Optimal Unconventional Monetary Policy

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Introduction

- Liquidity problems occur because there is a mismatch between assets and liabilities
- Rarely, such mismatches lead to financial crises
Observation:

- Federal Reserve began purchasing private securities in 2008 (for example, Maiden Lane I, II, and III)
- QE I: $1128 billion in purchase of mortgage-backed securities (February 2009 to July 2010) and $169 billion in purchase of agency securities (September 2008 to April 2010)
Introduction

Question:

- Is this "new" practice optimal?
- Our aim is to build a framework that can answer this question.
Related literature

- Unconventional monetary policy: Williamson (2010), Gertler and Karadi (2011)
- Framework draws on settlement friction a la’ Freeman (1996)
- Diamond-Dybvig (1983) type of liquidity shock
Environment
Time, locations and agents

- Infinite sequence of discrete time periods
- Three islands: creditor, debtor, settlement
- Both creditor and debtor live for 3 periods (OG)
- Continuum of measure one of each type born each period
Environment

Endowments

- Each young creditor endowed with $\kappa$ units of generation-specific capital
- Each young debtor endowed with 1 unit of labor
- Labor is a general input
Environment

Technologies

- Young debtor can instantaneously and linearly transform labor into units of any consumption good
- Young debtor employs capital in either a short term or long term production process
  - short term technology $f(k)$ with properties: one period maturity $f' > 0$, $f'' < 0$
  - long term technology $Af(k)$ with properties: two period maturity $A > 1$
Environment
Preferences

- **Debtor:** $-g(l_t) + u(x_{2,t+1} + x_{3,t+2})$
  - labor is costly and generation-specific consumption good enjoyed either when middle aged or old

- **Creditor:** $v(q_{2,t+1} + q_{3,t+2})$
  - enjoy generation-specific consumption good either when middle aged or old
  - perfect substitutes
Environment

Timing

- Young debtor travels to creditor island (acquires capital from young creditors)
- Young debtor returns and capital is employed in either short term or long term technology
- Young debtor’s labor is used to produce consumption good
- Young creditors stay home
Environment

Timing

- Middle-aged creditors arrive at settlement island
- Short-term production is completed
- All middle-aged debtors arrive at settlement island
- All middle-aged leave for debtor island
Environment

Timing

- Long-term production is completed
- All old debtors arrive at settlement island
- $1 - \alpha$ measure of old creditors arrive at settlement island
- All old leave for debtor island
Planner’s stationary allocation

Consider stationary allocations

\[
\max_{l_1, l_1^*, x_2, x_2^*, x_3, x_3^*, q_2, q_2^*, q_3, q_3^*, k, k^*, \lambda} \theta \left\{ \begin{array}{c}
\lambda \left[-g \left(l_1 \right) + u \left(x_2 + x_3 \right) \right] + \\
\left(1 - \lambda \right) \left[-g \left(l_1^* \right) + u \left(x_2^* + x_3^* \right) \right]
\end{array} \right\} \\
+ (1 - \theta) \left\{ (1 - \alpha) \nu \left(q_2 + q_3 \right) + \alpha \nu \left(q_2^* + q_3^* \right) \right\}
\]

s.t.

\[
\begin{align*}
\lambda \left[l_1 + f \left(k \right) \right] + (1 - \lambda) \left[l_1^* + Af \left(k^* \right) \right] &= \\
\left(1 - \alpha \right) \left(q_2 + q_3 \right) + \alpha \left(q_2^* + q_3^* \right) + \lambda \left(x_2 + x_3 \right) + (1 - \lambda) \left(x_2^* + x_3^* \right)
\end{align*}
\]

\[
\kappa = \lambda k + (1 - \lambda) k^*
\]

\[
0 \leq \lambda \leq 1
\]
Planner’s stationary allocation

The allocation is characterized by

\[
\hat{\lambda} = 0 \\
\hat{k}^* = \kappa \\
\hat{q}_2 + \hat{q}_3 = \hat{q}_2^* + \hat{q}_3^* \\
g' (\hat{l}_1^*) = u' (\hat{x}_2^* + \hat{x}_3^*) \\
g' (\hat{l}_1^*) = \frac{1 - \theta}{\theta} \nu' (\hat{q}_2^* + \hat{q}_3^*)
\]
Planner’s stationary allocation

- Planner allocates all capital to long-term technology
- Creditor’s consumption is independent of travel schedule
Additional issues

- Middle-aged and old agents are endowed with fiat money $M$
- Money is universally verifiable
- IOUs are verifiable but only on settlement island
- IOUs are state contingent with short term interest rate normalized to one and long-term rate equal to $\gamma \geq 1$
- IOUs redeemed by either money or goods
- There a secondary market for IOUs, the price of unredeemed IOUs sold here is denoted $\rho \leq 1$
Travel pattern

- Young debtor exchanges IOU for young creditor’s capital
- Young debtors accept money for labor-produced goods sold to middle-aged and old agents
- Short-term producers settle all debts with middle-aged creditors
- Non-returning (middle-aged) creditors sell unredeemed IOUs
  - potential buyers are short-term producers and returning creditors
- Long-term producers settle all debts with IOU holders
Creditor’s problem

\[
\max (1 - \alpha) \nu (q_2 + q_3) + \alpha \nu (q^*_2 + q^*_3)
\]

s.t.

\[
\left[ \rho (1 + \gamma) (1 - a) + a \right] p_k \kappa = p_x (q_2 + q_3)
\]
\[
\frac{\rho (1 + \gamma) (1 - a) + a}{\rho} p_k \kappa = p_x q^*_3
\]
Lemma 1

In equilibrium, \( \rho (1 + \gamma) = 1 \)

—the non-arbitrage condition
Short-term producer’s problem

\[ \max -g(l_1) + u(x_2 + x_3) \]

s.t

\[ f(k) + l_1 - \frac{p_k}{p_x} k = \rho (x_2 + x_3) \]

FOCs:

\[ f' (k) - \frac{p_k}{p_x} = 0 \]

\[ g' (l_1) - \frac{1}{\rho} u'(x_2 + x_3) = 0 \]
Long-term producer’s problem

\[
\max -g(l_1^*) + u(x_2^* + x_3^*)
\]

s.t.

\[
Af(k^*) + (1 + \gamma) \left( l_1^* - \frac{p_k}{p_x} k^* \right) = x_3^*
\]

FOCs:

\[
Af'(k^*) - \frac{1}{p} \frac{p_k}{p_x} = 0
\]

\[
g'(l_1^*) - \frac{1}{\rho} u'(x_3^*) = 0
\]
Equilibrium

Definition:
(i) debtors and creditors maximize expected lifetime utility, taking prices as given;
(ii) all markets clear; and
(iii) the subjective distribution of production types is equal to the objective distribution of production types
Market clearing conditions

- Usual goods market, capital market, money market and loan market clearing conditions.
- IOU resale market clearing condition:
  \[ \lambda f(k) + l_1^* \geq (1 - \alpha) \rho A f'(k^*) \kappa \]
- The inequality holds if and only if \( \rho = 1 \).
Proposition

**Proposition 1**: The equilibrium measure of long-term producers is strictly positive, or \( \lambda_t < 1 \). With \( \lambda_t > 0 \), both short-term producers and long-term producers choose \( k = k^* = \kappa \).

**Intuition**: If all debts are paid off in short term, there will be no profit in the IOU resale market. A debtor can be better off by choosing the long term production.
Stationary equilibrium

In stationary equilibrium,

\[ g'(l_1^*) = \frac{1}{\rho} u'(x_3^*) , \quad x_2^* = 0 \]

\[ l_1 = l_1^* , \quad x_2 = x_2^* , \quad x_3 = x_3^* \]

\[ k = k^* = \kappa \]

Three mutually exclusive and complementary cases:

- **Case 1:** \( \lambda = 0 , \rho = 1 \), and \( l_1^* \geq (1 - \alpha) A f' (\kappa) \kappa \). (ample liquidity)

- **Case 2:** \( \lambda = 0 , \rho \in (1/A, 1) \), and
  \( (1 - \alpha) f' (\kappa) \kappa \leq l_1^* < (1 - \alpha) A f' (\kappa) \kappa \). (scarce liquidity)

- **Case 3:** \( \lambda = \frac{(1 - \alpha) f'(\kappa) \kappa - l_1^*}{f(\kappa)} , \rho = \frac{1}{A} \), and \( l_1^* < (1 - \alpha) f' (\kappa) \kappa \).
  (very scarce liquidity)
**Stationary equilibrium – an example**

Utility fcns: 

\[ -g(l_1) + u(x_2 + x_3) = \phi \sqrt{1 - l^2} + \frac{(x_2 + x_3)^{1-\sigma}}{1-\sigma}, \]

\[ \psi(q_2 + q_3) = \frac{(q_2 + q_3)^{1-\sigma}}{1-\sigma}, \quad \sigma = 1.5, \quad \phi = 4. \]

Production fcn: 

\[ f(k) = k^{1/3}, \quad A = 1.5, \quad \kappa = 1. \]
Proposition 2: The stationary equilibrium in the decentralized economy achieves the planner’s allocation for some welfare weight $\theta$ if and only if the equilibrium is in case 1.
A three-step central bank operation

1. The central bank issues money to buy IOUs at $\rho = 1$.
2. IOUs are redeemed by long-term producers using consumption goods next period.
3. The central bank sells consumption goods on the debtor island to redeem money. Also
4. A lump-sum tax-transfer scheme to achieve the welfare weights.
A three-step central bank operation

With the central bank policy

- No profit in the IOU resale market. No short term production.
- No distortion in debtor’s intertemporal marginal substitution.
- The total money stock is constant.
- As long as $\hat{I} \leq (1 - \alpha) Af' (\kappa) \kappa$, there is no active central bank trading in the IOUs resale market.
Fisherian creditors

- The preference of the non-returning creditors is modified to $u(q_{2t+1})$. All others remain the same.
- Old-age consumption cannot substitute for middle age consumption.
- The stationary equilibrium in the decentralized economy without policy intervention does not change.
Planner’s allocation

- Additional constraint: $\lambda [l_1 + f(k)] + (1 - \lambda) l_1^* \geq (1 - \alpha) q_2$

Three mutually exclusive and complementary cases:

1. perfect risk sharing
2. rationing with all long-term production
3. mixed short-term and long-term production
Optimal policy

To implement the planner’s allocation, the central bank use the same 3-step operations,

- except that \( \rho = \frac{u' (\hat{x}_3^*)}{g' (\hat{l}_1^*)} \)
- the lump-sum tax-transfer scheme is bounded from implementing transfers to the early-settling creditors
- there is no active central bank trading if \( \hat{q}_2 \leq Af' (\kappa) \kappa \) in PA I and \( \hat{q}_2 = \frac{u' (\hat{x}_3^*)}{g' (\hat{l}_1^*)} Af' (\kappa) \kappa \) in PA II and III.
Conclusion

- We construct a model to think about unconventional monetary policy.
- Unconventional monetary policy alleviates liquidity problem in the private debt market.
- Monetary policy is not only about the total quantity of money (aggregate money growth rate), it is also about the amount of liquidity in a specific market.