Bubbles and Credit Constraints

Jianjun Miao\textsuperscript{1}  Pengfei Wang\textsuperscript{2}

\textsuperscript{1}Boston University
\textsuperscript{2}HKUST

November 2011
Motivation: US data
Motivation: US data

- **S&P Price Index**: The graph shows the S&P Price Index from 1990 to 2020, with significant fluctuations over time.
- **Net percentage of bank tightening standards**: The net percentage of bank tightening standards is depicted, with noticeable changes from 1990 to 2020.
- **St. Louis Financial Stress Index**: The St. Louis Financial Stress Index is illustrated, capturing stress periods over the years.

Miao and Wang (BU)
Motivation: International evidence

- Rapid increases in stock prices are linked to
  - heavy capital inflows
  - rapid credit growth
  - rapid investment growth

- Collapse in stock prices were accompanied by
  - capital outflow
  - decreased investment
  - tightened credit
  - recession

Motivation

- Bubbles and crashes are linked to booms and busts in credit market
- Is there any connection between rational bubbles/crashes and credit constraints?
- Provide a theory of credit-driven stock price bubble
Major Historical Examples

- Tulipmania (1634-1638)
- The Mississippi Bubble (1719-1720)
- The South Sea Bubble (1720)
- The Bull Market of the Roaring Twenties (1924-1929)
- dot-com bubble (1990s)
- China stock and property bubble (2007)
- US housing market bubbles
Types of Bubbles

- Credit-driven stock price bubbles
  - Bubbles are accompanied with credit booms
  - Bubbles are on productive assets
  - Bubbles occur in a sector or an industry (Miao and Wang (2011a,b))
  - Recurrent bubbles with firm entry (Miao and Wang (2011c))

- Bubbles in prices of land, gold, paintings, money, etc.

- Rational vs Irrational bubbles (DeLong et al (1990), Scheinkman and Xiong (2003), Xiong and Yu (2011))
Basic Intuition

- Asset pricing equation

\[ P_t = D_t + \frac{P_{t+1}}{1+r} \]

- Exogenous payoffs \( D_t \) and discount rate \( r \)

- Solving forward

\[ P_t = \sum_{s=0}^{\infty} \frac{D_{t+s}}{(1+r)^s} + B_t, \quad B_t = \frac{B_{t+1}}{1+r} \]

- Transversality condition rules out bubbles
Key Issues

- Rational bubbles are fragile (Santos and Woodford (1997))
- A necessary condition is $g \geq r$ or PV of consumption is infinity (Dynamically inefficient OLG)
- Existing theories typically study bubbles on assets with zero payoff or exogenously given payoffs
- What about bubbles on reproducible or productive assets? Dividends are endogenous!
- Do bubbles crowd out or crowd in capital?
- Welfare and policy implications of bubbles?
Firms face stochastic investment opportunities, and borrow to finance investment subject to endogenous credit constraints.

Pledge firm assets (capital) as collateral (borrow against firm value).

Collateral value may contain bubbles.

**Positive feedback loop**

Optimistic beliefs about asset values (bubbles) → raise collateral value → raise lending against these assets → raise investment → raise firm value or asset value → justify initial beliefs.

Bubbles are self-fulfilling.

Another equilibrium without bubble.

Stochastic bubbles, burst of bubbles → recession (No shocks to fundamentals).

Bubbles on both intrinsically useless assets and productive assets can coexist.
A Basic Model

- Discrete time $t = 0, dt, 2dt, \ldots$ Continuous time $dt \to 0$
- No aggregate uncertainty
- Risk neutral (can be relaxed) households supply labor inelastically (one unit).

$$\sum_{t \in \{0, dt, 2dt, \ldots\}} e^{-rt} C_t dt,$$

- Can introduce endogenous labor supply
- A continuum of firms indexed by $j \in [0, 1]$, with technology:

$$Y^j_t = (K^j_t)^\alpha (N^j_t)^{1-\alpha}, \quad \alpha \in (0, 1).$$

- Can introduce capacity utilization
- Solve static labor choice:

$$\max_{N^j_t} F \left( K^j_t, N^j_t \right) - w_t N^j_t = R_t K^j_t,$$
Investment opportunities arrive independently across firms and over time

\[ K_{t+dt}^j = \begin{cases} (1 - \delta dt) K_t^j + I_t^j & \text{with probability } \pi dt \\ (1 - \delta dt) K_t^j & \text{with probability } 1 - \pi dt \end{cases} \]

Can use idiosyncratic investment specific shocks with continuous distribution

\[ K_{t+dt}^j = (1 - \delta dt) K_t^j + \epsilon_t^j I_t^j dt \]

Can use idiosyncratic productivity shocks

\[ Y_t^j = (A_t K_t^j)^\alpha (N_t^j)^{1-\alpha} \]
Credit Constraints

- Intra-period loans (can be relaxed) \( L^j_t \),
  \[
  L^j_t \leq R^j_t K^j_t + L^j_t
  \]

- Assume no equity financing (can be relaxed)

- Let \( V_t (X^j) \) denote the market value of assets \( X^j \)

- Credit constraint:
  \[
  L^j_t \leq e^{-rdt} V_{t+dt}(\zeta K^j_t).
  \]

  - Pledge assets \( \zeta K^j_t \) as collateral (effectively firm value)
  - \( \zeta \) represents financial frictions

- Kiyotaki and Moore (1997) collateral constraint
  \[
  L^j_t \leq \zeta Q_t K^j_t.
  \]
Optimal Contract with Limited Commitment

- Borrow \( L^j_t \) and repay \( L^j_t \) only when investment opportunity arrives
- If firm defaults, lender seizes assets \( \xi K^j_t \)
- Assets are not specific to the owner
- Lender reorganizes the firm and obtains going-concern value \( e^{-rdt} V_{t+dt}(\xi K^j_t) \)
- The firm has all the bargaining power and the lender gets the threat value \( e^{-rdt} V_{t+dt}(\xi K^j_t) \)
- Incentive constraint:

\[
\begin{align*}
\begin{cases}
\underbrace{e^{-rdt} V_{t+dt}((1 - \delta dt) K^j_t + I^j_t) - L^j_t} & \text{Not default} \\
\geq & \\
\underbrace{e^{-rdt} V_{t+dt}((1 - \delta dt) K^j_t + I^j_t) - e^{-rdt} V_{t+dt}(\xi K^j_t)} & \text{Default}
\end{cases}
\end{align*}
\]
Firm’s Problem (Optimal Contract)

- Bellman equation

\[ V_t(K^j_t) = \max_{l^j_t, L^j_t} R_t K^j_t dt - \pi l^j_t dt + e^{-rdt} V_{t+dt}((1 - \delta dt) K^j_t + l^j_t) \pi dt \]
\[ + e^{-rdt} V_{t+dt}((1 - \delta dt) K^j_t)(1 - \pi dt), \]

subject to

\[ l^j_t \leq R_t K^j_t + L^j_t \]
\[ L^j_t \leq e^{-rdt} V_{t+dt}(\bar{\xi} K^j_t) \]

- Not a contraction mapping!
Competitive Equilibrium

- Aggregation: $K_t = \int_0^1 K_t^j dj$, $I_t = \int_0^1 I_t^j dj$, $N_t = \int_0^1 N_t^j dj$, and $Y_t = \int_0^1 Y_t^j dj$
- Households and firms optimize and markets clear

$$
N_t = 1, 
C_t + \pi I_t = Y_t, 
K_{t+dt} = (1 - \delta dt) K_t + l_t \pi dt.
$$
Firm value takes the form (Hayashi (1982)):

$$V_t(K_t^j) = v_t K_t^j$$

Verify:

$$v_t K_t^j = \max_{I_t^j} R_t K_t^j dt - \pi I_t^j dt + e^{-rdt} v_{t+dt} \pi I_t^j dt$$

$$+ e^{-rdt} v_{t+dt} (1 - \delta dt) K_t^j,$$

$$I_t^j \leq R_t K_t^j + e^{-rdt} v_{t+dt} \zeta K_t^j$$

Need $Q_t > 1$ for the investment and collateral constraint to bind
Bubble Solution

- Firm value takes the form:
  \[ V_t(K_t^j) = v_t K_t^j + b_t, \quad b_t > 0 \]
  fundamental  
  bubble

- Positive feedback loop:
  \[ v_t K_t^j + b_t = \max_{l_t^j} R_t K_t^j dt - \pi l_t^j dt + e^{-rdt} v_{t+dt} \pi l_t^j dt + e^{-rdt} v_{t+dt} (1 - \delta dt) K_t^j + e^{-rdt} b_{t+dt}, \]
  \[ Q_t \]
  \[ Q_t \]
  \[ B_t \]

- \( l_t^j \leq R_t K_t^j + e^{-rdt} v_{t+dt} \xi K_t^j + e^{-rdt} b_{t+dt} \)
  \[ Q_t \]
  \[ B_t \]

- Need \( Q_t > 1 \) for the investment and collateral constraint to bind
Continuous Time Equilibrium System

Suppose $Q_t > 1$. Then $(B_t, Q_t, K_t)$ satisfy:

\[
\dot{B}_t = rB_t - B_t \pi (Q_t - 1),
\]

\[
\dot{Q}_t = (r + \delta) Q_t - R_t - \pi (R_t + \xi Q_t)(Q_t - 1),
\]

\[
\dot{K}_t = -\delta K_t + \pi (R_t K_t + \xi Q_t K_t + B_t), \quad K_0 \text{ given},
\]

and the transversality condition:

\[
\lim_{T \to \infty} e^{-rT} Q_T K_T = 0, \quad \lim_{T \to \infty} e^{-rT} B_T = 0,
\]

- Bubbleless equilibrium: $B_t = 0$
- Bubbly equilibrium: $B_t \neq 0$
Why bubbles?

- Santos and Woodford condition? $r > 0$, zero economic growth
- Bubble dynamics

$$\frac{\dot{B}_t}{B_t} + \pi(Q_t - 1) = r$$

- capital gains
- dividend yields

- Bubble is productive!
- TV cannot rule out bubble
- Bubbleless SS is dynamically efficient because $K^* < K_{GR} = (\delta/\alpha)^{\frac{1}{\alpha-1}}$

$\implies$ Tirole (1985) condition does not apply
There exists \((B, Q_b, K_b)\) satisfying
\[
\frac{B}{K_b} = \frac{\delta}{\pi} - \frac{r + \delta + \xi r + \pi}{1 + r} > 0,
\]
\[
Q_b = \frac{r}{\pi} + 1 > 1,
\]
\[
\alpha (K_b)^{\alpha-1} = \frac{(1 - \xi)r + \delta}{1 + r} \left( \frac{r}{\pi} + 1 \right),
\]
if and only if
\[
0 < \xi < \frac{\delta (1 - \pi)}{r + \pi} - r. \tag{1}
\]

In addition, (i) \(Q_b < Q^*\), (ii) \(K_{GR} > K_E > K_b > K^*\), and (iii) the bubble-asset ratio \(B/K_b\) decreases with \(\xi\).
Suppose condition (1) holds. Then both the bubbly steady state \((B, Q_b, K_b)\) and the bubbleless steady state \((0, Q^*, K^*)\) are local saddle points for the nonlinear system for \((B_t, Q_t, K_t)\).

- Different from indeterminancy (Benhabib or Farmer)
Robustness

- Intertemporal Borrowing and Saving
- Idiosyncratic Investment-Specific Shocks: bubbles can arise even if capital can be fully pledgeable, e.g., $\xi = 1$
- Idiosyncratic Productivity Shocks: bubble can generate endogenous TFP through capital reallocation
Stochastic Bubbles

- Suppose a bubble exists initially, $B_0 > 0$.
- Between $t$ and $t + dt$, there is probability $\theta dt$ that the bubble bursts, $B_{t+dt} = 0$. Once it bursts, it will never be valued again in the future so that $B_{\tau} = 0$ for all $\tau \geq t + dt$.
- With probability $1 - \theta dt$, the bubble persists so that $B_{t+dt} > 0$.
- Take the continuous time limits as $dt \to 0$. 
Figure: This figure plots the dynamics of the stationary equilibrium with stochastic bubbles. Assume that the bubble bursts at time $t = 20$. Set the parameter values as follows:

- $r = 0.02$
- $\alpha = 0.4$
- $\delta = 0.025$
- $\theta = 0.05$
- $\pi = 0.01$
- $\xi = 0.2$

Miao and Wang (BU)  
Bubbles and Credit Constraints  
November 2011  
25 / 30
Capacity utilization

- **Production function**
  \[ Y_t^j = (u_t^j K_t^j)^{\alpha} \left( N_t^j \right)^{1-\alpha}, \]  
  where \( u_t^j \) represents the capacity utilization rate.

- **Deprecation rate**
  \[ \delta_t^j = \varphi(u_t^j), \]  
  where \( \varphi \) is an increasing and convex function.

- **At equilibrium**
  \[ u_t^j = u(Q_t), \]  
  where \( u'(Q_t) < 0 \)
Figure: This figure plots the dynamics of the stationary equilibrium with capacity utilization and stochastic bubbles. Set the parameter values as follows:

- $r = 0.02$,
- $\alpha = 0.4$,
- $\pi = 0.01$,
- $\theta = 0.05$,
- $\xi = 0.2$,
- $\gamma = 0.4$,
- $\delta_0 = 0.0075$, and 
- $\delta_1 = 0.0245$.

Miao and Wang (BU) Bubbles and Credit Constraints November 2011 27 / 30
Monetary policy: Lean vs clean debate

What types of bubbles? (Mishkin) What causes bubbles?

Inefficiency comes from credit constraints

Bubbles help relax these constraints, while the collapse of bubbles tightens them

Government can supply liquidity to the firms by issuing public bonds. These bonds are backed by lump-sum taxes.

\[ M_t P_t = T_t dt + M_{t+dt} P_t, \]

Bubble on unbacked public bonds can exist with household short sales constraint.
Suppose assumption (1) holds. Let the government issues a constant value $D = P_t M_t$ of government debt given by

$$D_t = D = K_E \left( \delta \frac{1 - \pi}{\pi} - r - \xi \right) > 0,$$

which is backed by lump-sum taxes $T_t = T = rD$ for all $t$. Then this credit policy will eliminate the bubble on firm assets and make the economy achieve the efficient allocation.

Unbacked public assets (intrinsically useless assets) can have a bubble value when households face short sales constraint. This bubble may coexist with stock price bubbles and boost the economy when stock market bubbles burst. The equilibrium real allocation is identical to that with stock price bubbles only.
Conclusion

- We have provided an infinite-horizon model of a production economy with bubbles on productive assets.
- Bubbles help relax collateral constraints and generate dividend yields.
- Collapse of bubbles have adverse impact on the economy.
- When public assets backed by lump-sum taxes are used as collateral, there exists a credit policy that can eliminate bubbles and make the economy achieve efficiency.