On the Welfare Effect of Credit Arrangements

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Allocation in a monetary economy is typically inefficient:

- inflation distortion,
- liquidity shock.

Some forms of credit arrangement can help improve efficiency.

However, the effect of credit arrangement and its interaction with money are less well understood.
Credit Arrangement: Nominal

- Facilitate inter-temporal trade of money balances,
  - required record-keeping technology: record financial history,
  - function of credit: a means of borrowing,
  - credit creation subject to liquidity constraints,
  - relationship with money: complement the use of money.

\[ (1 + i)^t \]

\[ (1 + i)^{t+1} \]
Credit Arrangement: Real

- Facilitate inter-temporal trade of goods
  - required record-keeping technology: record goods transaction history,
  \[\Rightarrow\text{credit creation is not subject to liquidity constraints,}\]
  - relationship with money: substitute the use of money.

![Diagram of credit arrangement]

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Research Questions

- Does increasing access to credit always improve welfare, given inefficient monetary allocations?
  - Economize on the usage of cash and reduce the impact of inflation.
  - Distort allocation through redistribution.

- Does the structure of credit arrangement matter?
  - Functions of credit.
  - Relationship with money.
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What have we done?

Construct two economies that differ by their record-keeping technology.

1. Arrangement 1: record financial history → "nominal loan" economy, e.g., Berentsen et al. (2007) or Chiu and Meh (forthcoming).

Compare welfare among real loan, nominal loan and monetary economy.

- Identify different channels that affect welfare.
- Examine the robustness of the results.
Increase credit usage \textit{may not} necessarily increase welfare.

Two effects

- \textit{quantity effect} (intensive margin effect)
- \textit{price effect} (redistribution effect)

The structure of credit arrangement \textit{matters}.

Robustness:

- linear v.s. convex cost,
- competitive pricing v.s. mechanism design,
- perfect v.s. limited enforcement.
Environment I

- Time is discrete and continues forever. In each period: Market 1 (Day) + Market 2 (Night). Discount factor: $\beta$
- Buyers: a fraction of $\alpha$ have access to credit (permanent); a fraction $\pi$ wants to consume during the day (i.i.d. shock realized at the beginning of the day)
- Sellers: all have access to credit; all can produce but do not want to consume during the day
- Monetary authority: $M_+ = \gamma M$. New money as lump-sum transfer (or tax) to buyers.
Night:
- all agents can consume and produce good $x$
- quasilinear preferences: $v(x) - y$
- production: $f(y) = y$

Day:
- anonymity + lack of double coincidence of wants $\rightarrow$ money is essential
- fraction of $\alpha$ can access credit
- buyers (fraction of $\pi$): $u(q)$
- sellers: $-c(q)$
Credit: banks

- record only financial history → make loans (credit) or take deposits in the form of money → money is the only means of payment during the day
- open at night and the beginning of the day
Model – Nominal Loan with Perfect Enforcement

Buyers

**Night**

\[
W^b(m, \ell, k) = \max_{x, y, \hat{m}_+} \left\{ v(x) - y + \beta V^b(\hat{m}_+) \right\}
\]

subject to

\[
y + \phi(m + \tau) + (1 + i^k)k = x + \phi \hat{m}_+ + (1 + i^\ell)\ell
\]

at total income

\[
V^b(\hat{m}) = \max_{q, \ell, k} \left\{ \pi \left[ u(q) + W^b(\hat{m} + \ell - pq, -\ell, 0) \right] \right. \\
+ \left(1 - \pi\right) W^b(\hat{m} - k, 0, k) \}
\]

subject to

\[
pq \leq \hat{m} + \ell \text{ and } \ k \leq \hat{m}
\]
Night

\[ W^s(m) = \max_{x,y,\hat{m}_+} \{ v(x) - y + \beta V^s(\hat{m}_+) \} \]

s.t. \( y + \phi m = x + \phi \hat{m}_+ \)

Day

\[ V^s(\hat{m}) = \max_q [-c(q) + W^s(\hat{m} + pq)] \]
- Free entry to the banking sector: $i^k = i^\ell$
- Loan market clearing: $(1 - \pi)k = \pi\ell$ in a symmetric equilibrium
In equilibrium, $i^k = i^l = i = \frac{\gamma}{\beta} - 1$ and $(q^1, q^0, q^s)$ solve

$$\frac{u'(q^1)}{c'(q^s)} = 1 + i,$$
$$\frac{u'(q^0)}{c'(q^s)} = 1 + \frac{i}{\pi},$$

$$q^s = \pi \left[ \alpha q^1 + (1 - \alpha) q^0 \right].$$

Note

1. credit as a means of borrowing $q^1 > q^0$, quantity effect
2. competitive market, $q^1$ and $q^0$ are related through $c'(q^s)$, price effect
Credit:

- record goods transaction history → credit can be used as a means of payment directly → no demand for loans and no deposit
Model – Real Loan with Perfect Enforcement
Buyers and Sellers during the Day

A buyer

\[ V^b(\hat{m}) = \max_{q,d,\ell} \pi \left[ u(q) + W^b(\hat{m} - d, -\ell) \right] + (1 - \pi)W^b(\hat{m}) \]

s.t. \( pq = d + \ell \) and \( d \leq \hat{m} \)

A seller

\[ V^s(\hat{m}) = \max_{q^s} \left[ -c(q) + W^s(\hat{m} + d, \ell) \right] \]

s.t. \( pq = d + \ell \)

Assume that sellers receive the same portfolio \((d, \ell)\)
In equilibrium, \( i = \frac{\gamma}{\beta} - 1 \) and \((q^2, q^0, q^s)\) solve

\[
\frac{u'(q^2)}{c'(q^s)} = 1, \\
\frac{u'(q^0)}{c'(q^s)} = 1 + \frac{i}{\pi}, \\
q^s = \pi \left[ \alpha q^2 + (1 - \alpha)q^0 \right].
\]

Note

1. credit as a means of borrowing \( q^2 > q^0 \), quantity effect
2. competitive market, \( q^2 \) and \( q^0 \) are related through \( c'(q^s) \), price effect
Aggregate welfare

\[ \mathcal{W} = \frac{1}{1 - \beta} \left\{ 2v(x^*) - 2x^* + \left[ \alpha \pi u(q^1) + (1 - \alpha) \pi u(q^0) - c(q^s) \right] \right\} \]

where \( \pi \alpha q^1 + \pi (1 - \alpha) q^0 = q^s \)
Linear production function $c''(q) = 0$, 
$\mathcal{W}(\text{real}) > \mathcal{W}(\text{nominal}) > \mathcal{W}(\text{money})$.
Intuition: no price effect; when $i > 0$, $q^1 < q^2$ and $q^{01} = q^{02}$ →
quantity effect
Convex Cost

- Convex production function $c''(q) > 0$, results depend on $(i, \pi, \alpha)$

**Proposition**

If $c'(0) > 0$, then $\mathcal{W}'(\alpha) < 0$ when $0 < \alpha < 1$, $i > 0$ and $\pi$ is small.
Price Effect

Figure: Initial $\alpha$

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Price Effect

\[ \phi_p \]

\[ u'(q^m) \]

\[ q^m_1 \quad q^m_0 \]

\[ q^c \]

\[ q^c_1 \quad q^c_0 \]

\[ q^s \]

\[ q^s_0 \quad q^s_1 \]

\[ \alpha \uparrow \]

\[ \Delta \alpha \]

**Figure:** Increase \( \alpha \) by \( \Delta \alpha \)

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Price Effect

\[ \phi_p \]

\[ u'(q^m) \]

\[ \phi_p \]

\[ u'(q^c) \]

\[ \phi_p \]

\[ AS \ c'(q^s) \]

Welfare loss \( \approx \pi(1 - \alpha) \times \)

Welfare gain \( \approx \pi \Delta \alpha \times \)

Figure: Welfare Change
Nominal Loan Dominates

**Proposition**

If \( c'(0) > 0 \), then \( \mathcal{W}(\text{real}) < \mathcal{W}(\text{nominal}) < \mathcal{W}(\text{money}) \) when \( 0 < \alpha < 1 \), \( i > 0 \) and \( \pi \) is sufficiently small.

![Figure: Nominal v.s. Real](image-url)
Mechanism Design

- The above results depending on competitive pricing may not appear in a bilateral bargaining.
- Mechanism design approach à la Hu et al. (2009) and Rocheteau (2011).
  - Abstract from all pricing inefficiencies, and focus on monetary frictions.
- All types are publicly observable except money holdings.
- A mechanism is a mapping:
  - \((\hat{m}, \chi_\alpha, \chi_\pi, \chi_s) \mapsto (q, d, \ell) \in \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}\).
- Implementation concept: immune to individual deviation (Nash).
  - immune to cooperative deviation (can be the same as our competitive equilibrium, Jiang (2011)).
Focus on the mechanism that maximize the social welfare subject to incentive constraints by different agents.

Finding: real loan dominates.

\[ A \]

\[ B \]

\[ C \]

\[ i^1(\alpha) \]

\[ i^2(\alpha) \]

\[ \alpha \]

\[ 1 \]

\[ 0 \]

\[ \pi \left( \frac{\pi u(q^*)}{c(q^*)} - 1 \right) \]

\[ \frac{\pi u(q^*)}{c(q^*)} - 1 \]
Numerical Examples

- Numerical analysis: \( u(q) = \frac{1}{\rho} q^{\rho} \) and \( c(q) = \frac{A}{\eta} q^{\eta} \)
  
  Let \( \rho = 0.5, \eta = 2, A = 0.1 \)
  
  Benchmark: \( \pi = 0.5, \alpha = 0.5, \gamma = 1.1 \)

- black – pure monetary economy; blue – nominal loan economy; red – real loan economy
Consumption: agents who can access credit

![Graph showing consumption with nominal/real loan money growth rate](chart.png)

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Consumption: agents who cannot access credit

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Figure: Left: $\pi = 0.5$, Right: $\pi = 0.1$
Welfare in Nominal Loan, Real Loan and Monetary Economies

money growth rate

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Inflation: Different $\alpha$

Figure: Left: $\alpha = 0.9$, Right: $\alpha = 0.1$
Suppose that repayment of credit cannot be enforced. There exists an endogenous credit limit. Add $l \leq \bar{l}$ to the buyer’s problem in CM1.

In both credit arrangements, there exists three types of equilibrium:

1. pure monetary equilibrium,
2. equilibrium with money and constrained credit,
3. equilibrium with money and unconstrained credit.

Inflation relaxes the endogenous credit limit. Welfare implications are similar to before.
Conclusion

- Interaction of money and credit:
  - quantity effect (improve welfare),
  - price effect (lower welfare).

- Increasing access to credit is not necessary welfare improving.

- The form of credit arrangement matters.

- Robustness:
  - production technology matters,
  - pricing mechanism may (not) matter depending on the equilibrium concept.
  - lack of enforcement does not matter.

- Future work: endogenize credit usage.
  - Over use of credit may happen.
Imperfect Enforcement: Nominal Loan

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Imperfect Enforcement: Welfare Comparison I

welfare and access to credit

fraction of agents with credit

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Figure: Left: $\alpha = 0.9$, Right: $\alpha = 0.1$