Information Aversion *

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Abstract

We propose a theory of inattention solely based on preferences, absent any cognitive limitations and external costs of acquiring information. Under disappointment aversion, information decisions and risk attitude are intertwined, and agents are intrinsically information averse. We illustrate this link between attitude towards risk and information in a standard portfolio problem, in which agents balance the costs, endogenous in our framework, and benefits of information. We show agents never choose to receive information continuously in a diffusive environment: they optimally acquire information at infrequent intervals only. We highlight a novel channel through which the optimal frequency of information acquisition decreases when risk increases, consistent with empirical evidence. Our framework accommodates a broad range of applications, suggesting our approach can explain many observed features of decision under uncertainty.

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1 Introduction

We propose a theory of inattention solely based on preferences, absent any cognitive limitations, or external costs of acquiring information. In our framework, agents are disappointment averse, in a dynamic extension to Gul (1991). Disappointment averse agents are disproportionately affected by bad news, so that a small negative signal, followed by an offsetting small positive one does not leave them indifferent: it lowers their utility. Sequences of repeated small news are particularly costly, and our agents are intrinsically averse to receiving signals. Formally, this feature of disappointment aversion leads to an unambiguous dislike for information. In this paper, we explore the widespread implications of this dislike for information for decision under uncertainty, with a focus on applications in financial economics.

Our first contribution is to characterizes the strength and properties of this *information aversion* for various types of risk. We show the endogenous costs of information resulting from our parsimonious model of preferences differ fundamentally both from the cognitive constraints and from the exogenous information costs that are commonly used in the inattention literature. In our setting, the preferences shape both the frequency of information acquisition as well as the structure of observed signals. In particular, we find a stark difference between diffusive and jump-driven information structure: agents behave as if much more risk averse in a diffusive environment. We further expand on the distinction with alternative models of inattention by revisiting a few standard puzzles of the finance literature.

Our second contribution is to study how information averse agents cope with their fear of information flows when information is necessary to make appropriate economic decisions. We focus our analysis on a standard dynamic portfolio problem in which agents continuously allocate their wealth between consumption and savings. Agents can invest their savings into both a risk-free and a risky asset. We find agents optimally observe their wealth (i.e. collect information) only at discrete points in time. They keep their eyes closed to the economic environment in between, even absent transaction costs and external information costs. Further, in our framework, changes in the characteristics of the economic environment affect not only the benefits, but also the costs of information. In particular, we highlight a new channel for the role of risk for inattention and savings. Because a more intense information flow is more stressful, the optimal frequency of information acquisition and risky investment decrease when risk increases, consistent with empirical evidence.

This consumption-saving problem illustrates the richness of our model for inattention, and is but one example in which we can expect endogenous information costs to play a distinctive role. In the last section of our paper, we outline how information aversion affect many other decisions under uncertainty, and how it sheds light on several existing puzzles. We focus on the value of diversification, the role of background risk, the evaluation of risky projects and the delegation of choice under uncertainty.

Let us more precisely describe our approach. We assume agents disappointment averse, as in Gul (1991). Under disappointment aversion, agents inflate the probabilities of outcomes that disappoint, i.e. fall under their ex-ante "fair value", or certainty equivalent.[1] This parsimonious model of preferences simply requires to specify the utility derived from realized outcomes, and a disappointment aversion parameter that measures how much to inflate the probabilities of disappointing outcomes. Disappointment aversion has been successfully implementated in the finance literature, where it has proven instrumental in explaining portfolio choices (Ang et al., 2005), equilibrium aggregate prices (Routledge and Zin (2010), Bonomo et al. (2011)), and the cross-section of expected returns (Ang et al. (2006), Lettau et al. (2013)). However, this literature left aside the important implications of these preferences for information choices. Disappointment averse agents ex-

[1]The "fair" value, or certainty equivalent, of risky outcomes is thus the unique solution to a fixed point problem.

hibit a preference for one-shot resolution of uncertainty (Dillenberger, 2010): they prefer not to receive any partial information about lottery outcomes if they cannot take any decision based on this information. As an illustration, consider the example of a disappointment averse investor having bought a stock that she has decided to sell after one year exactly. The investor has a lot of control over the information structure she faces. She can choose to follow the price of the stock continuously, each day, month, or ignore it altogether until the end of the year. This information is of no use to the investor as she will hold on to the asset anyways. Our investor unambiguously opts not to observe the price at all over the year. This unambiguous *information aversion* resulting from disappointment aversion is supported by a number of empirical and experimental findings.[2]

Our first contribution is to characterize the strength and properties of the endogenous costs of information implied by disappointment aversion. First, we show our information aversion model results in a distaste for receiving signals that differs fundamentally from both the exogenous information costs and the cognitive limitations that are commonly used in the inattention literature. Second, we analyze how the frequency of information observations impacts the certainty equivalents of lotteries whose payoffs correspond to the final value of a stochastic process. Across distributions, we find the certainty equivalent is increasing in the length of the interval between observations: the agent uniformly dislikes more frequent observations. The magnitude of this effect varies greatly across characteristics of the process. For a diffusion, we show the certainty equivalent converges to the worst-case outcome as observation frequency increases: the agent behaves as if infinitely risk averse when the flow of information becomes continuous. On the other hand, for

[2]Starting with Gneezy and Potters (1997) and Thaler et al. (1997), followed by Benartzi and Thaler (1999), Barron and Erev (2003), Gneezy et al. (2003), Bellemare et al. (2005), Haigh and List (2005), Fellner and Sutter (2009) and Anagol and Gamble (2011), experiments have consistently showed that subjects's valuations of risky outcomes diminish when they are given more detailed information.

a jump process, the certainty equivalent admits a finite limit within the domain of the distribution when the frequency increases. This result informs us that one should expect more inattention to information on smooth risky processes than to information on sudden large changes in utility. In both cases, the cost of information increases as risk increases or disappointment aversion increases.

Our second contribution is to analyze how agents balance the utility cost of paying attention to the economic environment with the benefits of making informed decisions. Beshears et al. (2012) highlights the centrality of this problem. They find they cannot replicate, in a natural setting, the results of the experiments cited above (starting with those of Gneezy and Potters (1997)): when agents use, as they arguably do in practice, the information they observe to make decisions, they are no longer unambiguously averse to receiving it. Going back to our example, assume now the investor manages her wealth in order to finance her consumption over time. She can invest in stocks that yield a high average return or in risk-free bonds. Following the evolution of stock prices is endogenously costly for the information averse investor, but also useful. Knowing her current wealth allows her to adjust her immediate consumption and savings, and thus to optimally smooth her consumption over time. We derive the optimal saving, consumption and attention policy in such a portfolio problem, in the case of i.i.d. returns to the risky asset. As in other models with infrequent transactions, the marginal cost of extending the observation interval is to forego some of the high returns of the risky asset. The novelty of our approach is to endogenize the marginal benefit of increasing the length of time between observations, driven by the lower "stress", or utility cost, to the agent as the frequency of information decreases. We show a sufficient statistic for this marginal benefit is the elasticity of the certainty equivalent with respect to the observation interval (see Proposition 6). We find higher risk, even compensated by higher expected returns, results in an increased optimal time between observations: riskier environments encourage more inattention. Such a result obtains because of the particular structure of the endogenous information costs, and is specific to our model. In particular, existing similar models with exogenous information costs do not typically yield the same implication. Consistent with our prediction, Karlsson et al. (2009) documents investors reduce the monitoring of their portfolios in riskier environments, displaying an "ostrich" behavior.

Finally, we outline a few extensions to this simple optimization problem, and show ours is a rich framework in which to analyze decisions under uncertainty. In our first extension, we augment the agent's investment opportunities, and show information aversion can significantly lower the benefits of diversification. In a second extension, we discuss how our framework provides a rationale for introducing intermediaries, i.e. firms or agents specialized in information diffusion, and the potential agency problems that it entails. Finally, we study how our information aversion model impacts investment decisions, when an agent has to choose among several projects. We show not only the risk and reward characteristics of the project matter, but also what type, and how much information on the project performance is revealed over time. These are but a few examples in which the endogenous costs of information due to disappointment aversion affect optimal decision making; they highlight the broad reach of our approach.

After a brief review of the related literature, Section 2 describes how dynamic disappointment aversion preferences result in endogenous information costs. In Section 3, we characterize the strength of information aversion across frequencies for various types of risk. Section 4 presents a standard consumption-savings problem and illustrates how agents optimally choose the frequency at which they acquire information. Section 5 proposes other examples in which our inattention model could provide novel justifications to existing puzzles. Section 6 concludes.

Related literature

Because losses below a reference point are more costly than comparable gains above the reference point, disappointment averse agents display loss aversion, one of the main components of the prospect theory model of Kahneman and Tversky (1979). The fact the frequency of utility evaluation plays an important role for the calibration of preferences with loss aversion is well known, at least since Benartzi and Thaler (1995). They show, numerically, the observed returns and risk of the market portfolio are consistent with the prospect theory model of Kahneman and Tversky (1979) only if agents evaluate their risky portfolio at yearly intervals, somewhat consistent with investors' observation intervals in the data. Our paper provides a framework to analyze simultaneously the endogenous determination of the information structure and of risk taking decisions. Disappointment aversion intertwines those two dimensions, providing a simple framework for such problems.

In a contemporaneous paper, Pagel (working paper) analyzes a consumption-savings problem similar to ours, under the reference-dependent preferences Kőszegi and Rabin (2009), a model with loss aversion that explicitly introduces flows of information in the utility function. Our approach, with disappointment aversion, is more parsimonious: risk aversion, information aversion and the endogenous reference point as a function of the distribution of future outcomes all derive from the same unique parameter. The relative simplicity of our model allows for a formal analysis of the endogenous information costs, and of our optimization problem, and can be extended to other economic applications. Disappointment averse preferences also have the appeal over standard loss aversion to be axiomatically funded. Finally, as mentioned above, they have been successfully implemented in the finance literature.

We contribute to the inattention literature, by introducing endogenous information costs that derive from agents' preferences. We revisit ones specific inattention framework, a consumption savings decision, through the lenses of our endogenous cost structure. Gabaix and Laibson (2002) show slow portfolio readjustment, as a result of inattention, can have a profound impact on equilibrium asset prices. In their work, the frequency of observation is exogenous or results from a fixed utility cost of information. Abel et al. (2007) derive optimal inattention period and portfolio decision with an exogenous monetary cost of information. Abel et al. (2013) add transaction costs. Along the same lines, Alvarez et al. (2012) analyze theoretically and provides micro-level empirical evidence on the consumption and savings decisions of investors facing information costs and transaction costs, in the case of durable consumption. We consider a setup similar to these last models, in which the investor optimally chooses how frequently she observes information, and also find a strictly positive optimal time interval between observations. However, our information costs, resulting from information aversion, respond differently to the economic environment, thus yielding novel implications for the sensitivity of the optimal frequency to changes in the underlying parameters of the model. Our analysis of the structure and strength of information aversion, in the case of disappointment averse agents, and in particular the fact that it is model equivalent to neither a limited ability to process information, or cognitive constraint (as in Sims (1998)), nor to exogenous costs of information, highlights how general interest our model potentially is to the inattention literature.

2 Disappointment Aversion and Information Aversion

We start by introducing dynamic disappointment preferences. Then, we show agents with such preferences have a motive for inattention we call *information aversion.*

2.1 Dynamic disappointment aversion

Under loss aversion, one of the main components of the seminal prospect theory model of Kahneman and Tversky (1979), agents value outcomes relative to a reference point, and losses relative to the reference create more disutility than comparable gains. Disappointment aversion, introduced by Gul (1991), provides a fully axiomatized model of preferences in which agents display loss aversion. In most of what follows, we focus on the piecewise linear case, to emphasize the role of the asymmetry between gains and losses rather than the curvature around the reference point. The certainty equivalent of a risky payoff X, with cumulative distribution function F, is given by

$$\mu(X) = \frac{\int x dF(x) + \theta \int_{x \leq \mu(X)} x dF(x)}{1 + \theta \int_{x \leq \mu(X)} dF(x)}. \tag{1}$$

The certainty equivalent is a weighted average of all possible future payoffs in which disappointing outcomes receive a higher weight $(1+\theta)$. Disappointing outcomes are defined with respect to the endogenous reference point $\mu(X)$, equal to the certainty equivalent. The parameter $\theta \geq 0$ captures the degree of disappointment aversion; larger values of θ correspond to more overweighting of disappointing events. The definition in Equation (1) is a fixed point problem in $\mu(X)$, and always admits a unique solution.

In a dynamic framework, we assume the agent values the lottery by taking into account the disappointments he can feel at each revelation of information. This assumption corresponds to a certainty equivalent evaluated recursively using Equation (1). Precisely, given certainty equivalents $\mu_{t+1}(s_{t+1})$ in each possible state s_{t+1} at date $t+1$ and transition c.d.f $F(s_{t+1}|s_t)$, the certainty equivalent for state s_t at date t is given by

$$\mu_t(s_t) = \frac{\int \mu_{t+1}(s_{t+1}) dF(s_{t+1}|s_t) + \theta \int_{\mu_{t+1}(s_{t+1}) \leq \mu_t(s_t)} \mu_{t+1}(s_{t+1}) dF(s_{t+1}|s_t)}{1 + \theta \int_{\mu_{t+1}(s_{t+1}) \leq \mu_t(s_t)} dF(s_{t+1}|s_t)}. \tag{2}$$

It is straightforward to check that introducing intermediate dates in which no new information is released does not affect the recursion. However, if one considers the operation of Equation (2) as a distorted expectation, no equivalent of the law of iterated expectations exists: the composition of the arrival of information does matter. We analyze how in the next section.

2.2 Information aversion

To clarify the role information plays on in the dynamic disappointment aversion model, let us focus on a setup with only three dates: 0, 1, and 2.

At date 2, the agent receives the final outcome X. At date 0, the agent knows the ex ante distribution of the final payoff, F. At date 1, the agent receives a signal i, $i \in \{1, N\}$, with probability α_i. Given this signal, the agent updates her belief on the distribution of X, from F to F_i. We are interested in comparing the certainty equivalent, at date $t = 0$, of the compound lottery with date $t = 1$ signals, $\mu(\{F_i, \alpha_i\})$, with that of a lottery without intermediate signal, $\mu(F)$. The distribution of final outcomes is the same for both lotteries: $F = \sum_i \alpha_i F_i$.

For an agent with dynamic disappointment aversion, we obtain the following result:

$$\forall F, \{F_i, \alpha_i\}_{i=1}^N \text{ s.t. } F = \sum_i \alpha_i F_i,$$
$$\mu(\{F_i, \alpha_i\}) \leq \mu(F)$$

The agent always weakly prefers not to receive intermediate information, and thus exhibits what we call *information aversion*. This result is a special case of the more general theorem of Dillenberger (2010), as disappointment averse preferences exhibit negative certainty independence.[3]

[3]Dillenberger (2010) describes this property as a preference for one-shot resolution of uncertainty.

Information aversion is generic: for most cases of partial information, the agent will strictly prefer not to receive the signal. The following corollary characterizes the particular cases for which there is indifference.

Corollary 1. *The agent is indifferent whether to receive information or not,* $\mu(\{F_i, \alpha_i\}) = \mu(F)$*, if and only if*

$$\forall i, \begin{cases} \mu(F_i) = \mu(F) \ \textit{ or} \\ F_i \textit{ is degenerate} \end{cases}$$

Receiving intermediate information is costless if and only if all second stage lotteries are either degenerate (taking only one possible value), or leave the certainty equivalent unchanged. This result shows information is strictly costly when it only partially reveals the final outcome, i.e. when some uncertainty remains (non degenerate lotteries), and it has changed the value of the certainty equivalent. Indeed, the agent is averse to information flows that can give rise to repeated disappointment, and prefers receiving all the bad news once.

Another useful insight from this corollary is that standard informativeness measures cannot quantify the endogenous information costs for disappointment averse agents.

Corollary 2. *For any level of mutual entropy at the first stage of the lottery, there exists a compounded lottery that provides as much utility as one-shot resolution.*

Exogenous costs of information, which are typically monotonically increasing in the quantity of information provided, are not model equivalent to the endogenous cost structure of our information averse agents either. In our framework, the costs are increasing when little information is provided, but decreasing above a threshold, with a zero cost limit for fully revealing information.

We have showed agents with dynamic disappointment aversion are also *information averse*. In situations when partial information is of no use to them, they prefer not to receive it. In the next section, we quantify the strength of this information aversion in specific cases.

3 Information Structure and Information Aversion

In the analysis so far, we have left the information structure completely free. In most practical situations, information is natually revealed progressively. For instance, for an investment in a stock for a period of a year, observing the value of a share closer and closer to the end of the year delivers an increasing amount of information.

In this section, we focus on lotteries with payoff corresponding to the final realization of a stochastic process. The agent receives information over time by observing the current value of the process. The certainty equivalent of the lottery depends on the frequency at which the agent observes the process. We find the characteristics of uncertainty significantly affect the strength of information aversion. In particular, we focus on two cases: Brownian motion and pure jump process.

3.1 Setting

The agent evaluates, at date $t = 0$, a lottery with payoff at date $t = 1$. The payoff corresponds to the final value of a stochastic process $X = \{X_t\}_{t\in[0,1]}$. We assume the process X exhibits i.i.d. growth rate. Without loss of generality $X_0 = 1$.

We are interested in understanding the role the frequency of information plays on valuing the certainty equivalent of the process. Assume the agent observes the process $\{X_t\}_{t\in[0,1]}$ at regular intervals of length T. We note

$V_t(T)$ the certainty equivalent of the lottery at any date $t \in [0,1]$. As no information is revealed between observations, the certainty equivalent is constant on those interval.[4] Taking advantage of the multiplicative structure of the process X, we obtain a simple expression for the valuation at each observation. Indeed, using the fact that the growth rate of the process is i.i.d., and the linearity of preferences, a simple recursion gives us

$$V_0(T) = V(T)^{1/T},$$

where $V(T)$ is the certainty equivalent at time 0 of a lottery with uncertain payoff X_T, at $t = T$, starting from $X_0 = 1$, and no intermediate observation. The certainty equivalent of the overall lottery with payoff at $t = 1$ is the certainty equivalent between each arrival of news, $V(T)$, compounded by the number of observations $1/T$.

Suppose the stochastic process $X = \{X_t\}_{t\in[0,1]}$ has average growth rate μ. In a risk-neutral framework, the value of the lottery with payoff X_T would simply be $V(T) = \exp(\mu T)$. We define the equivalent instantaneous rate under disappointment aversion as $v(T)$, such that:

$$V(T) = \exp(v(T)\,T) \tag{3}$$

Expected growth only adjusts multiplicatively the certainty equivalent. It does not interact with the risk-adjustment and information aversion adjustment coming from disappointment aversion: we could write the certainty equivalent rate $v(T)$ as μ minus a risk-adjustment term. In the examples that follow, we will analyze cases in which the expected growth μ is zero, to fully isolate the analysis of the risk and information adjustment term. Reverting to a process with non-zero growth simply shifts our results for $v(T)$ by μ.

We can extend this setup to longer horizons than 1. To consider longer

[4]Formally, $\forall \tau \in [0, T[,\ n \in [\![0, \frac{1}{T} - 1]\!], \quad V_{nT}(T) = V_{nT+\tau}(T)$.

horizons, one just needs compound the per-observation certainty equivalent for longer periods of time. Naturally, once we move to longer horizons, the observation frequency can be completely arbitrary, which allows us to define $v(T)$ for all possible positive values of the interval T.

With those results, we now turn to the behavior of the certainty equivalent for some precise dynamics of the process X. This approach informs us on the link between types of risks and information aversion.

3.2 Brownian payoffs

We fist consider the case of a geometric Brownian motion. Assume the following law of motion for X_t:

$$\frac{dX_t}{X_t} = \sigma dZ_t$$

where $\{Z_t\}$ is a standard Brownian motion, and the parameter σ is the volatility of X.

Proposition 3. *The certainty equivalent of the lottery with observations at intervals of length T, and payoff $X_{\mathcal{T}}$ at date $t = \mathcal{T}$, is $V_0(T) = \exp(v(T)\mathcal{T})$, where $v(T)$ is the unique solution of*

$$\exp\left(Tv\left(T\right)\right) = \frac{1+\theta\Phi\left(\frac{\sqrt{T}}{\sigma}\left(v\left(T\right)-\frac{1}{2}\sigma^2\right)\right)}{1+\theta\Phi\left(\frac{\sqrt{T}}{\sigma}\left(v\left(T\right)+\frac{1}{2}\sigma^2\right)\right)} < 1, \tag{4}$$

where Φ is the cumulative normal.

Note the certainty equivalent is always lower than one, the risk-neutral value of $X_{\mathcal{T}}$: disappointment averse agents are risk averse. The adjustment for risk is fully characterized by the instantaneous rate $v(T)$, and is function of the observation interval T, the volatility σ and aversion parameter θ. In the case of Brownian payoffs with volatility σ, the instantaneous rate $v(T)$ is

- increasing in the observation interval T,
- decreasing in volatility σ,
- decreasing in disappointment aversion θ.

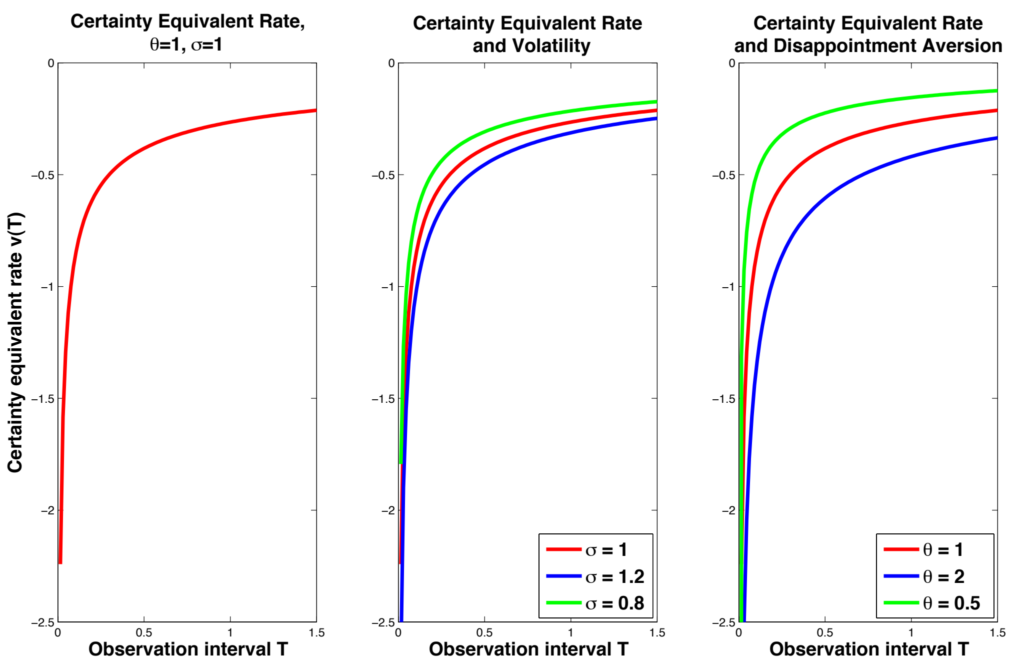

Figure 1: **Lottery values for a diffusion process: role of observation interval T, volatility σ and disappointment aversion θ.**

Figure 1 illustrates these results. We plot the instantaneous rate $v(T)$, as a function of the time interval T, across various values of the parameters θ and σ. When the frequency of information increases, the certainty equivalent sharply decreases, and the instantaneous rate becomes infinitively negative. This feature is specific to our framework and does not obtain with risk-neutral or CRRA preferences. Figure 1 also shows the certainty equivalent decreases with the volatility of payoffs and with the coefficient of disappointment aversion.

In what follows we characterize two interesting limits. We first ask what happens in the limit were the agent to observe information continuously. Then, we move to the other extreme where the agent observes the process extremely infrequently.

Continuous information limit We now study the limiting behavior of the certainty equivalent when frequency increases to infinity, and our framework converges towards continuous flows of information. As the period of observation T tends to 0, the instantaneous certainty equivalent rate $v(T)$ satisfies:

$$v(T) = \frac{-\kappa\left(\theta\right)\sigma}{\sqrt{T}} + o\left(-1/\sqrt{T}\right) \tag{5}$$

where $\kappa\left(\theta\right)$, the unique solution to:

$$1 = \theta\left(\frac{\Phi'(\kappa)}{\kappa} - \Phi(-\kappa)\right) \tag{6}$$

is positive, increasing in θ, with limit 0 in 0.

In particular, if $\theta > 0$:

$$\lim_{T\to 0} v(T) = -\infty$$

$$\lim_{T\to 0} V_0\left(T\right) = 0$$

As the frequency of information increases towards its continuous time limit, the value of the lottery converges to 0, the worst possible outcome for the final payoffs, with a faster convergence the higher the coefficient of disappointment aversion θ, or the underlying risk σ. Disappointment averse agents thus behave as though they were infinitely risk averse, when faced with continuous Brownian signals.

To understand this result, keep in mind that, even though the agent's time horizon does not change with the frequency of information, she evaluates her utility each time she observes a signal. At each arrival of news, she runs the risk of being disappointed, and, when the flow of news becomes continuous, the agent is almost surely disappointed in any time interval.

An alternative way to describe this phenomenon is through the lenses of the myopic risk aversion of Benartzi and Thaler (1995). The first-order risk aversion effect, inherent to preferences with kinks, results in agents who are more averse, comparatively, for small risks than for large risks. A frequent re-evaluation of the lottery value, when information arrives at small time intervals, corresponds to an accumulation of small risk taking. Because agents are first-order risk averse, a repetition of small risks is more costly for their utility than one large risk taking, and the lottery value decreases as the frequency increases. The results of Proposition 3.2 make formal this intuition.

Low frequency As the agent observes information at longer and longer intervals of time, her adjustment for information aversion decreases such that in the limit:

$$\lim_{T\to\infty} v(T) = 0 \tag{7}$$

Keep in mind, however, even at the infinite horizon limit, the agent never behaves as perfectly risk-neutral (she is disappointment averse, and thus risk averse), and the certainty equivalent $V_0(T)$, with payoff at horizon T, converges to $1/(1+\theta) < 1$.

3.3 Jump process

We now repeat our characterization of the certainty equivalent as a function of frequency for the case of a pure jump process. Even though the geometric Brownian motion and jump process have similar long-run behavior, their local evolution is sharply distinct. A Brownian motion is continuous, but has infinite variation on any time interval. In contrast, over the same time interval, a jump process is either constant with a given probability, or has large discontinuous changes. We show our information averse agents value these two processes very differently, most strikingly so in the limit case of continuous observation.

We keep the same type of lottery, but now assume the following law of motion for the process X_t:

$$\frac{dX_t}{X_{t-}} = \lambda\sigma dt - \sigma dN_t,$$

where N_t is the counting variable for a Poisson jump process with intensity λ, and $\sigma < 1$. At each realization of a jump, the current value of X_t is multiplied by $(1-\sigma)$. The expected instantaneous growth rate is zero.

Up to the compensating growth term, the distribution of the logarithm of the growth rate of the process after an interval T is Poisson, with parameter λT. We can therefore characterize the certainty equivalent.

Proposition 4. *The certainty equivalent of the lottery with payoff $X_{\mathcal{T}}$, at $t = \mathcal{T}$, determined by a jump process, is $V_0(T) = \exp(v(T)\mathcal{T})$, such that:*

$$\exp\left(Tv\left(T\right)\right) = \frac{1 - \frac{\theta}{1+\theta}\frac{\Gamma(k+1,(1-\sigma)\lambda T)}{k!}}{1 - \frac{\theta}{1+\theta}\frac{\Gamma(k+1,\lambda T)}{k!}}. \tag{8}$$

where $\Gamma(.,.)$ is the upper incomplete gamma function, and $k \in \mathbb{N}$ is the unique solution for:

$$\frac{\left(v\left(T\right) - \lambda\sigma\right)T}{\log\left(1-\sigma\right)} - 1 \leq k \leq \frac{\left(v\left(T\right) - \lambda\sigma\right)T}{\log\left(1-\sigma\right)}. \tag{9}$$

The certainty equivalent rate $v(T)$, for a lottery with final payoffs determined by a compensated geometric jump process, with negative jump size σ and intensity λ, is :

- increasing in the observation interval T

- decreasing in the jump size σ and the jump intensity λ
- decreasing in the loss aversion θ

As in the case of a Brownian motion, the certainty equivalent is increasing in the interval between observations, which we illustrate in Figure 2. Further, increases in loss aversion θ, the size of jumps σ or the intensity of jumps λ all correspond to decreases in the certainty equivalent: as risk increases, agents are less willing to observe information frequently.

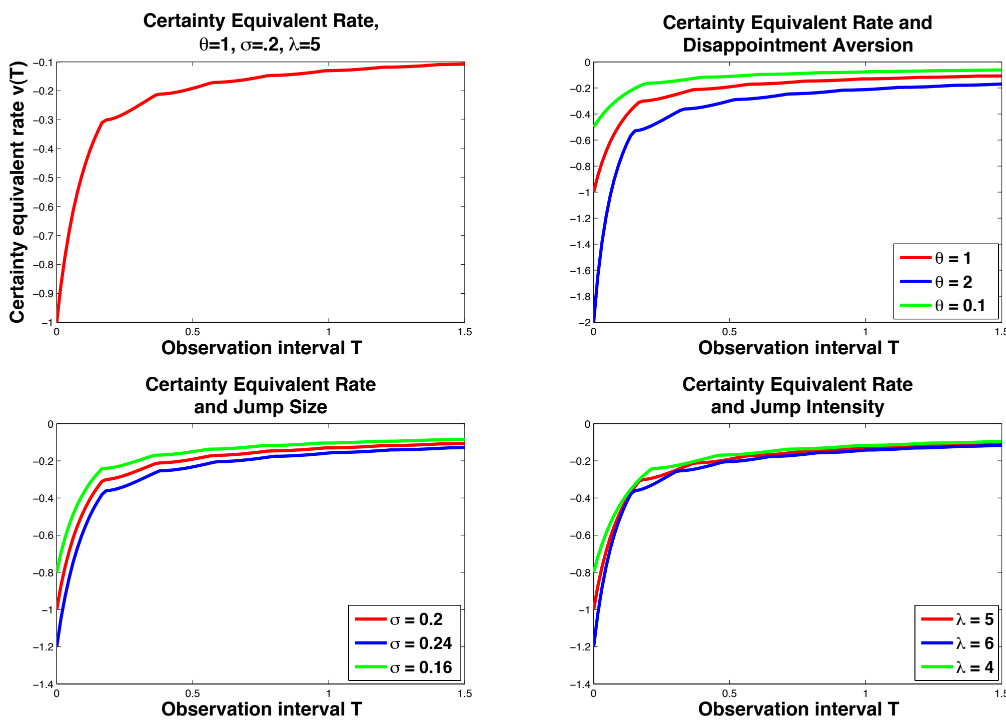

Figure 2: **Lottery values for a jump process: role of observation interval T, disappointment aversion θ, jump size σ and probability λ.**

All those results echo our previous analysis for the case of a Brownian motion. However, a striking difference can be noticed when looking at the limit when the observation interval converges towards 0. We now observe a strictly positive limit

Continuous information We can derive the limiting behavior of the certainty equivalent as we converge towards continuous information. As the observation interval T tends to 0, the certainty equivalent rate satisfies:

$$v(T) = -\theta\sigma\lambda + \mathcal{O}(T). \tag{10}$$

This result draws a sharp contrast between jump and diffusion processes. As the information flow becomes continuous, the agent behaves as if infinitely risk averse for the diffusion, but not for the jump process. The difference is intuitive. With continuous information under a diffusion process, the agent is constantly disappointed: in any interval of time, there is an infinity of negative draws of the process. Along the path of the jump process, there is only a finite number of negative draws and therefore of disappointment. The continuous flow of information is thus not as large a source of stress for the agent. Another way to comprehend this result is to think about the behavior of the certainty equivalent for risks over a small interval of time. For a brownian shock, the variation is localized closely to the certainty equivalent and the kink in preferences generates first-order risk aversion. In the case of a jump process, even for a small interval, the potential shocks are large. The kink in the preferences does not play as important a role as the distribution takes discrete values, both far from the kink.

This extreme differentiation in the limit is informative in terms of actual predictions. It suggests one should expect more inattention to signals for which the value moves continuously than those that display large sudden jumps. For instance, stock prices are subject to a lot of local variation, and our model implies, as is observed, most investors do not monitor them continuously. In contrast, while waiting for one important piece of news, agents are more willing to check regularly as the one-off nature of information limits its disutility impact.

Low frequency As in the case of the Brownian motion, in the limit:

$$\lim_{T \to \infty} v(T) = 0, \tag{11}$$

and $V_0(T)$ converges to a strictly less than one limit.

4 A consumption-savings problem

As shown in Section 3, observing signals is costly for information averse agents, the more so the higher the frequency of information. However, in most practical cases, agents must collect information so as to make appropriate choices. In this section, we focus on how agents optimally choose the frequency of observation in such situations. We study a portfolio problem, in which information is valuable in that it helps agents make better consumption and saving decisions. We study the tradeoff between the endogenous costs of information and its benefits, and characterize agents' optimal consumption, savings and information choices.

4.1 Setup

4.1.1 Preferences

We extend our definition of preferences to allow for intermediate consumption in a continuous time, infinite horizon framework. The value function $\mathcal{V}_t$ is defined as the limit:

$$\mathcal{V}_t = \lim_{\substack{\Delta t \to 0 \\ H \to \infty}} \mathcal{V}_t^{(H,\Delta t)}.$$

where, for a discrete time problem with time step Δt and horizon H, the value function $\mathcal{V}_t^{(\Delta t,H)}$, is solution to the recursion:

$$\frac{\left(\mathcal{V}_t^{(\Delta t,H)}\right)^{1-\alpha}}{1-\alpha} = \frac{C_t^{1-\alpha}}{1-\alpha}\Delta t + (1-\rho\Delta t)\frac{\left(\mu_\theta\left[\mathcal{V}_{t+\Delta t}^{(\Delta t,H)}|\mathcal{F}_t\right]\right)^{1-\alpha}}{1-\alpha}$$

with final condition:

$$\frac{\left(\mathcal{V}_H^{(\Delta t,H)}\right)^{1-\alpha}}{1-\alpha}=\frac{C_H^{1-\alpha}}{1-\alpha}.$$

and $\{C_t\}_{t\in[0,\infty[}$ is a consumption process adapted to a filtration $\{\mathcal{F}_t\}_{t\in[0,\infty[}$. This construction consists in adding the disappointment averse certainty equivalent operator $\mu_\theta(.)$ to an isoelastic specification of preferences. The parameter $\alpha>0$ controls the elasticity of intertemporal substitution between consumption at different times. The parameter $\rho>0$ controls the rate of time discount. The only source of instantaneous risk aversion is the disappointment aversion mechanism.

In our framework, we will show the agent optimally chooses deterministic consumption plans over intervals of length T along which no information is revealed, in which case the recursion for the instantaneous value function, $\mathcal{V}_t$, takes the simple form:

$$\frac{\mathcal{V}_t^{1-\alpha}}{1-\alpha}=\int_0^T e^{-\rho\tau}\frac{C_{t+\tau}^{1-\alpha}}{1-\alpha}d\tau+e^{-\rho T}\frac{\left(\mu_\theta\left[\mathcal{V}_{t+T}|\mathcal{F}_t\right]\right)^{1-\alpha}}{1-\alpha}. \tag{12}$$

This formula illustrates in a clear fashion the ingredients of the preferences: in the periods between observations of signals, intertemporal consumption choices are deterministic. When information is revealed, the continuation values are adjusted downwards using the disappointment averse certainty equivalent.

4.1.2 Opportunity sets

Investment opportunity set The agent has access to two investment accounts to allocate her wealth W_t over time. She can invest some of her wealth in a risk-free asset at a constant continuously compounded interest rate $r<\rho$, and/or she can place an investment in a risky asset with cumulative returns determined by the growth of a stochastic process X, with same properties as in Section 3. Note S_t the number of shares of the risky assets the agent owns at date t. The agent allocates the rest of her wealth between her instantaneous consumption needs, C_t, and the risk-free asset. The budget constraint is:

$$dW_t = -C_t dt + S_t dX_t + r(W_t - S_t X_t)dt$$

The agent can rebalance her wealth across assets at all time, at no transaction cost. We, however, do not allow for borrowing, so the agent must always ensure $W_t \geq 0$.

Information choice We allow for one specific information structure: at any time t, the agent must choose either to receive no information, or to observe the full value of her risky portfolio. She cannot receive partial signals related to the value of her wealth. In between observations, she is unaware of the exact value of her wealth, and makes her decision based on past information. Note this assumption does not correspond to limiting the cognitive ability of the agent, nor to assuming non-bayesian updating. The agent can always choose to access and process the maximal information available in the economic environment, and her expectations are driven by a standard probabilistic filtration.

Formally, noting $\{\bar{\mathcal{F}}_t\}$ the filtration generated by the process $\{X_t\}$ (appropriately completed) and $\{\mathcal{F}_t\}$ that of the agent, the constraint on information is:

$$\forall t, \mathcal{F}_t = \bar{\mathcal{F}}_{\tau(t)},$$

$$\tau(t) \leq t, \text{ increasing, càdlàg}$$

4.1.3 Optimization problem

Gathering the assumptions above, we define the optimization problem, given initial wealth W:

$$\max_{\{\tau(t), C_t, S_t\}_{t \in [0,\infty]}} \mathcal{V}(W)$$

$$\mathcal{F}_t = \bar{\mathcal{F}}_{\tau(t)}, \tau(t) \leq t, \ \tau(.) \text{ increasing càdlàg,}$$

$$(C_t, S_t) \ \mathcal{F}_t - \text{measurable}$$

$$dW_t = -C_t dt + S_t dX_t + r(W_t - S_t X_t) dt$$

$$W_0 = W, \ W_t \geq 0$$

where $\mathcal{V}$ is defined as in Equation (12).

Note the value function is homogenous of degree 1, and the opportunity set is linear in the total wealth. Let $\mathcal{V}(W_t)$ be the value function for total wealth W_t right after an observation of the current information. We can rewrite:

$$\mathcal{V}(W_t) = W_t \mathcal{V}_0 \tag{13}$$

where, because of the i.i.d growth of the risky asset, $\mathcal{V}_0$ is a constant, independent from the value of wealth, to be determined.

This result highlights information acquisition will optimally happen at constant time intervals. Indeed, at each observation, only the value of wealth changes, while the optimization problem for $\mathcal{V}_0$ remains the same.

Note also the adjustment for disappointment aversion of the continuation value, for a given wealth W_t, is given by: $\mu_\theta(\mathcal{V}(W_t)) = \mathcal{V}_0 \mu_\theta(W_t)$.

The agent does not make any change to her risky investments between observations. Indeed, because she is not allowed to hold negative positions on any investment, she cannot divest from her risky portfolio without first observing if she has sufficient funds. The restricted information choices she has (either no information, or full information) imply she has to observe the full value of her wealth before she can reduce her risky position. We also find, as we clarify later, she never optimally chooses to increase her risky position between observations.

The agent's optimization thus simplifies to the following Bellman problem, starting from initial wealth equals one:

$$\frac{\left(\mathcal{V}_0\left(T\right)\right)^{1-\alpha}}{1-\alpha} = \max_{T,S_0,\{C_t\}_{t=0}^{T}} \int_0^T e^{-\rho\tau}\frac{C_\tau^{1-\alpha}}{1-\alpha}d\tau + e^{-\rho T}\frac{\left(\mathcal{V}_0\left(T\right)\mu_\theta\left[W_T\right]\right)^{1-\alpha}}{1-\alpha} \tag{14}$$

$$\text{s.t.} \int_0^T e^{-r\tau}C_\tau d\tau = \mathcal{C}_0$$

$$S_0 + \mathcal{C}_0 < 1,\ S_0 > 0,\ \mathcal{C}_0 > 0$$

$$W_T = (1-\mathcal{C}_0 - S_0)\exp(rT) + S_0\frac{X_T}{X_0}.$$

T is the optimal time interval until next observation. $\mathcal{C}_0$ is the amount put in safe assets strictly to finance consumption between $t=0$ and $t=T$, i.e. until the next observation. S_0 is the amount invested in the risky asset until next observation. The remaining initial wealth, $1-\mathcal{C}_0-S_0$ is invested in the safe asset.

The recursive structure of the opportunity set and preferences guarantees time consistency in the optimal policy.

4.2 Optimal decisions

We start by deriving the optimal consumption policy between observation, for a given $\mathcal{C}_0$. We then focus on the savings policy from one observation to the next, and finally obtain the agent's optimal strategy.

4.2.1 Consumption between observations

Take as given the interval T between observations and the amount $\mathcal{C}_0$ put aside for consumption during this interval. The optimal consumption policy solves:

$$\max_{\{C_t\}_{t=0}^T} \quad \int_0^T e^{-\rho t} \frac{C_t^{1-\alpha}}{1-\alpha} dt$$

$$\text{s.t.} \int_0^T e^{-rt} C_t dt \leq \mathcal{C}_0$$

This problem admits the unique solution:

$$\forall t \in [0, T], \; C_t = \mathcal{C}_0 e^{-\frac{\rho - r}{\alpha} t} \frac{\frac{\rho}{\alpha} - \frac{1-\alpha}{\alpha} r}{1 - e^{-\left(\frac{\rho}{\alpha} - \frac{1-\alpha}{\alpha} r\right) T}}. \tag{15}$$

Consumption is proportional to $\mathcal{C}_0$, the amount initially put aside, and decays at rate $(\rho - r)/\alpha$. The decay rate reflects the tradeoff between time discount ρ and interest gains r, when the elasticity of substitution across periods is determined by α.

4.2.2 Optimal savings portfolio

Take as given the interval T between observations, and the amount $\mathcal{C}_0$ put aside for consumption during this interval. The optimal savings policy solves:

$$\max_{S_0} \quad \mu_\theta \left[W_T \right]$$

$$\text{s.t.}$$

$$W_T = (1 - \mathcal{C}_0 - S_0) \exp(rT) + S_0 \frac{X_T}{X_0}$$

$$0 \leq S_0 \leq 1 - \mathcal{C}_0$$

The investment in the safe asset has deterministic returns, so, using the linearity of the certainty equivalent μ_θ with respect to constants, we find:

$$\mu_\theta \left[W_T \right] = (1 - \mathcal{C}_0 - S_0) \exp(rT) + S_0 \mu_\theta \left[\frac{X_T}{X_0} \right]$$

and the optimisation problem admits a corner solution. In the notations of Section 3,

$$\mu_\theta \left[\frac{X_T}{X_0}\right] = \exp\left(v\left(T\right)T\right),$$

where $v(T)$ is the certainty equivalent rate, when observing the stochastic process X at interval T.

The optimal savings solution is:

$$\begin{cases} S_0 = 1 - \mathcal{C}_0 & \text{if } v(T) > r \\ S_0 = 0 & \text{if } v(T) \leq r. \end{cases} \tag{16}$$

This proposition provides some insight regarding the link between risk-taking decisions and information decision. In both examples we considered for the return process X, the agent perceives the asset as offering higher returns as the observation interval becomes longer: $v(.)$ is an increasing function of T. As T becomes infinitively large, $v(T)$ converges to μ, the instantaneous growth rate of X. Therefore, as long as $\mu > r$, there are observation intervals that make investing in the risky asset, rather than in the safe asset, optimal.

4.2.3 Optimal frequency and savings

We now characterize the optimal consumption $\mathcal{C}_0$ and observation interval T.

Suppose investing in the risky asset is never optimal, at any observation interval T (case $\mu \leq r$). Then, trivially, $\mathcal{C}_0 = 1$, and any $T \geq 0$ is an optimal solution. The agent only uses the risk-free asset and is completely unaffected by the information flow.

We focus on the more interesting case in which the investor optimally decides to invest in the risky asset. We show, for both the Brownian and jump cases, it is necessary and sufficient to have $\mu > r$.

Proposition 5. *For a given time interval T between observations, the opti-*

mal consumption and value function are given by:

$$\mathcal{C}_0\left(T\right) = 1 - \exp\left[\left(-\frac{\rho}{\alpha} + \frac{1-\alpha}{\alpha} v(T)\right) T\right] \tag{17}$$

$$\left(\mathcal{V}_0\left(T\right)\right)^{\frac{1-\alpha}{\alpha}} \left(\frac{\rho}{\alpha} - \frac{1-\alpha}{\alpha} r\right) = \frac{1 - \exp\left[\left(-\frac{\rho}{\alpha} + \frac{1-\alpha}{\alpha} r\right) T\right]}{1 - \exp\left[\left(-\frac{\rho}{\alpha} + \frac{1-\alpha}{\alpha} v(T)\right) T\right]} \tag{18}$$

The portfolio problem, across observations, is equivalent to having standard isoelastic utility and a deterministic rate of return $v(T)$. Hence optimal consumption takes the simple form of Equation (17). In particular, current consumption is increasing in the rate of return $v(T)$ if and only if the elasticity of intertemporal substitution $1/\alpha$ is lower than 1. In that case the income effect dominates: facing a better opportunity set the agent consumes more immediatly. Conversely, when $1/\alpha > 1$ the substitution effect dominates: the agent pushes her consumption towards the future.

We now maximize the value function by taking the first-order condition for the optimal frequency and obtain our main result.

Proposition 6. *If the instantaneous growth rate of the risky investment asset is greater than the risk-free rate, $\mu > r$, the agent's optimal strategy for information is to observe the value of her risky portfolio at constant time intervals, of length T^*, where T^* is the unique solution to:*

$$\begin{aligned} &\frac{\partial v(T)}{\partial \log(T)} f\left(v\left(T\right) - \frac{\rho}{1-\alpha}\right) = \\ &\left(r - \frac{\rho}{1-\alpha}\right) f\left(\left(r - \frac{\rho}{1-\alpha}\right)\right) - \left(v\left(T\right) - \frac{\rho}{1-\alpha}\right) f\left(v\left(T\right) - \frac{\rho}{1-\alpha}\right) \end{aligned} \tag{19}$$

where $f\left(x\right) = \exp\left(\frac{1-\alpha}{\alpha} xT\right) / \left(1 - \exp\left(\frac{1-\alpha}{\alpha} xT\right)\right)$.

The optimal length of time interval T^* verifies $v(T^*) > r$.

This condition characterizes the tradeoff agents face when deciding the frequency at which they observe the value of the risky asset. The right-hand side of Equation (19) is the marginal cost of increasing the interval T: the cost of financing some of the consumption stream at the risk-free rate r rather than at the superior risky rate $v(T)$. This marginal cost is increasing in the spread between $v(T)$ and r, and therefore in the observation interval T, and is standard to models with infrequent transactions à la Baumol-Tobin.

The novelty of our approach is to make the marginal benefit to inattention, the left-hand side of Equation (19), endogenous, rather than exogenously determined by a fixed cost of information. When observing returns less often, the agent considers the risky asset to be more attractive. The key quantity to determine the marginal benefit of increasing the time period between observations is the elasticity of the certainty equivalent rate $v(T)$, with respect to the observation interval T: $\partial v(T) / \partial \log(T)$. In both of our examples this elasticity is decreasing in the time interval T.

The behavior of this elasticity, and thus of the marginal benefit to increasing the time interval between observations, characterizes how the optimal policy responds to changes in the economic environment. In the next section, we derive such comparative statics for the case of a geometric Brownian motion.

4.3 With Brownian Risk

Consider the case of a geometric Brownian motion, with drift μ and volatility σ, as in Section 3. We suppose $\mu > r$, and the intertemporal elasticity of substitution, $1/\alpha$, larger than 1.

Figure 3 illustrates the behavior both of the value function and of the share of wealth allocated to the consumption account, as functions of the observation frequency. It is infinitely costly to hold the risky asset when the flow of information is continuous, and the value function $\mathcal{V}_0(T)$ converges to 0 when T goes to zero: for small values of the time interval, increasing the space between observations increases the value function. On the other hand, when the interval between observations becomes very large, the agent has to put aside more and more cash for consumption, which prevents her from fully benefiting from her investment opportunity in the risky asset. We find, for large time intervals, increasing the space between observations decreases the value function.

As investment opportunities deteriorate, and/or the agent is more disappointment averse, the value function becomes lower: the value function is decreasing in the risk σ and disappointment aversion θ, whereas it is increasing in the growth rates μ and r.

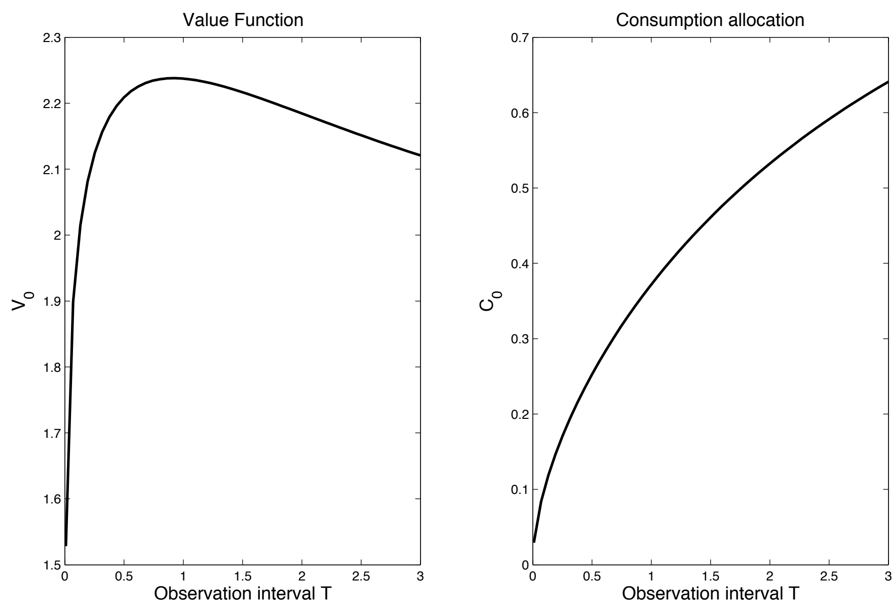

Figure 3: **Utility as a function of T, the time interval between observations**. For parameters values: $\theta = 1, \alpha = 0.5, \sigma = 1, \mu - r = 1, \rho = 0.1$.

We turn to the behavior of the unique optimal observation interval T^*. In particular we are interested in the behavior of this interval with respect to the volatility of risky asset returns, σ, and with respect to the disappointment aversion parameter, θ. Focusing first on the risk, σ, note both the cost and benefit of a higher observation interval are affected. Because the agent is risk averse, increasing σ makes the risky asset less attractive, so the agent has an incentive to move her holding towards the risk-free asset, and to lower the frequency of observation. This effect is standard to information costs model. In our model a second mechanism obtains: the marginal benefit of a higher time interval increases with the underlying risk: the elasticity of the certainty equivalent rate to the observation interval is increasing in the volatility σ. This second force also pushes the agent to increase the observation frequency. Figure 4 illustrates these results. Similar results obtain for the parameter of disappointment aversion θ.

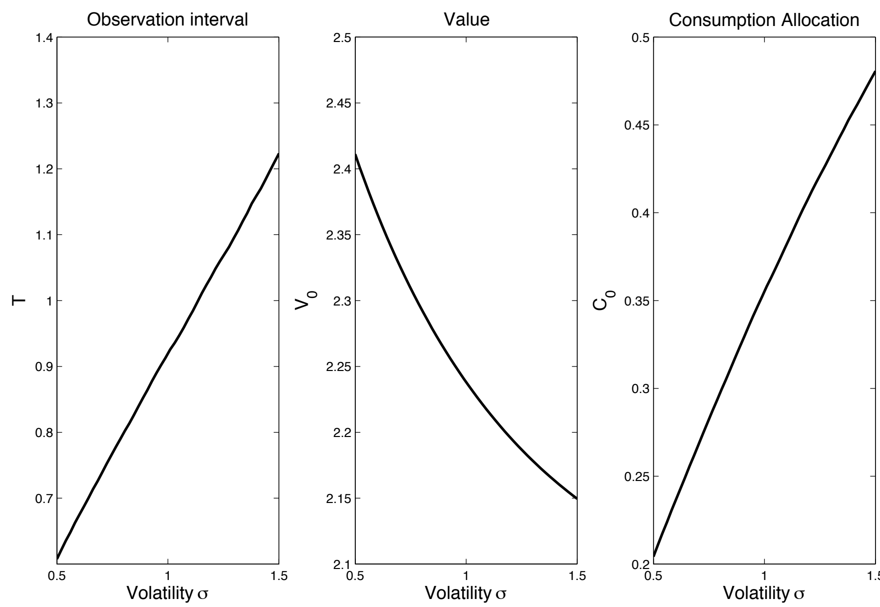

Figure 4: **Optimal frequency as a function of volatility** σ. For parameters values: $\theta = 1, \alpha = 0.5, \mu - r = 1, \rho = 0.1$.

Importantly, even in cases where the constant growth rate μ is augmented to directly compensate for an increase in the underlying risk σ, the net force towards longer time intervals is not suppressed. Suppose one increases μ to fully compensate for the increase in σ, in such a way that the instantaneous certainty equivalent rate, $v(T)$, remains unchanged at the initial optimal time interval T. This leaves the marginal cost of increasing T, which depends on

$v(T)$ only, unchanged. The marginal benefit of increasing T, which depends on the elasticity of the certainty equivalent rate, increases, on the other hand. Therefore, even if compensated for by an increases expected growth, an increase in volatility results in increased inattention. This result can provide a rationalization of the "ostrich" effect documented by Karlsson et al. (2009). To summarize those results, the elasticity of the certainty equivalent rate, $\partial v(T)/\partial \log(T)$ is

- decreasing in the observation interval T,
- increasing in the volatility σ and the disappointment aversion θ;

the optimal time interval T^* is

- increasing in σ and θ, even if compensated by a higher growth rate μ,
- decreasing in the growth rate μ.

5 Extensions

In section 4, we analyzed the portfolio choices of an investor who needs to optimally balance the costs of information, endogenously determined by her disappointment aversion, with the benefits of informed consumption and savings decisions. We showed, even in this relatively simple optimization problem, the endogenous information cost structure of our framework provides novel implications that are supported by the empirical evidence regarding the frequency of information acquisition in risky environments. This is but one example in which information aversion impacts and modifies the optimal decision making of agents facing uncertainty, relative to models with exogenous information costs or constraints. Our approach has pervasive implications for most decisions under uncertainty. In this section, we revisit several classic questions and stress the novel tradeoffs present with information averse investors.

5.1 Diversification and the multiplication of information flows

A robust insight of portfolio theory is that diversification is valuable. When presented with two assets with imperfectly correlated returns, it is optimal to invest in both. Because our disappointment averse agents are risk-averse, the rationale for diversification obtains. However, in our framework, not only does the distribution of the final payoffs matter, but also the structure of the information flow. It is plausible to think investing in a larger number of assets corresponds to more frequent arrivals of information. For instance, suppose an agent is faced with news reports at given intervals (daily headlines from national newspapers for instance), and she is invested in both tech stocks and automobile stocks. As long as the news report pertains to either one of those sectors of the economy, she will observe information relevant to the value of her portfolio. In contrast, if she were invested only in tech stocks (or automobile stocks), fewer news reports would have information content regarding her wealth. As disappointment averse agents fear high frequency information flows, this latter force might diminish and even overcome the benefit of diversification.

To characterize this tradeoff between the costs and benefits of diversification within our model, we focus on a simple example. Suppose the agent receives at date 1 the final value $\lambda X_1^{(1)} + (1-\lambda) X_1^{(2)}$, where the processes $X_t^{(1)}$ and $X_t^{(2)}$ are two arithmetic Brownian motions with volatility σ and correlation ρ. The fraction $\lambda \in [0,1]$ can be thought of as a portfolio share. We assume the agent observes alternatively the current value of each of the processes at intervals of length T. In the appendix, we derive a simple expression for the certainty equivalent of this lottery.[5]

Figure 5 represents the certainty equivalent of this lottery, as a function of the horizon, for three different cases. We consider (a) the case of investing

[5]Extending the horizon, as well as allowing for drifts, is easily done, as before.

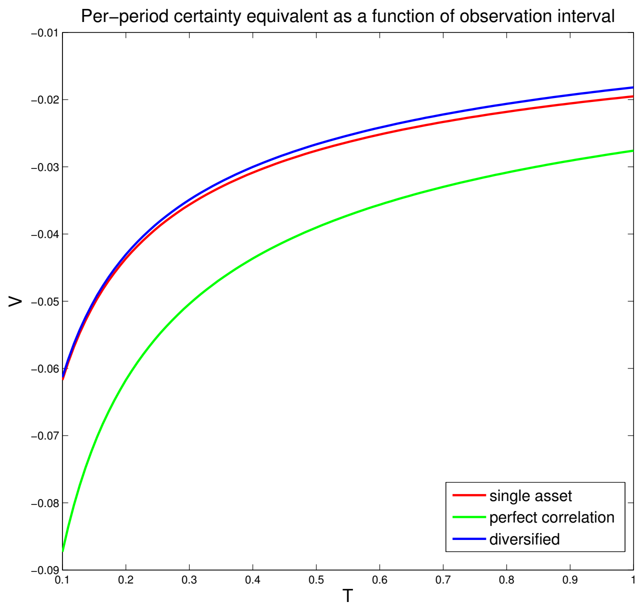

Figure 5: **Role of diversification**. Certainty equivalent as a function of the observation interval for a single asset portfolio (red), an equal-weight perfectly correlated portfolio (green), and an equal weight independent payoff portfolio (blue). Parameters values: $\theta = 1, \sigma = 0.1$.

in only one asset ($\lambda = 0$), (b) investing in two perfectly correlated assets ($\lambda = 50\%$, $\rho = 1$), and (c) investing equally in two fully independent assets ($\lambda = 50\%$, $\rho = 0$). Situations (b) and (c) allow us to decompose the impact of diversification between i) the risk reduction, and ii) the increase in information flows. Investments (a) and (b) share the exact same payoff structure, but have different information flows, with case (b) corresponding to observing the current value of the payoff process twice as frequently as case (a). Not surprisingly, the certainty equivalent of this "diversified" lottery is always lower than the undiversified one. Moving from case (b) to case (c), we now keep the frequency of information constant, with same interval $2T$ between observations for each individual asset, allow for diversification across the risks of the two assets, and find the certainty equivalence is now higher.

In this example, the net effect of full diversification is unambiguous: the agent always prefers the diversified portfolio with frequent information (c) over the portfolio with one asset and infrequent information (a). Further, the relative advantage of the diversified portfolio is increasing in the observation interval T. As the information flow becomes more infrequent, the relative risk of the two portfolios becomes the main concern, and the diversified portfolio dominates more and more the single-asset portfolio. For a continuous observation flow, on the other hand, we can show, at the limit, the two portfolios have same value. Indeed, the per observation certainty equivalent rate is approximately proportional to $-\sigma\sqrt{T}$, a first-order risk correction. The lottery value is therefore proportional to $-\sigma/\sqrt{T}$. The diversified portfolio has twice as many observations, but a variance divided by $\sqrt{2}$, so these two effects cancel out perfectly. In numerical calculations, we show this convergence is true for arbitrary values of λ. This continuous time limit delivers further intuition as to why the fully diversified portfolio always dominates, for $T > 0$.

To summarize, even when increasing the frequency of information flows, diversification is valuable in our framework, albeit with limited benefits. The correlation across assets plays a crucial role. Indeed if investing in more assets multiply the frequency of information without providing a complete diversification benefit, the agent might prefer to hold only a limited number of assets. Our framework could thus rationalize the under-diversification observed in the data.

5.2 Incentives for delegation

Another natural deviation from the core example of our paper would be to allow the agent to choose her information flows from a richer set of options than the all-or-nothing framework we used so far (the agent either fully observes her wealth or receives no information whatsoever). Consider again an investor with continuous consumption and same investment opportunities, risk-free bonds or risky asset, as before. She might be willing to forego a portion of her wealth if it allowed her to access useful information at a lower endogenously implied cost. For instance, rather than having to observe the full value of her portfolio each time she transfers funds to her consumption account, she could have an "alert" system that would be triggered only when her wealth reaches a certain value. Such a framework would significantly reduce the amount of information she is forced to receive and could be preferable to her.

Allowing the agent to optimize on the structure, or the type, of information she receives could provide a very natural rationale for delegation. Consider again the example just given: the "alert" system of information presupposes the existence of an intermediary who has access to the full information on the investor's risky assets, and who translates this continuous information flow into discrete time trigger signals. Of course, the intermediary would need to be compensated for the service provided. And, precisely because the investor pays the intermediary to hide information from her, agency problems quite naturally arise.

Enriching the model to let the agent optimally choose the flow and quality of the signals she observes intuitively results into settings with incentives for delegation. We leave for future research a more careful analysis of desirable contracts in such settings

5.3 The joint role of payoff distribution and information structure for investment decisions

So far, we focused mainly on the implications of information aversion for the time series of investor's decisions, with results on the frequency of signals in i.i.d environments. Our endogenous cost structure also informs us on the joint role of the distribution of payoffs and intermediate signals, for an agent who has to choose among a cross-section of investment projects. Indeed information averse agents care not only about the quantity of information, but also about how it shapes the conditional distribution of final payoffs.

As an illustration, consider the following simple setup. At date 0 an agent invests in a project, that can succeed (outcome G), with probability p, or fail (outcome B), with probability $1-p$, at date 2. We can assume, without loss of generality, the certainty equivalent of the project is 0.[6] The agent automatically observes a binary signal at date 1, and gets more informed about the quality of the project: with equal probability, the agent observes a good or a bad signal, such that the probability of the good outcome G becomes $p+x$ and $p-x$ respectively. x, which controls the strength of the signals, satisfies the constraint: $0 \leq x \leq 1-p$ and $0 \leq x \leq p$.

Because the agent is disappointment averse, observing the intermediate signal comes at a cost, except in the corner case $x=0$. However, let's allow for the agent to benefit from observing it: she can costlessly divest a portion α of her initial investment following a bad signal at time $t=1$.[7] At time $t=0$, the agent can choose among a cross-section of investment projects that vary in the quality, x, of the intermediate information. We can easily derive the following results:

1. If $\alpha > \frac{\theta}{1+\theta}$, the agent optimally chooses investments with the highest quality of intermediate information, $x = \min(p, 1-p)$

2. If $\alpha < \frac{\theta}{1+\theta}$, the agent optimally chooses investments with no interme-

[6] This corresponds to the constraint:

$$\mu_0 = \frac{pG + (1+\theta)(1-p)B}{1+\theta(1-p)} = 0$$

[7] We obtain the certainty equivalent of the project with partial information at time 1:

$$\tilde{\mu}_0 = \frac{x(G-(1+\theta)B)}{2+\theta}\left[\frac{(1+\theta(1-p))(1-(1+\theta)(1-\alpha)) + \theta x(1+(1+\theta)(1-\alpha))}{(1+\theta(1-p))^2 - \theta^2 x^2}\right]$$

diate information, $x = 0$, unless the uncertainty at time $t = 0$ is high and such that

$$p_1^* \leq p \leq 1 - p_2^*,$$

where, when $\alpha > 0$,

$$\frac{1}{2} > p_1^* = \frac{(1-\alpha)(1+\theta) - 1}{2(1-\alpha)\theta} > p_2^* = \frac{(1-\alpha)(1+\theta) - 1}{2\theta},$$

in which case, she chooses to observe the most possible intermediate information ($x = \min(p, 1-p)$).

In the first case, $\alpha > \frac{\theta}{1+\theta}$, the divestment option is high, and the agent is not too disappointment averse, so the benefits of information always outweigh its costs. In the second case, the agent is either highly disappointment averse, or has little divestment opportunity after observing information, in which case she chooses investments with intermediate signals only if her initial uncertainty is high (p around $\frac{1}{2}$). The set of probabilities p in which the agent chooses to observe the intermediate signals shrinks as θ increases (higher information aversion), and as α decreases (lower benefit of information). Strikingly, because $p_1^* > p_2^*$, information averse agents in this simple optimization problem are more likely to optimally observe intermediate signals when they are optimistic about the investment project, $p \geq 1/2$ (left-skewed distribution), than when they are pessimistic about the project, $p \leq 1/2$ (right-skewed distribution). These results complements our previous results: information averse agents dislike information even more in risky environments.

The somewhat complex results we obtain for the agent's optimal strategies in this very simple framework indicate information aversion should have rich implications for the analysis of investment strategies across projects that vary in their risk and information structures.

6 Conclusion

Because they run the risk of being disappointed each time they receive a signal, disappointment averse agents are intrinsically *information averse*. We propose a theory of inattention solely based on these preferences, absent any cognitive limitations, or external costs of acquiring information. We start by characterizing the strength and properties of the endogenous costs of information, implied by this model of preferences, and find them to differ fundamentally from both the cognitive constraints, and the exogenous costs commonly used in the inattention literature. We focus our analyzis on the impact of the frequency of information observations on the certainty equivalents of lotteries whose payoffs correspond to the final value of a stochastic process. We find agents behave as if infinitely risk averse when the flow of information becomes continuous, in the case of a diffusion, but not in the case of jumps. This result informs us that one should expect more inattention to information on smooth risky processes than to information on sudden large changes in utility. In both cases, the cost of information increases as risk increases or disappointment aversion increases. We then study how agents balance the utility cost of paying attention to the economic environment with the benefits of making informed decisions, in the case of a standard consumption-savings problem. In this setting, we find attention decreases in turbulent times: when there is more risk, information is more stressful. This endogenous cost-driven result is unique to our model of inattention, and is supported by the empirical evidence. Finally, we outline how information aversion impacts a number of decisions under uncertainty: i) information aversion can significantly reduce the benefits of diversification; ii) it creates a rationale for delegations; iii) it affects the evaluation and choice of investment projects. The simplicity of our approach, combined with its pervasive implications for decision under uncertainty, suggest a large avenue for future research, both to further clarify the theoretical predictions of this model, and to explore its empirical implications.

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A Disappointment Aversion and Information Aversion

Focus on a setup with three dates: 0, 1, and 2. At date 2, the agent receives a final outcome X with cumulative distribution function F. The certainty equivalent under disappointment aversion with linear realized utility and coefficient θ is $\mu(F)$. Define:

$$h(\mu) = \int_{x \geq \mu} (x - \mu)\, dF(x) + (1+\theta) \int_{x<\mu} (x-\mu)\, dF(x)$$

The function h is continuous, decreasing in μ. It admits limit $+\infty$ when μ tends to $-\infty$ and $-\infty$ when μ tends to $+\infty$. There exist as unique zero, the certainty equivalent $\mu(F)$.

If, at date 1, the agent receives a signal $i \in \{1, N\}$ with probability α_i, the agent updates her belief on the distribution of X from F to F_i. We are interested in comparing the certainty equivalent at date $t = 0$ of the compound lottery with date $t = 1$ signals, $\mu(\{F_i, \alpha_i\})$ with that of a lottery without intermediate signal, $\mu(F)$. Naturally, the distribution of final outcomes is the same for both lotteries: $F = \sum_i \alpha_i F_i$.

For all $i \in \{1, N\}$, the function

$$h_i(\mu) = \int_{x \geq \mu} (x - \mu)\, dF_i(x) + (1+\theta) \int_{x<\mu} (x-\mu)\, dF_i(x)$$

admits $\mu(F_i)$ as a unique zero. To simplify notations, we write $\mu(F_i) = \mu_i$ from now on.

Also, keep in mind $\mu(\{F_i, \alpha_i\})$ is the unique zero of

$$h_s(\mu) = \sum_{\mu_i \geq \mu} \alpha_i (\mu_i - \mu) + (1+\theta) \sum_{\mu_i < \mu} \alpha_i (\mu_i - \mu)$$

We write the certainty equivalent with intermediate signal $\mu(\{F_i, \alpha_i\}) = \mu_s$.

Let us compute $h\left(\mu\left(\{F_i,\alpha_i\}\right)\right)$:

$$\begin{aligned}
h\left(\mu_s\right) &= \int \left(x-\mu_s\right) dF\left(x\right) + \theta \int_{x<\mu_s} \left(x-\mu_s\right) dF\left(x\right) \\
&= \sum_i \alpha_i \left[\int \left(x-\mu_s\right) dF_i\left(x\right) + \theta \int_{x<\mu_s} \left(x-\mu_s\right) dF_i\left(x\right)\right] \\
&= \sum_i \alpha_i \left[\left(\mu_i-\mu_s\right) + \theta \int_{x<\mu_i} \left(\mu_i-x\right) dF_i\left(x\right) + \theta \int_{x<\mu_s} \left(x-\mu_s\right) dF_i\left(x\right)\right] \\
h\left(\mu_s\right) &= \theta \sum_{\mu_i<\mu_s} \alpha_i \left[\left(\mu_s-\mu_i\right) \int_{x\geq\mu_s} dF_i\left(x\right) + \int_{\mu_i\leq x<\mu_s} \left(x-\mu_i\right) dF_i\left(x\right)\right] \qquad (20) \\
&\quad + \theta \sum_{\mu_i\geq\mu_s} \alpha_i \left[\left(\mu_i-\mu_s\right) \int_{x<\mu_s} dF_i\left(x\right) + \int_{\mu_s\leq x<\mu_i} \left(\mu_i-x\right) dF_i\left(x\right)\right]
\end{aligned}$$

Observe all the terms on the right-hand side are positive, so that

$$h\left(\mu\left(\{F_i,\alpha_i\}\right)\right) \geq 0$$

Remember h is decreasing with $\mu\left(F\right)$ as its unique zero. Therefore we can conclude

$$\mu\left(\{F_i,\alpha_i\}\right) \leq \mu\left(F\right)$$

Let us now analyze under which condition $\mu\left(\{F_i,\alpha_i\}\right) = \mu\left(F\right)$, i.e. under which condition $h\left(\mu\left(\{F_i,\alpha_i\}\right)\right) = 0$. From equation 20, it is straightforward that if i_0 is such that $\mu_{i_0} = \mu_s$ then the positive terms in α_{i_0} are equal to zero. Suppose there is $j \in \{1, N\}$ such that $\mu_j \neq \mu_s$. If $\mu_j < \mu_s$, the positive contribution to $h\left(\mu\left(\{F_i,\alpha_i\}\right)\right)$ of the j term is:

$$\left(\mu_s-\mu_j\right) \int_{x\geq\mu_s} dF_j\left(x\right) + \int_{\mu_j\leq x<\mu_s} \left(x-\mu_j\right) dF_j\left(x\right)$$

The first term is zero iff $\forall x \geq \mu_s, F_j\left(x\right) = 0$, i.e. in the F_j distribution, all outcomes are below μ_s. Supposing that is the case, let us analyze the second term. From $\mu_j \leq \int x dF_j\left(x\right)$, we know the interval $\mu_j \leq x < \mu_s$ is not empty. Under these conditions, the second term $\int_{\mu_j\leq x<\mu_s} \left(x-\mu_j\right) dF_j\left(x\right)$ is null if and only if $x = \mu_j$, and the lottery under signal j is degenerate: F_j admits a unique non-zero, μ_j. A similar result obtains if $\mu_j > \mu_s$.

We have thus proven the result:

$$\mu\left(\{F_i, \alpha_i\}\right) = \mu\left(F\right)$$
$$\Leftrightarrow \forall i, \begin{cases} \mu\left(F_i\right) = \mu\left(F\right) \text{ or} \\ F_i \text{ is degenerate} \end{cases}$$

Finally, we prove using this result that for any level of mutual entropy at the first stage of the lottery, there exists a compounded lottery that provides as much utility as one-shot resolution. Indeed, consider the lottery that reveals the final outcome with probability p or nothing with probability $1 - p$. Clearly, such lottery satisfies the conditions above and is equivalent to one-shot resolution. One can choose p to attain any level of mutual entropy between the first stage outcome and the final outcome.

B Certainty Equivalent Rate

To be consistent with the notations in the body of the paper, note $V\left(T\right)$ the certainty equivalent at time 0 of a lottery with uncertain payoff X_T at $t = T$, starting from $X_0 = 1$, and no intermediate observation. Then we write

$$V\left(T\right) = \exp\left(v\left(T\right)T\right)$$

where $v\left(T\right)$ is the certainty equivalent rate.

B.1 Brownian motion

Assume

$$\frac{dX_t}{X_t} = \sigma dZ_t$$

so the log payoff is $\log\left(X_T\right) = -\frac{1}{2}\sigma^2 T + \sigma\sqrt{T}\epsilon$, where ϵ is distributed $\mathcal{N}\left(0,1\right)$. The certainty equivalent of payoff X_T is thus given by

$$V\left(T\right) = \frac{1 + \theta\int_{X_T < V(T)} X_T dF\left(X_T\right)}{1 + \theta\int_{X_T < V(T)} dF\left(X_T\right)}.$$

Expanding, we get

$$\exp\left(v\left(T\right)T\right) = \frac{1+\theta\int_{\epsilon<\left(v(T)+\frac{1}{2}\sigma^2\right)\frac{\sqrt{T}}{\sigma}}\exp\left(-\frac{1}{2}\sigma^2 T+\sigma\sqrt{T}\epsilon\right)\frac{\exp\left(-\frac{1}{2}\epsilon^2\right)}{\sqrt{2\pi}}d\epsilon}{1+\theta\int_{\epsilon<\left(v(T)+\frac{1}{2}\sigma^2\right)\frac{\sqrt{T}}{\sigma}}\frac{\exp\left(-\frac{1}{2}\epsilon^2\right)}{\sqrt{2\pi}}d\epsilon}$$

$$= \frac{1+\theta\int_{\epsilon-\sigma\sqrt{T}<\left(v(T)-\frac{1}{2}\sigma^2\right)\frac{\sqrt{T}}{\sigma}}\frac{\exp\left(-\frac{1}{2}\left(\epsilon-\sigma\sqrt{T}\right)^2\right)}{\sqrt{2\pi}}d\epsilon}{1+\theta\int_{\epsilon<\left(v(T)+\frac{1}{2}\sigma^2\right)\frac{\sqrt{T}}{\sigma}}\frac{\exp\left(-\frac{1}{2}\epsilon^2\right)}{\sqrt{2\pi}}d\epsilon}$$

and, finally,

$$\exp\left(Tv\left(T\right)\right) = \frac{1+\theta\Phi\left(\frac{\sqrt{T}}{\sigma}\left(v\left(T\right)-\frac{1}{2}\sigma^2\right)\right)}{1+\theta\Phi\left(\frac{\sqrt{T}}{\sigma}\left(v\left(T\right)+\frac{1}{2}\sigma^2\right)\right)} < 1,$$

where Φ is the cumulative distribution function of a standard normal distribution.

B.1.1 Continuous information limit

Result. $\sqrt{T}v\left(T\right) \to -\kappa\sigma$ where κ is the unique solution to

$$\kappa + \theta\kappa\Phi\left(-\kappa\right) = \theta\Phi'\left(-\kappa\right)$$

Derivation. Suppose $v\left(T\right)$ has a finite (negative) limit in zero. Then as T converges to 0, we have

$$1+Tv\left(T\right) = \frac{1+\theta\left(\frac{1}{2}+\frac{1}{\sqrt{2\pi}}\frac{1}{\sigma}\sqrt{T}\left(v\left(T\right)-\frac{1}{2}\sigma^2\right)\right)}{1+\theta\left(\frac{1}{2}+\frac{1}{\sqrt{2\pi}}\frac{1}{\sigma}\sqrt{T}\left(v\left(T\right)+\frac{1}{2}\sigma^2\right)\right)}$$

$$1+Tv\left(T\right) = \frac{\left(1+\frac{1}{\left(1+\frac{\theta}{2}\right)\sqrt{2\pi}}\frac{1}{\sigma}\sqrt{T}\left(v\left(T\right)-\frac{1}{2}\sigma^2\right)\right)}{\left(1+\frac{1}{\left(1+\frac{\theta}{2}\right)\sqrt{2\pi}}\frac{1}{\sigma}\sqrt{T}\left(v\left(T\right)+\frac{1}{2}\sigma^2\right)\right)}$$

$$1+Tv\left(T\right) \approx \left(1+\frac{1}{\left(1+\frac{\theta}{2}\right)\sqrt{2\pi}}\frac{1}{\sigma}\sqrt{T}\left(v\left(T\right)-\frac{1}{2}\sigma^2\right)\right)\left(1-\frac{1}{\left(1+\frac{\theta}{2}\right)\sqrt{2\pi}}\frac{1}{\sigma}\sqrt{T}\left(v\left(T\right)+\frac{1}{2}\sigma^2\right)\right)$$

$$\approx 1-\frac{1}{\left(1+\frac{\theta}{2}\right)\sqrt{2\pi}}\sqrt{T}\sigma.$$

This last approximation contradicts the existence of a finite limit.

Let us now look for a $-\infty$ limit (we still have to show that $v\left(T\right)$ is increasing but let us assume it for now). We know $Tv(T) \to 0$ so

$$\exp(Tv(T)) = \frac{1+\theta\Phi\left(\frac{1}{\sigma}\sqrt{T}\left(v(T)-\frac{1}{2}\sigma^2\right)\right)}{1+\theta\Phi\left(\frac{1}{\sigma}\sqrt{T}\left(v(T)+\frac{1}{2}\sigma^2\right)\right)}$$

becomes

$$1+Tv(T) = \frac{1+\theta\Phi\left(\frac{1}{\sigma}\sqrt{T}\left(v(T)-\frac{1}{2}\sigma^2\right)\right)}{1+\theta\Phi\left(\frac{1}{\sigma}\sqrt{T}\left(v(T)+\frac{1}{2}\sigma^2\right)\right)}.$$

Suppose $\sqrt{T}v(T) \to -\infty$, then

$$\begin{aligned}
1+Tv(T) &= \left(1+\theta\Phi\left(\frac{1}{\sigma}\sqrt{T}\left(v(T)-\frac{1}{2}\sigma^2\right)\right)\right)\left(1-\theta\Phi\left(\frac{1}{\sigma}\sqrt{T}\left(v(T)+\frac{1}{2}\sigma^2\right)\right)\right)\\
&= 1+\theta\left[\Phi\left(\frac{1}{\sigma}\sqrt{T}\left(v(T)-\frac{1}{2}\sigma^2\right)\right)-\Phi\left(\frac{1}{\sigma}\sqrt{T}\left(v(T)+\frac{1}{2}\sigma^2\right)\right)\right]\\
&= 1-\theta\frac{\sigma\sqrt{T}}{\sqrt{2\pi}}\exp\left(-\frac{1}{2\sigma^2}T(v(T))^2\right)
\end{aligned}$$

which again yields a contradiction.

If $\sqrt{T}v(T) \to 0$, then we can still write

$$\begin{aligned}
1+Tv(T) &= \left(1+\theta\Phi\left(\frac{1}{\sigma}\sqrt{T}\left(v(T)-\frac{1}{2}\sigma^2\right)\right)\right)\left(1-\theta\Phi\left(\frac{1}{\sigma}\sqrt{T}\left(v(T)+\frac{1}{2}\sigma^2\right)\right)\right)\\
&= 1+\theta\left[\Phi\left(\frac{1}{\sigma}\sqrt{T}\left(v(T)-\frac{1}{2}\sigma^2\right)\right)-\Phi\left(\frac{1}{\sigma}\sqrt{T}\left(v(T)+\frac{1}{2}\sigma^2\right)\right)\right]\\
&= 1-\theta\frac{\sigma\sqrt{T}}{\sqrt{2\pi}}\exp\left(-\frac{1}{2\sigma^2}T(v(T))^2\right)
\end{aligned}$$

and we obtain a contradiction.

Finally, suppose $\sqrt{T}v(T) \to -\kappa\sigma$. Then,

$$
\begin{aligned}
1 + Tv(T) &= \frac{1 + \theta\Phi\left(\frac{1}{\sigma}\sqrt{T}\left(v(T) - \frac{1}{2}\sigma^2\right)\right)}{1 + \theta\Phi\left(\frac{1}{\sigma}\sqrt{T}\left(v(T) + \frac{1}{2}\sigma^2\right)\right)} \\
&= \frac{1 + \theta\Phi(-\kappa) - \theta\frac{\sigma\sqrt{T}}{2\sqrt{2\pi}}\exp\left(-\frac{\kappa^2}{2}\right)}{1 + \theta\Phi(-\kappa) + \theta\frac{\sigma\sqrt{T}}{2\sqrt{2\pi}}\exp\left(-\frac{\kappa^2}{2}\right)} \\
&= \frac{1 - \theta\frac{\sigma\sqrt{T}}{2(1+\theta\Phi(-\kappa))\sqrt{2\pi}}\exp\left(-\frac{\kappa^2}{2}\right)}{1 + \theta\frac{\sigma\sqrt{T}}{2(1+\theta\Phi(-\kappa))\sqrt{2\pi}}\exp\left(-\frac{\kappa^2}{2}\right)}
\end{aligned}
$$

and

$$
\kappa = \frac{\theta}{(1 + \theta\Phi(-\kappa))\sqrt{2\pi}}\exp\left(-\frac{\kappa^2}{2}\right)
$$

so

$$
\kappa + \theta\kappa\Phi(-\kappa) = \theta\Phi'(-\kappa).
$$

We can show there is a unique solution for κ. Indeed, defining

$$
g(\kappa) = \kappa + \theta\kappa\Phi(-\kappa) - \theta\Phi'(-\kappa),
$$

we obtain the following properties:

$$
\begin{aligned}
&g(0) < 0 \\
&g(\kappa) \to_{+\infty} +\infty \\
&g'(\kappa) = 1 + \theta\left(\Phi(-\kappa) - \kappa\Phi'(-\kappa) + \kappa\Phi'(-\kappa)\right) > 0.
\end{aligned}
$$

These conditions guarantee the existence and uniqueness of a solution.

B.1.2 Very infrequent observation limit

Result.

$$
v(T) \sim_{+\infty} -\frac{\log(1+\theta)}{T}
$$

Derivation. Remember that

$$\exp\left(Tv\left(T\right)\right)=\frac{1+\theta\Phi\left(\frac{1}{\sigma}\sqrt{T}\left(v\left(T\right)-\frac{1}{2}\sigma^{2}\right)\right)}{1+\theta\Phi\left(\frac{1}{\sigma}\sqrt{T}\left(v\left(T\right)+\frac{1}{2}\sigma^{2}\right)\right)}.$$

So, unless $v\to 0$ in $+\infty$ (again, we're assuming that v is increasing and thus admits a limit in $+\infty$), $\exp\left(Tv\left(T\right)\right)\to 0$ which is impossible.Therefore $v\left(T\right)\to 0$ at least as fast as $\frac{1}{T}$. Hence, $\sqrt{T}v\left(T\right)\to 0$ and

$$\frac{1+\theta\Phi\left(\frac{1}{\sigma}\sqrt{T}\left(v\left(T\right)-\frac{1}{2}\sigma^{2}\right)\right)}{1+\theta\Phi\left(\frac{1}{\sigma}\sqrt{T}\left(v\left(T\right)+\frac{1}{2}\sigma^{2}\right)\right)}\to\frac{1}{1+\theta}$$

so that

$$Tv\left(T\right)\to-\log\left(1+\theta\right)$$

and

$$v\left(T\right)\sim-\frac{\log\left(1+\theta\right)}{T}$$

B.1.3 Role of observation interval T

Result. v is increasing in T.

Derivation.

$$\exp\left(Tv\left(T\right)\right)=\frac{1+\theta\Phi\left(\frac{1}{\sigma}\sqrt{T}\left(v\left(T\right)-\frac{1}{2}\sigma^{2}\right)\right)}{1+\theta\Phi\left(\frac{1}{\sigma}\sqrt{T}\left(v\left(T\right)+\frac{1}{2}\sigma^{2}\right)\right)}.$$

Let us define $\frac{1}{\sigma}\sqrt{T}v\left(T\right)=g\left(T,\sigma\right)$. We have

$$\exp\left(\sigma\sqrt{T}g\left(T,\sigma\right)\right)=\frac{1+\theta\Phi\left(g\left(T,\sigma\right)-\frac{1}{2}\sigma\sqrt{T}\right)}{1+\theta\Phi\left(g\left(T,\sigma\right)+\frac{1}{2}\sigma\sqrt{T}\right)}.$$

Writing $z=\frac{1}{2}\sigma\sqrt{T}$,

$$2zg\left(z\right)=\log\left(1+\theta\Phi\left(g\left(z\right)-z\right)\right)-\log\left(1+\theta\Phi\left(g\left(z\right)+z\right)\right).$$

Differentiating, we obtain

$$\begin{aligned}2g(z)+2zg'(z) &= \theta\left(\frac{(g'(\theta)-1)\,\Phi'(g(z)-z)}{1+\theta\Phi(g(z)-z)}-\frac{(g'(\theta)+1)\,\Phi'(g(z)+z)}{1+\theta\Phi(g(z)+z)}\right)\\ &= -2\theta\frac{\Phi'(g(z)-z)}{1+\theta\Phi(g(z)-z)}\\ &= -2\theta\frac{\Phi'(g(z)+z)}{1+\theta\Phi(g(z)+z)}.\end{aligned}$$

Let us define the function u by

$$u(x)=\log(1+\theta\Phi(x)).$$

We have

$$\begin{aligned}u'(x) &= \frac{\theta\Phi'(x)}{1+\theta\Phi(x)}>0\\ u''(x) &= \frac{-\theta\Phi'(x)\left[x(1+\theta\Phi(x))+\theta\Phi'(x)\right]}{(1+\theta\Phi(x))^2}\\ \left[x(1+\theta\Phi(x))+\theta\Phi'(x)\right]' &= (1+\theta\Phi(x))>0.\end{aligned}$$

So u'' is positive then negative, and u is increasing convex then concave with a unique inflection point x^*. Observe

$$u'(g(z)+z)=u'(g(z)-z),$$

so, $\forall z$, $g(z)-z\leq x^*\leq g(z)+z$. Because u is convex between $g(z)-z$ and x^*,

$$u(x^*)-u(g(z)-z)\geq(x^*-(g(z)-z))\,u'(g(z)-z).$$

Because u is concave between $g(z)+z$ and x^*,

$$u(g(z)+z)-u(x^*)\geq((g(z)+z)-x^*)\,u'(g(z)+z).$$

Putting these results together,

$$\begin{aligned}&u(g(z)+z)-u(g(z)-z)\geq((g(z)+z)-x^*)\,u'(g(z)+z)+(x^*-(g(z)-z))\,u'(g(z)-z)\\ &u(g(z)+z)-u(g(z)-z)\geq 2zu'(g(z)+z),\end{aligned}$$

and finally

$$-2zg(z) \geq 2zu'(g(z)+z)$$

which proves $g'(z)$ positive for all z.

We have

$$v(T) = \frac{\sigma}{\sqrt{T}} g\left(\frac{1}{2}\sigma\sqrt{T}\right)$$

so

$$v'(T) = \frac{\sigma}{2T\sqrt{T}}\left(zg'(z) - g\right),$$

and v' is positive for all T.

B.1.4 Role of the volatility σ

Result. v is decreasing in σ.

Derivation.

$$\exp(Tv(T)) = \frac{1+\theta\Phi\left(\frac{1}{\sigma}\sqrt{T}\left(v(T)-\frac{1}{2}\sigma^2\right)\right)}{1+\theta\Phi\left(\frac{1}{\sigma}\sqrt{T}\left(v(T)+\frac{1}{2}\sigma^2\right)\right)}.$$

Let us write $\frac{1}{\sigma}\sqrt{T}v(T) = g(T,\sigma)$, then

$$\exp\left(\sigma\sqrt{T}g(T,\sigma)\right) = \frac{1+\theta\Phi\left(g(T,\sigma)-\frac{1}{2}\sigma\sqrt{T}\right)}{1+\theta\Phi\left(g(T,\sigma)+\frac{1}{2}\sigma\sqrt{T}\right)}.$$

Let us write $z = \frac{1}{2}\sigma\sqrt{T}$, then

$$2zg(z) = \log(1+\theta\Phi(g(z)-z)) - \log(1+\theta\Phi(g(z)+z))$$

Differentiating, we obtain

$$\begin{aligned}2g(z)+2zg'(z)&=\theta\left(\frac{(g'(z)-1)\,\Phi'(g(z)-z)}{1+\theta\Phi(g(z)-z)}-\frac{(g'(z)+1)\,\Phi'(g(z)+z)}{1+\theta\Phi(g(z)+z)}\right)\\&=-2\theta\frac{\Phi'(g(z)-z)}{1+\theta\Phi(g(z)-z)},\end{aligned}$$

so

$$zg'(z)=-\frac{g(z)+\theta\left[g(z)\,\Phi(g(z)-z)+\Phi'(g(z)-z)\right]}{1+\theta\Phi(g(z)-z)}.$$

We have

$$v(\sigma)=\frac{\sigma}{\sqrt{T}}g\left(\frac{1}{2}\sigma\sqrt{T}\right),$$

so

$$\begin{aligned}\sqrt{T}v'(\sigma)&=g(z)+zg'(z)\\&=-\theta\frac{\Phi'(g(z)-z)}{1+\theta\Phi(g(z)-z)}<0.\end{aligned}$$

B.1.5 Role of disappointment aversion θ

Result. v is decreasing in θ.

Derivation.

$$\sigma\sqrt{T}g(\theta)=\log\left(1+\theta\Phi\left(g(\theta)-\frac{1}{2}\sigma\sqrt{T}\right)\right)-\log\left(1+\theta\Phi\left(g(\theta)+\frac{1}{2}\sigma\sqrt{T}\right)\right)$$

Let us differentiate:

$$\begin{aligned}\sigma\sqrt{T}g'(\theta)&=\left\{\begin{array}{l}\frac{\Phi\left(g(\theta)-\frac{1}{2}\sigma\sqrt{T}\right)+\theta g'(\theta)\Phi'\left(g(\theta)-\frac{1}{2}\sigma\sqrt{T}\right)}{1+\theta\Phi\left(g(\theta)-\frac{1}{2}\sigma\sqrt{T}\right)}-\\\frac{\Phi\left(g(\theta)+\frac{1}{2}\sigma\sqrt{T}\right)+\theta g'(\theta)\Phi'\left(g(\theta)+\frac{1}{2}\sigma\sqrt{T}\right)}{1+\theta\Phi\left(g(\theta)+\frac{1}{2}\sigma\sqrt{T}\right)}\end{array}\right.\\&=\frac{\Phi\left(g(\theta)-\frac{1}{2}\sigma\sqrt{T}\right)}{1+\theta\Phi\left(g(\theta)-\frac{1}{2}\sigma\sqrt{T}\right)}-\frac{\Phi\left(g(\theta)+\frac{1}{2}\sigma\sqrt{T}\right)}{1+\theta\Phi\left(g(\theta)+\frac{1}{2}\sigma\sqrt{T}\right)}\\&\quad+\frac{1}{\sqrt{2\pi}}\frac{\theta g'(\theta)\,e^{-\frac{1}{2}\left(g^2+\sigma^2T/4\right)}}{1+\theta\Phi\left(g(\theta)-\frac{1}{2}\sigma\sqrt{T}\right)}\left[e^{\frac{1}{2}g\sigma\sqrt{T}}-e^{-\frac{1}{2}g\sigma\sqrt{T}}\frac{1+\theta\Phi\left(g(T)-\frac{1}{2}\sigma\sqrt{T}\right)}{1+\theta\Phi\left(g(T)+\frac{1}{2}\sigma\sqrt{T}\right)}\right],\end{aligned}$$

and so

$$\sigma\sqrt{T}g'(\theta) = \frac{\Phi\left(g(\theta) - \frac{1}{2}\sigma\sqrt{T}\right) - \Phi\left(g(\theta) + \frac{1}{2}\sigma\sqrt{T}\right)}{\left(1 + \theta\Phi\left(g(\theta) - \frac{1}{2}\sigma\sqrt{T}\right)\right)\left(1 + \theta\Phi\left(g(\theta) + \frac{1}{2}\sigma\sqrt{T}\right)\right)} < 0.$$

B.2 Jumps

We conduct the same calculation for the case of a pure jump process. Write N_t the counting variable for a Poisson jump process with intensity λ. Define the process $\{X_t\}$ by the stochastic differential equation:

$$\frac{dX_t}{X_{t-}} = \lambda\sigma dt - \sigma dN_t,$$

where $\sigma < 1$. The value of X_t decreases geometrically at each jump. The drift term compensates for the average decrease, so that $\{X_t\}$ is a martingale. Solving this S.D.E. with initial condition $X_0 = 1$, we obtain

$$X_t = \exp\left(\lambda\sigma t + \log(1-\sigma) N_t\right).$$

We are interested in the certainty equivalent of a lottery paying X_T for various values of T.

B.2.1 Preliminaries

A few standard results on Poisson jump processes that will be useful:

$$\begin{aligned}
P[N_t = k] &= \frac{(\lambda t)^k}{k!}e^{-\lambda t} \\
P[N_t = 0] &= e^{-\lambda t} \\
P[N_t \leq k] &= e^{-\lambda t}\sum_{i=0}^{k}\frac{(\lambda t)^i}{i!} = \frac{\Gamma(k+1, \lambda t)}{k!} \\
\mathbb{E}[\exp(uN_t)] &= \exp\left(\lambda t\left(e^u - 1\right)\right) \\
\mathbb{E}[\exp(\log(1-\sigma) N_t)] &= \exp(-\lambda\sigma t),
\end{aligned}$$

where $\Gamma(.,.)$ is the incomplete gamma function.

Further, we can express the certainty equivalent in a more convenient way:

$$V = \frac{\mathbb{E}[y] + \theta\mathbb{E}\left[y 1_{y\leq V}\right]}{1 + \theta\mathbb{P}[y \leq V]}$$
$$V = \frac{(1+\theta)\mathbb{E}[y] - \theta\mathbb{E}\left[y 1_{y>V}\right]}{(1+\theta) - \theta\mathbb{P}[y > V]}.$$

B.2.2 Certainty equivalent

If the certainty equivalent is between the points of the distribution corresponding to k and $k+1$ jumps, we can compute it exactly. This corresponds to the condition:

$$(1-\sigma)^{k+1} \leq V \exp(-\lambda\sigma T) \leq (1-\sigma)^k.$$

Then, we get immediately

$$\exp\left(-\lambda\sigma T\right) V = \frac{(1+\theta)\exp\left(-\lambda\sigma T\right) - \theta\mathbb{E}\left[(1-\sigma)^{N_t} 1_{N_T\leq k}\right]}{(1+\theta) - \theta P\left[N_T \leq k\right]}.$$

Note that

$$\begin{aligned}
\mathbb{E}\left[(1-\sigma)^{N_t} 1_{N_T\leq k}\right] &= e^{-\lambda T}\sum_{i=0}^{k}(1-\sigma)^i \frac{(\lambda T)^i}{i!} \\
&= e^{-\lambda T + (1-\sigma)\lambda T} e^{-(1-\sigma)\lambda T}\sum_{i=0}^{k}\frac{((1-\sigma)\lambda T)^i}{i!} \\
&= e^{-\lambda\sigma T}\frac{\Gamma\left(k+1,(1-\sigma)\lambda T\right)}{k!}.
\end{aligned}$$

Therefore,

$$\exp\left(-\lambda\sigma T\right) V = \frac{\exp\left(-\lambda\sigma T\right)\left[(1+\theta) - \theta\frac{\Gamma(k+1,(1-\sigma)\lambda T)}{k!}\right]}{(1+\theta) - \theta\frac{\Gamma(k+1,\lambda T)}{k!}}$$
$$V = \frac{1 - \frac{\theta}{1+\theta}\frac{\Gamma(k+1,(1-\sigma)\lambda T)}{k!}}{1 - \frac{\theta}{1+\theta}\frac{\Gamma(k+1,\lambda T)}{k!}}.$$

As the certainty equivalent is unique, there is a unique k so that the corresponding V falls in the right interval.

Remark 1. In matlab, the incomplete gamma function is defined such that $\Gamma(k+1,x)/k! = \text{gammainc}(x,k+1)$.

Remark 2. At the points where we go from one k to the next, we have $V = (1-\sigma)^k \exp(\lambda\sigma T)$.

B.2.3 Continuous information limit

Result.

$$v(T) \to -\sigma\lambda.$$

Derivation. In the limit where T gets close to 0, the certainty equivalent falls in the region between 0 and 1 jumps. We guess and verify this result and obtain the limiting behavior of V as T converges to 0. In this case we have

$$V = \frac{1 - \frac{\theta}{1+\theta}\exp(-(1-\sigma)\lambda T)}{1 - \frac{\theta}{1+\theta}\exp(-\lambda T)},$$

which clearly converges to 1 as T converges to 0 so the guess is indeed verified. In the limit, we get:

$$\begin{aligned} V &\approx \frac{1 - \frac{\theta}{1+\theta}(1-(1-\sigma)\lambda T)}{1 - \frac{\theta}{1+\theta}(1-\lambda T)} \\ &\approx \frac{1+\theta(1-\sigma)\lambda T}{1+\theta\lambda T} \\ &\approx 1 - \theta\sigma\lambda T \\ V &\approx \exp(-\theta\sigma\lambda T). \end{aligned}$$

In particular it tells us that $V^{1/T}$ admits the finite limit $\exp(-\sigma\lambda)$ as $T \to 0$.

B.2.4 Infrequent observation limit

Result.

$$v(T) \to_{+\infty} 0.$$

Derivation.

B.2.5 Role of the observation interval T

Result. v is increasing in T.

Derivation. and v' positive in all T.

B.2.6 Role of the shock size σ

Result. v is decreasing in σ.

Derivation.

B.2.7 Role of the disappointment aversion θ

Result. v is decreasing in θ.

Derivation.

C Consumption-savings model

C.1 General case

Results.

$$\mathcal{V}_0^{\frac{1-\alpha}{\alpha}} = \frac{\left(\frac{1-\exp\left(-\frac{\rho+(\alpha-1)r}{\alpha}T\right)}{\frac{\rho+(\alpha-1)r}{\alpha}}\right)}{1-\left(\exp\left(-\frac{\rho}{\alpha}\right)\left(V_0\left(T\right)\right)^{\frac{1-\alpha}{\alpha}}\right)^{T}}$$

$$\mathcal{C}_0 = 1-\left(\exp\left(-\frac{\rho}{\alpha}\right)\left(V_0\left(T\right)\right)^{\frac{1-\alpha}{\alpha}}\right)^{T}.$$

T^* is the unique solution to

$$\frac{\partial v(T)}{\partial \log(T)} f\left(v\left(T\right)-\frac{\rho}{1-\alpha}\right) = \left(r-\frac{\rho}{1-\alpha}\right) f\left(r-\frac{\rho}{1-\alpha}\right) - \left(v\left(T\right)-\frac{\rho}{1-\alpha}\right) f\left(v\left(T\right)-\frac{\rho}{1-\alpha}\right)$$

where $f\left(x\right)=\exp\left(\frac{1-\alpha}{\alpha}xT\right)/\left(1-\exp\left(\frac{1-\alpha}{\alpha}xT\right)\right)$.

Derivation.

$$\mathcal{V}_0^{1-\alpha} = \left(\mathcal{C}_0\right)^{1-\alpha}\left[\left(\frac{1-\exp\left(-\frac{\rho+(\alpha-1)r}{\alpha}T\right)}{\frac{\rho+(\alpha-1)r}{\alpha}}\right)^{\alpha} + \exp\left(-\rho T\right)\left(\mathcal{V}_0 V\left(T\right)\right)^{1-\alpha}\left(\left(\mathcal{C}_0\right)^{-1}-1\right)^{1-\alpha}\right],$$

and

$$(\mathcal{C}_0)^{-1}\left(\frac{1-\exp\left(-\frac{\rho+(\alpha-1)r}{\alpha}T\right)}{\frac{\rho+(\alpha-1)r}{\alpha}}\right)=\left(\frac{1-\exp\left(-\frac{\rho+(\alpha-1)r}{\alpha}T\right)}{\frac{\rho+(\alpha-1)r}{\alpha}}\right)+\left(\exp\left(-\rho T\right)\left(\mathcal{V}_0 V\left(T\right)\right)^{1-\alpha}\right)^{\frac{1}{\alpha}}$$

so

$$\begin{aligned}
\mathcal{V}_0^{1-\alpha} &= \left((\mathcal{C}_0)^{-1}\left(\frac{1-\exp\left(-\frac{\rho+(\alpha-1)r}{\alpha}T\right)}{\frac{\rho+(\alpha-1)r}{\alpha}}\right)\right)^{\alpha-1} \\
&\quad\times\Bigg[\left(\frac{1-\exp\left(-\frac{\rho+(\alpha-1)r}{\alpha}T\right)}{\frac{\rho+(\alpha-1)r}{\alpha}}\right) \\
&\quad+\exp\left(-\rho T\right)\left(\mathcal{V}_0 V\left(T\right)\right)^{1-\alpha}\left((\mathcal{C}_0)^{-1}\left(\frac{1-\exp\left(-\frac{\rho+(\alpha-1)r}{\alpha}T\right)}{\frac{\rho+(\alpha-1)r}{\alpha}}\right)-\left(\frac{1-\exp\left(-\frac{\rho+(\alpha-1)r}{\alpha}T\right)}{\frac{\rho+(\alpha-1)r}{\alpha}}\right)\right)^{1-\alpha}\Bigg] \\
\mathcal{V}_0^{1-\alpha} &= \left((\mathcal{C}_0)^{-1}\left(\frac{1-\exp\left(-\frac{\rho+(\alpha-1)r}{\alpha}T\right)}{\frac{\rho+(\alpha-1)r}{\alpha}}\right)\right)^{\alpha-1}\times \\
&\quad\left[\left(\frac{1-\exp\left(-\frac{\rho+(\alpha-1)r}{\alpha}T\right)}{\frac{\rho+(\alpha-1)r}{\alpha}}\right)+\exp\left(-\rho T\right)\left(\mathcal{V}_0 V\left(T\right)\right)^{1-\alpha}\left(\exp\left(-\rho T\right)\left(\mathcal{V}_0 V\left(T\right)\right)^{1-\alpha}\right)^{\frac{1-\alpha}{\alpha}}\right] \\
\mathcal{V}_0^{1-\alpha} &= \left(\left(\frac{1-\exp\left(-\frac{\rho+(\alpha-1)r}{\alpha}T\right)}{\frac{\rho+(\alpha-1)r}{\alpha}}\right)+\left(\exp\left(-\rho T\right)\left(\mathcal{V}_0 V\left(T\right)\right)^{1-\alpha}\right)^{\frac{1}{\alpha}}\right)^{\alpha} \\
\mathcal{V}_0^{\frac{1-\alpha}{\alpha}} &= \left(\frac{1-\exp\left(-\frac{\rho+(\alpha-1)r}{\alpha}T\right)}{\frac{\rho+(\alpha-1)r}{\alpha}}\right)+\left(\exp\left(-\rho T\right)\left(\mathcal{V}_0 V\left(T\right)\right)^{1-\alpha}\right)^{\frac{1}{\alpha}},
\end{aligned}$$

and finally

$$\begin{aligned}
\mathcal{V}_0^{\frac{1-\alpha}{\alpha}} &= \frac{\left(\frac{1-\exp\left(-\frac{\rho+(\alpha-1)r}{\alpha}T\right)}{\frac{\rho+(\alpha-1)r}{\alpha}}\right)}{1-\exp\left(-\frac{\rho}{\alpha}T\right)\left(V\left(T\right)\right)^{\frac{1-\alpha}{\alpha}}} \\
\mathcal{V}_0^{\frac{1-\alpha}{\alpha}} &= \frac{\left(\frac{1-\exp\left(-\frac{\rho+(\alpha-1)r}{\alpha}T\right)}{\frac{\rho+(\alpha-1)r}{\alpha}}\right)}{1-\left(\exp\left(-\frac{\rho}{\alpha}\right)\left(V_0\left(T\right)\right)^{\frac{1-\alpha}{\alpha}}\right)^{T}}.
\end{aligned}$$

This is the optimal value function. Also,

$$\begin{aligned}
\mathcal{V}_0^{1-\alpha} &= \left(\left(\frac{1-\exp\left(-\frac{\rho+(\alpha-1)r}{\alpha}T\right)}{\frac{\rho+(\alpha-1)r}{\alpha}}\right) + \left(\exp\left(-\rho T\right)\left(\mathcal{V}_0 V\left(T\right)\right)^{1-\alpha}\right)^{\frac{1}{\alpha}}\right)^{\alpha} \\
&= \left(\left(\mathcal{C}_0\right)^{-1}\left(\frac{1-\exp\left(-\frac{\rho+(\alpha-1)r}{\alpha}T\right)}{\frac{\rho+(\alpha-1)r}{\alpha}}\right)\right)^{\alpha} \\
\mathcal{V}_0^{\frac{1-\alpha}{\alpha}} &= \left(\mathcal{C}_0\right)^{-1}\left(\frac{1-\exp\left(-\frac{\rho+(\alpha-1)r}{\alpha}T\right)}{\frac{\rho+(\alpha-1)r}{\alpha}}\right) \\
\left(\mathcal{C}_0\right)^{-1} &= \frac{1}{1-\left(\exp\left(-\frac{\rho}{\alpha}\right)\left(V_0\left(T\right)\right)^{\frac{1-\alpha}{\alpha}}\right)^{T}} \\
\mathcal{C}_0 &= 1-\left(\exp\left(-\frac{\rho}{\alpha}\right)\left(V_0\left(T\right)\right)^{\frac{1-\alpha}{\alpha}}\right)^{T}.
\end{aligned}$$

This is the optimal investment in the cash account.

Finally, let us turn to the fist order condition for the optimal observation interval $\frac{\partial \mathcal{V}_0}{\partial T}=0$. We have

$$\mathcal{V}_0^{\frac{1-\alpha}{\alpha}}\left(\frac{\rho+(\alpha-1)\,r}{\alpha}\right) = \frac{1-\exp\left(-\frac{\rho+(\alpha-1)r}{\alpha}T\right)}{1-\exp\left(-\frac{\rho+(\alpha-1)v}{\alpha}T\right)},$$

so

$$\begin{aligned}
\frac{\partial \mathcal{V}_0}{\partial T} &= 0 \\
\Leftrightarrow \frac{\partial}{\partial T}\left[\begin{cases}\log\left(1-\exp\left(-\frac{\rho+(\alpha-1)r}{\alpha}T\right)\right) \\ -\log\left(1-\exp\left(-\frac{\rho+(\alpha-1)v}{\alpha}T\right)\right)\end{cases}\right] &= 0 \\
\Leftrightarrow \begin{cases}\frac{\left(r-\frac{\rho}{1-\alpha}\right)\exp\left(\frac{1-\alpha}{\alpha}\left(r-\frac{\rho}{1-\alpha}\right)T\right)}{1-\exp\left(\frac{1-\alpha}{\alpha}\left(r-\frac{\rho}{1-\alpha}\right)T\right)} \\ -\frac{\left(v-\frac{\rho}{1-\alpha}\right)\exp\left(\frac{1-\alpha}{\alpha}\left(v-\frac{\rho}{1-\alpha}\right)T\right)}{1-\exp\left(\frac{1-\alpha}{\alpha}\left(v-\frac{\rho}{1-\alpha}\right)T\right)}\end{cases} &= v'\left(T\right)T\frac{\exp\left(\frac{1-\alpha}{\alpha}\left(v-\frac{\rho}{1-\alpha}\right)T\right)}{1-\exp\left(\frac{1-\alpha}{\alpha}\left(v-\frac{\rho}{1-\alpha}\right)T\right)}.
\end{aligned}$$

C.2 Case of a Brownian motion

C.2.1 Optimal investment in the cash account $\mathcal{C}_0$

Result. $\mathcal{C}_0$ is increasing in T, σ and θ.

Derivation.

$$\mathcal{C}_0 = 1 - \left(\exp\left(-\frac{\rho}{\alpha}\right)(V_0)^{\frac{1-\alpha}{\alpha}}\right)^T,$$

or equivalently

$$\mathcal{C}_0 = 1 - \exp\left[\left(-\frac{\rho}{\alpha} + \frac{1-\alpha}{\alpha} v(T)\right) T\right].$$

Role of the observation interval T.

$$\frac{d\mathcal{C}_0}{dT} = \left[\left(\frac{\rho}{\alpha} - \frac{1-\alpha}{\alpha}\left(v(T) + v'(T)\,T\right)\right)\right] \exp\left[\left(-\frac{\rho}{\alpha} + \frac{1-\alpha}{\alpha} v(T)\right) T\right].$$

Using the notations and results above,

$$v(T) = \mu + \frac{\sigma}{\sqrt{T}} g\left(\frac{1}{2}\sigma\sqrt{T}\right),$$

and

$$v'(T) = \frac{\sigma}{2T\sqrt{T}}\left(zg'(z) - g\right),$$

so

$$\begin{aligned}(v(T) - \mu) + v'(T)\,T &= \frac{\sigma}{2\sqrt{T}}\left(zg'(z) + g\right) \\ &= -\frac{\sigma}{2\sqrt{T}}\theta\frac{\Phi'(g(z) - z)}{1 + \theta\Phi(g(z) - z)} < 0.\end{aligned}$$

As long as $\rho - (1-\alpha)\mu > 0$, we have $\mathcal{C}_0 \to 1$ and $\frac{d\mathcal{C}_0}{dT} \to 0$ in $+\infty$ in zero. Therefore, $\mathcal{C}_0(0) = 0$,

$$\frac{d\mathcal{C}_0}{dT} \approx \frac{1-\alpha}{\alpha}\frac{\sigma\kappa}{2\sqrt{T}},$$

and if $1 - \alpha > 0$, $\mathcal{C}_0$ increasing everywhere in T. *Role of the volatility σ and the disappointment aversion θ.*

$$\frac{d\mathcal{C}_0}{d\sigma} = -\frac{1-\alpha}{\alpha} v'(\sigma) \exp\left[\left(-\frac{\rho}{\alpha} + \frac{1-\alpha}{\alpha} v(\sigma)\right) T\right].$$

If $1-\alpha>0$, $\mathcal{C}_0$ increasing everywhere in σ. The same result is valid for the dependence on θ.

C.2.2 Optimal value $\mathcal{V}_0$

Results.

$$\begin{aligned}\mathcal{V}_0^{\frac{1-\alpha}{\alpha}} &\sim_0 \frac{\sqrt{T}}{\kappa\sigma}\\ \mathcal{V}_0^{\frac{1-\alpha}{\alpha}} &\to_{+\infty} \frac{1}{\left(\frac{\rho}{\alpha}-\frac{1-\alpha}{\alpha}r\right)}\\ \mathcal{V}_0 &> 1\end{aligned}$$

when $v(T)>r$. Further, $\mathcal{V}_0$ has a unique maximum in T^*, $v(T^*)>r$, and $\mathcal{V}_0$ decreasing in σ and θ.

Derivation.

$$\mathcal{V}_0^{\frac{1-\alpha}{\alpha}}\left(\frac{\rho}{\alpha}-\frac{1-\alpha}{\alpha}r\right)=\frac{1-\exp\left(-\frac{\rho}{\alpha}+\frac{1-\alpha}{\alpha}r\right)T}{1-\left(\exp\left(-\frac{\rho}{\alpha}\right)\left(V_0\left(T\right)\right)^{\frac{1-\alpha}{\alpha}}\right)^{T}},$$

or equivalently

$$\mathcal{V}_0^{\frac{1-\alpha}{\alpha}}\left(\frac{\rho}{\alpha}-\frac{1-\alpha}{\alpha}r\right)=\frac{1-\exp\left(-\frac{\rho}{\alpha}+\frac{1-\alpha}{\alpha}r\right)T}{1-\exp\left(-\frac{\rho}{\alpha}+\frac{1-\alpha}{\alpha}\left(v\right)\right)T}.$$

Limits.

1. In $\theta=0$,

$$\left(\frac{1-\exp\left(-\frac{\rho}{\alpha}+\frac{1-\alpha}{\alpha}r\right)T}{1-\exp\left(-\frac{\rho}{\alpha}+\frac{1-\alpha}{\alpha}\mu\right)T}\right)=\left(\frac{1-\exp\left(-xT\right)}{1-\exp\left(-x+y\right)T}\right),$$

where $x=\frac{\rho}{\alpha}-\frac{1-\alpha}{\alpha}r$ and $y=\frac{1-\alpha}{\alpha}\left(\mu-r\right)<x$. We have

$$\left(\frac{1-\exp\left(-xT\right)}{1-\exp\left(-x+y\right)T}\right)'\propto\left(1-\exp\left(-xT\right)\right)y+x\left(\exp\left(-yT\right)-1\right).$$

In zero,

$$\left(1-\exp\left(-xT\right)\right)y+x\left(\exp\left(-yT\right)-1\right)\sim\frac{1}{2}xT^2y\left(y-x\right)<0.$$

In $+\infty$,

$$(1-\exp(-xT))\,y + x(\exp(-yT)-1) \sim (y-x) < 0,$$

and

$$[(1-\exp(-xT))\,y + x(\exp(-yT)-1)]^{'} = xy(\exp(-xT)-\exp(-yT)) < 0.$$

Therefore $\mathcal{V}_0^{\frac{1-\alpha}{\alpha}}\ |_{\theta=0}$ is decreasing everywhere in T. The agent optimally choses $T=0$ and is fully invested in the risky asset.

2. In $T=0$,

$$\mathcal{V}_0^{\frac{1-\alpha}{\alpha}}\left(\frac{\rho}{\alpha}-\frac{1-\alpha}{\alpha}r\right) \sim \frac{\left(\frac{\rho}{\alpha}-\frac{1-\alpha}{\alpha}r\right)\sqrt{T}}{\kappa\sigma}.$$

3. In $T=+\infty$, $\mathcal{V}_0^{\frac{1-\alpha}{\alpha}} \to \frac{1}{\left(\frac{\rho}{\alpha}-\frac{1-\alpha}{\alpha}r\right)}$.

4. For $\mu > r$, then, there is a $\hat{T}$, such that, if $T > \hat{T}$, $v(T) > R$ and $\mathcal{V}_0^{\frac{1-\alpha}{\alpha}} > \frac{1}{\left(\frac{\rho}{\alpha}-\frac{1-\alpha}{\alpha}r\right)}$. This proves the existence of an optimal value T^* satisfying $T^* > \hat{T}$.

Role of volatility σ and disappointment aversion θ.

$$\mathcal{V}_0^{\frac{1-\alpha}{\alpha}} = \frac{\left(\frac{1-\exp\left(-\frac{\rho+(\alpha-1)r}{\alpha}T\right)}{\frac{\rho+(\alpha-1)r}{\alpha}}\right)}{1-\exp\left[\left(-\frac{\rho}{\alpha}+\frac{1-\alpha}{\alpha}v\right)T\right]}$$

$$\frac{d\mathcal{V}_0^{\frac{1-\alpha}{\alpha}}}{d\sigma} = \frac{1-\alpha}{\alpha}v'(\sigma)\exp\left[\left(-\frac{\rho}{\alpha}+\frac{1-\alpha}{\alpha}v(\sigma)\right)T\right]\frac{\left(\frac{1-\exp\left(-\frac{\rho+(\alpha-1)r}{\alpha}T\right)}{\frac{\rho+(\alpha-1)r}{\alpha}}\right)}{1-\exp\left[\left(-\frac{\rho}{\alpha}+\frac{1-\alpha}{\alpha}v(\sigma)\right)T\right]}.$$

If $1-\alpha>0$, $\mathcal{V}_0$ is decreasing everywhere in σ. The same result applies to θ.

C.2.3 Optimal time period T^*

Results. T^* is decreasing in σ and θ.

Derivation.