1 Executive Summary

The Chicago Fed DSGE model resembles the models of Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2007), and Justiniano, Primiceri, and Tambalotti (2008). A family of individuals with preferences over nondurable consumption and leisure populates the economy. The preferences are consistent with the balanced-growth restrictions of King, Plosser, and Rebelo (1988); and they feature internal habit in nondurable consumption. Long-run growth arises from investment-specific and Hicks-neutral technological progress. Output can be used for either nondurable consumption, physical investment, or government spending.

Production begins with intermediate-good firms combining different varieties of labor into a labor aggregate and then combining these with capital services. Increasing capital utilization raises required maintenance costs but does not accelerate depreciation. Each intermediate-good firm produces a distinct variety and sells this to a competitive final-goods sector. Individuals’ wages and intermediate-goods firms’ prices are sticky in the sense of Calvo (1983). The prices of firms without an adjustment opportunity are indexed to a weighted average of the prior quarter’s inflation rate and the steady-state inflation rate. Workers without wage adjustment opportunities have their wages similarly indexed to a weighted average of the prior quarter’s wage inflation adjusted for labor productivity growth and nominal wage growth in the steady state. These primitives give rise to two Phillips curves, one for wage inflation and the other for price inflation. In these, indexation adds the lag of inflation to the usual expected future inflation term. The deviation of marginal cost from its steady-state value plays its usual role in the price Phillips curve. In the wage Phillips curve, the deviation of the wage from the marginal rate of substitution between consumption and leisure takes its place.

A Taylor rule governs monetary policy choices. In it, the Federal Funds rate responds to its value in the previous quarter, the four-quarter growth rate of GDP, the deviation of inflation’s four-quarter growth rate from a time-varying mean and a monetary policy shock.

In addition to the monetary policy shock and the time-varying inflation target, the model features shocks to both investment-specific and Hicks-neutral technology, the marginal efficiency of investment, the family’s rate of time preference and preference for leisure, government spending, and the elasticities of substitution in the aggregators of intermediate goods and labor varieties. This last disturbance and the monetary policy shock are white noise, while the others follow mutually independent AR(1) processes.

For estimation, we suppose that real GDP, consumption, investment, hours worked, and average wages equal their model counterparts. The estimation uses the implicit deflators for PCE on nondurable goods and services and for core PCE. We suppose that these each equal the model’s nominal price level for consumption plus a disturbance term that follows a first-order autoregression. Estimation also employs the implicit deflator
for investment — which we set equal to the model’s consumption price index minus the index of investment-specific technology — and the GDP deflator — which we match with a weighted average of the model’s consumption and investment deflators. We use Bayesian methods and data from 1984:Q1 through the present to calculate the empirically-relevant parameter values and forecasts.

2 Detailed Description of the Model

The description of the model economy proceeds in three stages. We first review the economy’s primitive tastes and technology, and we then present the model’s assumed market structure. The section concludes with the definition of an equilibrium.

2.1 Tastes and Technology

Seven types of agents inhabit the model economy; households, employment agencies, intermediate goods producers, final goods producers, capital installers, a fiscal authority, and a monetary authority. Since the monetary authority’s technology is intimately wound up with the opportunities for trade, we defer its presentation until the next subsection.

2.1.1 Households

Households value a composite nondurable consumption good and dislike working. Their preferences are given by the following utility function.

\[
E_t \left[ \sum_{\tau=0}^{\infty} \beta^\tau b_{t+\tau} \left( \log(C_{t+\tau} - hC_{t+\tau-1}) - \phi_{t+\tau} \frac{L_{t+\tau}^{1+\nu}}{1+\nu} \right) \right]
\]

Here, \(C_t\) is consumption and \(L_t\) is hours worked. The parameter \(h\) is the degree of habit formation. Our analysis does not (yet) use asset prices beyond the federal funds rate. In our framework, habit gives the economy’s fluctuations more persistence. Both the disutility from a given amount of work and the rate of discounting are stochastic. We label \(\phi_t\) the “labor-supply shock” and \(b_t\) the “discount rate” shock. First-order autoregressions govern their evolution.

\[
\ln \phi_t = \rho_\phi \ln \phi_{t-1} + \varepsilon_{\phi,t}
\]
\[
\ln b_t = \rho_b \ln b_{t-1} + \varepsilon_{b,t}
\]

Their innovations are mutually independent, \textit{i.i.d.} over time, and normally distributed

\[
\varepsilon_{\phi,t} \sim N(0, \sigma_{\phi}^2)
\]
\[
\varepsilon_{b,t} \sim N(0, \sigma_b^2)
\]

Note that these preferences are consistent with King, Plosser, and Rebelo’s (1988) requirements for balanced growth.

2.1.2 Employment Agencies

Each household provides a distinct labor service. It is the job of the employment agencies to package households together into teams. We use \(j \in [0, 1]\) to index the households and denote the time worked by
household $j$ for a particular employment agency with $L_t(j)$. The employment agency’s team output is

$$L_t = \left( \int_0^1 L_t(j) \frac{1}{1+\lambda_{w,t}} \, dj \right)^{1+\lambda_{w,t}}. \tag{1}$$

We call the team’s output “aggregate hours worked”. The elasticity of substitution between any two households’ hours worked is $-\lambda_{w,t}$. We assume that its logarithm follows a white-noise process.

$$\ln \lambda_{w,t} = \ln \lambda_w + \epsilon_{w,t}$$

$$\epsilon_{w,t} \sim N(0, \sigma^2_w)$$

### 2.1.3 Intermediate Goods Producers

There is a continuum of intermediate goods producers with names $i \in [0, 1]$. These use capital and labor services (provided by the employment agencies) to produce differentiated goods with a common technology.

$$Y_t(i) = \max \{0, A_t^{1-\alpha}K_t(i)^{\alpha}L_t(i)^{1-\alpha} - F_t \}$$

In this, $K_t(i)$ and $L_t(i)$ give the capital and labor services employed by producer $i$, $A_t$ is labor-augmenting technological change, and $F_t$ is a fixed cost of production. A first-order autoregression governs the growth rate of $A_t$, which we denote with $a_t \equiv \Delta \ln A_t$.

$$a_t = (1 - \rho_a)\gamma_a + \rho_a a_{t-1} + \epsilon_{a,t}$$

Here, $\epsilon_{a,t} \sim N(0, \sigma_a^2)$. This is our Hicks-neutral technology shock. We describe the evolution of $F_t$ below.

The intermediate goods producers create capital services through the utilization of capital. The capital services generated from utilizing $K$ units of capital at the rate $u$ equals $uK$. The cost of utilization, $a(u) \times K$, are paid in units of the consumption good.

### 2.1.4 Final Goods Producers

Final goods producers have access to a constant-returns-to-scale technology for combining the distinct goods of the intermediate goods producers into a homogeneous final good.

$$Y_t = \left( \int_0^1 Y_t(i) \frac{1}{1+\lambda_{p,t}} \, di \right)^{1+\lambda_{p,t}}$$

The elasticity of substitution between any two intermediate goods follows a first-order autoregression in logarithms

$$\ln \lambda_{p,t} = \ln \lambda_p + \rho_p \ln \lambda_{p,t-1} + \epsilon_{p,t}$$

$$\epsilon_{p,t} \sim N(0, \sigma_p^2)$$

Households can use the final good for nondurable consumption ($C_t$). Its alternative uses are for government consumption and physical capital accumulation.
2.1.5 Capital Installers

A capital installer using $I_t$ units of the final good can produce

$$Z_t \mu_t \left(1 - \chi \left(\frac{I_t}{I_{t-1}}\right)\right) I_t$$

units of installed capital. The function $S$ captures the presence of adjustment costs in investment. We assume that $\chi(1) = \chi'(1) = 0$ and $\chi'' > 0$. We call $Z_t$ the level of investment-specific technology, and we refer to $\mu_t$ as the marginal efficiency of investment. As its name suggests, exogenous technological development raises $Z_t$. Just as with $A_t$, its growth rate $z_t \equiv \Delta \ln Z_t$ follows a first-order autoregression.

$$z_t = (1 - \rho z) \gamma_z + \rho z z_{t-1} + \varepsilon_{z,t}$$

The logarithm of $\mu_t$ also follows a first-order autoregression.

$$\ln \mu_t = \rho \mu \ln \mu_{t-1} + \varepsilon_{\mu,t}$$

Here, $\varepsilon_{\mu,t} \sim N(0, \sigma^2_{\mu})$.

Installed capital depreciates at the constant rate $\delta$ per quarter.

2.1.6 Fiscal Authority

The fiscal authority demands and consumes an exogenous fraction of total final goods production, $g_t$. This follows a first-order autoregression.

$$\ln g_t = (1 - \rho_g) \ln g + \rho_g \ln g_{t-1} + \varepsilon_{g,t}$$

The disturbance $\varepsilon_{g,t} \sim N(0, \sigma^2_g)$. The fiscal authority finances these expenditures with lump-sum taxes that balance its budget quarter-by-quarter.

2.2 Market Structure

Market power and nominal rigidities pervade the economy’s trading environment. All prices are in units of money, which is intrinsically worthless paper. Denote the money price of the final good with $P_t$, the price of the labor aggregate with $W_t$, and their growth rates with $\pi^p_t = P_t / P_{t-1}$ and $\pi^w_t = W_t / W_{t-1}$. In the market for labor services, household $j$ is the sole supplier of its distinct labor variety, and so it has a limited ability to choose its wage. With probability $\xi_w$, the household sets its money wage to the value given by a simple indexation rule:

$$W_{t}(j) = W_{t-1}(j) (\pi_{t-1} S_{t-1})^{\gamma_w} (\pi \gamma)^{1-\iota_w}.$$ 

In this, $S_{t-1} = A_{t-1} Z_t^{\alpha/(1-\alpha)}$ is the source of long-run wage growth, $\gamma = \gamma_a + \alpha \gamma_z / (1 - \alpha)$ is its long-run growth rate, and $\pi$ is the steady-state inflation rate. With the complementary probability, the household’s choice of $W_{t}(j)$ is unconstrained.

The economy’s other price-setting agents are the intermediate-goods firms. With probability $\xi_p$, producer $i$ sets its price according to the indexation rule

$$P_{t}(i) = P_{t-1}(i) \pi_{t-1}^{\iota_p} \pi^{1-\iota_p}.$$ 

4
With the complementary probability, a firm’s price choice is unconstrained.

There is a market for one-period risk-free nominal bonds, with gross interest rate $R_t$. Denoting its steady-state value with $R$, the monetary authority sets this rate according to a Taylor rule.

$$R_t = \left( \frac{R_{t-1}}{R} \right)^{\rho_R} \left( \prod_{s=0}^{3} \phi_s \left( \frac{Y_t}{Y_{t-4}} \right)^{\frac{1}{4}} e^{-\gamma} \right)^{1-\rho_R} \times e^{\varepsilon_{R,t}}$$  \hspace{1cm} (2)

We call the disturbance $\varepsilon_{R,t} \sim N(0, \sigma^2_R)$ the “monetary-policy” shock. The Taylor rule depends on the monetary authority’s time-varying target inflation rate, $\pi^*_t$. A first-order autoregression governs its evolution.

$$\ln \pi^*_t = (1 - \rho^*) \ln \pi + \rho^* \ln \pi^*_{t-1} + \varepsilon_{\pi,t}$$  \hspace{1cm} (3)

The innovation $\varepsilon_{\pi,t} \sim N(0, \sigma^2_{\pi})$.

The markets for the labor aggregate and the final good are perfectly competitive. There also exist markets for claims on the final good contingent on the realizations of individual households’ wage-adjustment possibilities. The final markets to describe are those for capital and shares in intermediate-goods producers and capital installers. We follow Prescott and Mehra (1980) and presume without loss of generality that only households own capital between quarters. At the beginning of the quarter, households sell capital to the Intermediate Goods Producers. At the quarter’s end, the Intermediate Goods Producers and Capital Installers sell all capital available back to the households. Households own all shares in the intermediate goods producers. The households receive any of their profits as dividends each quarter. These shares have unlimited liability, so the households must concomitantly pay for any firm’s losses within the period.

2.3 Optimization and Equilibrium

The optimization of all agents but the fiscal and monetary authorities defines equilibrium. It is helpful to begin with the employment agencies’ and final goods producers’ optimization problems. We then consider households’ utility maximization and Intermediate Goods Producers’ profit maximization. We conclude with a discussion of the Capital Installers’ profit maximization.

2.3.1 Employment Agencies and Final Goods Producers

The problems’ of Employment Agencies and Final Goods Producers are similar, so we consider only the former explicitly. Given the nominal wage for “team” labor $W_t$ and the nominal prices of all households’ labor, $W_t(j)$ for all $j \in [0, 1]$, an Employment Agency chooses total team labor supplied $L_t$ and labor demanded from each household $(L_t(j))$ to maximize

$$W_t L_t - \int_0^1 W_t(j) L_t(j) dj$$

subject to the production function in (1). The first-order condition for the profit maximizing choice of $L_t(j)$ yields a labor demand curve for household $j$.

$$L_t(j) = L_t \left( \frac{W_t(j)}{W_t} \right)^{-\frac{\lambda_{w,t}}{\lambda_{w,t}}}$$  \hspace{1cm} (4)

Employment Agencies earn zero profits in equilibrium. We can combine this requirement with (4) to express $W_t$ as a function of $W_t(j)$.

$$W_t = \left( \int_0^1 W_t(j)^{-\frac{1}{\lambda_{w,t}}} dj \right)^{-\lambda_{w,t}}$$  \hspace{1cm} (5)
2.3.2 Households

If the economy begins with all households having identical financial wealth and consumption histories, then their optimal use of the available contingent claims markets ensures that this homogeneity will continue. Therefore, we will only need to consider the consumption and savings decisions of a representative household.

To begin, denote the marginal utility of expanding $C_t$ with $\Upsilon_t$.

$$\Upsilon_t \equiv \frac{b_t}{C_t} - \frac{h}{C_{t+1} - hC_t}$$

Households can save either by purchasing bonds (which are in zero net supply) or capital. Given the nominal interest rate $R_t$, the Euler equation for optimal bond purchases is

$$1 = \beta E_t \left[ \frac{b_{t+1}}{R_t} \left( \Upsilon_{t+1} + \Upsilon_t \right) \right].$$

Denote the nominal price of a unit of installed capital in quarter $t$’s beginning-of-period market with $Q^0_t$ and the analogous price for the end-of-period market with $Q^1_t$. With this notation, the Euler equation for optimal capital purchases can be expressed as

$$1 = \beta E_t \left[ \frac{b_{t+1} Q^0_{t+1}}{Q^1_t \pi_{t+1}} \right].$$

The obstacles to nominal wage adjustment substantially complicate a household’s labor supply decision, so it is helpful to first understand the “frictionless” decision in which $\xi_w = 0$ and $\lambda_{w,t} = \lambda_w$ always. The first-order conditions for this choice can be reduced to the usual monopoly pricing rule: Equate the percentage price-cost margin to the demand elasticity’s inverse.

$$\frac{W_t(j) P_t - \phi_t \Upsilon_t L_t(j) \nu_{W_t}}{W_t(j) P_t} = \frac{\lambda_w}{1 + \lambda_w}.$$  \hspace{1cm} (6)

Here, the “price” is household $j$’s real wage, and marginal cost is the marginal disutility of labor expressed in consumption units. When $\lambda_w$ equals zero, the employment agency’s elasticity of substitution between varieties becomes infinite and individual households have no market power. In that case, this condition reduces to the usual labor-supply condition.

We denote the ratio of the consumption-denominated marginal disutility of labor to the real wage with $X_t$, so the percentage price-cost margin in (6) equals $1 - X_t$. In the frictionless model, this real wage-marginal disutility “gap” always equals its steady state value. When $\xi_w > 0$, wages cannot adjust quickly enough to maintain this equality always. Knowing this, a household choosing its wage takes into account its possible impact on future labor demand and earnings. Substituting the first-order condition for this optimal choice and the wage indexation rule into Equation (5) and log linearizing the result yields the Wage Phillips Curve. To express it, we use small letters with hats to denote logarithmic deviations from steady-state values.

$$\hat{\pi}^w_t - \nu \hat{\pi}^{P}_{t+1} + \hat{s}_{t-1} = \kappa \times \left( \hat{x}_t + \hat{\lambda}_{w,t} / (1 + \lambda_w) \right) + \beta E \left[ \hat{\pi}^w_{t+1} - \nu \hat{\pi}^P_t + \hat{s}_t \right].$$  \hspace{1cm} (7)

In this,

$$\kappa = \frac{(1 - \xi_w)(1 - \beta \xi_w)}{\xi_w} \left( 1 + \nu (1 + \lambda_w) \right)^{-1}.$$  \hspace{1cm} (8)

This familiar equation states that wage inflation in excess of that expected from indexation equals a term increasing in the real wage-marginal disutility of labor gap plus its own discounted expected value in the next quarter. If we set $\xi_w$ to zero, then this reduces to $x_t = 0$. Setting $\nu W$ to zero removes any dependence of current wage inflation on past price inflation.
2.3.3 Intermediate Goods Producers

Each intermediate good producer’s profit maximization problem can be decomposed into cost-minimization and pricing problems. The first is simple: Given a required output $Y_t(i)$, choose $L_t(i)$ and $K_t(i)$, and the capital utilization rate to minimize the cost of production. It is not hard to show that the producer’s technology features a constant marginal cost and that the capital utilization rate is also independent of the scale of production. If we denote marginal cost (the Lagrange multiplier on the required output constraint) with $\Omega_t$, then the first order necessary conditions for cost minimization are

\[
\frac{W_t}{P_t} = \Omega_t A_t^{1-\alpha} \left( \frac{K_t(i)u_t}{L_t(i)} \right)^\alpha,  \\
a(u_t) + Q_t^l - (1 - \delta)Q_t^l = \Omega_t A_t^{1-\alpha} \left( \frac{K_t(i)u_t}{L_t(i)} \right)^{\alpha-1} u_t,  \\
a'(u_t) = \Omega_t A_t^{1-\alpha} \left( \frac{K_t(i)u_t}{L_t(i)} \right)^{\alpha-1}. 
\]

These together with the production function yield $K_t(i)$, $L_t(i)$, $u_t$ and $\Omega_t$ as functions of $Y_t(i)$.

Just as with wage-setting, it is helpful to begin with the “frictionless” problem with $\xi_p = 0$ and $\lambda_{p,t} = \lambda_p$ always. Such a firm would always equate the percentage price-cost markup to the inverse demand elasticity.

\[
\frac{P_t(i)/P_t - \Omega_t}{P_t(i)/P_t} = \frac{\lambda_p}{1 + \lambda_p} 
\]

Of course, a producer with limited opportunities for nominal price adjustment sets $P_t(i)$ with its future consequences in mind. Placing the first-order condition for this optimal dynamic choice into the final good producers’ zero-profit condition and log-linearizing the result yields the familiar new-Keynesian Phillips curve.

\[
\dot{\pi}^P = \lambda_{t+1} \pi^P_t = 1 + \beta \mathbb{E}_t \left[ \pi^P_{t+1} \right] + \kappa^P \times (\dot{\omega}_t + \lambda_{p,t}/(1 + \lambda_p)) 
\]

In this, $\dot{\omega}_t = \ln(\Omega_t/\Omega)$ and

\[
\kappa^P = \frac{(1 - \xi_p)(1 - \beta \xi_p)}{\xi_p}. 
\]

Since both $A_t$ and $Z_t$ are nonstationary, the producer’s surplus of each intermediate good producer is also nonstationary. To make each firm’s profits stationary, we set $F_t = F \times A_t Z_t^{\alpha/(1 - \alpha)}$.

2.3.4 Capital Installers

Because the past quarter’s input use influences the current quarter’s production possibilities, the profit-maximization problem of a capital installer is intrinsically dynamic. To characterize it, let $V_t(I_{t-1})$ denote the quarter $t$ value in units of the final good of a capital installer that used $I_{t-1}$ units of the final good in the previous quarter. This satisfies

\[
V_t(I_{t-1}) = \max_{I_t} \frac{Q_t^I}{P_t} \mu_t \left( 1 - \chi \left( \frac{I_t}{I_{t-1}} \right) \right) I_t - I_t + \beta \mathbb{E}_t I_{t+1} \mathbb{E}_t \left[ \frac{Q_{t+1}^I}{I_{t+1}} \right] V_{t+1}(I_{t+1}). 
\]

We can use the results of Benveniste and Scheinkman (1979) to derive $V_{t+1}(I_t)$. Combining this with the first-order condition for the optimal choice of $I_t$ yields

\[
\frac{Q_t^I}{P_t} \mu_t \left( 1 - \chi \left( \frac{I_t}{I_{t-1}} \right) \right) - \frac{Q_t^I}{P_t} \mu_t \chi' \left( \frac{I_t}{I_{t-1}} \right) I_t - 1 + \beta \mathbb{E}_t \left[ \frac{Q_{t+1}^I}{I_{t+1}} \frac{P_{t+1}}{I_{t+1}} \mu_{t+1} \chi' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right] = 0 
\]
If we begin with a state-contingent sequence for $I_t$ which satisfies (13) and multiply all of its elements by a positive scalar, then the new sequence also satisfies it. Therefore, the initial distribution of experience over capital installers is irrelevant, and we can proceed as if there is a single price-taking representative capital installer.

2.3.5 Equilibrium

The variable’s determining the model economy’s production possibilities and preferences at the beginning quarter $t$ are listed in Table 1. We call these the model’s *states*. Habit puts lagged nondurable consumption into the list, and investment adjustment costs put lagged investment there. The price indexing rules put lagged final good inflation there. Otherwise, the model’s state variables are standard.

The model’s endogenous variables are listed in Table 2. An equilibrium consists of state contingent sequences of these variables that satisfy the obvious optimality and market-clearing conditions.

### 3 Estimation

We employ a Bayesian estimation strategy that models each observable series of interest as the sum of its model counterpart with an “error” term that follows a first-order autoregression. We identify the model’s consumption aggregate with the sum of Personal Consumption Expenditures on Nondurable Goods and Services. Similarly, we suppose that the model’s investment aggregate represents the sum of Business Fixed

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Disappears if</th>
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<tbody>
<tr>
<td>$C_{t-1}$</td>
<td>Lagged Consumption</td>
<td>$h = 0$</td>
</tr>
<tr>
<td>$I_{t-1}$</td>
<td>Lagged Investment</td>
<td>$S(I_t/I_{t-1}) \equiv 0$</td>
</tr>
<tr>
<td>$\pi^p_{t-1}$</td>
<td>Lagged Price Inflation</td>
<td>$\tau_p = \tau_w = 0$</td>
</tr>
<tr>
<td>$K_t$</td>
<td>Stock of Installed Capital</td>
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</tr>
<tr>
<td>$A_t$</td>
<td>Hicks-Neutral Technology</td>
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</tr>
<tr>
<td>$a_t$</td>
<td>Growth rate of $A_t$</td>
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<tr>
<td>$a_{t-1}$</td>
<td>Lagged Growth Rate of $A_t$</td>
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<tr>
<td>$Z_t$</td>
<td>Investment-Specific Technology</td>
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<td>$z_t$</td>
<td>Growth rate of $Z_t$</td>
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<td>$\phi_t$</td>
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<td>$b_t$</td>
<td>Discount Rate Shock</td>
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<td>Employment Aggregator’s Elasticity of Substitution</td>
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<td>$\lambda_{p,t}$</td>
<td>Intermediate Good Aggregator’s Elasticity of Substitution</td>
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<td>$\mu_t$</td>
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<td>$\pi^*_t$</td>
<td>Target Inflation Rate</td>
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<td>$\varepsilon_{R,t}$</td>
<td>Monetary Policy Shock</td>
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Table 2: Model Variables Determined Endogenously

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$C_t$</td>
<td>Consumption</td>
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<td>$G_t$</td>
<td>Government Expenditure</td>
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<tr>
<td>$Y_t$</td>
<td>Gross Domestic Product</td>
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<td>$L_t(j)$</td>
<td>Household $j$’s Hours Worked</td>
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<td>Total Team Hours Worked</td>
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<td>$\pi_t^w$</td>
<td>Wage Inflation</td>
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<tr>
<td>$u_t$</td>
<td>Capital Utilization</td>
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<td>$\Omega_t$</td>
<td>Marginal Cost of Intermediate Good Production</td>
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<td>$Y_t(i)$</td>
<td>Intermediate Good Producer $i$’s Output</td>
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<td>$R_t$</td>
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<td>$Q_t^i/P_t$</td>
<td>Real Price of Capital in Opening Capital Market</td>
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<tr>
<td>$Q_t^i/P_t$</td>
<td>Real Price of Capital in Closing Capital Market</td>
</tr>
<tr>
<td>$K_{t+1}$</td>
<td>Stock of Installed Capital Available in Quarter $t+1$</td>
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</table>

Investment, Residential Investment, Personal Consumption Expenditures on Durable Goods, and Inventory Investment.

The estimation uses ten variables, measured from 1984:Q1 through the present:

- Growth of nominal per capita GDP,
- Growth of nominal per capita consumption, as defined above,
- Growth of nominal per capita investment, as defined above,
- Per capita hours worked in Nonfarm Business,
- Growth of nominal compensation per hour worked in Nonfarm Business,
- Growth of the implicit deflator for GDP,
- Growth of the implicit deflator for consumption, as defined above,
- Growth of the implicit deflator for investment, as defined above,
- Growth of the implicit deflator for core PCE, and
- The interest rate on Federal Funds.

We do not directly use data on either government spending or net exports. Their sum serves as a residual in the national income accounting identity. To construct series measured per capita, we used the civilian non-institutional civilian population 16 years and older. To eliminate level shifts associated with the decennial census, we project that series onto a fourth-order polynomial in time.
We gather the date \( t \) values of these series together in the vector \( y_t \), and we place the date \( t \) states into \( \zeta_t \). Given the vector of model parameters \( \theta \), the model’s log-linear solution can be described by a first-order autoregression

\[
\zeta_t = F(\theta)\zeta_{t-1} + \varepsilon_t
\]

\( \varepsilon_t \sim \mathcal{N}(0, \Sigma(\theta)) \)

Here, \( \varepsilon_t \) is a vector-valued innovation built from the model innovations described above. Many of its elements identically equal zero.

The model analogues to the elements of \( y_t \) can be calculated as linear functions of \( \zeta_t \) and \( \zeta_{t-1} \). We suppose that the data equal these model series plus a vector of “errors” \( v_t \).

\[
y_t = G(\theta)\zeta_t + H(\theta)\zeta_{t-1} + v_t
\]

\[
v_t = \Lambda(\varphi)v_{t-1} + e_t
\]

\( e_t \sim \mathcal{N}(0, D(\varphi)) \)

Here, the vector \( \varphi \) parameterizes the stochastic process for \( v_t \). In our application, the only non-zero elements of \( v_t \) correspond to the observation equations for the two consumption-based measures of inflation. These errors fit the high-frequency fluctuations in prices and thereby allow the price markup shocks to fluctuate more persistently. These two errors evolve independently of each other. In this sense, we follow Boivin and Giannoni (2006) by making the model errors “idiosyncratic”. The other notable feature of the observation equations concerns the GDP deflator. We model its growth as a share-weighted average of the model’s consumption and investment deflators. Since estimation incorporates observations of the investment deflator, our forecasts of GDP growth therefore take changes in the relative price of investment into account.

We denote the sample of all data observed with \( Y \) and the parameters governing data generation with \( \Theta = (\theta, \varphi) \). The prior density for \( \Theta \) is \( \Pi(\Theta) \), which we specify to be similar to that employed by Justiniano, Primiceri, and Tambalotti (2008). Given \( \Theta \) and a prior distribution for \( \zeta_0 \), we can use the model solution and the observation equations to calculate the conditional density of \( Y, F(Y|\Theta) \). To form the prior density of \( \zeta_0 \), we apply the Kalman filter using a four-year long “initialization sample” that begins in 1980:Q1. The actual estimation begins with 1984:Q1. Bayes rule then yields the posterior density up to a factor of proportionality.

\[
P(\Theta|Y) \propto F(Y|\Theta)\Pi(\Theta)
\]

We calculate our forecasts with the model’s parameter values set to this posterior distribution’s mode.

4 Model Responses to Shocks

To gain a sense of the model’s mechanics, we have calculated responses of three key observable variables, the Fed Funds Rate, Core PCE Inflation, and Real GDP, to several of the model’s shocks with the parameter vector set to the posterior distribution’s mode.

Figure 1 begins with the responses to a standard monetary policy shock, \( \varepsilon_{z,t} \) in the Taylor Rule (2). For clarity of presentation only, we have normalized the initial response of the Federal Funds to one percentage point.\(^1\) One quarter after the initial shock, the Fed Funds Rate increases another ten basis points, and thereafter it begins its descent. In the fourth quarter, it remains 54 basis points higher than otherwise, and by the eighth quarter it has essentially returned to its steady-state value. A strong contraction of real GDP

\(^1\)The response of the Fed Funds Rate to a one standard deviation monetary policy shock equals about 9 basis points.
follows the shock, and this persists long after its effects on the Fed Funds Rate have dissipated. Inflation responds only little.

The other shock to monetary policy is the inflation target shock, $\varepsilon_{\star,t}$ in Equation (3). Figure 2 presents the responses, which were normalized so that the shock produced a one-percent increase in Core PCE Inflation on impact. The shock induces an instantaneous rise in real GDP of over one percent, which then continues for almost four years before beginning a slow reversion. Clearly, this unexpected result is worthy of further scrutiny. The estimated value of $\rho_{\star}$ is 0.995, so the shock’s impact on inflation is very persistent. Indeed, it rises about 17 basis points during the three quarters following the shock and then begins a slow decline to its steady-state value. In the short run, the shock leaves the Fed Funds Rate basically unchanged. Hence, the real Fed Funds Rate drops by about the size of the shock. In this sense, a positive inflation target shock is accommodative. After two years, the nominal Fed Funds Rate has increased to eliminate any change in the real Fed Funds Rate.  

Two other shocks with substantial influence on inflation are the Price and Wage Markup shocks. Figures 3 and 4 present their impulse response functions. We normalized both shocks so that the peak response of Core PCE inflation was one percent. This peak occurs instantaneously for the Price Markup shock. However, this impact does not last. In the second quarter, inflation is only 29 basis points above its steady-state value, and it has essentially returned to that value by the third quarter. The Fed Funds Rate immediately responds by 14 basis points, and this response grows to 36 basis points in the fourth quarter. Over the next four quarters the Fed reverses this tightening. By the fourth quarter, Real GDP has fallen by 68 basis points. Therereafter it begins a gradual recovery.

In our model, nothing distinguishes the wage markup shock from the labor-supply shock, $\phi_t$, so we assume that the household’s utility from leisure is a constant. Consequentially, the wage markup shock is estimated to be much more persistent, $\rho_w = 0.92$. Upon impact, core PCE inflation rises 94 basis points and reaches its peak of 100 basis points in the second quarter. Thereafter, the inflation rate reverts to its steady state value at a measured pace. After 13 quarters, this transition is nearly completed. The Fed initially responds very little to the shock, but then raises the Fed Funds Rate by 52 basis points over six quarters. Since this response does not satisfy the Taylor principle, we might say that it is accommodative. The response of Real GDP gives some justification for this. The level of Real GDP falls 10 percentage points over the 13 quarters plotted.

As in most DSGE models, Hicks-neutral technology shocks contribute substantially to our model's output fluctuations. Figure 5 presents the responses to $\varepsilon_{a,t}$. We normalized the shock so that the response of Real GDP after 13 quarters equals one percent. The parameter estimates display substantial serial correlation in the growth rate of neutral technology, so the impact of a shock on GDP builds up over time. The shock has a very modest negative impact on inflation, and it leaves the Fed Funds Rate basically unchanged.

The final shock we examine is that to the marginal efficiency of investment, $\varepsilon_{\mu,t}$. Figure 6 presents the model’s responses to an expansionary MEI shock normalized to have a peak impact on Real GDP of one percent. The shock raises prices slightly and induces a small and gradual tightening by the Fed. On impact, the shock raises Real GDP by 37 basis points, and it reaches its peak impact in the seventh quarter. By the thirteenth quarter, Real GDP remains substantially elevated.

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2 We speculate that this “passive” accommodation substantially contributes to the sizeable expansion of GDP. Identifying the observations underlying this result remains on our research agenda.

3 The estimated value of $\rho_w$ is only 0.20

4 We estimate that a typical Wage Markup shock increases Core PCE inflation by only 4 basis points after two quarters, so the responses in Figure 4 are about 25 times as large as the typical response.
Figure 1: Responses to a Monetary Policy Shock

This figure plots the model responses to a monetary policy shock, $\varepsilon_{R,t}$, normalized to have a contemporaneous impact of one percent on the Fed Funds rate. The vertical axis is in percentage points, and the horizontal axis is in quarters. The shock occurs in quarter “1”.
Figure 2: Responses to an Inflation Target Shock

This figure plots the model responses to a shock to the inflation target, $\varepsilon_{\text{st},t}$, normalized to have a contemporaneous impact of one percent on core PCE inflation. The vertical axis is in percentage points, and the horizontal axis is in quarters. The shock occurs in quarter “1”.
Figure 3: Responses to a Price Markup Shock

Each panel plots the model responses to a price markup shock, normalized to have a peak impact on core PCE inflation of one percent. The vertical axes are in percentage points, while the horizontal axes are in quarters. The shock occurs in quarter “1”.
Figure 4: Responses to a Wage Markup Shock

Each panel plots the model responses to a wage markup shock, normalized to have a peak impact on core PCE inflation of one percent. The vertical axes are in percentage points, while the horizontal axes are in quarters. The shock occurs in quarter “1”.
Figure 5: Responses to a Neutral Technology Shock

This figure plots the model’s responses to a neutral technology shock, $\epsilon_{a,t}$, normalized to increase GDP by one percent after 13 quarters. The vertical axis is in percentage points, while the horizontal axis is in quarters. The shock occurs in quarter “1”.

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Figure 6: Responses to a Marginal Efficiency of Investment Shock

This figure plots the model’s responses to a shock to the marginal efficiency of investment, $\varepsilon_{\mu,t}$, normalized to increase GDP by one percent at the response’s peak. The vertical axis is in percentage points, while the horizontal axis is in quarters. The shock occurs in quarter “1”.
References


