Adverse Selection and Liquidity Distortion in Decentralized Markets

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Decentralized markets suffer massive illiquidity (buyers’ strike)

Question: Why markets remain illiquid even w/ positive gain from trade?

This paper:
  - An equilibrium model of illiquidity
  - Endogenous market segmentation
Introduction

Overview

- Decentralized markets suffer massive illiquidity (buyers’ strike)
- Question: Why markets remain illiquid even w/ positive gain from trade?
- This paper:
  - An equilibrium model of illiquidity
  - Endogenous market segmentation

- Liquidity: How fast a seller can find a buyer to cash his asset?
- Key feature: decentralized trading market with
  - Search frictions: eg, Over-the-Counter market (OTC)
  - Adverse Selection: sellers have private info about their asset quality
  - Example: Asset-backed securities, housing market, Corporate assets
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Two possible dimensions of market distortion:
  - price discount?
  - illiquidity?
Results

- Result 1 (Unobserved asset quality):
  - Liquidity is downward distorted
  - The higher the dispersion (range), the more illiquid the market
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- Result 2 (+Unknown motives for sale):
  - A submarket with price discount coexists with illiquid submarkets
- Predictions on price, liquidity (trade volume), market segmentation
Results

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  - The higher the dispersion (range), the more illiquid the market

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- **Predictions on price, liquidity (trade volume), market segmentation**

- **Relation to Guerrieri, Shimer and Wright (2010):**
  - a dynamic setting in asset trading market
  - a mechanism design approach
  - a semi-pooling Eq may arise
Related Literature

- **Asset market with search friction:**
  - Monetary Search: Williamson and Wright (1994), Trejos and Wright (1995)...

- **Asset market with adverse selection:** Akerlof (1970), Eisfeldt (2004)

- **Competitive Search Equilibrium:**
  - Complete information: Moen (1997), Mortensen and Wright (2002)
  - Decentralized price competition: Kircher (2010)
  - w/ Adverse Selection: Guerrieri, Shimer and Wright (2010)
Roadmap

- Setup
- Basic Model
- Generalization
- Obscure motives for sale
- Conclusion
Setup

- Players:
  - A continuum of sellers with asset quality $s, s \in S = [s_l, s_h]$ with $G(s)$
  - A continuum of homogenous buyers (more than sellers)

- Flow payoff of owning an asset $s$
  - Sellers: $s - c$
  - Buyers: $s$

- Setup: continuous time, risk-neutral, indivisible asset

- Competitive Search:
  - Buyers post trading prices $p$, at a flow cost $k > 0$
  - Sellers *direct* their search toward their preferred market
  - Traders meet randomly at each market
  - The meeting rate depends on buyer-seller ratio $\theta(p)$:
    - $m(\theta) = \theta^\rho$ for sellers ($\rho < 1$)
    - $\frac{m(\theta)}{\theta}$ for buyers
Competitive Search Equilibrium

- Each submarket is characterized by \((p, \theta(p))\)

**Equilibrium Conditions:**
- A Seller *directs* their search optimally, given \((p, \theta(p))\)
- A Buyer is indifferent among all submarkets \((p, \theta(p))\), expecting assets quality:
  \[
  \int \frac{\bar{s}}{r} \mu(\bar{s}|p) d\bar{s}
  \]
- No profitable deviation for buyers by posting a new price
Off-Path Belief of Buyers

- Opening new submarkets by posting $p'$:
  - Take $V^*(s)$ as given: Market utility property
  - Form a belief about $\theta(p')$ and the types he will attract $T(p')$
- $\theta(p')$: A lowest $\theta$ for which he can attract a seller
- $T(p')$: The types which are most likely to come
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No pooling
Equilibrium Characterization

- **Equilibrium:**
  - Sellers *direct* their search toward their preferred market
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Equilibrium Characterization

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- **A Mechanism Design Approach:** a market designer $\{\theta(\cdot), p(\cdot)\}$
  - **On the sellers' side:**
    - Promise a seller who reports his type $\hat{s} \in S$, with the pair $(p(\hat{s}), \theta(\hat{s}))$
  - **On the buyers' side:** *(In a matching environment)*
    - feasibility constraint (free-entry)
    - the recommended posting price $p$ must be optimal for buyers
Two Steps

- **Step 1:** Characterize the set of feasible mechanism \( \alpha = (p^\alpha, \theta^\alpha, V^\alpha) \in A \)
  - IC for sellers \( V^*(s) = \max \hat{s} V(\theta(\hat{s}), p(\hat{s}), s) \), IR, free entry
- **Proposition 1:** The pair of function \( \{\theta(\cdot), p(\cdot)\} \) satisfies sellers’ IC condition if and only if

\[
V^*(s) = \frac{1}{r + m(\theta^*(s))} \text{ is non-decreasing} \quad \text{(M)}
\]

\[
V^*(s) = \frac{s - c + p^*(s) \cdot m(\theta^*(s))}{r + m(\theta^*(s))} \quad \text{(ICFOC)}
\]

\[
= V^*(s_L) + \int_{s_L}^{s} V_s(\theta^*(\tilde{s}), \tilde{s}) d\tilde{s}
\]

(Milgrom and Segal (2002))
Basic Model

Buyers’ Optimality Condition

- **Step 2:** no profitable deviation by posting a new price, given $\alpha \in A$
- Lemma 1: pin down the type which is mostly likely to come
- The necessary condition for which $\alpha \in A$ can be decentralized
  - No pooling $\implies p(s) = \frac{s}{r} - \frac{k\theta(s)}{m\theta(s)}$
  - $V^*(s_L) = V^{FB}(s_L)$

Remarks:
- A least-cost separating equilibrium (Gale (1992), Guerrieri, et (2010)) only when buyers’ willingness to pay matches with sellers’ waiting preference
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Solution

- $\theta^*(s)$ is the solution to the differential equation $DE$

**Equilibrium $\theta^*(s)$**

- Initial condition: $\theta^*(s_L) = \theta^{FB}(s_L)$ & price schedule: $p^*(s) = \frac{s}{r} - \frac{k\theta^*(s)}{m(\theta^*(s))}$

- Downward Distorted market tightness (for better assets)
**Short Summary**

- **Endogenous market illiquidity**
  - A phenomenon of buyers’ strike
  - Liquidity works as a screening device (Guerrieri and Shimer (2011))
  - Independent of assumed distribution
  - $\theta^*(s)$ crucially depends on the range of underlying asset quality
Endogenous market illiquidity

A phenomenon of buyers’ strike
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Independent of assumed distribution
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Implications:

different severities of the adverse selection: \((y + \sigma_i s) \Rightarrow \theta^*(s; \sigma_i)\)
assets paying similar cash flow can differ significantly in their liquidity
capital reallocation is low when underlying dispersion is high

Can easily incorporate:

A general payoff function
Resale

\[ rJ(s) = s + \delta(V(s) - J(s)) \]

Heterogenous buyers
Obscure Motives for Sale

- Sellers have different liquidity position $c$ and it is unobserved by the market
- Two dimensions sellers’ type $(s_i, c_i)$
- The type who are willing to wait longer $\Rightarrow$ the more valuable assets
- The original screening mechanism must adjust
Obscure Motives for Sale

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- The original screening mechanism must adjust
- A Seller’s liquidity preference is determined by his flow payoff

$$x = s_i - c_i$$

- The key condition: is $E[s|x]$ monotonically increasing?
  - Yes $\implies$ can be nested in our general model $h(x) \equiv E[s|X = x]$
    - eg: $c_i \sim U[c_L, c_H]$
  - No $\implies$ Buyers’ willingness to pay doesn’t align with sellers’ liquidity preference
    - A semi-pooling equilibrium
An Example of Non-Monotonicity

- $s_i \in S$ and $c_i \in \{c_H, c_L\}$ and $P(c_H|s) = \lambda$

**Diagram:***

- **X-axis:** sellers’ value of holding the asset $x = s_i - c_i$
- **Y-axis:** how much is $x$ actually worth to buyers

**Equations:**

$E[s|X=x]$

$x + c_H$

$x + E[c]$

$x + c_L$

$s_{L-H}$

$s_{H-L}$

Buyers’ value $h: X \rightarrow R$
An Equilibrium with Fire Sale

- Constructing a semi-pooling EQ $x_1$

  $x < x_1$: a pooling market in which buyers get $E[s|x < x_1]$
  - Liquid market with price discount

  $x \geq x_1$: separated submarkets in which buyers get $x + c_L$
  - Liquidity distortion (as before)

- Apply Proposition 1 and Lemma 1
  - Key conditions: $V^*(x_1) = V^{FB}(x_1)$ and $V^*(x_L) \geq V^{FB}(c_H, s_L)$
Conclusion

- Two important dimensions in the trading market: \textit{Price} and \textit{Liquidity}
  - Standard Lemon Model: high types subsidize low types (pooling)
  - Basic Model: Liquidity Distortion (full separation)
  - Unobserved selling motives: Price + Liquidity distortion (semi-pooling)

- Different market distortion arise endogenously: Price discount? illiquid risk?
  - Sellers’ liquidity preference: asset quality (common value) + liquidity position (private value)
  - Buyers’ willingness to pay

- Jointly determination of price, liquidity and market segmentation
Off-Path

- Off-path: A buyer open up new submarkets by posting $p' \notin P^*$
- Market utility property (take $V^*(s)$ as given)
  - Belief about market tightness $\theta(p')$
    
    $$
    \theta(p', s) \equiv \inf \{ \tilde{\theta} > 0 : U(p', \tilde{\theta}, s) \geq V(s) \}
    $$
    
    $$
    \theta(p') \equiv \inf_s \theta(p', s)
    $$
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    \[
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- Expecting to attract the type \( s \)
  \[
  T(p') = \arg \inf_{s \in S} \{\theta(p', s)\}
  \]
  \[
  \mu(s|p') = 0 \text{ if } s \notin T(p')
  \]

There does not exist any \( p' \) such that \( U_b(p, \theta(p'), \mu_{p'}) > 0 \), where \( \theta(p') \) and \( \mu(s|p') \) satisfy restriction above above.
Substituting payment schedule \( p(s) = \frac{h(s)}{r} - \frac{k\theta(s)}{m(\theta(s))} \) into (ICFOC):

\[
V(s) = \frac{u(s) + \left( \frac{h(s)}{r} - \frac{k\theta}{m(\theta)} \right) m(\theta^*(s))}{r + m(\theta^*(s))} = V(s_l) + \int_{s_l}^{s} U_s(\theta^*(\tilde{s}), \tilde{s}) d\tilde{s}
\]

→ differential equation of \( \theta^*(s) \):

\[
\frac{d\theta^*(s)}{ds} = \frac{\frac{\theta h_s(s)}{\rho r} (r + m(\theta))}{[(h(s) - u(s)) + \frac{k}{\rho} ((\rho - 1)\theta - \frac{r\theta}{m(\theta)})]} \equiv f(\theta, s)
\]
Equilibrium

Definition

An equilibrium consists of $P^*$, a function of $V^* : S \rightarrow R_+$, a market tightness function $\theta(\cdot) : P \rightarrow [0, \infty]$, the conditional distribution of sellers in each submarket $\mu : S \times P^* \rightarrow [0, 1]$, such that the following conditions hold:

$E1$ (optimality for sellers): let

$$V^*(s) = \max \left\{ \frac{s - c}{r}, \max_{p' \in P^*} V(p', \theta(p'), s) \right\}$$

and for any $p \in P^*$ and $s \in S$, $\mu(s|p) > 0$ implies

$$p \in \arg \max_{p' \in P^* \cup \emptyset} V(p', \theta(p'), s)$$

$E2$ (optimality for buyers and free-entry): for any $p \in P^*$

$$0 = U_b(p, \theta(p), \mu_p)$$

;and there does not exist any $p' \in P$ such that $U_b(p', \theta(p'), \mu_{p'}) > 0$
The types which are most likely to come

Lemma

Given any mechanism $\alpha = (p^\alpha, \theta^\alpha, V^\alpha) \in A$, for any price $p' \notin \text{range of } p^\alpha$, the unique type $T(p')$ attracted by $p'$ is given by

$$T(p') = s^+ \cup s^-$$

where $s^- = \inf\{s \in S | p' < p^\alpha(s)\}$

$s^+ = \sup\{s \in S | p' > p^\alpha(s)\}$

![Diagram showing types and price relationship]
Constructing a Semi-pooling EQ

Reconstruct $h(\cdot)$ by bunching types: $h^*$ solves $\int_{\xi_L}^{\phi_1(h)} h(x) \, dx = h$
Also: \[ \mu(h) = \int_{\tilde{S}_L}^{\phi_1(h)} h(x) \, dx \geq h \]

The set of Eq: the marginal type \( x_1 \in (s_H - c_H, x^*) \)
The Marginal Type

The case when $\mu(h) > h$