Interest rates and inflation

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If a trader has lent wheat . . . at interest, then for every gur of wheat he shall take 100 qa as interest. If he has lent silver at interest, then for each shekel of silver he shall take a sixth part of a shekel, plus six grains, as interest.

The Hammurabi Code, No. 90, circa 2080 B.C.

The relation between interest rates and inflation has attracted much attention in recent years. Serious empirical research on this subject has resumed after a lapse of nearly four decades, from the early 1930s to the late 1960s. The point of departure of this work has been Irving Fisher's classic study, The Theory of Interest [5], published in 1930. Fisher found interest rates during the period 1890-1927 to respond slowly and incompletely to variations in inflation. The most common interpretation of these results is that inflationary expectations, which influence current interest rates, respond slowly to observations of past inflation.

The results of most recent studies have been consistent with Fisher's. But Eugene Fama [4] has presented results that contradict those of earlier writers. More important, Fama's work suggests that interest rates immediately and completely reflect inflationary expectations.

This article compares the results of Fisher and Fama and places these results in historical perspective. The small differences between Fisher and Fama appear to be due less to increases in financial market efficiency, improvements in statistical methods, and better data (as suggested by Fama) than to the special, relatively tranquil period chosen by Fama for his empirical work.

The years 1953-71 are unique in American history for their record of stable prices. Charts 1 and 2 make clear that there is not, among periods of similar length, a close competitor after 1894 with 1953-71 for the most-stable-price-period prize.

Variations in prices since 1971 have been more like those observed by Fisher than Fama. Therefore, an understanding of the connections between interest rates and inflation in the world in which we live, while not neglecting Fama's contributions, requires that we pay special attention to Fisher's.

Real and nominal interest rates

The distinction between interest rates in terms of money (e.g., silver) and interest rates in terms of goods (e.g., wheat) has long been recognized. It is useful to think of the former as nominal rates of interest, R, and of the latter as real rates of interest, r. In the United States nominal rates of interest measure returns in dollars. These are the rates of interest reported in newspapers and advertised by depository institutions. Real rates of interest, on the other hand, measure the productivity of investment goods (i.e., the rate of transformation of current goods into future goods) and the time preferences of households (i.e., the allocation of consumption between current and future goods). Differences between real and nominal interest rates ought to be due to expected rates of inflation, i.e., to expected rates of change in the value of money relative to goods. If the expected rate of inflation is denoted p, the equilibrium relation between R and r may be expressed as:

\[ (1 + R) = (1 + r)(1 + p). \]

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Suppose, for example, that the expected real return to an investment in a machine is \( r = 3 \) percent per annum. That is, the machine is expected to produce a net output each year worth 3 percent of the value of the machine. Further suppose that, due to inflation, the prices of the machine and its output are expected to rise 5 percent during the next year. That is, expected inflation is \( p = 5 \) percent. The expected nominal (dollar) rate of return to this real investment is therefore \((1.03)(1.05) - 1 = 8.15\) percent. An investor’s choice between the machine (or shares in the machine) and, say, a 52-week Treasury bill depends on the bill’s yield, or rate of return, \( R \). If \( R \) exceeds 8.15 percent, investors will be attracted to the bill, bidding its rate down until \( R \) equals the expected return on alternative investments of similar risk, including real investments.\(^1\)

If \( R \) is less than 8.15 percent, investors will avoid the bill until its rate becomes competitive with other investments. This line of argument suggests that, given \( r = .03 \) and \( p = .05 \), the equilibrium value of \( R \) is \( .0815 \). An alternative but equivalent way of looking at the real rate of interest, the gain in purchasing power from lending money, is presented in box 1.

Under the dubious assumption that the Babylonian bureaucracy fixed interest on silver and wheat at rates compatible with market forces, we can determine the expected rate of change of prices in that country in 2080 B.C. Since 1 gur = 300 qa of wheat and 1 shekel = 180 grains of silver, \( r = 100/300 = 33 \frac{1}{3} \) percent and \( R = 36/180 = 20 \) percent. Using equation (1), we can surmise that Hammurabi’s subjects expected an annual rate of deflation of 10 percent.\(^2\)

\(^1\)This argument follows Fisher and Fama in abstracting from complications associated with risk and taxes.

\(^2\)Unlike paper money, there is a real return to silver in productive activities. For the sake of completeness, therefore, \( r \) should be interpreted in our Babylonian example as the difference between the real returns to wheat and silver.

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**Chart 1. Inflation and real and nominal interest rates**

[Diagram showing inflation and real and nominal interest rates over time.]
Equation (1) is often expressed as the following linear approximation:

\[ R = r + p. \]

This approximation is fairly close for small \( p \) and \( r \). In the two examples given above, the approximate equilibrium nominal rates are 8 and -23 1/3 percent, compared with the exact equilibrium values of 8.15 and -20 percent.

Although the connections between interest and inflation have been understood for thousands of years, they have never been discussed more than during the past 250 years. This is due to the increased use of paper money and therefore to the increased volatility of inflation. William Douglass wrote in 1740 that large emissions of paper money "rise the interest to make good the sinking principal [2, p. 335]." For example, the Rhode Island issue of 1739, which caused a depreciation of paper money by 7 percent, required an increase in the rate of interest from 6 to 13 percent. Speaking in the House of Commons in 1811, during a period of wartime inflation, Henry Thornton pointed out that if a man borrowed money at a nominal interest rate of 5 percent and repaid the loan after a period of 2 percent or 3 percent inflation, "he would find that he had borrowed at 2 or 3 percent, and not at 5 percent as he appeared to do [11, p. 336]."

**Fisher’s results**

The first extensive statistical studies of the relations between real and nominal interest rates and inflation were carried out by Irving Fisher. His results were stated in their most complete form in *The Theory of Interest* [5, pp. 399-451]. Empirical tests of equation (1) are difficult for a variety of reasons. Most important, none of the three variables is directly observable. All are expectations of future events: \( p \) is the expected rate of inflation, \( r \) is the expected real return to productive activities, and \( R \) is the expected nominal
return to investments in debt to be repaid in dollars. Most researchers have simplified the problem by assuming the real return to be constant and by choosing high-grade short-term securities with the same maturity as the period of observation to ensure that observed nominal yields were virtually the same as expected nominal returns. For example, Fisher's most thorough tests used quarterly data and four- to six-month prime commercial paper. This is not a perfect solution to the problem, but later writers have been able to use Treasury bills.

This leaves expected inflation. It is conceivable that the market's expectation of inflation during the period beginning on date \( t \) is a weighted average of past inflation rates. Then using the linear approximation (2), our model may be written:

\[
(3) \quad R_t = r + w_{t-1} + w_{t-2} + \ldots + w_{n} p_{t-n} + e_t = r + \bar{p}_{nt} + e_t
\]

where \( R_t \) is the yield on date \( t \) on a security maturing in one period; the \( w \)'s are weights that indicate the importance of past rates of inflation \( (p_{t-1}, p_{t-2}, \ldots) \) in determining expectations of inflation for the coming period, \( p_t \); \( \bar{p}_{nt} \) is the weighted average of these past rates of inflation, with \( n \) being the length of the lag, i.e., the number of past rates included; and \( e_t \) is an unobserved random error term with a mean of zero. There is no time subscript on the real rate of interest because it is assumed to be constant.

Fisher did not estimate the \( w \)'s. Regressions in those days were too expensive for such a procedure. Rather, he tried different combinations of the \( w \)'s and \( n \) and ran correlations of the resulting \( \bar{p}_{n} \)'s and \( R_t \). Specifically, his weighting scheme was:

\[
(4) \quad \bar{p}_{nt} = \frac{np_{t-1} + (n-1)p_{t-2} + \ldots + p_{t-n}}{n + (n-1) + \ldots + 1}.
\]

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Chart 2. Inflation and real and nominal interest rates

percent

annual rates observed at 3-month intervals

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Economic Perspectives
Fisher’s correlations, using quarterly observations on the commercial paper rate and the wholesale price index, are shown in chart 3. For interest rates observed between 1890 and 1914, the highest correlation was achieved when rates of inflation lagged 30 quarters were used. The correlations fell as the lag was lengthened. But between 1915 and 1927, the correlation between interest rates and past inflation was continuously improved as the lag was lengthened—up to the maximum lag of 120 quarters tried by Fisher. He performed similar tests with annual data for the United States and Great Britain and the results were the same: assuming that (a) the real rate of interest is constant, (b) expectations of future inflation are determined by past inflation in the manner shown in equation (4), and (c) the approximate equilibrium relation (2) is satisfied, the current interest rate is apparently determined by expectations based on past inflationary experiences as distant as 30 years ago. Fisher wrote: “It seems fantastic, at first glance, to ascribe to events which occurred last century any influence affecting the rate of interest today. And yet that is what the correlations with distributed effects of p show [5, p. 428].”

Recent studies of interest rates and inflation

Studies published in 1969 and 1970 by William Gibson [6], Thomas Sargent [10], and William Yohe and Denis Karnosky [12] corroborated Fisher’s results. Based on models similar to equation (3) and data taken from periods both before and after World War II, these authors, like Fisher, found that interest rates responded slowly and incompletely to inflation and that long distributed lags of past inflation rates were useful in explaining interest rates. Sargent described his results as

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Fisher’s notation has been altered to conform to that used in this paper. For a discussion of Fisher’s speculations about the reasons for these results, see Rutledge [9, pp. 19-21].
The real rate of interest

Suppose you invest an amount of money, \( V \), at the rate of interest \( R \) prevailing on date 0. If \( P_0 \) is the average price of goods in your "normal consumption bundle" on that date, you have relinquished purchasing power over \( V/P_0 \) goods. A teenager existing exclusively on Big Macs priced at $1.25 gives up 80 units of current consumption when he deposits $100 in the local S&L. He does this, presumably, in order to be able to consume an even greater number of Big Macs in the future. At a rate of interest of 5 percent, his investment will grow to $105 by next year. This will enable him to consume \( 105/P_1 \) units if \( P_1 \) is the price of Big Macs next year. The nominal (money) return on his investment is the number of dollars gained as a proportion of the number of dollars invested (relinquished) and in this example is 5 percent. The real (hamburger) return is the number of Big Macs gained as a proportion of the number of Big Macs invested (relinquished). If prices are stable, i.e., \( P_1 = P_0 = $1.25 \), $105 will purchase 84 Big Macs next year and the real rate of return is \( 4/80 = 5 \) percent. But if a 20 percent per annum inflation has occurred so that \( P_1 = $1.50 \), $105 will be worth only \( 105/1.50 = 70 \) Big Macs and the real return will be \( -10/80 = -12.5 \) percent. He has gained money but lost goods.

In general, this real rate of interest, \( r \), may be expressed as follows:

\[
 r = \frac{V(1 + R) - P_1}{P_0} - \frac{V}{P_1} (1 + R) - 1
\]

Letting \( p = (P_1 - P_0)/P_0 \) denote the rate of inflation, the above equation may be rewritten as:

\[
(1 + r) = \frac{(1 + R)}{(1 + p)} \quad \text{or} \quad (1 + r)(1 + p) = (1 + R)
\]

which is identical to equation (1).

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Box 1

Economic Perspectives

The results are similar to Fisher's in a couple of ways. Not only do they confirm the importance of the distributed lag price expectations variable, but they imply a very long lag in the process of expectations formation [10].

Eugene Fama [4] followed a different method and obtained different results. He used Treasury bill yields and described his estimates as tests of the efficiency of the Treasury bill market. In efficient markets observed prices and interest rates correctly reflect all of the information available to market participants. This means that (a) observed interest rates correctly reflect the market's inflationary or deflationary expectations and (b) those expectations are unbiased; i.e., expectations, on average, are correct. If market expectations were biased and/or were not reflected in interest rates, there would exist opportunities to make substantial sums either by borrowing (if interest rates are "too low") in order to buy goods or by selling goods in order to lend (if interest rates are "too high"). To the statistician, runs of high or low observed real rates are indicated by high autocorrelations, meaning that real rates are highly correlated with their own past values. The concept of efficient markets, which is in turn closely related to rational expectations, is discussed in box 2. Autocorrelation is discussed in box 3.

In general, observed real rates are different from the expected real rates, \( r \), discussed above in connection with equations (1) and (2). This is true for several reasons. Probably the most important cause of differences between expected and realized (observed) real rates is the inability to forecast inflation, \( p \), accurately. For example, the nominal interest rate at time \( t \) might be \( R = .07 \) and be based on...
an expected inflation rate of $p = .03$. This means an expected real rate of approximately $r = .07 - .03 = .04$. But suppose inflation turns out to be 13 percent instead of 3 percent. The realized real rate of return to the nominal investment—the actual gain or loss in purchasing power—is approximately $r = .07 - .13 = -.06$. Runs of high or low observed real rates would suggest that the market was systematically overpredicting or underpredicting inflation. Under these circumstances nominal rates of interest would not, on average, be accurate predictors of inflation.

Fama's tests of efficiency were limited to observations between January 1953 and July 1971. He began his sample period with 1953 because before 1953 the Federal Reserve interfered with market efficiency by supporting government security prices. He excluded observations after July 1971 because queues, side payments, and increases in the various forms of nonprice rationing caused by price controls prevented stated prices from accurately measuring the costs of acquiring goods.

Fama's tests took two forms. First, he ran autocorrelations on realized real rates of interest. Even in efficient markets, predictions will almost always be wrong. But they will not be biased in the sense of being consistently too high or too low. An extreme example of an inefficient market is the period from May 1942 to July 1947, when the Federal Reserve pegged the discount rate on three-month bills at 0.375 percent. During this period the consumer and wholesale price indices rose at average annual rates of 6.2 percent and 7.9 percent, respectively. Monthly changes in the CPI and WPI, at annual rates,
Box 3
Autocorrelation

Suppose $x_t$ is the sales of a company in period $t$. The autocorrelations of this time series are the correlations of $x$ with its own past values. For example, the first-order autocorrelation, $\rho$, of $x$ is the correlation between $x_t$ and $x_{t-1}$. If the company's sales increase by some constant amount $c$ so that $x_t = x_{t-1} + c$, then $\rho = 1$. In this case, $x_t$ and $x_{t-1}$ are perfectly correlated; one consequence of this relation is that $x_t$ may be predicted with certainty if we know $x_{t-1}$. On the other hand, if $x$ varies in a completely random fashion such that a knowledge of $x_{t-1}$ conveys no information about $x_t$, then $\rho = 0$. Time series that are subject to smooth cyclical fluctuations have highly positive autocorrelations. Those which rapidly change direction have highly negative autocorrelations. Most American economic time series, including interest rates and inflation, belong to the former category.

exceeded 0.375 percent 40 and 42 times, respectively, during these 62 months. That is, realized real rates of interest were predominantly negative and, in addition, were highly autocorrelated.

Using the CPI and Treasury bill rates, Fama found, as expected, very different results for the 1953-71 period. The first-order autocorrelation coefficients of one-, two-, and three-month real returns on one-, two-, and three-month bills were .09, .15, and .00, respectively. Because these autocorrelations are "close to zero," Fama interpreted his results as consistent with the hypothesis that the Treasury bill market is efficient.

The second series of tests performed by Fama involved estimates of regression equations similar to the following:\footnote{Fama actually used the rate of change in the purchasing power of money, $\Delta_1$, which is approximately equal to $-p_t$ in his regressions. But his results are fully consistent with regressions of the form shown in equation (5), which has been used for ease of comparison with the work of Fisher and others.}

\[(5) \quad p_t = -r + R_t + e_t.\]

This is similar to Fisher's equation (3) except that (i) the actual rate of inflation, $p_t$, is used instead of a weighted average of past inflation rates, $\bar{p}_{it}$, and (ii) the positions of inflation and $R_t$ in the equation have been reversed. The nominal rate of interest is now the explanatory variable instead of the dependent variable. As in equation (3), $e_t$ is a random error term with mean zero. Equation (5) asserts that, given the assumed constant real rate, $r$, the market's expectation of the rate of inflation during the period beginning on date $t$, $p_t$, is fully reflected in the nominal rate of interest, $R_t$, observed on that date.

Fama reported regressions for the period 1953-71 on one- to three-month bill yields. In every case the coefficient of $R_t$ did not differ significantly from unity, as suggested by equation (5). His correlation coefficients (between $p_t$ and $R_t$) were statistically significant and ranged from .54 to .70. Fama also interpreted these results as consistent with the hypothesis that the Treasury bill market is efficient.

Earlier writers had observed that interest rates responded slowly to inflation, i.e., that $R_t$ did not fully reflect expected $p_t$. This suggests, if similar relations prevailed in the Treasury bill market during 1953-71, that the results obtained by means of equation (5) might be improved by adding past inflation rates as explanatory variables. Fama estimated regressions similar to (6), which he represented as tests of this hypothesis:

\[(6) \quad p_t = -r + R_t + W_1 p_{t-1} + e_t.\]

But the addition of $p_{t-1}$ failed to improve the correlations significantly and the estimates of the coefficient ($W_1$) of $p_{t-1}$ were statistically insignificant, leading Fama to claim these regressions as further evidence of the efficiency of the Treasury bill market. He dismissed the results of Fisher and others that suggested market inefficiency as probably having been caused by poor price data.

Back to Fisher

Fama's paper elicited critical comments by writers who combined his approach with...
Fisher's. Douglas Joines [7] and Charles Nelson and William Schwert [8] pointed out that regression (6) could not fairly be compared with Fisher's results, which depended on many—not one—past rates of inflation. Using Fama's data, they estimated regressions of the form:

\[
\begin{align*}
\begin{split}
7 \quad p_t = & r + bR_t + w_1 p_{t-1} + w_2 p_{t-2} + \ldots \\
& + w_n p_{t-n} + e_t
\end{split}
\end{align*}
\]

and found that past rates of inflation contained significant information about future inflation in addition to that reflected in \( R_t \). Furthermore, their estimates of the coefficient \( b \) of \( R_t \) were significantly different from unity. Apparently, during 1953-71 as during 1890-1927, interest rates responded slowly and incompletely to inflation.

Variations in inflation and interest rates during the Fisher and Fama periods of observation are shown in chart 1. The chart shows the rate of change in the wholesale price index and real and nominal returns on four- to six-month prime commercial paper beginning in June 1894, when four- to six-month prime commercial paper rates were first reported on a regular basis. Observations are at five-month intervals expressed in percentages at annual rates. One of the most striking characteristics of this chart is the stability of inflation in Fama's period, 1953-71, compared with the very large fluctuations before 1953 and after 1971. Apparently, Fama's sample is not typical of American experience. This conclusion is supported by chart 2, which shows the rate of change in the consumer price index and real and nominal returns on three-month Treasury bills, beginning with the regular reporting of bill yields in February 1930. Observations are at three-month intervals expressed in percentages at annual rates.

Now let's apply Fama's test of market efficiency to the data shown in the charts. The columns headed \( p \) in table 1 list the first-order autocorrelations of observed real rates during our complete sample periods and selected subperiods. We have separated the post-Fama period into observations falling within the period of price controls, August 1971 to April 1974, and those occurring after price controls were abandoned. The principal results shown in the table may be summarized as follows:

- The first-order autocorrelations are high and significant for both of the full sample periods (1894-1980 in table 1A and 1930-80 in table 1B) and for the subperiods dominated by the Great Depression (with runs of high real rates) and World War II (with runs of negative real rates).

<table>
<thead>
<tr>
<th>Time period</th>
<th>( \rho )</th>
<th>Standard deviations</th>
</tr>
</thead>
<tbody>
<tr>
<td>6/94 - 4/80</td>
<td>.317</td>
<td>13.79 2.65 16.31</td>
</tr>
<tr>
<td>6/94 - 6/29</td>
<td>.264</td>
<td>17.82 1.09 22.98</td>
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<tr>
<td>11/29 - 10/52</td>
<td>.522</td>
<td>13.45 1.10 12.28</td>
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<tr>
<td>3/53 - 2/71</td>
<td>.226</td>
<td>2.10 3.89 2.23</td>
</tr>
<tr>
<td>7/71 - 4/80</td>
<td>-.078</td>
<td>7.32 3.41 5.88</td>
</tr>
<tr>
<td>7/71 - 1/74</td>
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</tr>
<tr>
<td>6/74 - 4/80</td>
<td>-.555</td>
<td>6.78 3.69 5.10</td>
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<tr>
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<tbody>
<tr>
<td>2/30 - 5/80</td>
<td>.569</td>
<td>6.66 2.80 5.66</td>
</tr>
<tr>
<td>2/30 - 11/52</td>
<td>.570</td>
<td>8.78 .70 8.07</td>
</tr>
<tr>
<td>2/53 - 5/71</td>
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<td>8/71 - 2/74</td>
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<td>3.88 1.95 2.47</td>
</tr>
<tr>
<td>5/74 - 5/80</td>
<td>.279</td>
<td>3.59 2.54 1.73</td>
</tr>
</tbody>
</table>

\( p \) is the first-order autocorrelation of \( r \).
Standard deviations are in percentages at annual rates.
The 1974-80 subperiod yielded a positive autocorrelation in part B of the table but a negative autocorrelation in part A. This may be due to the frequent accelerations and decelerations of inflation during this period. Perhaps the five-month observational period used in A allows time for reversals in \( p \), and therefore a negative serial correlation of \( r \), not accounted for by the three-month period used in B. We should not put much stock in these results, however, due to the short period of observation.

Perhaps surprisingly, in view of the different results that have been associated with these subperiods, the first-order autocorrelation of \( r \) during 1894-1929, which is largely coincident with the period of Fisher’s analysis, is only slightly greater than that for 1953-71, the period of Fama’s analysis. Markets did remarkably well in forecasting the large fluctuations in inflation during the earlier period.

Notice, however, that the standard deviation of \( R \) during 1894-1929 was much less than the standard deviation of \( p \). This contrasts with the 1953-71 period, in which the volatility of \( R \) was only slightly less than that of \( p \). But this was due less to the greater responsiveness of \( R \) during the later period than to the smaller volatility of \( p \).

**Conclusions**

It should be stressed that much work remains to be done in this area and that none of the results presented in this paper—whether Fisher’s, Fama’s, or those in table 1—have justified any firm conclusions about the processes that determine observed relations between inflation and real and nominal rates of interest. About all that can be said on the basis of the available data is that, during most periods (excepting especially 1929-52), the Treasury bill and commercial paper markets have not appeared to be highly inefficient. Autocorrelations in real rates are not usually very high. Yet nominal interest rates persistently fail to respond fully to inflation when inflation is volatile. This was true before 1953 and after 1971. Apparently, like other reasonably effective but imperfect processes, the short-term securities markets perform well if not asked to do too much. They can keep up with inflation if the pace is not too fast or too variable.

**References**