Financial Risk Capacity

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June 27, 2011

Introduction

- Financial sector's capacity to intermediate $\Rightarrow$ growth
  - Capacity depends on bank net-worth

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- Function of banks $\Rightarrow$ mitigate asymmetric information
- Paper: risky financial intermediation + asymmetric information

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- Adverse-selection in fin. markets tied to bank net-worth
  - Propagation and spill-over of shocks

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- Adverse-selection in fin. markets tied to bank net-worth
  - Propagation and spill-over of shocks
- Explain why banks aren't quickly recapitalized during crisis
  - Persistence of financial crisis

Preview

- Growth model with financial intermediaries where:
  1. Collateral quality is private information
  2. Risky in process of intermediation
  3. Intermediation losses subject to limited liability equity

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  1. Collateral quality is private information
  2. Risky in process of intermediation
  3. Intermediation losses subject to limited liability equity
- Lab to analyze
  1. Shocks that affect financial sector's equity
  2. Analyze government policies

Environment

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  - Every period divided into two stages

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- Population:
  - Producers
  - Bankers

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- Preferences:

$$\mathbb{E}\left[\sum_{t\geq 0}\beta^t \log\left(c_t\right)\right]$$

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- Incomplete markets.

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- f_ϕ depends on shock-ϕ

Assumption

$\{f_\phi\}$ satisfies $E_\phi [\lambda(\tilde{\omega})|\tilde{\omega} < \omega]$ decreasing in $\phi, \forall \omega$.

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- Physical evolution:

$$\tilde{k} = k \int \lambda(\omega) f_{\phi}(\omega) \, d\omega.$$

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- Agents capital evolves:

$$k' = investment + purchases + k \int \lambda(\omega)(1 - \mathbb{I}(\omega)) f_{\phi}(\omega) d\omega$$

- $\mathbb{I}(\omega)$ sales of quality ω
  - ω private information.

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- Preferences

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- Intermediaries own banks

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  - Intermediation through banks
  - Only Net-worth n(j) is liable to intermediation losses

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  - Intermediation through banks
  - Only Net-worth n(j) is liable to intermediation losses
  - Can inject equity e to increase n
  - Can pay dividends d reducing n

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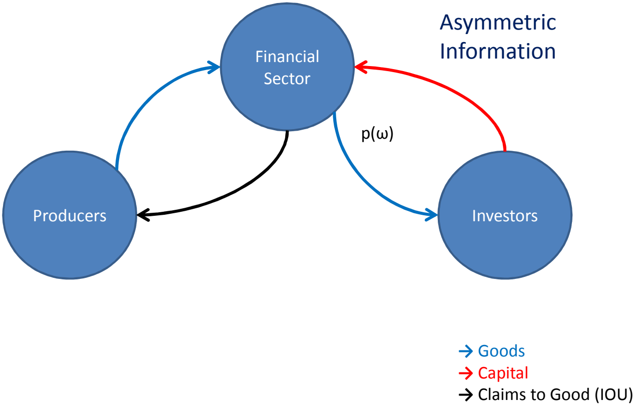
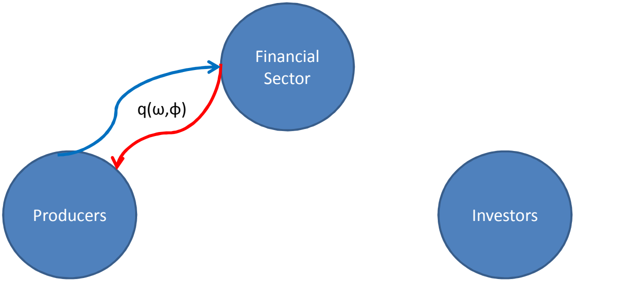
- Endogenous state financial sector size: κ
  - $\kappa = \frac{\int n'(j)dj}{\int k(z)dz}$
- State: $X = (A, \phi, \kappa)$

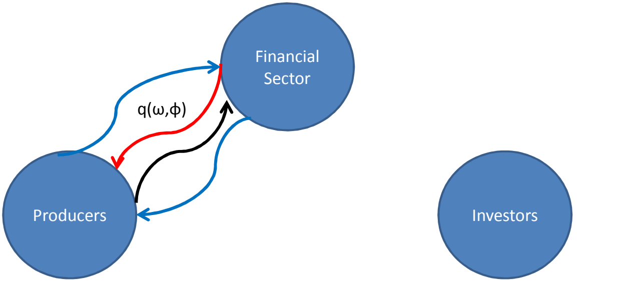
Stage 1: Capital Sales



Stage 2: Realization of Shock and Resale



Stage 2: Consumption Goods Settlements



Balance Sheets

- Stage 1 balance sheet:

Assets	Liability
n	
	Net-worth
	n

$\underbrace{\hphantom{xxxxxxxxxxxx}}_{\text{Initial Balance Sheet}}$

Assets	Liability
$n+e-d$	pQ
pQ	**Net-worth**
	$n+e-d$

$\underbrace{\hphantom{xxxxxxxxxxxx}}_{\text{Balance Sheet S1}}$

- Q amount of capital units purchased

Balance Sheets

- Stage 1 balance sheet:

Assets	Liability
n'	pQ
pQ	**Net-worth**
	n'

Balance Sheet S1

- $n' = n + e - d$

Balance Sheets

- Stage 1 balance sheet:

Assets	Liability
n'	pQ
pQ	**Net-worth**
	n'

Balance Sheet S1

- Stage 2 balance sheet:

Assets	Liability
n'	pQ
$q(\phi)\lambda(\phi)Q$	**Net-worth**
	$n' + [q(\phi)\lambda(\phi) - p]\,Q$

Balance Sheet S2

Balance Sheets

- Stage 1 balance sheet:

Assets	Liability
n'	pQ
pQ	**Net-worth**
	n'

Balance Sheet S1

- Stage 2 balance sheet:

Assets	Liability
n'	pQ
$q(\phi)\lambda(\phi)Q$	**Net-worth**
	$n' + \Pi Q$

Balance Sheet S2

- $\Pi = [q(\phi)\lambda(\phi) - p]$

Bankers - Problems

Problem (Stage 1)

$$V_1^f(n,X) = \max_{Q,e\in[0,\bar{e}],d\in[0,n]} c + \mathbb{E}\left[V_2^f(n' + \Pi(X,X')Q, X')|X\right]$$

$$\begin{aligned} s.t.\ -\Pi(X,X')Q &\leq n+e-d,\ \forall X' \\ c &= (\bar{e}-e) + (1-\tau)d \\ n' &= n+e-d \end{aligned}$$

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- Stage 2: $V_2^f(n, X) = \beta^F \mathbb{E}\left[V_1^f(R^b n, X')|X\right]$

C-goods producer's Problems

Problem (c-producer's stage 1)

$$V_p^1(k, X) = \mathbb{E}\left[V_p^2(k'(\phi'), x, X') \,|X\right]$$

$$s.t.\ x = Ak \ and\ k'(\phi') = k \int \lambda(\omega) f_{\phi'}(\omega)\, d\omega$$

Problem (c-producer stage 2)

$$V_p^2(k, x, X) = \max_{c \geq 0, i \leq 0, k^b \geq 0} \log(c) + \beta \mathbb{E}\left[V_j^1(k', X') \,|X\right], j \in \{i, p\}$$

$$c + i + qk^b = x \ and\ k' = k^b + i + k$$

k-producer Problems

Problem (k-producer's stage 1)

$$V_i^1(k, X) = \max_{\mathbb{I}(\omega)\in\{0,1\}} \mathbb{E}\left[V_i^2\left(k'\left(\phi'\right), x, X'\right)|X\right]$$

$$s.t.\ x = pk\int_0^1 \mathbb{I}(\omega)\,d\omega \ and\ k'\left(\phi'\right) = k\int \lambda(\omega)\left[1-\mathbb{I}(\omega)\right] f_{\phi'}(\omega)\,d\omega$$

Problem (k-producer's stage 2)

$$V_i^2(k, x, X) = \max_{c\geq 0, i, k^b\geq 0} \log(c) + \beta\mathbb{E}\left[V_j^1\left(k', X'\right)|X\right], j\in\{i, p\}$$

$$c + i + qk^b = x \ and\ k' = k^b + i + k$$

Recursive Competitive Equilibrium

Definition (RCE)

A RCE are policy functions, e,d,Q,ω,k^b,i and prices (q, p), and a l.o.m. for X s.t.:

1. Given p,q, and l.o.m., e,d and Q are solutions to intermediaries problem.
2. Given p and l.o.m., ω solves the i-problem in s1.
3. Given ϕ and l.o.m., policies solves producer problems in s2.
4. Markets clear in both stages.
5. L.o.m. is internally consistent.

Characterization - Stage 1 capital supply

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- Cut-off $\omega^*(X)$ for sales solves portfolio problem:

$$\omega^* = \arg\max_{\tilde{\omega}} \mathbb{E}\left[\log(\underbrace{p\tilde{\omega}}_{\text{Risk-free}} + \underbrace{\int_{\tilde{\omega}}^{1} \lambda(\omega) f_{\phi'}(\omega)\, d\omega}_{\text{Risky}})|X\right]$$

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- Defines $p(\omega^*)$ increasing supply schedule

Characterization - Stage 2 capital demand and profits

- S2 price of capital q :

$$q(X,X') = \left[\frac{\beta A}{\pi\omega^*(X)\,\mathbb{E}_\phi\left[\lambda(\omega)\,|\omega < \omega^*(X)\right] + (1-\pi)(1-\beta)\,\bar{\lambda}(X')}\right]$$

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- Profits:

$$\Pi(X, X') = q(X, X') \mathbb{E}_\phi [\lambda (\omega) | \omega < \omega^* (X)] - p(\omega^*(X))$$

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Characterization - Stage 1 Bankers' Policies

- $V_1^f(n, X) = v_1^f(X)n$ and $V_2^f(n, X) = v_2^f(X)n$
- Q, e and d linear in n

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- Reminder:

Problem

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Problem

$$Q = \arg\max_{\tilde{Q}} \mathbb{E}\left[v_2^f(X')\,\Pi(X, X')\,|X\right]\tilde{Q}$$

$$\textit{subject to}\ \underbrace{min_{X'} - \Pi(X, X')}_{\textit{Marginal Leverage}}\tilde{Q} \leq n'.$$

Characterization - Stage 1 Financial Policies

- Equity injections only if:

$$\beta^F \left[\underbrace{\mathbb{E}[v_2^f(X')]}_{\text{SDF}} + \mu(X) \right] \geq \underbrace{1}_{\text{Equity Cost}}$$

- Dividend payoffs only if:

$$\beta^F \left[\mathbb{E}[v_2^f(X')] + \mu(X) \right] \leq (1 - \tau).$$

Characterization - Stage 1 Financial Policies

- Equity injections only if:

$$\beta^F \left[\underbrace{\mathbb{E}[v_2^f(X')]}_{\text{SDF}} + \max \left\{ \frac{\overbrace{\mathbb{E}[v_2^f(X')\Pi(X,X')]}^{\text{Disc. Profit}}}{\underbrace{\min_{\tilde{X}} -\Pi\left(X,\tilde{X}\right)}_{\text{Leverage}}}, 0 \right\} \right] \geq \underbrace{1}_{\text{Equity Cost}}$$

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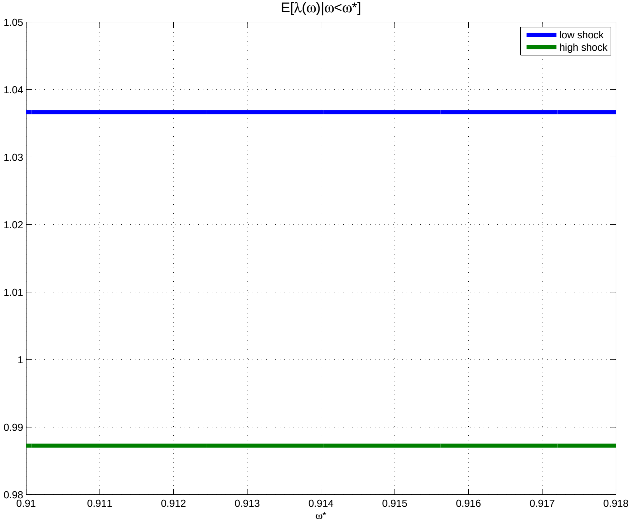
Characterization - Equilibria

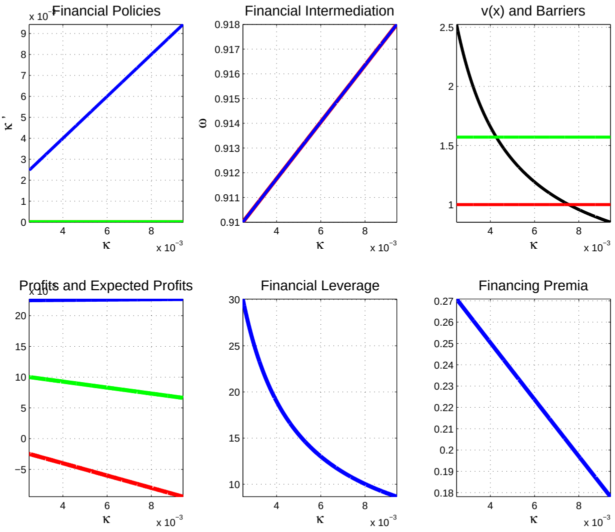
- Fixed point problem:
  - $\kappa \Rightarrow \omega^*$
  - $\omega^* \Rightarrow e, d \Rightarrow n' \Rightarrow \kappa'$

without Adverse Selection...

Example I - Risky intermediation without Adverse Selection



Financial Variables (before equity adjustments)



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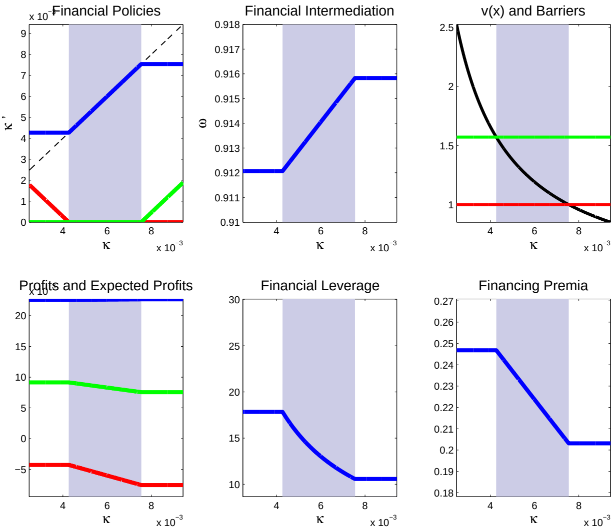
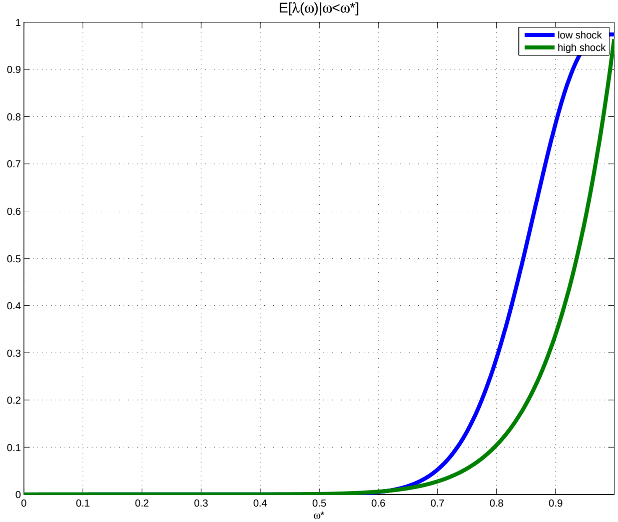


Lessons - Intermediation without A.I.

1. Intermediation, ω increasing in κ.
2. Profitability, Π decreasing in κ.
3. In equilibrium:
   - $\kappa' \in [\underline{\kappa}, \overline{\kappa}]$.
   - $\omega \in [\underline{\omega}, \overline{\omega}]$.

Adverse Selection...

Model with Asymmetric Information



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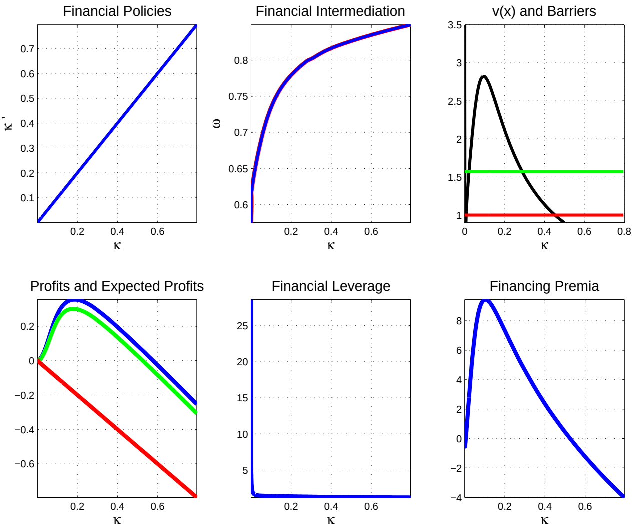

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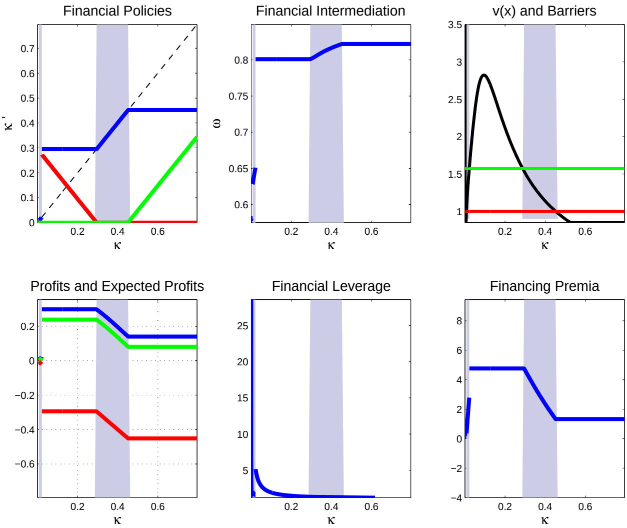
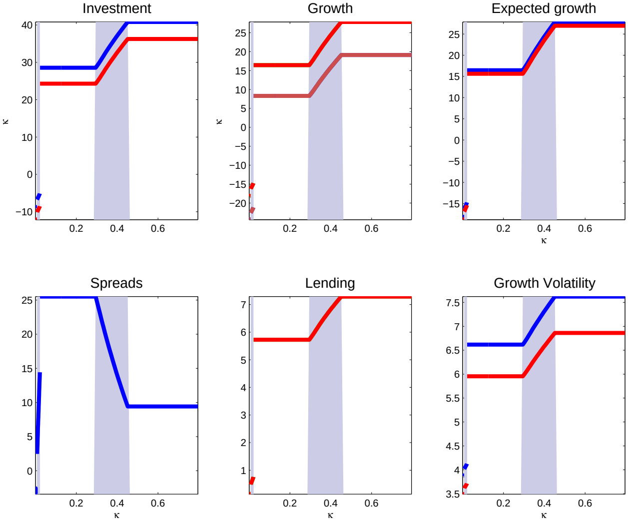


Real Side Variables



Lessons

1. Adverse selection $\Rightarrow$ non-monotone expected profits
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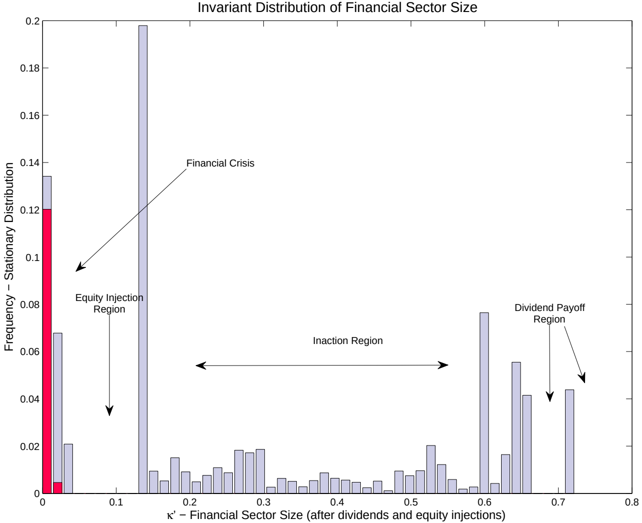
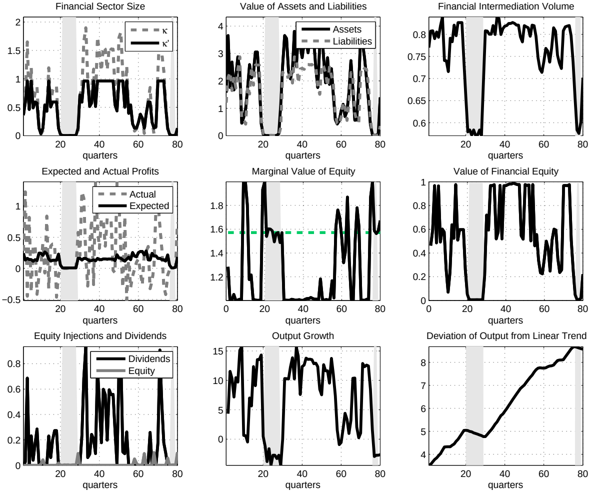
1. Adverse selection $\Rightarrow$ non-monotone expected profits
2. κ not in unique region
3. Adverse selection $\Rightarrow$ prevents recapitalization
4. κ grows only through retained earnings

In a richer version of the model...

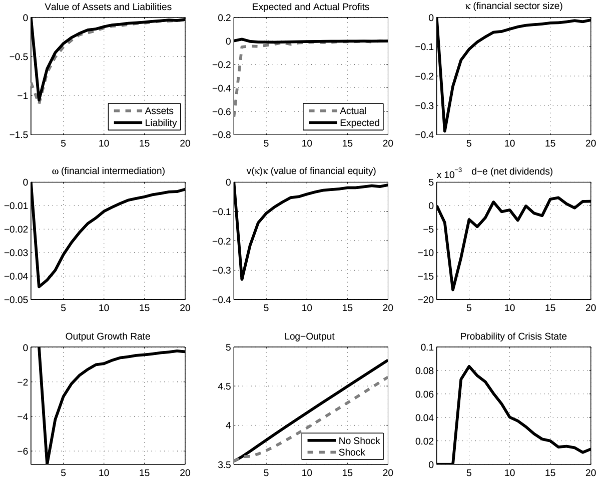
Invariant Distribution



Sample Path


Financial Sector Size
κ
κ'
Value of Assets and Liabilities
Assets
Liabilities
Financial Intermediation Volume
Expected and Actual Profits
Actual
Expected
Marginal Value of Equity
Value of Financial Equity
Equity Injections and Dividends
Dividends
Equity
Output Growth
Deviation of Output from Linear Trend
quarters

Dynamic Setup - Response to Dispersion Shock



Conclusions

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   - Easy to adapt to study spill-overs, fire-sales.

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2. Pecuniary externality: banks fail to internalize risk of triggering crisis
   - Capital requirements, dividend policies, government equity, CoCo.

Financial Risk Capacity

Saki Bigio
New York University

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