Liquidity Hoarding

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Motivation

- Current crisis associated with illiquidity and freeze in markets.
- Lack of liquidity in the interbank market.
- Banks hoard liquidity rather than lend.
- Rationing and rates reaching historic highs.
- Unprecedented government interventions.
- Introduction of many liquidity facilities.
Empirical evidence

- Afonso, Kovner, Schoar (2010): rates spiked and terms were sensitive to borrower risk, but volume of lending remained stable after Lehman’s collapse, possibly supply did not catch up with demand.
Motivation

- Liquidity hoarding: Lending vs. piling cash
- Idle cash
- Banks that demand cash cannot get it
- Inefficient early liquidations
- Inefficiently low level of lending (compared to a “benchmark”)
Motivation

- Liquidity hoarding
- No credit risk
- Uncertainty about future liquidity need and access to markets.
- Motives for hoarding:
  - Precautionary motive
  - Speculative motive
Motivation

- **Policy:**

- **Goodfriend & King (1988):** With efficient interbank markets only lend to the market (OMO).

- Interbank market will distribute the liquidity.

- Hoarding incentives create inefficiency in the interbank market.

- Lending to individual institutions.
Questions

- Efficient allocation of liquidity: Hoarding
- Efficient level of liquidity in the financial system: Portfolio choice
- Policies:
  - OMOs, Lender of Last Resort
- Liquidity Requirements
Related literature


- Our paper differs in several respects: precautionary motive for liquidity hoarding; initial portfolio choice and later decision to lend; policy options.

Outline

- The planner’s problem
  - constrained efficient outcome

- A laissez-faire equilibrium

- Constrained inefficiency of equilibrium
  - provision of liquidity: hoarding
  - level of liquidity: portfolio choice

- Policy analysis
  - LoLR
  - Other policies
The Model
Primitives I

- **Time:** Time is divided into four dates, indexed $t = 0, 1, 2, 3$

- **Assets:** Two assets:
  - liquid asset (‘cash’)
  - illiquid asset (‘the asset’)

- **Returns:**
  - cash pays a return of 1 at each date
  - asset pays a return of $R > 1$ at date 3
Primitives II

- **Bankers**: Ex ante identical, risk-neutral agents $i \in [0, 1]$
  
  - Has 1 unit of cash and 1 unit of asset at $t = 0$
  - Decide whether to hold cash or consume at $t = 0$
  - $U(c_0, c_3) = \rho c_0 + c_3$, with $\rho > 1$

- **Creditors**: Ex ante identical, risk-neutral agents $j \in [0, 1]$
  
  - Creditor $j$ has 1 unit of debt with face value 1 in bank $i = j$
  - Uncertain about when to consume $t = 1, 2, 3$
  - At each date $t = 1, 2$ a fraction $\theta_t$ of the creditors receive a liquidity shock (at most once)
  - $V(c_1, c_2, c_3) = \theta_1 c_1 + (1 - \theta_1)\theta_2 c_2 + (1 - \theta_1)(1 - \theta_2)c_3$
Primitives III

▷ *Liquidity shocks:* Creditors that receive a liquidity shock demand repayment from the bank

▷ *Default:* On receiving a shock, a bank must either pay one unit of cash to discharge debt or default and suffer a loss of 100% of the value of his portfolio

▷ *Distributions:* $\theta_1 \sim f_1(\theta_1)$ and $\theta_2 \sim f_2(\theta_2)$ and iid with full support, i.e., $[0, 1]$
The Planner’s Problem
The planner’s problem

- We assume the planner cannot transfer assets between agents.
- The planner can only accumulate and distribute liquidity at the first three dates and reallocate payoffs at the last date.
- The planner has complete information (for now).
- The planner’s policy consists of an cash balances $m_0, m_1(\theta_1), m_2(\theta_1, \theta_2)$ at date 0, at date 1 in state $\theta_1$ and at date 2 in state $(\theta_1, \theta_2)$, respectively.
- This defines the amounts $x_1(\theta_1) = m_0 - m_1(\theta_1)$ and $x_2(\theta_1, \theta_2) = m_1(\theta_1) - m_2(\theta_1, \theta_2)$ distributed at date 1 in state $\theta_1$ and at date 2 in state $(\theta_1, \theta_2)$, respectively.
The planner’s problem

- $t = 0$: $m_0$ units of cash

- $t = 1$:
  - $x_1$ units distributed
  - $m_1 = m_0 - x_1$ carried to $t = 2$

- $t = 2$:
  - $x_2$ units distributed
  - $m_2 = m_1 - x_2$ carried to $t = 3$
Use of cash

- One unit of cash is always consumed by creditors.
- One unit of cash can save one unit of the asset generating an output of $R$.
- Hence, one unit of cash, if used to save an asset, generates $R + 1$.
- Planner maximizes total expected output.
- Efficiency requires using cash to save as many assets as possible.
Feasible policies

- A policy $m_0, m_1(\theta_1), m_2(\theta_1, \theta_2)$ is feasible if
  \[
  m_0 \geq 0, \ x_1(\theta_1) \geq 0, \ x_2(\theta_1, \theta_2) \geq 0 \tag{1}
  \]
  and
  \[
  x_1(\theta_1) + x_2(\theta_1, \theta_2) \leq m_0, \tag{2}
  \]
  for any $(\theta_1, \theta_2)$.

- The planner chooses a feasible policy to maximize the total surplus
  \[
  E_0 \left[ R \{ x_1(\theta_1) + x_2(\theta_1, \theta_2) + (1-\theta_1)(1-\theta_2) \} + m_0(1-\rho) \right]
  \]
Efficiency

- Efficiency requires saving as many assets as possible.
- *Date 2*: Amount of cash at date 2 is $m_1$. The optimal policy is
  \[ x_2(\theta_1, \theta_2) = \min \{(1 - \theta_1) \theta_2, m_1\} \]
- *Date 1*: Amount of cash at date 1 is $m_0$. The optimal policy is
  \[ x_1(\theta_1) = \min \{\theta_1, m_0\} \]
- *Date 0*: There is an interior solution if $1 < \rho < R + 1$ and $m_0$ is characterized by the first-order condition
  \[
  R \left( 1 - \int_0^{m_0} F_2 \left( \frac{m_0 - \theta_1}{1 - \theta_1} \right) f_1(\theta_1) \, d\theta_1 \right) + 1 = \rho
  \]
  \[
  \Pr(\text{idle cash}) = \Pr(\theta_1 \leq m_0 \text{ and } (1 - \theta_1) \theta_2 \leq m_0 - \theta_1).
  \]
Figure 6a: Planner’s choice $m_0$ as a function of $\rho$ for $R=3$
Laisser-Faire Equilibrium
The laisser-faire economy

- At date 0, bankers decide whether to hold liquidity, that is, whether to become “liquid” bankers \((1 - \alpha)\) or remain “illiquid” \((\alpha)\)
- At date 1, there is a spot market on which the asset can be traded for cash
- Some bankers receive a liquidity shock \((\theta_1)\) that requires them to pay one unit of cash to creditors; failure to do so leads to default and liquidation
- At date 2, some of the bankers who have not already received a shock may receive a liquidity shock \(((1 - \theta_1)\theta_2)\)
- At date 3, solvent bankers receive the returns from the assets they hold and remaining debts are paid
1 - \( \alpha \) agents choose to become *liquid* agents; the remainder are *illiquid* agents.

- A fraction \( \theta_1 \) of agents are hit by a liquidity shock.
- Illiquid agents who receive a shock trade the asset for cash or default.
- Liquid agents who do not receive a shock become either *buyers* or *hoarders*.

- A fraction \( \theta_2 \) of agents are hit by a liquidity shock.
- Illiquid agents and 'buyers' who receive a shock, trade the asset for cash or default.
- Hoarders who do not receive a shock buy assets or hold cash.

- Asset returns are consumed.
Allocations I

- At date 0 a fraction $1 - \alpha$ of bankers decide to hold liquidity (one unit)
- At date 1, a fraction $\theta_1$ of the bankers receive a liquidity shock
- A measure $(1 - \alpha) \theta_1$ of liquid bankers use their own cash to discharge the debt; a measure $\alpha \theta_1$ of illiquid bankers must either sell $p_1$ assets for liquidity or default
- **Buyers:** $(1 - \alpha) (1 - \theta_1) \lambda$ of liquid bankers choose to buy assets
- **Hoarders:** $(1 - \alpha) (1 - \theta_1) (1 - \lambda)$ choose to hoard cash
Figure 2: Allocations at dates 0 and 1

\[ \begin{align*}
\alpha & \quad \text{(illiquid)} \\
1 - \alpha & \quad \text{(liquid)}
\end{align*} \]

\( (1, 0) \)

\[ \begin{align*}
\theta_1 & \quad \text{(shock)} \\
1 - \theta_1 & \quad \text{(no shock)}
\end{align*} \]

\( (1, 0) \)

\( (1, 0) \)

\[ \begin{align*}
\lambda & \quad \text{(buyer)} \\
1 - \lambda & \quad \text{(hoarder)}
\end{align*} \]

\( (1, 1) \)

\( (1, 1) \)
Allocations II

- At date 2, several types remain inactive:
  - those already received a shock at date 1;
  - hoarders who receive a shock at date 2,
  - buyers who do not receive a shock at date 2
  - illiquid bankers who do not receive a shock at date 2

- Demand for liquidity:
  - buyers who receive a shock at date 2
  - illiquid bankers who receive a shock at date 2

- Supply: Hoarders who do not receive a shock at date 2
Figure 3a: Allocations at date 2

- **Illiquid**
  - $\theta_2$
  - $1 - \theta_2$

- **Buyer**
  - $\theta_2$
  - $1 - \theta_2$

- **Hoarder**
  - $\theta_2$
  - $1 - \theta_2$

- $\theta_2$ paths:
  - $(\max\{1 - p_2, 0\}, 0)$
  - $(1 - p_2, 0)$
  - $(1, 0)$
  - $(1 + p_1, 0)$

- Non-\$\theta_2$ paths:
  - $(1 + p_2, 0)$
Figure 3b: Allocations at date 2

\[ \alpha \theta_i \]

\[ \alpha(1-\theta_i)\theta_2 \]

\[ \alpha(1-\theta_i)(1-\theta_2) \]

\[ (1-\alpha)\theta_i \]

\[ (1-\alpha)(1-\theta_i)\lambda\theta_2 \]

\[ (1-\alpha)(1-\theta_i)(1-\theta_2) \]

\[ (1-\alpha)(1-\theta_i)(1-\lambda)\theta_2 \]

\[ (1-\alpha)(1-\theta_i)(1-\lambda)(1-\theta_2) \]
Figure 4: Terminal Payoffs

\[
\begin{align*}
\alpha & \quad \text{(illiquid)} \\
\theta_1 & \quad \text{Shock} \\
1-\theta_1 & \quad \text{No Shock} \\
1-\alpha & \quad \text{(liquid)} \\
\theta_1 & \quad \text{Shock} \\
1-\theta_1 & \quad \text{No Shock} \\
\theta_2 & \quad \text{Shock} \\
1-\theta_2 & \quad \text{No Shock} \\
\lambda & \quad \text{Buy} \\
1-\lambda & \quad \text{No Shock} \\
1-\theta_2 & \quad \text{No Shock} \\
1-\theta_2 & \quad \text{No Shock} \\
\end{align*}
\]

- $R(1-p_1)$
- $\max\{0, R(1-p_2)\}$
- $R-1$
- $R-\rho$
- $R(1+p_1-p_2)-\rho$
- $R(1+p_1)-1-\rho$
- $R-\rho$
- $R(1+p_2)-1-\rho$
Market clearing I

- *Date 2:* Let $\theta_2^*$ and $\theta_2^{**}$ be defined by

$$\theta_2^* = (1 - \alpha)(1 - \lambda) \text{ and } \theta_2^{**} = 1 - \lambda.$$

- There are three demand-and-supply regimes:

$$\theta_2 > \theta_2^{**} \text{ and } p_2 = 1 + p_1 \text{ (only buyers)}$$

$$\theta_2^* < \theta_2 < \theta_2^{**} \text{ and } p_2 = 1 \text{ (buyers + some illiquid)}$$

$$\theta_2 < \theta_2^* \text{ and } p_2 = \frac{1}{R} \text{ (everyone)}$$
Figure 5A: Supply of cash at date 2

\[ \frac{1}{R} \]

\[ p_2 \]

\[ (1 - \alpha)(1 - \theta_1)(1 - \lambda)(1 - \theta_2) \]
Figure 5B: Demand for cash at date 2

\[ p_2 \]

\[ 1 + p_1 \]

\[ 1 \]

\[ (1 - \alpha)(1 - \theta_1) \lambda \theta_2 \]

\[ (1 - \alpha)(1 - \theta_1) \lambda \theta_2 + \alpha (1 - \theta_1) \theta_2 \]

Demand
Figure 5C: Different demand and supply regimes

(i) \( p_2 = \frac{1}{R} \)

(ii) \( p_2 = 1 \)

(iii) \( p_2 = 1 + p_1 \)
Market clearing II

- *Date 1*: For any $\theta_1$, $\lambda (\theta_1)$ is the fraction of *buyers* (and the complement *hoarders*)

- Buying is optimal iff $p_1 (\theta_1) \geq E [p_2 (\theta_1, \theta_2) | \theta_1]$

- Hoarding is optimal iff $p_1 (\theta_1) \leq E [p_2 (\theta_1, \theta_2) | \theta_1]$
Market clearing II

- Suppose $p_1 > E[p_2]$ and everyone is a buyer ($\lambda = 1$)
- No cash at $t = 2$, $p_2 = 1 + p_1$. CONTRADICTION!

- Suppose $p_1 < E[p_2]$ and everyone is a hoarder ($\lambda = 0$)
- $p_1 = 1$ and no buyer so $p_2 \leq 1$. CONTRADICTION!

- For every value of $\theta_1$,

$$0 < \lambda(\theta_1) < 1$$

in equilibrium at date 1, and hence,

$$p_1(\theta_1) = E[p_2(\theta_1, \theta_2)|\theta_1].$$
Market clearing III

► We know $p_2$:

$$\theta_2 > \theta_2^{**} \text{ and } p_2 = 1 + p_1$$

$$\theta_2^* < \theta_2 < \theta_2^{**} \text{ and } p_2 = 1$$

$$\theta_2 < \theta_2^* \text{ and } p_2 = \frac{1}{R}$$

► In equilibrium, we have $p_1 = E [p_2]$, so that we can derive $p_1$ as a function of $\lambda$:

$$\tilde{p} (\lambda) = \frac{1 + F_2 ((1 - \alpha) (1 - \lambda)) (1 - R^{-1})}{F_2 (1 - \lambda)}$$
Market clearing III

- In equilibrium, we have $p_1 = E[p_2]$.
- For low shocks $\theta_1$, $(1 - \alpha) (1 - \theta_1) \lambda = \alpha \theta_1$, and $p_1 = E[p_2]$.
- As $\theta_1$ increases, if everyone gets cash, little cash left for $t = 2$.
- $p_2$, therefore $E[p_2]$ and $p_1$ increase.
- At some point $p_1$ reaches the maximum value 1.
- If lending continues at $t = 1$, we cannot satisfy $p_1 = E[p_2]$ since $p_1 = 1$ but $p_2$ continues to increase.
- So lending at $t = 1$ has to stop.
- There is a unique value of $\lambda$, call it $\bar{\lambda} \in (0, 1)$, such that $\tilde{p}(\bar{\lambda}) = 1$. 
Hence, the equilibrium value of $\lambda(\theta_1)$ is given by

$$\lambda(\theta_1) = \min \left\{ \frac{\alpha \theta_1}{(1 - \alpha)(1 - \theta_1)}, \bar{\lambda} \right\},$$

for every value of $\theta_1$, and the equilibrium value of $p(\theta_1)$ is given by

$$p_1(\theta_1) = \min \left\{ \tilde{p} \left( \frac{\alpha \theta_1}{(1 - \alpha)(1 - \theta_1)} \right), 1 \right\},$$

for every value of $\theta_1$. 
Figure: Equilibrium $\lambda$ as a function of $\theta_1$

Region of rationing
Market clearing V

- **Date 0**: In equilibrium at date 0, $0 < \alpha < 1$, which implies that bankers must be indifferent between acquiring liquidity and not acquiring it.

- Bankers are indifferent if and only if

$$
\int_0^1 p_1 \left\{1 + (1 - \theta_1)(1 - F_2(\theta_2^{**}))E[\theta_2 | \theta_2 > \theta_2^{**}]\right\} f_1(\theta_1) d\theta_1 = \frac{\rho}{R}.
$$
Equilibrium

An equilibrium is described by the endogenous variables $\alpha$, $\lambda (\theta_1)$, $p_1 (\theta_1)$, and $p_2 (\theta_1, \theta_2)$ satisfying the following conditions:

- at date 2, for every value of $(\theta_1, \theta_2)$, $p_2 (\theta_1, \theta_2)$ is the market clearing price, given the values of $\alpha$, $\lambda (\theta_1)$ and $p_1 (\theta)$

- at date 1, for every value of $\theta_1$, $\lambda (\theta_1)$ and $p_1 (\theta)$ satisfy the market clearing conditions, given the value of $\alpha$

- at date 0, agents are indifferent between acquiring liquidity and not acquiring it
Liquidity insurance

- Let \( \{\alpha, \lambda(\theta_1), p_1(\theta_1), p_2(\theta_1, \theta_2)\} \) be an equilibrium and consider the effect of opening a market for liquidity insurance at date 0.
- At date 0, bankers enter into forward contracts to deliver or receive liquidity under specified conditions.
- Suppliers acquire one unit of liquidity at date 0; demanders do not.
- At dates \( t = 1, 2 \), each banker is required to report his type, that is, whether or not he has received a liquidity shock.
- Suppliers who report “shock” and demanders who report “no shock” do not trade.
Liquidity insurance II

- At date 1,
  - a supplier who reports “no shock” receives \((-1, \hat{p}_1(\theta_1))\)
  - a demander who reports “shock” receives \((1, -\hat{p}(\theta_1))\)

- At date 2,
  - a supplier who reports “no shock” for the second time and has not traded receives \((-1, \hat{p}_2(\theta_1, \theta_2))\)
  - a demander who reports “shock” for the first time receives \((1, -\hat{p}_2(\theta_1, \theta_2))\)
Incentive compatibility

- If $\hat{p}_1(\theta_1) > p_1(\theta_1)$, a demander who receives a shock will report “no shock” and buy on the spot market; if $\hat{p}_1(\theta_1) < p_1(\theta_1)$, a supplier who did receive a shock will report “shock” and sell on the spot market.

Thus, incentive compatibility at date 1 requires

$$\hat{p}_1(\theta_1) = p_1(\theta_1), \text{ for every } \theta_1$$

Similarly, incentive compatibility at date 2 requires

$$\hat{p}_2(\theta_1, \theta_2) = p_2(\theta_1, \theta_2), \text{ for every } (\theta_1, \theta_2)$$
Policy Analysis
Sources of inefficiency

- At $t = 2$, hoarders who receive a shock use their liquidity to discharge their own debt rather than the buyers’
- At $t = 1$, hoarders do not internalize the welfare losses resulting from early liquidations
- At $t = 0$, agents do not internalize the social value of paying off their debt
Central Bank sole provider of liquidity

- Can the central bank achieve the allocation from the planner’s problem?
- Suppose that Central Bank is the sole provider of liquidity \((\alpha = 1)\).
- Central Bank holds \(m_0\) units of liquidity and pursues the socially optimal.
- At date 2, the market-clearing price is denoted by \(p_2(\theta_1, \theta_2)\)
  and defined by

\[
p_2(\theta_1, \theta_2) = \begin{cases} 
1 & \text{if } (1 - \theta_1) \theta_2 > \max \left\{ m_0^* - \theta_1, 0 \right\} \\
R^{-1} & \text{if } (1 - \theta_1) \theta_2 < \max \left\{ m_0^* - \theta_1, 0 \right\}
\end{cases}
\]

- At date 1, the market clearing price is assumed to be

\[
p_1(\theta_1) = \begin{cases} 
1 & \text{if } \theta_1 > m_0^* \\
E [p_2(\theta_1, \theta_2) \mid \theta_1] & \text{if } \theta_1 < m_0^*
\end{cases}
\]

- We show that \(\alpha = 1\) is privately optimal.
Central Bank II

- An illiquid banker’s payoff is

\[
E \left[ \theta_1 R \left( 1 - p_1(\theta_1) \right) + (1 - \theta_1) \theta_2 R \left( 1 - p_2(\theta_1, \theta_2) \right) + (1 - \theta_1)(1 - \theta_2) R \right] = E \left[ R - (\theta_1 + (1 - \theta_1) \theta_2) p_2(\theta_1, \theta_2) R \right]
\]

- A liquid banker’s payoff is

\[
E \left[ R + (1 - \theta_1)(1 - \theta_2) p_2(\theta_1, \theta_2) R \right] - \rho
\]

- Then it is optimal to be illiquid if and only if

\[
E \left[ p_2(\theta_1, \theta_2) R \right] \leq \rho
\]
The first-order condition for the planner’s problem is

\[ R \left(1 - \int_0^{m_0} F_2 \left( \frac{m_0 - \theta_1}{1 - \theta_1} \right) f_1 (\theta_1) d\theta_1 \right) + 1 = \rho. \]

From the definition of \( p_2 (\theta_1, \theta_2) \),

\[
E [p_2 (\theta_1, \theta_2)] = R^{-1} F_2 \left( \frac{m_0^* - \theta_1}{1 - \theta_1} \right) + \left(1 - F_2 \left( \frac{m_0^* - \theta_1}{1 - \theta_1} \right) \right)
\]

\[ = 1 - (1 - R^{-1}) F_2 \left( \frac{m_0^* - \theta_1}{1 - \theta_1} \right). \]

\[
E [p_2 (\theta_1, \theta_2) R] = R - (R - 1) \int_0^{m_0^*} F_2 \left( \frac{m_0^* - \theta_1}{1 - \theta_1} \right) f_1 (\theta_1) d\theta_1
\]

\[ \leq R \left(1 - \int_0^{m_0^*} F_2 \left( \frac{m_0 - \theta_1}{1 - \theta_1} \right) f_1 (\theta_1) d\theta_1 \right) + 1
\]

\[ \leq \rho \]
Policy with private liquidity (date 1)

- Choose socially optimal $\lambda$ at $t = 1$ while allowing markets to clear at other dates

- Liquidity facilities

- The socially optimal level of $\lambda^{soc}$ has the same structure as the equilibrium $\lambda$ but is larger:

  $$\lambda^{soc} = \min \left\{ \frac{\alpha \theta_1}{(1 - \alpha)(1 - \theta_1)}, \tilde{\lambda} \right\}, \text{ where } \tilde{\lambda} > \bar{\lambda}$$

- Policy mitigates hoarding at $t = 1$. 

Figure 6b: Equilibrium and constrained efficient levels of $\lambda$ as a function of $\theta_1$ for $R=3$ and $\rho=2$

\[ \lambda^{eq} \text{ and } \lambda^{soc} \]
\[ \alpha^{eq} (\text{for } \lambda^{eq}) = 0.139, \alpha^{eq} (\text{for } \lambda^{soc}) = 0.136 \]
Policy with private liquidity (date 0)

- Choose the socially optimal $\alpha$ at $t = 0$ while allowing markets to clear at other dates
- Liquidity requirements (Basel III)
- The optimal value of $\alpha^{soc}$ is smaller than the equilibrium level
Figure 6c: Equilibrium and constrained efficient levels of $\alpha$ as a function of $\rho$ for $R=3$
Figure 6d: Equilibrium and constrained efficient levels of $\alpha$, and planner’s choice $(1-m_0)$ as a function of $\rho$ for $R=3$
Comparative statics I

- How do the distribution and the volatility of shocks change equilibrium and socially optimal liquidity, and the wedge between the two?
- More likely liquidity shocks at $t = 2$: $g_2(\theta_2)$ FOSD $f_2(\theta_2)$, $G_2(\theta_2) \leq F_2(\theta_2)$
- Equilibrium requires $p_1 = E[p_2]$

\[
F_2(1 - \bar{\lambda}_f) + F_2((1 - \alpha)(1 - \bar{\lambda}_f))(1 - R^{-1}) = 1 \\
G_2(1 - \bar{\lambda}_g) + G_2((1 - \alpha)(1 - \bar{\lambda}_g))(1 - R^{-1}) < 1
\]

- This gives us $\bar{\lambda}_f > \bar{\lambda}_g$
- We can also show $\tilde{\lambda}_f > \tilde{\lambda}_g$
Comparative statics II

- Suppose $\theta_2$ uniform over $[a, b]$.
- For $b' > b$, $f_2'(\theta_2) \text{ FOSD } f_2^b(\theta_2)$

\[
\frac{1}{b-a} \left[ (1 - \bar{\lambda}) - a + ((1 - \alpha)(1 - \bar{\lambda}) - a)(1 - R^{-1}) \right] = 1
\]

\[
\bar{\lambda} = 1 - \frac{bR + a(R - 1)}{R + (1 - \alpha)(R - 1)}
\]

\[
\frac{1}{b-a} \left[ (1 - \tilde{\lambda}) - a + (1 - \alpha)(1 - \tilde{\lambda}) - a \right] = 1
\]

\[
\tilde{\lambda} = 1 - \frac{b + a}{2 - \alpha}
\]

\[
\frac{d(\tilde{\lambda} - \bar{\lambda})}{db} = \frac{1 - \alpha}{(2 - \alpha)(R + (1 - \alpha)(R - 1))} > 0
\]

- The wedge increases as shocks become more likely.
Comparative statics II

- Effect of volatility of shocks
- Suppose $\theta_2$ uniform over $[a, b]$ with $a + b = 1$ (symmetric around 1/2)
- For $b' > b$, $f_b^b(\theta_2)$ is a mean-preserving spread of $f_b^b(\theta_2)$

\[
\tilde{\lambda} = 1 - \frac{R - 1 + b}{R + (1 - \alpha)(R - 1)}, \text{ decreasing in } b.
\]

\[
\check{\lambda} = 1 - \frac{1}{2 - \alpha}
\]

\[
\frac{d(\check{\lambda} - \tilde{\lambda})}{db} = \frac{1 - \alpha}{(2 - \alpha)(R + (1 - \alpha)(R - 1))} > 0
\]

- The wedge increases as volatility of shocks increases.
- Models using Knightian uncertainty.
Conclusion

- Goodfriend and King argued that it is sufficient to provide adequate liquidity to the system as a whole ...
- Yet, when agents are uncertain about future liquidity shocks, they hoard rather than lend.
- Inefficient (lack of) liquidity transfers.
- Freezes in markets.
- Reform of regulation of the financial sector.
- Role of Central Banks as LoLR.
- Liquidity requirements.