Really Uncertain Business Cycles
Nicholas Bloom, Max Floetotto and Nir Jaimovich*
Stanford University
September 2009

Abstract
We propose uncertainty shocks as a new impulse driving business cycles. First, we demonstrate that uncertainty, measured by a number of proxies is strongly countercyclical. Second, we build a dynamic stochastic general equilibrium model that extends the benchmark neoclassical growth model along two dimensions. It allows for the existence of heterogeneous firms with non-convex adjustment costs in both capital and labor and time-variation in uncertainty that is modeled as a change in the variance of innovations to productivity. We find that increases in uncertainty lead to large drops in economic activity. This occurs because a rise in uncertainty makes firms cautious, leading them to pause hiring and investment. It also reduces the reallocation of capital and labor across firms, leading to large falls in productivity growth. Finally, we show that because uncertainty makes firms cautious it significantly reduces the response of the economy to stimulative policy, leading to pro-cyclical policy multipliers.

Keywords: uncertainty, adjustment costs and business cycles.
JEL Classification: D92, E22, D8, C23.
Disclaimer: Any opinions and conclusions expressed herein are those of the authors and do not necessarily represent the views of the U.S. Census Bureau. All results have been reviewed to ensure that no confidential information is disclosed.

*We would like to thank our formal discussants Eduardo Engel at the AEA (2009) and Frank Smets at the ECB (2009) for their comments and suggestions. We thank Gadi Barlevy, Patrick Kehoe, and Naryana Kocherlakota for most helpful comments and suggestions. We would like to thank seminar participants at the AEA, ECB, UC-Irvine, IMF, Warwick University, Kellogg Business School of Management, NBER summer institute (2009). Correspondence: Nick Bloom, Department of Economics, Stanford University, Stanford, CA 94305, nbloom@stanford.edu.
1 Introduction

This paper studies the interaction between time-varying uncertainty and the business cycle. The notion of linking uncertainty to the business cycles is not new. John Maynard Keynes (1936) argued that changes in investor sentiments, the so-called ‘animal spirits’, could lead to economic downturns. While this can be interpreted as an argument for the role of uncertainty, it has not traditionally played a large role in the modern studies of business cycles. The study of uncertainty has received more attention during the recent economic downturn. As we document below, the US economy experienced a significant rise in measured uncertainty during the recession that started in 2007.

We start by addressing the empirical behavior of uncertainty over the business cycle. Evidence on the time series variation in uncertainty is scarce, as no good measure of uncertainty exists. To circumvent this difficulty we collect a wide range of proxies for uncertainty, which reveals two stylized facts. First, idiosyncratic uncertainty about the evolution of micro economic variables for individual establishments, firms or industries is strongly countercyclical. Second, aggregate uncertainty about the evolution of macro economic variables is also strongly countercyclical.

Given these two sources of uncertainty, our theoretical model considers both time varying micro and macro uncertainty. Specifically, we consider an environment where firms face both idiosyncratic and aggregate productivity shocks. At the micro level, we assume that production-units face stochastic volatility in the innovations to idiosyncratic productivity. In the model, this process is mainly manifested in time-varying cross-sectional measures of the production units’ performance (output, sales, stock returns etc.). We use US Census data to calibrate the time varying process of micro uncertainty to match the evolution of the cross-sectional dispersion of establishment output growth over time. Similarly at the macro level, we assume that the production units face stochastic volatility in the aggregate productivity shock. In the model this process is mainly manifested in a strong conditional heteroskedasticity of aggregate variables. We thus calibrate it to match the evolution of conditional heteroskedasticity in US gross domestic output (GDP) growth.

To quantify the effect of time-varying uncertainty on aggregate economic activity, we build a dynamic stochastic general equilibrium model. Various features of the model are specified to conform as closely as possible to the standard frictionless real business cycle (RBC) model as this greatly simplifies comparison with existing work. We deviate from this benchmark in three ways. First, uncertainty is time-varying. As stated above, the model includes shocks to both the level of technology (the first moment) and its variance (the second moment) at both the micro and macro level. Second, there are heterogeneous firms that are subject to idiosyncratic productivity shocks. Third, the model contains convex
and non-convex adjustment costs in both capital and labor. The non-convexities together with time variation in uncertainty imply that firms become more cautious in investing and hiring when uncertainty increases.

Simulations allow us to study the response of our model economy to an uncertainty shock. We show that a rise in uncertainty makes it optimal for each individual firm to wait, leading to a significant fall in aggregate economic activity. In addition, we show that time-varying uncertainty reduces productivity growth during times of high uncertainty because it lowers the extent of reallocation in the economy. When uncertainty rises productive firms expand less and unproductive firms contract less.\footnote{In the actual U.S. economy, reallocation is a key factor driving aggregate productivity. See, for example, Foster, Haltiwanger and Krizan (2000, 2006), who report that reallocation, broadly defined to include entry and exit, accounts for around 50% of manufacturing and 80% of retail productivity growth in the US.}

We then build on our theoretical model to investigate the effects of uncertainty on policy effectiveness. We use a simple illustrative example to show how time-varying uncertainty significantly dampens the effect of an expansionary policy. The key to this policy ineffectiveness is that a rise in uncertainty makes firms very cautious in responding to any stimulus and that includes the policy impulse. The impact of this stimulus is thus mitigated relative to its impact in low uncertainty times.

Our work is related to several strands in the literature. First, we add to the extensive literature building on the RBC framework that studies the role of productivity (TFP) shocks in causing business cycles. In this literature, recessions are generally caused by large negative technology shocks.\footnote{See, for example, the discussion in Rebelo (2005): “Most RBC models require declines in TFP in order to replicate the declines in output observed in the data.” For an excellent review of this literature see King and Rebelo (1999).} The reliance on negative technology shocks has proven to be controversial, as it suggests that recessions are times of technological regress.\footnote{This reasoning has lead many researchers to study models with other disturbances, which also mostly focus on first-moment (level) shocks. A partial list of these alternative shocks includes oil shocks, investment specific shocks, monetary shocks, government expenditure shocks, news shocks, and terms-of-trade shocks. Yet, in most models, negative technology shocks continue to be an important driver of economic downturns.} As discussed above, our work provides a rationale for falls in measured productivity. Countercyclical increases in uncertainty lead to a freeze in economic activity, substantially lowering productivity growth during recessions. In our model, however, the drop in productivity is not causing the recession, but rather an artifact of a recession that is caused in turn by an increase in uncertainty. Second, the paper relates to the literature on investment under uncertainty. A growing body of work has shown that uncertainty can directly influence firm-level investment and employment in the presence of adjustment costs.\footnote{See, for example; Bernanke (1983), Pindyck (1988), Dixit (1990), Bertola and Bentolilla (1990), Bertola and Caballero (1994), Dixit and Pindyck (1994), Abel and Eberly (1996), Hassler (1996), and Caballero and Engel (1999).} The most relevant paper is Bloom (2009) that solves a partial equilibrium model with stochastic
volatility and shows how high-frequency uncertainty shocks lead to drops in investment and hiring. Other linked papers included Justiniano and Primiceri’s (2008) work on lower frequency movements in volatility, Fernandez-Villaverde, Guerro, Rubio-Ramirez and Uribe’s (2009) paper on uncertainty and exchange rates, and Sims (2006) paper on uncertainty and business cycles. Third, the paper is builds upon a recent literature that studies role of micro-rigidities in general equilibrium macro models. Finally, the paper is related to the very recent literature trying to explain the recession starting in 2007, with our paper emphasizing the impact of uncertainty arising from the financial crisis.

Our empirical contribution to the existing literature is to measure the extent of time variation in uncertainty over the business cycle. We document a strong countercyclicality of uncertainty at the establishment, firm, industry and aggregate level. On the theoretical front, our paper has three key contributions. First, we show that in the presence of time-varying uncertainty, micro-rigidities of the type considered here (i.e., non-convex adjustment costs) have important general equilibrium effects. Second we show the significant quantitative impact of uncertainty shocks on the entire economy’s cyclical fluctuations. Finally, we use our work to show how policy effectiveness is reduced in the presence of time-varying uncertainty.

It is important to emphasize that we do not claim that other mechanisms that cause fluctuations in economic activity are irrelevant. Rather, we emphasize a specific mechanism in a model that encompasses the standard perfect competition RBC model, as this greatly simplifies comparison with existing work. Moreover, in the theoretical model we treat uncertainty as an exogenous shock. The advantage of this assumption is that it allows us to explore the effects of time variation in uncertainty in a tractable way, treating it similarly to the way the business cycle literature has analyzed other sources of shocks (e.g. TFP, government spending, monetary shocks).

The remainder of this paper is organized as follows. Section 2 discusses the behavior of uncertainty over the business cycle. In Section 3 we formally present the model, define the recursive equilibrium, and present our non-linear solution algorithm which builds on the work of Krusell and Smith (1998), Kahn and Thomas (2008) and Bachman, Caballero and Engel (2008). The model is calibrated and simulated in Section 4, where we study the role of uncertainty shocks in driving the business cycle. Section 5 studies the impact of policy shocks in the presence of time-varying uncertainty. Section 6 concludes.

5See for example, Thomas (2002), Veracierto (2002), Kahn and Thomas (2008), Bachman, Caballero and Engel (2008), and House (2008).
2 Measuring Uncertainty Over the Business Cycle

This section presents evidence on the cyclical behavior of uncertainty. Before presenting our empirical results, it is useful to briefly discuss what we mean by time-varying uncertainty in the context of our model.

A firm indexed by \( j \) produces output in period \( t \) according to the following production function

\[
y_{j,t} = A_t z_{j,t} f(k_{j,t}, n_{j,t})
\]

(1)

where \( k_{t,j} \) and \( n_{t,j} \) denote idiosyncratic capital and labor employed by the firm. Each firm’s productivity is a product of two separate processes: an aggregate component, \( A_t \), and an idiosyncratic component, \( z_{j,t} \).\(^6\) To match the empirical evidence for macro and micro uncertainty, we assume that both the aggregate and the firm-level components of productivity follow autoregressive processes:

\[
\log(A_t) = \rho^A \log(A_{t-1}) + \sigma^A_{t-1} \epsilon_{t}
\]

(2)

\[
\log(z_{j,t}) = \rho^Z \log(z_{j,t-1}) + \sigma^Z_{t-1} \epsilon_{j,t}
\]

(3)

We allow the variance of innovations to the productivity processes, \( \sigma^A_t \) and \( \sigma^Z_t \), to vary over time to generate periods of low and high macro and micro uncertainty.

There are two assumptions embedded in this formulation. First, the volatility in the idiosyncratic component of productivity, \( z_{j,t} \), implies that productivity dispersion across firms is time varying, while volatility in the aggregate component of productivity, \( A_t \), implies that all firms are affected by more volatile shocks. Second, given the timing assumption in (2) – (3), firms learn in advance that the distribution of shocks from which they will draw their productivity in the next period is changing. This timing assumption captures the notion of uncertainty that firms face.

As we argue in our theoretical section, these two shocks have different implications in terms of the statistics that are driven by them. Volatility in \( z_{t,j} \) implies that cross-sectional measures of firm performance (output, sales, stock market returns etc.) are time varying, while volatility in \( A_t \) induces conditional heteroskedasticity of aggregate variables. Next we turn to our cross-sectional and macro uncertainty measures, details of the construction of which are contained in Appendix A.

\(^6\)Note that we could also easily include demand shocks - most obviously at the micro level - into this specification by relabelling \( y_{j,t} \) as the revenue function and incorporating demand shocks into \( z_{j,t} \).
2.1 Cross-Firm and Cross-Industry Evidence

In this section we present a set of results showing that shocks at the establishment, firm and industry level all increase in variance during recessions. In our model in section 3 we focus on units of production, ignoring multi-establishment firms or industry level shocks to reduce computational burden. Nevertheless, we present data at these three different levels to demonstrate the generality of the increase in idiosyncratic shocks during recessions. We also show some data on forecaster disagreement, which can be taken as another proximate measure of uncertainty, despite again being something that is outside our formal model.

Establishment level evidence: The first measure combines data from the Census of Manufactures and the Annual Survey of Manufacturing, which yields data on annual establishment level shipments (output) from 1972 until 2006. To measure uncertainty we calculate the inter quartile range (IQR) of the growth rate of total value of shipments across establishments on a yearly basis.\(^7\) Row (1) of Table 1 reveals that this IQR of growth rates is 27.1% higher during quarters defined as recessionary by the NBER Business Cycle Dating Committee.\(^8\) This rise in the cross-sectional spread is also negatively correlated with real annual GDP growth, with a correlation of -0.570. Figure 1 depicts the time-series evolution of this cross-establishment shipments spread, plotted alongside grey bars reporting the share of quarters in a recession during each year. This clearly displays a large increase is cross-sectional spreads during the major recessions of the 1970s, 1980s and 2000s. Figure 2 reports the figure for establishments with 25+ years to control for potential sample composition effects, finding very similar results. Figure 2 also displays the IQR of shipments growth rates for establishments within their SIC 4-digit industry, again demonstrating a very similar counter-cyclical pattern. This reveals that most of the variance in establishment level shocks is within narrowly defined SIC 4-digit industries rather than due to large industry level shocks.\(^9\) Finally, in Figure 3 we plot the cross-sectional spread of estab-

\(^7\) Ideally we would use productivity data directly given that this is the underlying stochastic process driving our model. Unfortunately, the Census does not directly collect data on productivity. Productivity is instead estimated as the residual of shipments less factor-share weighted employment, materials and capital stocks, where capital stocks themselves are imputed from deflated capital expenditures using the perpetual inventory method. As an estimated residual productivity contains potentially substantial measurement error. So we use shipments growth rates as our cross-sectional proxy for productivity shocks. This is consistent with our model outlined in section 3 since productivity shocks drive output growth. Similarly, for firm and industry level data we also use sales or output data.

\(^8\) This quarterly increase is estimated by regressing the annual IQR on the share of quarters in a recessions within each year. The coefficient divided by the constant then reveals the average increase in uncertainty during a recessionary quarter.

\(^9\) This addresses the Abrahams and Katz (1986) critique of Lillien (1982). Lillien showed a strong correlation between cross-industry variation in unemployment and overall unemployment levels, arguing for a large role for structural unemployment during periods of rapid cross-industry movements of employment. Abrahams and Katz (1986) argued that it could instead be interpreted as a differential industry-level response to common negative macro shocks. Our results here show that most of the variation in the spread of establishment level output growth over the cycle is within SIC 4-digits.
lishments within firms, reported only for multi-establishment firms. Again this is clearly counter-cyclical, demonstrating that in recessions establishments experience a significantly higher level of within industry and firm output growth changes.

**Firm level evidence:** We move up one level of aggregation from establishments to firms, to examine the cross-sectional spread of firm-level sales growth rates. Row (2) of Table 1 shows that the cross-sectional spread of firm-level sales growth rates is 23.1% higher during recessionary quarters, in our sample of Compustat quoted firms spanning manufacturing and services. Figure 4 plots the evolution of the quarterly overall IQR, and the within SIC 2-digit IQR, again displaying that firm-level sales shocks increase in recessions and that this increase primarily occurs within rather than across industries.

To further evaluate whether this increase in firm-level sales spreads is an increase in predictable shocks or unpredictable shocks (i.e., uncertainty) we look at the cross-sectional spread of stock returns. The rationale behind this measure is the following. If the increase in sales is predictable, the stock-return dispersion should not change as shocks to sales would not contain news. If the sales shocks were news, however, the stock return variance should increase. Row (3) of Table 1 shows that the cross-sectional spread of firm-level stock returns is also clearly counter-cyclical, rising by 28.6% during recessionary quarters. Figure 5 plots the evolution of this quarterly stock returns and shows it rises steeply during recessions.

**Industry level evidence:** We move up another level of aggregation and look at the cross-sectional spread of industry-level sales growth rates. Specifically, we calculate the IQR of 3-month growth rates of industrial production in 196 manufacturing industries. Row (4) of Table 1 reveals that this measure is 66.9% higher during quarters defined as recessionary by the NBER Business Cycle Dating Committee. This rise in the cross-sectional spread is also negatively correlated with real GDP growth, with a correlation of −0.488. Figure 6 depicts the time-series evolution of this spread.

An issues that arises with this industry measure is that it may simply reflect differential responses of industries to a common macro shock. For example, during recessions luxury consumer good industries may see a larger decrease in sales than basic food industries. To address this concern, we regress each industry’s monthly industrial production growth on its 12 lags plus the aggregate industrial growth rate. We then calculate the cross-sectional dispersion of the regression residuals, to compute the "surprise" micro element of the shock. The rationale is that we are capturing the unforecastable part by analyzing the residuals from the forecasting equation, and excluding any predictable cyclical component. This, of course, hinges on the quality of the forecasting equations which indeed have an average $R^2$ of 0.79 across the different industry regressions. The resulting cross-sectional dispersion of the residuals from the forecasting equations is also highly countercyclical, rising by 41% during recessionary quarters and correlated with GDP growth at -0.478. This suggests that
most of this industry level variation in growth changes is not easily forecastable.

Staying at the industry level, Figure 7 plots the evolution of different percentiles from the distribution of the growth rates of industrial production within each quarter. Note that recessions are clearly characterized not only by a downward shift of the distribution of all firms, but also by a widening of the distribution. This is evident from the behavior of the 1st and 99th percentile.

Forecaster level evidence: Finally, two related but indirect measures we construct are the extent of disagreement between forecasters over future industrial production unemployment. These measures are relevant if they are due to different forecasters observing different signals which relate to different realizations of the $z_{j,t}$ shock. In rows (5) and (6) of Table 1 we see that the dispersion of professional forecasts over future industrial production and unemployment increases substantially during recessions, rising by 60.7% and 68.6%, respectively. The two measures of forecaster disagreement are shown in Figures 8 and 9.

2.2 Macroeconomic Measures of Uncertainty

The results discussed so far capture the idiosyncratic industry and firm level components. We now report measures that capture the common macroeconomic component of uncertainty. As we argued above, our theoretical model implies that it is mainly the volatility in the shocks to the aggregate component of productivity, $A_t$, that induces conditional heteroskedasticity in aggregate variables. We thus first analyze a $GARCH(1,1)$ estimator for quarterly log(GDP) including as many as four lags. In row (7) of Table 1 we see that the predicted standard deviation from the estimation rises by 37.5% during recessions on average. Figure 10 plots this measure.

Similarly, in row (8) of Table 1 we look at an index of stock market volatility, and find that recessionary quarters are associated with a 43.5% higher volatility of stock market returns. Figure 11 plots the evolution of this measure. To summarize, both output and

---

10 There is an extensive empirical literature that argues in favor of using disagreement among macro forecasts — as measured by mean forecast error — as a proxy for uncertainty (e.g. Zarnowitz and Lambros (1987), Bomberger (1996), and Giordani and Soderlind (2004)). Our model does not explicitly treat this issue and it is important to note that cross-sectional forecaster disagreement and uncertainty are not necessarily correlated in theoretical models (e.g. Amador and Weill (2008)).

11 We also estimated a $GARCH(1,1)$ for monthly industrial production, including as many as twelve lags and find very similar results. We also experimented with different specifications — such as ARCH(1) or using GDP growth rates — and results again were very similar.

12 The index is constructed from the S&P 100 implied volatility (the VXO) from 1987 onwards, and normalized realized volatility of actual S&P100 daily stock returns prior to 1986 (the VXO is not available prior to 1987). During the periods where both the VXO and the actual realized volatility are available they are correlated at 0.874.

Schwert (1989) and Hamilton and Lin (1996) both provide evidence that stock market volatility is much higher during recessions. Engle and Rangel (2006) look at data from 48 countries (developed and developing) and find similar results for this panel (stock market volatility is significantly higher when GDP growth is
stock market data suggest that macro uncertainty is substantially higher during recessions.

3 The General Equilibrium Model

We proceed by analyzing the quantitative impact of variation in uncertainty within a dynamic stochastic general equilibrium model where heterogeneous firms are subject to both first and second moment shocks. In the model, each firm uses capital and labor to produce a final good. Firms that adjust their capital stock and employment incur non-convex adjustment costs. As is standard in the RBC literature, firms are subject to an exogenous process for productivity. We assume that the productivity process has an aggregate and an idiosyncratic component. In addition to these first-moment shocks, we allow the second moment of the innovations to productivity to vary over time. That is, shocks to productivity can be fairly small in normal times, but become potentially large when uncertainty is high.

3.1 Firms

3.1.1 Technology

The economy is populated by a large number of heterogeneous firms that employ capital and labor to produce a single final good. We assume that each firm operates a diminishing returns to scale production function with capital and labor as the variable inputs.\footnote{An alternative model has a setup of monopolistically competitive firms in which each firm produces a differentiated good. Note that the assumption of decreasing returns to scale implies that there is a fixed factor of production that pins down firm size.}

Specifically, a firm indexed by $j$ produces output according to

$$y_{j,t} = A_t z_{j,t} k_{j,t}^{\alpha} n_{j,t}^{\nu}, \quad \alpha + \nu < 1.$$  \hfill (4)

Each firm’s productivity is a product of two separate processes: aggregate productivity, $A_t$, and an idiosyncratic component, $z_{j,t}$. Both the macro- and firm-level components of productivity follow autoregressive processes as noted in equations (2) and (3). We depart from the benchmark RBC model in that we allow the variance of innovations to the productivity processes, $\sigma^4_t$ and $\sigma^2_t$, to vary over time as noted in equations (??) and (??).
3.1.2 Capital and Labor Adjustment Costs

We allow for the presence of various types of convex and non-convex adjustment costs in capital and labor.\(^\text{14}\) With respect to capital, we assume that a firm’s capital stock evolves according to the standard law of motion

\[
\gamma k_{j,t+1} = (1 - \delta_k)k_{j,t} + i_{j,t}
\]

(5)

where the \(\gamma - 1\) is the trend growth rate of output and \(\delta_k\) is the rate of capital depreciation.

The first adjustment cost we allow for involves a non-convexity — conditional on undertaking an investment, a fixed cost \(F^K\) is incurred independently of the scale of investment. The second capital adjustment cost we consider is a partial irreversibility. Resale of capital occurs at a price that is only a share \((1 - S)\) of its purchase price.

Similarly, we assume that the law of motion for hours worked is governed by

\[
n_{t,t} = (1 - \delta_n)n_{j,t-1} + s_{j,t}.
\]

(6)

At each period a constant fraction \(\delta_n\) of hours worked is exogenously destroyed due to retirement, illness, maternity leave, exogenous quits, etc. Whenever the firm chooses to adjust its stock of hours relative to \((1 - \delta_n)n_{j,t-1}\), it incurs a fixed cost \(F^L\) independently of the size of the change in hours. We also allow for hiring and firing costs which represent, for example, variable interviewing and training costs or severance packages. In our model, we assume that this cost is identical for hiring and firing and expressed as a share \(H\) of the annual wage bill per worker.

3.1.3 The Firm’s Value Function

We denote by \(V(k, n_{-1}, z; A, \sigma^A, \sigma^Z, \mu)\) the value function of a firm. The seven state variables are given by (1) a firm’s capital stock \(k\), (2) a firm’s hours stock from the previous period \(n_{-1}\), (3) the firm’s idiosyncratic productivity \(z_{j,t}\), (4) aggregate productivity \(A_t\), (5) macro uncertainty \(\sigma^A_t\), (6) micro uncertainty \(\sigma^Z_t\) and (7) the joint distribution of idiosyncratic productivity and firm-level capital stocks and hours worked in the last period \(\mu_t\), which is defined for the product space \(S = Z \times R_+ \times R_+\).

The dynamic problem of the firm consists of choosing investment and hours to maximize

\(^{14}\)See the literature focused on estimating labor and capital adjustment costs, including, Nickell (1986), Caballero and Engel (1999), Ramey and Shapiro (2002), Hall (2004), Cooper and Haltiwanger (2006), Merz and Yashiv (2007), and Bloom (2009).
the present discounted value of future profits

\[
V(k, n_{-1}, z; A, \sigma^A, \sigma^Z, \mu) = \\
\max_{i,n} \left\{ \begin{array}{l}
y - w(A, \sigma^A, \sigma^Z, \mu)n - i \\
-AC^k(k', k) - AC^n(n_{-1}, n) \\
+ \mathbb{E} \left[ m \left( A, \sigma^A, \sigma^Z, \mu; A', \sigma^A, \sigma^Z, \mu' \right) V(k', n, z'; A', \sigma^A, \sigma^Z, \mu') \right] \end{array} \right\}
\]  

(7)

given a law of motion for the joint distribution of idiosyncratic productivity, capital and
hours,

\[
\mu' = \Gamma(A, \sigma^A, \sigma^Z, \mu),
\]  

(8)

and the stochastic discount factor, \( m \). We denote by \( AC^k(k, k') \) and \( AC^n(n_{-1}, n) \) the
capital and labor adjustment cost functions, respectively. \( K(k, n_{-1}, z; A, \sigma^A, \sigma^Z, \mu) \) and
\( N^d(k, n_{-1}, z; A, \sigma^A, \sigma^Z, \mu) \) denote the policy rules associated with the firm’s choice of capital
for the next period and current demand for hours worked.

### 3.2 Households

The economy is populated by a large number of identical households that we normalize to
a measure one. Households choose paths of consumption, labor supply, and investments in
firm shares to maximize lifetime utility. We use the measure \( \phi \) to denote the one-period
shares in firms. The dynamic problem of the household is given by

\[
W(\phi, A, \mu) = \max_{\{C, N, \phi\}} \left\{ U(C, N) + \beta \mathbb{E} [W(\phi', A', \mu')] \right\}
\]  

(9)

subject to the law of motion for \( \mu \) and a sequential budget constraint

\[
C + \int q(k', n, z; A, \sigma^A, \sigma^Z, \mu)\phi'(dkdn) \leq w(A, \sigma^A, \sigma^Z, \mu)N + \int \rho(k, n_{-1}, z; A, \sigma^A, \sigma^Z, \mu)\phi(dkdn).
\]  

(10)

Households receive labor income as well as the sum of dividends and the resale value of
their investments, \( V(k, n_{-1}, z; A, \sigma^A, \sigma^Z, \mu) \). With these resources the household consumes
and buys new shares at a price \( q(k, n_{-1}, z; A, \sigma^A, \sigma^Z, \mu) \) per share of the different firms in
the economy. We denote by \( C(\phi, A, \mu), N^s(\phi, A, \mu), \Psi(k', n, z; A, \sigma^A, \sigma^Z, \mu) \) the policy rules
determining current consumption, time worked, and quantities of shares purchased in firms
that begin the next period with a capital stock that equals \( k' \) and who currently employ \( n \)
hours, respectively.
3.3 Recursive Competitive Equilibrium

A recursive competitive equilibrium in this economy is defined by a set of quantity functions \( \{C, N^s, \Psi, K, N^d\} \), pricing functions \( \{w, q, \rho, m\} \), and lifetime utility and value functions \( \{W, V\} \). \( V \) and \( \{K, N^d\} \) are the value function and policy functions solving (7) while \( W \) and \( \{C, N^s, \Psi\} \) are the value function and policy functions solving (9). There is market clearing in the asset markets

\[
\Psi(k', n, z; A, \sigma^A, \sigma^Z, \mu) = \mu(z, k', n) \text{ for every triplet } (z, k', n) \in S,
\]

the goods market

\[
C(\phi, A, \mu) = \int_S \left[ A\alpha N^\rho(k, n_{-1}, z; A, \sigma^A, \sigma^Z, \mu)^\rho - (K(k, n_{-1}, z; A, \sigma^A, \sigma^Z, \mu) - (1 - \delta_k)k) \right] \mu(dkdndz),
\]

and the labor market

\[
N^s(\phi, A, \mu) = \int_S [N^d(k, n_{-1}, z; A, \sigma^A, \sigma^Z, \mu)] \mu(dkdndz).
\]

Finally, the evolution of the joint distribution of \( z, k \) and \( n \) is consistent. That is, \( \Gamma(A, \sigma^A, \sigma^Z, \mu) \) is generated by \( K(k, n_{-1}, z; A, \sigma^A, \sigma^Z, \mu) \), \( N^d(k, n_{-1}, z; A, \sigma^A, \sigma^Z, \mu) \), and the exogenous stochastic evolution of \( A, z, \sigma^Z \) and \( \sigma^A \) with the appropriate summation of firms’ optimal choices of capital and hours worked given current state variables.

3.4 Sketch of the Numerical Solution

The model can be simplified substantially if we combine the firm and household problems into a single dynamic optimization problem as in Kahn and Thomas (2008). From the household problem we get

\[
w = \frac{U_N(C, N)}{UC(C, N)} \quad (11)
\]

\[
m = \beta \frac{UC'(C', N')}{UC(C, N)} \quad (12)
\]

where equation (11) is the standard optimality condition for labor supply and equation (12) is the standard expression for the stochastic discount factor. To ease the burden of computation it is useful to assume that the momentary utility function for the household
is separable across consumption and hours worked,

\[ U(C_t, N_t) = \frac{C_t^{1-\eta}}{1-\eta} - \theta \frac{N_t^\chi}{\chi}, \tag{13} \]

implying that the wage rate is a function of the marginal utility of consumption,

\[ w_t = \phi N_t^{\chi-1} \frac{\theta}{C_t^{1-\eta}}. \tag{14} \]

Kahn and Thomas (2008) and Bachmann, Caballero and Engel (2008) define the intertemporal price of consumption goods as \( p(A, \sigma^Z, \sigma^A, \mu) \equiv UC(C, N) \). Using this approach, we can redefine the firm problem in terms of marginal utility, denoting the new value function as \( \bar{V} \equiv pV \). The firm problem can then be expressed as

\[
\bar{V}(k, n_{-1}, z; A, \sigma^A, \sigma^Z, \mu) = \\
\max_{\{i,n\}} \left\{ \begin{array}{c}
p(A, \sigma^A, \sigma^Z, \mu) \left( y - w(A, \sigma^A, \sigma^Z, \mu) - i - AC^k(k, k') - AC^n(n_{-1}, n) \right) \\
+ \beta \mathbb{E} \left[ \bar{V}(k', n', z'; A', \sigma^{A'}, \sigma^{Z'}, \mu') \right] \end{array} \right\} \tag{15} 
\]

We employ non-linear techniques that build upon Krusell and Smith (1998). Specifically, we summarize \( \mu \) with a small set of moments of the firm distribution which we denote by \( \Omega \). Specifically, in each iteration we preform four steps. We first forecast the intertemporal price \( \hat{p} \) and next period’s moments \( \hat{\Omega}' \) as functions of the current aggregate state:

\[
\hat{p} = f_1^{(l)}(A, \sigma^A, \sigma^Z, \Omega) \\
\hat{\Omega}' = f_2^{(l)}(A, \sigma^A, \sigma^Z, \Omega) 
\]

Assuming that \( \chi = 1 \) (we discuss the choice of this parameter value below) we get for a given forecast of \( \hat{p} \), the current period wage \( w \) from (14). We can then find the value function \( \bar{V}^t \) associated with those forecasting functions by solving (15) substituting the approximated state \( \hat{\Omega} \) for the joint distribution \( \mu \) and \( f_2^{(l)} \) for the law of motion \( \Gamma \). We then simulate the economy for many periods during which the forecasting rule for the intertemporal price is not used. Rather, in each period the market clearing price \( p_t \) is calculated as the price that combines firm optimization and goods market clearing. For a given price, the simplified firm optimization problem becomes

\[
\max_{\{i,n\}} \left\{ p \left( y - wn - i - AC^k(k, k') - AC^n(n_{-1}, n) \right) + \beta \mathbb{E} \left[ \bar{V}^t(k', n, z'; A', \sigma^{A'}, \sigma^{Z'}, \hat{\Omega}') \right] \right\} 
\]

which uses the value function calculated in the second step and the moment forecasting
function from the first step. Market clearing is achieved when aggregation of the optimal policies from this problem yield market clearing in the goods market

\[ C = \int \left( y + i - AC^h - AC^a \right) \mu(dkdndz). \]

This simulation yields sequences of exogenous states \( \{A_t, \sigma_t^A, \sigma_t^Z\} \) states, prices \( \{p_t\} \) and moments \( \{\Omega_t\} \). As the final step we then update the forecasting functions \( f_1^{(l+1)} \) and \( f_2^{(l+1)} \) from the observed moments and equilibrium prices and restart the algorithm at the first step. We iterate until the forecasting functions converge.\(^{15}\)

4 Simulation

This section motivates the choice of parameter values used in the simulations (see Table 2) and also presents simulation results for our preferred specification.

4.1 Calibration

4.1.1 Frequency and Preferences

We set the time period to equal a quarter and the household’s discount rate, \( \beta \), is calibrated to 0.985. \( \eta \) is set equal to one which implies that the momentary utility function features an elasticity of intertemporal substitution of one. Following Kahn and Thomas (2008) and Bachmann, Caballero and Engel (2008) we make the simplifying assumption that the Frisch labor supply elasticity is infinite, corresponding to \( \chi = 1 \). This assumption implies that we do not need to forecast the wage rate in addition to the forecast of \( p \) because when \( \chi = 1 \) we get

\[ w_t = \frac{\theta}{C_t} = \frac{\theta}{p} \]

Hence, once we construct a forecast for \( p \), we immediately obtain a forecast for \( w \), eliminating the need to forecast it separately and simplifying the computational problem. We set the parameter \( \theta \) such that households spend a third of their time working in the non-stochastic steady state. The trend growth rate of per capita output is set to equal 1.6% annually.

\(^{15}\)For the forecasting functions, we use the aggregate productivity state, the first moments of the distribution over capital and labor as well as the aggregate uncertainty state. Interestingly, this provides a very good fit and an \( R^2 \) of above 0.985. Additional moments of the distributions over capital, labor and idiosyncratic productivity could be added to the forecasting functions. We are currently working on extending the model in this way, but the computational burden quickly becomes very large.
4.1.2 Production Function, Depreciation, and Adjustment Costs

We set $\delta_k$ to match a 10% annual capital depreciation rate. The annual exogenous quit rate of labor is a key parameter set to 15%. This estimate is based on the quit rate reported in the Bureau of Labor Statistic JOLTS data.\footnote{JOLTS stands for Job Openings and Labor Turnover Data, which the BLS has been collecting since January 2001. Hence, this data spans two NBER defined recessions. It distinguishes between quits, layoffs, and other separations. Our figures are seasonally adjusted for total private employment. In JOLTS, the monthly quit figure varies between 1.6% and 2.4%, with the lowest value occurring in November 2008 during the depths of the recent recession. Annualizing the November 2008 quit rate we get a value of 19.2%. Our calibration using a lower value of 15% is thus a conservative calibration.} We set the exponents on capital and labor in the firm’s production to be $\alpha = 0.25$ and $\nu = 0.5$, consistent with a capital cost share of $1/3$ and a 33% markup when the firm faces an iso-elastic demand curve.

The existing literature provides a wide range of estimates for capital and labor adjustment costs.\footnote{See, for example, Hayashi (1982), Nickel (1986), Shapiro (1986), Caballero and Engel (1999), Ramey and Shapiro (2001), Hall (2004), Cooper, Haltiwanger and Willis (2004), Cooper and Haltiwanger (2006) as well as Mertz and Yashiv (2007).} We set our adjustment cost parameters to match Bloom (2009), which to our knowledge is the only paper that jointly estimates capital and labor convex and non-convex adjustment costs. Fixed costs of capital adjustment are set to 1.5% of annual sales, and the resale loss of capital amounts to 40%. The fixed cost of adjusting hours, is set to 2.1% of annual wages, and the hiring and firing costs equal 1.8% of annual wages.

4.1.3 Aggregate and Idiosyncratic TFP Processes

Productivity both at the aggregate and the idiosyncratic level is determined by AR1 processes as specified in equations (2) and (3). The serial autocorrelation is taken directly from Khan and Thomas (2008) and adjusted to the quarterly frequency. Hence, $\rho^A$ and $\rho^Z$ are set to yield an annual persistence parameter of 0.859. In our model, the variance of the innovations to these processes is time-varying. The exact calibration is presented in some detail in the subsequent paragraph, but on average, $\sigma_t^A$ and $\sigma_t^Z$ are set to 1.59% and 8.50%. We approximate the autoregressive processes with Markov chains. The support for the processes are set to include three standard deviations on either side of the mean.

4.1.4 The Calibrated Process for Uncertainty

In the benchmark calibration we assume that the uncertainty process is independent of the first-moment shocks. This implies that we are not artificially creating the drop in economic activity following a second-moment shock by correlating it with the first-moment shock.

We assume for simplicity that the stochastic volatility processes, $\sigma_t^A$ and $\sigma_t^Z$, each follow...
a two-point Markov chain

\[
\begin{align*}
\sigma^A_t &\in \{\sigma^A_L, \sigma^A_H\} \quad \text{where } \Pr(\sigma^A_{t+1} = \sigma^A_j | \sigma^A_t = \sigma^A_k) = \pi^A_{k,j} \\
\sigma^Z_t &\in \{\sigma^Z_L, \sigma^Z_H\} \quad \text{where } \Pr(\sigma^Z_{t+1} = \sigma^Z_j | \sigma^Z_t = \sigma^Z_k) = \pi^Z_{k,j}
\end{align*}
\]

(16) (17)

Since we cannot directly observe the stochastic process of uncertainty in the data the calibration has to be guided the impact of uncertainty on observable cross sectional and aggregate time series moments. There are eight parameters that need to be calibrated: \(\sigma^A_L, \sigma^A_H, \sigma^Z_L, \sigma^Z_H, \sigma^A_{L,H}, \sigma^A_{H,L}, \sigma^Z_{L,H}\) and \(\sigma^Z_{H,L}\). The empirical section (2) suggests that uncertainty at the micro and at the macro level is highly correlated. So as a simplification to ease computational constraints we assume in the benchmark calibration that a single process determines the economy’s uncertainty regime. This reduces the number of parameters to six: \(\sigma^A_L, \sigma^A_H, \sigma^Z_L, \sigma^Z_H, \sigma^A_{L,H} \) and \(\sigma^Z_{H,L}\) since \(\sigma^A\) and \(\sigma^Z\) follow the same Markov process (with different levels, of course).

With these six parameters we try to match eight moments. The first four are based on the cross-sectional IQR of sales growth rates in the establishment data, which is our largest and most representative micro dataset. We calculate the mean, standard deviation, skewness and serial correlation of this IQR time series, which characterizes how much micro-uncertainty changes over time. These four moments are reported in the first column in the top panel of Table 3. The second set of moments is based on the GARCH(1,1) estimated conditional heteroskedasticity of GDP growth shown in Figure 10. We again calculate the mean, standard deviation, skewness and serial correlation of this series, which characterizes how much macro-uncertainty changes over time. The first column in the lower panel of Table 3 reports these four moments.

We calibrate our uncertainty parameters by aligning these eight moments from the actual data with the simulated data. In particular, in order to get the first four micro moments we run the simulation and then aggregate the firm-level values to an annual value. Using this we construct the cross sectional IQR of these growth rates, and then generate the mean, standard-deviation, skewness and serial correlation of this series. Similarly, in order to get the four macro moments we simulate the model, estimate a GARCH(1,1) process on log(GDP) and four lags, and generate the mean, standard-deviation, skewness and serial correlation of this. We thus try to match the simulated counterpart of each empirical moment to calibrate the underlying uncertainty process. In order to understand how robust the estimates are, we recalculate the exact same eight moments using slightly different data samples. Specifically, the second and third column in Table 3 show the minimum and maximum value that was recorded for each moment when either the first or last five years are dropped from the sample. This immediately highlights that the skewness is obviously
difficult to measure in short samples and varies significantly in between variations of the sample.

Based on our preferred calibration we find that periods of high uncertainty occur with a quarterly probability of slightly below 5%. The period of heightened uncertainty is quite persistent with a quarterly probability of 88.5% of staying in the high uncertainty state. Idiosyncratic volatility is set to a lower value of 6.7%, but almost doubles in the heightened uncertainty state. Aggregate volatility is at a low of 0.81%, but more than quadruples when an uncertainty shock hits. The baseline calibration yields moments that are shown in column four of Table 3. We are obviously constrained by having to match eight moments with only six parameters. As a result, some of our moments are difficult to be matched both at the micro and macro level. Specifically, the skewness is too low at the micro and too high at the macro level. Conversely, the serial autocorrelation is too high for the simulated micro process at the micro too low at the macro level. Even though the fit is not perfect we find the results to be surprisingly in accordance with the data.

The last two columns of Table 3 report the simulated moments for two alternative specifications of the model. In column five, $\sigma^A$ does not vary and is instead set to its long run average value. Interestingly, the resulting micro moments are very similar to those obtained in the benchmark calibration. Turning off macro uncertainty, for instance, only slightly reduces the estimated standard deviation of the micro measure from 3.53% to 3.34%. Analogously, the calibration in column six sets $\sigma^Z$ to its long run average and again there seems to be only a small effect on the resulting macro moments. While the two channels are not completely independent of each other, the cross effects are rather small. This suggests that our approach of linking the process of micro uncertainty to dispersion measures from establishment data and macro uncertainty to the conditional heteroskedasticity of aggregate output is reasonable.

### 4.2 The Effects of an Uncertainty Shock

We first study the effects of an isolated increase in uncertainty. We simulate the model economy 500 times with 4000 firms, and let each simulation run for 250 periods to initialize the distribution over $z$, $k$ and $n$. We then force uncertainty to be low for 10 periods. Finally we induce an uncertainty shock in time period zero. This is to mimic a typical business cycle shock to uncertainty that occurs after a period of low uncertainty. We average over the 500 simulated economies to yield an average effect of a business cycle sized increase in uncertainty.

Before showing the simulation results it is important to note that the unweighted average of firm level productivity $\sum_j N (A_t z_{j,t}) / N$ does not vary in this experiment as shown in Figure 12. Thus, the results shown on the figures below are driven entirely by changes in
uncertainty. This uncertainty shock leads to three phases of activity:

*The drop:* When uncertainty rises in period zero, the investment and hiring thresholds move out as the real options effect leads firms to defer spending and hiring projects. That is higher uncertainty makes firms cautious as they don’t want to make a costly hiring or investment mistake. So because most firms pause hiring there is an immediate fall in aggregate hours worked. The fall in aggregate hours worked manifests itself into a drop in aggregate output, as can be seen in Figure 13. Output falls by almost 2% upon impact and continues to fall for another quarter to a low of about 2.1% from the effects of further falls in hours and the drop in capital from the pause in investment in period 0. Both hours and capital falls are driven by exogenous quits and depreciation - that is workers leaving (for retirement, sickness, maternity etc.) and capital depreciating that are not replaced.

*The rebound:* By the second quarter uncertainty has fallen sufficiently, and firms built up enough pent-up demand for hiring and investment that output begins to rebound. During this transition the thresholds slowly begin to move back in as more economies leave the high uncertainty state. At the same time the distribution of firm specific productivity fans out, so that more and more firms begin hitting the new, wider thresholds, accelerating the rebound.

*The overshoot:* By the fourth quarter the economy has rises above its long-run trend for a few quarters before returning to it’s long-run average. The reason for this overshooting is that many firms are bunched near their $S_s$ investment and hiring thresholds due to depreciation, labor attrition and trend growth. So small increases in productivity causes firms to hit those hiring and investment thresholds, while small decreases in productivity move them towards the interior of their $(S, s)$ bands. As a result, the increased variance of idiosyncratic productivity shocks induced by higher uncertainty increases aggregate medium-run hiring and investment. That is, most firms that receive a positive productivity shocks hit their $(S, s)$ bands and invest and hire, while most firms that receive a negative shock move to the interior of the $(S, s)$ bands and do nothing.

In the top left panel of Figure 14 we show hours over the cycle which fall immediately after the shock arrives because we assume hiring happens instantaneously (capital has one quarter time to build). So hours fall by about 2.4% on impact, begin to recover after two periods and overshoot from 4 onwards. The uncertainty shock also induces a similar drop and subsequent rebound in investment, as shown in the top right panel of Figure 14.

The lower right hand panel plots the time profile of consumption. When the uncertainty shock occurs in period zero, consumption jumps up immediately and then falls below trend for about three quarters. The reason for the initial spike in consumption is that the freeze in investment and hiring reduces the resources spent on capital and labor adjustment. Since the interest rate drops upon impact of the shock, consumers are signaled that consumption
is cheap, which leads to an increase in consumption in period zero. In other words, even though consumers know they face higher uncertainty in the future and they would like to save more, they do not increase savings in the first period because the returns to saving have become (temporarily) low and very risky.\textsuperscript{18}

Finally, the lower left hand panel plots the value of aggregate productivity, defined as \( \sum_j^N (A_t z_{j,t} n_{j,t}) / \sum_j^N n_{j,t} \). Productivity also has a clear drop and subsequent rebound following the uncertainty shock, despite the fact that the average micro and macro productivity shocks are unchanged, as shown in Figure 12. The reason is that uncertainty freezes the reallocation of capital and labor from low- to high-productivity firms. In normal times, unproductive firms contract and productive firms expand, helping to maintain high productivity levels. When uncertainty is high, firms reduce expansion and contraction, shutting off much of this productivity-enhancing reallocation, leading to a fall in productivity growth rates. When uncertainty reverts back to normal, firms rapidly address their pent-up demand for reallocation so that productivity returns to its long-run trend.

A second-moment shock induces a fall in investment, hours, and output. Intriguingly, aggregate productivity falls even though this is an effect rather than a cause of the drop in economic activity. Consumption also exhibits a fall from quarter two onwards, although there is an initial one-period jump in the basic model.

### 4.3 Model Simulations

#### 4.3.1 Business cycle statistics

We have shown that our model can generate expansions and contractions in response to an increase in uncertainty. One natural question is whether this success comes at a cost of the model’s ability to generate empirically recognizable business fluctuations. That is, can the model, when calibrated with the same parameters as used in the experiments discussed so far, generate levels of comovement and volatility of macroeconomic aggregates that are empirically plausible? To answer this question, we simulate our model for 4000 periods and compute the standard set of business cycle statistics.

Table 4 illustrates that the calibration generates second-moment statistics (panel 2) that resemble their empirical counterparts in U.S. data (panel 1). Investment is more volatile than output, while consumption is less volatile. Investment and hours comove with output. Similar to many variants of the RBC model, hours are not as volatile relative to output in

\textsuperscript{18} This logic suggests that if we extended the model to allow for some alternative savings technology – for example, inventories or savings abroad – this initial spike in consumption would disappear as the representative consumer would just increase savings through this channel when the uncertainty shock hit to reduce the subsequent drop in consumption. Due to computational constraints we cannot currently increase the state space of the model in this way.
the model as they are in the data. Panel 3 of the same table reports the same statistics for an alternative calibration of the model without aggregate productivity shocks and an otherwise identical calibration. While this is only a crude experiment, it is interesting to note that time-variation in micro uncertainty can produce about 30% of the volatility in output that we find using the benchmark model.

4.3.2 Establishment Level Moments

Time averaged moments of establishment level investment rates provide an alternative way to evaluate the empirical realism of our model. Table 5 illustrates this idea. The first row contains a set of moments that are taken directly from previous work by Cooper and Haltiwanger (2006). They construct annual investment rates using data on capital retirements and investment for a balanced panel of plants from the Longitudinal Research Database (LRD). This table groups plants as investing (investment rate greater or equal than 0.01), showing an investment spike (i/k greater than 0.2) or inactive (absolute i/k < 0.01).

The rest of the table shows those same moments from simulations of our baseline model aggregated to the annual frequency. The four rows within each panel refer to different levels of unit aggregation. For example, the second row in panel 2 shows results for plants that contain 10 independent production lines. The reason for displaying various levels of aggregation is that the establishments in the Cooper and Haltiwanger (2006) are a balanced panel of large continuing plants with an average of almost 600 employees, compared to about 10 employees for the average of all Census establishments. Hence, in some senses these moments are for more aggregated production units.

Our model can match those moments relatively well. Depending on the exact degree of unit aggregation, our model can easily match the share of plants that are inactive, investing and showing an investment spike. As alternatives to further improve the fit the model could be extended to allow small adjustments to the capital and labor stock without adjustment costs as in Khan and Thomas (2008) or by introducing maintenance investment as in Bachmann, Caballero and Engel (2008).

4.4 Inspecting the Mechanism

We report in this subsection five experiments that are meant to highlight the different forces at work in our model. First, we consider alternative specifications of adjustment costs. Here, we simulate models with only capital capital adjustment costs, only labor adjustment costs and no adjustment costs at all. Second, we are interested in the differential effect of uncertainty at the macro and micro level. We thus simulate economies with time variation in only one of the two.
Adjustment Costs: To highlight the differential effects of adjustment costs in capital and labor, we use alternative calibrations that turn off adjustment costs in either capital or labor. Figure 15 illustrates the effects of an uncertainty shock for those alternative cases. In the model with adjustment costs in capital only, there is no impact of an uncertainty shock in period 0 because of the time-to-built assumption in capital. Investment, however, falls immediately and output thus falls in the subsequent period. The drop in output is smaller at a maximum of almost 1% as opposed to 2% in the baseline model. A model with adjustment costs in labor only, produces a slightly bigger fall in aggregate output by about 1.3%. In this case, the drop occurs in the period the uncertainty hits as the hiring freeze affects the labor input in production in the same period.

Figure 15 also shows that when there are no adjustment costs of any type in the economy, economic activity actually increases following an uncertainty shock. The reason for this result is related to the Hartman (1976) and Abel (1983) effect whereby a higher variance of productivity increases investment, hiring and output because the marginal revenue product of capital and labor is convex in productivity.\(^\text{19}\)

Only Micro or Only Macro Uncertainty: We perform two additional experiments where we consider an economy that exhibits either only micro or only macro uncertainty. In those experiments, we turn off time variation in one of the two uncertainty processes.\(^\text{20}\) Those experiments are reported in Figure 16. As the figure suggests the presence of both macro and micro uncertainty magnifies the effect of the uncertainty shock. Note however that each shock separately induces a similar effect on output.

The main difference between micro and macro uncertainty shocks is in the recovery and overshoot. The recovery is faster and the overshoot larger in the case of only micro uncertainty relative to the only macro uncertainty case. The reason for this is that is it the cross-sectional spread of shocks that generates the overshoot for the baseline uncertainty simulation arises primarily from the micro-uncertainty. So with macro uncertainty there is very little overshoot.

5 Policy in the Presence of Uncertainty

In this section, we analyze the effects of stimulative policies in the presence of uncertainty shocks. It is important to emphasize that any such policy is not optimal within the context

\(^{19}\)To be precise if \(Y = AK^aL^b\) and the per period rental cost of capital is \(r\) and labor is \(w\), then the without adjustment costs the optimal choice of \(K\) and \(L\) are \(K^* = \phi_1 A^{-\frac{1}{a+b}}\) and \(L^* = \phi_2 A^{-\frac{1}{a+b}}\) where \(\phi_1\) and \(\phi_2\) are functions of \(a, b, r\) and \(w\). Hence, it is clear that \(K^*\) and \(L^*\) are convex in \(A\) so that higher variance in \(A\) will increase the average levels of \(K\) and \(L\), which is commonly known as the Hartman-Abel effect after it was pointed out in Hartman (1972) and refined by Abel (1983).

\(^{20}\)Those experiments are indeed identical to the ones mentioned in column five and six of Table 3 in the calibration section.
of our model as the competitive equilibrium is Pareto optimal. Rather, we see our policy experiment as a means of documenting that in this framework a given policy can be less effective in times of heightened uncertainty. The reason is that during times of increased uncertainty, firms are far away from their hiring and investment thresholds, making them less responsive to the policy stimulus. Our quantitative model thus allows us to shed some light on the effectiveness of the policy as opposed to its desirability.

We are interested in a policy that attempts to temporarily stimulate hiring by reducing the effective wage paid by firms. More specifically, the policy consists of an unanticipated 1% wage bill subsidy paid for a period of two quarters. We simulate this policy impulse once during an uncertainty shock and also in an economy that is not hit by an uncertainty shock. By comparing the effect in those two cases, we can attempt to identify the dampening effect of uncertainty on policy effectiveness.\footnote{In this version of the experiment, we abstract from balanced budget considerations. Those can obviously have important general equilibrium effects, but we are focusing here on the relative effectiveness of a stimulative policy in normal and highly uncertain times and not on the absolute size of the policy multiplier.}

Figure 17 illustrates our experiment and depicts the response of output to a temporary wage subsidy that takes place at period zero. The green line (marked with a cross) refers to the case with no accompanying uncertainty shock. Not surprisingly, the artificially reduced wage stimulates hiring and increases output for two quarters which then gradually returns to its long run trend. The red line (squares) shows the response of output to such a policy when the policy is introduced at the exact time that the uncertainty shock hits the economy. The uncertainty shock will still result in an immediate drop in output and a subsequent rebound an overshot. To ease comparison, the figure also shows the effect of an uncertainty shock on output in the absence of any policy (blue, circles). The difference between the second and the third line can be interpreted as the policy’s impact policy during an uncertainty shock.

Figure 18, compares the policy’s impact on the same scale. The blue line (marked with a circle) shows the marginal impact of the wage subsidy with no accompanying shock and the green line (crosses) shows the marginal impact with uncertainty. As it is clear from the figure, the presence of uncertainty substantially reduces the effects of such a policy relative to an economy that is in the normal, or low uncertainty state. For example, in this particular case the period 2 increase in hours is 1.2% with low uncertainty and 0.5% with high uncertainty, so that uncertainty reduces the impact of the policy stimulus by almost 60%.

To better understand the mechanism that is causing the policy ineffectiveness result, it is useful to look at Figure 19. This plots the hiring thresholds and the evolution of the cross-sectional distribution of firm level TFP for a given combination of \( k \) and \( n \) following an uncertainty shock. It is clear that on impact the hiring threshold jumps out.\footnote{We find that the firing thresholds are not as reactive. This is due to the presence of exogenous labor attrition} As a
result no firms are near the hiring threshold in period 1, so that any policies to reduce the cost of hiring - for example from a hiring subsidy - will have very little impact since no firms are close to their hiring threshold. That is uncertainty makes firms cautious so that they do not respond to policy stimulus. But, slowly over time however, these thresholds slowly fall back as uncertainty drops. At the same time the distribution of firm-level TFP fans out towards the thresholds. So that over time as uncertainty falls the responsiveness to policy rises.

Two messages arise from this experiment. First, in order for such a policy to have any effect on hiring (or investment) in the presence of uncertainty it has to be larger than the equivalent policy that would be implemented during normal times - in this particular simulation, for example, it would need to be 140% larger. Second, to avoid overshooting once uncertainty falls, the policy stimulus has to be abandoned as uncertainty reduces the short-run impact of policy much more than the medium and long run impact.

6 Conclusions

This paper proposes time variation in uncertainty as a new impulse driving business cycles. First, we demonstrate that macro and micro uncertainty, measured by a number of proxies, appears to be strongly countercyclical. We then study a dynamic stochastic general equilibrium model that allows for shocks to both the level of technology (the first moment) as well as uncertainty (the second moment). We use establishment level data from the US Census to calibrate the time variation in uncertainty at the micro level and the conditional heteroskedasticity in aggregate output to calibrate uncertainty at the macro level. We find that increases in uncertainty lead to large drops in aggregate economic activity. This occurs because uncertainty makes firms cautious, leading them to pause hiring and investment. This freezing in activity also reduces the reallocation of capital and labor across firms, leading to a fall in productivity growth. We then conclude by using our model to investigate the effects of uncertainty on policy effectiveness. We use a simple illustrative example to show that the mechanism emphasized in this paper can significantly dampen the effect of an expansionary policy in times of increased economic uncertainty.

in the model that assures the firms that it can get "for free" firing in the model without having to pay the adjustment cost. Hence, absent very big shocks we find the firing thresholds to be more stable than those of hiring. Interestingly this result is consistent with the claims in Hall (2005) and Shimer (2005) that firing is acylical and that the majority of the cyclical adjustment in the labor force is done through a reduction in hiring.
References


Appendix: Uncertainty Data

This section outlines the details behind the construction of the eight uncertainty measures.

Establishment level shipments growth spread: We obtained access to the Census of Manufacturing (CM) and the Annual Survey of Manufacturing (ASM) from the US Census Bureau, which was combined to generate an establishment level panel with an average of 45,196 establishments spanning 1972 to 2006. We define shipments growth in establishment $i$ in year $t$ as $\Delta s_{i,t} = (s_{i,t+1} - s_{i,t-1})/(0.5 \times s_{i,t+1} + 0.5 \times s_{i,t-1})$ where $s_{i,t}$ is shipments. We then generate the interquartile range of this from 1973 to 2005 to generate the date in Figure 1. In Figure 2 we keep only the establishments with 25+ years of data, which represents 9,753 establishments accounting for 56% of total shipments (as longer-lived establishments are much larger on average). We do this because the ASM has a 5 year rotating panel which could potentially bias our measures of volatility, so we check our results using this panel of continuing establishments. If Figure 2 we also report values for the IQR within industries, which is calculated by demeaning the sales growth rate within each SIC 4-digit industry in each year. In Figure 3 we report the IQR within firms which is generated by demeaning the sales growth rate within each firm in each year, so is constructed from the deviations of establishment level shipment growth rates from the parent firm mean.

Firm sales growth spread: Calculated from the complete Compustat panel from 1965 to 2009 of quarterly accounts data. Growth rate of sales defines as $\Delta s_{i,t} = (s_{i,t+2} - s_{i,t-2})/(0.5 \times s_{i,t+2} + 0.5 \times s_{i,t-2})$ where the difference is taken over 4 quarters to eliminate quarterly effects. The IQR is then generated for all firms with 25+ years of data - to eliminate the type of compositional problems with Compustat highlighted in Davis et al. (2006) - and then quarters are only kept with 500+ observations to ensure adequate sample size. Values for within SIC 2-digit are calculated after demeaning sales growth rates by industry within each quarter, with values only reported for industry-year cells with 25+ observations.

Firm stock returns spread: Calculated from the complete CRSP panel of stock-returns from 1965 to 2009. The IQR of quarterly stock-returns is generated for all firms with 25+ years of data, and only quarters kept with 1000+ observations to ensure adequate sample size. Values for within SIC 2-digit are calculated after demeaning returns by industry within each quarter, with values only reported for industry-year cells with 25+ observations.

Cross-industry growth spread: Calculated from Federal Reserve Board’s G17 database on monthly output for 196 NAICS manufacturing industries. Quarterly output growth defined from monthly data as $\Delta s_{i,t} = (s_{i,t+2} - s_{i,t-2})/(0.5 \times s_{i,t+2} + 0.5 \times s_{i,t-2})$. The IQR of industry growth rates is then generated from the (balanced) industry panel.

Forecaster unemployment and industrial production dispersion: Calculated from the survey of professional forecasters downloaded from the Philadelphia Federal Reserve Bank. Forecasts used for 4 quarters ahead with an average of 41 forecasters in each cross-section over the period. For the unemployment dispersion the cross-sectional dispersion was normalized by the mean to prevents rises in unemployment alone driving the results (without the normalization the dispersion measure is much more countercyclical).

\footnote{http://www.federalreserve.gov/datadownload/Build.aspx?rel=G17}
**GDP growth volatility:** Calculated from a GARCH(1,1) specification with \( \log(\text{GDP}) \) regressed on its 4 quarterly lagged values. The conditional standard-deviation is plotted. Results for an ARCH(1) process look similar (in fact display an even larger increase in recessions) as does a specification using the growth rate of GDP. Data from BEA NIPA tables, calculated from 1955 to avoid the impact of the wage and price controls from the Korean War.

**Stock-market volatility:** CBOE VXO index of \% implied volatility, on a hypothetical at the money S&P100 option 30 days to expiration, from 1986 to 2009. Pre 1986 the VXO index is unavailable, so actual monthly returns volatilities calculated as the monthly standard-deviation of the daily S&P500 index normalized to the same mean and variance as the VXO index when they overlap (1986-2006). Actual and VXO are correlated at 0.874 over this period. The market was closed for 4 days after 9/11, with implied volatility levels for these 4 days interpolated using the European VX1 index, generating an average volatility of 58.2 for 9/11 until 9/14 inclusive.
<table>
<thead>
<tr>
<th>Table 1: The Increase in Measures of Uncertainty During Recessions</th>
</tr>
</thead>
<tbody>
<tr>
<td>% increase during recessions, mean (standard deviation)</td>
</tr>
<tr>
<td>Establishment level shipments growth spread</td>
</tr>
<tr>
<td>(1) (annual cross-sectional interquartile range)</td>
</tr>
<tr>
<td>Firm sales growth spread</td>
</tr>
<tr>
<td>(2) (quarterly cross-sectional interquartile range)</td>
</tr>
<tr>
<td>Firm stock returns spread</td>
</tr>
<tr>
<td>(3) (quarterly cross-sectional interquartile range)</td>
</tr>
<tr>
<td>Industry output growth spread</td>
</tr>
<tr>
<td>(4) (quarterly cross-sectional interquartile range)</td>
</tr>
<tr>
<td>Forecaster predicted industrial production spread</td>
</tr>
<tr>
<td>(5) (quarterly interquartile range / mean)</td>
</tr>
<tr>
<td>Forecaster predicted unemployment spread</td>
</tr>
<tr>
<td>(6) (quarterly interquartile range / mean)</td>
</tr>
<tr>
<td>Macro output growth volatility</td>
</tr>
<tr>
<td>(7) (quarterly average conditional standard deviation)</td>
</tr>
<tr>
<td>Macro stock returns volatility</td>
</tr>
<tr>
<td>(8) (quarterly standard deviation of daily stock returns)</td>
</tr>
</tbody>
</table>

Notes: The first column reports the percentage increase of the variable during a recession, with standard errors in ( ). For example, row (2) reports that the interquartile range of quarterly firm sales growth is 23.1% higher during quarters in a recession. Recessions defined using the NBER dates. The second column shows the correlation with GDP growth (all correlations significant at the 5% level). Data sources as are follows: Row (1) the Annual Survey of Manufacturing combined with the Census of Manufacturing where we have used “shipments” as a measure of “output”; Row (2) Compustat, Row (3) CRSP, Row (4) The Federal Reserve Board, Rows (5) and (6) the Survey of Professional Forecasters, Row (7) BEA NIPA and Row (8) CRSP actual CBOE’s VXO index from 1987 onwards, and the standard deviation of daily returns on the S&P 500 from CRSP prior to 1987. Full data details for all series in Appendix A.
<table>
<thead>
<tr>
<th>Table 2: Parameters in the Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Preferences and Technology</strong></td>
</tr>
<tr>
<td>$\beta$</td>
</tr>
<tr>
<td>$\eta$</td>
</tr>
<tr>
<td>$\theta$</td>
</tr>
<tr>
<td>$\gamma$</td>
</tr>
<tr>
<td>$\alpha$</td>
</tr>
<tr>
<td>$\nu$</td>
</tr>
<tr>
<td>$\rho^A$</td>
</tr>
<tr>
<td>$\rho^Z$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Adjustment Costs</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_k$</td>
</tr>
<tr>
<td>$\delta_n$</td>
</tr>
<tr>
<td>$F^K$</td>
</tr>
<tr>
<td>$S$</td>
</tr>
<tr>
<td>$F^L$</td>
</tr>
<tr>
<td>$H$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Uncertainty Process</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^A_L$</td>
</tr>
<tr>
<td>$\sigma^A_H$</td>
</tr>
<tr>
<td>$\sigma^Z_L$</td>
</tr>
<tr>
<td>$\sigma^Z_H$</td>
</tr>
<tr>
<td>$\pi^q_L,H$</td>
</tr>
<tr>
<td>$\pi^q_H,H$</td>
</tr>
</tbody>
</table>

Notes: The calibration procedure of the uncertainty process is explained in detail in the main text.
Table 3: Moments in Calibration

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Full Sample</td>
</tr>
<tr>
<td></td>
<td>Value</td>
</tr>
<tr>
<td><strong>Micro Moments</strong></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.296</td>
</tr>
<tr>
<td>Standard Deviation * 100</td>
<td>3.230</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.989</td>
</tr>
<tr>
<td>Serial Correlation</td>
<td>0.625</td>
</tr>
<tr>
<td><strong>Macro Moments</strong></td>
<td></td>
</tr>
<tr>
<td>Mean * 100</td>
<td>0.753</td>
</tr>
<tr>
<td>Standard Deviation * 100</td>
<td>0.342</td>
</tr>
<tr>
<td>Skewness</td>
<td>1.279</td>
</tr>
<tr>
<td>Serial Correlation</td>
<td>0.911</td>
</tr>
</tbody>
</table>

Notes: The micro moments are calculated using the cross sectional IQR annual sales growth rates at the establishment level. The data is from the Census of Manufacturers and the Annual Survey of Manufacturing from 1972 to 2006. The macro moments are calculated from the predicted standard deviation from a GARCH(1,1) estimation using log GDP with four lags for the same time period. Column I shows the moments for the full sample. Columns II and II show the minum and maximum value for each moment when five years are dropped either at the beginning or end of the sample. Column IV shows the corresponding moments in simulated data from the model. Column V and VI refer to alternative specification with a fixed level of macro and micro uncertainty respectively.
Table 4: Business Cycle Statistics

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Baseline Model</th>
<th>No 1st Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma(x)$</td>
<td>$\sigma(x)$</td>
<td>$\sigma(x)$</td>
</tr>
<tr>
<td></td>
<td>$\sigma(y)$</td>
<td>$\rho(x,y)$</td>
<td>$\sigma(y)$</td>
</tr>
<tr>
<td>Output</td>
<td>1.56</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Investment</td>
<td>7.49</td>
<td>4.80</td>
<td>0.92</td>
</tr>
<tr>
<td>Consumption</td>
<td>1.27</td>
<td>0.81</td>
<td>0.86</td>
</tr>
<tr>
<td>Hours</td>
<td>1.87</td>
<td>1.20</td>
<td>0.88</td>
</tr>
</tbody>
</table>

Notes: The first panel contains business cycle statistics for US data. The data on output, investment and consumption is taken from the NIPA accounts. The hours series refers to the nonfarm private business sector (series LBMNU in Global Insight Database). The period is chosen to correspond to the availability of Census data, 1972 to 2006. The second panel contains results for the benchmark model. The third panel refers to an alternative specification without aggregate productivity shocks, but with time-varying uncertainty at the micro level. Within each panel, the three columns show for the hp filtered logarithm of each series (i) absolute standard deviation, (ii) the standard deviation relative to the standard deviation of output and (iii) the correlation with output.
Table 5: Plant Level Investment Rates

<table>
<thead>
<tr>
<th>Shares of firms with...</th>
<th>Positive Investment</th>
<th>Investment Spike</th>
<th>Inaction</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data</strong></td>
<td>0.815</td>
<td>0.186</td>
<td>0.081</td>
</tr>
<tr>
<td><strong>Model (annual)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single establishment</td>
<td>0.303</td>
<td>0.290</td>
<td>0.697</td>
</tr>
<tr>
<td>10 establishments</td>
<td>0.972</td>
<td>0.098</td>
<td>0.028</td>
</tr>
<tr>
<td>25 establishment</td>
<td>1.000</td>
<td>0.022</td>
<td>0.000</td>
</tr>
<tr>
<td>250 establishment</td>
<td>1.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Notes: This table shows the share of firms that invest (i/k greater or equal 0.01), have an investment spike (i/k greater than 0.2) or are inactive (|i/k|<0.01) within in a given period. The first row shows empirical moments taken from Cooper and Haltiwanger (2006) data from the Longitudinal Research Database, which is a similar establishment-level dataset to the one we used to create Figure 1. The second panel uses our simulated data that is first aggregated to the annual frequency. The four rows within each panel refer to different levels of unit aggregations, for that for example the figures for 10+ establishments refer to figures for groups of 10+ establishments. Aggregation is important because Cooper and Haltiwanger (2006) use a balanced panel of very large establishments with an average of almost 600 employees.
Figure 1: Cross-establishment output growth spread

Notes: Constructed from the Census of Manufacturers and the Annual Survey of Manufacturing establishments (45,196 establishments on average per year). The grey shaded columns are the number of quarters in recession (left axis) within each year.

Figure 2: Cross-establishment output growth spread, in long lived establishments and within SIC 4-digit industry

Notes: Annual Survey of Manufacturing establishments with 25+ years of observations (9,673 establishments) kept to control for sample composition changes. Output growth spread (right axis) is the inter-quartile range across of all establishments. Within industry output growth spread calculated after the 4-digit SIC yearly average removed. The grey shaded columns are the number of quarters in recession (left axis) within each year.
Figure 3: Cross-establishment output growth spread, within firms

Notes: Annual Survey of Manufacturing establishments with 25+ years of observations kept to control for sample composition changes. Within firm output growth spread calculated after the firm yearly average removed, calculated only for multi-establishment firms. The grey shaded columns are the number of quarters in recession (left axis) within each year.

Figure 4: Cross firm sales growth spread

Interquartile range of sales growth rate

Interquartile range of sales growth (Compustat firms). Only firms with 25+ years of accounts, and quarters with 500+ observations. SIC2 only cells with 25+ obs. SIC2 is used as the level of industry definition to maintain sample size. The grey shaded columns are recessions according to the NBER.
Figure 5: Cross-firm stock-returns spread

Interquartile range of stock returns (CRSP firms). Only firms with 25+ years of accounts, and quarters with 1000+ observations. SIC2 only cells with 25+ obs. SIC2 is used as the level of industry definition to maintain sample size.

Figure 6: Cross-industry growth spread

Plots the IQR of the monthly industry growth rates within each quarter across the 196 NAICS manufacturing industries.
1st, 5th, 10th, 25th, 50th, 75th, 90th, 95th and 99th percentiles of 3-month growth rates of industrial production within each quarter. All 196 manufacturing NAICS sectors in the Federal Reserve Board database.

Figure 7: Cross industry output growth distribution

99th percentile, 2.2% higher in recessions
50th percentile, 1.3% lower in recessions
1st percentile 7.4% lower in recessions

Figure 8: Forecaster unemployment dispersion

Notes: Interquartile range of cross-sectional forecasts divided by average of cross-sectional forecasts. 4 quarters ahead unemployment rates from the Survey of Professional Forecasters. Forecasts collected quarterly with an average of 41 forecasters per period. The grey shaded columns are recessionary quarters defined according to the NBER.
Figure 9: Forecaster industrial production dispersion

Notes: Interquartile range of cross-sectional forecasts of 4 quarters ahead industrial production growth from the Survey of Professional Forecasters. Forecasts collected quarterly with an average of 41 forecasters per period. The grey shaded columns are recessionary quarters defined according to the NBER.

Figure 10: GDP growth volatility

Notes: Predicted volatility from a GARCH(1,1) estimation of current quarterly log(GDP) on its four lagged values.
Figures 11 and 12: Graphs depicting stock-market volatility and unweighted TFP, respectively.

Figures 11 and 12: Graphs depicting stock-market volatility and unweighted TFP, respectively.

Notes: Stock market volatility used actual quarterly standard deviation of daily returns until 1987, and average quarterly implied volatility from 1987 onwards.
Figure 13: Effect of rise in uncertainty on output

Figure 14: Labor, investment, weighted TFP, consumption
Figure 15: Output – alternative AC specifications

Figure 16: Output – only micro / macro uncertainty
Figure 17: Output – policy effectiveness

Figure 18: Output – differential policy effect
Figure 19: Policy - thresholds