Dynamic Adverse Selection: A Theory of Illiquidity, Fire Sales, and Flight to Quality

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August, 2011
Summer Workshop on Money, Banking, Payments and Finance
adverse selection as a source of illiquidity

- sellers can always sell an asset for a low price
- owners of good assets demand a high price in an illiquid market

one possible explanation for fire sales in asset markets in 2007–2008

asset purchase program can raise prices and alleviate illiquidity

contrast this with a more standard “pooling” equilibrium
Some Literature

- adverse selection with pooling:

- adverse selection with separation:
  - De Marzo and Duffie (1999), Guerrieri, Shimer and Wright (2010), Chang (2010)

- illiquidity and search frictions:
  - Duffie, Garleanu and Pederson (2005), Weill (2008), Lagos and Rocheteau (2009)
Model
Model

- unit measure of risk-neutral, infinitely-lived consumers
  - stochastic discount factor, i.i.d.
  - \( \beta_s \) with probability \( \pi_s \), \( s \in \{l, h\} \)
  - later we allow for a Markov process

- fixed supply of heterogeneous trees
  - type \( j \in \{1, \ldots, J\} \) tree produces \( \delta_j \) units of fruit per period
  - \( \delta_{j+1} > \delta_j > 0 \), measure \( K_j \) of type \( j \) trees

- fruit is perishable

- low \( \beta \) consumers sell trees to high \( \beta \) consumers

- the owner of a tree knows its type \( j \), but no one else does
each agent owns a portfolio of trees \( \{k_j\} \)
trees produce fruit
discount factors are realized
buyers and sellers choose prices \( p \in \mathbb{R} \)
trade occurs
agents consume their remaining fruit
Key Equilibrium Objects

- $\Theta(p) \in [0, \infty]$: buyer-seller ratio at price $p$
  - sell a tree at $p$ with probability $\min\{\Theta(p), 1\}$
  - buy a tree at $p$ with probability $\min\{\Theta(p)^{-1}, 1\}$

- $\Gamma(p) \in \Delta^J$: probability distribution over types at price $p$
  - $\gamma_j(p)$ is the fraction of type $j$ trees offered at price $p$

- $\mathbb{P}$: set of prices with trade

- $F$: cumulative distribution of prices
Equilibrium
can solve everything on a per-tree basis (Proposition 1)

- $v_{s,j}$: value of a type $j$ tree to a consumer in preference state $s$
- $\bar{u}_j = \pi_h v_{h,j} + \pi_l v_{l,j}$: continuation value

equilibrium is a vector $(v_h, v_l, \Theta, \Gamma, P, F)$
buyers’ optimality:

\[
\nu_{h,j} = \max_p \left( \min\{\Theta(p)^{-1}, 1\} \frac{\delta_j}{p} \beta_h \sum_{j'} \gamma_{j'}(p) \bar{v}_{j'} + (1 - \min\{\Theta(p)^{-1}, 1\}) \delta_j \right) + \beta_h \bar{v}_j
\]
buyers’ optimality:

\[ v_{h,j} = \delta_j \max_p \left( \min\{\Theta(p)^{-1}, 1\} \frac{\beta_h \sum_{j'} \gamma_{j'}(p) \bar{v}_{j'}}{p} + (1 - \min\{\Theta(p)^{-1}, 1\}) \right) + \beta_h \bar{v}_j \]
Equilibrium

buyers’ optimality:

\[ v_{h,j} = \delta_j \lambda + \beta_h \bar{v}_j, \]

where

\[ \lambda \equiv \max_p \left( \min\{\Theta(p)^{-1}, 1\} \frac{\beta_h \sum_{j'} \gamma_{j'}(p) \bar{v}_{j'}}{p} + (1 - \min\{\Theta(p)^{-1}, 1\}) \right) \]
Equilibrium

-buyers’ optimality:

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where

\[ \lambda \equiv \max_p \left( \min\{\Theta(p)^{-1}, 1\} \frac{\beta_h \sum_{j'} \gamma_{j'}(p) \bar{v}_{j'}}{p} + (1 - \min\{\Theta(p)^{-1}, 1\}) \right) \]

-active markets: \( p \in \mathbb{P} \Rightarrow p \) solves the above problem
sellers’ optimality:

\[ v_{l,j} = \delta_j + \max_p \left( \min\{\Theta(p), 1\} p + (1 - \min\{\Theta(p), 1\}) \beta_l \bar{v}_j \right) \]
sellers’ optimality:

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rational beliefs: if \( \Theta(p) < \infty \) and \( \gamma_j(p) > 0 \),

\[ v_{l,j} = \delta_j + \min\{\Theta(p), 1\} p + (1 - \min\{\Theta(p), 1\}) \beta_l \bar{v}_j \]
all sellers’ trees are offered for sale at some price $p \in \mathbb{P}$:

$$\frac{K_j}{\sum_{j'} K_{j'}} = \int_{\mathbb{P}} \gamma_j(p) dF(p)$$

fruit market clears:

$$\pi_h \sum_j \delta_j K_j = \pi_l \sum_j K_j \int_{\mathbb{P}} \Theta(p) p dF(p)$$
Characterization

- equilibrium exists and is unique
- equilibrium is separating
- algorithm for finding an equilibrium
  - fix $\lambda \in [1, \beta_h/\beta_l]$
  - find a “partial equilibrium”
  - check if fruit-market clears

- next: algorithm to find a partial equilibrium
Buyers’ Indifference Curves

\[ p_j \lambda = \beta_h \bar{v}_j \]

sale probability \( \min\{\theta, 1\} \)

price \( p \)

\( \delta_1 \), \( \delta_2 \), \( \delta_3 \)
Sellers’ Indifference Curves

\[ \theta_j p_j + (1 - \theta_j) \beta_l \bar{v}_j = \theta' p' + (1 - \theta') \beta_l \bar{v}_j \]
Sellers’ Indifference Curves

\[ \theta_j p_j + (1 - \theta_j)\beta_l \bar{v}_j = \theta p' + (1 - \theta')\beta_l \bar{v}_j \]
Sellers’ Indifference Curves

\[ \theta_j p_j + (1 - \theta_j) \beta_l \bar{v}_j = \theta' p' + (1 - \theta') \beta_l \bar{v}_j \]
Continuous Types and Continuous Time
Continuous Types

![Graph showing the relationship between price (p) and sale probability (θ). The sale probability decreases sharply as the price increases.]
Continuous Types

sale probability $\theta$

price $p$

Dynamic Adverse Selection
Continuous Types

Dynamic Adverse Selection

Sale probability $\theta$ vs price $p$.
Closed-Form Solution

- lowest price:  
  \[ P(\delta) = \frac{\delta \beta_h (\pi_l + \pi_h \lambda)}{\lambda - \beta_h (\pi_l + \pi_h \lambda)} \]

- sale probability:  
  \[ \Theta(p) = \left( \frac{P(\delta)}{p} \right)^{\frac{\beta_h}{\beta_h - \beta_l \lambda}} \]

- rate of return decreasing in \( \Theta \), hence increasing in \( \delta \):  
  \[ \frac{\delta + P(\delta)}{P(\delta)} = \frac{\lambda + (\beta_h - \lambda \beta_l) (1 - \Theta(P(\delta))) (1 - \pi_h)}{\beta_h (\pi_l + \lambda \pi_h)} \]
Persistent Types and Continuous Time

- allow preferences to follow a first order Markov process: $\pi_{ss'}$
- useful for taking a continuous time limit of the model
  - $q_{hl}$ and $q_{lh}$ are transition rates for preferences
  - $\rho_h < \rho_l$ are discount rates
- in continuous time, buyers contact sellers at a Poisson rate $\alpha(p)$
- for example, if tree types are dense on $[\delta, \bar{\delta}]$ and $\lambda = 1$:
  \[
  \alpha(p) = \frac{q_{hl} + q_{lh} + \rho_l}{\left(\frac{p}{P(\delta)}\right)^{\frac{q_{hl}+q_{lh}+\rho_l}{\rho_l-\rho_h}} - 1}
  \]
- real trading delays even if trading opportunities are abundant
  - contrast with search theoretic models of illiquidity
Firesales, Flight to Quality, and Asset Purchase Programs
possible explanation for fire sales in asset markets in 2007–2008

“The crisis that can occur with debt is due to the fact that the debt is not riskless. A bad enough shock can cause information insensitive debt to become information sensitive, make the production of private information profitable, and trigger adverse selection. Instead of trading at the new and lower expected value of the debt given the shock, agents trade much less than they could or even not at all. There is a collapse in trade. The onset of adverse selection is the crisis.”
– Dang, Gorton, and Holmström (2009)
suppose initially everyone believes all trees are worth $\delta_0$

suddenly we learn there is dispersion in tree quality

- expected value is $\delta_0$, but $\delta < \delta_0$
- value function $\nu_{s,j}$ is convex, so everyone wants to learn $\delta$
- trees become illiquid, possibly reducing all tree prices
Firesale

sale probability $\min\{\theta, 1\}$

price $p$

0 5 10 15 20

0 1

"Dynamic Adverse Selection"
Firesale

The graph shows the sale probability \( \min\{\theta, 1\} \) as a function of the price \( p \). The probability decreases sharply as the price increases, indicating adverse selection in a dynamic setting.
Imagine there are two types of trees:

- Potential adverse selection problem for type $a$ trees
- No adverse selection problem for type $b$ trees
- All fruit are perfect substitutes

Emergence of adverse selection reduces $\lambda$ if originally $\lambda > 1$:

- The price of type $b$ trees increases
- Interpret this as a flight to quality
suppose “government” can offer to pay $\bar{p} > p$ for any tree

new equilibrium:

- minimum price in private market is $\bar{p}$
- government buys trees which, if completely liquid, are worth less
- other trees stay in the private market, prices & liquidity increase
- price of another type of tree (without adverse selection) falls

if government previously owned some trees, can even be profitable
Asset Purchase Program

![Graph showing sale probability min{\(\theta, 1\)} against price \(p\). The graph is a decreasing curve starting from 1 at price 0 and approaching 0 as price increases.]
Asset Purchase Program

sale probability $\min\{\theta, 1\}$

price $p$
Asset Purchase Program

sale probability $\min\{\theta, 1\}$

price $p$

$\bar{p}$
Asset Purchase Program

sale probability $\min\{\theta, 1\}$

price $p$

$0$ $5$ $10$ $15$ $20$

$0$ $1$

$p$
Pooling Environment
focus with assets dense on $[\delta, \bar{\delta}]$

all trades occur at a common price $p$

value functions: $v_h(\delta) = \delta \lambda + \beta_h \bar{v}(\delta)$ and $v_l(\delta) = \delta + \max\{p, \beta_l \bar{v}(\delta)\}$

seller’s optimality: $\zeta(\delta) = \begin{cases} 1 & \text{if } p > \beta_l \bar{v}(\delta) \\ 0 & \text{if } p < \beta_l \bar{v}(\delta) \end{cases}$

buyers’ optimality: $p \lambda = \beta_h \frac{\int_{\delta}^{\bar{\delta}} \zeta(\delta) \bar{v}(\delta) d\Phi(\delta)}{\int_{\delta}^{\bar{\delta}} \zeta(\delta) d\Phi(\delta)}$

market clearing: $\pi_h \int_{\delta}^{\bar{\delta}} \delta d\Phi(\delta) = \pi_l p \int_{\delta}^{\bar{\delta}} \zeta(\delta) d\Phi(\delta)$
Key Outcome

- trees with $\delta < \delta^*$ are liquid, $\delta > \delta^*$ are illiquid

\[
\delta^* = \frac{\beta_h (1 - \pi_h \beta_h - \pi_l \beta_l)}{\beta_l (\lambda (1 - \pi_h \beta_h) - \pi_l \beta_h)} \frac{\int_{\delta}^{\delta^*} \delta d\Phi(\delta)}{\int_{\delta}^{\delta^*} d\Phi(\delta)}
\]

- possible nonuniqueness

- but see Chari, Shourideh, and Zetlin-Jones
- or assume $\int_{\delta}^{\delta^*} \Phi(\delta) d\delta$ is log concave
Results

- notion of liquidity is dichotomous
- no link between price, dividend, and liquidity
- firesales: dispersion in tree quality weakly reduces the price
- asset purchase program
  - private market price must be $\bar{p}$
  - size of private market after intervention is indeterminate
  - odd behavior if the government caps the size of the program
Results

- notion of liquidity is dichotomous
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- asset purchase program
  - private market price must be $\bar{p}$
  - size of private market after intervention is indeterminate
  - odd behavior if the government caps the size of the program
- using the correct notion of equilibrium matters
solve the following sequence of problems (P-\(j\)):

\[
\nu_{l,j} = \delta_j + \max_{p,\theta} \left( \min\{\theta, 1\} p + (1 - \min\{\theta, 1\}) \beta_l \bar{v}_j \right)
\]

s.t. \( p \leq \beta_h \bar{v}_j \),
\[
\nu_{l,j'} \geq \delta_{j'} + \min\{\theta, 1\} p + (1 - \min\{\theta, 1\}) \beta_l \bar{v}_{j'} \quad \text{for all} \quad j' < j
\]
\[
\bar{v}_j = \pi_h (\delta_j + \beta_h \bar{v}_j) + \pi_l \nu_{l,j}
\]

solution is unique, except \(\theta_1 \geq 1\) (Lemma 1)

pin down \(\theta_1\) to ensure fruit market clears

\[
\pi_h \sum_j \delta_j K_j = \pi_l \sum_j \theta_j p_j K_j
\]

if this defines \(\theta_1 < 1\), look for a different type of equilibrium