Financial Risk Capacity

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June 27, 2011
Introduction

- Financial sector’s capacity to intermediate ⇒ growth
  - Capacity depends on bank net-worth
Financial sector’s capacity to intermediate ⇒ growth
   ▶ Capacity depends on bank net-worth

Function of banks ⇒ mitigate asymmetric information
Introduction

- Financial sector’s capacity to intermediate ⇒ growth
  - Capacity depends on bank net-worth

- Function of banks ⇒ mitigate asymmetric information

- Paper: risky financial intermediation + asymmetric information
Why study this? - Reasons

- Adverse-selection in fin. markets tied to bank net-worth
  - Propagation and spill-over of shocks
Why study this? - Reasons

- Adverse-selection in fin. markets tied to bank net-worth
  - Propagation and spill-over of shocks

- Explain why banks aren’t quickly recapitalized during crisis
  - Persistence of financial crisis
Growth model with financial intermediaries where:

1. Collateral quality is private information
2. Risky in process of intermediation
3. Intermediation losses subject to limited liability equity
Growth model with financial intermediaries where:
1. Collateral quality is private information
2. Risky in process of intermediation
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Lab to analyze
1. Shocks that affect financial sector’s equity
2. Analyze government policies
Environment

- Infinite horizon:
  - Every period divided into two stages
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- Commodity Space:
  - Consumption good (perishable numeraire)
  - Capital
Environment

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  - Every period divided into two stages

- **Commodity Space:**
  - Consumption good (perishable numeraire)
  - Capital

- **Population:**
  - Producers
  - Bankers
Unit continuum, $z \in [0, 1]$

Start with capital stock: $k(z)$
Environment - Producers

- Unit continuum, $z \in [0, 1]$
- Start with capital stock: $k(z)$
- Preferences:

$$\mathbb{E} \left[ \sum_{t \geq 0} \beta^t \log (c_t) \right]$$
Segmentation of activities

- $\pi \rightarrow$ produce capital goods (k-producers)
  - consumption $\Leftrightarrow$ capital one for one

Linear technology: $y = A_k$
Segmentation of activities

- \( \pi \rightarrow \) produce capital goods (k-producers)
  - consumption \( \leftrightarrow \) capital one for one

- \( (1 - \pi) \rightarrow \): produce consumption goods (c-producers)
  - Linear technology: \( y = Ak \)
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Need for Trade

- investors lack consumption goods as input for investment
- producers lack investment technology
Segmentation of activities

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Need for Trade

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Incomplete markets.
Capital stock *divisible* into *continuum*
Environment - Heterogeneous Capital

- Capital stock divisible into continuum
- Each unit identified with quality $\omega \in [0, 1]$
- Efficiency of units $\lambda(\omega)$
Environment - Heterogeneous Capital

- Capital stock *divisible into continuum*
- Each *unit* identified with *quality* $\omega \in [0, 1]$
- Efficiency of *units* $\lambda(\omega)$
- Distribution over qualities given by $f_\phi$
- $f_\phi$ depends on shock-$\phi$
Capital stock divisible into continuum

Each unit identified with quality $\omega \in [0, 1]$
Efficiency of units $\lambda(\omega)$

Distribution over qualities given by $f_\phi$
$f_\phi$ depends on shock-$\phi$

Assumption

$\{ f_\phi \}$ satisfies $E_\phi [\lambda(\tilde{\omega}) | \tilde{\omega} < \omega]$ decreasing in $\phi$, $\forall \omega$. 
Physical evolution:

\[
\tilde{k} = k \int \lambda(\omega) f_\phi(\omega) d\omega.
\]
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Agents capital evolves:

\[ k' = \text{investment} + \text{purchases} + k \int \lambda(\omega)(1 - \mathbb{I}(\omega)) f_\phi(\omega) d\omega \]

\( \mathbb{I}(\omega) \) sales of quality \( \omega \)

\( \omega \) private information.
Continuum of intermediaries: $j \in [0, 1]$
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Big and risk neutral

Preferences

$$\mathbb{E} \left[ \sum_{t \geq 0} (\beta^f)^t c_t \right]$$
Continuum of intermediaries: $j \in [0, 1]$

- Big and risk neutral
- Preferences

$$\mathbb{E} \left[ \sum_{t \geq 0} (\beta^f)^t c_t \right]$$

- Intermediaries own banks
Environment - Bankers

- Role: *intermediate* in capital market
  - Buy $k$ from investors $\Leftrightarrow$ exchange for *goods*
    - Under *asymmetric information*
Role: intermediate in capital market
- Buy k from investors ⇔ exchange for goods
  - Under asymmetric information
- Sell k to producers ⇔ exchange for goods
  - Sell pool of qualities bought
  - Intermediation risky ⇒ expected ≠ realized quality
Environment - Bankers

- **Role**: intermediate in capital market
  - **Buy** $k$ from investors $\Leftrightarrow$ exchange for goods
    - Under asymmetric information
  - **Sell** $k$ to producers $\Leftrightarrow$ exchange for goods
    - Sell pool of qualities bought
    - Intermediation risky $\Rightarrow$ expected $\neq$ realized quality

- Periodic endowment of goods $\bar{e}(j)$ and $n(j)$ stored in banks
  - Intermediation through banks
  - Only Net-worth $n(j)$ is liable to intermediation losses
Role: intermediate in capital market

- Buy k from investors ⇔ exchange for goods
  - Under asymmetric information
- Sell k to producers ⇔ exchange for goods
  - Sell pool of qualities bought
  - Intermediation risky ⇒ expected ≠ realized quality

Periodic endowment of goods $\bar{e}(j)$ and $n(j)$ stored in banks

- Intermediation through banks
- Only Net-worth $n(j)$ is liable to intermediation losses
- Can inject equity $e$ to increase $n$
- Can pay dividends $d$ reducing $n$
- $\phi$-shock distribution of quality,
  - Affects $f_\phi$
  - Shock after purchase but before resell
Environment - Aggregate State

- $\phi$-shock distribution of quality,
  - Affects $f_\phi$
  - Shock after purchase but before resell

- Endogenous state financial sector size: $\kappa$
  - $\kappa = \frac{\int n'(j) dj}{\int k(z) dz}$
Environment - Aggregate State

- $\phi$-shock distribution of quality,
  - Affects $f_\phi$
  - Shock after purchase but before resell

- Endogenous state financial sector size: $\kappa$
  - $\kappa = \int \frac{n'(j) dj}{\int k(z) dz}$

- State: $X = (A, \phi, \kappa)$
Stage 1: Capital Sales

- Goods → Capital → Claims to Good (IOU)
- Producers
- Investors
- Financial Sector
- Asymmetric Information
- $p(\omega)$
Stage 2: Realization of Shock and Resale

- Goods
- Capital
- Claims to Good (IOU)

Producers → Financial Sector → Investors

$q(\omega, \phi)$
Stage 2: Consumption Goods Settlements

- Goods → Financial Sector
- Capital → Claims to Good (IOU)
- Producers
- Investors

$q(\omega,\phi)$
Balance Sheets

- Stage 1 balance sheet:

\[
\begin{array}{|c|c|}
\hline
\text{Assets} & \text{Liability} \\
\hline
n & \text{Net-worth} \\
\hline
\end{array}
\]

Initial Balance Sheet

- Stage 2 balance sheet:

\[
\begin{array}{|c|c|}
\hline
\text{Assets} & \text{Liability} \\
\hline
n + e - d & pQ \\
\hline
pQ & \text{Net-worth} \\
\hline
n + e - d & \\
\end{array}
\]

Balance Sheet S1

- Q amount of capital units purchased
Stage 1 balance sheet:

\[
\begin{array}{c|c}
\text{Assets} & \text{Liability} \\
\hline
n' & pQ \\
pQ & \text{Net-worth} \\
\hline
\end{array}
\]

\[n' = n + e - d\]
### Balance Sheets

#### Stage 1 balance sheet:

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n'$</td>
<td>$pQ$</td>
</tr>
<tr>
<td>$pQ$</td>
<td><strong>Net-worth</strong></td>
</tr>
<tr>
<td></td>
<td>$n'$</td>
</tr>
</tbody>
</table>

Balance Sheet S1

#### Stage 2 balance sheet:

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n'$</td>
<td>$pQ$</td>
</tr>
<tr>
<td>$q(\phi)\lambda(\phi)Q$</td>
<td><strong>Net-worth</strong></td>
</tr>
<tr>
<td></td>
<td>$n' + [q(\phi)\lambda(\phi) - p] Q$</td>
</tr>
</tbody>
</table>

Balance Sheet S2
Balance Sheets

- **Stage 1 balance sheet:**
  
  \[
  \begin{array}{|c|c|}
  \hline
  \text{Assets} & \text{Liability} \\
  \hline
  n' & pQ \\
  pQ & \text{Net-worth} \\
  \hline
  \end{array}
  \]

  Balance Sheet S1

- **Stage 2 balance sheet:**
  
  \[
  \begin{array}{|c|c|}
  \hline
  \text{Assets} & \text{Liability} \\
  \hline
  n' & pQ \\
  q(\phi)\lambda(\phi)Q & \text{Net-worth} \\
  \hline
  \end{array}
  \]

  Balance Sheet S2

- \[\Pi = [q(\phi)\lambda(\phi) - p]\]
### Problem (Stage 1)

\[
V_1^f (n, X) = \max_{Q, e \in [0, \bar{e}], d \in [0, n]} c + \mathbb{E} \left[ V_2^f (n' + \Pi (X, X') Q, X') | X \right]
\]

\[
s.t. \quad - \Pi (X, X') Q \leq n + e - d, \quad \forall X'
\]

- \[
c = (\bar{e} - e) + (1 - \tau) d
\]

- \[
n' = n + e - d
\]
Problem (Stage 1)

\[ V_1^f (n, X) = \max_{Q, e \in [0, \bar{e}], d \in [0, n]} \quad c + \mathbb{E} \left[ V_2^f (n' + \Pi (X, X') Q, X') | X \right] \]

\[ s.t. \quad - \Pi (X, X') Q \leq n + e - d, \quad \forall X' \]
\[ c = (\bar{e} - e) + (1 - \tau) d \]
\[ n' = n + e - d \]

- Stage 2: \[ V_2^f (n, X) = \beta^F \mathbb{E} \left[ V_1^f (R^b n, X') | X \right] \]
C-goods producer’s Problems

Problem (c-producer’s stage 1)

\[ V_p^1 (k, X) = \mathbb{E} \left[ V_p^2 (k' (\phi'), x, X') \mid X \right] \]

s.t. \( x = Ak \) and \( k' (\phi') = k \int \lambda (\omega) f_{\phi'} (\omega) d\omega \)

Problem (c-producer stage 2)

\[ V_p^2 (k, x, X) = \max_{c \geq 0, i \leq 0, k^b \geq 0} \log (c) + \beta \mathbb{E} \left[ V_j^1 (k', X') \mid X \right], j \in \{i, p\} \]

\( c + i + qk^b = x \) and \( k' = k^b + i + k \)
k-producer Problems

Problem (k-producer’s stage 1)

\[ V^1_i(k, X) = \max_{\mathbb{I}(\omega) \in \{0, 1\}} \mathbb{E} \left[ V^2_i(k'(\phi'), x, X') \mid X \right] \]

s.t. \( x = pk \int_0^1 \mathbb{I}(\omega) \, d\omega \) and \( k'(\phi') = k \int \lambda(\omega) [1 - \mathbb{I}(\omega)] \, f_{\phi'}(\omega) \, d\omega \)

Problem (k-producer’s stage 2)

\[ V^2_i(k, x, X) = \max_{c \geq 0, i, k^b \geq 0} \log(c) + \beta \mathbb{E} \left[ V^1_j(k', X') \mid X \right], j \in \{i, p\} \]

\[ c + i + qk^b = x \text{ and } k' = k^b + i + k \]
Recursive Competitive Equilibrium

Definition (RCE)

A RCE are policy functions, $e, d, Q, \omega, k^b, i$ and prices $(q, p)$, and a l.o.m. for $X$ s.t.:

1. Given $p, q$, and l.o.m., $e, d$ and $Q$ are solutions to intermediaries problem.
2. Given $p$ and l.o.m., $\omega$ solves the i-problem in $s1$.
3. Given $\phi$ and l.o.m., policies solves producer problems in $s2$.
4. Markets clear in both stages.
5. L.o.m. is internally consistent.
Policy functions are linear in $k$
Policy functions are linear in $k$

Cut-off $\omega^*(X)$ for sales solves portfolio problem:

$$\omega^* = \arg \max_{\tilde{\omega}} \mathbb{E} \left[ \log(p\tilde{\omega}) + \int_{\tilde{\omega}}^{1} \lambda(\omega) f_{\phi'}(\omega) d\omega \right] | X$$
Policy functions are linear in $k$

Cut-off $\omega^*(X)$ for sales solves portfolio problem:

$$\omega^* = \arg \max_{\tilde{\omega}} \mathbb{E} \left[ \log(\tilde{p}\tilde{\omega}) + \int_{\tilde{\omega}}^{1} \lambda(\omega) f_{\phi'}(\omega) d\omega | X \right]$$

Defines $p(\omega^*)$ increasing supply schedule
Characterization - Stage 2 capital demand and profits

- S2 price of capital $q$:

$$ q(X, X') = \left[ \frac{\beta A}{\pi \omega^* (X) \mathbb{E}_\phi [\lambda(\omega) | \omega < \omega^* (X)] + (1 - \pi) (1 - \beta) \bar{\lambda}(X')} \right] $$
Characterization - Stage 2 capital demand and profits

- S2 price of capital $q$:

$$q(X, X') = \left[ \frac{\beta A}{\pi \omega^*(X) \mathbb{E}_\phi [\lambda(\omega) | \omega < \omega^*(X)] + (1 - \pi)(1 - \beta) \bar{\lambda}(X')} \right]$$

- Profits:

$$\Pi(X, X') = q(X, X') \mathbb{E}_\phi [\lambda(\omega) | \omega < \omega^*(X)] - p(\omega^*(X))$$
Characterization - Stage 2 capital demand and profits

- **S2 price of capital** $q$:

$$q(X, X') = \frac{\beta A}{\pi \omega^*(X) \mathbb{E}_\phi [\lambda(\omega) | \omega < \omega^*(X)] + (1 - \pi)(1 - \beta)\bar{\lambda}(X')}$$

- **Profits**:

$$\Pi(X, X') = \left[ \frac{\beta A}{\pi \omega^*(X) + (1 - \pi)(1 - \beta)\bar{\lambda}(X')} \right] - p(\omega^*(X))$$
Characterization - Stage 1 Bankers’ Policies

- $V_1^f(n, X) = v_1^f(X)n$ and $V_2^f(n, X) = v_2^f(X)n$
- $Q, e$ and $d$ linear in $n$
Characterization - Stage 1 Bankers' Policies

- \( V_1^f(n, X) = v_1^f(X)n \) and \( V_2^f(n, X) = v_2^f(X)n \)
- \( Q, e \) and \( d \) linear in \( n \)

Reminder:

**Problem**

\[
V_1^f(n, X) = \max_{Q, e \in [0, \bar{e}], d \in [0, n]} c + \mathbb{E} \left[ v_2^f(X')(n' + \Pi(X, X')Q) \mid X \right]
\]

s.t. \(-\Pi(X, X')Q \leq n', \forall X'\)

\[
c = (\bar{e} - e) + (1 - \tau)d
\]
Characterization - Stage 1 Bankers’ Policies

- $V_1^f(n, X) = v_1^f(X)n$ and $V_2^f(n, X) = v_2^f(X)n$
- $Q$, $e$ and $d$ linear in $n$

Reminder:

Problem

$$Q = \arg \max_{\tilde{Q}} \mathbb{E} \left[ v_2^f(X') \Pi(X, X') | X \right] \tilde{Q}$$

subject to $\min_{X'} - \Pi(X, X') \tilde{Q} \leq n'$.

*Marginal Leverage*
Characterization - Stage 1 Financial Policies

- Equity injections only if:
  \[
  \beta^F \left[ \mathbb{E}[v^f(X')] + \mu(X) \right]_{\text{SDF}} \geq 1
  \]

- Dividend payoffs only if:
  \[
  \beta^F \left[ \mathbb{E}[v^f(X')] + \mu(X) \right] \leq (1 - \tau).
  \]
Characterization - Stage 1 Financial Policies

- **Equity injections only if:**

  \[
  \beta^F \left[ \mathbb{E}[v_2^f(X')] \right] + \max \left\{ \frac{\mathbb{E}[v_2^f(X') \Pi(X, X')]}{\text{SDF}}, 0 \right\} \geq \frac{1}{\text{Equity Cost}}
  \]

- **Dividend payoffs only if:**

  \[
  \beta^F \left[ \mathbb{E}[v_2^f(X')] \right] + \max \left\{ \frac{\mathbb{E}[v_2^f(X') \Pi(X, X')]}{\min \tilde{X} - \Pi(X, \tilde{X})}, 0 \right\} \leq (1 - \tau).
  \]
Fixed point problem:

- $\kappa \Rightarrow \omega^*$
- $\omega^* \Rightarrow e, d \Rightarrow n' \Rightarrow \kappa'$
without Adverse Selection...
Example I - Risky intermediation **without** Adverse Selection
Financial Variables (before equity adjustments)

- **Financial Policies**
  - $\mathbf{\kappa}$
  - $\mathbf{\kappa}'$

- **Financial Intermediation**
  - $\mathbf{\Theta}$

- **$v(x)$ and Barriers**
  - $2.5$
  - $2.0$
  - $1.5$

- **Profits and Expected Profits**

- **Financial Leverage**
  - $0.18$
  - $0.19$
  - $0.20$
  - $0.21$
  - $0.22$
  - $0.23$
  - $0.24$
  - $0.25$
  - $0.26$

- **Financing Premia**
  - $0.27$
  - $0.26$
  - $0.25$
  - $0.24$
  - $0.23$
  - $0.22$
  - $0.21$
  - $0.20$
  - $0.19$
  - $0.18$
Financial Variables (after equity adjustments)

Financial Policies

Financial Intermediation

v(x) and Barriers

Profits and Expected Profits

Financial Leverage

Financing Premia
1. Intermediation, $\omega$ increasing in $\kappa$.
2. Profitability, $\Pi$ decreasing in $\kappa$.
3. In equilibrium:
   - $\kappa' \in [\kappa, \bar{\kappa}]$.
   - $\omega \in [\omega, \bar{\omega}]$. 
Adverse Selection...
Model with Asymmetric Information

$$E[\lambda(\omega)|\omega<\omega^*]$$
Financial Variables (before equity adjustments)

Financial Policies

Financial Intermediation

v(x) and Barriers

Profits and Expected Profits

Financial Leverage

Financing Premia
Financial Variables (after equity adjustments)

- Financial Policies
- Financial Intermediation
- \(v(x)\) and Barriers
- Profits and Expected Profits
- Financial Leverage
- Financing Premia
Real Side Variables

- Investment
- Growth
- Expected growth
- Spreads
- Lending
- Growth Volatility

Graphs showing the relationship between $\kappa$ and various variables. The $\kappa$ values range from 0.2 to 0.6, and the y-axes represent the corresponding values of each variable.
Lessons

1. Adverse selection $\Rightarrow$ non-monotone expected profits
2. $\kappa$ not in unique region
Lessons

1. Adverse selection $\Rightarrow$ non-monotone expected profits
2. $\kappa$ not in unique region
3. Adverse selection $\Rightarrow$ prevents recapitalization
4. $\kappa$ grows only through retained earnings
In a richer version of the model...
Invariant Distribution

Invariant Distribution of Financial Sector Size

- Financial Crisis
- Equity Injection Region
- Inaction Region
- Dividend Payoff Region
Dynamic Setup - Response to Dispersion Shock

- Value of Assets and Liabilities
- Expected and Actual Profits
- \( \kappa \) (financial sector size)
- \( \omega \) (financial intermediation)
- \( v(\kappa) \kappa \) (value of financial equity)
- \( d-e \) (net dividends)
- Output Growth Rate
- Log–Output
- Probability of Crisis State

Graphs and charts illustrating the dynamic setup's response to dispersion shock.
Conclusions

1. Study a.i. and financial intermediation
   - Easy to adapt to study spill-overs, fire-sales.
Conclusions

1. Study a.i. and financial intermediation
   ▶ Easy to adapt to study spill-overs, fire-sales.

2. Pecuniary externality: banks fail to internalize risk of triggering crisis
   ▶ Capital requirements, dividend policies, government equity, CoCo.
Financial Risk Capacity

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June 27, 2011