# The Fundamental Surplus in Matching Models

Lars Ljungqvist Thomas J. Sargent*

August 21, 2014

**Abstract**

To generate big responses of unemployment to productivity shocks in a business cycle context or to productivity changes in a welfare state context, researchers have reconfigured matching models in various ways, e.g., by elevating the utility of leisure, by making wages sticky, by assuming alternating-offer wage bargaining, by introducing fixed matching costs, by modelling costly acquisition of credit, or by positing government mandated unemployment benefits, layoff costs and taxes. All of these redesigned matching models increase responses of unemployment to movements in productivity through a common intermediating channel that we call the *fundamental surplus* fraction. The fundamental surplus fraction is an upper bound on the fraction of a job's output that the invisible hand can allocate to vacancy creation. All reconfigured matching models generate larger unemployment responses to movements in productivity by making the fundamental surplus fraction smaller.

Key words: Matching model, market tightness, fundamental surplus, unemployment, volatility, business cycle, welfare state.

*Ljungqvist: Stockholm School of Economics and New York University (email: lars.ljungqvist@hhs.se); Sargent: New York University and Hoover Institution (email: thomas.sargent@nyu.edu). We are grateful for excellent research assistance by Isaac Baley, Cecilia Parlatore Siritto, and Jesse Perla. Comments by participants at the conference on Recursive Methods in Economic Dynamics, in Honor of 25 Years of [the treatise by] Stokey, Lucas, and Prescott, at the Federal Reserve Bank of Minneapolis, as well as by Nicolas Petrosky-Nadeau, were most useful.

# 1 Introduction

Because it summarizes outcomes of labor market frictions without explicitly modelling them, Petrongolo and Pissarides (2001) call the matching function a black box. Under constant returns to scale in matching, a widely used assumption for which Petrongolo and Pissarides (2001) find ample empirical support, a ratio of vacancies to unemployment – 'market tightness' – becomes a key variable that drives unemployment dynamics. Workers' job finding rates and firms' vacancy filling rates are both functions of market tightness. These two rates, together with exogenously specified or endogenously determined job separation rates, determine unemployment and the joint probability distribution of waiting times to filling vacancies and finding jobs in equilibrium models.

To get big responses of unemployment to movements in productivity, a matching model requires a high elasticity of market tightness with respect to productivity. In this paper, we isolate a single intermediating channel through which all economic forces that generate a high elasticity of market tightness with respect to productivity must operate. Understanding how a variety of apparently disparate reconfigured matching models all work through this same channel sheds light on what features are needed to produce big unemployment responses to movements in productivity.

With exogenous separation, comparative steady state analysis shows that the elasticity of market tightness in a variety of matching models can be decomposed into two multiplicative factors, both of which are bounded from below by unity. In a matching model of variety $j$, let $\eta^j_{\theta,y}$ be the elasticity of market tightness $\theta$ with respect to productivity $y$:

$$\eta^j_{\theta,y} \;=\; \Upsilon^j \, \frac{y}{y - x^j}. \tag{1}$$

The first factor $\Upsilon^j$ has an upper bound inherited from a consensus in the literature about what constitutes a reasonable range for the elasticity of matching with respect to unemployment. The second factor, $y/(y - x^j)$, is the inverse of what we define as the 'fundamental surplus' fraction. Unlike $\Upsilon^j$, the fraction $y/(y-x^j)$ has no widely agreed upon upper bound. To get a high elasticity of market tightness requires that $y/(y - x^j)$ be large, i.e., that what we call the fundamental surplus fraction be small.[1]

The fundamental surplus $y - x$ equals a quantity that deducts from productivity $y$ a value $x$ that the 'invisible hand' cannot allocate to vacancy creation in an equilibrium.

[1] We call $y - x$ the fundamental surplus and $\frac{y-x}{y}$ the fundamental surplus fraction.

Table 1: Elasticities of market tightness and fundamental surpluses

| Business cycle context | Elasticity | Key variables |
|---|---|---|
| Nash bargaining (Hagedorn and Manovskii 2008) | $\Upsilon^{\text{Nash}} \dfrac{y}{y-z}$ | $z$, value of leisure |
| Sticky wage (Hall 2005) | $\Upsilon^{\text{sticky}} \dfrac{y}{y-\hat{w}}$ | $\hat{w}$, sticky wage |
| . . . and a financial accelerator (Wasmer and Weil 2004) | $\Upsilon^{\text{sticky}} \dfrac{y}{y-\hat{w}-a}$ | $a$, annuitized value of credit search costs |
| Alternating-offer bargaining (Hall and Milgrom 2008) | $\Upsilon^{\text{alternating}} \dfrac{y}{y-z-\beta(1-s)\gamma}$ | $\gamma$, firm's cost of delay in bargaining$^\star$ |
| Welfare state context$^\dagger$ | | |
| Unemployment insurance | $\Upsilon^{\text{Nash}} \dfrac{y}{y-z-b}$ | $b$, unemployment benefit |
| Layoff costs | $\Upsilon^{\text{Nash}} \dfrac{y}{y-z-\beta s\tau}$ | $\tau$, layoff tax$^\star$ |

$^\star$ The associated term in the elasticity also involves the discount factor $\beta$ and the separation rate $s$.

$^\dagger$ Theories that attribute high European unemployment to productivity changes include a widened earnings distribution in Mortensen and Pissarides (1999), higher capital-embodied technological change in Hornstein et al. (2007), and shocks to human capital in Ljungqvist and Sargent (2007).

Across reconfigured matching models, many details differ, but what matters in the end is the fundamental surplus. In the standard matching model with Nash bargaining, the fundamental surplus is simply what remains after deducting the worker's value of leisure from productivity, $x = z$. Workers have to receive at least the value of leisure to be willing to work; hence, the invisible hand cannot allocate that value to vacancy creation.

In other specifications of matching models appearing in Table 1, the fundamental surplus emerges after making other deductions from productivity. In a model with a sticky wage $\hat{w}$, the deduction is simply the wage itself, $x = \hat{w}$, since the invisible hand cannot allocate so many resources into vacancy creation that there remains too little to pay the wage. If there is also costly acquisition of credit, as in Wasmer and Weil's (2004) model of a financial accelerator, an additional deduction needed to arrive at the fundamental surplus is the annuitized value $a$ of the average search costs for the formation of a unit that can post a vacancy, $x = \hat{w} + a$. Similarly, in the case of a layoff tax $\tau$ for which liability arises after the formation of an employment relationship, the fundamental surplus under Nash bargaining is obtained by deducting the value of leisure and also a value reflecting the eventual payment of the layoff tax, $x = z + \beta s\tau$, where the product of the discount factor $\beta$, match destruction probability $s$, and the layoff tax $\tau$, is an annuity payment that has the same expected present value as the future layoff tax. In Hall and Milgrom's (2008) model of alternating-offer wage bargaining in which firms incur a cost of delay $\gamma$, we deduct both the value of leisure and a quantity measuring the worker's ability to impose delay costs on the firm coming from the bargaining timing protocol. Under the assumption that firms make the first wage offer, the fundamental surplus is obtained by making deducting $x = z + \beta(1 - s)\gamma$ (when it is assumed that a bargaining firm-worker pair faces the same separation rate $s$ before and after an agreement is reached). Other formulas for the deductions pertinent for forming the fundamental surplus emerge if instead workers make the first wage offer in Hall and Milgrom's model or, in the case of the layoff tax analysis, if instead firms are liable for paying layoff taxes after simply encountering unemployed workers (regardless of whether or not any employment relationships were formed).

This paper emphasizes the key findings (1) that as a fraction of productivity, the fundamental surplus must be small to produce high unemployment volatility during business cycles, and (2) that a small fundamental surplus fraction also explains high unemployment in the context of adverse welfare state dynamics. Within several matching models, we characterize the forces at work by presenting closed-form solutions for steady-state comparative statics, and also by reporting numerical simulations that confirm how those forces shape outcomes in stochastic models.

The following mechanical intuition underlies our findings. The fundamental surplus is an upper bound on what the invisible hand can allocate to vacancy creation. A given change in productivity translates into a larger percentage change in the fundamental surplus when the fundamental surplus fraction is small. That induces the invisible hand to make resources allocated to vacancy costs comove strongly with changes in productivity. The relationship is immediate in a matching model with a sticky wage because a free-entry condition in vacancy creation makes the expected cost of filling a vacany equal the expected present value of the difference between a job's productivity and the sticky wage. Consequently, a change in productivity has a direct impact on resources devoted to vacancy creation and if those resources are small relative to output, a percentage change in productivity translates into a much larger percentage changes in resources used for vacancy creation and hence big responses of unemployment to movements in productivity. The relationship is subtler but similar in other matching models, whereby changes in productivity that have large effects on the fundamental surplus must spill over to the equilibrium amount of resources devoted to vacancy creation.

Failure to reason in terms of the fundamental surplus fraction has led many researchers to mistaken imputations of the sources of a heightened elasticity of market tightness to the elasticity of the wage with respect to productivity and to the suppression of a worker's outside value in some particular bargaining protocol. We will discuss some of those misinterpretations and advocate instead recognizing that all ways of attaining a high elasticity of market tightness must work by severely diminishing the fundamental surplus fraction.

After laying out the preliminaries of a standard matching model in section 2, section 3 elaborates further on the fundamental surplus as an object unifying seemingly disparate matching models. We derive steady-state comparative-statics expressions for the elasticity of market tightness with respect to productivity in models of welfare states and models of business cycles in sections 4 and 5, respectively. Stochastic versions of the latter business cycle models are simulated in section 6. Section 7 offers some concluding remarks.

# 2 Preliminaries

To set the stage, we review key equations and equilibrium relationships of a basic discrete time matching model.[2] Let there be a continuum of identical workers with measure normalized to 1. Workers are infinitely lived and risk neutral with discount factor $\beta = (1+r)^{-1}$. A worker's objective is to maximize the expected discounted value of leisure and labor income. The leisure enjoyed by an unemployed worker is denoted $z$, while an employed worker derives utility from the equilibrium wage rate $w$.

The production technology has constant returns to scale with labor as the sole input. Each employed worker produces $y$ units of output. Suppose that each firm employs at most one worker. A firm entering the economy incurs a vacancy cost $c$ in each period when looking for a worker, and in a subsequent match the firm's per-period earnings are $y-w$. All matches are exogenously destroyed with per-period probability $s$. Free entry implies that a new firm's expected discounted stream of vacancy costs and earnings equals zero.

The measure of successful matches in a period is governed by a matching function $M(u, v)$, where $u$ and $v$ are aggregate measures of unemployed workers and vacancies. The matching function is increasing in both arguments, concave, and homogeneous of degree 1. By the homogeneity assumption, we can write the probability of filling a vacancy as $q(v/u) \equiv$

[2] For a textbook exposition, see Ljungqvist and Sargent (2012, section 28.3), or, in continuous time, Pissarides (2000).

$M(u,v)/v$. The ratio between vacancies and unemployed workers is called *market tightness*, $\theta \equiv v/u$. The probability that an unemployed worker will be matched in a period is $\theta q(\theta)$.

A firm's value of a filled job $J$ and a vacancy $V$ are

$$J = y - w + \beta\left[sV + (1-s)J\right], \tag{2}$$

$$V = -c + \beta\big\{q(\theta)J + [1-q(\theta)]V\big\}. \tag{3}$$

After invoking the zero profit condition $V = 0$, equation (3) implies

$$J = \frac{c}{\beta q(\theta)} \equiv J(\theta), \tag{4}$$

which we can substitute into equation (2) to arrive at

$$w = y - \frac{r+s}{q(\theta)}c. \tag{5}$$

A worker's value $E$ as employed and as $U$ unemployed are

$$E = w + \beta\big[sU + (1-s)E\big], \tag{6}$$

$$U = z + \beta\big\{\theta q(\theta)E + [1-\theta q(\theta)]U\big\}. \tag{7}$$

In the standard matching model, the match surplus, $S \equiv J + E - U$, is split between a matched firm and worker according to Nash bargaining. The maximization of Nash product $(E-U)^{\phi}J^{1-\phi}$ has the solution

$$E - U = \phi S \quad \text{and} \quad J = (1-\phi)S, \tag{8}$$

where $\phi \in [0,1)$ measures the worker's 'bargaining power'. After solving equations (2) and (6) for $J$ and $E$, respectively, then substituting them into equations (8), we get a second expression for the wage rate

$$w = \frac{r}{1+r}U + \phi\left(y - \frac{r}{1+r}U\right). \tag{9}$$

The annuity value of being unemployed, $rU/(1+r)$, can be obtained by solving equation (7) for $E - U$ and substituting this expression and equation (4) into equations (8):

$$\frac{r}{1+r} U \;=\; z \,+\, \frac{\phi \, \theta \, c}{1-\phi}\,. \tag{10}$$

Substituting equation (10) into equation (9) yields

$$w \;=\; z \,+\, \phi(y - z \,+\, \theta c)\,. \tag{11}$$

The two expressions (5) and (11) for the wage rate jointly determine the equilibrium value of $\theta$:

$$y - z \;=\; \frac{r \,+\, s \,+\, \phi \, \theta \, q(\theta)}{(1-\phi) q(\theta)} \, c\,. \tag{12}$$

The assumptions of identical workers/vacancies, risk neutrality, and constant returns to scale in production and in matching imply that equilibrium market tightness $\theta$ does not depend on distributions of workers across employment and unemployment. In a steady state, $\theta$ determines unemployment by the condition that the measure of laid-off workers in a period, $s(1-u)$, must equal the measure of unemployed workers gaining employment, $\theta q(\theta) u$, which implies

$$u \;=\; \frac{s}{s \,+\, \theta q(\theta)}\,. \tag{13}$$

We proceed under the assumption that the matching function has the Cobb-Douglas form, $M(u,v) = A u^{\alpha} v^{1-\alpha}$, where $A > 0$, and $\alpha \in (0,1)$ is the constant elasticity of matching with respect to unemployment.

# 3 Fundamental surplus as essential object

The derivative of steady-state unemployment in equation (13) with respect to market tightness is

$$\frac{d\,u}{d\,\theta} \;=\; -\frac{s\,[q(\theta) \,+\, \theta\, q'(\theta)]}{[s \,+\, \theta\, q(\theta)]^2} \;=\; -\frac{u\, q(\theta)}{s \,+\, \theta\, q(\theta)} \left[1 \,+\, \frac{\theta\, q'(\theta)}{q(\theta)}\right] \;=\; -\frac{u\, q(\theta)}{s \,+\, \theta\, q(\theta)}\,(1-\alpha)\,,$$

where the second equality uses equation (13) and factors $q(\theta)$ from the expression in square brackets of the numerator, and the third equality is obtained after invoking the constant elasticity of matching with respect to unemployment, $\alpha = -q'(\theta)\,\theta / q(\theta)$. So the elasticity of unemployment with respect to market tightness becomes

$$\begin{aligned}
\eta_{u,\theta} \quad &\equiv \quad \frac{d\,u}{d\,\theta}\,\frac{\theta}{u} \;=\; -(1-\alpha)\,\frac{\theta\,q(\theta)}{s\,+\,\theta\,q(\theta)} \\
&= \quad -(1-\alpha)\left(1\,-\,\frac{s}{s\,+\,\theta\,q(\theta)}\right) \;=\; -(1-\alpha)\,(1-u)\,, \qquad (14)
\end{aligned}$$

where the second equality is obtained after adding and subtracting $s$ to the numerator, and the last third equality invokes expression (13).

To shed light on what can cause significant volatility in unemployment, we explore what forces can make market tightness $\theta$ highly elastic with respect to productivity.

## 3.1 A decomposition of the elasticity of market tightness

To compute the elasticity of market tightness $\theta$ with respect to productivity $y$, we begin by noting that the equilibrium expression (12) for market tightness can be rewritten as

$$\frac{1-\phi}{c}(y-z) \;=\; \frac{r+s}{q(\theta)} \,+\, \phi\,\theta\,. \qquad (15)$$

Implicit differentiation yields

$$\begin{aligned}
\frac{d\,\theta}{d\,y} \quad &= \quad -\frac{\dfrac{1-\phi}{c}}{-\left(\dfrac{-q'(\theta)\,(r+s)}{q(\theta)^2}\,+\,\phi\right)} \;=\; -\frac{\left(\dfrac{r+s}{q(\theta)}\,+\,\phi\,\theta\right)\dfrac{1}{y-z}}{-\left(\dfrac{\alpha(r+s)}{\theta q(\theta)}\,+\,\phi\right)} \\
&= \quad \frac{(r+s)\,+\,\phi\,\theta\,q(\theta)}{\alpha(r+s)\,+\,\phi\,\theta\,q(\theta)}\;\frac{\theta}{y-z} \;\equiv\; \Upsilon\big(\phi\,\theta q(\theta)\big)\,\frac{\theta}{y-z}\,, \qquad (16)
\end{aligned}$$

where the second equality is obtained after using equation (15) to make a substitution in the numerator, while in the denominator, we invoke the constant elasticity of matching with respect to unemployment, $\alpha = -q'(\theta)\,\theta/q(\theta)$; the third equality follows from multiplying and dividing by $\theta\,q(\theta)$. We can then compute the elasticity of market tightness as

$$\eta_{\theta,y} \;\equiv\; \frac{d\,\theta}{d\,y}\,\frac{y}{\theta} \;=\; \Upsilon\big(\phi\,\theta q(\theta)\big)\,\frac{y}{y-z}\,. \qquad (17)$$

This multiplicative decomposition of the elasticity of market tightness is at the centre of our analysis. Similar decompositions will arise for a variety of matching model setups.

The first factor $\Upsilon\big(\phi\,\theta q(\theta)\big)$ in expression (17), has counterparts in other setups. A consensus about what constitute reasonable calibrations limits its contribution to the elasticity of market tightness. Hence, the magnitude of the elasticity of market tightness depends mostly on the second factor in expression (17), i.e., the inverse of what we in section 1 defined to be the fundamental surplus fraction. As a useful preliminary, it is useful to revisit a critical observation of Shimer (2005), who asserted that common calibrations of the standard matching model make the elasticity of market tightness with respect to productivity too low to explain business cycle fluctuations.

## 3.2 The Shimer critique

Shimer (2005, p. 25) concluded that the "matching model cannot generate the observed business-cycle-frequency fluctuations in unemployment and job vacancies in response to shocks of a plausible magnitude. In the United States, the standard deviation of the vacancy-unemployment ratio is almost 20 times as large as the standard deviation of average labor productivity, while the [matching] model predicts that the two variables should have nearly the same volatility." In terms of expression (17), the following mechanics express Shimer's logic. Common calibrations of matching models assign a substantial bargaining weight $\phi$ to workers and assume small values of the net interest rate and the separation rate, $(r+s)$, as compared to their endogenously determined and much higher probability of a worker finding a job, $\theta\, q(\theta)$. So the first factor in expression (17), $\Upsilon\big(\phi\,\theta q(\theta)\big)$, is not far away from its lower bound of unity. The second factor in expression (17), $y/(y-z)$, exerts an amplifying effect. But it would attain its lowest value of unity when the fundamental surplus fraction is equal to one, i.e., if there is no value of leisure. Hence, a zero value of leisure and otherwise a common calibration of the matching model, imply an elasticity of market tightness with respect to productivity that is close to its lower bound of unity. Raising the value of leisure but confining it to a commonly assumed range that is well below productivity increases the elasticity but still leaves it small. This is the heart of Shimer's critique.

Shimer (2005, pp. 39-40) also documented that steady-state comparative statics for the elasticity of market tightness in a nonstochastic economy, as given by expression (17), serve as a good approximation to outcomes from simulations of an economy subject to aggregate productivity shocks. Provisionally expecting similarly good approximations to prevail in other matching frameworks, we will derive some closed-form solutions for steady-state comparative statistics in those other setups in the hope that they will to shed light on the findings from stochastic simulations to be reported in section 6.

### 3.3 A small fundamental surplus is key to a high elasticity

The multiplicative decomposition (17) of the elasticity of market tightness with respect to productivity is complicated by the fact that the first factor, $\Upsilon\big(\phi\,\theta q(\theta)\big)$, depends on the endogenous market tightness $\theta$ that is a function of all parameters, including those comprising the second factor, $y/(y-z)$. However, the first factor is bounded from above by the inverse of the elasticity of matching with respect to unemployment, $1/\alpha$, an upper bound that would be attained if workers' bargaining weight $\phi$ were zero. Since reasonable estimates of the elasticity $\alpha$ restrict the magnitude of the first factor in expression (17), the second factor, $y/(y-z)$ is what is critical in generating movements in market tightness. The second factor goes to infinity as the fundamental surplus goes to zero, i.e., as the value of leisure approaches productivity.

We shall describe how researchers in the business cycle literature use one of several devices to increase the magnitude of the second factor, either by assuming a high value of leisure (including any home production and unemployment compensation) or by replacing Nash bargaining with sticky wages or with an alternative bargaining protocol. Not fully appreciating the importance of the fundamental surplus fraction has sometimes caused misplaced imputations of the source for high elasticities to the sensitivity of the wage to productivity changes and to the suppression of a worker's outside value $U$ in an alternative bargaining protocol. We aim to correct those misinterpretations and show that all approaches to attaining a high elasticity must work through the same intermediating variable – a fundamental surplus fraction that is somehow severely diminished.

That same avenue gives rise to high unemployment in matching models of adverse welfare state dynamics, because of how welfare state policies suppress the fundamental surplus. But here the diminished fundamental surplus might apply only to a subset of workers, and the effects on unemployment can depend on the number of matching functions assumed and the assignments of different types of workers to separate matching functions. We turn first to such models of welfare state dynamics, and thereafter to alternative ways of modelling business cycle fluctuations in matching models. We begin with a few general remarks about how the fundamental surplus provides a new perspective on the matching model framework, and what the relationship is to standard concepts of outside value and match surplus.

## 3.4 Fundamental surplus and concepts it supplements

The concepts of outside value and match surplus are central to the mechanics of matching models. By introducing the notion of the fundamental surplus, our purpose is not to deny the importance of the outside value and the match surplus but instead to provide a complementary perspective that isolates essential forces at work across a variety of matching models.

The match surplus is the capitalized surplus accruing to a firm and a worker in the current match. It is the difference between the present value of the match and the sum of the worker's outside value and, if there is any, the firm's outside value. The worker's outside value is the sum of three parts. First, a worker's outside value can never fall below the capitalized value of enjoying leisure in all future periods. Second, the outside value includes the sum of the discounted values of the worker's share of match surpluses in his or her as yet unformed future matches. Third, and key to our new perspective, a worker's outside value consists of those parts of fundamental surpluses from future employment matches that are not allocated to match surpluses.

A superficial distinction is that we express the fundamental surplus as a flow value while the match surplus is known as a capitalized value. Instead, significantly, the fundamental surplus is an upper bound on resources that the invisible hand can allocate to vacancy creation, and its magnitude as a fraction of output is the prime determinant of the elasticity of market tightness with respect to productivity. In contrast, although it measures resources that are used for vacancy creation, the size of the match surplus relative to output has no direct bearing on the elasticity of market tightness. Recall that in the standard matching model, the zero-profit condition for vacancy creation implies that the expected present value of a firm's share of match surpluses equals the average cost of filling a vacancy. Since common calibrations award firms a significant share of match surpluses and vacancy cost expenditures are calibrated to be relatively small, it follows that equilibrium match surpluses form a small part of output across various matching models, regardless of the elasticity of market tightness in any particular model.

As noted by Shimer (2005), common calibrations of the matching model are associated with low elasticities of market tightness with respect to productivity. This reflects a fundamental surplus so big that it is easy for the 'invisible hand' to realign the workers' outside value in a way that keeps the match surplus almost unchanged when productivity changes. In contrast, in alternative calibrations with a high value of leisure, the fundamental-surplus component of workers' outside value is so small that there is little room for the invisible hand to realign things in that way, making the equilibrium amount of resources allocated to vacancy creation respond sensitively to variations in productivity. That results in a high elasticity of market tightness with respect to productivity. Put differently, since the fundamental surplus is a part of productivity, it follows that a given change in productivity becomes a greater percentage change in the fundamental surplus by a factor of $y/(y-z)$, i.e., the inverse of the fundamental surplus fraction. Thus, the small fundamental surplus fraction in the alternative calibrations with a high value of leisure implies large percentage changes in the fundamental surplus. Such large changes in the amount of resources that could potentially be used for vacancy creation cannot be offset by the invisible hand and hence, variations in productivity lead to large variations in vacancy creation, resulting in a high elasticity of market tightness with respect to productivity.

As we will learn in various contexts, dynamics that are intermediated through the fundamental surplus transcend the standard matching model and apply in other interesting setups, including those with sticky wages and alternative bargaining protocols. For example, it does not matter much if the reason for a diminished fundamental surplus fraction is Hagedorn and Manovskii's (2008) high value of leisure for workers, Hall's (2005) sticky wage, or Hall and Milgrom's (2008) cost of delay for firms that participate in alternating-offer bargaining. A small fundamental surplus fraction causes variations in productivity to have large effects on resources devoted to vacancy creation, because of workers who insist on being compensated for their losses of leisure, firms that have to pay the sticky wage, or workers who strategically exploit the firm's cost of delay under an alternating-offer bargaining protocol. Likewise, the fundamental surplus fraction can be diminished because of welfare state policies such as unemployment compensation and government-imposed layoff costs, as we turn to next.

# 4 Fundamental surplus and welfare states

## 4.1 Unemployment compensation

We begin by considering a simplified version of Mortensen and Pissarides' (1999) matching model of workers who are heterogeneous in their skills and enjoy a welfare state safety net. Mortensen and Pissarides make technological assumptions that guarantee the existence of skill-specific matching functions – unemployed workers enter a skill-specific matching function to match with vacancies targetting their skill levels. A critical assumption for Mortensen and Pissarides is that the value of leisure including unemployment compensation does not vary proportionately with workers' skills. We incorporate these key features in our standard matching model by assuming that workers have different productivities $y$ but a common value of leisure $z$, defined as a value that includes unemployment compensation.

We let workers' productivities span the range $[0.6, 1]$ with a value of leisure $z = 0.6$. At the high-end of these productivities, the value of leisure is in a typical range for common calibrations of matching models. Following Mortensen and Pissarides (1999), we assume that $\phi = \alpha = 0.5$, which is also a common calibration: workers' bargaining weight $\phi$ falls mid-range and equals the elasticity of matching with respect to unemployment, so that the Hosios efficiency condition is satisfied. We retain our assumption of an exogenous separation rate $s$, and defer discussing Mortensen and Pissarides's analysis of layoff taxes and endogenous separations to section 4.2.

The model period is a day and the discount factor is $\beta = 0.95^{1/360}$, i.e., an annual interest rate of 5 percent.[3] A daily separation rate of $s = 0.001$ means that jobs last on average 2.8 years. For the highest productivity level $y = 1$, we target an unemployment rate of 5 percent, which by equation (13) implies that unemployed workers face a daily job finding probability equal to $\theta\, q(\theta) = 0.019$, i.e., the probability of finding a job within a month is 44 percent. According to equilibrium expression (15), two parameters remain to be set to attain this equilibrium outcome: the vacancy cost $c$ and the efficiency parameter $A$ of the matching function. Without any targets for vacancy statistics, this is just an issue of choice of normalization, as noted by Shimer (2005).[4] Therefore, we set $c = 0.1$ and let

[3]A short model period helps vacancy creation take place even at low values of the fundamental surplus fraction. Note that the lower bound on the average recruitment cost is attained when market tightness has dropped so low that a vacancy encounters an unemployed worker with probability one, i.e., the recruitment cost becomes certain and equals the one-period vacancy cost.

[4]Given a calibration $c$ and $A$ with an associated market tightness $\theta$ determined by equation (15), the same equilibrium unemployment rate and job finding probability can be attained with an alternative parameterization $\hat{c} = \xi c$ and $\hat{A} = \xi^{1-\alpha} A$, for any $\xi \in (0, \theta^{\alpha}/A]$. To verify this claim, solve for the new market tightness $\hat{\theta}$ from an appropriate version of equation (15),

$$\frac{1-\phi}{[\xi c]}(y-z) \;=\; \frac{r+s}{[\xi^{1-\alpha}A]\hat{\theta}^{-\alpha}} \;+\; \phi\,\hat{\theta}\,.$$

After multiplying both sides by $\xi$ and comparing to equilibrium expression (15) for the initial parameterization, the new solution satisfies $\hat{\theta} = \theta/\xi$. At this new market tightness $\hat{\theta}$, the unemployment rate is unchanged because the job finding probability is unchanged, $\hat{\theta}[\xi^{1-\alpha}A]\hat{\theta}^{-\alpha} = \theta\, A\theta^{-\alpha} = \theta\, q(\theta)$. The new probability of filling a vacancy is $[\xi^{1-\alpha}A]\hat{\theta}^{-\alpha} = \xi A\theta^{-\alpha} = \xi\, q(\theta) \in (0, 1]$, where the bounds follow from the range for $\xi$. The value of a filled job remains unchanged, as can be verified from expression (4), the no-profit condition in vacancy creation

$$\hat{J} \;=\; \frac{\hat{c}}{\beta[\xi^{1-\alpha}A]\hat{\theta}^{-\alpha}} \;=\; \frac{\xi c}{\beta[\xi^{1-\alpha}A][\theta/\xi]^{-\alpha}} \;=\; \frac{c}{\beta A\theta^{-\alpha}} \;=\; J\,,$$

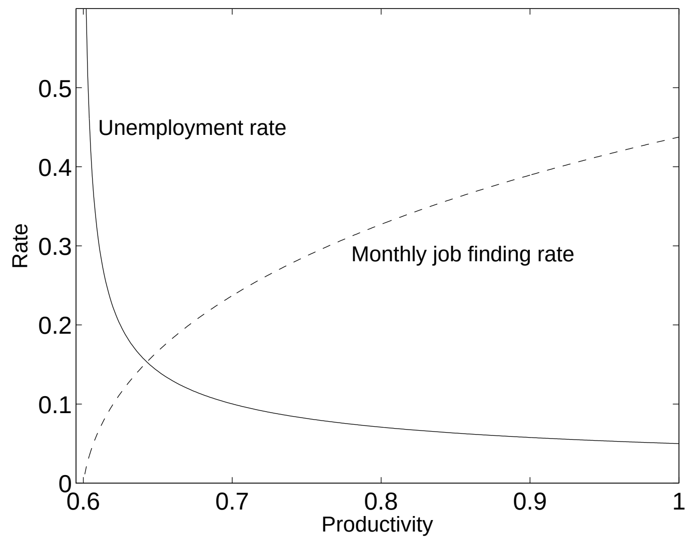


Figure 1: Unemployment rate and monthly job finding rate.

the parameter $A$ adjust so that the unemployment target of 5 percent is attained.

The solid line in Figure 1 depicts the unemployment rate for different productivity levels. Unemployment spikes at low productivities when the fundamental surplus fraction, $(y-z)/y$, becomes ever smaller, in accordance with formula (17) for the elasticity of market tightness with respect to productivity. Specifically, at an ever diminished fundamental surplus fraction, the elasticity of market tightness with respect to additional decreases in productivity is magnified and hence, the increments in unemployment become successively larger as we move to ever lower productivity levels.

The solid line in Figure 2 shows how the annuitized value of average recruitment costs becomes an ever larger fraction of the fundamental surplus. The annuitized value is just the difference between productivity and the wage rate, $y - w$, since vacancy creation breaks even when the expected present value of firms' share of match surpluses are pledged against average recruitment costs. In spite of being an increasing fraction of the fundamental surplus, recruitment costs fall in absolute terms when productivity declines, reflected in the falling probability of workers finding jobs in Figure 1. The dashed line in Figure 2 shows what the fraction of recruitment costs to the fundamental surplus would have had to be for these costs to have remained constant in absolute terms.

where the second equality invokes the alternative parameterization $\hat{c} = \xi c$ and the equilibrium outcome $\hat{\theta} = \theta/\xi$.

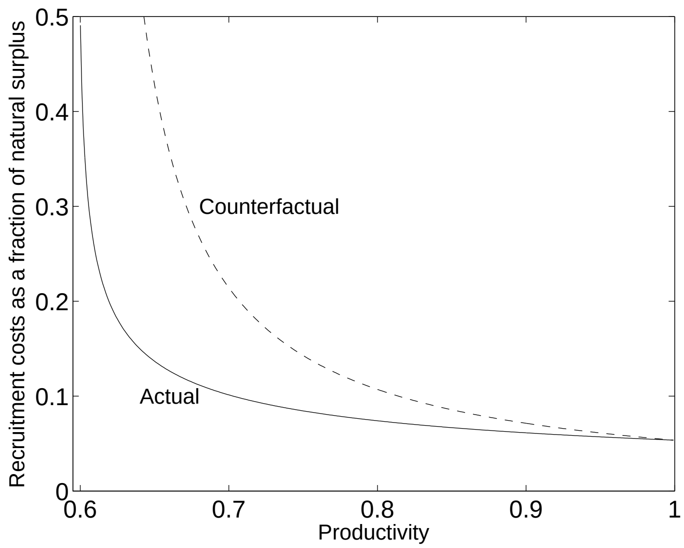


Figure 2: Annuitized recruitment costs as a fraction of the fundamental surplus. The dashed counterfactual curve depicts what the fraction would be if recruitment costs were constant and equal to those prevailing at the highest productivity, $y = 1$.

Given our $\phi = 0.5$ calibration (meaning equal bargaining weights for workers and firms), the annuitized match surplus as a fraction of the fundamental surplus is twice that of the solid line in Figure 2, so it ranges from a mere 10 percent at high productivities to 100 percent in the limit when productivity approaches the value of leisure. Hence, in the lowest range of productivities, recruitment costs are bound to fall with a drop in productivity – the annuitized match surplus is then comprising almost the entire fundamental surplus, so without being able to impinge on the value of workers' leisure, the "invisible hand" has to let resources allocated to vacancy creation move with productivity. Together with the fact that a given percentage change in productivity becomes so much larger as a percentage change of a small fundamental surplus (of which, as mentioned, now almost all is allocated to the match surplus), it is inevitable that the elasticity of market tightness with respect to productivity explodes as productivity approaches the value of leisure.

Mortensen and Pissarides (1999, p. 258) compare the unemployment schedule in Figure 1 to that for an economy with a lower unemployment compensation (a lower value of $z$) – 'Europe' versus the 'US' – and conclude that "the relationship between the unemployment rate and worker productivity is *much more convex* in the 'European' case than in the 'US'." As we have shown, this outcome is a necessary consequence of a much smaller fundamental surplus fraction in the economy with a higher value of $z$.

Next, Mortensen and Pissarides hypothesize that the widening unemployment differential between Europe and the US after the late 1970s can be explained by 'skill-biased' technology shocks, modelled as a mean preserving increase in the spread of the distribution of productivity across workers. Figure 1 shows that those workers who are moved to a lower range of productivities cause a larger increase in unemployment than the decrease that is causeed by workers who are moved in the opposite direction. Mortensen and Pissarides (1999, p. 259) add that such skill-biased shocks "induce reductions in the participation rate like those observed in the major European economies." In our language, this is because the fundamental surplus becomes too small or perhaps even negative, making vacancy creation shut down and market tightness become zero in the matching functions for workers with low productivities.

## 4.2 Single versus multiple matching functions

In contrast to Mortensen and Pissarides (1999) who assume that individual workers are permanently attached to their productivity levels, Ljungqvist and Sargent (2007) formulate a matching model in which workers can move back and forth between two skill levels. On the one hand, after each period of working, a worker faces a probability of becoming experienced by moving up to the higher skill level (unless the worker has already attained that level). On the other hand, in the event of an exogenous separation from a job, a worker is exposed to a probability $\pi^d$ of moving down to the lower skill level (unless the worker already is at that level). A constant entry rate of new workers who start at the lower skill level, and an exogenous retirement rate of the same magnitude keep the population of workers constant over time.

Both low- and high-skilled employed workers are subject to idiosyncratic productivity shocks. A high-skilled worker is better off in the sense of facing better initial productivity draws in new jobs and also the productivity process in ongoing employment that stochastically dominates those of a low-skilled worker. Those shocks generate endogenous separations whenever productivity realizations are so dismal that it is better to break up matches, in addition to the assumed exogenous separation rate.

Two government policies are unemployment compensation as a replacement rate on a worker's past earnings and a layoff tax on all separations of workers from jobs. For computational simplicity, the unemployment compensation is based not on an individual worker's own past earnings but rather on the average earnings of the skill group to which the worker belonged just before his job separation. Hence, there are two unemployment benefit levels, low and high. Under the assumption that newborn workers are entitled to low benefits (even before having held a job), it follows that there are three distinct classes of unemployed workers in an equilibrium: high-skilled with high benefits, low-skilled with low benefits, and low-skilled with high benefits, where latter are unemployed workers who were high-skilled before the separation but lost their skills when experiencing an exogenous separation.

Mortensen and Pissarides (1999) attributed the rise in European unemployment to a widening of the distribution of workers' permanent earnings. Ljungqvist and Sargent (2007) instead explored an increase in 'turbulence' modeled as a higher value of the probability $\pi^d$, the probability of losing skills when an exogenous separation occurs. This creates more workers having low skills and high benefits, causing higher equilibrium unemployment for the following reason. Everything else equal, firms meeting low-skilled workers having high benefits make smaller profit than if those same low-skilled workers had low benefits, since the former workers have a higher threat point, i.e., outside value, because of their high benefits. By lowering market tightness, the 'invisible hand' then restores firms' profitability enough for them to break even when they create vacancies. Lower market tightness improves firms profitability directly via a shorter average time to fill a vacancy (which saves on vacancy costs), as well as indirectly when the correspondingly longer average duration of unemployment spells reduces unemployed worker's outside values in wage bargaining (which reduces wage costs).

While these forces lead to higher unemployment, their magnitudes depend critically on the number of matching functions in the economy and how unemployed workers are assigned to them. Ljungqvist and Sargent examine four possible configurations: (1) one single matching function, (2) two matching functions, where the unemployed are assigned according to their current skill levels, (3) two matching functions, where the unemployed are assigned according to their benefit levels, and (4) three matching functions, where the unemployed are sorted both by their current skill and benefits levels (i.e., high-skilled with high benefits, low-skilled with low benefits, and low-skilled with high benefits). Figure 3 shows steady-state unemployment rates for different parameterizations of $\pi^d$ in economies with different numbers of matching functions.[5]

The adverse dynamics in response to higher turbulence in a welfare state come from the presence of unemployed workers with low skills and high benefits. If these workers are assigned to their own separate matching function, it would be the analogue in Mortensen

[5]Figure 3 is a reproduced version of Figure 13 in Ljungqvist and Sargent (2007) but for one change. Instead of a model period equal to half a quarter, we now assume a daily frequency.

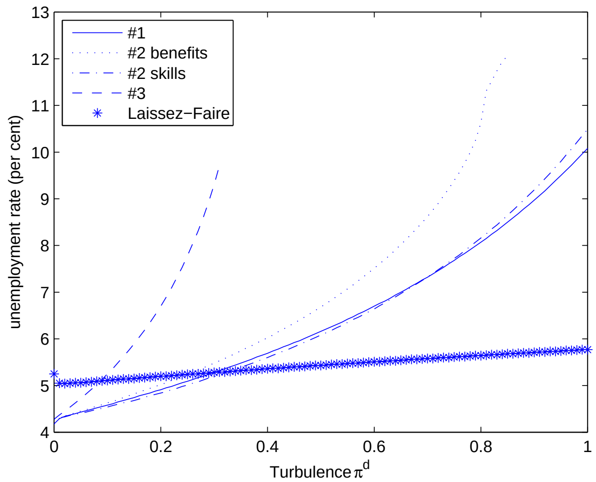


Figure 3: Unemployment for different levels of turbulence, $\pi^d$, in economies with different number of matching functions.

and Pissarides (1999) to a worker type towards the left end of Figure 1. Specifically, the more skill loss erodes workers' productivity relative to their benefit entitlement, the farther back and up along the convex relationship between a depressed fundamental surplus and unemployment. The associated economy-wide unemployment rate, as a function of the degree of turbulence, is drawn as a dashed line in Figure 3, and reflects primarily how an increase in turbulence mechanically generates more laid-off workers with skill loss and how their type-specific elevated unemployment drives the aggregate unemployment rate.

The adverse consequences of a set of low-skilled, high-benefit unemployed workers are diluted when these workers are mixed with other unemployed in a shared matching function, as shown in Figure 3: mixed with all the unemployed (solid line), with all low-skilled unemployed (dash-dotted line), or with all unemployed who are entitled to high benefits (dotted line). But in any case, the unemployment rate of a welfare state will eventually, for a high enough degree of turbulence, be considerably higher than that of a laissez-faire economy without any unemployment benefit (or for that matter, any layoff tax), as depicted by the starred line in Figure 3.

Yet another lesson from Figure 3 is that the unemployment rate in the welfare state is lower than that of the laissez-faire economy when there is little or no turbulence. As in Mortensen and Pissarides (1999), the unemployment rate in tranquil times in the welfare state is supressed because of the layoff tax. One might ask why we have focused on the role of unemployment benefits when discussing the outbreak of high unemployment in the welfare state in response to turbulence, and ignored the layoff tax. We shall see why next as we move on to analyze how layoff taxes affect the fundamental surplus fraction.

### 4.3 Layoff taxes under Nash product $(E-U)^{\phi}J^{1-\phi}$

We assume that the government imposes a layoff tax $\tau$ on each layoff. Tax revenues are returned as lump sum transfers to the workers, but these transfers do not show up in the expressions below because they do not affect behavior. To make our inquiry as transparent as possible, we retain the assumption of exogenous job destruction and focus on how layoff taxes affect the elasticity of market tightness with respect to productivity. We will again find that the elasticity of market tightness with respect to productivity depends on size of the fundamental surplus fraction.

A critical assumption is whether liability for the layoff tax occurs only after the formation of employment relationships or earlier when firms match with unemployed workers. The first assumption is the most common, and the one that we adopt here (we defer studying the consequences of the alternative assumption to section 4.4). Under this assumption, Nash bargaining solution (8) continues to hold, i.e., a worker and a firm split match surpluses, including a negative match surplus at a separation, $S=-\tau$, according to their respective bargaining powers

$$(1-\phi)(E-U) \;=\; \phi J\,. \tag{18}$$

Hence, a firm's value of a filled job and the value of an employed worker in expressions (2) and (6) are modified in the presence of layoff taxes to become

$$\begin{aligned} J &= y - w + \beta\big[-s(1-\phi)\tau + (1-s)J\big] \\ E &= w + \beta\big[s(U-\phi\tau) + (1-s)E\big]\,, \end{aligned}$$

where we have imposed $V=0$ so that vacancies break even in an equilibrium. These two expressions can be rewritten as

$$J \;=\; \frac{y - w - \beta s(1-\phi)\tau}{1-\beta(1-s)}\,, \tag{19}$$

$$E \;=\; \frac{w + \beta s(U-\phi\tau)}{1-\beta(1-s)}\,. \tag{20}$$

The no-profit condition for vacancies from expression (4) and the value of an unemployed worker from expression (7) remain the same.

After equating the right sides of expressions (4) and (19) and then rearranging, we find that the equilibrium wage must satisfy

$$w \;=\; y \;-\; \frac{r+s}{q(\theta)}c \;-\; \beta s(1-\phi)\tau\,. \tag{21}$$

As compared to wage expression (5) in an economy without layoff taxes, the expression (21) has an additional negative term involving the layoff tax, namely, $-\beta s(1-\phi)\tau$. However, the shared negative term, $-(r+s)c/q(\theta)$, becomes less negative with a layoff tax because, as we will show, market tightness falls, making the probability $q(\theta)$ of filling a job increase. But as we shall also show, the former negative effect outweighs the latter positive one, so the equilibrium wage falls when there are layoff taxes.

To obtain another useful equation for the equilibrium wage, use expressions (19) and (20) to eliminate $J$ and $E$ from equation (18):

$$(1-\phi)\left\{\frac{w \,+\, \beta s(U-\phi\tau)}{1 \,-\, \beta(1-s)} \,-\, U\right\} \;=\; \phi\,\frac{y \,-\, w \,-\, \beta s(1-\phi)\tau}{1 \,-\, \beta(1-s)}\,. \tag{22}$$

After multiplying both sides by $(1-\beta(1-s))$ and simplifying, we find that the equilibrium wage satisfies

$$w \;=\; (1-\beta)U \,+\, \phi\big(y \,-\, (1-\beta)U\big)\,. \tag{23}$$

As a side calculation, we seek an expression for $(1-\beta)U$. First, solve for $E-U$ from expression (7):

$$E-U \;=\; \frac{(1-\beta)U \,-\, z}{\beta\,\theta\,q(\theta)}\,. \tag{24}$$

Use this expression and equation (4) to eliminate $(E-U)$ and $J$ from expression (18),

$$(1-\phi)\,\frac{(1-\beta)U \,-\, z}{\beta\,\theta\,q(\theta)} \;=\; \phi\,\frac{c}{\beta\,q(\theta)}\,, \tag{25}$$

which after simplifications yields

$$(1-\beta)U \;=\; z \,+\, \frac{\phi\,\theta\,c}{1-\phi}\,. \tag{26}$$

After using expression (26) to eliminate $(1-\beta)U$ in expression (23), and simplifying, we obtain our second wage equation:

$$w = z + \phi(y - z + \theta c)\,. \tag{27}$$

While this expression for the wage is identical to the corresponding expression (11) for a model without layoff taxes, it is now evaluated at a lower market tightness $\theta$, and hence, the equilibrium wage rate is lower with the imposition of layoff taxes. To confirm this, we equate the right hand sides of (21) and (27), and after rearranging, we obtain the following equation for equilibrium market tightness $\theta$:

$$y - z - \beta s\tau = \frac{r + s + \phi\,\theta\, q(\theta)}{(1-\phi)q(\theta)}\, c\,. \tag{28}$$

Since left side of (28) is lower than the left hand side of (12), it follows that market tightness $\theta$ must be lower on the right side of the former expression as compared to the latter expression. Thus, layoff taxes suppress market tightness and therefore increases unemployment in the present model.

Following the steps in section 3.1, implicit differentiation of expression (28) yields

$$\frac{d\theta}{dy} = \Upsilon\big(\phi\,\theta q(\theta)\big)\,\frac{\theta}{y - z - \beta s\tau}\,. \tag{29}$$

We can then compute the elasticity of market tightness as

$$\eta_{\theta,y} = \Upsilon\big(\phi\,\theta q(\theta)\big)\,\frac{y}{y - z - \beta s\tau}\,. \tag{30}$$

The only difference between the elasticity of market tightness (30) here with layoff taxes and the earlier expression (17) for an economy without layoff taxes is that the fundamental surplus has an additional deduction of $\beta s\tau$. So long as the firm continues to operate, this is an annuity payment $a$ with the same present value as the future layoff tax:

$$\sum_{t=0}^{\infty}\beta^t(1-s)^t a = \sum_{t=1}^{\infty}\beta^t(1-s)^{t-1}s\,\tau \qquad \Longrightarrow \qquad a = \beta s\tau\,, \tag{31}$$

where the flow of annuity payments on the left side of the first equation starts in the first period of operating and ceases when the job is destroyed, while the future layoff tax on the right side occurs first after the initial period of operation. Since the “invisible hand” can never allocate those resources to vacancy creation, it is appropriate to subtract this annuity value when computing the fundamental surplus.

Our conclusion from expression (30) is, once again, that a necessary condition for a high elasticity of market tightness is a small fundamental surplus fraction.

### 4.4 Layoff taxes under Nash product $(E-U)^{\phi}(J+\tau)^{1-\phi}$

An alternative assumption in the analysis of layoff taxes is that firms are liable for the layoff tax immediately upon being matched with unemployed workers regardless of whether or not any employment relationships are eventually formed, see e.g. Millard and Mortensen (1997). Under this assumption, the firm's threat point is $-\tau$ and the Nash product to be maximized is $(E-U)^{\phi}(J+\tau)^{1-\phi}$ implying the solution

$$E - U \;=\; \phi\big(S+\tau\big) \quad \text{and} \quad J \;=\; (1-\phi)S - \phi\tau\,, \tag{32}$$

i.e., a worker and a firm split match surpluses so that

$$(1-\phi)(E-U-\phi\tau) \;=\; \phi(J+\phi\tau)\,. \tag{33}$$

Hence, as compared to expression (2), a firm's value of a filled job is modified, but the value of an employed worker continues to be the same as expression (6), i.e.,

$$\begin{aligned} J \;&=\; y - w + \beta\big[-s\,\tau + (1-s)J\big] \\ E \;&=\; w + \beta\big[s\,U + (1-s)E\big]\,, \end{aligned}$$

where we have imposed that $V=0$ so that vacancies break even in an equilibrium. The preceding two expressions can be rewritten as

$$J \;=\; \frac{y - w - \beta s\tau}{1-\beta(1-s)} \tag{34}$$

$$E \;=\; \frac{w + \beta sU}{1-\beta(1-s)}\,. \tag{35}$$

The no-profit condition for vacancies and the value of an unemployed worker continue to be as in expressions (4) and (7).

After equating the right sides of expressions and (4) and (34) and rearranging, it follows that the equilibrium wage satisfies

$$w \;=\; y \,-\, \frac{r+s}{q(\theta)}c \,-\, \beta s\tau\,. \tag{36}$$

To obtain another equation for the equilibrium wage, use expressions (34) and (35) to eliminate $J$ and $E$ from equation (33),

$$(1-\phi)\left\{\frac{w \,+\, \beta sU}{1 \,-\, \beta(1-s)} \,-\, U \,-\, \phi\tau\right\} \;=\; \phi\,\frac{y \,-\, w \,-\, \beta s\tau}{1 \,-\, \beta(1-s)} \,+\, \phi\tau\,. \tag{37}$$

After multiplying both sides by $(1-\beta(1-s))$, and simplifying, we find that the equilibrium wage satisfies

$$w \;=\; (1-\beta)U \,+\, \phi\Big(y \,-\, (1-\beta)U \,+\, (1-\beta)\tau\Big)\,. \tag{38}$$

To obtain an expression for $(1-\beta)U$, we can as before use expressions (4) and (24) to eliminate $(E-U)$ and $J$ in expression (33),

$$(1-\phi)\left\{\frac{(1-\beta)U \,-\, z}{\beta\,\theta\,q(\theta)} \,-\, \phi\tau\right\} \;=\; \phi\left\{\frac{c}{\beta\,q(\theta)} \,+\, \phi\tau\right\}, \tag{39}$$

which after simplifications, yields

$$(1-\beta)U \;=\; z \,+\, \frac{\phi}{1-\phi}\big[\theta\,c \,+\, \beta\,\theta\,q(\theta)\,\tau\big]. \tag{40}$$

After using expression (40) to eliminate $(1-\beta)U$ in expression (38), and simplifying, we obtain our second equation for the equilibrium wage:

$$w \;=\; z \,+\, \phi\big\{y - z \,+\, \theta\,c \,+\, \big[1-\beta\,(1-\theta\,q(\theta))\big]\tau\big\}\,. \tag{41}$$

Next, we equate the right sides of (36) and (41). After moving all terms that involve $\theta$ to one side and collecting the remaining terms on the other side, we obtain the following expression for equilibrium market tightness $\theta$:

$$(1-\phi)\left\{y - z \,-\, \frac{\phi(1-\beta)+\beta s}{1-\phi}\,\tau\right\} = \frac{r+s}{q(\theta)}c \,+\, \phi\,\theta\,c \,+\, \phi\,\beta\,\theta\,q(\theta)\,\tau\,. \tag{42}$$

Implicit differentiation of expression (42) yields

$$
\begin{aligned}
\frac{d\,\theta}{d\,y} &= -\frac{1-\phi}{-\left(\dfrac{-q'(\theta)\,(r+s)\,c}{q(\theta)^2} + \phi\,c + \phi\,\beta\,q(\theta)\,\tau + \phi\,\beta\,\theta\,q'(\theta)\,\tau\right)} \\
&= \frac{\dfrac{r+s}{q(\theta)}c + \phi\,\theta\,c + \phi\,\beta\,\theta\,q(\theta)\,\tau}{\dfrac{\alpha\,(r+s)\,c}{\theta\,q(\theta)} + \phi\,c + (1-\alpha)\phi\,\beta\,q(\theta)\,\tau} \left(y - z - \frac{\phi(1-\beta)+\beta s}{1-\phi}\tau\right)^{-1} \\
&= \frac{r+s + \phi\,\theta\,q(\theta)\big[1+\tau/J(\theta)\big]}{\alpha\,(r+s) + \phi\,\theta\,q(\theta)\big[1+(1-\alpha)\tau/J(\theta)\big]} \; \frac{\theta}{y - z - \dfrac{\phi(1-\beta)+\beta s}{1-\phi}\tau} \\
&\equiv \hat{\Upsilon}\big(\phi\,\theta q(\theta), \tau/J(\theta)\big) \; \frac{\theta}{y - z - \dfrac{\phi(1-\beta)+\beta s}{1-\phi}\tau}\,,
\end{aligned} \tag{43}
$$

where the second equality is obtained after using equation (42) to eliminate $(1-\phi)$ in the numerator, while in the denominator, we twice invoke the constant elasticity of matching with respect to unemployment, $\alpha = -q'(\theta)\,\theta/q(\theta)$; and the third equality follows from multiplying and dividing by $\theta\,q(\theta)/c$, and invoking equation (4), $\beta\,q(\theta)/c = J(\theta)^{-1}$. We can compute the elasticity of market tightness to be

$$
\eta_{\theta,y} \;=\; \hat{\Upsilon}\big(\phi\,\theta q(\theta), \tau/J(\theta)\big) \; \frac{y}{y - z - \dfrac{\phi(1-\beta)+\beta s}{1-\phi}\tau}\,. \tag{44}
$$

The factor $\hat{\Upsilon}\big(\phi\,\theta q(\theta), \tau/J(\theta)\big)$ is bounded from above by $\max\{\alpha^{-1}, (1-\alpha)^{-1}\}$. Therefore, and as before, a high elasticity of market tightness requires that the second factor in expression (44) is large, i.e., that the fundamental surplus fraction be small.

If workers' bargaining weight $\phi$ is zero, the fundamental surplus in expression (44) equals that in expression (30) for the earlier Nash product specification, i.e., the fundamental surplus is reduced by the annuitized layoff tax, $\beta s\tau$, as described by expression (31). But if workers' bargaining weight were to be raised, the fundamental surplus would be further reduced in the present setting where workers by assumption can threaten firms with the layoff tax even while bargaining as job applicants. Ljungqvist (2002) showed that this assumption has a dramatic impact on equilibrium unemployment. However, for common calibrations of the matching model, expression (44) suggests that the impact on the elasticity of market tightness with respect to productivity might not be too substantial. The dashed line in Figure 4 depicts

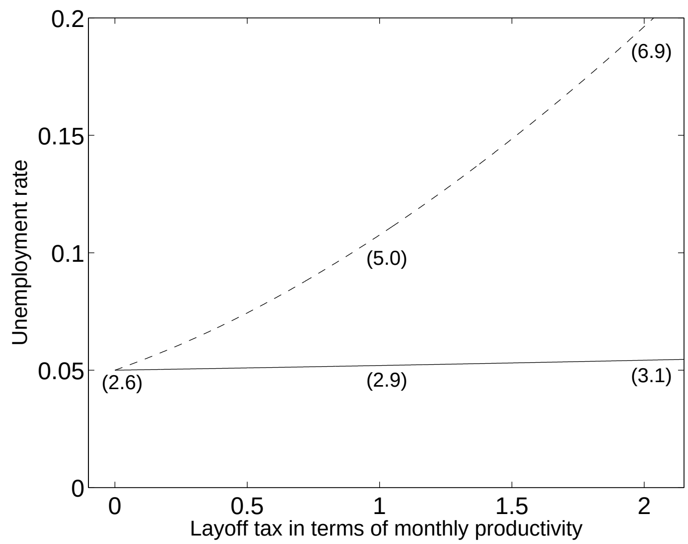


Figure 4: Unemployment rates for different layoff taxes with elasticities of market tightness with respect to productivity within parantheses (at layoff taxes 0, 1 and 2, respectively). The dashed and solid lines refer to Nash products $(E-U)^{\phi}(J+\tau)^{1-\phi}$ and $(E-U)^{\phi}J^{1-\phi}$, respectively.

unemployment for different layoff taxes with elasticities of market tightness in parentheses, given our parameterization in section 4.1. While the layoff tax is associated with dramatic deteriorations of firms' bargaining position so that the invisible hand must engineer large drops in market tightness and a correspondingly large increase in unemployment in order to create vacancies that let firms break even, it does not follow that the elasticity of market tightness becomes very high, because the fundamental surplus fraction fails to fall to very low levels. The solid line in Figure 4 depicts outcomes under the Nash product specification in the previous section 4.3, and layoff taxes then have a small impact.[6]

## 4.5 Alliance of policies determines fundamental surplus

Hornstein et al. (2007, hereafter HKV) use a matching model with vintage capital to attribute the increase in European unemployment to an acceleration of capital-embodied technological change. The formation of a job coincides with investing in a one-worker machine

[6]To compare with Ljungqvist's (2002) simulations, please keep in mind that the separation rate is exogenous in the present parameterization and hence a layoff tax can no longer suppress unemployment by causing firms to lower reservation productivities in their layoff decisions. Still, the outcomes in Figure 4 confirm the very different employment implications of the two alternative Nash product specifications.

with a capital-embodied productivity that is determined by the location of an exogenously moving technology frontier at the time of investment. The two inputs into a single matching function are homogeneous unemployed workers and vacant machines of different vintages. Matched worker-machine pairs engage in Nash-bargaining to set vintage-specific wage rates. All worker-machine pairs are subject to exogenous separation shocks sending separated workers and machines back to the matching function. There are no other costs of posting vacancies, so the upfront investment cost in a machine constrains job creation. When the capital-embodied productivity of a vintage has fallen too far behind the current technology frontier, machines are scrapped because there no longer exists any match surplus to be shared.

In Figure 5, the dashed and solid lines, meant to refer to stylized versions of Europe and the U.S., respectively, are almost replicas of those lines in panels A and B of HKV's figure 4. They depict steady-state unemployment rates and average durations of unemployment for different assumed rates of capital-embodied technological change, $\gamma$. HKV suggest that pre-1970 and post-1990 are characterized by $\gamma = 0.04$ and $\gamma = 0.077$, respectively, i.e., the leftmost ends of the panels vis-à-vis the dotted vertical lines. Hence, starting from the same unemployment rate of 4% in pre-1970, panel A shows that the unemployment rate increases in post-1990 by over four percentage points Europe but by just one percentage point in the U.S., with corresponding changes in average unemployment duration in panel B. These different outcomes are due to HKV's assumptions about government policies and also about exogenous separation rates that differ across Europe and the U.S. but remain fixed over time.

HKV calibrate government policies to be more stringent in Europe as compared to the U.S.: unemployment benefits $b^{\mathrm{EU}} = 0.33$ versus $b^{\mathrm{US}} = 0.05$, which correspond to replacement rates of 75% and 10% of average wages in pre-1970 (when $\gamma = 0.04$) in Europe and the U.S., respectively; a layoff tax $\tau^{\mathrm{EU}} = 0.45$ in Europe, which is equivalent to one year of average wages in pre-1970 (when $\gamma = 0.04$) versus no layoff tax in the U.S.; and a 2-tuple of European income and payroll taxes $\{24\%, 21\%\}$ versus a U.S. 2-tuple $\{17\%, 8\%\}$.[7] To attain the same 4% unemployment rate in pre-1970 (when $\gamma = 0.04$), HKV assume a significantly lower exogenous separation rate in Europe than in the U.S. Based on these calibrations, the steady-state outcomes in Figure 5 emerge when varying $\gamma$. HKV (2007, p. 1110) also solve the model for Europe with one government policy at a time and conclude that "the technology–policy interaction is much starker when the three policies are considered together;

[7] To study balanced growth paths, HKV assume that unemployment benefits $b$ and layoff taxes $\tau$ (as well as the investment cost for machines and the value of leisure) change at the economy's growth rate.

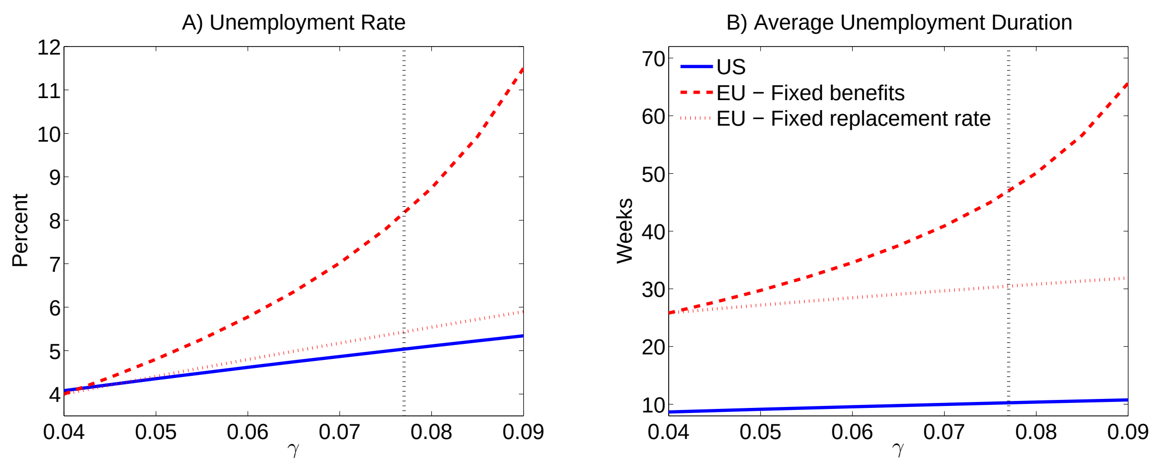


Figure 5: Unemployment rates and average durations of unemployment for different rates of capital-embodied technological change, $\gamma$, where the dashed and solid lines refer to Europe and the U.S., respectively. The dotted line depicts European outcomes if the replacement rate in unemployment compensation had been kept constant (rather than the quantity $\hat{b}^{\mathrm{EU}}$).

as $\gamma$ increases, if one estimated the total role of policy by merely summing the effects of the individual policies, one would only account for less than one-third of the total technology–policy interaction predicted by the model with all policies jointly considered."

To understand these outcomes, we turn to the fundamental surplus and draw analogies to the forces at work earlier in Figure 1. A higher $\gamma$ acts as an obsolence shock because it shortens the equilibrium lifespan of machines, leaving less time to recover the investment cost. Everything else equal, a shorter lifespan of a machine also implies that the annuity value of the layoff tax increases and hence, the fundamental surplus falls (in line with the discussion of expression (30)). However, not everything else is equal – the fundamental surplus now shrinks faster with the age of a machine since both the unemployment benefit and the layoff tax grow at the economy's growth rate that has risen as a consequence of a higher $\gamma$ (whereas the vintage-dependent output evolves over a machine's lifespan in the same way as before). To understand the response of the steady-state unemployment rate, the implied reduction in the fundamental surplus fraction can be thought of as a leftward movement in a graph like Figure 1. Thus, HKV's finding of a large increase in European unemployment is a manifestation of how welfare state policies had already diminished the initial fundamental surplus fraction, i.e., in the earlier analogue to Figure 1, Europe was positioned farther to the left in that graph before the onslaught of higher capital-embodied technological change.

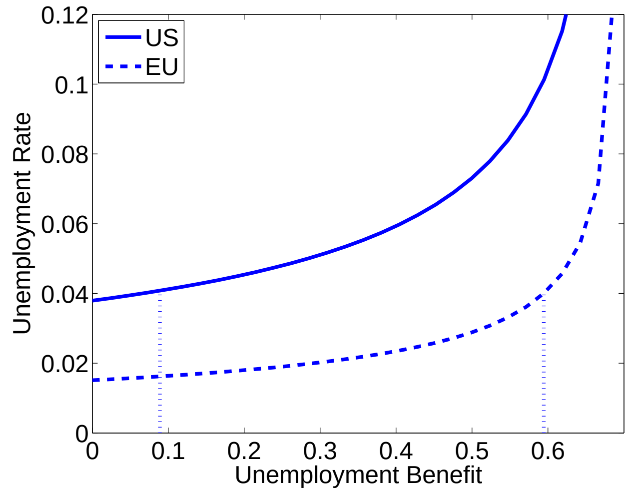


Figure 6: Unemployment rates for different unemployment benefits in pre-1970 (when $\gamma = 0.04$) in the U.S. (solid line) and Europe (dashed line).

To make things more precise, we explain how we constructed Figure 5. Selecting one of HKV's possible government policies, we consider only unemployment benefits, and find that our alternative setting of $\hat{b}^{\text{EU}} = 0.594$ and $\hat{b}^{\text{US}} = 0.089$, can reproduce the outcomes of HKV's bundle of policies (as mentioned, the dashed and solid lines in Figure 5 are virtually the same as those of HKV's figure 4).[8] It is instructive to examine how the unemployment rate depends on benefits in pre-1970 (when $\gamma = 0.04$), as depicted in Figure 6. Following HKV we assume a much smaller exogenous separation rate in Europe than in the U.S. and hence the dashed line for Europe lies much below the solid line for the U.S. The vertical dotted lines mark our benefits, $\hat{b}^{\text{US}}$ and $\hat{b}^{\text{EU}}$, respectively, at which the U.S. and Europe attain the same unemployment rate of 4% in pre-1970. A change to a higher $\gamma$ with its implied decline in the fundamental surplus fraction pushes the graph in in Figure 6 rightward. Evidently, before 1970 Europe was poised to experience a larger increase in unemployment because a higher benefit level had decreased its fundamental surplus fraction. Thus, what matters is how far an economy is situated to the right along the curve in Figure 6 relative to where the

[8]With unemployment benefits as the sole policy, our algorithm for reproducing HKV's unemployment outcomes is as follows. First, for each value of unemployment benefits, we find an exogenous separation rate that produces HKV's targeted unemployment rate of 4% in pre-1970. Next, among all such pairs of benefits and exogenous separation rates, we select the pair that best reproduces HKV's relationship between unemployment and the exogenous rate of capital-embodied technological change (exhibited in panel A of HKV's figure 4). While this algorithm induces us to lower HKV's parameterization of the exogenous separation rate in Europe from 0.0642 to 0.0356, it leaves HKV's value for the U.S. unchanged at 0.2117.

unemployment relationship becomes *much more convex*. This is a way of expressing HKV's observation above that "the technology–policy interaction is much starker when [all] policies are considered together."

In their conclusion, HKV (2007, p. 1113) perceptively note that there are important connections to be made with business-cycle studies using matching models; "[o]ne lesson from these studies (Hagedorn and Manovskii, [2008], in particular) is that significant volatility requires large values of a worker's utility when unemployed – the value of leisure and unemployment benefits ... Although we focus on steady states rather than short-run dynamics, we also emphasize that in the presence of strict labour-market policy (e.g. high unemployment benefits $b$), productivity changes (here, growth in capital-embodied technology) have a significant impact on quantities ... Thus, our interpretation of the recent European experience builds on similar mechanisms as those discussed in the recent literature on short-run dynamics." Here we interpet HKV as pointing to the combination of effects of welfare state policies on the fundamental surplus. Specifically, for unemployment to rise sharply with a higher pace of capital-embodied technological change, the fundamental surplus fraction must be small.

It is instructive to investigate the role of a contributing factor to the diminished post-1990 fundamental surplus fraction in HKV's theory of the outbreak of high European unemployment. Our HKV version in Figure 5 with a single policy of unemployment benefits, $\hat{b}^{\mathrm{EU}} = 0.594$, implies a replacement rate of 85% in pre-1970, while the same fixed quantity $\hat{b}^{\mathrm{EU}}$ implies a higher replacement rate of more than 93% in post-1990. If we instead let $\hat{b}^{\mathrm{EU}}$ vary with $\gamma$ so as to keep the replacement rate constant at the value that prevails in pre-1970, we obtain the equilibrium relationship presented by the dotted curve in Figure 5. The resulting much smaller rise in European unemployment in post-1990 suggests that HKV's explanation of higher European unemployment is not just about higher capital-embodied technological change but also comes from their implicit assumptions that unemployment benefits became more generous (and also that the layoff tax in HKV's analysis became more onerous).[9] If HKV had wanted to keep the unemployment benefit and the layoff tax policy constant over time in relation to the average wage, HKV would have had to calibrate an even smaller

[9] In HKV's analysis of Europe, the unemployment benefit, $b^{\mathrm{EU}} = 0.33$, corresponds to a replacement rate of 75% (83%), and the layoff tax, $\tau^{\mathrm{EU}} = 0.45$, is equal to 1.0 (1.14) annual average wages in pre-1970 (post-1990). To keep European policy unchanged over time in terms of the average wage, post-1990 values of the unemployment benefit and the layoff tax would have to be $b^{\mathrm{EU}} = 0.295$ and $\tau^{\mathrm{EU}} = 0.40$, respectively, resulting in a post-1990 European unemployment rate of merely 5.4%. Once again, unemployment outcomes in our augmented HKV analysis are very close to those of our HKV version with only unemployment benefits.

fundamental surplus fraction for pre-1970 Europe in order to generate a larger response of unemployment to higher capital-embodied technological change in post-1990. Mechanically, HKV would have had to increase the unemployment benefit and/or the layoff tax to move pre-1970 Europe closer to the more convex region in Figure 6, and to lower further the exogenous separation rate in Europe to attain an unemployment rate of 4% in pre-1970.

The remainder of this paper studies the role of the fundamental surplus in business cycles models with matching functions.

# 5 Fundamental surplus and business cycles

## 5.1 Alternative calibration

To confront the Shimer critique described above in section 3.2, Hagedorn and Manovskii (2008) propose an alternative calibration of the standard matching model that effectively places the economy at the left end of our Figure 1. To illustrate such alternative calibrations, let the productivities 0.61, 0.63 and 0.65 in Figure 1 represent three different economies with homogeneous workers. For each economy, we renormalize the efficiency parameter $A$ in the matching function to make the unemployment rate be 5 percent at the economy's postulated productivity level. Figure 7 shows how the steady-state unemployment rate would change if we were to perturb productivity around each economy's productivity level. Specifically, the elasticity of market tightness with respect to productivity would be 65, 22 and 14 in an economy with productivity 0.61, 0.63 and 0.65, respectively. According to formula (14) for the elasticity of unemployment with respect to market tightness (evaluated at our calibration with $\alpha = 0.5$), the corresponding elasticities of unemployment with respect to productivity are roughly negative half those numbers for the elasticity of market tightness with respect to productivity.

The relationship between unemployment and productivity in Figure 7 foreshadows our numerical simulations of economies with aggregate productivity shocks in section 6. There we replicate earlier findings that the empirical volatility of unemployment can be reproduced under the Hagedorn-Manovskii alternative calibration but not under a 'common' calibration of the matching model, i.e., the Shimer critique in section 3.2. As we have demonstrated, what accounts for these different outcomes is the size of the fundamental surplus fraction. However, as reported but also questioned by Hagedorn and Manovskii (2008, p. 1695), another "prominent explanation of the findings in Shimer (2005) is that the elasticity of wages

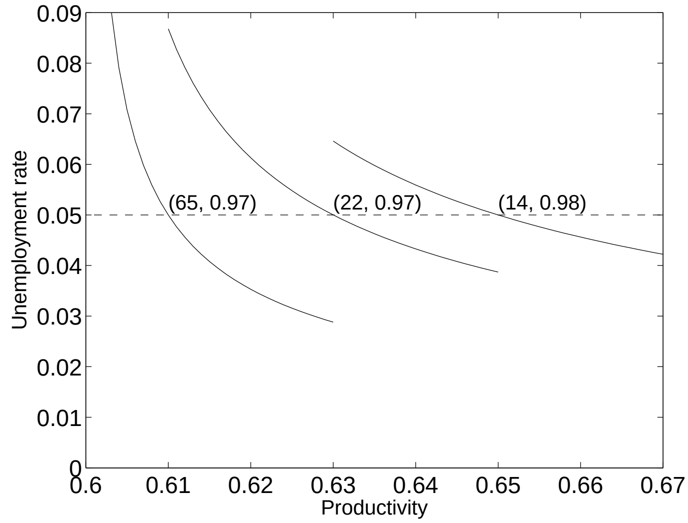


Figure 7: Unemployment for three segments of the schedule in Figure 1, normalized to generate 5 percent unemployment at productivities 0.61, 0.63 and 0.65, respectively, with the elasticities of market tightness and of the wage rate, respectively, within parentheses $(\eta_{\theta,y},\, \eta_{w,y})$.

is too high in his model (0.964). The argument is then that an increase in productivity is largely absorbed by an increase in wages, leaving profits (and, thus, the incentives to post vacancies) little changed over the business cycle." An accompanying counterposition, as argued e.g. by Rogerson and Shimer (2011, p. 660), is that wages are rigid under the calibration of "Hagedorn and Manovskii (2008), although it is worth noting that the authors do not interpret their paper as one with wage rigidities. They calibrate ... a small value for the workers' bargaining power [$\phi = 0.052$]. This significantly amplifies productivity shocks ..." A problematic aspect of the Rogerson and Shimer's argument is that the elasticity of the wage with respect to productivity is around 0.97 for all three of our economies in Figure 7, where we assume a much higher workers' bargaining weight $\phi = 0.5$, yet still obtain Hagedorn and Manovskii's high elasticity of market tightness and unemployment with respect to productivity.

Hagedorn and Manovskii (2008) make the same point when they perturb their own calibration by raising the workers' bargaining weight in a way that generates the same high wage elasticity as in a common calibration of the matching model, while retaining the outcome that unemployment is very sensitive to changes in productivity because of the small fundamental surplus fraction in the Hagedorn-Manovskii calibration. Hagedorn and Manovskii

(2008) also perturb a common calibration of the matching model by lowering the workers' bargaining weight, in a way that yields the same low wage elasticity as in the Hagedorn-Manovskii calibration, while now retaining the outcome that unemployment is insensitive to changes in productivity because of the high fundamental surplus fraction in a common calibration of the matching model. We perform a similar exercise in Figure 8 where we recompute our relationship between unemployment and productivity for different values of workers' bargaining weight $\phi \in \{0.05, 0.1, 0.2, 0.5, 0.8\}$, where the solid line with $\phi = 0.5$ is the same as in Figure 1. The corresponding elasticities of wages with respect to productivity are depicted in Figure 9.[10] We conclude from Figure 8 that high responses of unemployment to changes in productivity occur only at low fundamental surplus fractions. Figure 9 confirms that the Shimer critique of common calibrations does not hinge on a high wage elasticity and that the Hagedorn-Manovskii result is not predicated on a low wage elasticity.

In conclusion, we do not deny that common calibrations of the matching model and the specific alternative calibration of Hagedorn and Manovskii (2008) are characterized by high and low wage elasticities, respectively; but we do assert that these wage outcomes are not essential in activating the main force at work, which is the size of the calibrated fundamental surplus fraction.

**Wage elasticity**

To study the determinants of the elasticity of the wage with respect to productivity, we compute the derivative of the wage rate $w$ with respect to productivity $y$. First, we differentiate wage expression (11) with respect to $w$, $y$ and $\theta$,

$$dw \;=\; \phi\, dy \,+\, \phi\, c\, d\theta$$

or

$$\frac{d\,w}{d\,y} \;=\; \phi \left[1 \,+\, c\,\frac{d\,\theta}{d\,y}\right]. \tag{45}$$

Together with the derivative of market tightness with respect to productivity in (16), we arrive at

$$\frac{d\,w}{d\,y} \quad=\quad \phi \left[1 \,+\, c\,\Upsilon\big(\phi\,\theta q(\theta)\big)\;\frac{\theta}{y-z}\right] \;=\; \phi \left[1 \,+\, \frac{(1-\phi)\,q(\theta)}{r+s\,+\,\phi\,\theta\,q(\theta)}\Upsilon\big(\phi\,\theta q(\theta)\big)\;\theta\right]$$

[10]For an explanation to why the wage elasticities in Figure 7 are higher than those for the solid line ($\phi = 0.5$) in Figure 9, see the last paragraph of this section under subheading "Wage elasticity."

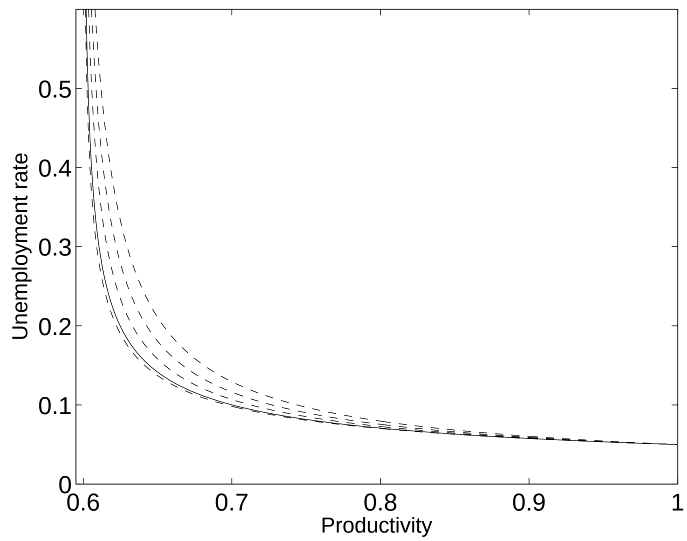


Figure 8: Unemployment for different workers' bargaining weight, $\phi \in \{0.05, 0.1, 0.2, 0.5, 0.8\}$ (from top to bottom curve).

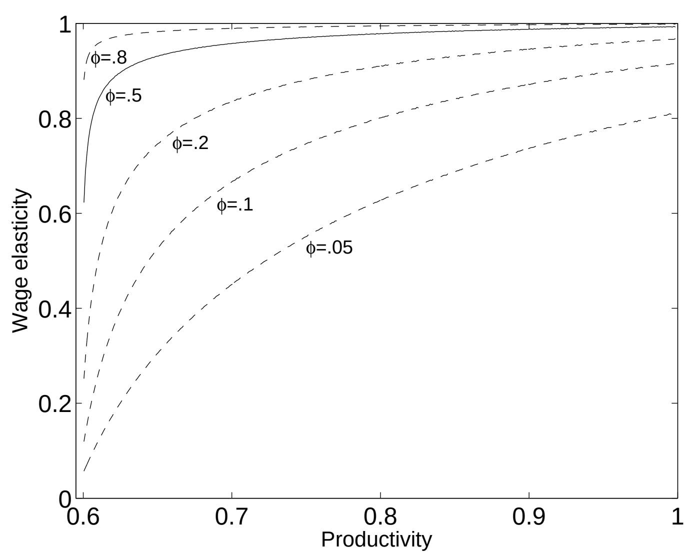


Figure 9: Elasticity of wage with respect to productivity for different workers' bargaining weight, $\phi \in \{0.05, 0.1, 0.2, 0.5, 0.8\}$ (from bottom to top curve).

$$= \quad \phi \left[ 1 + \frac{(1-\phi)\,\theta\, q(\theta)}{\alpha(r+s) \,+\, \phi\,\theta\, q(\theta)} \right] \;=\; \phi\, \frac{\alpha(r+s) \,+\, \theta\, q(\theta)}{\alpha(r+s) \,+\, \phi\,\theta\, q(\theta)}\,, \tag{46}$$

where the second equality uses expression (15) to eliminate $c/(y-z)$, and the third equality invokes the definition of $\Upsilon(\cdot)$ in (16).[11] The derivative (46) varies from zero to one as $\phi$ varies from zero to one. At one extreme, we know that if workers have a zero bargaining weight, $\phi = 0$, the equilibrium wage is equal to the value of leisure, $w = z$, and hence, the wage does not respond to changes in productivity, $dw/dy = 0$. At the other extreme, if firms have a zero bargaining weight, $\phi = 1$, workers reap all gains from productivity, $w = y$, and hence, the wage responds one-for-one to changes in productivity, $dw/dy = 1$. But of course, in the latter case, there would be no vacancy creation in the first place so no one would be employed.

Next, we use equation (5) to derive an expression for the wage as a fraction of productivity:

$$\begin{aligned} \frac{w}{y} \quad &= \quad 1 - \frac{r+s}{q(\theta)}\,\frac{c}{y} \;=\; 1 - \frac{(1-\phi)(r+s)}{r+s \,+\, \phi\,\theta\, q(\theta)}\,\frac{y-z}{y} \\ &= \quad \frac{\big[\phi + (1-\phi)\, z/y\big](r+s) \,+\, \phi\,\theta\, q(\theta)}{r+s \,+\, \phi\,\theta\, q(\theta)}\,, \end{aligned} \tag{47}$$

where the second equality uses equation (15) to eliminate $c/q(\theta)$. In the Hagedorn-Manovskii calibration with an extremely small fundamental surplus fraction, $(y-z)/y \approx 0$, it follows immediately from the second equality of expression (47) that $w/y \approx 1$. This outcome is a necessary consequence of the fact that the wage rate cannot fall below $z$ if workers are to be willing to work, nor can it exceed $y$ if firms are to be motivated to post vacancies. Within more common calibrations of the matching model, as discussed in section 3.2, recall that $\phi\theta q(\theta)$ is usually high relative to $(r+s)$, and hence, the third equality of expression (47) explains why $w/y$ can still be close to one even when the value of leisure is not close to productivity.

Using expressions (46) and (47), the elasticity of the wage with respect to productivity,

[11] Another approach to compute $dw/dy$ (and $d\theta/dy$) is to express the total differential of wage equations (5) and (11) in matrix form,

$$\begin{bmatrix} q(\theta) & (w-y)q'(\theta) \\ 1 & -\phi\, c \end{bmatrix} \begin{bmatrix} dw \\ d\theta \end{bmatrix} \;=\; \begin{bmatrix} q(\theta)\, dy \\ \phi\, dy \end{bmatrix},$$

and then apply Cramer's rule, followed by substitutions analogous to those above. As an alternative to any one of the two wage equations, we can also use equation (15) in this calculation.

$\eta_{w,y} \equiv (dw/dy)/(w/y)$, becomes

$$\eta_{w,y} \;=\; \phi\, \frac{\alpha(r+s) \,+\, \theta\, q(\theta)}{\alpha(r+s) \,+\, \phi\,\theta\, q(\theta)}\; \frac{r+s \,+\, \phi\,\theta\, q(\theta)}{\big[\phi + (1-\phi)\, z/y\big](r+s) \,+\, \phi\,\theta\, q(\theta)}\,. \tag{48}$$

As just discussed, under the Hagedorn-Manovskii calibration, the second fraction in expression (48) must approximately equal one, and hence, the wage elasticity coincides with the derivative of the wage with respect to productivity in expression (46). This observation sheds light on why and how Hagedorn and Manovskii (2008) designed their calibration. The choice of the workers' bargaining weight $\phi$ in equation (46) is used to attain a target wage elasticity, while the choice of the fundamental surplus fraction, $(y-z)/y$, in equation (17) is used to attain a particular elasticity of market tightness that via equation (14) is linked to a target elasticity of unemployment. Therefore, as also argued by Hagedorn and Manovskii, the choice of wage elasticity is incidental to the outcome that unemployment is highly sensitive to productivity changes, which is driven by their calibration of a small fundamental surplus fraction.

As mentioned above, Hagedorn and Manovskii proceed to argue that the high wage elasticity in a common calibration of the matching model can be lowered without changing its implication that unemployment is not very sensitive to productivity changes. Such a perturbed calibration also involves modifying the wage elasticity by altering the worker's bargaining weight $\phi$, though the complication is that both fractions in expression (48) are now at play when $z << y$. Nevertheless, within some bounds, the workers' bargaining power can be lowered in a common calibration of the matching model to reduce the wage elasticity substantially, as argued by Hagedorn and Manovskii and illustrated by our Figure 9. And as long as the fundamental surplus fraction is kept high, the elasticity of market tightness remains low in equation (17) so that unemployment does not respond much to productivity, as illustrated in Figure 8.

Finally, for a given workers' bargaining weight $\phi$, we take the limit of the wage elasticity when $y$ approaches $z$, which is also associated with a worker's probability of finding a job $\theta q(\theta)$ going to zero:

$$\lim_{y\to z,\theta q(\theta)\to 0} \eta_{w,y} \;=\; \lim_{\theta q(\theta)\to 0} \left[\phi\, \frac{\alpha(r+s) \,+\, \theta\, q(\theta)}{\alpha(r+s) \,+\, \phi\,\theta\, q(\theta)}\right] \;=\; \phi\,. \tag{49}$$

This limit is discernable as productivity approaches $z = 0.6$ in Figure 9. In Figure 7, we temporarily arrest this convergence by recalibrating three unemployment schedules for $\phi = 0.5$ to increase the efficiency parameter $A$ in the matching function so that the unemployment rate is 5 percent at productivities 0.61, 0.63 and 0.65, respectively. This recalibration increases a worker's probability of finding a job $\theta q(\theta)$, which by the logic of our limiting calculation in expression (49) arrests the convergence to the limit described in (49), and explains why all three wage elasticities in Figure 7 are approximately 0.97 as compared to 0.83, 0.91 and 0.93 for productivities 0.61, 0.63 and 0.65, respectively, along the solid line for $\phi = 0.5$ in Figure 9.

## 5.2 Sticky wages

Another response to the Shimer critique is Hall's (2005) analysis of sticky wages, or more precisely, a constant wage in his main analysis. Hall notes that a constant wage in a matching model can be consistent with no private inefficiencies in contractual arrangements. Specifically, matching frictions imply a range of wages that the firm and worker both prefer to breaking a match. Hence, the standard assumption of Nash bargaining in matching models is just one way to determine the wage that prevails within a match. As an alternative to bargaining, Hall considers a 'wage norm' that stipulates a particular wage $\hat{w}$ inside the Nash bargaining set that must be paid to workers. How would assuming such a constant wage change the elasticity of market tightness with respect to productivity? The answer again hinges on the size of the appropriately defined fundamental surplus fraction that determines the elasticity of market tightness.

Given a constant wage $w = \hat{w}$, an equilibrium is again characterized by the zero-profit condition for vacancy creation in expression (5) of the standard matching model,

$$\hat{w} \;=\; y - \frac{r+s}{q(\theta)} c \,. \tag{50}$$

There exists an equilibrium for any constant wage $\hat{w} \in [z, y-(r+s)c]$, where the lower bound is a worker's utility of leisure and the upper bound is determined by the zero-profit condition for vacancy creation when the probability of a firm filling a vacancy is at its maximum value of one, $q(\theta) = 1$.

Rearrange expression (50) to get

$$q(\theta)\,(y - \hat{w}) \;=\; (r+s)\,c$$

and implicitly differentiate to get

$$\frac{d\,\theta}{d\,y} \;=\; -\frac{q(\theta)}{q'(\theta)(y-\hat{w})} \;=\; \frac{\theta}{\alpha(y-\hat{w})}\,, \tag{51}$$

where the second equality follows from invoking the constant elasticity of matching with respect to unemployment, $\alpha = -q'(\theta)\,\theta/q(\theta)$. We can then compute the elasticity of market tightness as

$$\eta_{\theta,y} \;=\; \frac{1}{\alpha}\;\frac{y}{y-\hat{w}}\,. \tag{52}$$

This expression resembles the earlier one in (16). Not surprisingly, if the constant wage is equal to the value of leisure, $\hat{w} = z$, the present elasticity (52) is equal to that earlier elasticity of market tightness in the standard matching model with Nash bargaining if the workers have a zero bargaining weight, $\phi = 0$. Because under such lopsided bargaining, the equilibrium wage would indeed be the constant value $z$ of leisure.

From this similarity, we are reminded that the first factor in expression (52) plays a limited role in magnifying the elasticity $\eta_{\theta,y}$ because it is bounded from above by, and here actually equal to, the inverse of the elasticity of matching with respect to unemployment, $\alpha$. So, as before, it is the second factor, the inverse of the fundamental surplus fraction, that tells whether the elasticity of market tightness is high or low. The appropriate definition of the fundamental surplus is now the difference between productivity and the stipulated constant wage.

Our earlier discussion of the significance of a small fundamental surplus fraction becomes highlighted in the present context where all of the fundamental surplus goes to vacancy creation (as when the workers' bargaining weight is zero in the standard matching model with Nash bargaining). A given percentage change in productivity is magnified to become a larger percentage change in the fundamental surplus by a factor of $y/(y-\hat{w})$. Because all of the fundamental surplus now goes to vacancy creation, there is a correspondingly large impact on unemployment. Our interpretation is born out in the numerical simulations of economies with aggregate productivity shocks in section 6.

## 5.3 Alternating-offer wage bargaining

Hall and Milgrom (2008) proposed yet another response to the Shimer critique. They replaced standard Nash bargaining with alternating-offer bargaining. A firm and a worker take turns making wage offers. The threat is not to break up and receive outside values, but instead to continue to bargain because that choice has a strictly higher payoff than accepting the outside option. After each unsuccessful bargaining round, the firm incurs a cost of delay $\gamma > 0$ while the worker enjoys the value of leisure $z$. There is a probability $\delta$ that the job opportunity is exogenously destroyed between bargaining rounds, sending the worker to the unemployment pool.

It is optimal for either bargaining party to make a barely acceptable offer to the other side. The firm always offers $w^f$ and the worker always offers $w^w$. Consequently, in an equilibrium, the first wage offer is accepted. Hall and Milgrom assume that firms make the first wage offer.

In their concluding remarks, Hall and Milgrom (2008, p. 1673) choose to emphasize that "the limited influence of unemployment [the outside value of workers] on the wage results in large fluctuations in unemployment under plausible movements in [productivity]." However, we think that it is more enlightening to emphasize that once again the key force is that an appropriately defined fundamental surplus fraction is calibrated to be small. Without a small fundamental surplus fraction, it matters little that the outside value has been removed from the bargaining. We illustrate this idea by computing the elasticity of market tightness with respect to productivity.

After a wage agreement, a firm's value of a filled job, $J$, and the value of an employed worker, $E$, remain given by expressions (2) and (6) in the standard matching model. These can be rearranged to become

$$E = \frac{w + \beta\, s\, U}{1 - \beta(1-s)}\,, \tag{53}$$

$$J = \frac{y - w}{1 - \beta(1-s)}\,, \tag{54}$$

where we have imposed a zero-profit condition on vacancy creation, $V = 0$, in the second expression. Thus, using expression (53), the indifference condition for a worker who has just received a wage offer $w^f$ from the firm and is choosing whether to decline the offer and wait until the next period to make a counteroffer $w^w$ is

$$\frac{w^f + \beta\, s\, U}{1 - \beta(1-s)} = z + \beta\left[(1-\delta)\frac{w^w + \beta\, s\, U}{1 - \beta(1-s)} + \delta\, U\right]. \tag{55}$$

Using expression (54), the analogous condition for a firm contemplating a counteroffer from the worker is

$$\frac{y - w^w}{1 - \beta(1-s)} \;=\; -\gamma \,+\, \beta(1-\delta)\,\frac{y - w^f}{1 - \beta(1-s)}\,. \tag{56}$$

After collecting and simplifying the terms that involve the worker's outside value $U$, expression (55) becomes

$$\frac{w^f}{1 - \beta(1-s)} \;=\; z \,+\, \beta(1-\delta)\,\frac{w^w}{1 - \beta(1-s)} \,+\, \beta\,\frac{1-\beta}{1 - \beta(1-s)}\,(\delta - s)\,U. \tag{57}$$

As emphasized by Hall and Milgrom, the worker's outside value $U$ has a small influence on bargaining; when $\delta = s$, the outside value disappears from expression (57). That is, under continuing bargaining that ends only either with an agreement or with destruction of the job, the outside value will matter only if the job destruction probability differs before and after reaching an agreement. To strengthen Hall and Milgrom's (2008) observation that the outside value has at most a small influence under their bargaining protocol, we proceed under the assumption that $\delta = s$, so the two indifference conditions (57) and (56) become

$$w^f \;=\; (1-\tilde{\beta})\,z \,+\, \tilde{\beta}\,w^w, \tag{58}$$

$$y - w^w \;=\; -(1-\tilde{\beta})\,\gamma \,+\, \tilde{\beta}\,(y - w^f)\,, \tag{59}$$

where $\tilde{\beta} \equiv \beta(1-s)$. Solve for $w^w$ from (59) and substitute into (58) to get

$$w^f \;=\; (1-\tilde{\beta})\,z \,+\, \tilde{\beta}\left[(1-\tilde{\beta})(y+\gamma) \,+\, \tilde{\beta}\,w^f\right], \tag{60}$$

which can be rearranged to read

$$w^f \;=\; \frac{(1-\tilde{\beta})\left[z \,+\, \tilde{\beta}(y+\gamma)\right]}{1 - \tilde{\beta}^2} \;=\; \frac{z \,+\, \tilde{\beta}(y+\gamma)}{1+\tilde{\beta}}\,. \tag{61}$$

This is the wage that the firm would immediately offer the worker when first matched and the offer would be accepted.[12] In an equilibrium, this wage must also be consistent with the no-profit condition in vacancy creation. Substitution of $w = w^f$ from expression (61) into the no-profit condition (5) of the standard matching model results in the following expression

[12] When firms make the first wage offer, a necessary condition for an equilibrium is that $w^f$ in expression (61) is less than productivity $y$, i.e., the parameters must satisfy $z + \tilde{\beta}\gamma < y$.

for equilibrium market tightness:

$$\frac{z + \tilde{\beta}(y+\gamma)}{1+\tilde{\beta}} = y - \frac{r+s}{q(\theta)}\, c. \tag{62}$$

After multiplying both sides by $(1+\tilde{\beta})\, q(\theta)$ and rearranging, expression (62) can be written as

$$q(\theta)\left[y - z - \tilde{\beta}\,\gamma\right] = (1+\tilde{\beta})(r+s)\, c.$$

Implicit differentiation yields

$$\frac{d\,\theta}{d\,y} = -\frac{q(\theta)}{q'(\theta)\left[y - z - \tilde{\beta}\,\gamma\right]} = \frac{\theta}{\alpha\left[y - z - \tilde{\beta}\,\gamma\right]}\,, \tag{63}$$

where the second equality follows from invoking the constant elasticity of matching with respect to unemployment, $\alpha = -q'(\theta)\,\theta/q(\theta)$. We can then compute the elasticity of market tightness as

$$\eta_{\theta,y}\Big|_{\text{first wage offer by firms}} = \frac{1}{\alpha}\;\frac{y}{y - z - \tilde{\beta}\,\gamma}\,. \tag{64}$$

Under the alternative assumption that workers formulate the first wage offer, we substitute expression (61) for $w^f$ into equation (59)

$$w^w = \frac{\tilde{\beta}z + y + \gamma}{1+\tilde{\beta}}\,, \tag{65}$$

and then use this wage $w = w^w$ in the no-profit condition (5) to obtain an alternative to equilibrium expression (62).[13] Following the steps above, we arrive at the alternative elasticity of market tightness,

$$\eta_{\theta,y}\Big|_{\text{first wage offer by workers}} = \frac{1}{\alpha}\;\frac{y}{y - z - \gamma/\tilde{\beta}}\,. \tag{66}$$

If the discount factor $\tilde{\beta} \equiv \beta(1-s)$ is close to one, expressions (64) and (66) yield approximately the same value, i.e., it is irrelevant whether the firm or the worker makes the first wage offer. In both cases, the fundamental surplus is the productivity that remains after making deductions for the value of leisure $z$ and a firm's cost of delay $\gamma$. That is, in

[13]When workers make the first wage offer, a necessary condition for an equilibrium is that $w^w$ in expression (65) is less than productivity $y$, i.e., the parameters must satisfy $z + \gamma/\tilde{\beta} < y$.

addition to deducting the value of leisure, we must also deduct a term that captures the worker's prospective gains from his inalienable right in bargaining to exploit the cost that delay imposes on the firm. What remains of productivity is the fundamental surplus that could potentially be extracted by the 'invisible hand' and devoted to sustaining vacancy creation in an equilibrium.

Note that for low values of $\tilde{\beta}$ (e.g., due to a high job destruction rate $s$), the alternating-offer bargaining protocol awards a substantial advantage to the party that makes the first wage offer. On one side, in the limit with $\tilde{\beta} = 0$, the elasticity of market tightness in expression (64) when firms make the first wage offer is the same as in expression (17) for the standard matching model with Nash bargaining when a worker's bargaining weight $\phi$ is zero. At the other extreme, for sufficiently low values of $\tilde{\beta}$ when workers make the first wage offer, the fundamental surplus in expression (66) turns negative. This indicates a degenerate equilibrium with no employment because firms would not be able to recover the vacancy costs. (See footnote 13.)

While Hall and Milgrom (2008, p. 1670) notice that their "sum of $z$ and $\gamma$ is ... not very different from the value of $z$ by itself in ... Hagedorn and Manovskii's calibration" (as studied in our section 5.1), they downplay this similarity and instead emphasize differences in mechanisms across the two frameworks. But from our perspective of the fundamental surplus, it is really this similarity that should be stressed – the frameworks are united in requiring a small fundamental surplus fraction to generate high unemployment volatility over the business cycle.

To summarize, we do not doubt that the alternative bargaining protocol of Hall and Milgrom (2008) suppresses the influence of the worker's outside value during bargaining. But this outcome would be inconsequential had Hall and Milgrom not calibrated a small fundamental surplus fraction.

## 5.4 A financial accelerator

Wasmer and Weil (2004) explore how a financial accelerator affects the elasticity of labor market tightness with respect to productivity. They assume that a process of matching in a credit market precedes matching in the labor market. Credit matching determines equilibrium measures $e$ and $f$ of entrepreneurs and financiers, respectively, the two inputs into a matching function for the credit market. Matched entrepreneur-financier pairs then post vacancies in a matching function for labor. As before, the labor market matching function matches vacancies with workers, and filled jobs are exogenously destroyed with per-period probability $s$. Though, when jobs are now destroyed, so are the entrepreneur-financier matches that helped to create them.

The credit market matching function has constant returns to scale and credit market tightness $\sigma \equiv e/f$ determines the probability $p(\sigma)$ ($\sigma p(\sigma)$) that an entrepreneur (a financier) finds a counterparty. Per-period credit market search costs of an entrepreneur and a financier are denoted $\epsilon > 0$ and $\kappa > 0$, respectively. A successfully matched entrepreneur-financier pair immediatelely posts one vacancy in the matching function for the labor market. An entrepreneur-financier pair shares the value of a vacancy according to Nash bargaining, with $\psi$ and $1-\psi$ being the bargaining power of the entrepreneur and the financier, respectively. In their main setup, Wasmer and Weil (2004) assume a sticky wage $\hat{w}$ in the labor market. The key question ultimately to be studied is how the elasticity of labor market tightness differs from that of Hall's (2005) sticky wage model described in section 5.2.

In an equilibrium, costly search in the credit market assumes a strictly positive value of a vacancy in the labor market, $V > 0$. Nash bargaining splits that value so that the entrepreneur and the financier receive $\psi V$ and $(1-\psi)V$, respectively. Moreover, free entry on both sides of the credit market ensures that the two parties expect to break even, so that the per-period search costs of an entrepreneur and a financier equal their respective expected payoffs:

$$\epsilon \;=\; p(\sigma)\psi V \quad \text{and} \quad \kappa \;=\; \sigma p(\sigma)(1-\psi)V. \tag{67}$$

The zero expected profits conditions (67) imply that the equilibrium value of a vacancy in the labor market equals total search costs in the credit market divided by the number of entrepreneur-financier pairs formed, i.e., the average search cost incurred for the formation of an entrepreneur-financier pair in the credit market:

$$V \;=\; \frac{\epsilon}{p(\sigma)} + \frac{\kappa}{\sigma p(\sigma)} \;\equiv\; K(\sigma). \tag{68}$$

The zero expected profits conditions (67) also imply that equilibrium credit market tightness $\sigma$ is solely a function of relative bargaining powers and relative per-period search costs,

$$\sigma \;=\; \frac{1-\psi}{\psi}\frac{\kappa}{\epsilon} \;\equiv\; \sigma^{\star}. \tag{69}$$

The value of a vacancy continues to be given by equation (3), which can be solved for the value of a filled job

$$J \;=\; \frac{1}{\beta q(\theta)}\Big[c \,+\, \Big(1-\beta[1-q(\theta)]\Big)K(\sigma^\star)\Big]\,, \tag{70}$$

where we have invoked equilbrium outcomes (68) and (69); $V = K(\sigma^\star)$. Another expression for the value $J$ of a filled job is obtained by solving present version of equation (2),[14]

$$J \;=\; y \,-\, \hat{w} \,+\, \beta(1-s)J\,,$$

"forward" to obtain

$$J \;=\; \frac{y \,-\, \hat{w}}{\beta(r+s)}\,. \tag{71}$$

The two expressions (70) and (71) for the value $J$ of a filled job determine the equilibrium value of labor market tightness $\theta$:

$$q(\theta)\Big[y-\hat{w}-(r+s)\beta K(\sigma^\star)\Big] \;=\; (r+s)\Big[c \,+\, (1-\beta)K(\sigma^\star)\Big]\,. \tag{72}$$

Implicit differentiation yields

$$\frac{d\,\theta}{d\,y} \;=\; -\frac{q(\theta)}{q'(\theta)\big[y-\hat{w}-(r+s)\beta K(\sigma^\star)\big]} \;=\; \frac{\theta}{\alpha\big[y-\hat{w}-(r+s)\beta K(\sigma^\star)\big]}\,, \tag{73}$$

where the second equality follows after invoking the constant elasticity of matching with respect to unemployment, $\alpha = -q'(\theta)\,\theta/q(\theta)$. We can then compute the elasticity of market tightness as

$$\eta_{\theta,y} \;=\; \frac{1}{\alpha}\;\; \frac{y}{y-\hat{w}-(r+s)\beta K(\sigma^\star)}\,. \tag{74}$$

Comparing (74) to the elasticity (52) that emerges from Hall's (2005) sticky wage model, we observe that (74) contains another deduction that diminishes the fundamental surplus in the labor market, namely, $(r+s)\beta K(\sigma^\star)$, which is the annuitized value of the average search costs incurred in the credit market for the formation of an entrepreneur-financier pair.[15] In the Wasmer and Weil setup, the invisible hand cannot use these resources to pay for vacancy costs in the labor market, since they are required to assure that entrepreneur-financier pairs

[14]Note that upon the destruction of a job in the Wasmer and Weil setup, the entrepreneur-financier pair also breaks up so that the value of a vacancy $V$ vanishes in the present version of equation (2).

[15]To have the same expected present value as the average search costs in the credit market, $K(\sigma^\star)$, we compute an annuity $a$ with a stream that starts when an entrepreneur-financier pair matches with a worker,

on average earn zero expected profits from investing in credit market search costs.

## 5.5 Alliance of forces determines the fundamental surplus

Mortensen and Nagypál (2007) make observations about alternative approaches to getting unemployment to respond sensitively to productivity changes that we read as being related to ours. Regarding calibrations with a high value of leisure, they say that "[f]rankly, the Hagedorn-Manovskii value for $z$ seems implausibly large. For example, the flow surplus enjoyed by an employed worker in the model for these parameter values is minuscule." Likewise, they note that the Hall-Milgrom model of alternating-offer wage bargaining requires that "the worker's benefit from delay [value of leisure] plus the employer's cost [of delay], which is the opportunity cost of a match in their model, is equal to the required value derived by Hagedorn and Manovskii [2008] for the standard model." In the case of sticky wages, they emphasize that "a rigid wage is not enough, its level must also be sufficiently high."

To overcome criticisms about particular parameters associated with any single approach being deemed to be too high, Mortensen and Nagypál (2007) favor combining several of them; moderately higher value of leisure, alternating-offer wage bargaining as well as the inclusion of turnover costs – "allowing for hiring and firing costs increase the opportunity cost of the match without resorting to very high values of opportunity cost of employment for the worker."[16] The recommendation to combine several forces is reminiscent of HKV's emphasis in section 4.5 on the potency of the interaction of multiple government policies. Thus, as in our earlier conclusion that it is the *alliance of policies* that determines the fundamental surplus, we can say here that it is the *conjunction of forces* that determines the fundamental surplus. In both cases, what really matters is the outcome that ultimately the resulting fundamental surplus fraction is small.

---

and ends at the job's stochastic destruction:

$$\sum_{t=0}^{\infty} \beta^t (1-s)^t a \; = \; K(\sigma^\star) \qquad \Longrightarrow \qquad a \; = \; (r+s)\beta K(\sigma^\star)\,.$$

[16]The introduction of a fixed hiring cost operates in the same way as the credit-market search cost of Wasmer and Weil (2004) in section 5.4, i.e., an additional deduction needed to arrive at the fundamental surplus is the annuitized value of the fixed hiring cost. Pissarides (2009) makes the same general point that a fixed matching cost has the effect of increasing the elasticity of market tightness, and we just add, that its quantitative importance is inversely related to the ultimate size of the fundamental surplus fraction.

# 6 Simulations of business cycle fluctuations

To illustrate how a small fundamental surplus fraction is essential for generating ample unemployment volatility over the business cycle in matching models, we turn to Hall's (2005) specification with discrete time and a discrete random productivity process. Following Hall (2005), productivity takes on five discrete values $y_s$ uniformly spaced around a mean of one on the interval $[0.9935, 1.00565]$ and is governed by a monthly transition probability matrix

$$\Pi \equiv \begin{bmatrix} 1-2(1-\rho) & 2(1-\rho) & 0 & 0 & 0 \\ 2(1-\rho) & 1-5(1-\rho) & 3(1-\rho) & 0 & 0 \\ 0 & 3(1-\rho) & 1-6(1-\rho) & 3(1-\rho) & 0 \\ 0 & 0 & 3(1-\rho) & 1-5(1-\rho) & 2(1-\rho) \\ 0 & 0 & 0 & 2(1-\rho) & 1-2(1-\rho) \end{bmatrix},$$

where the parameter $\rho = 0.9899$ is also the serial correlation of productivity. The worker's value of leisure is $z = 0.40$, and the monthly discount factor $\beta$ corresponds to a 5-percent annual rate. The elasticity of matching with respect to unemployment is $\alpha = 0.235$, and the exogenous monthly separation rate is $s = 0.034$.

However, we alter Hall's value of the vacancy cost because, regrettably, his calibration implies that the job filling probability exceeds unity for all productivity levels except for the highest one.[17] By lowering the vacancy cost to $c = 0.1$, and making a corresponding adjustment of the efficiency parameter $A$ of the matching function, as described in footnote 4, we can preserve the same equilibrium unemployment outcomes reported by Hall.

To facilitate our sensitivity analysis, we also alter Hall's model period from one month to one day because a shorter model period fosters the existence of equilibria with vacancy creation. (See footnote 3.) To accomplish our conversion from a monthly to a daily frequency, we compute a daily version $\hat{\Pi}$ of the monthly transition probability matrix $\Pi$ above, by minimizing the sum of squared elements from the matrix operation $(\hat{\Pi}^{30} - \Pi)$ so that the monthly transition probabilities implied by $\hat{\Pi}$ are close to those of $\Pi$. Because of the high persistence in productivity, this target is achieved well with a daily value of $\hat{\rho} = 0.9996$. Other parameters that need to be converted are the efficiency parameter of the matching

[17] When eyeballing Hall's (2005) Figure 2, there are two ways of inferring market tightness at e.g. the lowest productivity level. Firstly, measures of vacancies and unemployment are $v \approx 0.025$ and $u \approx 0.082$. Secondly, the probability of finding a job is $\theta q(\theta) \approx 0.38$. Given Hall's calibration of the efficiency parameter of the matching function, $A = 0.947$, both ways imply a market tightness of around 0.30 at the lowest productivity level, which in turn implies a probability of filling a vacancy of $q(\theta) \approx 1.25$.

function and the separation rate, now with daily values of $A/30$ and $s/30$, and also the discount factor that becomes $\beta^{1/30}$ at a daily frequency.

### 6.1 Hall's (2005) sticky wage

As in Hall's (2005) analysis of a constant wage, we postulate a fixed wage $\bar{w} = 0.9657$, which equals the flexible wage that would prevail at the median productivity level under standard Nash bargaining (with equal bargaining weights, $\phi = 0.5$). Figure 10 reproduces Hall's figures 2 and 4 for those two models. The solid line and the upper dotted line depict unemployment rates at different productivities for the sticky-wage model and the standard Nash-bargaining model, respectively.[18] Unemployment is almost invariant to productivity under Nash bargaining but responds sensitively under the sticky wage. The outcomes are explained by differences in job-finding rates as shown by the dashed line and the lower dotted line for the sticky-wage model and the standard Nash-bargaining model, respectively, expressed at our daily frequency.[19] Under the sticky wage, high productivity causes firms to post a lot of vacancies, making it easy for unemployed workers to finds jobs, while the opposite is true when productivity is low.

Given this verification of our conversion of Hall's (2005) model into a daily frequency, we conduct a sensitivity analysis of the choice of the fixed wage. The solid line in Figure 11 shows how the average unemployment rate varies with the fixed wage $\bar{w}$. Note how a small range of wages spans outcomes from a very low to a very high average unemployment rate. Small variations in a fixed wage close to productivity represent large changes in the fundamental surplus fraction, $(y - \bar{w})/y$, which by free entry of firms map directly into the amount of resources devoted to vacancy creation. The dashed line in Figure 11 traces out the implications for the volatility of unemployment. The standard deviation of unemployment is nearly zero at the left end of the graph when the job-finding probability is almost one for all productivity levels. Unemployment volatility then increases for higher constant wages until, outside of the graph at the right end, vacancy creation becomes so unprofitable that average unemployment converges to its maximum of 100 percent, and there are no more fluctuations.

At Hall's fixed wage $\bar{w} = 0.9657$, Figure 11 shows a standard deviation of unemployment

[18]Unemployment is a state variable that is not just a function of the current productivity, as are all of the other variables, but depends on the history of the economy. But, given the persistence of productivity and the high job-finding rates, the unemployment rate to be observed at a productivity is well approximated by expression (13) evaluated at the market tightness $\theta$ prevailing at that productivity (see Hall (2005, p. 59)).

[19]Our daily job-finding rates are roughly 1/30 of the monthly rates in Hall (2005, figures 2 and 4), validating our conversion from a monthly to a daily frequency.

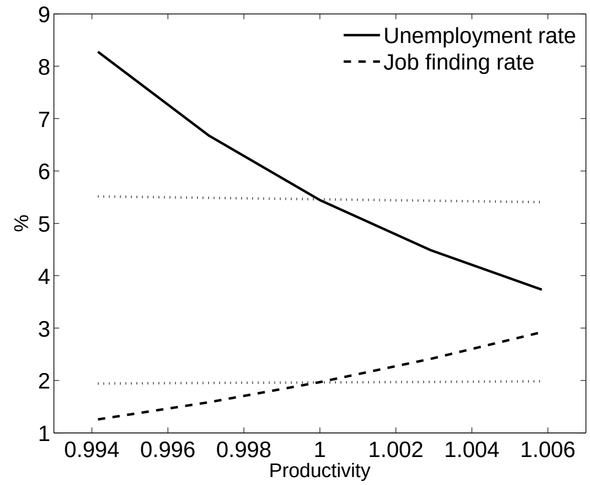


Figure 10: Unemployment rates (solid curve) and daily job-finding rates (dashed curve) at different productivities in the sticky-wage model. The dotted line that crosses respective curve is the counterpart in a standard Nash-bargaining model.

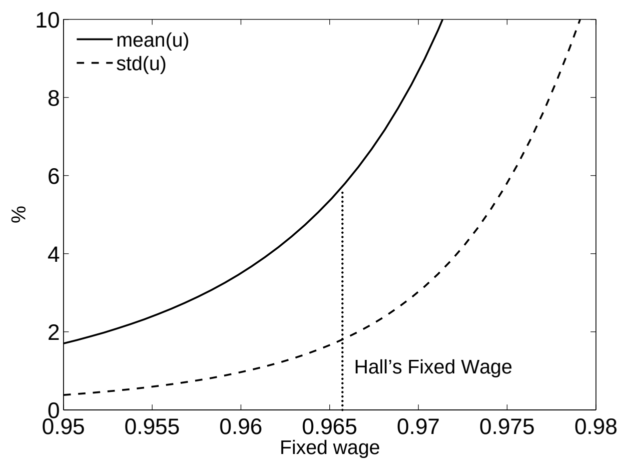


Figure 11: The average unemployment rate (solid curve) and the standard deviation of unemployment (dashed curve) in the sticky-wage model for different postulated values of the fixed wage.

equal to 1.80 percentage points that is close to the target of 1.54 to which Hall (2005) calibrated his model. Next, we will see how the standard Nash-bargaining model can attain the same volatility of unemployment by elevating the value of leisure.

## 6.2 Hagedorn and Manovskii's (2008) high value of leisure

Using Hall's (2005) parametric environment but assuming standard Nash wage bargaining, we revisit the analysis of Hagedorn and Manovskii (2008) by postulating a high value of leisure, $z = 0.960$, and a low bargaining power of workers, $\phi = 0.0135$. For those parameter values, we do indeed obtain a high standard deviation of 1.4 percentage points for unemployment. To shed light on the sensitivity of the choice of parameters, Figure 12 depicts outcomes for different constellations of $z \in [0.4, .99]$, and $\phi \in [0.001, 0.5]$. For each pair $(z, \phi)$, we adjust the efficiency parameter $A$ of the matching function to make the average unemployment rate be 5.5 percent. As we would expect, a high value of leisure, i.e., a small fundamental surplus fraction, is essential for obtaining a high volatility of unemployment, i.e., large variations in market tightness.

As detailed in section 5.1, Hagedorn and Manovskii (2008) need a low bargaining power of workers to match the elasticity of wages with respect to productivity. When we now revisit their argument based on $(z, \phi) = (0.960, 0.0135)$ in the otherwise parametrically defined environment of Hall (2005), the resulting wage elasticity of 0.44 is approximately the value to which Hagedorn and Manovskii calibrated their original analysis. Following the computational approach in Figure 12, Figure 13 provides a sensitivity analysis for the choice of parameter values $(z, \phi)$. The figure verifies the centrality of a low $\phi$ to obtain a low wage elasticity.

Taken together, Figures 12 and 13 show how the low wage elasticity of Hagedorn and Manovskii (2008) is coincidental to and neither sufficient nor necessary for obtaining a high volatility of unemployment. Hence, earlier attributions to the importance of a rigid wage can be set aside and instead, the key requirement is that the fundamental surplus fraction must be small.

## 6.3 Hall and Milgrom's (2008) alternating-offer bargaining

Finally, we turn to Hall and Milgrom's (2008) model of alternating-offer wage bargaining, as another way to increase unemployment volatility in a matching framework. Besides the wage formation process, their environment is the same as that of Hall (2005) but quite differently

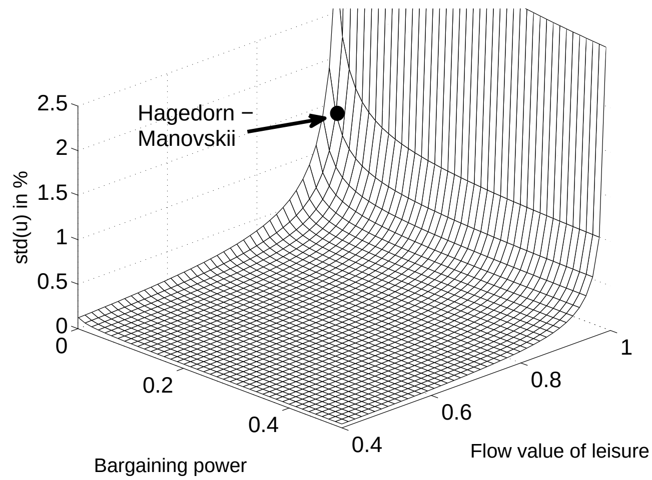


Figure 12: The standard deviation of unemployment in percentage points for different constellations of the value of leisure ($z$) and the bargaining power of workers ($\phi$) in the Nash-bargaining model.

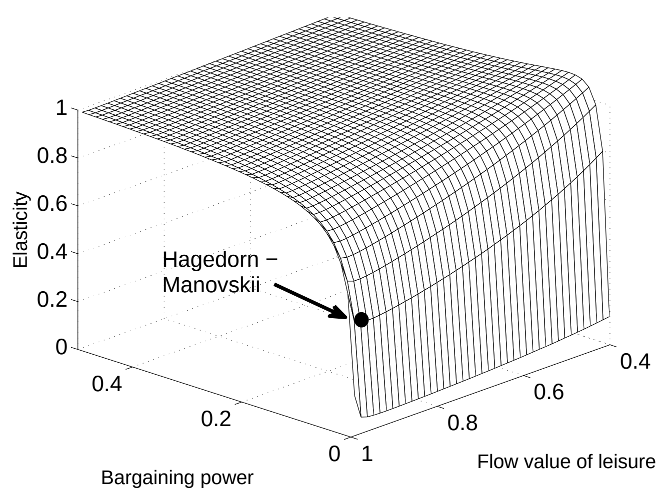


Figure 13: The wage elasticity with respect to productivity for different constellations of the value of leisure ($z$) and the bargaining power of workers ($\phi$) in the Nash-bargaining model.

parameterized. When now adopting Hall and Milgrom's parameterization, we just mention one difference that is central from the perspective of the fundamental surplus: the value of leisure is raised to $z = 0.71$. As we learnt in section 5.3, the value of leisure combined with firms' cost of delay in bargaining ($\gamma$) are likely to be critical for the magnitude of the elasticity of market tightness with respect to productivity and hence, for the volatility of unemployment.

Hall and Milgrom (2008) choose instead to emphasize how much the outside value of unemployment is suppressed in alternating-offer wage bargaining since disagreement does no longer lead to unemployment but rather to another round of bargaining. Hence, a key parameter from Hall and Milgrom's perspective is the exogenous rate $\delta$ at which parties break up between bargaining rounds. Figure 14 shows how different constellations of $(\gamma, \delta)$ affect the standard deviation of unemployment. For each pair $(\gamma, \delta)$, we adjust the efficiency parameter $A$ of the matching function to make the average unemployment rate be 5.5 percent. For the record, Hall and Milgrom (2008) argue that productivity shocks are not the sole source for unemployment fluctuations and consequently, set a lower target of explaining a standard deviation of unemployment equal to 0.68 percentage points – a target that is attained with their parameterization $(\gamma, \delta) = (0.27, 0.0055)$ and reproduced in Figure 14.

Figure 14 supports our earlier finding that the cost of delay $\gamma$ (plus the value of leisure $z$) is paramount for generating higher volatility of unemployment. Without a sufficiently high cost of delay that reduces the fundamental surplus fraction, it does not matter much what is the exogenous separation rate between bargaining rounds.[20]

# 7 Concluding comments

That firms allocate resources to create vacancies is central to all matching models of the labor market. Those resources are used before firms meet workers and therefore are not contractable. In calibrating key objects in matching models, most applied studies impute vacancy costs that in equilibrium amount to a small fraction of labor productivity. A key question is how small vacancy costs make unemployment sensitive to small productivity shocks in some matching models, but not in others? Our answer is that the high sensitivity of unemployment to productivity shocks requires that the fundamental surplus fraction must

[20] To be specific, our formula (64) for the steady-state comparative statics is an approximation of the elasticity of market tightness at the rear end of Figure 14 where the exogenous rate $\delta$ at which parties break up between bargaining rounds is equal to Hall and Milgrom's (2008) assumed job destruction rate of 0.0014 per day.

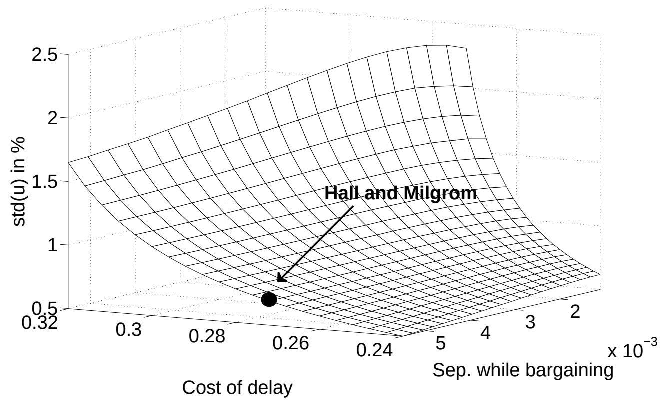


Figure 14: The standard deviation of unemployment in percentage points for different constellations of firms' cost of delay in bargaining ($\gamma$) and the exogenous separation rate while bargaining ($\delta$) in the alternating-offer bargaining model.

be small.

Take the output of a job, then deduct the value of leisure, annuitized values of layoff costs and training costs, worker's inalienable right to exploit a firm's cost of delay under alternating-offer wage bargaining and any other items that must be set aside to arrive at the fundamental surplus – an upper bound on what the "invisible hand" could allocate to vacancy creation. If that fundamental surplus constitutes a small fraction of a job's output, it means that a change in productivity translates into a much larger percentage change in the fundamental surplus. Such large movements in the amount of resources that could potentially be used for vacancy creation cannot be offset by the invisible hand, so significant variations in market tightness ensue, causing large movements in unemployment.

In the standard matching model under commonly used calibrations, unemployment reponds weakly to productivity shocks because the fundamental surplus fraction is large. Note that there is no way to glean this information from the match surplus because, as mentioned earlier, matching models are usually calibrated to be consistent with observations indicating that expenditures that can be construed as vacancy costs are small; hence, the implied calibrated match surpluses must be small because firms are also commonly calibrated to receive a significant share of match surpluses, a share that firms use to finance the small vacancy costs. Consequently, given a small match surplus in a worker's current job as well as the discounted values of match surpluses expected to accrue from future jobs, it follows that a worker's outside value must mostly be made up of something else. It is then crucial whether or not this something else also belongs to the fundamental surplus. When it does, as in the standard matching model under common calibrations, the invisible hand can, by diverting resources from workers' outside values, mitigate the impact of productivity changes on match surpluses and hence, on resources allocated to vacancy creation. This explains why a large fundamental surplus fraction mutes the effects of productivity changes on unemployment and why the usual manifestation of a small match surplus is uninformative about this issue.

The concept of the fundamental surplus unites analyses of welfare state dynamics such as Hornstein et al. (2007), who mix various welfare state policies to diminish the fundamental surplus fraction, with analyses of business cycle dynamics such as Mortensen and Nagypál (2007), who combine different approaches effectively to diminish what we call the fundamental surplus fraction. Generating large movements in unemployment in a welfare state context, or for causing large unemployment flucuations over the business cycle requires the same thing: that the fundamental surplus fraction must be small.

Since models of business cycle fluctuations and models of adverse welfare state dynamics both operate through the same intermediating channel, namely, the fundamental surplus fraction, tensions will arise. Commenting on the approach that recalibrates the standard matching model to have a high value of leisure, Costain and Reiter (2008, p. 1120) identify such a tension between the quest for a high elasticity of market tightness over the business cycle, while also having the model give reasonable responses of unemployment to variations in unemployment benefits. They conclude that the "standard model can generate sufficiently large cyclical fluctuations in unemployment, or a sufficiently small response of unemployment to labor market policies, but it cannot do both." Likewise, theories of high European unemployment that operate through a diminished fundamental surplus fraction in a population of *homogeneous* workers would imply that productivity-driven business cycles are more volatile in the welfare states as compared to a corresponding calibration of the U.S. with less stringent government policies and hence, a larger fundamental surplus fraction. But the business-cycle sensitivity would be less in *heterogeneous*-agent models such as Mortensen and Pissarides (1999) and Ljungqvist and Sargent (2007) that ignite adverse welfare state dynamics for only a subset of workers. We have learned that it will then matter if there are multiple matching functions and how unemployed people are assigned to those matching functions. In particular, do all of those workers who experience adverse circumstances enter a separate matching function of their own characterized by a severely diminished fundamental surplus fraction, or are they assigned to matching functions with other workers associated with larger a fundamental surplus?

The fundamental surplus fraction is the single intermediating channel through which economic forces generating a high elasticity of market tightness with respect to productivity must operate. Differences in the fundamental surplus explain why unemployment responds sensitively to movements in productivity in some matching models but not in others. The role of the fundamental surplus in explaining that response sensitivity transcends diverse matching models having very different outcomes along other dimensions that include the elasticity of wages with respect to productivity, the size of match surpluses, and whether or not outside values affect bargaining outcomes.

# References


Costain, James S. and Michael Reiter. 2008. Business Cycles, Unemployment Insurance, and the Calibration of Matching Models. *Journal of Economic Dynamics and Control* 32:1120–1155.

Hagedorn, Marcus and Iourii Manovskii. 2008. The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited. *American Economic Review* 98 (4):1692–1706.

Hall, Robert E. 2005. Employment Fluctuations with Equilibrium Wage Stickiness. *American Economic Review* 95 (1):50–65.

Hall, Robert E. and Paul R. Milgrom. 2008. The Limited Influence of Unemployment on the Wage Bargain. *American Economic Review* 98 (4):1653–1674.

Hornstein, Andreas, Per Krusell, and Giovanni L. Violante. 2007. Technology–Policy Interaction in Frictional Labour-Markets. *Review of Economic Studies* 74:1089–1124.

Ljungqvist, Lars. 2002. How Do Layoff Costs Affect Employment? *Economic Journal* 112:829–853.

Ljungqvist, Lars and Thomas J. Sargent. 2007. Understanding European Unemployment with Matching and Search-Island Models. *Journal of Monetary Economics* 54:2139–2179.

———. 2012. *Recursive Macroeconomic Theory,* Third Edition. Cambridge, Mass.: MIT Press.

Millard, Stephen P. and Dale T. Mortensen. 1997. The Unemployment and Welfare Effects of Labor Market Policy: A Comparison of the USA and the UK. In *Unemployment Policy: Government Options for the Labor Market*, edited by Dennis Snower and Guillermo de la Dehesa. Cambridge, UK: Cambridge University Press.

Mortensen, Dale T. and Éva Nagypál. 2007. More on Unemployment and Vacancy Fluctuations. *Review of Economic Dynamics* 10:327–347.

Mortensen, Dale T. and Christopher A. Pissarides. 1999. Unemployment Responses to 'Skill-Biased' Technology Shocks: The Role of Labor Market Policy. *Economic Journal* 109:242–265.

Petrongolo, Barbara and Christopher A. Pissarides. 2001. Looking into the Black Box: A Survey of the Matching Function. *Journal of Economic Literature* 39 (2):390–431.

Pissarides, Christopher A. 2000. *Equilibrium Unemployment Theory,* Second Edition. Cambridge, Mass.: MIT Press.

———. 2009. The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer? *Econometrica* 77 (5):1339–1369.

Rogerson, Richard and Robert Shimer. 2011. Search in Macroeconomic Models of the Labor Market. In *Handbook of Labor Economics, Vol. 4A*, edited by Orley Ashenfelter and David Card. Amsterdam: Elsevier.

Shimer, Robert. 2005. The Cyclical Behavior of Equilibrium Unemployment and Vacancies. *American Economic Review* 95 (1):25–49.

Wasmer, Etienne and Philippe Weil. 2004. The Macroeconomics of Labor and Credit Market Imperfections. *American Economic Review* 94 (4):944–963.