Asset Pricing when 'This Time is Different'

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Empirical evidence on formation of macro beliefs

Malmendier and Nagel (2011, 2013): Experiential learning

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Quantitative asset pricing implications?

1. Wisdom of crowds: Median macro beliefs from surveys very good forecasters (Ang, Bekaert, and Wei (2007))
   - Small aggregate effects?

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Quantitative asset pricing implications?

1. Wisdom of crowds: Median macro beliefs from surveys very good forecasters (Ang, Bekaert, and Wei (2007))
   - Small aggregate effects?

2. Heterogeneous beliefs and risk-sharing: optimists insure pessimists (Chen, Joslin, and Tran (2012))
   - Reduce average Sharpe ratios/risk premiums?

This paper

- Embed heterogeneous agents and experiential learning (calibrated to evidence in Malmendier and Nagel (2013)) in standard macro-finance model
  - Understand fundamental economic mechanisms, quantitative asset pricing and macro implications of such bias

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Features of model(s):

- Consider workhorse exchange and production economies
- Recursive preferences, two agents (Young and Old), Bayesian learning within a life
- Young update more as prior more dispersed (more volatile expectations)
  - Mean beliefs inherited from 'parents'
  - 'Old' more confident in economic environment (data generating process)

Main take-aways

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- Small changes in beliefs, large multiplier for asset prices

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- Accompanied by persistent over-/underinvestment

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Agents' mean beliefs excellent forecasters of consumption/GDP growth

- I.e., 'small' departure from rational expectations

Related literature

- Lots of papers on heterogeneous beliefs
- Most related: Ehling, Graniero, Heyerdahl-Larsen (2014), Choi and Mertens (2013), Garleanu and Pedersen (2014), Borovicka (2013), Chen, Joslin, and Tran (2012), Bansal and Shaliastovich (2010), Baker, Hollifield, and Osambela (2014), Hirshleifer and Yu (2013), Collin-Dufresne, Johannes, and Lochstoer (2013)
- Marginal contribution:
  - Uncertain heterogeneous beliefs and learning with recursive preferences
  - Both exchange and production economy analyses

Aggregate Dynamics

- First consider *exchange economy* with iid aggregate consumption:

$$\Delta c_t = \mu + \sigma \varepsilon_t + d_t,$$

$\varepsilon_t \overset{i.i.d.}{\sim} N(0,1)$, $d_t = \underline{d}$ with prob p, 0 otherwise

  - μ and p unknown.

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- We calibrate:
  - $\underline{d} = -18\%$ (U.S. Great Depression consumption drop), $p = 1.7\%$ *p.a.* (as in Barro (2006))
  - In all models: $E[\Delta c_{Annual}] = 1.8\%$, $\sigma[\Delta c_{Annual}] = 2.2\%$ as in U.S. sample from 1929-2013.

Aggregate Dynamics and Learning

Agent i born with prior $\mu \sim N\left(m_{i,0}, A_{i,0}\sigma^2\right)$ and $p \sim \beta\left(a_{i,0}, A_{i,0}^{-1} - a_{i,0}\right)$.

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- Bayesian within-life learning implies subjective consumption dynamics:

$$\begin{aligned}
\Delta c_{t+1} &= m_{i,t} + E_t^i\left[p\right]\underline{d} + \sqrt{A_{i,t}+1}\sigma\tilde{\varepsilon}_{t+1} + \left(d_{t+1} - E_t^i\left[p\right]\underline{d}\right) \\
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A_{i,t+1}^{-1} &= A_{i,t}^{-1} + 1 \\
a_{i,t+1} &= a_{i,t} + \mathbf{1}_{d_{t+1}=\underline{d}}
\end{aligned}$$

$$E_t^i\left[p\right] = a_{i,t}A_{i,t}, \quad var_t^i\left(p\right) = \tfrac{A_{t,t}}{1+A_{t,t}}E_t^i\left[p\right]\left(1 - E_t^i\left[p\right]\right).$$

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Thus, if i lives forever, $A_{i,\infty} = 0$ and $m_{i,\infty} = \mu$, $E_\infty^i\left[p\right] = p$; $var_\infty^i\left[p\right] = 0$

The Experiential Learning Bias

Each agent lives for T years

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Inherit 'parents' mean beliefs, but increase dispersion of beliefs:

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m_{Young,0} &= m_{Old,T} \\
E_0^{Young}[p] &= E_T^{Old}[p] \\
A_{Young,0} &= \mathbf{k} A_{Old,T}.
\end{aligned}
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where $k > 1$

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If $k = 1$, economy converges to one-agent, rational expectations case

The Model: OLG setup

- Two agents concurrently alive
  - Agents are Young for $T/2$ years and Old for $T/2$ years
  - Set $k = \left(A_0^{-1} + T\right) A_0$ such that $A_{i,0} = A_0$ for all cohorts (stationary learning dynamics)

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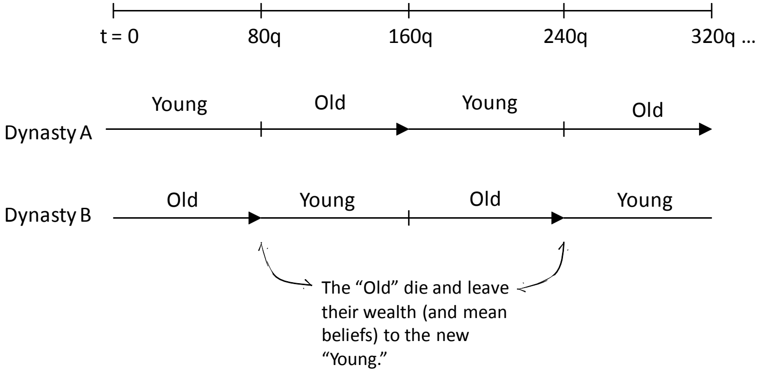
  - Set $k = \left(A_0^{-1} + T\right) A_0$ such that $A_{i,0} = A_0$ for all cohorts (stationary learning dynamics)
- E-Z utility with Dynasty form of bequest function: maximize offspring's utility, leave ex-consumption wealth for them.
  - Thus, two dynasties, A and B, where at each point in time the representative agent for Dynasty A (B) is either Young (Old) or Old (Young)
  - Stylized OLG to keep number of state variables low

The Model: OLG setup



The model: Equilibrium

- Complete markets, so for any asset j, for $i \in \{A, B\}$

$$E^i \left[M^i_{t+1} R^j_{t+1} | X_t \right] = 1$$

where $X_t = [m_{A,t},\ m_{B,t},\ a_{A,t},\ a_{B,t},\ A_{A,t},\ A_{B,t},\ W_{A,t}/W_{B,t}]$

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and

$$\pi^A\left(\Delta c_{t+1}|X_t\right) M^A\left(\Delta c_{t+1}, X_t\right) = \pi^B\left(\Delta c_{t+1}|X_t\right) M^B\left(\Delta c_{t+1}, X_t\right).$$

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and

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- Note the endogenous state variable (the relative wealth of the agents)
  - Complicates model solution as evolution equation depends on value functions which are what we are trying to solve for...
- Consider 'uncertain mean' and 'uncertain probability' models separately
  - Also, note that $A_{A,t}$, $A_{B,t}$ deterministic, so each model has 4 state variables

The model: Calibration

- Learning channel: choosing A_0 (or equivalently k)
- Use numbers estimated by Malmendier and Nagel (2009):
  - The sensitivity of Young's (about 30yo) updates in beliefs to macro shock: 0.025
  - The sensitivity of Old's (about 70yo) updates in beliefs to macro shock: 0.01
- In our case, amounts to calibration of $A_0 = 0.025$.
  - Implies sensitivity of old of 0.005, a little on the conservative side
- How 'wrong' are investors' expectations wrt macroeconomic quantities in model?
  - Takes on average 175 years of time-series data to reject the model of consumption growth used by a Dynasty
  - Bias is, however, immediately identified in the cross-section of beliefs

Calibration (cont'd)

Preference parameters:	**Uncertain mean**	**Uncertain Prob**
γ (risk aversion parameter)	10	5
ψ (elasticity of intertemporal substitution)	1.5	1.5
β (quarterly time discounting)	0.994	0.994
Priors:		
A_0 (dispersion parameter for prior at birth)	0.025	0.025
$\overline{m}$, $\overline{p}$ (upper truncation point of prior)	1.45%	0.01%
$\underline{m}$, $\underline{p}$ (lower truncation point of prior)	−0.55%	4.00%

- Equity claim to exogenous dividends (as in, e.g., Campbell and Cochrane (1999))

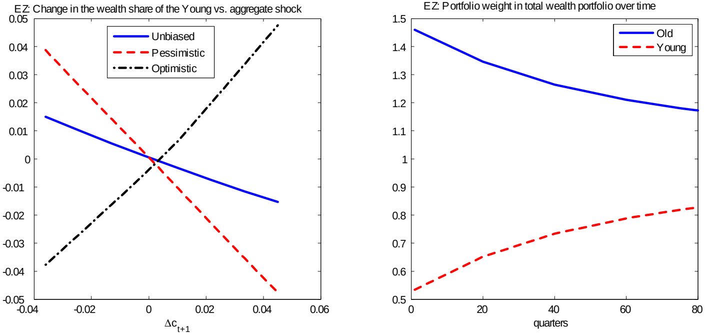
$$\Delta d_{t+1} = \lambda \Delta c_{t+1} + \sigma_d \eta_{t+1}$$

where $\lambda = 3$, $\sigma_d = 5\%$

Risk-sharing: EZ strong motive

'Uncertain Mean' case

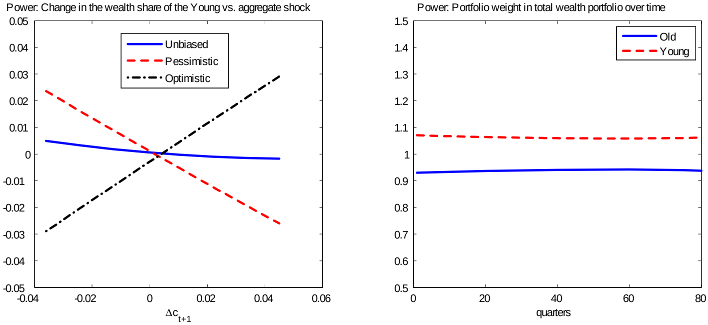
- Agents in the 'middle' of their generations (10yo and 30yo)
- The 'Old' have conditional unbiased beliefs



- More optimistic allocate more to total wealth (consumption) claim
- Agent who perceives more confidence risk (parameter uncertainty) holds less

Risk-sharing: Power Utility, not strong motive

- Same parameters, except $\psi = 0.1 = 1/\gamma$



- Less pronounced wealth-dynamics
- Agent who perceives more model risk holds *more* in wealth claim (low EIS)

Why? More heterogeneity in long-run risk

Recall subjective consumption dynamics (**case:** uncertain mean, no disaster):

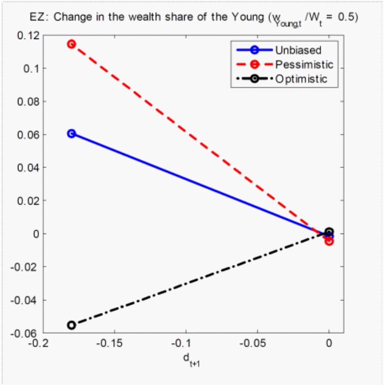
$$\begin{aligned}
\Delta c_{t+1} &= m_{i,t} + \sqrt{A_{i,t}+1}\,\sigma\tilde{\varepsilon}_{t+1} \\
m_{i,t+1} &= m_{i,t} + \frac{A_{i,t}}{\sqrt{A_{i,t}+1}}\sigma\tilde{\varepsilon}_{t+1} \\
A_{i,t+1}^{-1} &= A_{i,t}^{-1} + 1
\end{aligned}$$

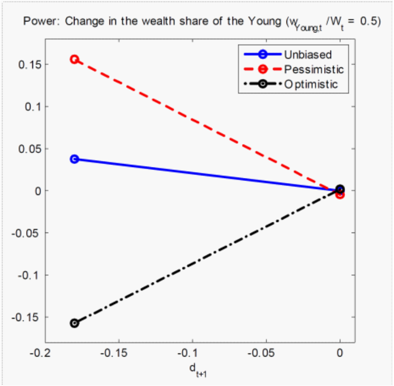
In the middle of their respective generations (ages 10 and 30):

$$\begin{aligned}
\frac{\text{Short-run risk (Young)}}{\text{Short-run risk (Old)}} - 1 &= \frac{\sqrt{A_{Young,t}+1}\,\sigma}{\sqrt{A_{Old,t}+1}\,\sigma} - 1 = 0.3\% \\
\frac{\text{Long-run risk (Young)}}{\text{Long-run risk (Old)}} - 1 &= \frac{\frac{A_{Young,t}}{\sqrt{A_{Young,t}+1}}\sigma}{\frac{A_{Old,t}}{\sqrt{A_{Old,t}+1}}\sigma} - 1 = 67\%
\end{aligned}$$

Risk-sharing: 'Uncertain Probability'-case

- Plots correspond to agents in the 'middle' of their generations (10yo and 30yo)
- The 'Old' have conditional unbiased beliefs

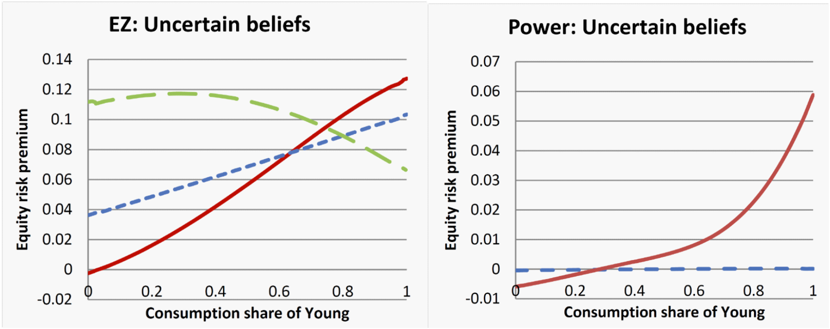




- More optimistic allocate more to total wealth (consumption) claim
- Agent who perceives more confidence risk (parameter uncertainty) holds less
- 'Power'-case: portfolio allocation more responsive to mean beliefs

Wealth dynamics and the risk premium

- Uncertain probability, disaster case



- Dashed blue: both agents have unbiased mean beliefs
- Solid red: Young optimists, Old pessimists
- Dashed green: Young pessimists, Old optimists

Standard moments – 'Uncertain Mean'

- Unknown mean, no disasters

	Data	'This Time is Different'		Known mean
	1929 – 2011	$EZ: \ \gamma = 10$ $\psi = 1.5$ $\beta = 0.994$	$Power: \ \gamma = 10$ $\psi = 1/10$ $\beta = 0.994$	$EZ: \ \gamma = 10$ $\psi = 1.5$ $\beta = 0.994$
$E_T[r_m - r_f]$	5.1	5.2	0.1	1.5
$\sigma_T[r_m - r_f]$	20.2	16.6	10.5	12.9
$SR_T[r_m - r_f]$	0.25	0.31	0.01	0.12
$\sigma_T[M_{t+1}] / E_T[M_{t+1}]$	-	0.51	0.20	0.27
$\gamma \times \sigma_T[\Delta c_{t+1}]$	-	0.27	0.27	0.27

- E-Z TTiD model performs well!
  - Power case suffers from all the usual puzzles...
- 2 stdev range of P/D-ratio: 27 to 54, highly persistent
  - Benchmark case: P/D ratio is constant

Standard moments – 'Uncertain Probability'

- Known mean, unknown disaster probability

	Data	'This Time is Different'		Known probability
	1929 – 2011	$EZ: \gamma = 5$, $\psi = 1.5$, $\beta = 0.994$	$Power: \gamma = 5$, $\psi = 1/5$, $\beta = 0.994$	$EZ: \gamma = 5$, $\psi = 1.5$, $\beta = 0.994$
$E_T[r_m - r_f]$	5.1	4.9	0.2	1.7
$\sigma_T[r_m - r_f]$	20.2	16.7	12.1	13.2
$SR_T[r_m - r_f]$	0.25	0.30	0.01	0.13
$\sigma_T[M_{t+1}]/E_T[M_{t+1}]$	-	0.69	0.17	0.20
$\gamma \times \sigma_T[\Delta c_{t+1}]$	-	0.135	0.135	0.135

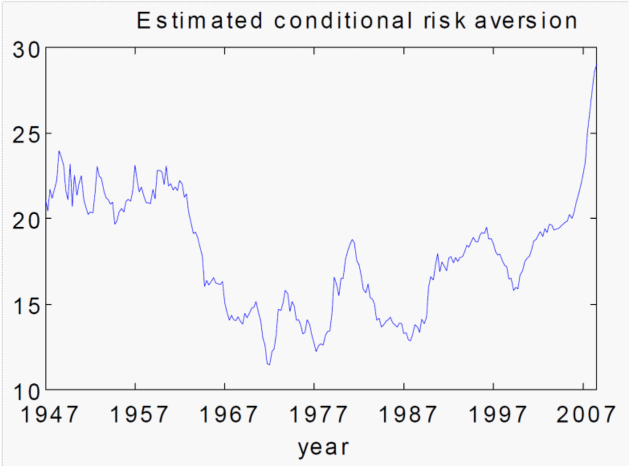
- E-Z TTiD model performs well!
    - Power case suffers from all the usual puzzles...
- 2 stdev range of P/D-ratio: 25 to 45, highly persistent
    - Benchmark case: P/D ratio is constant

Econometrician's 'risk aversion estimate'

'Uncertain Mean'-case

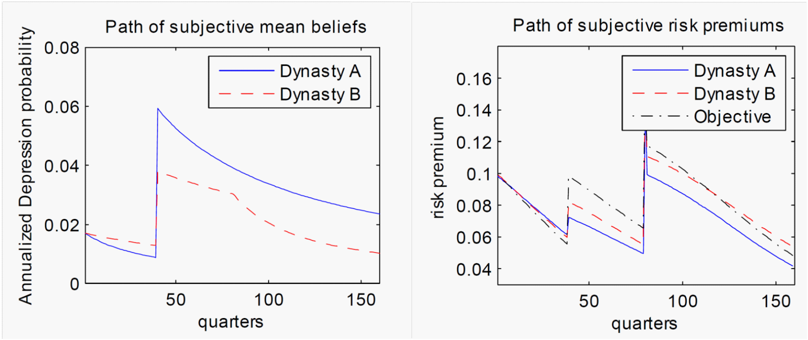
Sensitivity of pricing kernel to aggregate shocks estimated by econometrician using a (very) long sample:

$$\gamma_t \equiv \frac{1}{\sigma_{\Delta c}} \frac{\sigma_t^P\left[M_{t+1}\right]}{E_t^P\left[M_{t+1}\right]}$$



'Depression babies'

'Uncertain Probability'-case



- The 'Old', who experienced disaster perceive a lower risk premium (by about 1.5% p.a.) than the 'Young' that did not live through disaster.

A production economy and endogenous consumption

Log technology growth (unknown μ case, only):

$$\Delta z_t = \mu + \sigma_z \varepsilon_{t+1},$$

Production function:

$$Y_t = (Z_t N_t)^{1-\alpha} K_t^{\alpha},$$

where $N_t = 1$.

Capital accumulation equation is:

$$K_{t+1} = (1 - \delta) K_t + \phi (I_t / K_t) K_t,$$

where δ is the depreciation rate, $\phi(\cdot)$ is a concave function, and I_t is gross investment.

Resource constraint:

$$Y_t = C_t + I_t.$$

Investment and Equity Returns

Adjustment costs (Zhang (2005)):

$$\phi\left(x_t\right) = x_t - \frac{a_t}{2} x_t^2,$$

Return to investment and unlevered stock return:

$$R_{I,t+1} = \phi'\left(I_t/K_t\right)\left(\alpha\left(K_t/Z_t\right)^{\alpha-1} + \frac{1-\delta+\phi\left(I_{t+1}/K_{t+1}\right)}{\phi'\left(I_{t+1}/K_{t+1}\right)} - \frac{I_{t+1}}{K_{t+1}}\right),$$

Firm's FOC yields:

$$E_t^A\left[M_{t+1}^A R_{I,t+1}\right] = E_t^B\left[M_{t+1}^B R_{I,t+1}\right] = 1.$$

Levered equity returns:

$$R_{E,t+1} = \left(1 + B/E\right) R_{I,t+1}$$

Calibration

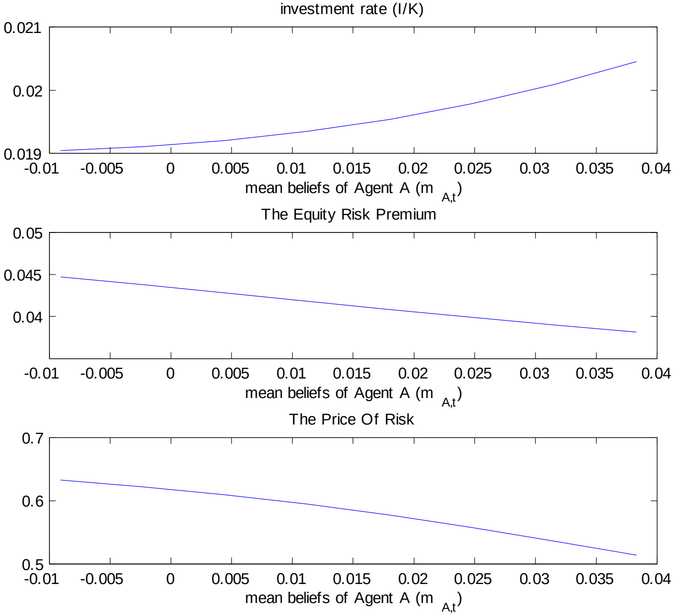
Preference parameters:	**EZ – Production**
γ (risk aversion parameter)	5
ψ (elasticity of intertemporal substitution)	2
β (quarterly time discounting)	0.994
Priors:	
A_0 (dispersion parameter for prior at birth)	0.025
$\overline{m}$ (upper truncation point of prior)	1.5%
$\underline{m}$ (lower truncation point of prior)	−0.5%
Technology and Production Parameters:	
α (the capital share)	0.36
μ (true mean growth rate)	0.5%
σ_z (volatility of technology shocks)	3.8%
δ (capital depreciation rate)	2.5%
a_- (downside adjustment costs parameter)	20
a_+ (upside adjustment costs parameter)	6

Moments

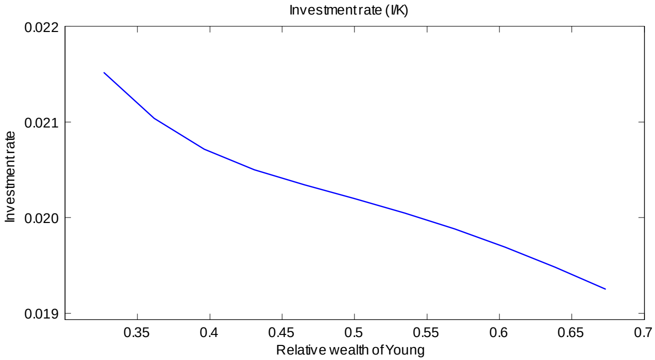
- Excess investment volatility due to experiential learning
  - 10.8% versus 7.8%
- Close to order of magnitude increase in return volatility
  - 6% versus 0.9%

	Data 1929 – 2011	**Model**
$\sigma_T\left[\Delta y\right](\%)$	4.9	4.9
$\sigma_T\left[\Delta c\right]/\sigma_T\left[\Delta y\right]$	0.52	0.47
$\sigma_T\left[\Delta i\right]/\sigma_T\left[\Delta y\right]$	2.3	2.20
$E_T\left[r_m - r_f\right](\%)$	5.1	3.1
$\sigma_T\left[r_m - r_f\right](\%)$	20.2	6.0
$SR_T\left[R_M - R_f\right]$	0.36	0.51

Conditional moments versus beliefs

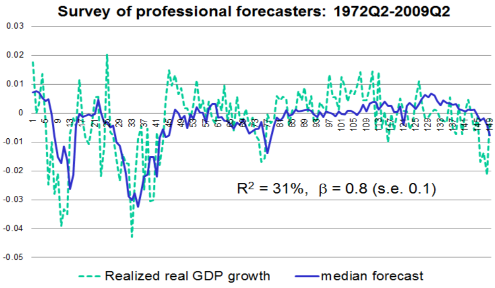


Investment versus uncertainty



- High model uncertainty, less investment

Investor expectations about real GDP growth



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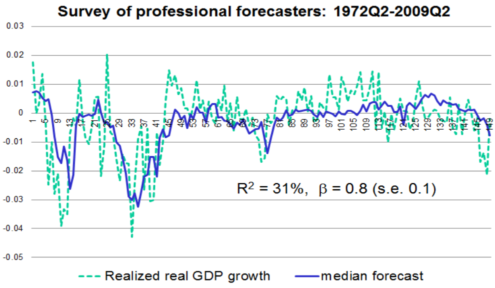
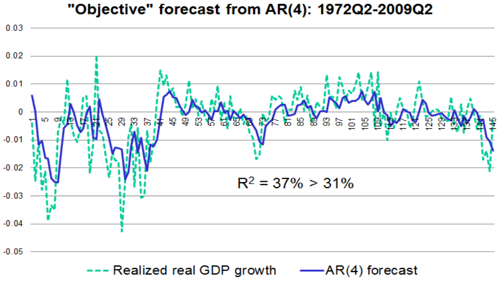
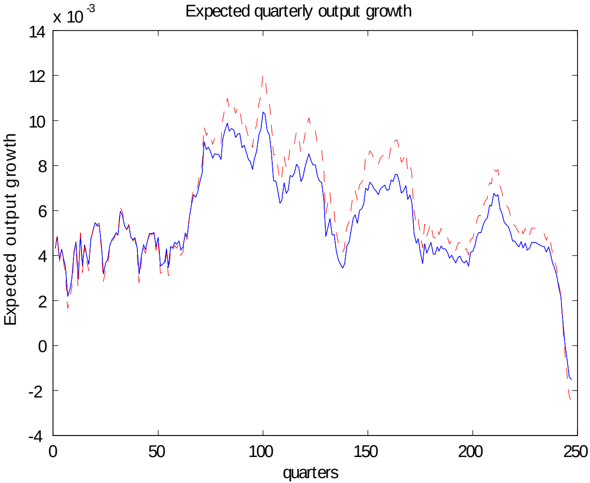




Expected subjective and objective GDP growth

- Close relation subjective and true GDP growth
  - Agents *act* on their beliefs: high (low) perceived growth rate, high (low) investment
  - Agents' models very close to true model in a statistical sense



Conclusion

- With 'This Time is Different'-bias, model uncertainty persists indefinitely
  - Calibration consistent with micro estimates of age-dependent updating rule
  - Still, 'small' departure from Rational Expectations; median belief excellent forecaster
- With E-Z preferences, extra risk factor arises (amplification of macro risks)
  - Large effect on investment and asset prices
  - Optimal risk-sharing amplifies business cycle and asset price fluctuations through relative wealth fluctuations
  - Endogenously counter-cyclical uncertainty premia
  - Helps account for the standard macro-finance 'puzzles'
- Extended periods of prices and investment reflecting 'irrational exuberance' or 'irrational fear'

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- Assume economy ends at $\tilde{T}$, make relative consumption of agent A the endogenous state variable, $c_A = C_A / C$.

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$$

$$
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- Proceed backwards until transversality condition 'kicks in': reached solution for infinite horizon problem.