A Model of Commodity Money with Minting and Melting

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August 18, 2011
Any study of the money supply [of medieval Europe] needs to take account not only of the total face value of the currency, but also of the metals and denominations of which it is composed.

(Mayhew 2004)
What were these denominations?

- 800-1200 A.D. most European states issued only one coin type - a penny containing \( \sim 1.7 \) gms fine silver
Two major changes to European monetary systems:

- Debasement of the penny - to a varying extent across mints
  - In England - in 1160, still \( \sim 1.4 \text{ gms} \)
  - In Venice - in 1160, \( \sim 0.10 \text{ gms} \)

- Introduction of a larger coin - at different times across mints
  - In Venice - grosso 1194: 2.18 gms (26d)
  - in England - groat 1351: 4.66 gms (4d)
What drove the changes?

- **Conventional view - debasement:**
  - debasements were revenue generators
  - debasements created more units of money so facilitated more exchange

- **Conventional view - larger coins:**
  - large coins were needed to pay urban workers

- **Our view**
  - changes in coin types were consistent with welfare increasing responses to change in the economic environment
What drove the changes?

- we build a random matching model to assess these views

- the paper extends existing search models:
  - to allow for multiple coins
  - to allow for an endogenous quantity of money
We find that:
- the size of a coin affects social welfare
- the size of a coin has distributional consequences
- the frequency of trade affects the optimal coin size
- the stock of monetary metal affects optimal coin size
- permitting minting of two types of coin *may* raise social welfare
We use these results to reconsider the motives for coinage changes:

- debasement may have been a response to urbanization rather than (only) generating revenue or making 'more' units of the medium of exchange

- large coins may have been a response to silver discoveries rather than a response to urbanization
Outline of talk

- Model
- Results
- Apply model to historical choices of denomination
- Conclude/further research
Environment

- Time discrete and infinite
- One nonstorable, perfectly divisible consumption good
- One storable metal (silver) in **fixed supply** \((m)\)
Silver can be held as coins or jewelry (bullion)

Silver coins are indivisible, but can be minted or melted

Silver coin contains $b_1$ ounces of silver
  - possible second silver coin contains $b_2 = \eta b_1$ ounces of silver
Agents hold

- $s_1$ small silver coins
- $s_2$ large silver coins
- $j$ units silver jewelry

⇒ Only coins can be used in trade

⇒ Only jewelry yields utility (similar to Velde-Weber)
Agents

- [0, 1] continuum, infinitely-lived
- Preferences:
  
  \[ u(c) - q + \mu(b_1) - \gamma(s_1 + s_2) \]

  \[ u(0) = 0, \quad u' > 0, \quad u'(0) = \infty, \quad u'' < 0 \]

  \( \gamma \) utility cost of holding a coin

- Maximize expected discounted (\( \beta \)) lifetime utility
- \( \theta \) prob of a being a buyer or seller in a single coincidence match
Each period has two subperiods

1. First subperiod: decentralized trade in bilateral matches
   - Preference assumption rules out double coincidence matches
   - Past trading histories private (no monitoring or commitment technology) - rules out gift-giving equilibrium
   - Agents are anonymous - rules out credit

2. Second subperiod: agents can alter coin/jewelry portfolio by minting or melting
   - Can change how metal stocks held – no change in quantity of silver
Choices

1st sub period

- Single coincidence matches: potential consumer makes TIOLI offer \((q, p_1, p_2)\)
- Buyer ‘sees’ seller’s portfolio

2nd sub period

- Agents make portfolio adjustment after trade \((z_1, z_2)\)
- \(z_i\) is the amount of coins minted (melted if negative)
Model: Value functions

- Expected value of holding $y_t = (s_{1t}, s_{2t}, j_t)$ beginning second subperiod

$$v_t(y_t) = \max_{z_{1t}, z_{2t}} \left\{ \beta w_{t+1}(s_{1t} + z_{1t}, s_{2t} + z_{2t}, j^s_t - z_{1t} - \eta z_{2t}) - S(z_{1t}, z_{2t}; j_t) \right\}$$

$S(z_{1t}, z_{2t}; j_t)$ is seigniorage
Model: Value functions

- Expected value of holding $y_t$ beginning of first subperiod

$$w_t(y_t) = \theta \sum_{\tilde{y}_t} \pi_t(\tilde{y}_t) \max_{\Lambda} [u(q_t) + \nu_t(s_{1t} - p_{1t}, s_{2t} - p_{2t}, j_t)] + (1 - \theta) \nu_t(y_t) + \mu(b_1j_t) - \gamma(s_{1t} + s_{2t})$$

where:

- $\Lambda = \text{set of all feasible TIOLI offers}$
- $\pi_t(y_t) = \text{fraction of agents with } y_t \text{ beginning first subperiod}$
- $\tilde{y}$ denotes seller portfolios
Model: Equilibrium

- Steady state symmetric equilibrium:
  - Value functions $w, v$; asset holdings $\pi$; and quantities $p_1, p_2, z_1, z_2, q$ that satisfy
    - 1 Bellman equations
    - 2 asset transitions
    - 3 market clearing
Results

- Numerical – analytic results not possible

Assume:

\[ \beta = 0.9 \]
\[ \sigma = 0.04 \]
\[ \gamma = 0.001 \]
\[ u(q) = q^{1/4} \]
\[ \mu(b_1j) = 0.05(b_1j)^{1/2} \]

Base case:

\[ \theta = \frac{1}{3} \]
\[ m = 0.1 \]
Results

- Social Welfare depends on coin size
- Welfare distribution depends on coin size
- Optimal coin size depends on trading frequency
- Optimal coin size depends on quantity of metal
- Adding a second coin type may increase welfare
Social Welfare depends on coin size
Welfare distribution depends on coin size
Optimal coin size depends on trading frequency
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Single coin: Welfare effect of changing coin size

Commodity money
Social Welfare depends on coin size
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Single coin: Distribution of coin and jewelry holdings

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Social Welfare depends on coin size
Welfare distribution depends on coin size
Optimal coin size depends on trading frequency
Optimal coin size depends on quantity of metal
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Distribution of welfare
Results

- Social Welfare depends on coin size
- Welfare distribution depends on coin size
- **Optimal coin size depends on trading frequency**
- Optimal coin size depends on quantity of metal
- Adding a second coin type may increase welfare
Optimal coin size depends on trading frequency
Results

- Social Welfare depends on coin size
- Welfare distribution depends on coin size
- Optimal coin size depends on trading frequency
- **Optimal coin size depends on quantity of metal**
- Adding a second coin type may increase welfare
Optimal coin size depends on quantity of metal

![Diagram showing the relationship between ex ante welfare and silver coin size for different values of m (metal proportion). The curves peak at different points, indicating the optimal coin size for each m value.]
Results

- Social Welfare depends on coin size
- Welfare distribution depends on coin size
- Optimal coin size depends on trading frequency
- Optimal coin size depends on quantity of metal
- Adding a second coin type may increase welfare
Adding a second coin type *may* increase welfare.
Shift to smaller coins

- Pennies in 800 A.D. were $\sim 1.7$ gms of fine silver

- By 1160
  - In England still $\sim 1.4$ gms
  - In Venice $\sim 0.10$ gms
Motives for smaller coins

- The model suggests that optimal coin size depends on trading frequency

- Venice urbanized earlier and much more than England
  - Venice urbanized from 1000 AD
  - English market towns grew especially after 1250

- This difference in debasement policy is consistent with a social welfare maximizing response to urbanization
Introduction of grossi and groats

- In 1194 Venice introduced large silver coins
  - grossi weighing 2.18 gms of 96.5% fine silver
  - contained the same fine silver as about 26 denari

- Not until 1351 did the English produce large silver coins
  - groats weighing 4.66 gms of 92.5% fine silver
  - contained the same fine silver as 4 pence.
Silver flows

- The model suggests that larger stocks of silver imply larger optimal coin size

- The late 12th century saw large increases in silver
  - 1160-1320 known for the large amounts of silver mined
  - Flows (from Saxony) went first to Venice
  - in England inflows came later

- The introduction of grossi and groats may have been motivated by the larger silver stocks
Money stock in England

![Graph showing the money stock in England from 1240 to 1360. The y-axis represents Pence per capita, ranging from 0 to 100, and the x-axis represents years, from 1240 to 1360. The graph shows a trend where the money stock increases from around 20 Pence per capita in 1240 to a peak of around 80 Pence per capita in 1330, before decreasing to around 40 Pence per capita by 1360.](image-url)
Two coins with varying metal stocks

Commodity money
Conclusion

- Coin size/type affects welfare in the economy
- Debasement of the penny is consistent with a monetary policy that valued social welfare
- Silver inflows in the 13th century give a rationale for increasing coin sizes
Next direction - outstanding issues

- Why debase rather than introduce a second (smaller) coin?
- Build a model where agents benefit from a large gold coin