Liquidity, Productivity and Efficiency

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Introduction

- Efficiency of private liquidity provision:
  - Liquidity $\equiv$ Pledgeability
  - Existing literature abstracts from:
    - Investment heterogeneity
    - Endogeneity of the assets’ liquidity choice
  - Investment heterogeneity $\Rightarrow$ Endogenous liquidity choice
  - Farhi and Tirole (2011)
Introduction

- Key features:
  - Limited pledgeability:
    - Limited commitment
    - Limited enforcement
    - Moral hazard
  - Higher returns $\Rightarrow$ Lower pledgeability
Outline

• Model Setup

• Competitive Equilibrium

• Steady States

• Efficiency and Welfare
Model Setup: Preferences and Technology

- OLG economy of entrepreneurs
  - Young, middle aged and old
  - Unit measure of each for $t \geq 0$
  - Young receives $e > 0$ perishable consumption goods
  - Middle aged invest in:
    - Type $i$, return and pledgeability $(R_i, \theta_i R_i)$
    - Type $\ell$, return and pledgeability $(R_\ell, \theta_\ell R_\ell)$
  - Consume only when old
Model Setup: Problem of the Middle Aged

- Middle aged at $t > 0$ solves:

$$\max_{i_t, x_{it}, x_{\ell t} \geq 0} R_i x_{it} + R_\ell x_{\ell t} - (1 + r_t) i_t$$

s.t.

$$x_{it} + x_{\ell t} \leq (1 + r_{t-1}) e + i_t$$

$$(1 + r_t) i_t \leq \theta_i R_i x_{it} + \theta_\ell R_\ell x_{\ell t}$$

- $x_{it}$ and $x_{\ell t}$ investment in type $i$ and $\ell$

- $i_t$, funds borrowed from young at $t$

- $(1 + r_{t-1})e$ is return to past investment

**Assumption.** $R_i > R_\ell > 1$ and $\theta_i R_i < \theta_\ell R_\ell < 1$. 
Lemma. If $1 + r_t < R_i$, borrowing constraint of the middle aged entrepreneurs binds at $t$.

- If borrowing constraint binds at $t > 0$, middle aged solves:

$$
\max_{i_t} \quad \Lambda(\theta, R; r_t)i_t + \Phi(\theta, R; r_{t-1})e \\
\text{s.t.} \quad \left( \frac{\theta_i R_i (1 + r_{t-1})}{1 + r_t - \theta_i R_i} \right) e \leq i_t \leq \left( \frac{\theta_\ell R_\ell (1 + r_{t-1})}{1 + r_t - \theta_\ell R_\ell} \right) e .
$$

Original problem:

$$
\max_{i_t, x_{it}, x_{\ell t} \geq 0} \quad R_i x_{it} + R_\ell x_{\ell t} - (1 + r_t)i_t \\
\text{s.t.} \quad x_{it} + x_{\ell t} \leq (1 + r_{t-1})e + i_t \\
(1 + r_t)i_t \leq \theta_i R_i x_{it} + \theta_\ell R_\ell x_{\ell t}
$$
Competitive Equilibrium

- $\Lambda(\theta, R; r_t)$ is net return of increase in $i_t$ while FC binds:

$$
\Lambda(\theta, R; r_t) \equiv \left( \frac{(\theta_\ell - \theta_i)R_i R_\ell}{\theta_\ell R_\ell - \theta_i R_i} \right) - \left( \frac{(1 - \theta_i)R_i - (1 - \theta_\ell)R_\ell}{\theta_\ell R_\ell - \theta_i R_i} \right) (1 + r_t)
$$

- To increase $i_t$ by $\epsilon > 0$

- Borrowing constraint binds: $(1 + r_t)i_t = \theta_i R_i x_{it} + \theta_\ell R_\ell x_{\ell t}$
  - $x_{it} \downarrow$ by $\delta > 0$
  - $x_{\ell t} \uparrow$ by $\epsilon + \delta$

- Investment size $\uparrow$, return $\downarrow \Rightarrow$ net gain $\Lambda(\theta, R; r_t) \epsilon$

- $\Lambda(\theta, R; r_t)$ can be positive or negative
Demand and Supply of Fund

\[ i_t = \psi_1(\psi_2(w_{t-1}; r_t)) \]
Demand and Supply of Fund

\[ i_t^d = \psi(w_{t-1}; r_t) \]

\[ i_t^s = e \]
Demand and Supply of Fund

\[ r_t \]

\[ i_t^d = \psi(w_{t-1}; r_t) \]

\[ i_t^s = e \]
Competitive Equilibrium

**Definition.** Let $1 + r_\Lambda(\theta, R)$ be the gross interest rate that makes $\Lambda(\theta, R; r_t)$ zero:

$$1 + r_\Lambda(\theta, R) \equiv \frac{(\theta_\ell - \theta_i)R_iR_\ell}{(1 - \theta_i)R_i - (1 - \theta_\ell)R_\ell}$$

- Market clearing $i_t = e$
- Market clearing + optimal policy of middle aged:

$$\begin{cases} 1 + r_t = \theta_\ell R_\ell (2 + r_{t-1}) & \text{If } \theta_\ell R_\ell (2 + r_{t-1}) < 1 + r_\Lambda(\theta, R) \\ 1 + r_t = \theta_i R_i (2 + r_{t-1}) & \text{If } \theta_i R_i (2 + r_{t-1}) > 1 + r_\Lambda(\theta, R) \\ 1 + r_t = 1 + r_\Lambda(\theta, R) & \text{Otherwise} \end{cases}$$
**Definition.** A competitive equilibrium is a sequence of 
\[ \{i_t, x_{it}, x_{\ell t}, r_t\}_{t=0}^{\infty} \] and initial wealth \( w_{-1} = (1 + r_{-1})e \) that maximize consumption for the old entrepreneur, satisfy the interest rate conditions above and \( 1 + r_t < R_i \) for all \( t > 0 \).
Steady State

- $F$ set of $(\theta, R)$ that:
  - $R_i > R_\ell > 1$ and $\theta_i R_i < \theta_\ell R_\ell < 1$
  - Financing constraint binds at SS

- $F = F_\ell \cup F_m \cup F_i$ such that at SS:
  - Only liquid in $F_\ell$
  - Mix of both in $F_m$
  - Only illiquid in $F_i$
Lemma. For any $\left(\theta, R\right) \in F$, there is a unique and stable steady state equilibrium where:

$$1 + r_{ss}^\ell = \frac{\theta \ell R \ell}{1 - \theta \ell R \ell} \text{ if } (\theta, R) \in F\ell$$

$$1 + r_{ss}^m = 1 + r_A(\theta, R) \text{ if } (\theta, R) \in F_m$$

$$1 + r_{ss}^i = \frac{\theta i R i}{1 - \theta i R i} \text{ if } (\theta, R) \in F_i$$

Proposition. Given any $\left(\theta, R\right) \in F$, and an initial condition $1 + r_{-1} < R_i$, there exists a unique competitive equilibrium that converges to the steady state corresponding to $\left(\theta, R\right)$ which is given in the above lemma.
$R_i = 4$, $R_\ell = 3$
Steady State: Properties of Equilibria

- $1 + r^{ss}$: non-monotone in $\theta_i$/ monotone in $\theta_\ell$
  - Investment demand $\uparrow$
  - Substitution effect

  Decline in the real interest rates after 90s $\rightarrow \theta_i \uparrow$

- $\frac{x^{ss}_\ell}{x^{ss}_i + x^{ss}_\ell}$: monotone in $\theta_i$/ non-monotone in $\theta_\ell$
  - Partial Eqm: $\theta_\ell \uparrow \Rightarrow \frac{x^{ss}_\ell}{x^{ss}_i + x^{ss}_\ell} \uparrow$  \hspace{1cm} $\theta_i \uparrow \Rightarrow \frac{x^{ss}_\ell}{x^{ss}_i + x^{ss}_\ell} \downarrow$
  - General Eqm: $\theta_\ell \uparrow \Rightarrow r^{ss} \uparrow$  \hspace{1cm} $\theta_i \uparrow \Rightarrow r^{ss} \downarrow \uparrow$
Steady State: Interest Rate Contour

\[ R_i = 4, \ R_\ell = 3 \]
Steady State: High and Low $\theta_i$

\[ R_i = 4, \quad R_\ell = 3 \]

The diagrams illustrate the behavior of $x^{ss}_i$, $x^{ss}_\ell$, and $x_i^{ss} + x_\ell^{ss}$ as a function of $\theta_\ell$. The shaded areas represent different regions of the parameter space.
Steady State: Properties of Equilibria

\[ R_i = 4, \ R_\ell = 3 \]

\[ \arg\max_{\theta_\ell} \frac{x_\ell^{ss}}{x_i^{ss} + x_\ell^{ss}} \]

\[ F_\ell \]

\[ F_m \]

\[ F_i \]
Efficiency and Welfare

**Definition.** A competitive equilibrium is **constrained Pareto efficient** if a social planner cannot make at least someone strictly better off while keeping all others at least as well off by a reallocation that respects the pledgeability constraint.

- Constrained Pareto efficient $\iff \{c_t^*, x_{it}^*, x_{\ell t}^*\}_{t=0}^\infty$ solves:

$$
\max_{\{c_t, x_{it}, x_{\ell t}\}_{t=0}^\infty} \sum_{t=0}^\infty \lambda_t c_t
$$

$$
c_t + x_{it} + x_{\ell t} \leq R_i x_{it-1} + R_{\ell} x_{\ell t-1} + e
$$

$$
x_{it} + x_{\ell t} \leq \theta_i R_i x_{it-1} + \theta_{\ell} R_{\ell} x_{\ell t-1} + e
$$

$\lambda_t > 0$ are Pareto weights.
Efficiency and Welfare: A Reallocation

• Let \((\theta, R) \in F_{\ell} \cup F_m \Rightarrow x_{ss}^\ell > 0\)

• Planner reduces \((1 + r^{ss})e\), by \(\delta > 0\):
  
  ○ FC slack \(\Rightarrow x_i \uparrow\), by \(\epsilon > 0\)/ \(x_{\ell} \downarrow\), by \(\epsilon + \delta\)

  ○ Maximum \(\epsilon\) when FC binds:

\[
\delta = (\theta_{\ell} R_{\ell} - \theta_i R_i)\epsilon + \theta_{\ell} R_{\ell}\delta
\]

\[
\epsilon = \frac{1 - \theta_{\ell} R_{\ell}}{\theta_{\ell} R_{\ell} - \theta_i R_i}\delta
\]
Efficiency and Welfare: A Reallocation

• Change in utility:

\[ \Delta V^{ss} = \left( \frac{1 - \theta \ell R \ell}{\theta \ell R \ell - \theta i R i} R i - (1 + \frac{1 - \theta \ell R \ell}{\theta \ell R \ell - \theta i R i}) R \ell + 1 \right) \delta \]

**Proposition.** For any \((\theta, R) \in F_\ell \cup F_m\), one has \(\Delta V^{ss} \geq 0\), and consequently the steady state is constrained Pareto inefficient, if and only if \(r_\Lambda(\theta, R) \leq 0\).

• Outside steady state:

**Lemma.** Given \((\theta, R) \in F_\ell \cup F_m\), if \(r_\Lambda(\theta, R) \leq 0\) any competitive equilibrium is constrained Pareto inefficient.
Inefficient Equilibria

\[ R_i = 4, \ R_\ell = 3 \]

\( \theta_i \) vs. \( \theta_\ell \)

- Ineff. liquid
- Ineff. illiquid

\( r_\Lambda = 0 \)
Efficiency and Welfare: A Reallocation

**Proposition.** All competitive equilibria in $F_\ell$ are constrained Pareto inefficient if and only if:

$$\frac{R_i - R_\ell}{R_\ell - 1} > 1$$

- Pareto frontier of $F$?

**Proposition.** Given $(\theta, R) \in F$, if $r_\Lambda(\theta, R) > 0$ any allocation satisfying resource and pledgeability constraints with equality for $t \geq 0$, including all competitive equilibria, is constrained Pareto efficient. Moreover, any allocation satisfying resource and pledgeability constraints with equality for $t \geq 0$ such that $x_{\ell t} = 0, t \geq T$ for some $T \geq 0$, including all equilibria in $F_i$, is constrained Pareto efficient.
Inefficient Equilibria

\[ R_i = 4, \ R_\ell = 2 \]
Efficiency and Welfare: A Reallocation

- Reinterpret OLG → 3 infinitely lived agents:

\[
\begin{align*}
0 & \quad 1 & \quad 2 & \quad 3 & \quad 4 & \quad \ldots \\
(y, m, o), \quad & (y, m, \ldots) \quad (i) \\
m, o), \quad & (y, m, o), \quad \ldots \quad (ii) \\
o), \quad & (y, m, o), \quad (y, \ldots) \quad (iii)
\end{align*}
\]

- Discount factor \( \beta \in (0, 1) \)

**Proposition.** A competitive equilibrium for \((\theta, R) \in F_m \cup F_i\) such that \(r_A(\theta, R) > 0\) reinterpreted as above is constrained Pareto inefficient, if \(\bar{\beta}(\theta, R) \leq \beta < 1\) for some threshold \(\bar{\beta}(\theta, R) \in (0, 1)\).
Inefficient Equilibria

\[ R_i = 4, \ R_\ell = 3 \]

\[ r_\Lambda = 0 \]
Efficiency and Welfare: Regulated Economy

- Pareto reallocation $\leftrightarrow$ regulating $\alpha_t = \frac{x_{\ell t}}{x_{it} + x_{\ell t}}$:
  - Inefficiently liquid $\implies \alpha_t \downarrow$
  - Inefficiently illiquid $\implies \alpha_t \uparrow$

**Proposition.** A Pareto improving reallocation for a small enough $\delta > 0$ can be implemented by a regulation that sets $\alpha_t = \alpha_t^*$ for $t \geq T$ where $T \geq 0$ for the OLG as well as the reinterpreted economy. For inefficiently liquid equilibria, this regulation can result in an allocation on the Pareto frontier.
Efficiency and Welfare: Regulated Economy

- \((\theta, R) \in F_\ell\) and \(w_{t-1} = (1 + r_{t-1})e\) low:

\[
\max_{i_t, x_{it}, x_{\ell t} \geq 0} R_i x_{it} + R_\ell x_{\ell t} - (1 + r_t)i_t
\]

s.t.
\[
x_{it} + x_{\ell t} \leq w_{t-1} + i_t
\]

\[
(1 + r_t)i_t \leq \theta_i R_i x_{it} + \theta_\ell R_\ell x_{\ell t}
\]

Unregulated Eqm:

\[
\begin{align*}
i_t &= e \\
1 + r_t &= \theta_\ell R_\ell (2 + r_{t-1}) \\
V &= (1 - \theta_\ell) R_\ell (w_{t-1} + e)
\end{align*}
\]

Regulated Eqm \((\alpha_t = 0)\):

\[
\begin{align*}
\tilde{i}_t &= e \\
1 + \tilde{r}_t &= \theta_i R_i (2 + r_{t-1}) \\
\tilde{V} &= (1 - \theta_i) R_i (w_{t-1} + e)
\end{align*}
\]

- \(\tilde{V} > V \iff (1 - \theta_i) R_i > (1 - \theta_\ell) R_\ell\)

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Efficiency and Welfare: Regulated Economy

- $r_{A}(\theta, R) \leq 0 \iff \frac{(1-\theta_i)R_i}{1-\theta_i R_i} \geq \frac{(1-\theta_\ell)R_\ell}{1-\theta_\ell R_\ell}$

- Note that $V^{ss}(\theta, R) =$
  - $\frac{(1-\theta_i)R_i}{1-\theta_i R_i}$, investing only in type $i$
  - $\frac{(1-\theta_\ell)R_\ell}{1-\theta_\ell R_\ell}$, investing only in type $\ell$

- Low $w_{t-1} = (1 + r_{t-1})e \to$ low $\theta$
Efficiency and Welfare: Regulated Economy

- Inefficiently illiquid $\rightarrow \alpha \uparrow$
  - $r_t^{CE} > 0$, Pareto reallocation $\rightarrow r_t^{PO} > r_t^{CE} > 0$
  - More traditional

- Inefficiently liquid $\rightarrow \alpha \downarrow$
  - $r_t^{CE} \leq 0$, Pareto reallocation $\rightarrow r_t^{PO} < r_t^{CE} < 0$
  - Overinvestment $\rightarrow r_t^{CE} < 0$, $r_t^{CE} < r_t^{PO} < 0$
  - Sign of $r_t^{CE}$ can be a misleading indicator
Conclusion

- Endogenous liquidity choice → Investment heterogeneity

- Positive implications:
  - Share of liquid type → non-monotone in $\theta_\ell$
  - Interest rate → non-monotone in $\theta_i$

- Normative implications:
  - Endogenous liquidity → pecuniary externality → inefficiency
  - Inefficiently liquid / Inefficiently illiquid (more traditional)
  - Pareto reallocation $\equiv$ regulating share of liquid investment
  - Sign of interest rate → misleading indicator of inefficiency

- Effect of bubbles or public liquidity?