Central Bank Liquidity Policies and Interbank Markets: A Quantitative Analysis

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Motivation

- During the recent financial crisis, a substantial amount of liquidity was injected into the banking system to ensure the smooth functioning of payment systems and interbank and other core funding markets.
- In many cases, policy makers were forced to carry out intervention and implement new policy frameworks without guidance provided by formal, quantitative economic models.

Objectives

- This project aims to develop a quantitative model of payment systems and interbank markets to perform theory based policy analysis on these issues.
- Identify equilibrium effects of different policies, shocks, and frictions on allocation and welfare
  - Interest rate (e.g. bounds and target of channel system)
  - Supply of settlement balances
  - Fundamental payment liquidity shocks
  - Frictions in interbank market

Model Framework

- Two Main Components:
  1. Channel Systems for Monetary Policy Implementation:
     e.g. Whitesell (2006), Berentsen and Monnet (2009), Berentsen and Waller (2010), Martin and Monnet (2011)
  2. OTC Interbank Markets:
     e.g. Ashcraft and Duffie (2007), Ashcraft, McAndrews, and Skeie (2009), Afonso and Lagos (2011)

- Banks manage liquidity by:
  1. managing payment inflows/outflows
  2. trading liquidity among themselves in the interbank market
  3. trading liquidity with the central bank standing facility

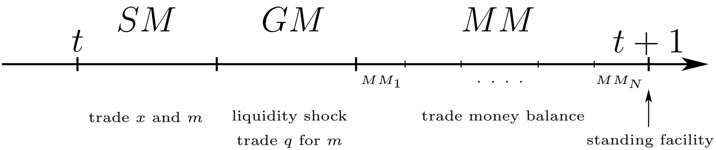
Model

Environment

- Infinite horizon: $t = 1, 2, 3, ...$
- Measure 1 of agents $i \in [0, 1]$
- Two goods: consumption good q and a settlement good x
- m: holding of reserve balances (normalized by total supply)
- β: discount factor

Environment

- 3 sub-periods:
  - SM: Settlement mkt to trade x and m → repay overnight loans
  - GM: Goods market to trade q and m → payment flows
  - MM: N rounds of money markets → interbank overnight loans
- End of period: central bank facilities to deposit/borrow



(1) (Centralized) Settlement Market

$$
\begin{aligned}
W(m_1, \ell, L) &= \max_{m_2, x} x + Z(m_2) \\
\text{s.t. } x + \phi\ell + \phi L &= \phi m_1 - \phi m_2 + \phi T,
\end{aligned}
$$

where

- m_1 : money brought to SM, m_2 : money brought to GM
- ℓ : outstanding interbank loan (lending if $\ell < 0$)
- L : outstanding central bank loan (lending if $L < 0$)
- T : transfer from the central bank (growth rate μ)
- ϕ : real price of money
- $Z(m_2)$: value function in GM

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Linear Preference $\Rightarrow$ W_m=-W_ℓ=-W_L=ϕ

(2) (Centralized) Goods Market

$$Z(m_2) = \int \{\max_{q_b, q_s} \varepsilon u(q_b) - c(q_s) + V_1(m_3)\} d\Omega_b(\varepsilon)$$
$$\text{s.t. } m_3 = m_2 - p(q_b - q_s)$$

where

ε : preference shock

$V_1(m_3)$: value function in the first round of MM

Note:

- $m_3 = m_2 - p(q_b - q_s) \in \mathbb{R}$ (there is no CIA constraint)
- money balance goes up/down according to net payment flow

(3) (Decentralized) Money Market

- Pairwise random matching with probability α
- Consider a match in the n-th money market:
  i borrows d dollars from j and repays ℓ in the next SM.
- Terms of trade (d, ℓ) determined by proportional bargaining:

$$\max_{d,\ell} S_i + S_j,$$

s.t.

$$S_i = S_j$$

borrower i's surplus: $S_i = V_{n+1}(m_i + d, \ell_i + \ell) - V_{n+1}(m_i, \ell_i)$

lender j's surplus: $S_j = V_{n+1}(m_j - d, \ell_j - \ell) - V_{n+1}(m_j, \ell_j)$

- Assumption: equal bargaining weight.

(4) Central Bank Lending Facility

Settlement at the end of a day:

$$V_{N+1}(m,\ell) = \beta W(0, \frac{\ell}{\mu}, -\frac{m}{\mu} \cdot (1 + r(m)))$$

where the overnight rate is

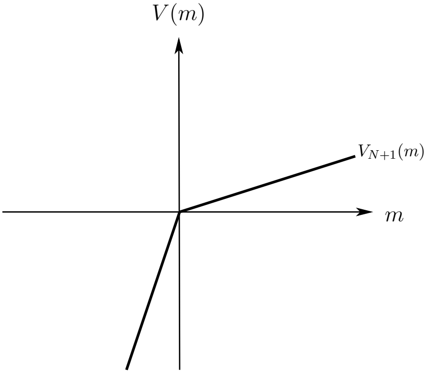
$$r(m) = \begin{cases} r^D & \text{if } m \geq 0 \\ r^L & \text{if } m < 0 \end{cases} .$$

Interest Policy: r^D, r^L

Solving the model

Value of Money at Settlement

$$V_{N+1}(m,\ell) = \bar{V}_{N+1}(m) - \beta\phi\ell/\mu = \begin{cases} \beta\phi/\mu[m(1+r^D)-\ell] & \text{if } m \geq 0 \\ \beta\phi/\mu[m(1+r^L)-\ell] & \text{if } m < 0 \end{cases}$$

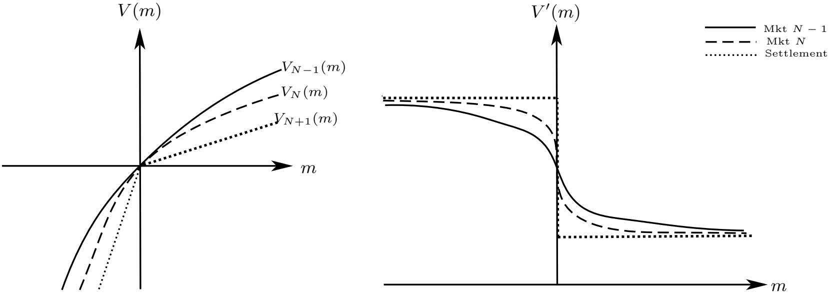


Value of Money in Money Markets

- For all $n = 1, ..., N+1$, the value function is

$$V_n(m, \ell) = \bar{V}_n(m) - \beta\phi\ell/\mu$$

- $\bar{V}_n(m)$ is strictly increasing and weakly concave.



Trading in Money Markets

Bargaining solution in money markets

$$d_n(m_i, m_j) = \frac{m_j - m_i}{2}$$

$$\ell_n(m_i, m_j) = \frac{\bar{V}_{n+1}(m_j) - \bar{V}_{n+1}(m_i)}{2\beta\phi/\mu}$$

Interest rate in a match

$$r_n(m_i, m_j) = \frac{\bar{V}_{n+1}(m_j) - \bar{V}_{n+1}(m_i)}{[m_j - m_i]\beta\phi/\mu}$$

is decreasing in the money holdings (m_i, m_j).

Dynamics of Value Functions and Money Distribution

Evolution of money distribution:

$$f_{n+1}(m) = (1-\alpha_n) f_n(m) + 2\alpha_n \int_{-\infty}^{\infty} f_n(\hat{m}) f_n(2m-\hat{m}) d\hat{m}.$$

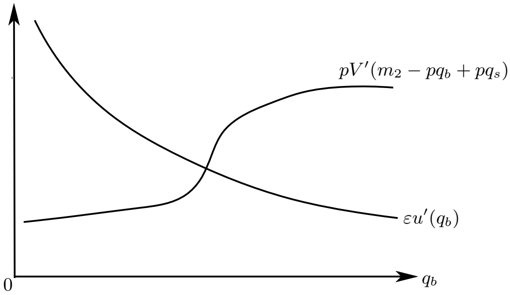
Evolution of value function:

$$\begin{aligned}\bar{V}_n(m) \quad &= \quad \left(1-\frac{\alpha_n}{2}\right)\bar{V}_{n+1}(m) + \\ &\frac{\alpha_n}{2}\int_{-\infty}^{\infty}[2\bar{V}_{n+1}(\frac{m+\hat{m}}{2}) - \bar{V}_{n+1}(\hat{m})] f_n(\hat{m}) d\hat{m}.\end{aligned}$$

Goods Market Trading

FOC in Centralized Good Market:

$$\varepsilon u'(q_b) = c'(q_s) = pV'(m_2 - pq_b + pq_s)$$



Goods Market Trading (Cont'd)

FOC in Centralized Good Market:

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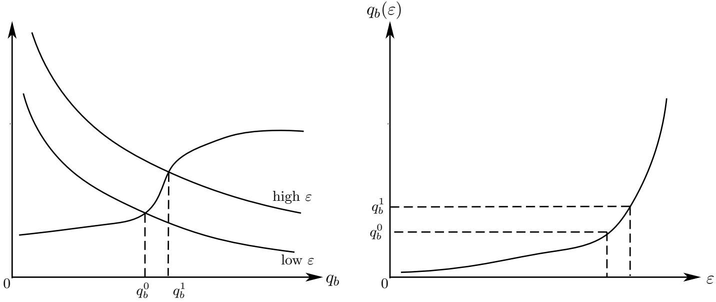
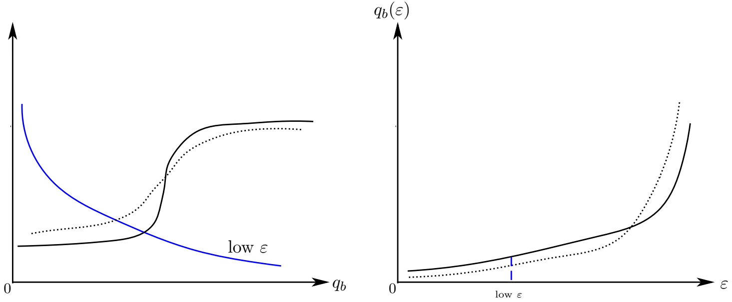


Goods Market Trading (Cont'd)

Changes in interest policies (r^D, r^L) or money market frictions (N, α) affect goods allocation.



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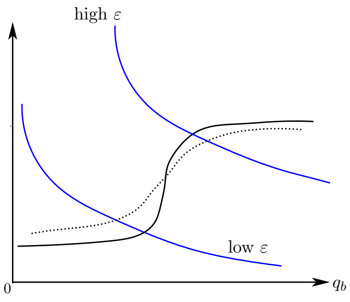
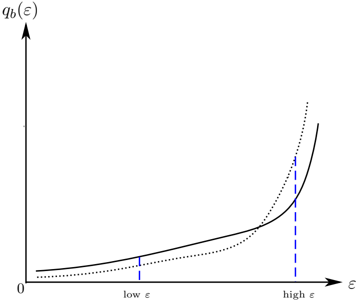




A Numerical Example

Parameter Values for Numerical Example

Preferences and Technology:

- $\beta = 0.9999$
- $u(q) = \frac{\varepsilon(365q_b)^{1-\sigma}}{1-\sigma}$
- $c(q) = \frac{(365q_s)^{1+\chi}}{1+\chi}$
- $\sigma = 0.2$, $\chi = 2$
- $\varepsilon \sim beta(2,2)$ on $[0.1, 2]$

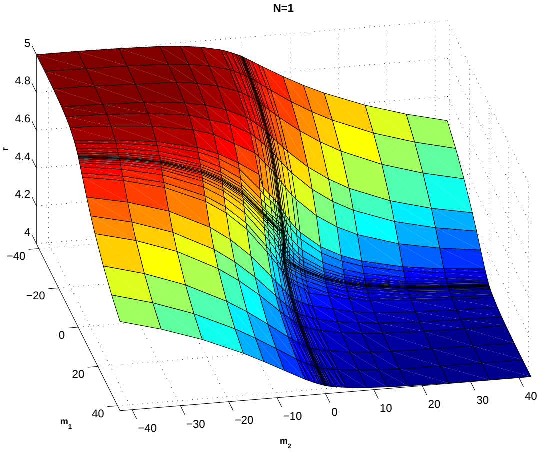
Money Market Frictions:

- $N = 4$, $\alpha = 1$

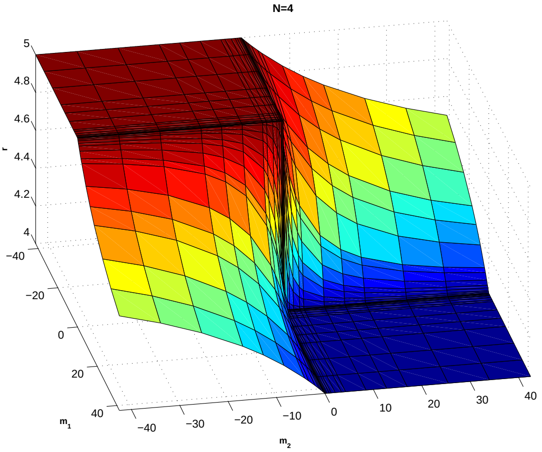
Interest Rate Policy:

- $r^D = 4\%$, $r^L = 5\%$

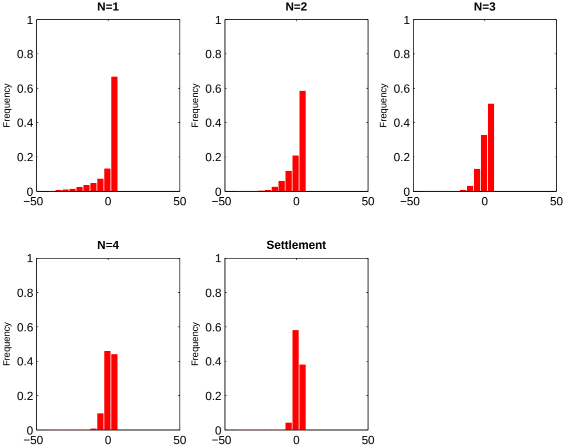
Bargaining Solution in Mkt 1



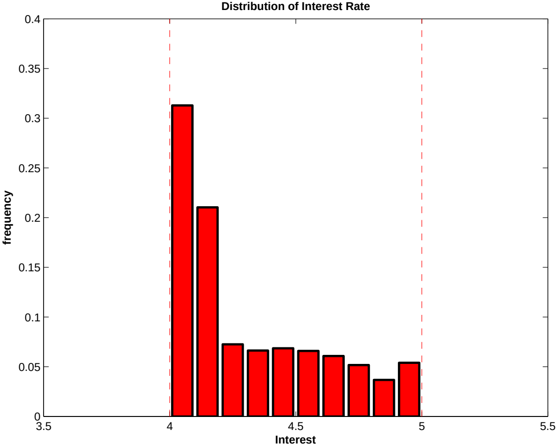
Bargaining Solution in Mkt N



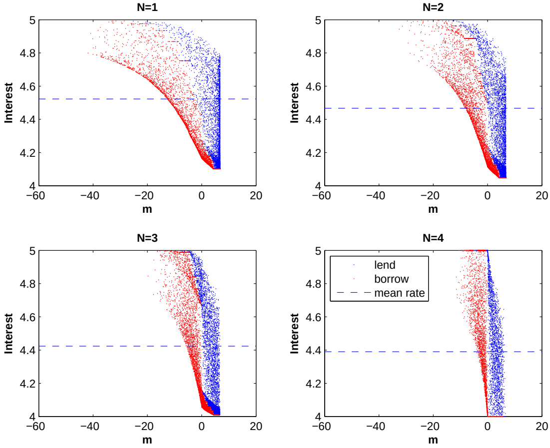
Evolution of Money Distribution



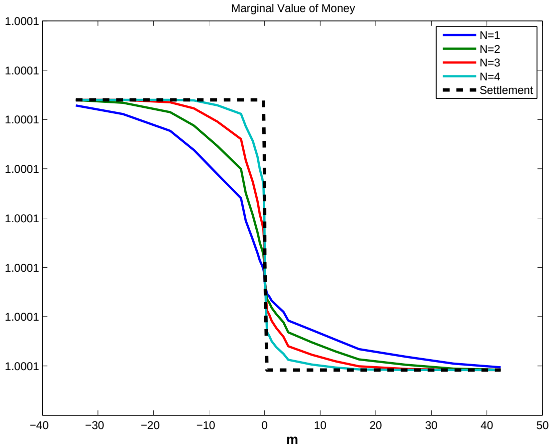
Interest Rates in Money Markets



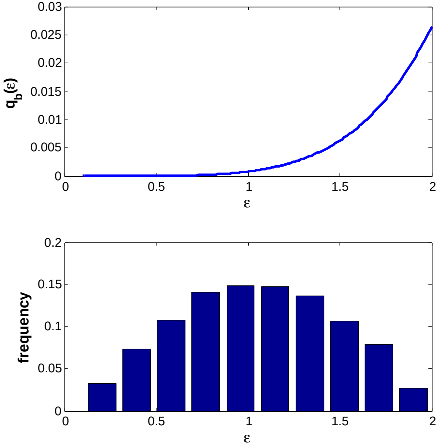
Interest Rates in Money Markets



Evolution of Money Demand



Trading in Goods Market



Equilibrium Outcome

Benchmark: $r^D = 4\%, r^L = 5\%, \alpha = 1, \varepsilon \sim [0.1, 2]$

	Benchmark
Goods Market	
Output	$3.0882e-3$
Welfare	1.3128
Money Market:	
Average Interest Rate	4.4530%
Std. Dev of m (before)	8.2808
Std. Dev of m (after)	2.0821
Liquidity Facility:	
Loan/Deposit Ratio	0.3040

Ex.1: Effects of Interest Rate Policy

Benchmark: $r^D = 4\%, r^L = 5\%$

	$r^L = 5.5\%$	$r^D = 4.5\%$
Goods Market		
Output	$\downarrow$	$\uparrow$
Welfare	$\downarrow$	$\uparrow$
Money Market:		
Average Interest Rate	$\uparrow$	$\uparrow$
Std. Dev of m (before)	$\downarrow$	$\uparrow$
Std. Dev of m (after)	$\downarrow$	$\uparrow$
Liquidity Facility:		
Loan/Deposit Ratio	$\downarrow$	$\uparrow$

Ex.2: Effects of Money Market Frictions

Benchmark: $\alpha = 1$

	$\alpha = 0.1$
Goods Market	
Output	$\downarrow$
Welfare	$\downarrow$
Money Market:	
Average Interest Rate	$\uparrow$
Std. Dev of m (before)	$\downarrow$
Std. Dev of m (after)	$\uparrow$
Liquidity Facility:	
Loan/Deposit Ratio	$\uparrow$

Ex.3: Effects of Liquidity Shocks

Benchmark: $\varepsilon \sim [0.1, 2]$

	$\varepsilon \sim [0.1, 2.5]$
Goods Market:	
Output	$\uparrow$
Welfare	$\uparrow$
Money Market:	
Average Interest Rate	$\uparrow$
Std. Dev of m (before)	$\uparrow$
Std. Dev of m (after)	$\uparrow$
Liquidity Facility:	
Loan/Deposit Ratio	$\uparrow$

Ex.4: Temporary Increase in Settlement Balance

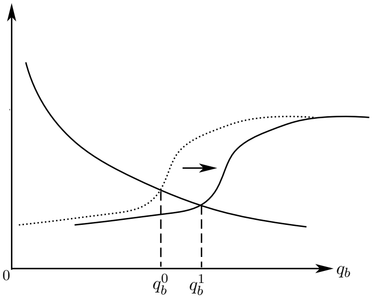
- An unanticipated lump-sum money transfer at the beginning of the GM, with commitment to re-absorb it in the next SM.
- Temporary increase in settlement balance, with inflation expectation unchanged.

Goods Market	
Output	$\uparrow$
Welfare	$\uparrow$
Money Market:	
Average Interest Rate	$\downarrow$
Std. Dev of m (before)	$\uparrow$
Std. Dev of m (after)	$\uparrow$
Liquidity Facility:	
Loan/Deposit Ratio	$\downarrow$

Ex.4: Temporary Increase in Settlement Balance (Cont'd)

FOC in Centralized Good Market:

$$\varepsilon u'(q_b) = c'(q_s) = pV'(m_2 - pq_b + pq_s)$$



Next Step

- Calibrate the model to Canadian Data
- Identify equilibrium effects of different forces on allocation and welfare during the crisis (Interest rate Policy, settlement balances, liquidity shocks, Frictions in interbank market)
- Perform experiments
  - Evaluate effects of counterfactual policies
  - Equilibrium responses to other shocks

Conclusions

- We develop a framework to quantitatively evaluate the effects of central bank liquidity policies during the recent crisis.
- Useful for decomposing the effects of policy and other fundamental changes.
- Useful for evaluating alternative policies.

Appendix

B. Extension: Introducing Collateral

Introduce an asset with return rate $R < 1/\beta$:

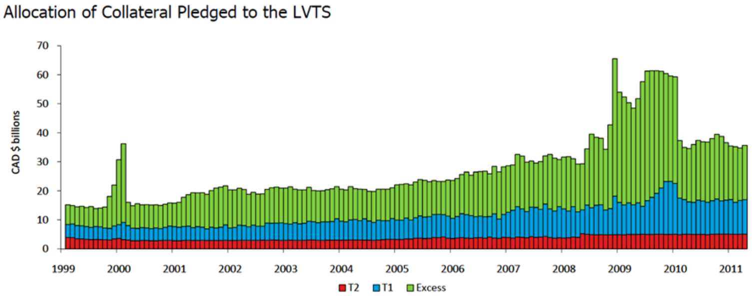
$$\begin{aligned} W(m_1, A_1, \ell, L) &= \max_{m_2, x} x + Z(m_2, A_2) \\ \text{s.t. } x + \phi\ell + \phi L + A_2 &= \phi m_1 - \phi m_2 + \phi T + A_1 R, \end{aligned}$$

Central bank loan subject to collateral constraint. Uncollaterallized overdraft subject to penalty rate ρ.

$$r(\ell, A) = r^L + \rho \max\{\ell - RA, 0\}$$

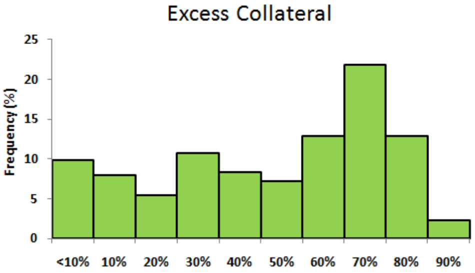
Collateral Constraint not Binding in LVTS

Overall, LVTS participating banks have large excess collateral holdings.



Collateral Constraint not Binding in LVTS (Cont'd)

Individual banks excess collateral, as a fraction of total collateral pledged to the LVTS.



- On average: banks have a 53% collateral buffer.
- 90% of time: banks have at least a 10% collateral buffer.

C. Extension: Introducing Default Risk

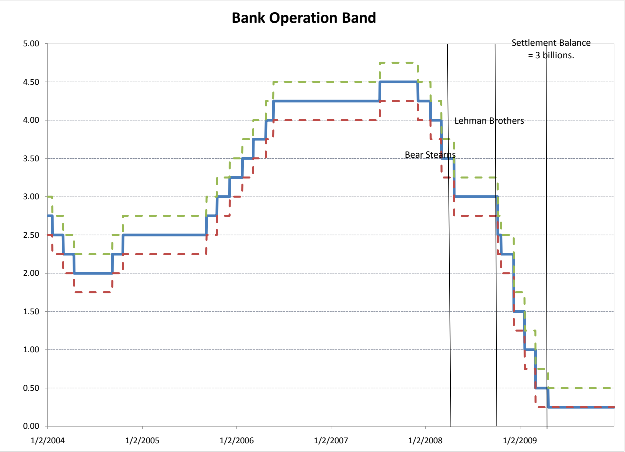
- Suppose an agent will die with probability $1 - \delta$ (replaced by new agents), implying a default on loans.
- So effective discount factor is $\hat{\beta} = \beta\delta$
- SM problem:

$$\begin{aligned} W(m_1, \ell, L) &= \max_{m_2, x} x + Z(m_2) \\ \text{s.t. } x + \phi\delta\ell + \phi L + A_2 &= \phi m_1 - \phi m_2 + \phi T + A_1 R, \end{aligned}$$

Application to Recent Crisis (Preliminary)

Policy Change: Interest Rate Channel

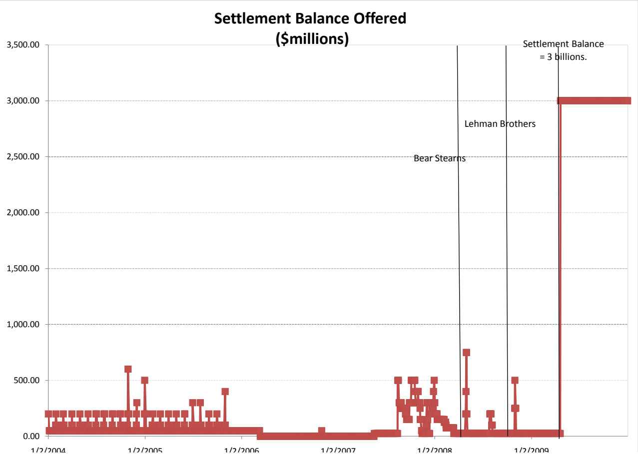
(back)



- Before Crisis: $r^L = r^D + 0.50\%$, target$= 0.5(r^L + r^D)$.
- During Crisis: $r^L = r^D + 0.25\%$, target$= r^D$.

Policy Change: Settlement Balance

(back)



- Before Crisis: net balance = 25 millions.
- During Crisis: net balance = 3 billions.

A. Interbank Loan Data

- Use Furfine algorithm (1999) to identify interbank overnight loans using LVTS payment transaction data between 15 banks from 2004 to 2010.
- Potential loans: a pair of payments between bank i and j
  - payment from i to j on day t greater than \$10 million and rounded to the nearest dollar
  - payment from j to i on day $t+1$, with a reasonable implicit overnight interest rate ($r^D - 0.1\% < r < r^L + 0.1\%$)
- Tie-breaking rule:
  - select the repayment with an implied overnight rate closer to the target.
  - "first-loan-to-first-repayment" algorithm

A. Interbank Loan Data (Contd.)

- Identified 52720 loans initiated between 4:00pm and 6:30pm over the sample period.

- Average 31 loans per day:
  - average size=$ 130 mil
  - min size=$ 10 mil
  - max size=$1.8 bil
  - average deviation from target = -0.89bps

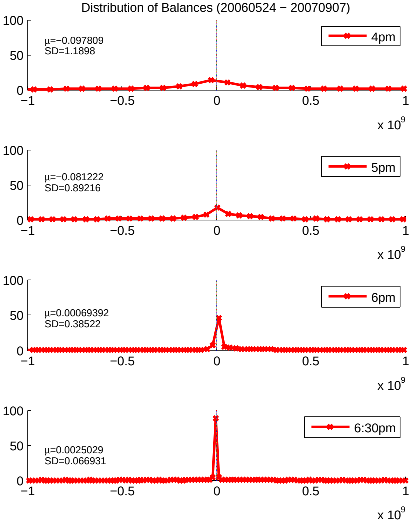
Interbank Activities

Liquidity Balance at 4pm

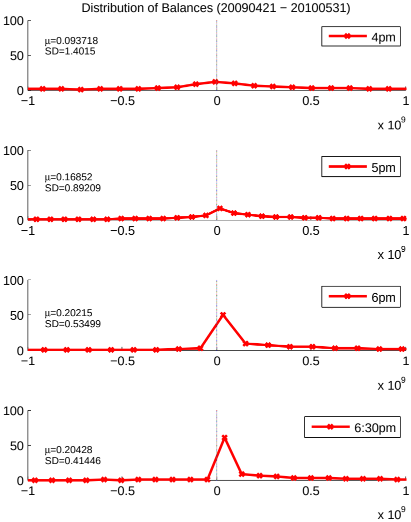
	10%	50%	90%	*Std.Dev*
Before Crisis	-1.218	-0.0057	0.9544	1.1898
During Crisis	-1.3387	0.0523	1.4153	1.4015

- Overall, dispersion of liquidity holdings increased.
- For 77% of banks, standard deviation of individual liquidity holdings increased. Average increase is 52%.

Evolution of Liquidity Distribution (Before Crisis)



Evolution of Liquidity Distribution (During Crisis)

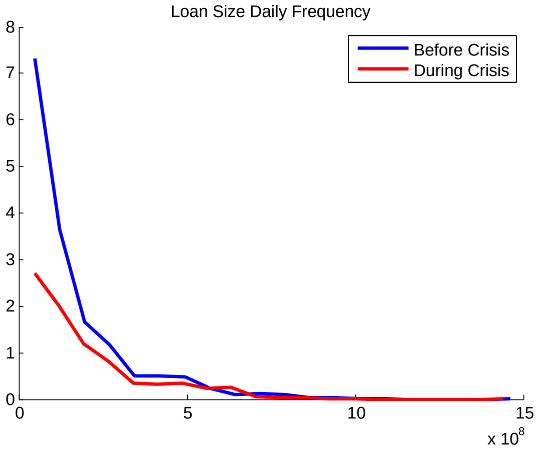


Interbank Lending (4pm-6:30pm)

Time	Lending (in number)	Activity (in value)	Avg. Spread (bps)
Before Crisis	36.67	5.02	0.01
During Crisis	24.97	3.53	-2.29

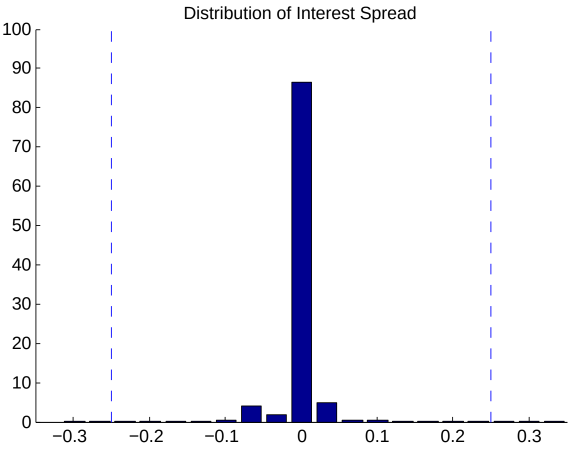
- Interbank lending activities dropped
  - by 32% in number
  - by 30% in value
- Average interest spread dropped by 2.3 bps.

Interbank Lending: Loan Size Distribution (4pm-6:30pm)



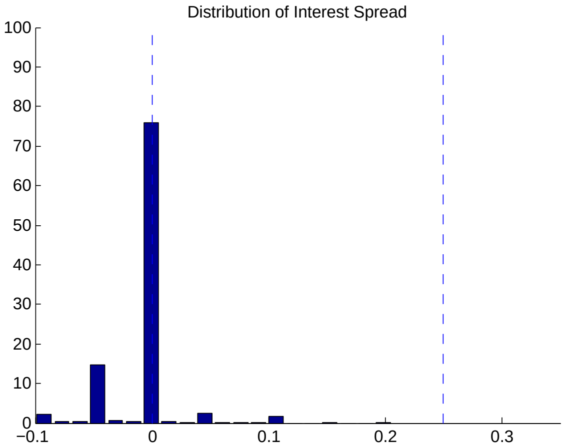
Lending of small loans dropped.

Interbank Lending: Interest Distribution



Before Crisis: cluster in the middle of the band

Interbank Lending: Interest Distribution



During Crisis: cluster at the bottom of the band

Central Bank Facilities (6:30pm)

Period	Deposit		Borrow	
	Freq.	Average Size	Freq.	Average Size
Pre-Crisis	90%	7 mil.	10%	37 mil.
Crisis	98%	213 mil.	2%	164 mil.

- Before Crisis:
  - Use deposit facility more often than lending facility.

Central Bank Facilities (6:30pm)

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	Freq.	Average Size	Freq.	Average Size
Pre-Crisis	90%	7 mil.	10%	37 mil.
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- Before Crisis:
  - Use deposit facility more often than lending facility.
- During Crisis:
  - Usage of deposit facility increased.
    Average size of deposit increased a lot.
  - Usage of lending facility dropped.
    But average size of loans increased a lot.

(back to conclusion)