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Bursting Bubbles: Consequences and Cures*

Narayana R. Kocherlakota

FRB-Minneapolis
University of Minnesota
NBER

ABSTRACT

I construct a model in which infinitely-lived entrepreneurs cannot borrow more than the value of their collateral holdings. I show how this constraint leads naturally to an equilibrium in which the collateral's price has a bubble. I demonstrate that bursting bubbles in collateral prices may have dramatic and persistent distributional and aggregate effects. I discuss appropriate and inappropriate policy interventions in the wake of a bubble collapse.

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Housing and housing price derivatives are important sources of collateral for loans in the United States. From July 2006 to October 2008, the twenty-city Case-Shiller house price index fell by just under 25%. This price decline is often interpreted as representing the bursting of a house price *bubble*. The decline is blamed for significant changes in credit markets that began in the second half of 2007, and to a recession that is now dated as having begun in December 2007. There has been a massive and varied government response to these events.

Motivated by these observations, in this paper I construct a model in which collateralized borrowing plays an essential role in re-allocating capital to its efficient uses. I show that collateral scarcity can generate a stochastic bubble in the price of collateral. I discuss the implications of this bubble's bursting for aggregates and for welfare. Using the model, I assess several ongoing policy initiatives and propose a specific superior intervention.

The structure of my model closely resembles models described in Angeletos (2007), Kartashova (2008), and Kiyotaki-Moore (2008). A fraction of entrepreneurs have productive investment opportunities, while others do not. The arrival of the desirable projects is i.i.d. over entrepreneurs and over time. Efficient production requires the re-allocation of physical capital from entrepreneurs without good projects to those with good projects. This re-allocation is accomplished via loans. Markets are incomplete in the sense that these loans cannot be made contingent on whether a given entrepreneur gets a good project.

The novel feature of my model is that all entrepreneurs are endowed with an initial stock of a good called *collateral*. (I am agnostic about what that good is, but for concreteness, one can think about housing.) If the borrower defaults, a lender can seize a borrower's collateral but no other borrower resources. Hence, a borrower's repayment is bounded from above by the value of his collateral. Like physical capital, collateral depreciates over time and agents can augment the stock of collateral through the investment of consumption goods. However (somewhat extremely), I assume that collateral cannot be used to produce consumption. In this sense, collateral is intrinsically worthless.

Given this set of assumptions, it is not surprising that there is a set of initial conditions (on collateral and capital) consistent with a balanced-growth-path (BGP) equilibrium in which collateral has zero value. In this BGP equilibrium, no borrowing and lending takes place and the stock of collateral eventually disappears. However, there is another set of initial conditions that induce a BGP equilibrium in which the collateral price is positive. I interpret this positive price as being a bubble in collateral prices. In this latter bubbly equilibrium, the market value of the aggregate stock of collateral grows over time. This growth requires collateral investment at every date, and so agents consistently give up socially useful resources to produce more (intrinsically) useless collateral.

The intuition behind the existence of the bubble is simple, robust, but often ignored. All entrepreneurs face occasionally binding borrowing constraints. Collateral, even though it is intrinsically worthless, may have value because it serves to relax this constraint (see Kocherlakota (1992)). In this way, the bubble allows entrepreneurs to re-allocate physical capital more efficiently, which leads to higher wages, output, and consumption for the economy. Remarkably, this re-allocation is so useful that (under some weak conditions) the economy generates these higher aggregates in the bubbly BGP equilibrium using *less* physical capital.

I use these two equilibria to construct another one with a stochastic bubble. (Such equilibria can be quite complicated; I deliberately focus on one that is simple.) In this third kind of equilibrium, there is some small probability of the bubble's bursting at each date.

Before the bubble bursts, the collateral price is positive and constant. After the bubble bursts, the collateral price reverts to zero forever. Entrepreneurs can exchange any kind of financial contract that is explicitly contingent on the price of collateral. However, this kind of financial market completeness is not all that helpful, given the aggregate nature of the shock.

Immediately after the bubble bursts, the entrepreneurs with good projects have little capital available for investment. Macroeconomic aggregates fall dramatically. Entrepreneurs have to self-finance their projects, and begin to accumulate physical capital for this purpose. In any given date, much of the accumulated capital in society is used inefficiently because the owners do not have useful projects. The economy transits to a new lower level of economic activity. Entrepreneurs and workers alike mourn the collapse of the bubble. Nonetheless, from an ex-ante perspective, a positive stochastic bubble expands the social pie.

I discuss a range of possible interventions in the wake of the bubble collapse. The bubble's collapse creates two related but distinct problems. First, entrepreneurs have lost wealth. Second, entrepreneurs without good projects are accumulating wealth via a low-return savings vehicle. Successful interventions must cure both problems. I argue that several of the current policy moves (including bailing out financial intermediaries) are poorly designed to meet these objectives.

My preferred post-policy intervention is based on an insight of Caballero and Krishnamurthy (2006). They argue that, given the conditions that generate a bubble, the government's ability to tax ensures that it can always credibly roll over its debt. With that in mind, I construct a two-part intervention. First, the government compensates owners of collateral for their losses by giving them government debt. This debt can be seized by creditors, and injects a new source of collateral into the economy. Second, the government commits to paying a high real interest rate on its debt. I show that this intervention completely eliminates the adverse ex-post impact of the bubble's collapse on aggregate outcomes, without creating undesirable ex-ante incentive effects. Moreover, the government can create new collateral by simply rolling over its existing debt. Such a rollover uses no resources, and so is more efficient than the collateral accumulation technology available to private citizens. Hence, this intervention actually generates a flow of extra goods for the government.

This paper is closely related to that of Kraay and Ventura (2007). They set up a two-period overlapping generations model in which external firm finance involves a social loss. They use that model to argue that the 1990's dot.com stock price bubble may have led to an improvement in societal welfare by crowding out inefficient investments. They interpret the expansion of government debt in the 2000's as a way to re-create the desirable effects of the bubble in the wake of its collapse. Thus, the policy implications of the papers are somewhat similar.

I make several modelling contributions relative to their work. My model is an infinite horizon setup in which borrowing is limited by collateral. The structure of the model is designed to mimic the workhorse growth model used in macroeconomics.[1] It is desirable to be able to work with bubbles in such frameworks, because they are more readily mapped into macroeconomic data. This framework allows me to show that there is a direct connection between an asset's role as collateral and its price having a bubble. I demonstrate too that (in the absence of government interventions), it may be socially and individually optimal to

[1] There are a number of papers in which borrowing constraints create bubbles in economies with infinitely-lived agents. (See, among many others, Hellwig and Lorenzoni (forthcoming) and Scheinkman and Weiss (1986).) In these papers, bubbles allow agents to achieve better intertemporal allocations. However, as far as I know, the papers do not show a tight connection between the existence of bubbles and productive efficiency.

invest resources in the physical accumulation of collateral assets, even if they are intrinsically worthless.

1. Constant Bubbles

In this section, I set up a version of the model without any aggregate shocks. The basic framework closely resembles that in Kiyotaki-Moore (2008). The main difference is that there is a second form of capital called collateral. This kind of capital is intrinsically useless and is costly to produce. The analysis demonstrates that balanced-growth-path (BGP) equilibria with bubbles are always better in terms of output, consumption, wages, and welfare than steady-states without bubbles. Nonetheless, physical capital may well be *lower* in the bubbly BGP equilibrium.

A. Model Economy

The economy is discrete time and infinite horizon. There is a unit measure of workers. Workers play little role in this analysis, except to soak up the returns to labor. More specifically, each worker supplies one unit of labor inelastically at each date. The workers simply consume their labor income at every date; they do not borrow or lend.

There is also a unit measure of entrepreneurs. Entrepreneurs maximize the expectation of:

$$(1) \qquad \sum_{t=1}^{\infty} \beta^{t-1} \ln(c_t), 0 < \beta < 1$$

where c_t is consumption at date t. Each entrepreneur is initially endowed with k_1 units of capital goods..Entrepreneurs have a technology that converts k_{t+1} units of capital installed at date t and n_{t+1} units of labor hired at date $t+1$ into y_{t+1} units of output, according to the production function:

$$(2) \qquad y_{t+1} = f(k_{t+1}, n_{t+1}, A_{t+1})$$

Here, the productivity disturbance A_t is a random variable that is i.i.d. over entrepreneurs and is i.i.d. over time. It equals 1 with probability π and 0 with probability $(1-\pi)$. I denote the history of productivity shocks in period t by A^t. Capital depreciates at rate δ_k per period. I parameterize the production function to be of the form:

$$(3) \qquad \begin{aligned} f_{t+1}(k_{t+1}, n_{t+1}, A_{t+1}) &= (1+g_{TFP})^t k_{t+1}^{\alpha} n_{t+1}^{1-\alpha} \text{ if } A_{t+1} = 1 \\ &= \delta_k k_{t+1} \text{ if } A_{t+1} = 0 \end{aligned}$$

Thus, entrepreneurs with $A_{t+1} = 1$ have a Cobb-Douglas production function in which total factor productivity grows at an exogenous constant rate g_{TFP}. For entrepreneurs with $A_{t+1} = 0$, capital is simply a way to store consumption. (This latter storage technology could be made available to entrepreneurs with $A_{t+1} = 1$; they will not use it.) Either entrepreneur can convert capital one-for-one into consumption and vice-versa.

Each entrepreneur is also initially endowed with h_1 units of a *collateral* good. Collateral is intrinsically useless: it provides no direct utility, cannot be used in production, and cannot be converted into consumption. Collateral depreciates at rate $\delta_h \geq 0$, but its stock can be augmented. An entrepreneur with h_t units of collateral goods at the end of period $(t-1)$ can transform x_t units of consumption goods in period $(t-1)$ into $\phi x_t^{\psi} h_t^{1-\psi}$ units of additional collateral goods in period t. This accumulation technology is not reversible.

At the beginning of a given period t, the entrepreneur learns A_{t+1} (the realization of his productivity in the following period). The entrepreneur can borrow or save using one-period risk-free bonds. However, he cannot borrow against the proceeds of his project. Instead, the entrepreneur's repayment in period $(t+1)$ is bounded from above by the value of his collateral goods in period $(t+1)$.

More precisely, suppose that the entrepreneur faces a collateral price sequence $(p_t)_{t=1}^{\infty}$, an interest rate sequence $(r_t)_{t=1}^{\infty}$, and a wage sequence $(w_t)_{t=1}^{\infty}$. At each date, the entrepreneur chooses consumption c_t, collateral goods h_{t+1}, investment x_{t+1} into collateral, installed capital k_{t+1}, bond-holdings b_{t+1}, and hired labor n_t. The entrepreneur's budget set consists of (c,h,x,b,k,n) that satisfy:

$$\begin{aligned}
& c_t(A^{t+1}) + p_t h_{t+1}(A^{t+1}) + x_{t+1}(A^{t+1}) + b_{t+1}(A^{t+1}) + k_{t+1}(A^{t+1}) + w_t n_t(A^{t+1}) \\
\leq\ & b_t(A^t)(1+r_t) + f_t(k_t(A^t), n_t(A^{t+1}), A_t) + (1-\delta_k)k_t(A^t) \\
& + p_t(1-\delta_h)h_t(A^t) + \phi x_t(A^t)^{\psi} h_t(A^t)^{1-\psi} \text{ for all } t, A^{t+1}
\end{aligned}$$

$$\begin{aligned}
& b_t(A^t)(1+r_t) \geq -p_t[(1-\delta_h)h_t(A^t) + \phi x_t(A^t)^{\psi} h_t(A^t)^{1-\psi}] \text{ for all } A^t, t \geq 2 \\
& c_t(A^{t+1}), x_{t+1}(A^{t+1}), k_{t+1}(A^{t+1}), h_{t+1}(A^{t+1}) \geq 0 \text{ for all } t, A^{t+1} \\
& b_1 = 0 \\
& k_1, h_1 \text{ given}
\end{aligned}$$

The second set of inequality constraints captures the assumption that an entrepreneur's debt repayment in period $(t+1)$ is bounded from above by the market value of his collateral at that date.

A specification of prices (p,w,r) and entrepreneurial quantities (c,h,x,b,k,n) forms an equilibrium if (c, h, x, b, k, n) maximizes the entrepreneur's utility among all allocations in his budget set, and markets clear:

$$\sum_{A^{t+1}} \Pr(A^{t+1}) n_t(A^{t+1}) = 1 \tag{4}$$

$$\sum_{A^{t+1}} \Pr(A^{t+1}) c_t(A^{t+1}) + \sum_{A^{t+1}} \Pr(A^{t+1}) x_{t+1}(A^{t+1}) + \sum_{A^{t+1}} \Pr(A^{t+1}) k_{t+1}(A^{t+1}) \tag{5}$$

$$= (1-\delta_k) \sum_{A^t} \Pr(A^t) k_t(A^t) + \sum_{A^{t+1}} \Pr(A^{t+1}) f_t(k_t(A^t), n_t(A^{t+1}), A_t)$$

$$\sum_{A^{t+1}} \Pr(A^{t+1}) h_{t+1}(A^{t+1}) = \sum_{A^t} \Pr(A^t)[h_t(A^t)(1-\delta_h) + \phi_t x_t(A^t)^{\psi} h_t(A^t)^{1-\psi}] \tag{6}$$

$$\sum_{A^{t+1}} \Pr(A^{t+1}) b_{t+1}(A^{t+1}) = 0 \tag{7}$$

Here, $\Pr(A^{t+1})$ is the probability of a given sequence A^{t+1} occurring. I assume that a law of large numbers applies in the population, so that the fraction of entrepreneurs in period t with history A^{t+1} is the same as the unconditional probability of that history.

Throughout this paper, I am agnostic about what the collateral good represents. However, the analysis is motivated by the fall in the price of residential housing, and so the collateral good is designed to resemble housing. Like housing, collateral depreciates. And, as with housing, agents can expend consumption to expand the stock of collateral.

It is true that few entrepreneurs literally collateralize their loans using housing. However, consider the following chain of transactions, given that the interest rate is 0. First, homeowner X borrows \$800000 from bank Y, using a \$1 million home as collateral. Bank Y identifies an investment opportunity with return $r > 0$. Bank Y uses its loan to X as collateral to borrow \$800000 from bank Z, and then invests that \$800000 in the high-return project. In this story, X and Y are jointly operating as an entrepreneur in the model does when $A_{t+1} = 1$. Bank Z is operating like an entrepreneur with $A_{t+1} = 0$. Thus, in the real world, there are many layers of "paper" collateralization that are ultimately grounded in a physical asset. The model abstracts from these multiple layers.

B. Two Balanced Growth Path Equilibria

In this section, I construct two balanced-growth-path (BGP) equilibria in which prices are constant and *aggregate* quantities are growing at a constant rate over time. (In both equilibria, the distribution of wealth across entrepreneurs is spreading over time.) In the first BGP, collateral prices equal zero, and in the second, collateral prices are positive.

The construction of these equilibria follows that in KM. The basic idea is that there are two kinds of entrepreneurs at any date t. The first kind knows that his realization of A_{t+1} is 1. His production technology has a high return (assuming aggregate capital is sufficiently low), and so he wants to borrow as much capital as possible to invest in it. Following KM, I'll label these entrepreneurs "investors". The second kind of entrepreneurs knows that his production technology has a low return, because $A_{t+1} = 0$. I'll label these entrepreneurs "savers".

A critical feature of the model is that the investors can freely adjust labor demand. This means that any investor who installed capital k_t at date $t-1$ and faces wage w_t in period t will choose n_t so that:

(8) $$n_t = (1+g_{TFP})^{(t-1)/\alpha} k_t [w_t/(1-\alpha)]^{-1/\alpha}$$

Thus, all investors have the same capital-labor ratio. It follows that this capital-labor ratio is the same as that set by the average investor, so that the equilibrium wage:

(9) $$w_t = (1-\alpha)(1+g_{TFP})^{t-1}\overline{k}_t^{\alpha}(1/\pi)^{-\alpha}$$

where $\overline{k}_t$ is the per-capita level of installed capital. We can conclude that an investor who installs k_t in period $(t-1)$ gets a payoff equal to:

$$(10) \quad (1 - \delta_k + \alpha(1 + g_{TFP})^{t-1}(\overline{k}_t \pi)^{\alpha-1})k_t$$

Both investors and savers face standard consumption-savings problems. The gross rate of return of investors is governed by the marginal product of capital for a representative investor. The gross rate of return for savers is 1.

No Bubbles

I solve first for an balanced growth path (BGP) equilibrium in which $p_t = 0$ for all t. In a BGP equilibrium, wages w_t and per-capita[2] wealth $\overline{W}_t$, consumption $\overline{c}_t$ capital $\overline{k}_{t+1}$ and output $\overline{y}_t$ all grow at the same endogenous rate g. As is standard in the neoclassical growth model, this endogenous growth rate is determined by the growth rate of total factor productivity:

$$(11) \quad (1 + g) = (1 + g_{TFP})^{1/(1-\alpha)}$$

Only one specification of initial per-capita capital k_1 is consistent with a BGP equilibrium in which $p_t = 0$, and I will need to solve for this value in what follows. In contrast, given a BGP equilibrium with $p_t = 0$ for all t, it is consistent with any specification of (h_1, x_1). If $p_t = 0$, no agent will expend consumption to build new collateral. Without loss of generality, we can simply assume that there is no trade in collateral, so that any existing collateral depreciates at rate δ_h.

[2] Throughout, I use the term "per-capita" to refer to "per-entrepreneur".

Without collateral, neither investors nor savers can borrow. Define:

$$(12) \quad MPK_{NB} = \alpha(1 + g_{TFP})^{t-1}(\overline{k}_t\pi)^{\alpha-1}$$

to be the marginal product of capital in period t. Because $\overline{k}_t$ grows at rate g, MPK_{NB} is constant in a BGP equilibrium. Consider an entrepreneur with wealth W_t defined to be:

$$\begin{aligned}(13) \quad W_t &= (1 - \delta_k + MPK_{NB})k_t \text{ if } A_t = 1 \\ &= k_t \text{ if } A_t = 0\end{aligned}$$

(this is the right-hand side of his flow budget constraint in period t). All entrepreneurs have log utility and face standard consumption-savings problems. They follow "myopic" decision rules: they choose $c_t = (1 - \beta)W_t$ and $k_{t+1} = \beta W_t$. Savers earn a gross rate of return equal to 1 on this investment. Investors earn a gross rate of return equal to $(1 - \delta_k + MPK_{NB})$, where MPK_{NB} is the marginal product of capital for the representative investor.

Since per-capita wealth grows at rate g, we know that:

$$(14) \quad (1 + g) = \pi\beta(MPK_{NB} + 1 - \delta_k) + (1 - \pi)\beta$$

This restriction pins down MPK_{NB} to be:

$$(15) \quad MPK_{NB} = [(1 + g) - (1 - \pi)\beta]\pi^{-1}\beta^{-1} - 1 + \delta_k$$

Let $\overline{W}_{NB}$ be period 1 per-capita wealth. The marginal product of capital in period 2 is determined by the amount of capital ($\beta\overline{W}_{NB}$) installed by investors in period 1. We can then solve for $\overline{W}_{NB}$ in period 1 using the restriction:

$$(16) \quad MPK_{NB} = \alpha(1 + g_{TFP})(\beta\overline{W}_{NB}\pi)^{\alpha-1}$$

where we exploit the equilibrium condition that investors hire $1/\pi$ units of labor each.

It is now straightforward to solve for the rest of the equilibrium. The specification of k_1 consistent with a BGP equilibrium is $\overline{k}_{NB} = \beta\overline{W}_{NB}/(1+g)$. Period 1 wages are given by period 2 wages discounted by $(1+g)$:

$$(17) \quad \overline{w}_{NB} = (1-\alpha)(1+g_{TFP})(1+g)^{-1}(\beta\overline{W}_{NB})^{\alpha}(1/\pi)^{-\alpha}$$

and per-capita period 1 output $\overline{y}_{NB}$ equals $\overline{w}_{NB}/(1-\alpha)$. Per-capita period 1 consumption $\overline{c}_{NB}$ equals $(1-\beta)\overline{W}_{NB}$.

Entrepreneurs' wealths evolve over time in response to their idiosyncratic shocks, according to the rule:

$$(18) \quad \begin{aligned} W_{t+1} &= \beta(1-\delta_k + MPK_{NB})W_t \text{ if } A_{t+1} = 1 \\ &= \beta W_t \text{ if } A_{t+1} = 0 \end{aligned}$$

Then, at each date, they set k_{t+1} and c_t as above. Note that the cross-sectional variance of logged entrepreneurial consumptions/wealths grows over time.

There is no borrowing in equilibrium. Because bonds are in zero net supply, any non-positive interest rate clears markets. If bonds were in slightly positive net supply, the interest rate would necessarily equal 0. For this reason, I simply set the non-bubbly BGP interest rate equal to 0.

Constant Positive Bubble

I now choose initial conditions so that there is an equilibrium in which collateral prices are constant at a positive level p^*, per-capita collateral is positive and growing at a constant rate, and the equilibrium interest rate r^* is constant. As was argued above, in a balanced growth path the common growth rate g of per-capita consumption, output, and capital must satisfy $(1+g) = (1+g_{TFP})^{1/(1-\alpha)}$. I restrict attention to parameter settings such that:

$$(1-\psi)g + \psi(-\delta_h) > 0 \tag{19}$$

As we shall see, this restriction (19) guarantees that the equilibrium interest rate is sufficiently high that savers prefer to invest in bonds than hold capital.

In the balanced growth path, collateral investment x_{t+1} grows at rate g. Collateral grows at a constant rate g_h, which satisfies the accounting relationship:

$$(1+g_h) = (1-\delta_h) + \phi h_t^{-\psi} x_t^{\psi} \tag{20}$$

Since g_h is constant, h_t must grow at the same rate as x_t, and so $g_h = g$. Let h^* denote the ratio of collateral h_t to collateral investment x_t. This ratio equals:

$$h^* = (g_h + \delta_h)^{-1/\psi} \phi^{-1/\psi} \tag{21}$$

In equilibrium, savers and investors can transfer consumption from period t into period $(t+1)$ in three possible ways: buying bonds (b_{t+1}), buying collateral (h_{t+1}), or producing additional collateral (x_{t+1}). It must be true that the interest rate equals the marginal return to buying collateral:

$$\begin{aligned}(1+r^*) &= (1-\delta_h) + \phi(1-\psi)h^{*-\psi} \\ r^* &= \psi(-\delta_h) + (1-\psi)g \qquad (22)\end{aligned}$$

The restriction (19) ensures that r^* is positive. Note that r^* cannot exceed g.

We can use a similar marginal indifference condition to solve for the collateral price p^*. At the margin, entrepreneurs are indifferent between creating additional collateral goods and buying bonds. Hence the collateral price p^* satisfies:

(23) $$(1 + r^*) = p^* \phi \psi h^{*1-\psi}$$

and so:

(24) $$p^* = (\phi \psi h^{*1-\psi})^{-1}(1 + r^*)$$

We can now turn to the task of solving for aggregate quantities. At each date, define entrepreneurial wealth W_t to be:

(25) $$W_t = f_t(k_t, n_t, A_t)k_t + b_t(1 + r^*) + p^* h_t(1 - \delta_h) + p^* \phi x_t^{\psi} h_t^{1-\psi}$$

With this change of adding $p^* h_t(1 - \delta_h) + p^* \phi x_t^{\psi} h_t^{1-\psi} + b_t(1 + r^*)$ to wealth, entrepreneurial decision rules are similar to the non-bubbly BGP. At date t, investors have an investment opportunity with a gross rate of return equal to $(1 - \delta_k + MPK_{BUB})$, where MPK_{BUB} is the time-invariant marginal product of capital. In response, investors choose $k_{t+1} = \beta W_t$, set consumption equal to $(1 - \beta)W_t$, and choose some $(b_{t+1}, x_{t+1}, h_{t+1})$ so that their borrowing constraint is satisfied with equality:

(26) $$b_{t+1}(1 + r^*) + \{h_{t+1}(1 - \delta_h) + \phi x_{t+1}^{\psi} h_{t+1}^{1-\psi}\}p^* = 0$$

In contrast, it is suboptimal for savers to choose a positive level of k_{t+1}, because other assets earn a positive rate of return r^*. Instead, savers set consumption equal to $(1 - \beta)W_t$ and choose $(b_{t+1}, x_{t+1}, h_{t+1})$ so that:

(27) $$b_{t+1} + x_{t+1} + h_{t+1}p^* = \beta W_t$$

Along a BGP, wealth grows at rate g. Given the above decision rules, this growth rate requires:

$$(28) \quad (1+g) = \pi\beta(1-\delta_k + MPK_{BUB}) + (1-\pi)\beta(1+r^*)$$

which implies that:

$$(29) \quad MPK_{BUB} = [1+g-(1-\pi)\beta(1+r^*)]\pi^{-1}\beta^{-1} - 1 + \delta_k$$

We can then solve for period 1 per-capita wealth as before; in particular, $\overline{W}_{BUB}$ satisfies:

$$(30) \quad MPK_{BUB} = \alpha(1+g_{TFP})(\beta\overline{W}_{BUB}\pi)^{\alpha-1}$$

Initial per-capita consumption $\overline{c}_{BUB}$ equals $(1-\beta)\overline{W}_{BUB}$. Period 1 wages equal period 2 wages, discounted at $(1+g)$:

$$(31) \quad \overline{w}_{BUB} = (1+g)^{-1}(1+g_{TFP})(1-\alpha)(\beta\overline{W}_{BUB}\pi)^{\alpha}$$

Finally, per-capita period 1 output $\overline{y}_{BUB}$ equals $\overline{w}_{BUB}/(1-\alpha)$ or:

$$(32) \quad \overline{y}_{BUB} = (\beta\pi\overline{W}_{BUB}/(1+g))^{\alpha}$$

We need also to solve for the initial levels of per-capita collateral and capital that are consistent with this BGP. In terms of period 1 capital k_1, it equals:

$$(33) \quad \overline{k}_{BUB} = (1+g)^{-1}(\beta\overline{W}_{BUB}\pi)$$

In terms of period 1 per-capita h_1 and x_1, bond market-clearing implies that the market value of collateral in period 3 equals the payoff from investing $\beta(1-\pi)\overline{W}_{BUB}$ in collateral. Mathematically, the initial h_1 equals $\overline{h}_{BUB}$, which solves:

$$(34) \quad p^*\overline{h}_{BUB}(1+g)^2 = \beta(1+r^*)(1-\pi)\overline{W}_{BUB}$$

The initial x_1 equals $\overline{x}_{BUB}$, which is simply $\overline{h}_{BUB}/h^*$.

Entrepreneurs' wealths evolve over time in response to their idiosyncratic shocks, according to the rule:

$$
\begin{aligned}
(35)\quad W_{t+1} &= \beta(1-\delta_k + MPK_{BUB})W_t \text{ if } A_{t+1}=1 \\
&= \beta(1+r^*)W_t \text{ if } A_{t+1}=0
\end{aligned}
$$

However, entrepreneurs have indeterminate portfolios: they are indifferent between holding bonds or collateral. Without loss of generality, I resolve this indeterminacy by assuming that there is no heterogeneity in the holding of collateral, only in bonds. Thus, I set:

$$
\begin{aligned}
(36)\quad h_{t+1}(A^{t+1}) &= \overline{h}_{BUB}(1+g)^t \\
(37)\quad x_{t+1}(A^{t+1}) &= (\overline{h}_{BUB}/h^*)(1+g)^t
\end{aligned}
$$

for all A^{t+1}. Hence, savers and investors buy and make the same amount of collateral at each date. These decision rules for collateral imply that:

$$
\begin{aligned}
(38)\quad b_{t+1}(A^{t+1}) &= -p^*\overline{h}_{BUB}(1+g)^{t+1}/(1+r^*) \text{ if } A_{t+1}=1 \\
&= \beta W_t - p^*\overline{h}_{BUB}(1+g)^{t+1}/(1+r^*) \text{ if } A_{t+1}=0
\end{aligned}
$$

Note that b_{t+1} might be negative for some savers, so that some savers might actually be borrowers (against the values of their collateral).

I refer to collateral's having a bubble in this equilibrium because it has positive value even though it is intrinsically worthless in terms of generating consumption. However, the present value of collateral's future dividends does equal its price. In particular, a unit of collateral in period t generates a future dividend flow equal to:

$$(39) \quad \{p^* \phi(1-\psi) h^{*-\psi} (1-\delta_h)^{s-1}\}_{s=1}^{\infty}$$

The present value of this stream, using r^* as a discount rate, equals p^*. There is a circularity within this equilibrium. Collateral's price does equal the present value of collateral's future dividends. But these dividends have positive value only because collateral has a positive price.

Model Ingredients and Their Implications

In this subsection, I discuss how the various ingredients of the model translate into features of the BGP equilibrium with a positive bubble.

Importance of Borrowing Constraints The existence of the positive bubble hinges on the nature of the borrowing constraint faced by entrepreneurs. With a positive bubble, the equilibrium interest rate r^* cannot exceed the equilibrium growth rate g. This restriction implies that the present value of entrepreneurial future income is infinite at each date. The entrepreneur's decision problem only has a solution if entrepreneurs cannot access that full future present value because of binding borrowing constraints.

Woodford (1986) offers one way to understand this connection between bubbles and borrowing constraints. Models with borrowing constraints resemble overlapping generations (OG) economies. In the current model, investors want to dump all of their financial assets. In this sense, they resemble the old agents in an overlapping generations framework. (The difference is that investors want to use their financial assets to fund investment, not consumption.) In contrast, savers are happy to hold financial assets. In this sense, they resemble the young agents in an OG setting. In the no-bubble BGP, the equilibrium interest rate 0 is less than the growth rate of the economy. As in an OG economy, this dynamic inefficiency suggests that a bubble equilibrium may exist. (The same calculation also suggests that the bubbly BGP itself is dynamically inefficient. I will return to this point later.)

Collateral Accumulation Technology It is important to note that the ability to accumulate collateral is inessential for the existence of bubbles in this economy. Bubbly BGP equilibria exist even if the stock of collateral grows at some exogenous rate (no larger than g). I introduce the possibility of collateral accumulation only to emphasize that bubbles exist even if agents must expend valuable resources to keep the bubble going.

In the bubbly BGP equilibrium, collateral prices are constant and the quantity of collateral grows at rate g. However, these features of the equilibrium depend critically on the assumption that ϕ (the productivity term in the collateral accumulation technology) is constant. If ϕ grows over time at g_ϕ, then there is a BGP equilibrium in which collateral prices rise at a rate equal to $(1+g_\phi)^{-1/\psi}-1$ and the quantity of collateral grows at rate $g_h = (1+g)(1+g_\phi)^{1/\psi}-1$. In this BGP equilibrium, the equilibrium interest rate equals $\psi(-\delta_h)+(1-\psi)g_h$.

Zero-Dividend Collateral? In this model, collateral pays dividends in terms of future collateral. However, collateral pays no dividend in terms of consumption itself. If collateral did pay a positive dividend in terms of consumption, the growth rate of this dividend would have to be less than r^*, or the price of collateral would be infinite. At the same time, we have seen that the growth rate of the economy cannot be smaller than r^* if a bubble is to exist. It follows that the collateral asset can have a bubble and a positive consumption dividend only if that dividend grows at a rate less than (and not equal to) g. It would be possible to include such dividends in an analysis of non-stationary equilibria. However, in analyses of BGPs, dividends disappear (because they grow so slowly).[3]

C. Balanced Growth Path Comparisons

Aggregates grow at the same rate whether or not there is a bubble. However, the levels of aggregates are determined by the levels of wealth in the two BGPs. It is easy to see that:

$$(40) \quad MPK_{NB} > MPK_{BUB}$$

which implies in turn that:

$$(41) \quad \overline{W}_{BUB} > \overline{W}_{NB}$$

There is more wealth with bubbles, which means that per-capita consumption, output, and wages are all higher in the bubble steady-state.

It is true that investors receive a lower return in the bubble BGP, because the marginal product of capital is lower. However, it is simple to exploit the concavity of the log function to show that:

$$(42) \qquad \pi \ln(1-\delta+MPK_{BUB}) + (1-\pi)\ln(1+r^*)$$

$$> \quad \pi \ln(1-\delta+MPK_{NB}) + (1-\pi)$$

[3] Santos and Woodford (1997) provide a highly general argument along these lines.

The average return across investors and savers is necessarily the same in the two BGPs. However, the variance of the returns is lower in the bubbly BGP. Any entrepreneur, with a given level of wealth, would be happier in the bubbly BGP.

Of course, these are two BGP equilibria, with different initial levels of physical capital. (The initial level of collateral is pinned down in the bubble BGP, but is essentially irrelevant in the non-bubble BGP.) It is useful to understand to what extent these differences in outcomes can be attributed to these different levels of capital. Toward that end, note that in the non-bubble BGP, all entrepreneurs hold capital and so per-capita capital is equal to $\overline{k}_{NB} = \beta\overline{W}_{NB}/(1+g)$. We can write $\overline{k}_{NB}$ in terms of primitives as:

$$\text{(43)} \quad \overline{k}_{NB}(1+g) = \frac{[MPK_{NB}(1+g_{TFP})^{-1}/\alpha]^{1/(\alpha-1)}}{\pi} = \frac{\{[1+g-(1-\pi)\beta]\pi^{-1}\beta^{-1} - 1 + \delta_k\}^{1/(\alpha-1)}}{(1+g_{TFP})^{1/(\alpha-1)}\alpha^{1/(\alpha-1)}\pi}$$

In contrast, in the bubble steady-state, only investors hold capital and so per-capita capital is equal to $\overline{k}_{BUB}(1+g) = \beta\pi\overline{W}_{BUB}$. We can write $\overline{k}_{BUB}$ in terms of primitives as:

$$\text{(44)} \quad \overline{k}_{BUB}(1+g) = \frac{\{[1+g-(1-\pi)\beta(1+r^*)]\pi^{-1}\beta^{-1} - 1 + \delta_k\}^{1/(\alpha-1)}}{(1+g_{TFP})^{1/(\alpha-1)}\alpha^{1/(\alpha-1)}}$$

We can use these expressions to derive the following rather remarkable result. We have seen that all agents are better off in the bubbly BGP. Nonetheless, the following proposition shows that there are natural conditions under which initial capital is actually *lower* in the bubbly BGP.

PROPOSITION 1. *Suppose* $(g_{TFP}, \beta, \alpha, \delta_k, \phi, \psi)$ *are such that* $(1-\alpha) > \beta r^*/(1+g-\beta)$, *where* r^* *is the equilibrium interest rate and* g *is the equilibrium growth rate. Then* $\overline{k}_{BUB} < \overline{k}_{NB}$.

Proof. The definitions of initial capitals implies that:

$$(45) \quad \begin{aligned} \overline{k}_{NB}^{\alpha-1}/\overline{k}_{BUB}^{\alpha-1} &= \frac{[1+g-(1-\pi)\beta]-\beta\pi+\beta\pi\delta_k}{[1+g-(1-\pi)\beta(1+r^*)]-\beta\pi+\beta\pi\delta_k}\pi^{1-\alpha} \\ &\leq \frac{1+g-\beta}{1+g-\beta-(1-\pi)\beta r^*}\pi^{1-\alpha} \end{aligned}$$

The RHS is less than 1 if:

$$(46) \quad \pi^{1-\alpha} < 1-\beta(1-\pi)r^*/(1+g-\beta)$$

Note that r^* and g are independent of π. Define $\xi(\pi) = \pi^{1-\alpha} - 1 + \beta(1-\pi)r^*/(1+g-\beta)$. It is clear that:

$$(47) \quad \xi(1) = 0$$

$$(48) \quad \xi'(1) = (1-\alpha) - \beta r^*/(1+g-\beta) > 0$$

$$(49) \quad \xi''(\pi) < 0$$

It follows that $\xi(\pi) < 0$ for all $\pi < 1$, which proves the proposition. QED

This result underscores the productive efficiency role of the collateral bubble in this economy. Without a bubble, entrepreneurs are forced to self-finance by storing capital. When π is small, in many periods they are unable to exploit their capital effectively because they don't have a good project. They end up accumulating a lot of (wasted) capital just to take advantage of those dates in which their project is actually operational. With a collateral bubble, investors can borrow, and so resources flow readily from savers to investors. There is no need for entrepreneurs to accumulate as much capital.

Thus, agents are better off in a bubbly BGP, even though there is less physical capital in such a BGP if π is sufficiently low. This result is reminiscent of the standard comparison of monetary versus non-monetary steady-states in overlapping generations settings. However, in this setting, agents have to expend real resources to keep the bubble going. Despite this need for collateral investment, the agents are still better off with a bubble, given the sufficient condition in Proposition 1.

2. Stochastic Bubble

I now consider the behavior of the economy in an equilibrium in which the collateral price follows a stochastic bubble of the following kind. At each date, the coin is flipped which has a probability ε of coming up heads. If the outcome of the coin flip is tails, the collateral price equals a positive number p^+; if the outcome is heads, then the collateral price equals zero at date and thereafter.

A. Model Economy With A Sunspot

I change the above economy in two ways. First, there is a Markov chain z with support $\{0, 1\}$ such that:

$$
\begin{aligned}
z_1 &= 1 \\
\Pr(z_t &= 1 | z_{t-1} = 1) = (1 - \varepsilon), \varepsilon > 0 \\
\Pr(z_t &= 1 | z_{t-1} = 0) = 0
\end{aligned}
\tag{50}
$$

Thus, z_t equals one until it switches to being equal to zero forever. The probability of a switch is ε.

Second, in period $(t-1)$, entrepreneurs can buy and sell *two* kinds of one-period assets that are available in zero net supply. The first asset is a risk-free asset that pays off one unit of consumption regardless of the realization of z. The second asset is an Arrow security that pays off one unit of consumption if $z_t = 1$ and pays off zero units of consumption if $z_t = 0$. In this market structure, financial markets are complete with respect to the shocks involved in the evolution of z. Note that the Arrow security is valueless if $z_{t-1} = 0$.

It is straightforward to extend the definition of an equilibrium in light of these changes in the economy. Instead of a single risk-free rate, we need to price z_{t+1}-contingent consumption. Let $q_t(z^{t+1})$ be the price of z_{t+1}-contingent consumption in history z^t. Then, the budget set of the entrepreneur in this equilibrium, given (p, w, q), is defined by two key constraints. The first is the flow constraint:

$$
\begin{aligned}
(51) \qquad & c_t(A^{t+1}, z^t) + p_t(z^t)h_{t+1}(A^{t+1}, z^t) + x_{t+1}(A^{t+1}, z^t) \\
& + \sum_{z_{t+1} \in \{0,1\}} q_t(z^{t+1}) b_{t+1}(A^{t+1}, z^{t+1}) + k_{t+1}(A^{t+1}, z^t) + w_t(z^t) n_t(A^{t+1}, z^t) \\
\leq\ & b_t(A^t, z^t) + f_t(k_t(A^t, z^{t-1}), n_t(A^t, z^t), A_t) + (1-\delta_k) k_t(A^t, z^{t-1}) \\
& + p_t(z^t) h_t(A^t, z^{t-1})(1-\delta_h) + p_t(z^t)\phi x_t(A^t, z^t)^{\psi} h_t(A^t, z^t)^{1-\psi} \text{ for all } t, A^{t+1}, z^t \\
& b_1 = 0 \\
& k_1, h_1 \text{ given}
\end{aligned}
$$

The second is the borrowing constraint:

$$
\begin{aligned}
(52) \qquad b_{t+1}(A^{t+1}, z^{t+1}) \geq & -p_{t+1}(z^{t+1}) h_{t+1}(A^{t+1}, z^t)(1-\delta_h) \\
& -p_{t+1}(z^{t+1})\phi x_{t+1}(A^{t+1}, z^t)^{\psi} h_{t+1}(A^{t+1}, z^t)^{1-\psi} \text{ for all } t, A^{t+1}, z^t
\end{aligned}
$$

$$(53) \qquad c_t(A^{t+1}, z^t), x_{t+1}(A^{t+1}, z^t), k_{t+1}(A^{t+1}, z^t), h_{t+1}(A^{t+1}, z^t) \geq 0 \text{ for all } t, A^{t+1}, z^t$$

An equilibrium is a specification of $(p, w, q, c, x, h, k, b, n)$ such that (c, x, h, k, b, n) solves the entrepreneur's problem given (p, w, q) and markets clear for all (t, z^t).

B. A Simple Stochastic Equilibrium

In this subsection, I find initial conditions so that there is an equilibrium in which $p_t(z^t) = 0$ if $z_t = 0$ and in which if $z_t = 1$, the economy is on a balanced growth path with a positive constant collateral price p^+. The interpretation of this equilibrium is that there is a bubble in collateral prices when $z_t = 1$ and this bubble bursts when $z_t = 0$. The analysis depends on the assumption that ε is sufficiently small that savers are still willing to accumulate risky collateral.

On any loan from one entrepreneur to another, repayment is bounded from above by the market value of collateral. Since this market value is stochastic, the loan itself is intrinsically risky. I refer to these loans as being *collateralized debt.* These loans pay off zero when $z_t = 0$ and pay off a positive amount when $z_t = 1$. Hence, they are isomorphic to the Arrow security in the market structure described above. As we shall see, while entrepreneurs can trade risk-free assets, their borrowing constraints imply that no such trade takes place in equilibrium.

Before the Bubble Bursts

I begin by analyzing what happens during the initial periods in which $z_t = 1$. As above, along a BGP, quantities must grow at rate g where $(1 + g) = (1 + g_{TFP})^{1/(1-\alpha)}$. As before, let h^* represent the constant ratio of h_t/x_t along this BGP. Then:

(54) $\quad (1+g) = (1-\delta_h) + \phi h^{*-\psi}$

From a one-period perspective, both collateral itself and collateralized debt are equivalent to an Arrow security that pays off only when $z_{t+1} = 1$. The price of this security is constant at some $\overline{q}_1$ while $z_t = 1$; it equals 0 if $z_t = 0$. Its price is such that agents are marginally indifferent between holding the Arrow security and collateral:

(55) $\quad 1 = \overline{q}_1(1 - \delta_h + \phi(1-\psi)h^{*-\psi})$

As well, for the collateral stock to grow, at least some agents must make new collateral goods. Manufacturing goods is also equivalent to buying Arrow securities that pay off only when $z_{t+1} = 1$. Hence, we can readily solve for the collateral price p^+ using the fact that agents are marginally indifferent between producing more collateral goods and buying an Arrow security:

(56) $\quad 1 = p^+ \overline{q}_1 \phi \psi h^{*\psi-1}$

Together these imply that:

(57) $\quad 1/\overline{q}_1 - 1 = r^* = (1-\psi)g + \psi(-\delta_h)$

(58) $\quad p^+ = p^*$

where r^* is the equilibrium interest rate from the deterministic bubbly BGP and p^* is the collateral price from the deterministic bubbly BGP.

As before, let W_t represent the right-hand side of the entrepreneur's budget constraint in date t. Investors (who have $A_{t+1} > 0$) act much like they do in a deterministic bubbly

BGP. They set $c_t = (1-\beta)W_t$, $k_t = \beta W_t$ and set (b, x, h) so that in history (A^{t+1}, z^{t+1}):

(59) $b_{t+1}(A^{t+1}, z^{t+1}) + p_{t+1}(z^{t+1})\{h_{t+1}(A^{t+1}, z^t)(1-\delta_h) + \phi x_{t+1}(A^{t+1}, z^t)^{\psi} h_{t+1}(A^{t+1}, z^t)^{1-\psi}\} = 0$

Because collateral has zero value when $z_{t+1} = 0$, the borrowing constraint implies that $b_{t+1}(A^{t+1}, z^t, 0) = 0$. In words, investors can only borrow by issuing the Arrow security - i.e. through *collateralized* debt.

The savers also set $c_t = (1-\beta)W_t$, but their portfolio problem is more complicated than in the deterministic bubbly BGP. Saving through collateral alone delivers no payoff if $z_{t+1} = 0$. Hence, savers now want to hedge themselves either by buying some claims that pay off when $z_{t+1} = 0$ or by holding physical capital. However, the investors cannot provide any claims that pay off when $z_{t+1} = 0$ because their collateral is worthless in that state. In equilibrium, savers can only hedge themselves by holding some physical capital.

More specifically, savers invest $(1-\gamma)\beta W_t$ by buying collateral, making collateral, and buying Arrow securities that pay off when $z_t = 1$. They also invest $\gamma\beta W_t$ in physical capital. Here, γ satisfies the first-order condition:

(60) $$\frac{(1-\varepsilon)r^*}{(1-\gamma)(1+r^*)+\gamma} - \frac{\varepsilon}{\gamma} = 0$$

It follows that:

(61) $$\gamma = \frac{\varepsilon(1+r^*)}{r^*}$$

I impose the restriction that ε is sufficiently low:

(62) $$\varepsilon < \frac{r^*}{1+r^*}$$

to ensure that γ is less than one.

With these decision rules in hand, we can solve for per-capita quantities much as we did in the deterministic bubbly BGP. The constant MPK_{stoch} for investors must satisfy:

$$(1+g) = \beta\pi(1-\delta_k + MPK_{stoch}) + \beta(1-\pi)[1+(1-\gamma)r^*] \tag{63}$$

We can then solve for period 1 per-capita wealth $\overline{W}_{stoch}$:

$$MPK_{stoch} = \alpha(1+g_{TFP})(\beta\pi\overline{W}_{stoch})^{\alpha-1} \tag{64}$$

(Note that $\overline{W}_{stoch} < \overline{W}_{bub}$, because for $\varepsilon > 0$, entrepreneurs hold some wealth in the form of storage.) Given $\overline{W}_{stoch}$, we can solve for the initial levels of the other aggregate quantities as in the deterministic bubbly BGP case:

$$\begin{aligned}
\overline{c}_{stoch} &= (1-\beta)\overline{W}_{stoch} && (65)\\
\overline{y}_{stoch} &= (1+g_{TFP})(1+g)^{-1}(\beta\pi\overline{W}_{stoch})^{\alpha-1} && (66)\\
\overline{w}_{stoch} &= (1-\alpha)\overline{y}_{stoch} && (67)\\
\overline{k}_{stoch} &= \beta\pi\overline{W}_{stoch}/(1+g) && (68)\\
p^*\overline{h}_{stoch} &= \beta(1-\pi)\overline{W}_{stoch}(1+r^*)/(1+g)^2 && (69)\\
\overline{x}_{stoch} &= \overline{h}_{stoch}/h^* && (70)
\end{aligned}$$

As in the deterministic bubbly BGP case, I assume that all entrepreneurs hold the per-capita levels of collateral and make the per-capita level of additional collateral. Intertemporal trade is conducted through the trade of collateralized debt.

These calculations imply that if ε is near zero, equilibrium per-capita quantities when $z_t = 1$ are well-approximated by the deterministic bubbly steady-state. In contrast, the deterministic steady-state is a poor guide to the behavior of asset prices. When $z_t = 1$, the price of a risk-free bond is given by:

$$\text{(71)} \quad \begin{aligned} \overline{q}_1 + \overline{q}_0 &= 1/(1+r^*) + q_t(z^t, 0) \\ &= 1/(1+r^*) + \varepsilon/\gamma \\ &= 1 \end{aligned}$$

Intuitively, the savers hold a positive amount of physical capital and are not borrowing-constrained. For them, physical capital is a risk-free asset that has gross return equal to 1, and so arbitrage pins down the net risk-free rate at 0.

However, in this stochastic equilibrium, *collateralized* debt is not risk-free. Collateral is worthless when $z_t = 0$, and so is any debt that is backed by collateral. Thus, collateralized debt's expected gross return is given by $(1-\varepsilon)(1+r^*)$. It follows that there is a spread in expected returns between collateralized debt and risk-free debt equal to:

$$\text{(72)} \quad (1-\varepsilon)(1+r^*) - 1 = r^*(1-\varepsilon) - \varepsilon$$

Note that as the probability of the bubble's bursting becomes larger, the expected return on collateralized debt falls and the risk-free rate stays unchanged. The risk premium on collateralized debt is *decreasing* as a function of ε.

Moreover, the risk premium is not continuous with respect to ε. If $\varepsilon = 0$, collateralized debt is risk-free and its risk premium is zero. For ε positive but near zero, the gap in expected returns is close to $r^* > 0$.

After the Bubble Bursts

Now suppose that z^t is a history in which $z_t = 0$ and $z_{t-1} = 0$. Along this sample path, the bubble bursts at date t. Collateral is worth zero at date t. Entrepreneurs who were investors in period $(t-1)$ are unaffected by this realization of z; their obligations are wiped out but so is the collateral backing those loans. The savers from period $(t-1)$ are greatly affected. They have invested $(1-\gamma)\beta\overline{W}_{stoch}(1+g)^{t-1}$ by buying and making collateral, and in collateralized debt. All of this wealth is wiped out.

Despite this massive redistribution, there is no immediate impact on aggregate output. The investors at date t take their accumulated capital, hire workers, and produce output. Wages in period t are unaffected by the bubble's bursting, because they are fully pinned down by the fixed quantities of capital and labor. However, there *is* an immediate effect on aggregate consumption. Recall that consumption is equal to a fraction $(1-\beta)$ of wealth. A fraction $p^*\overline{h}_{stoch}(1+g)/\overline{W}_{stoch}$ of per-capita wealth vanishes because collateral wealth has vanished. Hence, consumption also falls by this same fraction.[4]

From period $(t+1)$ onwards, the situation with respect to production changes dramatically. The economy's dynamics are determined by the evolution of per-capita wealth $\overline{W}_{t+s}$, which is governed by the (nonlinear) difference equation:

$$\overline{W}_{t+s} = \beta\pi(1 + MPK_{t+s} - \delta_k)\overline{W}_{t+s-1} + \beta(1-\pi)\overline{W}_{t+s-1}, s \geq 1 \tag{73}$$

$$MPK_{t+s} = \alpha(\beta\pi\overline{W}_{t+s})^{\alpha-1}(1+g_{TFP})^{t+s-1} \tag{74}$$

[4]This fall in consumption may seem puzzling. Installed capital and output don't change. Entrepreneurs are now investing less into collateral. So what happens to all of the previously consumed output? The answer is that period $(t+1)$ savers hold a lot more physical capital than in period t. This increase in physical capital holdings is enough to rationalize the fall in consumption.

$$(75) \qquad \overline{W}_t = \overline{W}_{stoch}(1+g)^{t-1}[1-\beta(1-\pi)(1+r^*)(1-\gamma)/(1+g)]$$

It is useful to consider detrended per-capita wealth $\widehat{W}_{t+s} = \overline{W}_{t+s}(1+g)^{1-s-t}$. (This detrended per-capita wealth is constant at $\overline{W}_{stoch}$ before the bubble bursts.) The law of motion of $\widehat{W}$ is given by:

$$(76) \qquad \widehat{W}_{t+s} = \beta(1+g)^{-1}\widehat{W}_{t+s-1}[\pi(1+MPK_{t+s}-\delta_k)+(1-\pi)], s \geq 1$$

$$(77) \qquad MPK_{t+s} = \alpha(\beta\pi\widehat{W}_{t+s-1})^{\alpha-1}(1+g_{TFP})^{-1}$$

$$(78) \qquad \widehat{W}_t = \overline{W}_{stoch}[1-\beta(1-\pi)(1+r^*)/(1+g)]$$

Note that the post-bubble path of detrended per-capita wealth does not depend on exactly when the bubble bursts.

Once we know the detrended per-capita level of wealth at each date, it is straightforward to compute similarly detrended versions of other per-capita variables in each period. Their time paths are given by:

$$(79) \qquad \widehat{c}_{t+s} = (1-\beta)\widehat{W}_{t+s}$$

$$(80) \qquad \widehat{w}_{t+s} = (1-\alpha)(1+g_{TFP})(1+g)^{-1}(\beta\pi\widehat{W}_{t+s})^{-\alpha}$$

$$(81) \qquad \widehat{y}_{t+s} = (\beta\pi\widehat{W}_{t+s})^{\alpha}(1+g_{TFP})(1+g)^{-1}$$

All of these detrended variables fall sharply from period t to period $(t+1)$ as a result of the bubble collapse, and then transit to a new, lower, steady-state level.

After the bubble collapses, the interest rate on risk-free debt is still (bounded from above by) 0. However, the interest rate on collateralized debt changes. An issuer of collateralized debt cannot raise any funds today, regardless of what the issuer promises to repay next period (because the collateral is now worthless). Hence, the interest rate on collateralized debt is now (essentially) infinite (its price is zero). The spread between collateralized debt and risk-free debt spikes greatly in the wake of the bubble's collapse.

If $z_{t-1} = 1$, investors who issue collateralized debt at that date make positive payments if $z_t = 1$ and make zero payments if $z_t = 0$. It is natural to interpret the latter state as being one in which these investors *default* on their obligations. After these wide-scale defaults, no further borrowing takes place. All projects are fully self-financed. There is a very real sense in which financial markets shut down after a bubble collapse.

Intermediation is direct in this model. Suppose instead that savers lent and investors borrowed from a common, zero-profit, intermediary. At date t in a bubbly BGP, the investors each owe $p^*\overline{h}_1(1+g)^t$ to the intermediaries in the form of debt backed by collateral. After the bubble collapses, the investors will give this (worthless) collateral to the intermediaries. The intermediaries are now insolvent: they owe $p^*\overline{h}_1(1+g)^t$ to savers and have no resources with which to make this repayment.

After the bubble collapses, no further collateral investment takes place. Collateral goods cannot be transformed into consumption. Instead, the previously valuable collateral is allowed to slowly depreciate over time.

C. Would Society Have Been Better Off Without the Bubble?

The bursting of the bubble leads to a collapse in output, consumption, and wages. These effects suggest a natural question: given that with probability one the bubble was going to burst at some point, would the society have been better off without the bubble at all? Wages are an increasing function of entrepreneurial wealth. Hence, the workers are certainly better off with the stochastic bubble. The key issue is the welfare of the entrepreneurs.

We can use familiar dynamic programming techniques to compute this welfare. After the bubble bursts, an entrepreneur's utility is a linear function of his wealth. The intercept A^* and slope B^* satisfy the equation:

$$\begin{aligned} & A^* + B^* \ln W \\ = \; & \ln\{(1-\beta)W\} + \beta[A^* + \pi B^* \ln(\beta R_{NB} W) + (1-\pi)B^* \ln(\beta(1+r^*)W)) \end{aligned}$$

for all W, where $R_{NB} = (1 - \delta + MPK_{NB})$. Note that A^* and B^* are independent of ε. Now consider an entrepreneur with wealth W when $z_t = 1$, before he knows that he's an investor or a saver. His welfare is also a linear function of $\ln W$, where the intercept $A'(\varepsilon)$ and slope $B'(\varepsilon)$ satisfy the functional equation:

$$\begin{aligned} & A'(\varepsilon) + B'(\varepsilon) \ln W \\ = \; & \ln\{(1-\beta)W\} + \beta(1-\varepsilon)[A'(\varepsilon) + \pi B'(\varepsilon) \ln(\beta R_{BUB}(\varepsilon) W) + (1-\pi) B'(\varepsilon) \ln(\beta(1+r^*)W) \\ & + \beta\varepsilon[A^* + \pi B^* \ln(\beta R_{BUB}(\varepsilon) W) + (1-\pi) B^* \ln(\beta\varepsilon W(1+r^*)/r^*)] \end{aligned}$$

for all W. Here, I use the notation $R_{BUB}(\varepsilon)$ to represent $(1 - \delta_k + MPK_{stoch}(\varepsilon))$. Note that

$\varepsilon \ln(\varepsilon)$ converges to zero as ε converges to 0. We can conclude that, when ε is near zero, the utility of an entrepreneur with wealth W in the stochastic bubbly equilibrium is well approximated by his welfare in the deterministic bubbly BGP.

Now suppose all entrepreneurs begin with the same level of $(k_1, h_1, x_1) = (\overline{k}_{stoch}, \overline{h}_{stoch}, \overline{x}_{stoch})$. If ε is small, then $\overline{k}_{stoch}$ is approximately equal to $\overline{k}_{bub}$. Suppose that $\overline{k}_{stoch} < \overline{k}_{nb}$ (as would be true under the condition described in Proposition 1). We know from our earlier analysis of BGPs that all agents are worse off in a non-bubbly BGP in which all entrepreneurs begin life with $\overline{k}_{nb}$ than in a bubbly BGP. Of course, all agents are even worse off in a non-bubbly equilibrium if all entrepreneurs begin life with the lower initial level of capital $\overline{k}_{stoch}$. Thus, if ε is sufficiently low, and all entrepreneurs begin life with $(\overline{k}_{stoch}, \overline{h}_{stoch}, \overline{x}_{stoch})$, we know that agents are better off with the stochastic bubble.

D. Numerical Example

In this subsection, I numerically simulate the results of a bubble collapse in this model, assuming that ε is small (10^{-7}). I set $g = 0.02$, $\alpha = 1/3$, $\delta_k = 0.1$, and $\beta = 0.95$. These settings are standard in annual macroeconomic models. I set $r^* = 0.015$, which is halfway between usual measures of the risk-free interest rate and g. However, the results are largely unaffected by this parameter. Finally, I set $\pi = 0.8$, so that 20% of the entrepreneurs have useless projects in any given year. (I pick this last parameter somewhat arbitrarily, so as to generate a fall in entrepreneurial wealth of about 20%.) I plot the effects of the shock on logged detrended output; the effects on wages, consumption, and per-capita capital are quite similar.

The computed path is depicted in Figure 1. In this figure, detrended output falls

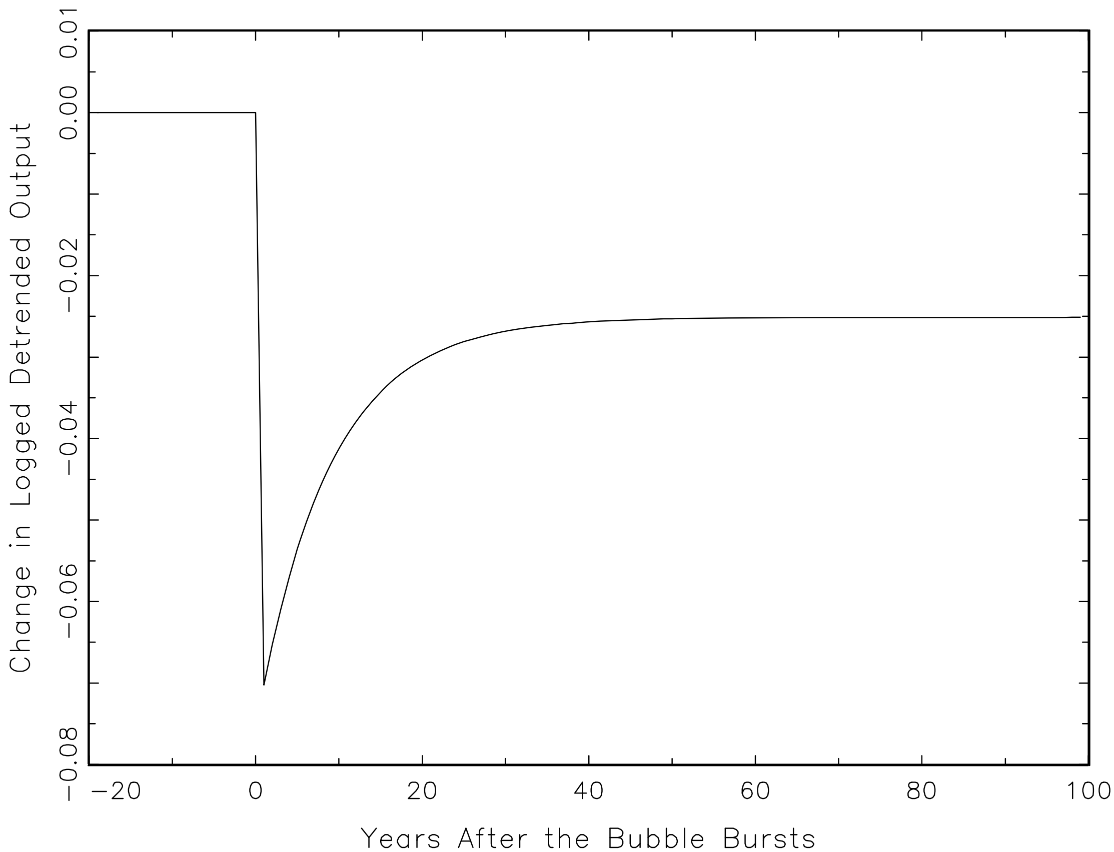

Figure 1: **Output Before and After A Bubble Burst**

by 7% in the first year after the bubble bursts. It rises thereafter, but the shock is highly persistent. In the long run, detrended output remains 1.5% below its original steady-state.

The initial fall in output is attributable to the share of wealth that disappears when the bubble collapses. Given these parameter settings, around 20% of per-capita wealth in the initial, bubbly, steady-state is in the form of collateral. This fall in wealth is translated one-for-one into a fall in the capital that gets used by investors. Because capital share is 1/3, this 20% fall in capital translates into a 7% fall in output. The law of motion (76) of $\ln \overline{W}$ is basically a unit root, and so this shock is highly persistent.

The long-run impact of the shock is shaped by a different force. Before the bubble bursts, savers' wealths grow at $r^* > 0$ (if ε is small). After the bubble bursts, savers have an interest rate equal to zero. This change in the savers' rates of return is responsible for the long run fall in wealth and output.

3. Government Interventions

In this economy, bubble years are good years. The collapse of the bubble triggers a precipitous fall in output which has permanent adverse consequences. Suppose the economy is in the first period after a bubble collapse. What can the government do, if anything, to restore better long-run economic health? I first discuss a class of desirable interventions, and then compare them to what is currently (late 2008-early 2009) being done in the United States to deal with the ongoing financial crisis.

A. Government Debt as Collateral

In the model economy, the bubble is useful because it expands borrowing capacity. Once the bubble collapses, there is no source of collateral, and entrepreneurs are forced to self-finance their projects. To help the economy, the government must provide some other source of collateral to the entrepreneurs.

Caballero and Krishnamurthy (2006) (CK) provide useful insights about what these other forms of collateral might be. They analyze an overlapping generations model of an incomplete markets open economy. They use the overlapping generations model to generate bubbles in real estate prices, and study the consequences of these bubbles. My model and theirs differ in important respects, but their discussion of government debt is highly relevant.

CK emphasize the role of government debt as an extra source of collateral. They first contemplate unbacked government debt, in which the government re-finances existing debt simply by rolling it over. Such debt is isomorphic to the fiat money in KM. and to housing in my model. Suppose that, immediately after the bubble collapses, the government gives each entrepreneur promises to future consumption. This promise is unbacked, in the sense that the government will simply default on this promise if it cannot roll it over. There is an equilibrium in which this debt is valued and held. However, there are also other equilibria in which this debt is priced at zero (just like collateral is). Once debt is completely unbacked, its ability to operate as collateral is up to the self-fulfilling beliefs of private agents. There is no way for the government to guarantee that its debt will function as a form of collateral.

CK then consider debt that is explicitly backed by the taxation authority of the government. The fundamental premise here is that the government is a superior collection agency than are private agents. From a strict theoretical point of view, this premise is hard to defend (why not just use those great collection powers on behalf of private lenders?). But in reality, it is true that people who do not pay their taxes can go to jail, while debtors cannot. The government seems to have collection powers that it is unwilling to let private creditors use.

It is clear that if the government commits to using taxes to repay its bonds, the bonds are fundamentally different from intrinsically useless collateral. More subtly, CK argue that, in equilibrium, the government will never need to collect the taxes. Instead, the government can commit to a strategy under which it commits to rolling over the existing debt, and then levies taxes if agents fail to buy the issued debt. Given such a commitment, there is a unique equilibrium in which agents are always willing to buy the issued debt, without any taxes ever being levied. Note though that the government's ability to collect the taxes is necessary to its being able to rule out collapses in the value of this debt. Any limitations on this ability to collect taxes (say, because of distortions) curtail the effectiveness of this proposed policy.

B. A Desirable Intervention

The previous subsection argues that the government bonds can serve as collateral. In this subsection, I describe how the government can intervene after a bubble's collapse to restore the economy's health. Throughout, I assume that ε is near zero, so that the path of aggregate variables before the bubble's collapse is well-approximated by the deterministic bubbly BGP.

Bail Out Collateral Owners And Raise Interest Rates

There are two related, but distinct, problems in the economy after the bubble's collapse. First, entrepreneurs have lost wealth. This fall is responsible for the *immediate* adverse impact on aggregate variables. Second, the equilibrium interest rate has fallen from r^* to 0. This fall is responsible for the *long-term* adverse impact on aggregate variables. The government can readily fix both these problems with a two-part intervention.

Suppose the bubble bursts at date t. The per-capita wealth loss of the entrepreneurial sector equals:

$$p^*\overline{h}_{stoch}(1+g)^t$$

The government hands out bonds to entrepreneurs. The distribution of this handout across entrepreneurs is really irrelevant, as long as the bonds promise to pay:

$$p^*\overline{h}_{stoch}(1+g)^t(1+r^*)$$

next period per-capita. Assuming that the interest rate is r^*, this injection restores per-capita entrepreneurial wealth to what it would have been in the absence of the bubble's bursting.

The above injection of wealth cures the first problem created by the bubble's collapse. The second problem is that savers are accumulating wealth through a low rate of return vehicle (storable capital goods). The government can cure the second problem by committing itself to borrow at the real interest rate r^*. The savers will lend to the government at this rate r^*, but the investors will not. Now the law of motion of per-capita wealth in the economy mimics that in the deterministic bubbly BGP:

$$W_{t+s} = \beta\pi W_{t+s-1}(1 - \delta_k + \alpha(\beta\pi W_{t+s-1})^{\alpha-1}(1 + g_{TFP})^{t-1}) + \beta(1 - \pi)W_{t+s-1}(1 + r^*)$$

Assuming ε is small (so that $\overline{W}_{stoch}$ is close to $\overline{W}_{bub}$), this law of motion implies that, as in the deterministic bubbly BGP, wealth grows at rate g.

To implement this policy, the government has to repay $\beta(1 - \pi)W_{t+s}(1 + r^*)$ at each date $(t + s)$. It can readily afford this repayment because in period $(t + s)$, it raises $\beta(1 - \pi)W_{t+s}(1+g)$ in new funds by its debt issue. Because $r^* < g$, the government's plan generates extra resources available at each date. It could simply discard these, give them to the workers, or buy public goods. Why do these extra resources appear? The government has the ability to costlessly create collateral by simply printing pieces of paper (government debt). In contrast, the private sector has to expend consumption goods in order to create new collateral goods. It is exactly these consumption goods that were used to create collateral that accrue to the government under this plan.

This policy of raising interest rates may strike some readers as counterintuitive. It is in fact standard in economies with borrowing constraints. By their very nature, borrowing constraints choke off the demand for loans and thereby force down interest rates. A supply of outside government debt allows agents to avoid borrowing constraints by accumulating enough saving. However, this extra supply of loans also necessarily raises interest rates. (See KM (2008) for a similar argument.)

Ex-Ante Effects

I motivated the above intervention (bailout plus interest rate increases) through its desirable post-bubble effects. In this subsection, I discuss the effect on the economy *before* the bubble bursts if asset traders anticipate a collateral bailout and a real interest rate peg at r^* once $z_t = 0$.

The ex-post benefits of injecting wealth did not depend on the distribution of this injection across the entrepreneurs because their decision rules are linear. However, there is a particular distribution that has desirable ex-ante effects. Consider an entrepreneur who has $h_t(1-\delta_h) + \phi x_t^{\psi} h_t^{1-\psi}$ units of collateral in period t when the bubble bursts. The government gives that entrepreneur bonds that promise $(p^* h_t(1-\delta_h) + p^* \phi x_t^{\psi} h_t^{1-\psi})(1+r^*)$ next period. Thus, the bailout is proportional to the entrepreneur's holdings of collateral.

This proportional intervention has desirable ex-ante properties. In the stochastic bubble equilibrium, collateral is risky. As a hedge, savers hold a portfolio of both collateral and risk-free storable physical capital. This portfolio behavior implies that per-capita wealth is lower in the stochastic bubble equilibrium, even before the bubble bursts, than in the deterministic bubbly BGP. Savers *under-accumulate* collateral because the bubble is risky.

Now suppose that people are aware of the government's bailout plan. Holders of collateral can now expect it to pay a rate of return equal to r^* regardless of the realization of z. Collateral is risk-free[5], and so savers no longer hold any physical capital. The law of motion

[5] Kocherlakota (2001) and Kocherlakota and Shim (2007) consider economies in which contracts are enforced solely through the loss of collateral. As in the model in this paer, they show that ex-ante optimal

of entrepreneurial wealth is exactly the same as in the deterministic bubbly BGP throughout the lifetime of the economy, not just after the bubble bursts. As a result, aggregate variables and welfare are higher before the bubble bursts because of the government's post-bubble intervention.

In reaching this conclusion, it is important to keep in mind two critical aspects of the intervention. First, the government explicitly allows the bonds used for the bailout to be seized by creditors when the bubble bursts. This feature of the intervention means that the lenders face no ex-ante risk in making their loans against the borrower's collateral. Second, the government only compensates owners of the physical collateral good. The government does not bail out all holders of Arrow securities.

The equilibrium under the government's post-bubble intervention is actually even better than the deterministic bubbly BGP. When z_t jumps to 0, the economy switches to government debt as a form of collateral. As stressed above, unlike the collateral good h, government debt is costless to create. As a result, the switch to government debt frees up resources for the government to spend.

C. Current Interventions

As of this writing (early 2009), the federal government is intervening in a massive way in financial markets. The model is clearly simplistic in a number of important ways. Nonetheless, it is useful to think through the effects of the current federal interventions in the context of the model.

One current government policy is to hand out backed debt to banks. The model does

contracts may well require collateral insurance.

not speak directly to the efficacy of this policy. However, if entrepreneurs cannot credibly commit to repaying their loans to banks, then giving backed debt to banks is useless. The policy needs to get extra wealth into the hands of entrepreneurs with desirable projects if it is to be effective.

The Federal Reserve has also adopted a policy by which it will purchase *unsecured* commercial paper. In the context of the model, suppose the government sells backed debt to each entrepreneur in exchange for that entrepreneur's promise to make a repayment next period. This policy has no intrinsic impact on collateral prices and so collateral prices may well remain zero. In this equilibrium, there is no viable collateral, and the government will get *nothing* back for its loan. This sounds like a bad policy. But it is essentially an indirect way for the government to give each entrepreneur extra wealth. If the government also pegs the real interest rate at r^*, then this policy has equivalent ex-post outcomes to my preferred intervention described above.[6]

The Federal Reserve is currently *lowering* interest rates. This policy does not work well in the model economy. Lowering interest rates increases the demand for loans. But all of the potential borrowers are already on their borrowing constraint. As discussed above, the correct solution is to *raise* interest rates after bailing out holders of collateral. In this fashion, the savers are encouraged to lend to the government, and the government can serve as an intermediary between savers and investors.[7]

[6] The ex-ante effects of this policy are not the same as mine though. My preferred intervention explicitly insures collateral owners against the fall in the value of collateral. This policy makes collateral risk-free from an ex-ante perspective. Buying unsecured commercial paper does not do so.

[7] This policy seems counter-intuitive to those used to working with representative agent models. However, models with liquidity constraints typically imply that central banks should raise interest rates whenever those liquidity constraints get tighter (see Kocherlakota (2003) and KM).

Finally, there are many economic actors who have issued securities that pay off contingent on house price movements, without owning an actual physical object called a house. In the context of the model, these agents have issued an Arrow security that pays off when $z_t = 0$. With the collapse in the price of collateral, the agents are not able to make their commitments. At least in some instances, the government has provided sufficient funding to such agents to allow them to make their commitments. Note that this kind of bailout is distinct from the bailout proposed above, in which only owners of collateral goods receive compensation for their losses.

Within the model, insuring all owners of Arrow securities is problematic from both an ex-post and ex-ante point of view. From an ex-post perspective, there is no natural cap on how much the government will be injecting into the economy. This difficulty can be solved (with sufficient political will) by the government's restricting the aggregate size of the bailout to match the total size of the fall in the value of collateral. The ex-ante deficiency is insurmountable though. Suppose agents anticipate that the government will fully insure all issuers of Arrow securities. Their optimal response is to create enormous amounts of such securities. (It is definitely tempting to speculate that these motives were responsible for the creation of the giant credit default swap market.)

The collapse of a bubble creates two problems. First, it robs entrepreneurs of wealth. Second, it lowers the real interest rate. A successful intervention needs to resolve both of these difficulties. Not all current government interventions do so.

D. The Role for Government: Assessing Dynamic Efficiency Revisited

We have seen that there is a role for a welfare-improving government intervention in the stochastic bubble equilibrium. One benefit of the government is that its ability to tax ensures that there will never be a collapse in the value of its debt, as opposed to what can happen to a bubble in the price of private sector collateral. However, the government has another benefit: it can create new collateral costlessly. This superior technology implies that the government can actually improve on the *deterministic* bubbly BGP, even given the same initial conditions.

Here's how the government accomplishes this improvement. First, it initially endows entrepreneurs with government debt worth $p^*\overline{h}_{bub}(1+g)$. It then commits to borrow at interest rate r^* forever. This policy implies that savers earn r^* and investors earn $MPK - \delta$. The dynamics of entrepreneurial wealth and per-capita variables is the same as in the deterministic bubbly BGP (except that with the government intervention, the value of private sector collateral is zero). Since wealth grows at rate g, the government's debt issue in period $(t+1)$ is $(1+g)$ times larger than its debt in period t. Hence, because $r^* < g$, the government's period $(t+1)$ debt issue raises enough money to pay its obligations in period t, with resources left over to be used at its discretion.

This analysis is reminiscent of the discussions of dynamic efficiency in overlapping generations economies pioneered by Diamond (1965). However, there is a key difference. In Diamond's setup, the interest rate equals the marginal product of capital net of depreciation $(MPK - \delta)$. Hence, one can check for dynamic efficiency by comparing the interest rate or $(MPK - \delta)$ to the growth rate. In the deterministic bubbly BGP in my model, the existence of financing constraints mean that $MPK - \delta$ is larger than the interest rate. To assess dynamic efficiency, one has to use a comparison of interest rates and growth rates.[8] In United States data from 1989-2005, the average real return on Treasury bills is about 1.3%. The average growth rate of nondurable consumption and services is about 1.8% The gap is considerably larger in the 1934-2005 period, because the average interest rate falls to zero. (See Mehra and Prescott (2008).)

4. Conclusions

In this paper, I examine a model economy in which capital re-allocation is critical. This re-allocation is accomplished via collateralized lending. However, collateral is scarce and all entrepreneurs face borrowing constraints that bind infinitely often into the future. These two ingredients imply that equilibrium bubbles naturally emerge in the price of the collateral. The resulting bubbles expand entrepreneurial borrowing capacity and generate more output, consumption, and welfare. In this framework, the collapse of a bubble has a dramatic and immediate adverse impact on aggregate variables, which thereafter never fully recover.

The model provides a justification for interventions similar to (but definitely distinct from) those that have been considered and implemented in recent months. When π is small, the economy requires a large amount of capital re-allocation on an annual basis. If these re-allocations are disrupted, then aggregate output can be surprisingly severely affected within the course of only one year. In this way, the model provides a *possible* justification for the

[8] Abel, Mankiw, Summers, and Zeckhauser (1989) consider dynamic efficiency in overlapping generations economies with aggregate shocks. They show that in these models, one cannot reliably evaluate dynamic efficiency by comparing average risk-free interest rates to average growth rates. This conclusion applies to my model as well. Their solution is to use the sign of net inflows into firms. This solution is valid in their context, but is unreliable in my model with financial constraints. (Kraay and Ventura (2007) make a similar point.)

need for speed apparently perceived by the federal government. On the other hand, the model also makes clear that the details of interventions (like whether to raise or lower interest rates!) matter a great deal.

Other recent papers draw connections between collateral scarcity and the existence of bubbles (Caballero and Krishnamurthy (2006) and Araujo, et al (2005)). However, to gauge the empirical relevance of these theoretical connections, we need to have good measures of entrepreneurial risk and collateral scarcity. As KM point out, models like theirs (and Angeletos (2007)) are specifically constructed to mimic standard macroeconomic frameworks. For this reason, I believe that it will be relatively easy to augment the model in this paper so that it is well-suited for a serious quantitative analysis of bubbles in the macroeconomy.

Real estate prices fell dramatically in Japan in the 1990's and in the United States in the 2000's. In both settings, the fall in an asset's price had a significant impact on collateralized lending. The theme of this paper is that these kinds of bubble collapses may well be an inevitable part of private sector collateral provision. The taxation power of the government (in the United States and other developed countries) means that its debt is a less risky form of collateral than what is available to the private sector. Moreover, if collateral is costly to accumulate, government debt is also a more socially efficient form of collateral. The government can provide better outcomes if it is able to give this debt to people with investment opportunities, and is willing to pay a high rate of return.

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