit
- Period: 2004 to June 2010
# Introduction

Step 1: learn about $P(A_i \cap A_j)$

Step 2: construct bounds

## Results

### Extra Slides

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## Data

### Table 1

<table>
<thead>
<tr>
<th>Avg valid bonds</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
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<td>Abn Amro</td>
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<td>1.8</td>
<td>2.2</td>
<td>4.0</td>
<td>4.4</td>
<td>3.0</td>
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<td>32.3</td>
<td>17.5</td>
<td>25.4</td>
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<td>7.2</td>
<td>9.8</td>
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<td>15.3</td>
<td>15.4</td>
<td>-</td>
</tr>
<tr>
<td>Bnp Paribas</td>
<td>7.0</td>
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<td>9.1</td>
<td>7.7</td>
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Note: first column reports average number of bonds for each institution that are used for the estimation of marginal default probabilities. Columns 2-8 break this number down by year.
## Data

### Table 2

<table>
<thead>
<tr>
<th>Bank</th>
<th>Avg CDS spread</th>
<th>Std CDS spread</th>
<th>Min spread</th>
<th>Max spread</th>
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Data

Table 2

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</table>
Symmetry

- $p$ is *symmetric* if it does not depend on the ordering of $A_i$'s.
- A LP problem

\[
\begin{align*}
\max & \quad c'p \\
\text{s.t.} & \quad Ap \leq b
\end{align*}
\]

is *symmetric* if $c'p$ and all constraints do not depend on the ordering of $A_i$'s.

- Example: union of all events, average of probabilities
- **Proposition 3**: problem is symmetric $\implies \exists$ symmetric solution
- **Corollary**: symmetric systems have the widest bounds given average probabilities
Symmetry

The graph shows the trend of liquidity for nonfinancials and nonneg liq from 2007 to 2010. The data is represented using two lines:

- **Average bonds and CDS** (dashed line)
- **All bonds and CDS** (solid line)

The x-axis represents the years (2007 to 2010), and the y-axis represents the liquidity values (0 to 0.015 for the upper graph and 0 to 0.015 for the lower graph). The graph indicates fluctuations in liquidity over the years.
Robustness: R and S

- **Dependence on R**
  - Two-period case:
    
    \[ p_i = 1 - (1 - R)P(A_i) \]
    
    \[ z_{ji} = (1 - R)P(A_i) - (1 - S)(1 - R)P(A_i \cap A_j) \]
  
  - Higher R ⇒ lower yield ⇒ bond-implied probability scales up
  - Higher R ⇒ lower CDS spread ⇒ cds-implied probabilities scale up
  - Bounds scale up
Robustness: R and S

- Dependence on S
  - Depends on whether the basis can be all explained by counterparty risk
  - Remember the constraints:

\[
P(A_i) \leq a_i(\gamma_i)
\]

\[
P(A_i) - (1 - S) \frac{\sum_{j \neq i} P(A_i \cap A_j)}{N - 1} = \bar{p}_i(\bar{z}_i)
\]

- S=1 ⇒ For each \( i \), \( P(A_i) = \bar{p}_i \)
- Decrease S ⇒ \( P(A_i) > \bar{p}_i \) : counterparty risk
- But if S large, \( P(A_i) - \bar{p}_i \) requires high counterparty risk
Robustness: R and S

\[ P(A_i) \leq a_i(\gamma_i) \]

\[ P(A_i) - (1 - S) \frac{\sum_{i \neq j} P(A_i \cap A_j)}{N - 1} = \bar{p}_i(\bar{z}_i) \]

- S decreases more ⇒ \( P(A_i \cap A_j) \) can fill a larger gap ⇒ systemic risk increases
- This *ignores* constraints from bonds
- Once \( P(A_i) \) hits the upper bound \( a_i(\gamma_i) \), \( P(A_i \cap A_j) \) has to decrease
Robustness: $R$ and $S$

- **Example:** June 25, 2008. Bank of America, Citigroup, GS. Probabilities are average monthly risk-neutral probabilities in bp.

<table>
<thead>
<tr>
<th>Bank</th>
<th>$a_i(0)$</th>
<th>$\bar{p}_i$</th>
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<tr>
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<tr>
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<tr>
<td>3</td>
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Robustness: R and S

Graph showing the robustness of R and S in different scenarios.
Robustness: $R$ and $S$

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<th>$a_i(0)$</th>
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<td>18.5</td>
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<td>3</td>
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Stefano Giglio (Harvard University)  
May 3, 2011
Robustness: $R$ and $S$

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# Robustness: R and S

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Stefano Giglio  (Harvard University)  May 3, 2011
Robustness: R and S

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<td>33.8</td>
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</tbody>
</table>
Time-varying recovery rates

- R could be lower in bad times
  - Adjusting $R \downarrow$ would imply bounds $\downarrow$

- S could be lower in bad times
  - lower S $\rightarrow$ joint default risk has greater effect on basis
  - in peak episodes, basis is small $\rightarrow$ joint default risk even smaller
Stochastic Recovery Rates

- Bonds and CDSs price in stochastic recovery rate
- Recovery rate depends on number of defaults
- Simple case: $R_H$ if 1 bank defaults, $R_L$ if more banks default
- Call $B(R_H, R_L)$ the price of a bond, $z(R_H, R_L)$ the price of a CDS
**Stochastic Recovery Rates**

- Show that:
  
  $$B(R_H, R_L) = B(R_L, R_L) + Y_{bond}(R_H, R_L)$$

- And:
  
  $$\sum_{s=1}^{T} \delta(0, s-1)(1 - P(A_i \cup A_j))^{s-1} z_{ji}(R_H, R_L) =$$

  $$= \sum_{s=1}^{T} \delta(0, s-1)(1 - P(A_i \cup A_j))^{s-1} z_{ji}(R_L, R_L) - Y_{cds}(R_H, R_L)$$

- with $Y_{bond} \approx Y_{cds}$
Stochastic Recovery Rates

- Yields and CDS spreads are
  - Rescaled as if $R = R_L$
  - Shifted by a constant
- Adding $Y$ to both bonds and CDSs does not change the basis
- The relevant rate is $R_L$
Other robustness tests

<table>
<thead>
<tr>
<th>Model</th>
<th>Max P1</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>2007</td>
</tr>
<tr>
<td><strong>Baseline</strong></td>
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<tr>
<td>Using swap rates</td>
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<tr>
<td>US banks</td>
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<tr>
<td>Reweight top 5 banks</td>
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<td>Reweight, decreasing</td>
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<td>Alternative bond model</td>
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## Other robustness tests

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Other robustness tests

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</table>

Stefano Giglio (Harvard University)
Other robustness tests - derivations

- Alternative Pricing Model
- Using Swap Rates
- Different weighting in CDS
- Different currencies
- TRACE, larger transactions
Robustness: Pricing model

- Hazard rate deterministic but not constant:
  \[ h_{t+s} = (1 - \rho_t) \overline{h}_t + \rho_t h_{t+s-1} \]

- CDS: assume joint default risk inherits \( \rho_t \) and \( \overline{h}_t / h_t \) from reference entity

- Approximate around \( h_t = 0 \)
Robustness: Swap rates

- Interest Rate Swaps
  - contain counterparty risk
  - are not indexed to a risk-free short rate

- Swap rates are higher than Treasuries -> lower systemic risk

- However, partly offset by calibrated liquidity process
Robustness: Weighting scheme

- If not all dealers post quotes every day, observed average will overrepresent more active banks.

- Assume CDS spread is:

\[
\bar{z}_i = \left[ P(A_i) - (1 - S) \left( \sum_{i \neq j} w_j P(A_i \cap A_j) \right) \right] \frac{[\sum_{s=t+1}^{T} \delta(t, s)] (1 - R)}{[\sum_{s=t}^{T-1} \delta(t, s)]} \\
\text{with } w_j \neq \frac{1}{N-1}
\]
Robustness: Weighting scheme

- Obtain list of top 5 counterparties by trade count
- Two schemes (call $w$ the weight of banks 6-15):
  1. $5w, 5w, 5w, 5w, 5w$
  2. $10w, 8w, 6w, 4w, 2w$
Robustness: Currencies

- Bonds and CDSs denominated in different currencies
- What assumptions do we need to mix them?
- Two bonds, same firms, different currencies
- $s = 0$ or $i$, default state
- $e$ exchange rate
- $m_{se}$ SDF
- Joint distribution of $s$ and $e$

$$f(s, e) = \pi_s f_s(e)$$
Robustness: Currencies

\[ p_i^\$ = \pi_0 E[m_{se}|s = 0] + R\pi_i E[m_{se}|s = i] \]
\[ = E[m_{se}] - (1 - R)\pi_i E[m_{se}|s = i] \]

\[ p_i^E e_0 = E[e \cdot m_{se}] - (1 - R)\pi_i E[e \cdot m_{se}|s = i] \]
\[ t^\$ = E[m_{se}] \]
\[ t^E e_0 = E[e \cdot m_{se}] \]
Robustness: Currencies

\[ P(A_i) = \pi_i \frac{E[m_{se}|s = i]}{E[m_{se}]} \]

From Euro bonds, we obtain

\[ \pi_i \frac{E[m_{se}|s = i]}{E[m_{se}]} \]

So we can mix if:

\[ \frac{E[e \cdot m_{se}|s = i]}{E[m_{se}|s = i]} = \frac{E[e \cdot m_{se}]}{E[m_{se}]} \]
Robustness: Currencies

- Now take Euro-denominated CDS for \( i \). Counterparty \( j \) American.

- \( s \in \{ i, j, ij, 0 \} \)

\[
\begin{align*}
  z_{ji} e_0 &= (1 - R) \pi_i E [ e \cdot m_{se} | s = i ] + (1 - R) S \pi_{ij} E [ e \cdot m_{se} | s = ij ] \\
  &= E [ e \cdot m_{se} ] \left( (1 - R) \pi_i \frac{E [ e \cdot m_{se} | s = i ]}{E [ e \cdot m_{se} ]} + (1 - R) S \pi_{ij} \frac{E [ e \cdot m_{se} | s = ij ]}{E [ e \cdot m_{se} ]} \right)
\end{align*}
\]

- So: condition is for every \( s \)

\[
\frac{E [ e \cdot m_{se} | s ]}{E [ m_{se} | s ]} = \frac{E [ e \cdot m_{se} ]}{E [ m_{se} ]}
\]
Robustness: TRACE, larger transactions

- Quoted data might have lags and matrix prices
- Small trades might be less reflective of credit risk
- Results using
  - only US banks
  - TRACE trades \( \geq \$100,000 \)
Discussion: Arora et al.

- Arora, Gandhi and Longstaff (2010) run the regression for bond $k$:

$$z_{jk,t} = a_{k,t} + bz_{j,t-1} + e_{jk,t}$$

- Find that $b$ is negative but small
- Default probability of the counterparty little cross-sectional effect on price.

First point:
- The starting point of my paper is the difference between $P(A_j)$ and $P(A_i \cap A_j)$
- Only $P(A_i \cap A_j)$ is priced in the CDS, not $P(A_j) \approx z_{j,t}$.

Second point:
- $a_{k,t}$ removes all average counterparty risk
- This paper is based only on the pricing of average counterparty risk
- Even if for some reason there is compression of quotes
Discussion: Arora et al.

- Third point:
  - Cross-sectional difference in $S$ (collateralization) might induce lower dispersion of quotes.
  - If $S_j$ is different by $j$ the average quote reflects a *weighted average* of $P(A_i \cap A_j)$

\[
\frac{z_1i + z_2i}{2} = P(A_i) - \frac{(1 - S_1)}{2} P(A_i \cap A_1) - \frac{(1 - S_2)}{2} P(A_i \cap A_2)
\]

\[
= P(A_i) - (1 - S)\left[\frac{(1 - S_1)}{(1 - S)^2} P(A_i \cap A_1) + \frac{(1 - S_2)}{(1 - S)^2} P(A_i \cap A_2)\right]
\]

where $S = \frac{S_1 + S_2}{2}$

- Lower collateral requirement -> lower $S$ -> higher weight
- Robustness: biggest dealers (Goldman, DB, JPM) safer
  -> less collateral
- Smaller dealers (Lehman, Merrill) -> more collateral
Explore: ETF binary puts

- Loss of 10%
- Loss of 15%
- Loss of 20%

- Black: binary put
- Red: P4
- Blue: P1