Inflation and Welfare with Search and Price Dispersion

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Motivation

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- There are two major sources of the welfare cost of inflation:
  - Opportunity Cost of Holding Money (monetary friction)
    - The social cost of creating non-interest-bearing money is zero, while the private cost of holding money is positive under inflation.
  - Relative Price Dispersion
    - Price dispersion creates a gap between marginal utility and marginal cost, thus distorting production and consumption away from efficiency.
- Study the interaction of two channels and its implication on welfare cost.
- Burstein and Hellwig (2008): menu cost model with money-in-utility
- Aruoba and Schorfheide (2010): Lagos-Wright with Calvo pricing
- Small welfare cost of inflation and no interaction of two channels.
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  - The circulation of money and the existence of price dispersion are only driven by consumer search.

- Calibrate the model to the U.S. data on money demand and price dispersion and estimate the welfare cost of inflation.
  - The welfare cost of 10\% annual inflation is worth 3.23\% of the consumption in the economy with zero inflation.
  - The cost is more than the sum of the effects through each individual channel.
  - Decompose the welfare effect of inflation by different channels, identify the source of inefficiency, and understand the interaction of different channels.

- Three channels: real balance channel, price posting channel, and search channel.
  - The coexistence of the price posting channel and the real balance channel generates an amplifying negative effect on welfare.
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Related Literature

- **Welfare Cost of Inflation:**

- **Endogenous Price Dispersion:**
  - Burdett and Judd (1983), Head, Liu, Menzio, and Wright (2011)
THE MODEL
The Model

- A continuum of homogeneous buyers and sellers, each with measure 1.

- Time is discrete. Each period is divided into two subperiods:
  - 1\textsuperscript{st} subperiod: bilateral trade, decentralized market (DM)
  - 2\textsuperscript{nd} subperiod: Walrasian trade, centralized market (CM)

- Money is the essential medium of exchange for bilateral trades in the DM, and it is perfectly storable and divisible.
  - Money supply follows $M_{t+1} = (1 + \gamma)M_t$. New money is injected (withdrawn) equally to each buyer in a lump-sum transfer at the beginning of the CM.
  - In a stationary equilibrium, $\phi_t M_t = \phi_{t+1} M_{t+1}$ and $\phi_t / \phi_{t+1} = 1 + \gamma$, $\phi_t$ is the price of money in terms of the CM consumption goods in period $t$. 
Timeline

DM

↑

New money is injected or withdrawn.

↑

Sellers and buyers adjust money holdings, produce and consume the CM goods.

CM

β

↑

Sellers post prices for the DM goods, and buyers choose search intensities.

↑

DM

↑

Buyers choose the amount of money to spend.

↑

CM

Sellers produce, buyers consume; then they return to the CM.
Buyer’s Problem
Centralized Market (CM)

- Buyer’s value function in the CM is

\[ W^b(z) = \max_{x,h,\hat{z}} \left\{ v(x) - h + \beta V^b(\hat{z}) \right\} \]

\[ s.t. \ h + z + T = x + (1 + \gamma)\hat{z} \]

- \( x \): CM consumption; \( h \): CM labor; \( z \) and \( \hat{z} \): real balance of the current and the next period; \( T = \gamma \phi M \) is the lump-sum transfer.
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  - \( x \): CM consumption; \( h \): CM labor; \( z \) and \( \hat{z} \): real balance of the current and the next period; \( T = \gamma \phi M \) is the lump-sum transfer.

- The CM value function is linear in \( z \), \( W^b(z) = W^b(0) + z \).

  - The optimal choice of \( x \) and \( \hat{z} \) are independent of \( z \),

  \[ v'(x^*) = 1, \ \beta \frac{\partial V^b(\hat{z}^*)}{\partial \hat{z}} = (1 + \gamma). \]
Buyer’s Problem
Decentralized Market (DM)

- The buyer takes as given the price distribution $F(p)$ with support $\mathcal{Z}_F$, chooses search intensity $\alpha \in [0, 1]$, and pays a real search cost $\alpha k$. 

More
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  - With probability $\alpha$, the buyer samples two prices.
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  - With probability $\alpha$, the buyer samples two prices.

- The buyer’s optimization problem in the DM is

\[
V^b(z) = \max_{\alpha \in [0,1]} \left\{ \int_{\mathcal{Z}_F} \left[ u \left( \frac{d^*(p; z)}{p} \right) + W^b [z - d^*(p; z)] \right] dG(p; \alpha) - \alpha k \right\}
\]

$d^*(p; z)$ is the buyer’s optimal shopping rule for the DM goods, and $G(p; \alpha)$ is the distribution of transaction prices, defined as

\[
G(p; \alpha) = (1 - \alpha)F(p) + \alpha \left[ 1 - (1 - F(p))^2 \right]
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$$G(p; \alpha) = (1 - \alpha)F(p) + \alpha \left[ 1 - (1 - F(p))^2 \right]$$

- $\alpha^* \in (0, 1)$ only if $\int_{\mathcal{Z}_F} \left[ u\left( \frac{d^*(p; z)}{p} \right) - d^*(p; z) \right] [1 - 2F(p)]dF(p) = k.$
In the DM, the buyer’s optimal shopping rule $d^*(p; z)$ is the solution to

$$\max_d u \left( \frac{d}{p} \right) - d,$$

$$s.t. \ d \leq z$$
Buyer’s Problem
Optimal Shopping Rule in the DM

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\[
\max_d u\left(\frac{d}{p}\right) - d,
\]

s.t. \( d \leq z \)

If the DM utility function \( u(q) \) has the CRRA form with coefficient \( \sigma < 1 \), then

\[
d^*(p; z) = \begin{cases} 
  z, & \text{if } p < \hat{p} \\
  d^*(p), & \text{otherwise}
\end{cases}
\]
Buyer’s Problem
Optimal Shopping Rule in the DM

\[ d^*(p;z) \]

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Buyer’s Problem
Optimal Shopping Rule in the DM

\[ d^*(p; z) \]

\[ z \]

\[ p \]
Seller’s Problem
Profit Function in the DM

Given $z, \alpha, d, \text{ and } F$, the seller's profit function in the DM is

$$\pi(p) = \left[1 - \alpha + 2\alpha(1 - F(p))\right] \left(d(p; z) - c \frac{d(p; z)}{p}\right).$$

The trade-off between profit per trade and expected trade volume generates endogenous price dispersion.
Given $z$, $\alpha$, $d$, and $F$, the seller's profit function in the DM is

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- The trade-off between profit per trade and expected trade volume generates endogenous price dispersion.

At the upper limit $\bar{p}$, the profit function becomes

$$\pi(\bar{p}) = (1 - \alpha) \left[d(\bar{p}; z) - c \frac{d(\bar{p}; z)}{\bar{p}}\right].$$

- The seller chooses $\bar{p}$ to maximize $\pi(\bar{p})$, and $\bar{p}^* = \max\{\hat{p}, \bar{p}\}$.

  * Recall that $\hat{p}$ is the cut-off price in the buyer’s optimal shopping rule.
Seller’s Problem

Upper Limit of Price Distribution
For any price $p$ on the support $\mathcal{Z}_F$, $p \in \arg\max_p \pi(p)$, in particular,

$$\pi(p) = [1 - \alpha + 2\alpha (1 - F(p))] \left( d(p; z) - c \frac{d(p; z)}{p} \right) = \pi(\bar{p})$$

Given the buyer’s real balance $z$, search intensity $\alpha$, and shopping rule $d$, $F(p)$ in the decentralized market is uniquely characterized as

- if $\alpha = 0$, $F(p)$ is concentrated at $\bar{p}$.
- if $\alpha = 1$, $F(p)$ is concentrated at $c$.
- if $\alpha \in (0, 1)$, $F(p)$ is nondegenerate and continuous with a connected support $\mathcal{Z}_F = [\underline{p}, \bar{p}]$, and it is given by

$$F(p) = 1 - \frac{1 - \alpha}{2\alpha} \left[ \frac{d(\bar{p}; z)(1 - c \bar{p})}{d(p, z)(1 - c \bar{p})} - 1 \right].$$
In the centralized market, the seller’s value function is

$$W^S(z) = \max_{x,h,\hat{z}} v(x) - h + \beta [\pi^* + W^S(\hat{z})]$$

subject to

$$h + z = x + (1 + \gamma)\hat{z}$$

In equilibrium sellers do not carry any money into the decentralized market, i.e., $\hat{z}^* = 0$. 
A symmetric stationary monetary equilibrium (SSME) is a profile \( \{ F^*, z^*, x^*, h^*, d^*, \alpha^* \} \) satisfying the following conditions:

1. Given \( d^*, z^*, \) and \( \alpha^* \), \( F^* \) is consistent with the seller’s optimal price posting strategy in the DM.
2. Given \( F^*, d^*, \) and \( \alpha^*, z^*, x^*, \) and \( h^* \) solve the buyer’s optimization problem in the CM.
3. Given \( F^* \) and \( z^* \), the buyer’s optimal shopping rule in the DM is characterized by \( d^* \).
4. Given \( F^*, z^*, \) and \( d^*, \alpha^* \) solves the buyer’s search problem in the DM.
Equilibrium

- There exists no SSME with $\alpha = 1$ or $\alpha = 0$ if $1 + \gamma > \beta$. 
Equilibrium

- There exists no SSME with $\alpha = 1$ or $\alpha = 0$ if $1 + \gamma > \beta$.

- If $1 + \gamma > \beta$ and the buyer’s decentralized-market utility function has the CRRA form with coefficient $\sigma < 1$, SSME exists with nondegenerate price distribution for $k \leq \bar{k}$. 
Equilibrium
Three Channels

- Real Balance Channel
  - If buyers carry a smaller real balance, welfare decreases.

- Price Posting Channel
  - If sellers post higher real prices, welfare decreases.

- Search Channel
  - If buyers search harder, welfare increases.
Equilibrium
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- Real Balance Channel
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  - If $\gamma$ gets bigger, buyers bring less money, price distribution increases in the sense of first order stochastic dominance, and they will search less intensively.

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Buyer’s Optimal Shopping Rule in the DM
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- **Search Channel**
  - If buyers search harder, welfare increases.
  - If a higher search intensity is chosen, buyers will carry more money, and the price distribution decreases in the sense of first order stochastic dominance.
Equilibrium
Buyer’s Optimal Shopping Rule in the DM

The diagram illustrates the relationship between the price $p$ and the demand $d^*(p)$, with a horizontal line at $z$ indicating a constant level of demand. The arrows and dashed lines highlight the changes in demand as the price varies. The points $p$, $\hat{p}$, and $\bar{p}$ are significant price levels, with $\hat{p}$ being a critical threshold. The curve $d^*(p)$ shows the inverse relationship between price and demand.
CALIBRATION
Calibration

Functional Forms and Calibration Strategy

Centralized Market (CM): \( v(x) = A \log x - h \)

Decentralized Market (DM): \( u(q) = \frac{q^{1-\sigma}}{1-\sigma} \) and \( c(q) = q \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta ) discount factor</td>
<td>annual real interest rate 4%</td>
</tr>
<tr>
<td>( k ) search cost</td>
<td>magnitude of price dispersion</td>
</tr>
<tr>
<td>( A ) CM preference</td>
<td>money demand</td>
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<td>( \sigma ) elasticity of demand</td>
<td>money demand</td>
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Model-generated money demand function

\[ L(i) = \frac{M/P}{Y} = \frac{z}{2A + \int_p^\bar{p} \frac{d(p;z)}{p} dG(p;\alpha)}, \]

where \( i \) is nominal interest rate.

U.S. annual data on nominal GDP, M1, and short-term (6-month) commercial paper rate from 1900 to 2000. (Lucas 2000, Lagos & Wright 2005)
Relative Price Variability (RPV)

\[ RPV = \left[ \int_p^{\bar{p}} (R_i - \bar{R})^2 dF \right]^{\frac{1}{2}} \]

where \( R_i = \log(p_i / \bar{p}) \).

Targets from empirical studies on inflation and price dispersion based on annual U.S. price data of the retail sector.

- Average RPV 0.035 at annual inflation rate of 4.3% (Debelle & Lamont, 1997)
- Average RPV 0.0923 at 5.3% (Parsley, 1996)
Calibration
Results and Model Fitness

- Benchmark: match RPV target 0.035 and money demand.

<table>
<thead>
<tr>
<th>$\beta$</th>
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<th>$\mu_{DM}$</th>
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- $\mu_{DM}$ and $\mu$ stand for markup in the decentralized market and average markup, respectively.

- Model Fitness:
  - Model-generated money demand function vs. money demand data.
Benchmark: match RPV target 0.035 and money demand.

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Model Fitness:

- Model-generated money demand function vs. money demand data.
- Price dispersion is regressed against inflation.
  - In the model, the coefficient of inflation is 0.2784.
  - In Debelle and Lamont (1997) the range of coefficients is (0.12, 0.39) with an average of 0.21.
WELFARE ANALYSIS
Welfare Cost of Inflation

- Social welfare is the total surplus from trade in the centralized market and the decentralized market minus the cost of search.

- Welfare cost of inflation is measured by compensated consumption
  - How much agents would be willing to increase or decrease their consumption in the benchmark equilibrium with zero inflation in order to be indifferent to the economy with $\tau$ percent inflation?

- The welfare cost of 10% annual inflation is worth 3.23% of consumption.

- Compare with previous studies:

<table>
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<th>Model</th>
<th>Cooley &amp; Hansen</th>
<th>Lucas</th>
<th>Lagos &amp; Wright</th>
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<td>0.52%</td>
<td>&lt;1%</td>
<td>4.6%</td>
<td>1.31%</td>
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Welfare Cost of Inflation

Welfare Cost of Inflation, 0% inflation as benchmark

Compensating Consumption %

Inflation 1+γ
Welfare Cost Decomposition

1. Solve the equilibrium real balance, search intensity, and price distribution of the model at different levels of inflation.

2. Keep two endogenous objects (channels) at their equilibrium levels, and change the values of the third object (channel) by holding it constant at a benchmark level.

3. Using the welfare of the original economy at zero inflation as the benchmark, calculate the welfare cost of the artificial economy at different levels of inflation.

4. Compare the new welfare cost of inflation with the original value, and the difference represents the contribution of the third channel.
The welfare cost of 10% annual inflation decreases to 0.04% of consumption.
The welfare cost of 10% annual inflation decreases to 0.15% of consumption.
The welfare cost of 10% annual inflation decreases by 0.1% of consumption.
Welfare Cost Decomposition

Definition of Social Welfare: Review

Social welfare is the total surplus from trade in the centralized market and in the decentralized market minus the cost of search

\[(1 - \beta)\mathcal{W}(\tau) = \int_{p}^{\bar{p}} \left[ u \left( \frac{d^*(p; z^*)}{p} \right) - c \frac{d^*(p; z^*)}{p} \right] dG(p; \alpha^*) - \alpha^* k + 2 [v(x^*) - x^*] \]

and \(G(p; \alpha^*)\), the distribution of transaction prices is

\[G(p; \alpha^*) = (1 - \alpha^*) F(p) + \alpha^* \left[ 1 - (1 - F(p))^2 \right].\]
Welfare Cost Decomposition
Shut Down Search Channel

The welfare cost at 10% annual inflation increases to 6.99% of consumption.
Welfare Cost Decomposition

Summary

- If either the real balance or the price posting channel is held constant, the welfare cost significantly decreases from 3.23% to less than 0.15% of consumption.

- The main source of inefficiency resides in the interaction of the real balance channel and the price posting channel.
  
  - The coexistence of these two channels generates an amplifying effect.
  - The total negative effect on welfare exceeds the positive effect due to the search channel.
  - The search channel mostly affects welfare indirectly by changing the distribution of posted prices.
  - The search cost alone generates a negligible welfare loss.
Conclusion

- In this paper, I develop a general equilibrium model with money, price dispersion, and consumer search to study the welfare cost of inflation.
  - Search friction is the only driving force for price dispersion and the circulation of money.

- The welfare cost of 10% annual inflation is worth 3.23% of consumption, and it decreases to less than 0.15% of consumption if either real balance or price distribution is held constant.
  - Calibration exercises with alternative targets and different sample periods generate similar results.
  - Welfare cost is positively correlated with the value of the price dispersion target.

- Inflation has a non-monotonic effect on social welfare, and the optimal monetary policy is above the Friedman rule.
Conclusion

- In this paper, I develop a general equilibrium model with money, price dispersion, and consumer search to study the welfare cost of inflation.
  - Search friction is the only driving force for price dispersion and the circulation of money.

- The welfare cost of 10% annual inflation is worth 3.23% of consumption, and it decreases to less than 0.15% of consumption if either real balance or price distribution is held constant.
  - The price posting channel amplifies the welfare-diminishing effect of the real balance channel, and the aggregated negative effect exceeds the positive effect due to the search channel.
  - The search cost alone generates a negligible welfare loss.
  - Calibration exercises with alternative targets and different sample periods generate similar results.

- Inflation has a non-monotonic effect on social welfare, and the optimal monetary policy is above the Friedman rule.
Recalibrate the model to match different targets of relative price variability (RPV) at 4.3% inflation:

<table>
<thead>
<tr>
<th>RPV</th>
<th>k</th>
<th>A</th>
<th>σ</th>
<th>μ</th>
<th>1 – Δ₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.035</td>
<td>0.0043</td>
<td>0.4916</td>
<td>0.1181</td>
<td>2.47%</td>
<td>3.2%</td>
</tr>
<tr>
<td>0.06</td>
<td>0.0072</td>
<td>0.5396</td>
<td>0.3256</td>
<td>10.65%</td>
<td>12.7%</td>
</tr>
<tr>
<td>0.1</td>
<td>0.0362</td>
<td>1.0373</td>
<td>0.6091</td>
<td>12.38%</td>
<td>15.4%</td>
</tr>
</tbody>
</table>

Welfare cost gets higher with a bigger target of RPV, which implies a higher search cost and less competition in the decentralized market.
Alternative Calibration Targets

- Target for search cost:
  - Average RPV 0.0923 at the annual inflation rate of 5.3%. (Parsley, 1996)
  - DM markup 30% at an annual inflation of 5.46%. (Faig and Jerez 2005)

- Target for elasticity of demand: interest elasticity of money demand

\[
\log z_t = b_0 + b_1 \log i_t + b_2 \log y_t + b_3 \log z_{t-1} + u_t
\]

- assume \( u_t = \rho u_{t-1} + \varepsilon_t \) and apply Cochrane-Orcutt procedure to correct first-order serial correlation. (Goldfeld and Sichel 1990; Aruoba, Waller, and Wright 2010)
## Alternative Calibration Targets

### Calibration Results and Welfare Cost

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Target 2</th>
<th>Target 3</th>
<th>Target 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>0.0043</td>
<td>0.0328</td>
<td>0.0057</td>
<td>0.003</td>
</tr>
<tr>
<td>$A$</td>
<td>0.4916</td>
<td>1.0134</td>
<td>0.5064</td>
<td>0.3309</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.1181</td>
<td>0.5326</td>
<td>0.2111</td>
<td>0.1005</td>
</tr>
<tr>
<td>$\mu_{DM}$</td>
<td>9.72%</td>
<td>47.13%</td>
<td>25.64%</td>
<td>12.21%</td>
</tr>
<tr>
<td>$\mu$</td>
<td>2.4%</td>
<td>8.75%</td>
<td>5.52%</td>
<td>2.95%</td>
</tr>
<tr>
<td>$1 - \Delta_0$</td>
<td>3.23%</td>
<td>8.31%</td>
<td>7.23%</td>
<td>3.25%</td>
</tr>
</tbody>
</table>
## Shorter Sample Period

### 1959-2000

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Target 2</th>
<th>Target 3</th>
<th>Target 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>0.0089</td>
<td>0.033</td>
<td>0.0061</td>
<td>0.0062</td>
</tr>
<tr>
<td>$A$</td>
<td>1.1841</td>
<td>1.5488</td>
<td>0.6175</td>
<td>0.8702</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.1861</td>
<td>0.5262</td>
<td>0.2200</td>
<td>0.1441</td>
</tr>
<tr>
<td>$\mu_{DM}$</td>
<td>8.34%</td>
<td>45.14%</td>
<td>35.32%</td>
<td>9.96%</td>
</tr>
<tr>
<td>$\mu$</td>
<td>1.42%</td>
<td>5.78%</td>
<td>4.79%</td>
<td>1.67%</td>
</tr>
<tr>
<td>$1 - \Delta_0$</td>
<td>2.34%</td>
<td>5.08%</td>
<td>6.68%</td>
<td>2.69%</td>
</tr>
</tbody>
</table>
If $\sigma > 1$, the buyer’s optimal spending rule is

$$d^*(p; z) = \begin{cases} 
  d^*(p), & \text{if } p < \hat{p} \\
  z, & \text{if } \hat{p} \leq p \leq p^R \\
  0, & \text{otherwise}
\end{cases}$$

where $p^R$ satisfies $u(d^*(p^R; z) / p^R) = d^*(p^R; z)$, $\partial p^R / \partial z > 0$.

If $\sigma = 1$, the buyer’s optimal spending rule is

$$d^*(p; z) = \begin{cases} 
  \min\{\tilde{d}, z\}, & \text{if } p \leq p^R \\
  0, & \text{otherwise}
\end{cases}$$

where $\tilde{d}$ is a constant satisfying $u'(\frac{\tilde{d}}{p}) = p$. 

Buyer’s Problem

Optimal Shopping Rule in the DM
Buyer’s Problem
Optimal Shopping Rule in the DM

\[
d^*(p; z)
\]

\[
d^*(p)
\]

Parsley (1996): quarterly survey data from Cost of Living Index published by the American Chamber of Commerce Researchers Association, a panel from 1975 to 1992 with 48 cities and 32 kinds of goods and services, mostly from the retail sector.
Money Demand Data

- Nominal GDP is taken from the Historical Statistics of the United States, Colonial Times to Present (1970) and the GDPA series from the Citibase database.

- Money supply is M1, as of December of each year, and is not seasonally adjusted. It is from the Historical Statistics of the United States (1960), Friedman and Schwartz (1963), and the FRED II database of the Federal Reserve Bank of St. Louis.

The welfare cost of 10% annual inflation decreases to 0.04% of consumption.
The welfare cost of 10% annual inflation is equal to 0.02% of consumption.
Buyer’s Problem: An Alternative Setup

- In the CM, buyers first choose whether they will sample one price or two prices in the next DM

\[ W^b(z) = \max \left\{ W^b_1(z), W^b_2(z) \right\}, \]

and

\[ W^b_i(z) = \max_{x, h, \hat{z}_i} v(x) - h + \beta V^b_i(\hat{z}_i) - ik \]

s.t. \( h + z + T = x + (1 + \gamma)\hat{z}_i \)

for \( i = 1 \) or 2.

- The buyer’s value functions in the DM are

\[ V^b_1(z_1) = \int_{Z_F} \left[ u\left( \frac{d^*(p; z_1)}{p} \right) + W^b [z_1 - d^*(p; z_1)] \right] \, dF(p; \alpha) \]

\[ V^b_2(z_2) = \int_{Z_F} \left[ u\left( \frac{d^*(p; z_2)}{p} \right) + W^b [z_2 - d^*(p; z_2)] \right] \, dG(p; \alpha) \]

where \( \alpha \) is the fraction of total buyers that sample two prices in the DM.
Seller’s Problem: An Alternative Setup

- If buyers first choose the number of prices to sample in DM, the seller’s profit function in the DM now becomes

\[
\pi(p) = (1 - \alpha) \left[ d(p; z_1) - c \frac{d(p; z_1)}{p} \right] + 2\alpha (1 - F(p)) \left[ d(p; z_2) - c \frac{d(p; z_2)}{p} \right]
\]

- For \( \alpha \in (0, 1) \), the nondegenerate price distribution in the DM is characterized as

\[
F(p) = 1 - \frac{(1 - \alpha) \left[ \left( d(\bar{p}; z_1) - c \frac{d(\bar{p}; z_1)}{p} \right) - \left( d(p; z_1) - c \frac{d(p; z_1)}{p} \right) \right]}{2\alpha \left[ d(p; z_2) - c \frac{d(p; z_2)}{p} \right]}
\]

and \( \bar{p} = \max\{\hat{p}(z_1), \tilde{p}\} \).