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The starting point for our model is the standard normal probit linking an indicator variable for U.S. recessions (R_t) to the current change ($\Delta(u_t - u_t^*)$) and lagged level ($u_{t-1} - u_{t-1}^*$) of the unemployment rate gap since 1960. The estimated recession probability from this model is

$$\Pr(R_t = 1) = \Phi(-2.05 + 7.79\Delta(u_t - u_t^*) - 0.61(u_{t-1} - u_{t-1}^*)),$$

where Φ is the standard normal cumulative distribution and u_t^* is the natural rate of unemployment. We assess its predictive accuracy using receiver operating characteristic (ROC) analysis based on the area under the curve shown in figure A1.

Our model also conditions this probability indirectly on the Phillips curve correlation between the unemployment rate gap and detrended four-quarter inflation according to the Price Index for Personal Consumption Expenditures (PCE), i.e., the inflation gap, $\pi_t - \pi_t^*$. We detrend PCE inflation with the Hodrick–Prescott filter. However, once the long-term trend (π_t^*) settles at 2% in 2002:Q4, we hold it fixed at this level to reflect the inflation target of 2% that was formally adopted by the Federal Reserve in early 2012. The time series of PCE inflation and its trend can be seen in figure A2.

Our full model is an endogenous regime-switching regression, where the level and slope of the Phillips curve are allowed to vary by regime. We equate expansions and recessions with the two model regimes and use the probit specification described previously to capture them where $R_t = 0$ when the latent index $r_t \leq 0$ and $R_t = 1$ when $r_t > 0$. The full model and its estimated coefficients are summarized by the following equations:

$$\begin{aligned} \pi_t - \pi_t^* &= -0.23 - 0.10(u_t - u_t^*) + \varepsilon_{0t} \text{ if } r_t \leq 0 \\ &= \ \ 1.00 - 0.40(u_t - u_t^*) + \varepsilon_{1t} \text{ if } r_t > 0, \end{aligned}$$

$$r_t = -2.05 + 7.79\Delta(u_t - u_t^*) - 0.61(u_{t-1} - u_{t-1}^*) + \nu_t,$$

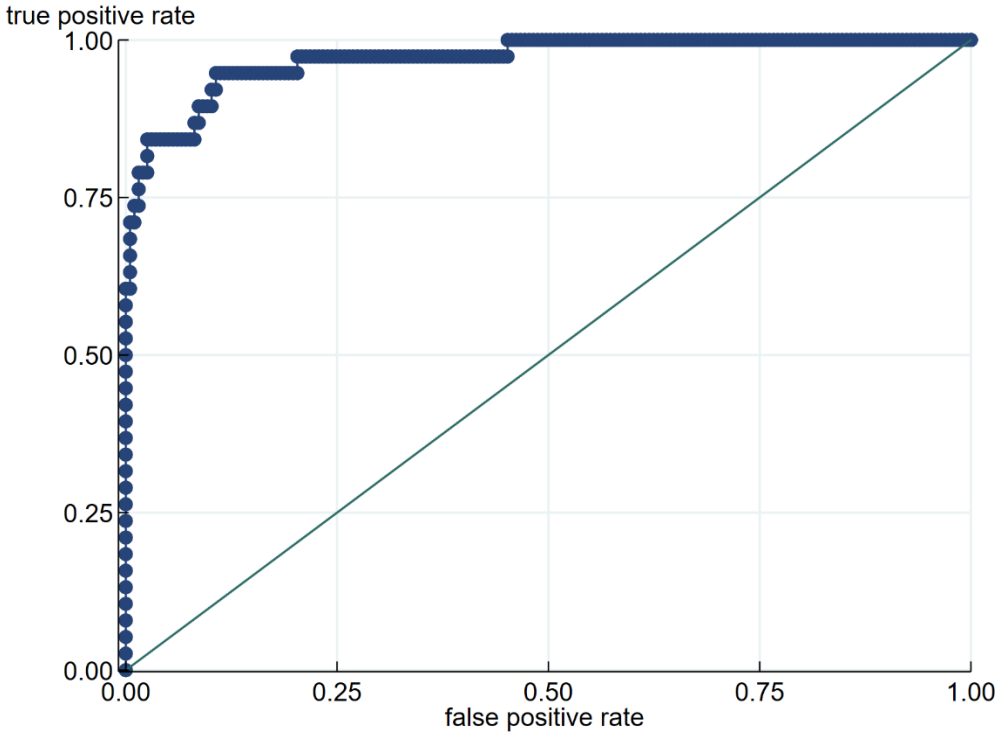
$$(\varepsilon_{0t}, \quad \varepsilon_{1t}, \quad \nu_t) \sim N(0, \Sigma),$$

$$\Sigma = \begin{bmatrix} 0.65 & . & -0.52 \\ . & 1.60 & 0.05 \\ -0.52 & 0.05 & 1 \end{bmatrix}.$$

It is possible within this framework to test whether or not the shocks to the probit equation and Phillips curves are jointly independent. The associated Wald test rejects this null hypothesis at the 95% confidence level. In other words, we cannot reject that the unobserved drivers of the Phillips curve and recessions are endogenously related. We tend to think of these drivers as

reflecting various types of aggregate supply shocks not captured in the unemployment rate gap. This form of endogeneity is addressed in estimation with the use of a "control function," which assumes that the shocks to the Phillips curves (ε_{0t} and ε_{1t}) and the probit equation (ν_t) are trivariate normally distributed with mean zero and covariance matrix Σ.

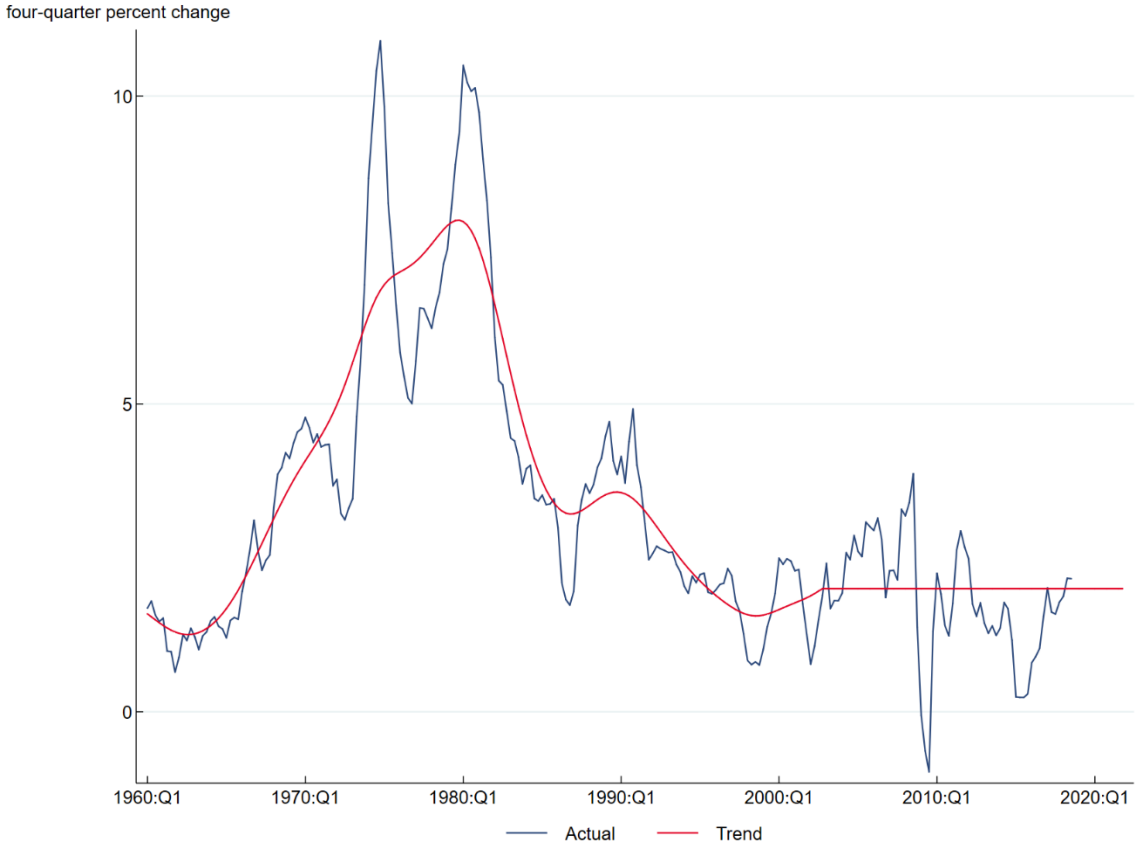
A1. ROC curve for probit model of U.S. recessions



Notes: The blue dots correspond to the receiver operating characteristic (ROC) curve for our probit model of U.S. recessions. The ROC curve captures the trade-off between minimizing type I (false positive) and type II (false negative) errors. The thin solid green line is a 45-degree line from the origin with an area under the line equal to 0.5, and is used to judge the statistical significance of the area under the ROC curve. The area under the ROC curve shown in the figure is 0.9686, reflecting roughly 97% accuracy in correctly classifying U.S. expansions and recessions since 1960, and is statistically significant at standard confidence levels.

Source: Authors' calculations based on data from Haver Analytics.

A2. PCE inflation



Notes: This figure plots inflation according to the Price Index for Personal Consumption Expenditures (PCE), along with its long-run trend. See the text for further details.

Source: Authors' calculations based on data from Haver Analytics.