A Network-Based Analysis of Over-the-Counter Markets

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Research questions

- Over-the-counter markets are different from a centralized exchange:
  - Trading relationships between market participants (trust, netting agreements, geography, etc.)
  - One or more intermediaries between a seller and a buyer
  - Bilateral trades, prices are determined by bilateral bargaining or auctions
- How does intermediation in over-the-counter markets affect the efficiency of resource allocation?
- What is the relationship between market structure and efficiency?
- What is the cost and the benefit of large interconnected financial institutions?

Global OTC Markets

- The size of OTC markets is large:
  - OTC derivatives: $604 trillion
  - Bonds: $91 trillion
  - Asset-Backed-Securities (ABS): $10+ trillion

  Sources: BIS (2008-2009), afme/esf (Q4:2009)
- Average bank trades with a very small number of counterparties
  - Bech and Atalay (2008) - Federal Funds Market (1000 banks, 3.3 counterparties on average, max 165)
  - Cajueiro and Tabak (2007) - Brazilian Interbank Market
  - Toivanen (2009) - Finnish Interbank Market
  - Upper and Worms (2002) - German Interbank Market
  - Wells (2004) - UK Interbank Market

Main results

- When intermediation is required and intermediaries cannot extract the full surplus in each trade, efficiency is not guaranteed
- Markets that are more interconnected can be less efficient
- A large interconnected financial institution can improve efficiency

Outline

- Examples of inefficient allocations using efficient bargaining or auctions
- A general model of trading in a financial network with endogenous intermediation
- Comparison of different financial architectures
- Costs and benefits of large interconnected financial institutions

Bilateral Bargaining

Figure: Two buyer-seller pairs bargain over the surplus

Bilateral bargaining with intermediation

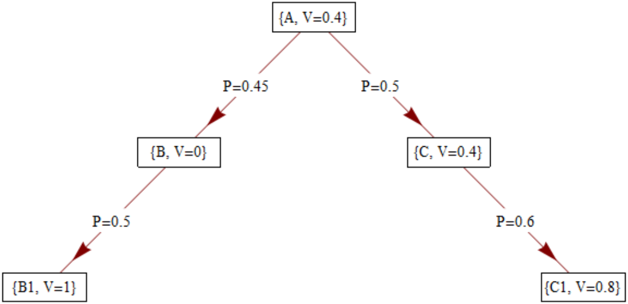


Figure: Bargaining friction with intermediated trades

Inefficient Sequential Auctions

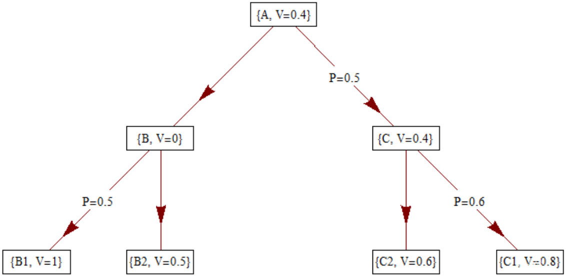


Figure: Inefficient Allocation with Sequential Second-price Sealed-bid Auctions

- TAF allocated $1 trillion dollars using auctions
- NY Fed deals with 19 primary dealers using auctions
- Corporate bond markets: "Trading occurred by successive auctions during any given day." (Saunders et. al. 2002)

Related literature

- **Financial and economic networks**
  - Take-it-or-leave-it offers (Gale and Kariv 2007, Blume, Easley, Kleinberg, and Tardos 2009, Condorelli 2009 - incomplete information)
  - Auctions without intermediation (Kranton and Minehart 2001)
- **Search-based models of OTC markets**
  - Bilateral bargaining (Duffie, Garleanu, and Pedersen 2005; Afonso and Lagos 2010; Wong and Wright 2011)
- **My paper**:
  - A model that accounts for trading relationships and bargaining
  - Algorithm to compute bilateral prices, allocations and profits for any financial network
  - Comparison of different financial architectures in terms of trading efficiency

The Model

- **Agents**: $N = \{1, ..., n\}$
- **A trading network** is an undirected graph g, a set of all trading relationships.
- **A trading relationship** between i and j exists if i and j can trade directly.
- **Assumption**: g is ***connected***, everyone can trade directly or indirectly with everyone else.
- **Definition**: g is ***complete*** if everyone can trade directly
- **Endowment vector**: $E = \{0, \ldots, E_i = 1, \ldots, 0\}$, one good: $\sum E_i = 1$
- **Private valuations**: $V = \{V_i\}_{i=1}^{n}$, $V_i \in [0, 1]$ (hedging demands, liquidity needs, taxation, etc.)

Trading protocol

- Each agent decides whether to hold the good or sell it to one of his trading partners.
- Trading ends when someone prefers to keep the good
- P_i - **equilibrium valuation** of agent i. $P_i = V_i$ if he keeps. $P_i = V_i + B_i(P_j - V_i)$ if he sells to $j \in N(i, g)$.
- **A bargaining ability** of seller i is $B_i \in (0, 1)$; an agent buying from seller i receives a share $1 - B_i$ of the surplus.
- B is a vector of the **bargaining abilities** of all agents.
- **Complete information** about B, V, E, and g

Equilibrium

Definition (Equilibrium)

i. Agent i's equilibrium valuation is given by:

$$P_i = \max\left\{V_i, \max_{j \in N(i,g)} V_i + B_i\left(P_j - V_i\right)\right\}.$$

ii. Agent i's equilibrium trading decision is given by:

$$\sigma_i = \arg\max_{j \in N(i,g) \cup i} P_j$$

Equilibrium Properties

- Two algorithms to compute equilibrium prices and trading decisions for any network structure: recursive backward induction and contraction mapping. For each vector of valuations and endowment solve for the equilibrium allocation.
- There are no trading cycles in equilibrium (no bubbles).
- Equilibrium valuations are unique, trading decisions generically unique: a unique trading path from the initial seller to the final buyer.
- Prices are increasing along the equilibrium trading path.

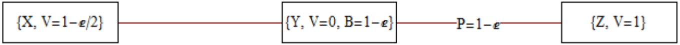
Efficiency

Definition (Efficiency of an equilibrium allocation)

The equilibrium is (Pareto)-efficient if the final buyer has the highest private valuation for the good.

Proposition

(i) If g is complete then the equilibrium allocation is efficient for any B, E, and V.
(ii) If g is incomplete then for any B, there exist vectors E and V such that the equilibrium allocation is inefficient.



Market Structure and Efficiency

- Adding more trading relationships can decrease efficiency because it creates asymmetry in the number of intermediaries
- Policy implication: decreasing costs to establish or maintain trading relationships (CCP to decrease counterparty risk) can decrease efficiency.

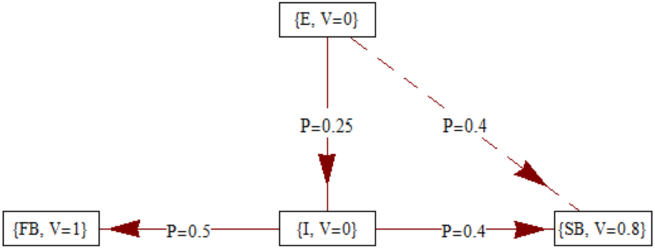


Figure: Adding a trading relationship between *E* and *SB* decreases efficiency

A Simple Economy

- Bargaining ability is the same for all agents: $b \in (0,1)$
- Private valuations: $V^s = \{0,\ldots,0,0 < v < 1,1\}$
- $n-2$ agents with the **third-best** valuation (*TB*) always sell
- 1 agent with the **first-best** valuation (*FB*) only buys, never sells
- 1 agent with the **second-best** valuation (*SB*): if this agent keeps the good, the welfare loss is $1-v$

Proposition

In the simple economy, the equilibrium allocation in a trading network g is always efficient if and only if any pair of agents requires at most $\hat{d} = \left\lfloor \frac{\log(v)}{\log(b)} \right\rfloor$ *intermediaries to trade.*

$$\boxed{\{SB, v\}} \xrightarrow{P=v+b(b^2-v)} \boxed{\{j, 0\}} \xrightarrow{P=b^2} \boxed{\{k, 0\}} \xrightarrow{P=b} \boxed{\{FB, 1\}}$$

Homogeneous trading network

- p - probability of a trading relationship between any pair of agents
- Complete network: $p = 1$.
- If $p(n) > O(log(n)/n)$ then the probability that g is connected tends to one as the network size increases (Erdos and Renyi 1961)

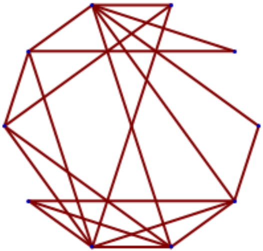


Figure: Number of traders: $n = 10$, $K = p(n-1) = 3.15$ (average number of trading partners)

Homogeneous trading network

Proposition

In the simple economy the probability that the equilibrium allocation is always efficient is at most $1-(1-p)^{K^{\hat{d}}}$, *where* $\hat{d} = \left\lfloor \frac{\log(v)}{\log(b)} \right\rfloor$, *and* $K = (n-1)p$ *is the expected number of trading partners of each agent.*

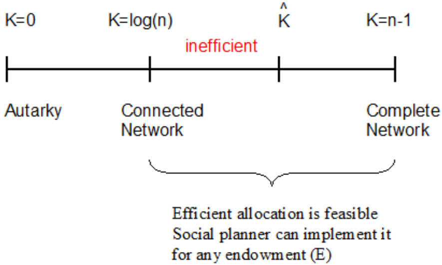
- Compute the probability that SB and FB require more than $\hat{d}$ intermediaries
- $F(d)$ - probability that any two agents require at least d intermediaries to trade. $F(1) = 1 - p$
- Agent *SB* requires at least d intermediaries to trade with *FB* if all trading partners of *SB* require at least $d-1$ intermediaries.
- $F(d) \geq (1-p)^{K^{d-1}}$

Homogeneous trading network

Theorem

In the simple economy if $p(n) < \hat{p} = O(n^{-\frac{\hat{d}}{\hat{d}+1}})$ then the probability that the equilibrium allocation is always efficient tends to zero as the size of the trading network increases.

- Threshold on the expected number of trading partners: $\hat{K} = n^{1/(\hat{d}+1)}$
- If $v > b$ then $\hat{K} = O(n)$, if $b^2 < v < b$ then $\hat{K} = O(n^{1/2})$



Homogeneous trading network with heterogeneous private valuations

- $V_i \sim U[0,1]$ (numerical solution, 1000 draws of g for each p)

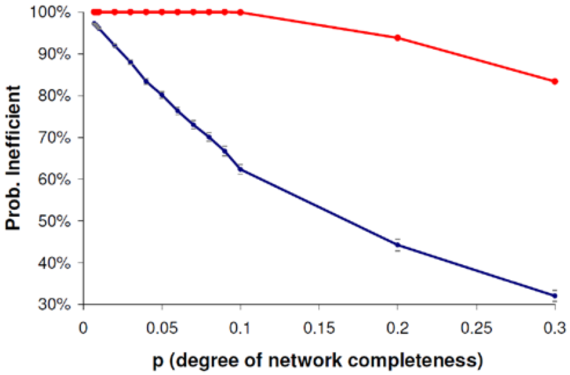


Figure: $b = 0.5$, $n = 1000$, uniform endowment (blue line)

Expected welfare loss

- Expected welfare loss = (prob. eq. allocation is inefficient) x (welfare loss)
- Integrate out uncertainty about endowment, valuations and amount of intermediation.

Steps to compute expected welfare loss in a simple economy:

- Probability that second-best keeps the good in equilibrium
- Share of agents with third-highest valuation who sell directly or through intermediaries to the second-best
- Account for the probability that endowment is second-best or third-best.
- Integrate over possible values of v

Expected welfare loss: comparative statics

The expected welfare loss is given by:

$$\sum_{l=0}^{D^h-1} F(l+1|n,p)\left(q_{SB}+q_{TB}\sum_{j=1}^{D^h+2-l} f(j|n,p)F(j+l)\right)\int_{b^{l+1}}^{b^l}(1-v)dG(v)$$

- The expected welfare loss is decreasing in b.
- The expected welfare loss is non-monotonic in p when the initial allocation is third-best.
- The expected welfare loss is monotonic in p when the initial allocation is second-best.
- If $n = 1000$ and $q_{TB} = (n-2)/n$ then adding randomly 125,000 trading relationships (increasing p from 8% to 33%) doubles the expected welfare loss (from 0.9% to 1.8%)

Expected welfare loss and the degree of network completeness

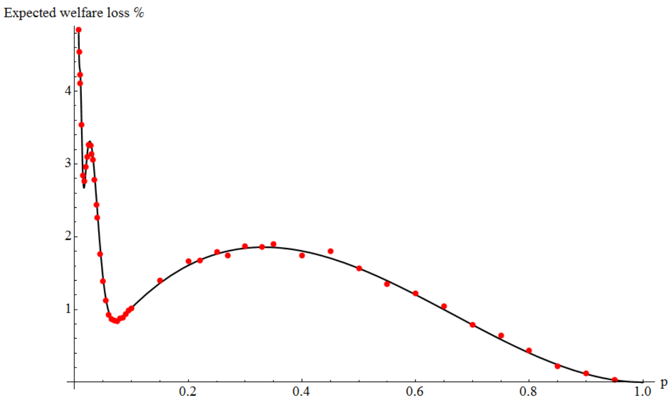


Figure: $n = 1000$, $b = 0.5$, $v \sim U[0, 1]$, uniform endowment. Red dots represent the results of the numerical solution for the expected welfare loss.

One large interconnected financial institution

- One agent with $n-1$ trading relationships
- Other $n-1$ agents can trade directly with each other with probability p
- Star: $p=0$, Complete network: $p=1$

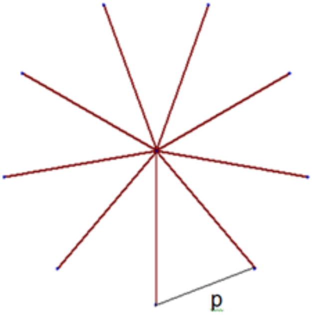
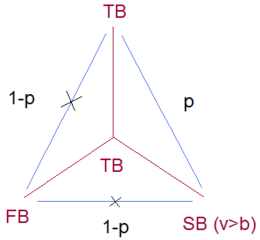


Figure: Number of traders: $n=10$, one large interconnected institution

Financial architecture with one large intermediary

- In a simple economy, the expected welfare loss is 0 if the large interconnected institution is second-best or first-best
- If this institution is TB (0 valuation), v is uniform, endowment uniform, then the expected welfare loss is:

$$EWL^{ih} = \underbrace{\left(0.5 - b + 0.5b^2\right)}_{\text{loss}} \underbrace{(1-p)\left(\frac{1}{n} + \frac{n-3}{n}(1-p)p\right)}_{\text{prob. inefficient}}$$



Financial architecture with one large intermediary

Large interconnected financial institution is a pure intermediary ($v = 0$)

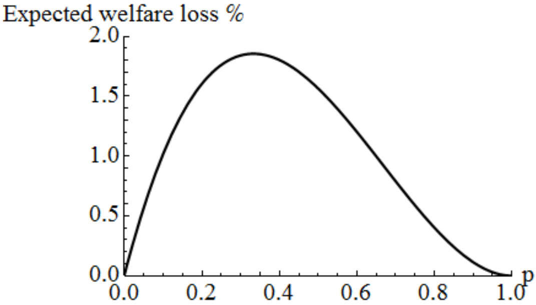


Figure: $n = 1000$, $b = 0.5$, $v \sim U[0, 1]$, uniform endowment

Efficiency gains from a large interconnected intermediary

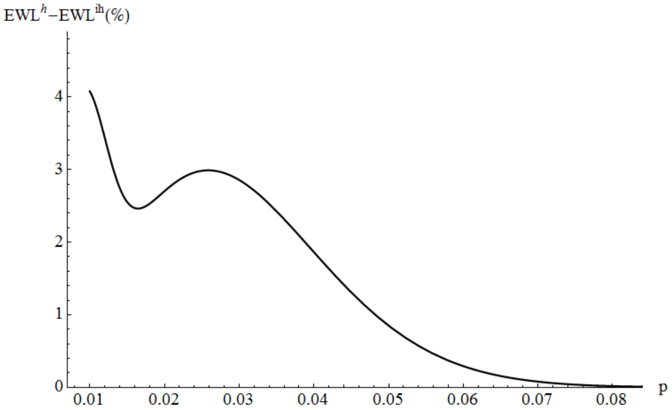


Figure: $b = 0.5$, $n = 1000$, uniform endowment, $v \sim U[0, 1]$

Costs of a large interconnected institution

- **Assumption**: The private valuation of the large interconnected institution increases by $0 < \alpha < 1$ if it anticipates to be bailed out.
- **Motivation**: Private valuations for a financial asset depend on future cash flows; if a large interconnected institution is bailed out in some future states when the cash flows from holding an asset are negative then the bailout increases its private valuation.

Large interconnected institution (second-best)

- If the large interconnected institution is second-best then when $v + \alpha > 1$, it buys the asset from everyone and the resulting welfare loss is $1 - v$.

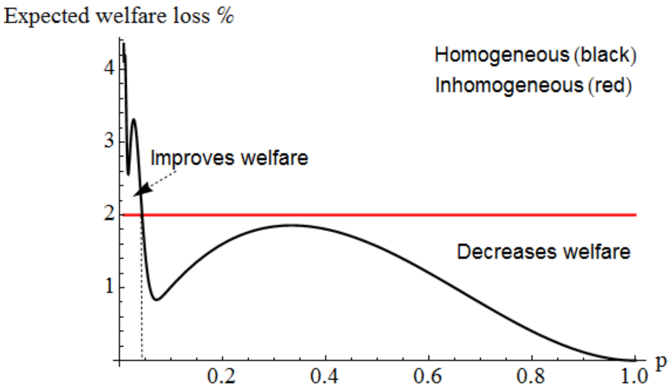
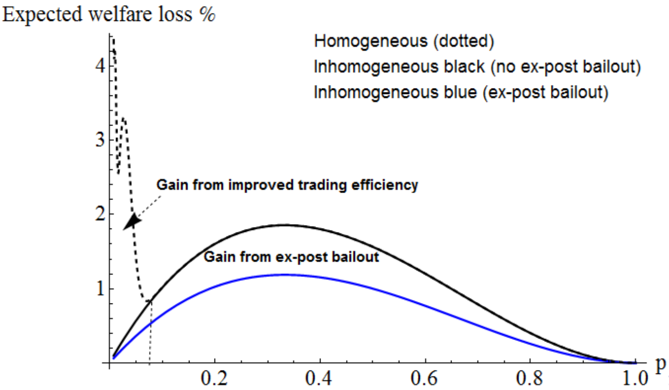


Figure: $\alpha = 0.2$, $b = 0.5$, $n = 1000$, $v \sim U[0, 1]$, uniform endowment

Large interconnected intermediary (third-best)

- The intermediary sells to the agent with the highest valuation for $b + (1 - b)\alpha$. There is a welfare gain from a bailout put. The intermediary is more likely to buy from the agent with the second-highest valuation because the resale price increases.



Policy Implications

- A policy that decreases the cost of establishing or maintaining trading relationships in OTC markets can decrease welfare. For example, if establishing a CCP decreases the cost of some bilateral trades, then CCP can decrease trading efficiency.
- Restricting the number of trading partners of each agent can mitigate the too interconnected to fail problem but can decrease the efficiency of resource allocation. This result addresses the questions in the Dodd-Frank Financial Reform Act, Sec. 123.
- A bailout put does not always decrease trading efficiency.

Conclusion

- Over-the-counter markets are vulnerable to a bargaining friction that can result in a substantial welfare loss.
- A financial architecture with a large interconnected institution improves welfare but the trading decisions of this institution can decrease welfare if it expects to be bailed out.
- This theoretical framework allows us to analyze the costs and benefits of large interconnected institutions and has implications for the desired structure of the financial system.

Future Research

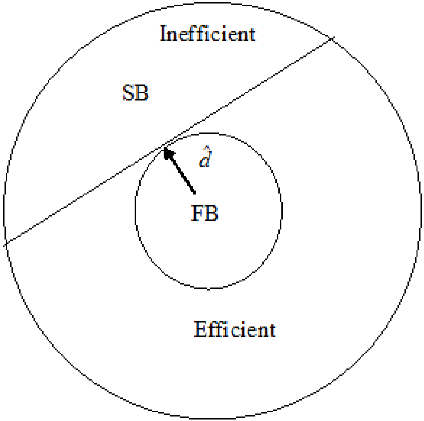
- Market freeze and myopic pricing
- Positive analysis of OTC markets
- Merger waves
- Contracting in supplier-customer networks
- Authority in hierarchical organizations

Expected welfare loss in a homogeneous trading network

- Example: $v \sim U[0,1]$, endowment uniform ($q_{SB} = \frac{1}{n}$, $q_{TB} = \frac{n-2}{n}$), $b = 0.5$

Traders (n)	Trading Partners (K)	Expected Welfare Loss	Prob. of Inefficiency	Expected Welfare Loss ($q_{SB} = 1$)
100	5	5.6%	18.6%	27.3%
100	10	3.5	13.5	16.9
1000	10	4.2	15.3	30.4
1000	30	3.2	12.7	18.5
1000	100	1	4.1	11.2
1000	300	1.8	7.4	8.7
10000	20	3.6	13.8	32.1
10000	40	1.7	6.5	25.8
10000	100	3	11.9	18.1
10000	200	**0.5**	1.8	12.5
10000	3000	**1.8**	7.4	8.7

Intuition for Computing the Expected Welfare Loss



Heterogeneous valuations

- $V_i \sim U[0, 1]$ (numerical solution, 100 draws of g for each p)
- Private valuation of the large interconnected intermediary is 0, 0.5, 0.8.
- The expected welfare loss can be as high as 14%
- The expected welfare loss is non-monotonic in p

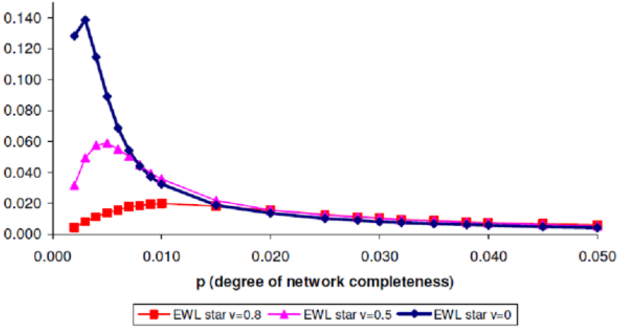
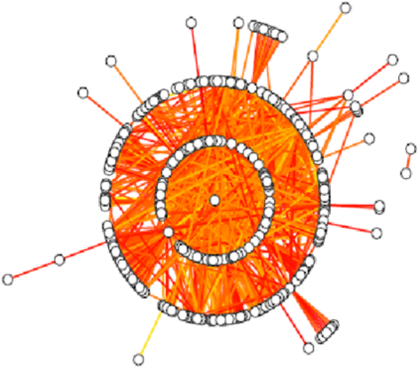


Figure: $b = 0.5$, $n = 1000$, uniform endowment

Federal Funds Market



Source: “The Topology of the Federal Funds Market” by Bech and Atalay (2008)