Asset Pricing when 'This Time is Different'

Pierre Collin-Dufresne, Michael Johannes, and Lars A. Lochstoer

University of Lausanne and Columbia Business School

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Empirical evidence on formation of macro beliefs

Malmendier and Nagel (2011, 2013): Experiential learning

"When forming macroeconomic expectations, individuals put a higher weight on realizations of macroeconomic data experienced during their life-times compared with other available historical data."
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   - Small aggregate effects?
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Quantitative asset pricing implications?
1. Wisdom of crowds: Median macro beliefs from surveys very good forecasters (Ang, Bekaert, and Wei (2007))
   ▶ Small aggregate effects?

2. Heterogeneous beliefs and risk-sharing: optimists insure pessimists (Chen, Joslin, and Tran (2012))
   ▶ Reduce average Sharpe ratios/risk premiums?
This paper

- Embed heterogeneous agents and experiential learning (calibrated to evidence in Malmendier and Nagel (2013)) in standard macro-finance model
  - Understand fundamental economic mechanisms, quantitative asset pricing and macro implications of such bias
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Features of model(s):

- Consider workhorse exchange and production economies
- Recursive preferences, two agents (Young and Old), Bayesian learning within a life
- Young update more as prior more dispersed (more volatile expectations)
  - Mean beliefs inherited from 'parents'
  - 'Old' more confident in economic environment (data generating process)
Main take-aways

Experiential learning, heterogeneity in updating, strongly affects asset prices
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- Trading due to differences in mean beliefs (optimists vs pessimists) less pronounced when beliefs not dogmatic
  - Disaster risk premiums 'survives' introduction of optimists
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- Wealth-weighted confidence risk increases in bad times—new channel for counter-cyclical risk prices
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Generational nature of bias leads to persistent beliefs

- Small changes in beliefs, large multiplier for asset prices
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Generational nature of bias leads to persistent beliefs

- Small changes in beliefs, large multiplier for asset prices
- P/D ratio features persistent over-/undervaluations of order ±30%
- Accompanied by persistent over-/underinvestment
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Agents’ mean beliefs excellent forecasters of consumption/GDP growth

- I.e., 'small' departure from rational expectations
Related literature

- Lots of papers on heterogeneous beliefs


- Marginal contribution:
  - Uncertain heterogeneous beliefs and learning with recursive preferences
  - Both exchange and production economy analyses
First consider *exchange economy* with iid aggregate consumption:

\[ \Delta c_t = \mu + \sigma \varepsilon_t + d_t, \]

\[ \varepsilon_t \sim i.i.d. N(0, 1), \quad d_t = d \text{ with prob } p, \ 0 \text{ otherwise} \]

\[ \mu \text{ and } p \text{ unknown}. \]
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- \( \mu \) and \( p \) unknown.

We calibrate:

- \( d = -18\% \) (U.S. Great Depression consumption drop), \( p = 1.7\% \) p.a. (as in Barro (2006))
- In all models: \( E[\Delta c_{Annual}] = 1.8\%, \sigma[\Delta c_{Annual}] = 2.2\% \) as in U.S. sample from 1929-2013.
Aggregate Dynamics and Learning

Agent $i$ born with prior $\mu \sim N(m_i,0,A_i\sigma^2)$ and $p \sim \beta(a_i,1,A_i^{-1} - a_i)$. 
Agent *i* born with prior $\mu \sim N(m_{i,0}, A_{i,0}\sigma^2)$ and $p \sim \beta(a_{i,0}, A_{i,0}^{-1} - a_{i,0})$.

- Bayesian within-life learning implies subjective consumption dynamics:

\[
\Delta c_{t+1} = m_{i,t} + E_t^i[p] d + \sqrt{A_{i,t}} + 1\sigma \bar{e}_{t+1} + (d_{t+1} - E_t^i[p] d)
\]
\[
m_{i,t+1} = m_{i,t} + \frac{A_{i,t}}{\sqrt{A_{i,t} + 1}} \sigma \bar{e}_{t+1}
\]
\[
A_{i,t+1}^{-1} = A_{i,t}^{-1} + 1
\]
\[
a_{i,t+1} = a_{i,t} + 1_{d_{t+1} = d}
\]

$E_t^i[p] = a_{i,t} A_{i,t}$, $\text{var}_t^i(p) = \frac{A_{t,t}}{1 + A_{t,t}} E_t^i[p] (1 - E_t^i[p])$. 
Agent $i$ born with prior $\mu \sim N(m_{i,0}, A_{i,0}\sigma^2)$ and $p \sim \beta(a_{i,0}, A_{i,0}^{-1} - a_{i,0})$.

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$$m_{i,t+1} = m_{i,t} + \frac{A_{i,t}}{\sqrt{A_{i,t} + 1}}\sigma\tilde{\varepsilon}_{t+1}$$

$$A_{i,t+1}^{-1} = A_{i,t}^{-1} + 1$$

$$a_{i,t+1} = a_{i,t} + 1_{d_{t+1} = d}$$

$$E_t^i[p] = a_{i,t}A_{i,t}, \quad var_t^i(p) = \frac{A_{t,t}}{1 + A_{t,t}}E_t^i[p] (1 - E_t^i[p]).$$

Thus, if $i$ lives forever, $A_{i,\infty} = 0$ and $m_{i,\infty} = \mu$, $E_{\infty}^i[p] = p$; $var_{\infty}^i[p] = 0$.
The Experiential Learning Bias

Each agent lives for $T$ years.
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Each agent lives for $T$ years

Inherit 'parents’ mean beliefs, but increase dispersion of beliefs:

\[
\begin{align*}
    m_{\text{Young},0} &= m_{\text{Old},T} \\
    E_{0}^{\text{Young}}[p] &= E_{T}^{\text{Old}}[p] \\
    A_{\text{Young},0} &= kA_{\text{Old},T}.
\end{align*}
\]

where $k > 1$
The Experiential Learning Bias

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- Thus, the Young update ’more’ that the Old
- Sensitivity of update in expectations to shock \( \approx A_{i,t} \)
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- Sensitivity of update in expectations to shock $\approx A_{i,t}$

If $k = 1$, economy converges to one-agent, rational expectations case
The Model: OLG setup

- Two agents concurrently alive
  - Agents are Young for $T/2$ years and Old for $T/2$ years
  - Set $k = \left( A_0^{-1} + T \right) A_0$ such that $A_{i,0} = A_0$ for all cohorts (stationary learning dynamics)
The Model: OLG setup

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- E-Z utility with Dynasty form of bequest function: maximize offspring’s utility, leave ex-consumption wealth for them.
  - Thus, two dynasties, \( A \) and \( B \), where at each point in time the representative agent for Dynasty \( A \) (\( B \)) is either Young (Old) or Old (Young)
  - Stylized OLG to keep number of state variables low
The Model: OLG setup

The “Old” die and leave their wealth (and mean beliefs) to the new “Young.”
The model: Equilibrium

- Complete markets, so for any asset $j$, for $i \in \{A, B\}$

\[
E^i \left[ M_{t+1}^i R_{t+1}^i | X_t \right] = 1
\]

where $X_t = [m_{A,t}, m_{B,t}, a_{A,t}, a_{B,t}, A_{A,t}, A_{B,t}, W_{A,t} / W_{B,t}]$
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$$M_{t+1}^i = \beta \left( \frac{C_{i,t+1}}{C_{i,t}} \right)^{\alpha-1} \left( \frac{V_{i,t+1} / C_{i,t+1}}{E_t^i \left[ (V_{i,t+1} / C_{i,t+1})^\alpha (C_{i,t+1} / C_{i,t})^{\alpha} \right]^{1/\alpha}} \right)^{\alpha-\rho}$$
The model: Equilibrium

- Complete markets, so for any asset \( j \), for \( i \in \{A, B\} \)

\[
E^i \left[ M^i_{t+1} R^j_{t+1} | X_t \right] = 1
\]

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\[
M^i_{t+1} = \beta \left( \frac{C_{i,t+1}}{C_{i,t}} \right)^{\alpha-1} \left( \frac{V_{i,t+1}/C_{i,t+1}}{E^i_t \left[ (V_{i,t+1}/C_{i,t+1})^\alpha \left( C_{i,t+1}/C_{i,t} \right)^\alpha \right]^{1/\alpha}} \right)^{\alpha-\rho}
\]

and

\[
\pi^A (\Delta c_{t+1} | X_t) M^A (\Delta c_{t+1}, X_t) = \pi^B (\Delta c_{t+1} | X_t) M^B (\Delta c_{t+1}, X_t).
\]
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\]

- Note the endogenous state variable (the relative wealth of the agents)
  - Complicates model solution as evolution equation depends on value functions which are what we are trying to solve for...

- Consider 'uncertain mean' and 'uncertain probability' models separately
  - Also, note that \( A_{A,t}, A_{B,t} \) deterministic, so each model has 4 state variables
The model: Calibration

- Learning channel: choosing $A_0$ (or equivalently $k$)

- Use numbers estimated by Malmendier and Nagel (2009):
  - The sensitivity of Young’s (about 30yo) updates in beliefs to macro shock: 0.025
  - The sensitivity of Old’s (about 70yo) updates in beliefs to macro shock: 0.01

- In our case, amounts to calibration of $A_0 = 0.025$.
  - Implies sensitivity of old of 0.005, a little on the conservative side

- How 'wrong' are investors' expectations wrt macroeconomic quantities in model?
  - Takes on average 175 years of time-series data to reject the model of consumption growth used by a Dynasty
  - Bias is, however, immediately identified in the cross-section of beliefs
Preference parameters:

<table>
<thead>
<tr>
<th></th>
<th>Uncertain mean</th>
<th>Uncertain Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma ) (risk aversion parameter)</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>( \psi ) (elasticity of intertemporal substitution)</td>
<td>1.5</td>
<td>1.5</td>
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<tr>
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<td>0.994</td>
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Priors:

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<td>0.025</td>
<td>0.025</td>
</tr>
<tr>
<td>( m, p ) (upper truncation point of prior)</td>
<td>1.45%</td>
<td>0.01%</td>
</tr>
<tr>
<td>( m, p ) (lower truncation point of prior)</td>
<td>-0.55%</td>
<td>4.00%</td>
</tr>
</tbody>
</table>

- Equity claim to exogenous dividends (as in, e.g., Campbell and Cochrane (1999))

\[
\Delta d_{t+1} = \lambda \Delta c_{t+1} + \sigma_d \eta_{t+1}
\]

where \( \lambda = 3 \), \( \sigma_d = 5\% \)
Risk-sharing: EZ strong motive

'Uncertain Mean' case

- Agents in the 'middle' of their generations (10yo and 30yo)
- The 'Old' have conditional unbiased beliefs

More optimistic allocate more to total wealth (consumption) claim
Agent who perceives more confidence risk (parameter uncertainty) holds less
Risk-sharing: Power Utility, not strong motive

- Same parameters, except $\psi = 0.1 = 1/\gamma$

Less pronounced wealth-dynamics

Agent who perceives more model risk holds *more* in wealth claim (low EIS)
Why? More heterogeneity in long-run risk

Recall subjective consumption dynamics (case: uncertain mean, no disaster):

\[
\begin{align*}
\Delta c_{t+1} &= m_{i,t} + \sqrt{A_{i,t} + 1}\sigma \tilde{\varepsilon}_{t+1} \\
m_{i,t+1} &= m_{i,t} + \frac{A_{i,t}}{\sqrt{A_{i,t} + 1}}\sigma \tilde{\varepsilon}_{t+1} \\
A_{i,t+1}^{-1} &= A_{i,t}^{-1} + 1
\end{align*}
\]

In the middle of their respective generations (ages 10 and 30):

\[
\begin{align*}
\frac{\text{Short-run risk (Young)}}{\text{Short-run risk (Old)}} - 1 &= \frac{\sqrt{A_{\text{Young},t} + 1}\sigma}{\sqrt{A_{\text{Old},t} + 1}\sigma} - 1 = 0.3\% \\
\frac{\text{Long-run risk (Young)}}{\text{Long-run risk (Old)}} - 1 &= \frac{A_{\text{Young},t}}{\sqrt{A_{\text{Young},t} + 1}\sigma} - \frac{A_{\text{Old},t}}{\sqrt{A_{\text{Old},t} + 1}\sigma} - 1 = 67\% 
\end{align*}
\]
Risk-sharing: 'Uncertain Probability'-case

- Plots correspond to agents in the 'middle' of their generations (10yo and 30yo)
- The 'Old' have conditional unbiased beliefs

More optimistic allocate more to total wealth (consumption) claim
- Agent who perceives more confidence risk (parameter uncertainty) holds less
- 'Power'-case: portfolio allocation more responsive to mean beliefs
Wealth dynamics and the risk premium

- Uncertain probability, disaster case

- Dashed blue: both agents have unbiased mean beliefs
- Solid red: Young optimists, Old pessimists
- Dashed green: Young pessimists, Old optimists
Standard moments – ’Uncertain Mean’

- Unknown mean, no disasters

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>‘This Time is Different’</th>
<th>Known mean</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>EZ</td>
<td>Power</td>
<td>EZ</td>
</tr>
<tr>
<td></td>
<td>$\gamma = 10$</td>
<td>$\gamma = 10$</td>
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</tr>
<tr>
<td></td>
<td>$\psi = 1.5$</td>
<td>$\psi = 1/10$</td>
<td>$\psi = 1.5$</td>
</tr>
<tr>
<td>1929 – 2011</td>
<td>$\beta = 0.994$</td>
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- $E_T [r_m - r_f]$ 5.1 5.2 0.1 1.5
- $\sigma_T [r_m - r_f]$ 20.2 16.6 10.5 12.9
- $SR_T [r_m - r_f]$ 0.25 0.31 0.01 0.12
- $\sigma_T [M_{t+1}] / E_T [M_{t+1}]$ - 0.51 0.20 0.27
- $\gamma \times \sigma_T [\Delta c_{t+1}]$ - 0.27 0.27 0.27

- E-Z TTiD model performs well!
  - Power case suffers from all the usual puzzles...

- 2 stdev range of P/D-ratio: 27 to 54, highly persistent
  - Benchmark case: P/D ratio is constant
Standard moments – ‘Uncertain Probability’

- Known mean, unknown disaster probability

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- E-Z TTiD model performs well!
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- 2 stdev range of P/D-ratio: 25 to 45, highly persistent
  - Benchmark case: P/D ratio is constant
Econometrician’s 'risk aversion estimate'

'Uncertain Mean'-case

Sensitivity of pricing kernel to aggregate shocks estimated by econometrician using a (very) long sample:

\[ \gamma_t \equiv \frac{1}{\sigma_{\Delta c}} \frac{\sigma^P_t [M_{t+1}]}{E^P_t [M_{t+1}]} \]
The 'Old', who experienced disaster perceive a lower risk premium (by about 1.5% p.a.) than the 'Young' that did not live through disaster.
A production economy and endogenous consumption

Log technology growth (unknown $\mu$ case, only):

$$\Delta z_t = \mu + \sigma_z \varepsilon_{t+1},$$

Production function:

$$Y_t = (Z_t N_t)^{1-\alpha} K_t^\alpha,$$

where $N_t = 1$.

Capital accumulation equation is:

$$K_{t+1} = (1 - \delta) K_t + \phi \left( \frac{I_t}{K_t} \right) K_t,$$

where $\delta$ is the depreciation rate, $\phi (\cdot)$ is a concave function, and $I_t$ is gross investment.

Resource constraint:

$$Y_t = C_t + I_t.$$
Investment and Equity Returns

Adjustment costs (Zhang (2005)):

\[ \phi(x_t) = x_t - \frac{a_t}{2} x_t^2, \]

Return to investment and unlevered stock return:

\[ R_{I,t+1} = \phi'(I_t/K_t) \left( \alpha (K_t/Z_t)^{\alpha-1} + \frac{1 - \delta + \phi(I_{t+1}/K_{t+1})}{\phi'(I_{t+1}/K_{t+1})} - \frac{I_{t+1}}{K_{t+1}} \right), \]

Firm’s FOC yields:

\[ E_t^A [M_{t+1}^A R_{I,t+1}] = E_t^B [M_{t+1}^B R_{I,t+1}] = 1. \]

Levered equity returns:

\[ R_{E,t+1} = (1 + B/E) R_{I,t+1} \]
## Calibration

### Preference parameters: EZ – Production

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### Technology and Production Parameters:

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<td>( \alpha ) (the capital share)</td>
<td>0.36</td>
</tr>
<tr>
<td>( \mu ) (true mean growth rate)</td>
<td>0.5%</td>
</tr>
<tr>
<td>( \sigma_z ) (volatility of technology shocks)</td>
<td>3.8%</td>
</tr>
<tr>
<td>( \delta ) (capital depreciation rate)</td>
<td>2.5%</td>
</tr>
<tr>
<td>( a_- ) (downside adjustment costs parameter)</td>
<td>20</td>
</tr>
<tr>
<td>( a_+ ) (upside adjustment costs parameter)</td>
<td>6</td>
</tr>
</tbody>
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Moments

- Excess investment volatility due to experiential learning
  - 10.8% versus 7.8%
- Close to order of magnitude increase in return volatility
  - 6% versus 0.9%

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<td>$\sigma_T [\Delta c]/\sigma_T [\Delta y]$</td>
<td>0.52</td>
<td>0.47</td>
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<td>2.20</td>
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<td>$SR_T [R_M - R_f]$</td>
<td>0.36</td>
<td>0.51</td>
</tr>
</tbody>
</table>
Conditional moments versus beliefs

- **Investment Rate (I/K)**
  - The relationship shows an upward trend with increasing mean beliefs of Agent A.

- **The Equity Risk Premium**
  - The premium decreases as mean beliefs increase.

- **The Price Of Risk**
  - The price decreases as mean beliefs increase.

*Note: High investment > low future returns (Cochrane, 1991)*
Investment versus uncertainty

- High model uncertainty, less investment
Investor expectations about real GDP growth

Survey of professional forecasters: 1972Q2-2009Q2

$R^2 = 31\%$, $\beta = 0.8$ (s.e. 0.1)

- Green dashed line: Realized real GDP growth
- Blue solid line: Median forecast
Investor expectations about real GDP growth

Survey of professional forecasters: 1972Q2-2009Q2

$R^2 = 31\%, \ \beta = 0.8 \ (s.e. \ 0.1)$

"Objective" forecast from AR(4): 1972Q2-2009Q2

$R^2 = 37\% > 31\%$
Expected subjective and objective GDP growth

- Close relation subjective and true GDP growth
  - Agents act on their beliefs: high (low) perceived growth rate, high (low) investment
  - Agents’ models very close to true model in a statistical sense
Conclusion

- With 'This Time is Different'-bias, model uncertainty persists indefinitely
  - Calibration consistent with micro estimates of age-dependent updating rule
  - Still, 'small' departure from Rational Expectations; median belief excellent forecaster

- With E-Z preferences, extra risk factor arises (amplification of macro risks)
  - Large effect on investment and asset prices
    - Optimal risk-sharing amplifies business cycle and asset price fluctuations through relative wealth fluctuations
  - Endogenously counter-cyclical uncertainty premia
    - Helps account for the standard macro-finance 'puzzles'

- Extended periods of prices and investment reflecting 'irrational exuberance' or 'irrational fear'
Assume economy ends at $\bar{T}$, make relative consumption of agent $A$ the endogenous state variable, $c_A = C_A / C$. 

Final period value functions as boundary condition, imposing resource constraint: 

$$V_A, \bar{T} = \left(1 - \beta \right) \frac{1}{\rho} c_A, \bar{T}C_{\bar{T}}$$

Use complete markets assumption to find evolution equation from $C_A, \bar{T}$ to $C_A, 1$ (state variables): 

$$\pi_A \Delta c_{\bar{T}} - \Delta X_{\bar{T}}$$

$$\pi_A \Delta c_{\bar{T}} - \Delta X_{\bar{T}}$$

$$E_A, \bar{T} \left(1 - \alpha \right) \frac{1}{\alpha} C_A \alpha \rho = \ldots$$

$$\pi_B \Delta c_{\bar{T}} - \Delta X_{\bar{T}}$$

$$\pi_B \Delta c_{\bar{T}} - \Delta X_{\bar{T}}$$

$$E_B, \bar{T} \left(1 - \alpha \right) \frac{1}{\alpha} C_A \alpha \rho$$
The model: Solution Method (PCD, MJ, and LL (2014))

- Assume economy ends at $\tilde{T}$, make relative consumption of agent $A$ the endogenous state variable, $c_A = C_A / C$.

- Final period value functions as boundary condition, imposing resource constraint:

$$V_{A,\tilde{T}} = (1 - \beta)^{1/\rho} c_{A,\tilde{T}} C_{\tilde{T}}$$
$$V_{B,\tilde{T}} = (1 - \beta)^{1/\rho} (1 - c_{A,\tilde{T}}) C_{\tilde{T}}$$
The model: Solution Method (PCD, MJ, and LL (2014))

- Assume economy ends at $\tilde{T}$, make relative consumption of agent $A$ the endogenous state variable, $c_A = C_A / C$.

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$$V_{A,\tilde{T}} = (1 - \beta)^{1/\rho} c_{A, \tilde{T}} C_{\tilde{T}}$$

$$V_{B,\tilde{T}} = (1 - \beta)^{1/\rho} (1 - c_{A, \tilde{T}}) C_{\tilde{T}}$$

- Use complete markets assumption to find evolution equation from $C_{A, \tilde{T} - 1}$ to $C_{A, \tilde{T}}$ ($X_t$ - state variables):

$$\pi^A (\Delta c_{\tilde{T}} | X_{\tilde{T} - 1}) \left( \frac{c_{A, \tilde{T}}}{c_{A, \tilde{T} - 1}} \right)^{\alpha - 1} \left( \frac{v_{A, \tilde{T}} C_{\tilde{T}} / C_{\tilde{T} - 1}}{E^A_{\tilde{T} - 1} [v_{A, \tilde{T}} (C_{\tilde{T}} / C_{\tilde{T} - 1})^{\alpha}]^{1/\alpha}} \right)^{\alpha - \rho} = ...$$

$$\pi^B (\Delta c_{\tilde{T}} | X_{\tilde{T} - 1}) \left( \frac{1 - c_{A, \tilde{T}}}{1 - c_{A, \tilde{T} - 1}} \right)^{\alpha - 1} \left( \frac{v_{B, \tilde{T}} C_{\tilde{T}} / C_{\tilde{T} - 1}}{E^B_{\tilde{T}} [v_{B, \tilde{T}} (C_{\tilde{T}} / C_{\tilde{T} - 1})^{\alpha}]^{1/\alpha}} \right)^{\alpha - \rho}$$
Use complete markets assumption to find evolution equation from \( C_{A,\tilde{\tau}} \) to \( C_{A,\tilde{\tau}} (X_t - \text{state variables}) \):

\[
\pi^A (\Delta c_{\tilde{\tau}} | X_{\tilde{\tau}-1}) \left( \frac{c_{A,\tilde{\tau}}}{c_{A,\tilde{\tau}-1}} \right)^{\alpha-1} \left( \frac{v_{A,\tilde{\tau}} C_{\tilde{\tau}} / C_{\tilde{\tau}-1}}{E^A_{\tilde{\tau}-1} \left[ v_{A,\tilde{\tau}} (C_{\tilde{\tau}} / C_{\tilde{\tau}-1})^{\alpha} \right]^{1/\alpha}} \right)^{\alpha-\rho} = \ldots
\]

\[
\pi^B (\Delta c_{\tilde{\tau}} | X_{\tilde{\tau}-1}) \left( \frac{1 - c_{A,\tilde{\tau}}}{1 - c_{A,\tilde{\tau}-1}} \right)^{\alpha-1} \left( \frac{v_{B,\tilde{\tau}} C_{\tilde{\tau}} / C_{\tilde{\tau}-1}}{E^B_{\tilde{\tau}} \left[ v_{B,\tilde{\tau}} (C_{\tilde{\tau}} / C_{\tilde{\tau}-1})^{\alpha} \right]^{1/\alpha}} \right)^{\alpha-\rho}
\]
Use complete markets assumption to find evolution equation from $C_{A,\bar{T}-1}$ to $C_{A,\bar{T}}$ ($X_t$ - state variables):

\[
\pi^A (\Delta c_{\bar{T}} | X_{\bar{T}-1}) \left( \frac{c_{A,\bar{T}}}{c_{A,\bar{T}-1}} \right)^{\alpha-1} \left( \frac{v_{A,\bar{T}} C_{\bar{T}} / C_{\bar{T}-1}}{E_{\bar{T}-1}^A \left[ v_{A,\bar{T}}^\alpha (C_{\bar{T}} / C_{\bar{T}-1})^\alpha \right]^{1/\alpha}} \right)^{\alpha-\rho} = \ldots
\]

\[
\pi^B (\Delta c_{\bar{T}} | X_{\bar{T}-1}) \left( \frac{1 - c_{A,\bar{T}}}{1 - c_{A,\bar{T}-1}} \right)^{\alpha-1} \left( \frac{v_{B,\bar{T}} C_{\bar{T}} / C_{\bar{T}-1}}{E_{\bar{T}}^B \left[ v_{B,\bar{T}}^\alpha (C_{\bar{T}} / C_{\bar{T}-1})^\alpha \right]^{1/\alpha}} \right)^{\alpha-\rho}
\]

By solving fixed point for $k_{\bar{T}-1} (X_{\bar{T}-1}) = \frac{E_{\bar{T}}^B \left[ v_{B,\bar{T}}^\alpha (C_{\bar{T}} / C_{\bar{T}-1})^\alpha \right]^{1/\alpha}}{E_{\bar{T}-1}^A \left[ v_{A,\bar{T}}^\alpha (C_{\bar{T}} / C_{\bar{T}-1})^\alpha \right]^{1/\alpha}}$, we obtain the evolution equation from $c_{A,\bar{T}-1}$ to $c_{A,\bar{T}}$. Next, find value functions at $\bar{T} - 1$. 
Use complete markets assumption to find evolution equation from $C_{A,\bar{T}-1}$ to $C_{A,\bar{T}}$ ($X_t$ - state variables):

\[
\pi^A (\Delta c_{\bar{T}} | X_{\bar{T}-1}) \left( \frac{c_{A,\bar{T}}}{c_{A,\bar{T}-1}} \right)^{\alpha-1} \left( \frac{v_{A,\bar{T}} C_{\bar{T}} / C_{\bar{T}-1}}{E_{\bar{T}-1}^A \left[ v_{A,\bar{T}}^\alpha (C_{\bar{T}} / C_{\bar{T}-1})^\alpha \right]^{1/\alpha}} \right)^{\alpha-\rho} = \ldots
\]

\[
\pi^B (\Delta c_{\bar{T}} | X_{\bar{T}-1}) \left( \frac{1 - c_{A,\bar{T}}}{1 - c_{A,\bar{T}-1}} \right)^{\alpha-1} \left( \frac{v_{B,\bar{T}} C_{\bar{T}} / C_{\bar{T}-1}}{E_{\bar{T}}^B \left[ v_{B,\bar{T}}^\alpha (C_{\bar{T}} / C_{\bar{T}-1})^\alpha \right]^{1/\alpha}} \right)^{\alpha-\rho}
\]

By solving fixed point for $k_{\bar{T}-1} (X_{\bar{T}-1}) = \frac{E_{\bar{T}}^B \left[ v_{B,\bar{T}}^\alpha (C_{\bar{T}} / C_{\bar{T}-1})^\alpha \right]^{1/\alpha}}{E_{\bar{T}-1}^A \left[ v_{A,\bar{T}}^\alpha (C_{\bar{T}} / C_{\bar{T}-1})^\alpha \right]^{1/\alpha}}$, we obtain the evolution equation from $c_{A,\bar{T}-1}$ to $c_{A,\bar{T}}$. Next, find value functions at $\bar{T} - 1$.

Proceed backwards until transversality condition 'kicks in': reached solution for infinite horizon problem.