Liquidity, Productivity and Efficiency

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Introduction

- Efficiency of private liquidity provision:
    - Liquidity $\equiv$ Pledgeability
    - Existing literature abstracts from:
        - Investment heterogeneity
        - Endogeneity of the assets' liquidity choice
    - Investment heterogeneity $\Rightarrow$ Endogenous liquidity choice
    - Farhi and Tirole (2011)

Introduction

- Key features:
    - Limited pledgeability:
        - Limited commitment
        - Limited enforcement
        - Moral hazard
    - Higher returns $\Rightarrow$ Lower pledgeability

Outline

- Model Setup
- Competitive Equilibrium
- Steady States
- Efficiency and Welfare

Model Setup: Preferences and Technology

- OLG economy of entrepreneurs
  - Young, middle aged and old
  - Unit measure of each for $t \geq 0$
  - Young receives $e > 0$ perishable consumption goods
  - Middle aged invest in:
    - Type i, return and pledgeability $(R_i, \theta_i R_i)$
    - Type ℓ, return and pledgeability $(R_\ell, \theta_\ell R_\ell)$
  - Consume only when old

Model Setup: Problem of the Middle Aged

- Middle aged at $t > 0$ solves:

$$\begin{aligned} \max_{i_t, x_{it}, x_{\ell t} \geq 0} \quad & R_i x_{it} + R_\ell x_{\ell t} - (1 + r_t) i_t \\ s.t. \quad & x_{it} + x_{\ell t} \leq (1 + r_{t-1}) e + i_t \\ & (1 + r_t) i_t \leq \theta_i R_i x_{it} + \theta_\ell R_\ell x_{\ell t} \end{aligned}$$

- x_{it} and $x_{\ell t}$ investment in type i and ℓ
- i_t, funds borrowed from young at t
- $(1 + r_{t-1})e$ is return to past investment

Assumption. $R_i > R_\ell > 1$ and $\theta_i R_i < \theta_\ell R_\ell < 1$.

Competitive Equilibrium

Lemma. If $1 + r_t < R_i$, borrowing constraint of the middle aged entrepreneurs binds at t.

- If borrowing constraint binds at $t > 0$, middle aged solves:

$$\max_{i_t} \quad \Lambda(\boldsymbol{\theta}, \boldsymbol{R}; r_t) i_t + \Phi(\boldsymbol{\theta}, \boldsymbol{R}; r_{t-1}) e$$

$$s.t. \quad \left(\frac{\theta_i R_i (1 + r_{t-1})}{1 + r_t - \theta_i R_i}\right) e \leq i_t \leq \left(\frac{\theta_\ell R_\ell (1 + r_{t-1})}{1 + r_t - \theta_\ell R_\ell}\right) e\,.$$

Original problem:

$$\max_{i_t, x_{it}, x_{\ell t} \geq 0} \quad R_i x_{it} + R_\ell x_{\ell t} - (1 + r_t) i_t$$

$$s.t. \quad x_{it} + x_{\ell t} \leq (1 + r_{t-1}) e + i_t$$

$$(1 + r_t) i_t \leq \theta_i R_i x_{it} + \theta_\ell R_\ell x_{\ell t}$$

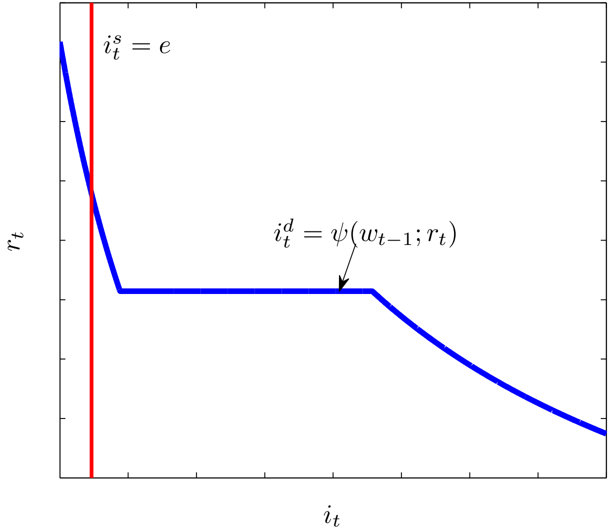
Competitive Equilibrium

- $\Lambda(\boldsymbol{\theta}, \boldsymbol{R}; r_t)$ is net return of increase in i_t while FC binds:

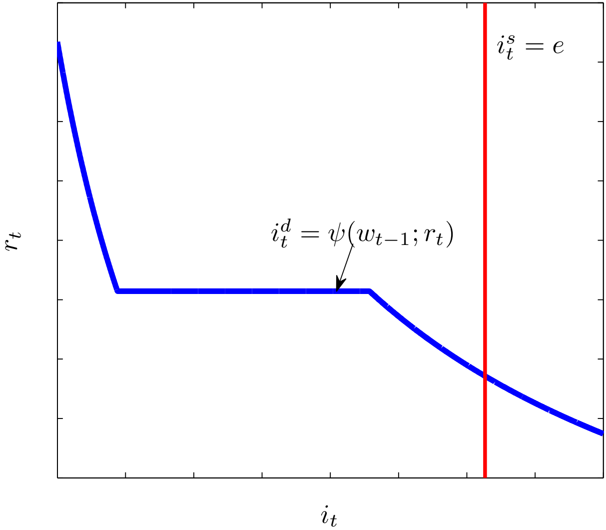
$$\Lambda(\boldsymbol{\theta}, \boldsymbol{R}; r_t) \equiv \left(\frac{(\theta_\ell - \theta_i) R_i R_\ell}{\theta_\ell R_\ell - \theta_i R_i} \right) - \left(\frac{(1-\theta_i) R_i - (1-\theta_\ell) R_\ell}{\theta_\ell R_\ell - \theta_i R_i} \right) (1 + r_t)$$

  - To increase i_t by $\epsilon > 0$
  - Borrowing constraint binds: $(1 + r_t) i_t = \theta_i R_i x_{it} + \theta_\ell R_\ell x_{\ell t}$
    - $x_{it} \downarrow$ by $\delta > 0$
    - $x_{\ell t} \uparrow$ by $\epsilon + \delta$
  - Investment size $\uparrow$, return $\downarrow$ $\Rightarrow$ net gain $\Lambda(\boldsymbol{\theta}, \boldsymbol{R}; r_t)\epsilon$
  - $\Lambda(\boldsymbol{\theta}, \boldsymbol{R}; r_t)$ can be positive or negative

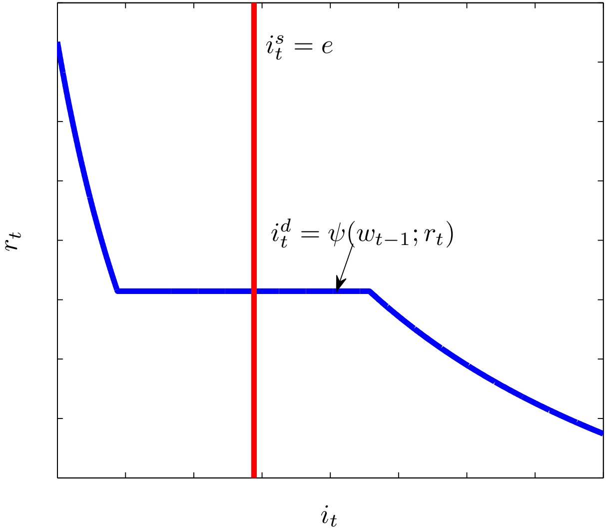
Demand and Supply of Fund



Demand and Supply of Fund



Demand and Supply of Fund



Competitive Equilibrium

Definition. Let $1 + r_\Lambda(\boldsymbol{\theta}, \boldsymbol{R})$ be the gross interest rate that makes $\Lambda(\boldsymbol{\theta}, \boldsymbol{R}; r_t)$ zero:

$$1 + r_\Lambda(\boldsymbol{\theta}, \boldsymbol{R}) \equiv \frac{(\theta_\ell - \theta_i) R_i R_\ell}{(1-\theta_i)R_i - (1-\theta_\ell)R_\ell}$$

- Market clearing $i_t = e$
- Market clearing + optimal policy of middle aged:

$$\begin{cases} 1 + r_t = \theta_\ell R_\ell (2 + r_{t-1}) & \text{If } \theta_\ell R_\ell (2 + r_{t-1}) < 1 + r_\Lambda(\boldsymbol{\theta}, \boldsymbol{R}) \\ 1 + r_t = \theta_i R_i (2 + r_{t-1}) & \text{If } \theta_i R_i (2 + r_{t-1}) > 1 + r_\Lambda(\boldsymbol{\theta}, \boldsymbol{R}) \\ 1 + r_t = 1 + r_\Lambda(\boldsymbol{\theta}, \boldsymbol{R}) & \text{Otherwise} \end{cases}$$

Competitive Equilibrium

Definition. A competitive equilibrium is a sequence of $\{i_t, x_{it}, x_{\ell t}, r_t\}_{t=0}^{\infty}$ and initial wealth $w_{-1} = (1 + r_{-1})e$ that maximize consumption for the old entrepreneur, satisfy the interest rate conditions above and $1 + r_t < R_i$ for all $t > 0$.

Steady State

- F set of $(\boldsymbol{\theta}, \boldsymbol{R})$ that:
  - $R_i > R_\ell > 1$ and $\theta_i R_i < \theta_\ell R_\ell < 1$
  - Financing constraint binds at SS
- $F = F_\ell \cup F_m \cup F_i$ such that at SS:
  - Only liquid in F_ℓ
  - Mix of both in F_m
  - Only illiquid in F_i

Steady State

Lemma. For any $(\boldsymbol{\theta}, \boldsymbol{R}) \in F$, there is a unique and stable steady state equilibrium where:

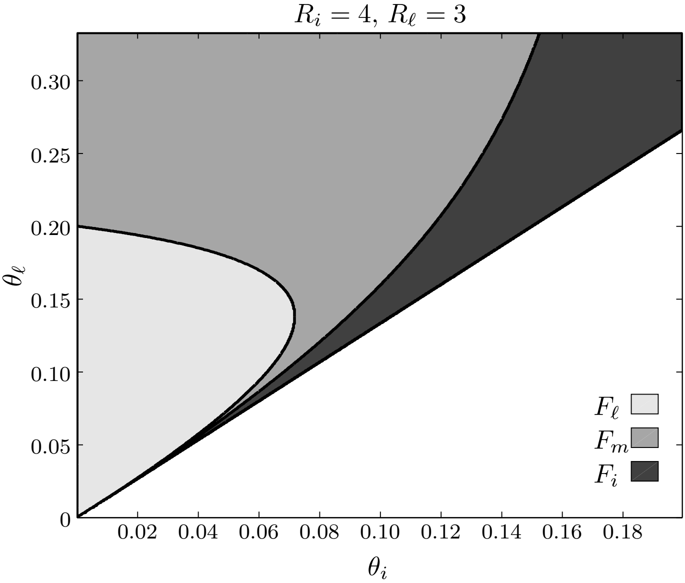
$$1 + r_\ell^{ss} = \frac{\theta_\ell R_\ell}{1 - \theta_\ell R_\ell} \text{ if } (\boldsymbol{\theta}, \boldsymbol{R}) \in F_\ell$$

$$1 + r_m^{ss} = 1 + r_\Lambda(\boldsymbol{\theta}, \boldsymbol{R}) \text{ if } (\boldsymbol{\theta}, \boldsymbol{R}) \in F_m$$

$$1 + r_i^{ss} = \frac{\theta_i R_i}{1 - \theta_i R_i} \text{ if } (\boldsymbol{\theta}, \boldsymbol{R}) \in F_i$$

Proposition. Given any $(\boldsymbol{\theta}, \boldsymbol{R}) \in F$, and an initial condition $1 + r_{-1} < R_i$, there exists a unique competitive equilibrium that converges to the steady state corresponding to $(\boldsymbol{\theta}, \boldsymbol{R})$ which is given in the above lemma.

Steady State



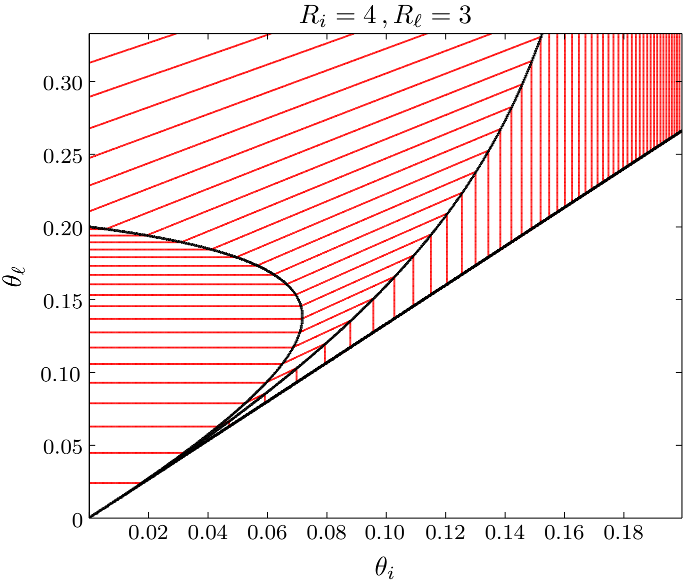
Steady State: Properties of Equilibria

- $1 + r^{ss}$: non-monotone in θ_i/ monotone in θ_ℓ
  - Investment demand $\uparrow$
  - Substitution effect

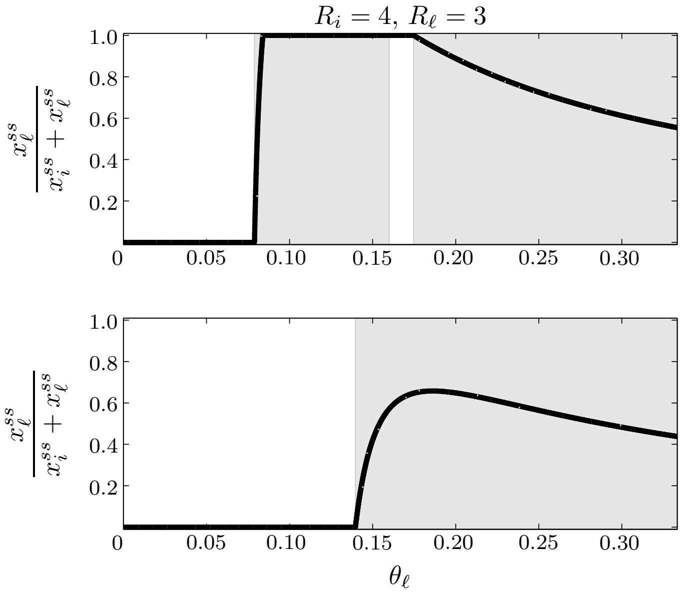
Decline in the real interest rates after 90s $\rightarrow \theta_i \uparrow$

- $\frac{x_\ell^{ss}}{x_i^{ss}+x_\ell^{ss}}$: monotone in θ_i/ non-monotone in θ_ℓ
  - Partial Eqm: $\theta_\ell \uparrow \Rightarrow \frac{x_\ell^{ss}}{x_i^{ss}+x_\ell^{ss}} \uparrow$ $\quad \theta_i \uparrow \Rightarrow \frac{x_\ell^{ss}}{x_i^{ss}+x_\ell^{ss}} \downarrow$
  - General Eqm: $\theta_\ell \uparrow \Rightarrow r^{ss} \uparrow$ $\quad \theta_i \uparrow \Rightarrow r^{ss} \downarrow\uparrow$

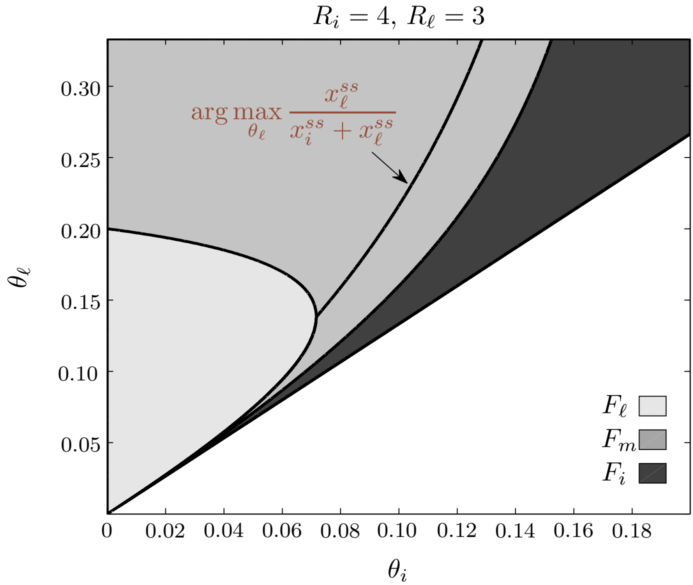
Steady State: Interest Rate Contour



Steady State: High and Low θ_i



Steady State: Properties of Equilibria



Efficiency and Welfare

Definition. A competitive equilibrium is **constrained Pareto efficient** if a social planner cannot make at least someone strictly better off while keeping all others at least as well off by a reallocation that respects the pledgeability constraint.

- Constrained Pareto efficient $\Leftrightarrow \{c_t^*, x_{it}^*, x_{\ell t}^*\}_{t=0}^{\infty}$ solves:

$$\max_{\{c_t, x_{it}, x_{\ell t}\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \lambda_t c_t$$

$$c_t + x_{it} + x_{\ell t} \leq R_i x_{it-1} + R_\ell x_{\ell t-1} + e$$

$$x_{it} + x_{\ell t} \leq \theta_i R_i x_{it-1} + \theta_\ell R_\ell x_{\ell t-1} + e$$

$\lambda_t > 0$ are Pareto weights.

Efficiency and Welfare: A Reallocation

- Let $(\boldsymbol{\theta}, \boldsymbol{R}) \in F_\ell \cup F_m \Rightarrow x_\ell^{ss} > 0$
- Planner reduces $(1 + r^{ss})e$, by $\delta > 0$:
  - FC slack $\Rightarrow x_i \uparrow$, by $\epsilon > 0 /\ x_\ell \downarrow$, by $\epsilon + \delta$
  - Maximum ϵ when FC binds:

$$\delta = (\theta_\ell R_\ell - \theta_i R_i)\epsilon + \theta_\ell R_\ell \delta$$

$$\epsilon = \frac{1 - \theta_\ell R_\ell}{\theta_\ell R_\ell - \theta_i R_i}\delta$$

Efficiency and Welfare: A Reallocation

- Change in utility:

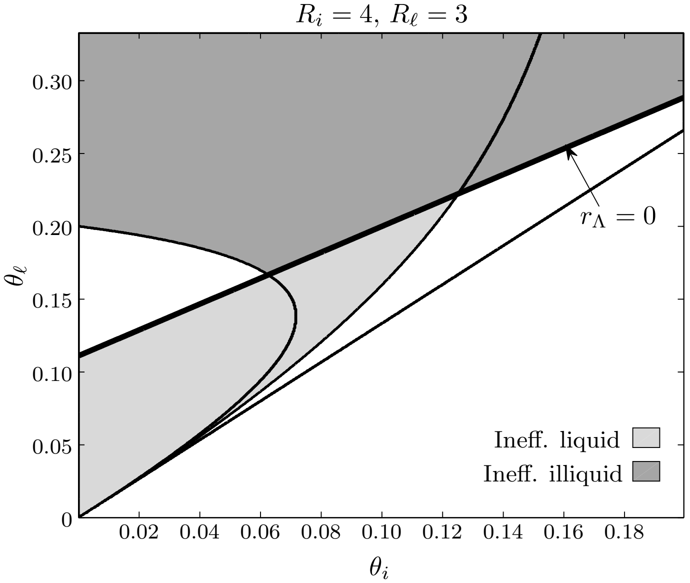
$$\Delta V^{ss} = \left(\frac{1-\theta_\ell R_\ell}{\theta_\ell R_\ell - \theta_i R_i} R_i - (1 + \frac{1-\theta_\ell R_\ell}{\theta_\ell R_\ell - \theta_i R_i}) R_\ell + 1 \right) \delta$$

Proposition. For any $(\boldsymbol{\theta}, \boldsymbol{R}) \in F_\ell \cup F_m$, one has $\Delta V^{ss} \geq 0$, and consequently the steady state is constrained Pareto inefficient, if and only if $r_\Lambda(\boldsymbol{\theta}, \boldsymbol{R}) \leq 0$.

- Outside steady state:

Lemma. Given $(\boldsymbol{\theta}, \boldsymbol{R}) \in F_\ell \cup F_m$, if $r_\Lambda(\boldsymbol{\theta}, \boldsymbol{R}) \leq 0$ any competitive equilibrium is constrained Pareto inefficient.

Inefficient Equilibria



Efficiency and Welfare: A Reallocation

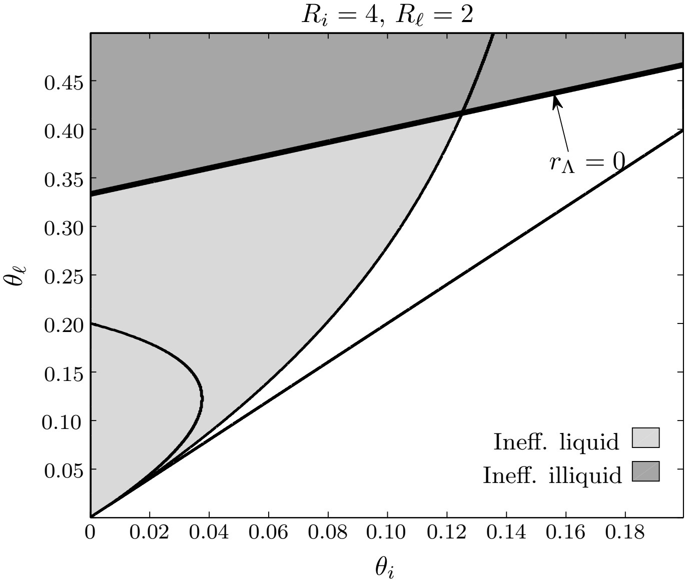
Proposition. All competitive equilibria in F_ℓ are constrained Pareto inefficient if and only if:

$$\frac{R_i - R_\ell}{R_\ell - 1} > 1$$

- Pareto frontier of F?

Proposition. Given $(\boldsymbol{\theta}, \boldsymbol{R}) \in F$, if $r_\Lambda(\boldsymbol{\theta}, \boldsymbol{R}) > 0$ any allocation satisfying resource and pledgeability constraints *with equality for* $t \geq 0$, including all competitive equilibria, is constrained Pareto efficient. Moreover, any allocation satisfying resource and pledgeability constraints *with equality for* $t \geq 0$ such that $x_{\ell t} = 0, t \geq T$ for some $T \geq 0$, including all equilibria in F_i, is constrained Pareto efficient.

Inefficient Equilibria



Efficiency and Welfare: A Reallocation

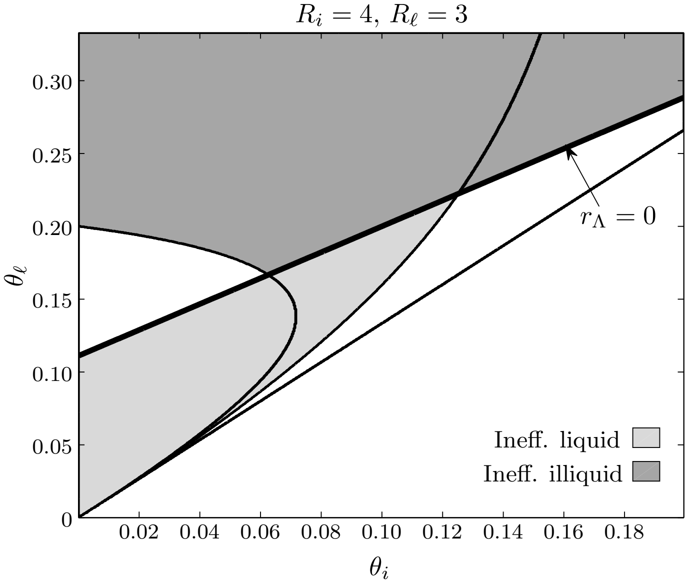
- Reinterpret OLG $\rightarrow$ 3 infinitely lived agents:

$$\begin{cases} \begin{array}{llllllr} 0 & 1 & 2 & 3 & 4 & \ldots & \\ (y, & m, & o), & (y, & m, & \ldots & \text{(i)} \\ m, & o), & (y, & m, & o), & \ldots & \text{(ii)} \\ o), & (y, & m, & o), & (y, & \ldots & \text{(iii)} \end{array} \end{cases}$$

- Discount factor $\beta \in (0, 1)$

Proposition. A competitive equilibrium for $(\boldsymbol{\theta}, \boldsymbol{R}) \in F_m \cup F_i$ such that $r_\Lambda(\boldsymbol{\theta}, \boldsymbol{R}) > 0$ reinterpreted as above is constrained Pareto inefficient, if $\bar{\beta}(\boldsymbol{\theta}, \boldsymbol{R}) \leq \beta < 1$ for some threshold $\bar{\beta}(\boldsymbol{\theta}, \boldsymbol{R}) \in (0, 1)$.

Inefficient Equilibria



Efficiency and Welfare: Regulated Economy

- Pareto reallocation $\leftrightarrow$ regulating $\alpha_t = \frac{x_{\ell t}}{x_{it}+x_{\ell t}}$:
    - Inefficiently liquid $\rightarrow \alpha_t \downarrow$
    - Inefficiently illiquid $\rightarrow \alpha_t \uparrow$

Proposition. A Pareto improving reallocation for a small enough $\delta > 0$ can be implemented by a regulation that sets $\alpha_t = \alpha_t^*$ for $t \geq T$ where $T \geq 0$ for the OLG as well as the reinterpreted economy. For inefficiently liquid equilibria, this regulation can result in an allocation on the Pareto frontier.

Efficiency and Welfare: Regulated Economy

- $(\boldsymbol{\theta}, \boldsymbol{R}) \in F_\ell$ and $w_{t-1} = (1 + r_{t-1})e$ low:

$$\max_{i_t, x_{it}, x_{\ell t} \geq 0} \quad R_i x_{it} + R_\ell x_{\ell t} - (1 + r_t) i_t$$

$$s.t. \quad x_{it} + x_{\ell t} \leq w_{t-1} + i_t$$

$$(1 + r_t) i_t \leq \theta_i R_i x_{it} + \theta_\ell R_\ell x_{\ell t}$$

Unregulated Eqm:

$$\begin{cases} i_t = e \\ 1 + r_t = \theta_\ell R_\ell (2 + r_{t-1}) \\ V = (1 - \theta_\ell) R_\ell (w_{t-1} + e) \end{cases}$$

Regulated Eqm ($\alpha_t = 0$):

$$\begin{cases} \widetilde{i}_t = e \\ 1 + \widetilde{r}_t = \theta_i R_i (2 + r_{t-1}) \\ \widetilde{V} = (1 - \theta_i) R_i (w_{t-1} + e) \end{cases}$$

- $\widetilde{V} > V \Leftrightarrow (1 - \theta_i) R_i > (1 - \theta_\ell) R_\ell$ ✔

Efficiency and Welfare: Regulated Economy

- $r_\Lambda(\boldsymbol{\theta}, \boldsymbol{R}) \leq 0 \Leftrightarrow \frac{(1-\theta_i)R_i}{1-\theta_i R_i} \geq \frac{(1-\theta_\ell)R_\ell}{1-\theta_\ell R_\ell}$

- Note that $V^{ss}(\boldsymbol{\theta}, \boldsymbol{R}) =$
  - $\frac{(1-\theta_i)R_i}{1-\theta_i R_i}$, investing only in type i
  - $\frac{(1-\theta_\ell)R_\ell}{1-\theta_\ell R_\ell}$, investing only in type ℓ

- Low $w_{t-1} = (1 + r_{t-1})e \rightarrow$ low $\boldsymbol{\theta}$

Efficiency and Welfare: Regulated Economy

- Inefficiently illiquid $\rightarrow \alpha \uparrow$
  - $r_t^{CE} > 0$, Pareto reallocation $\rightarrow r_t^{PO} > r_t^{CE} > 0$
  - More traditional
- Inefficiently liquid $\rightarrow \alpha \downarrow$
  - $r_t^{CE} \leq 0$, Pareto reallocation $\rightarrow r_t^{PO} < r_t^{CE} < 0$
  - Overinvestment $\rightarrow r_t^{CE} < 0\,,\;\; r_t^{CE} < r_t^{PO} < 0$
  - Sign of r_t^{CE} can be a misleading indicator

Conclusion

- Endogenous liquidity choice → Investment heterogeneity
- Positive implications:
  - Share of liquid type → non-monotone in θ_ℓ
  - Interest rate → non-monotone in θ_i
- Normative implications:
  - Endogenous liquidity → pecuniary externality → inefficiency
  - Inefficiently liquid / Inefficiently illiquid (more traditional)
  - Pareto reallocation ≡ regulating share of liquid investment
  - Sign of interest rate → misleading indicator of inefficiency
- Effect of bubbles or public liquidity?