

# Measuring and managing interest rate risk: A primer

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Losses from unexpected changes in interest rates have become an increasing problem at depository institutions over the past decade, as interest rates have become more volatile and have climbed to unprecedented levels. Such losses occur when unexpected increases in interest rates decrease the market value of an institution's assets more quickly than the market value of its liabilities—deposits and other borrowed funds. This differential change in market values occurs if the institution's assets are less *interest sensitive* than its deposits, that is, if the earnings rate on assets adjusts more slowly to market changes in interest rates than does the payout—the coupon or contract rate—on deposits. Under the same balance sheet condition, the institution experiences a gain when interest rates decline unexpectedly.

The more quickly an asset or liability adjusts to market rate changes, the more interest sensitive it is said to be. Institutions expose themselves to interest rate risk whenever the interest sensitivity of the two sides of their balance sheet is not equal.

The problems of interest rate risk are well-known, but accurate measurement of risk exposure is not easy. And, without such measurements, reliable management of this risk is not possible. This article describes a new technique for measuring in one number or factor the degree of risk exposure an institution assumes, and develops simple hypothetical examples to demonstrate the implications of various interest rate changes for depository institutions. The article also discusses alternative strategies for

managing or controlling interest rate risk and the pros and cons of the new technique relative to more commonly used procedures.

## A hypothetical bank balance sheet

The implications of interest rate changes may be analyzed most easily with a simplified bank balance sheet. The same principles apply to more complex and realistic situations. Here, we describe an institution that has only three types of assets:

1. Cash reserves (C)
2. 2½-year business loans, amortized monthly (BL), and
3. 30-year mortgage loans, amortized monthly (ML).

It also has only two types of deposits (P):

1. 1-year single payment certificate of deposit (CD1) and
2. 5-year single payment certificate of deposit (CD5).

These deposits make no coupon payments and may not be redeemed before maturity. The remaining item on the right side of the balance sheet is net worth or capital (K). The balance sheet shown in Figure 1 describes an institution with total footings of \$1,000. All accounts are valued at market. Cash is \$100; business loans are \$400; mortgage loans are \$500; one-year CDs are \$600; and five-year CDs are \$300. The bank's capital is valued at \$100, and its capital-to-asset ratio is 10 percent.

For the sake of simplicity, interest rates are assumed to be the same for all terms to maturity for all securities and deposits of a given default

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risk class. That is, the yield curve is assumed to be flat. All interest rates are compounded monthly. All payments are to be made on schedule; there are no assumed defaults, prepayments, or early deposit withdrawals. The interest rate on all business loans is initially assumed to be 13 percent and on all deposits, 11 percent. Cash reserves are assumed not to bear interest initially. The projected net income of the bank for the year may be computed by multiplying the market value of each account by the appropriate interest yield. This is shown in the summary income statement in Figure 1.<sup>1</sup> The bank's initial net income (NI) projected for the year is 1.8 percent on assets. This income will be realized if interest rates do not change during the year. The expected return on capital is 18 percent.

If interest rates change, they are assumed to change by equal percentage points (basis points) for all securities. After a change in interest rates, all bank accounts are marked to their new market (present) values—the price for which the accounts could be sold, if necessary. The balance sheet is designed so that accounts are not equally sensitive to interest rate changes.

### Interest rates increase by 200 basis points

Now let interest rates increase 200 basis points across the board. This reduces the market value of all accounts. The new balance sheet and income statements are shown in Figure 2. It is obvious that the accounts do not change by equal amounts. Longer-term accounts decline more in value than shorter-term accounts. For example, the market value of the business loan declines from \$400 to \$390, while that of the longer-term mortgage loan declines from \$500 to \$437. Total assets decline to \$927. Likewise, the market value of the one-year CD declines from \$600 to \$589, the five-year CD from \$300 to \$272, and total deposits from \$900 to \$861. The

<sup>1</sup>This definition of net income reflects economic income. All gains and losses due to changes in interest rates are recognized immediately. In practice, banks do not use this accounting system and do not recognize these gains and losses unless securities are sold before maturity. Nevertheless, economic income provides the most meaningful picture of a bank's income condition.

**Figure 1**  
**Initial conditions**

Balance sheet					
Assets	Dollars * D (yrs.)†		Liabilities	Dollars * D (yrs.)†	
Cash	100	0	CD (1 yr.)	600	1.0
BL (2½ yr.)	400	1.25	CD (5 yr.)	300	5.0
ML (30 yr.)	500	7.0	Net worth (K)	100	5.5
Total	1,000	4.0	Total	1,000	2.65

**Deposit duration**

$$D_p = \frac{600(1) + 300(5)}{900} = 2.33 \text{ years}$$

**Projected annual income statement for year**

	Interest yield	Market value ÷ total assets	Interest ÷ total assets (percent)	
<b>Revenues</b>				
Cash	0	.1	0	
Loans	13	.9	11.7	11.7
<b>Expenses</b>				
Deposits	11	.9	9.9	9.9
Net income				1.8

K = \$100  
K/A = 10%  
NI = 1.8%

**Summary accounts**

\*All accounts are valued at market (present value).  
†Approximate, using Equations 2 and 4.

value of capital, which is the difference between the value of total assets and deposits, declines 34 percent from \$100 to only \$66. Capital as a ratio of total assets declines from 10 percent to 7.1 percent. The increase in interest rates also decreases the projected annual net income by increasing the interest cost of deposits more than the revenue from assets, even though cash is now assumed to yield a small interest return. This occurs because deposits now account for proportionately more of total footings than before, so that interest expense has increased in relative importance. The projected net income declines to 1.5 percent of total assets from 1.8 percent. It is evident that the increase in interest rates has harmed the institution.

An equal decrease in interest rates of 200 basis points would have opposite effects. As can be seen from Figure 3, the values of all bank accounts except cash increase. Capital increases

to \$147, the capital to asset ratio to 13.5 percent, and net projected income to 2.0 percent of total assets. The institution is better off than it was before.

### Duration analysis

Changes in a bank's financial position due to interest-rate changes can be looked at with the help of *duration analysis*. Duration is a measure of the average life of a security. In its simplest form, it is computed by multiplying the length of time to each scheduled payment by the ratio of the present value of that payment to the total present value or price of the security and summing, or

$$D = \frac{\sum_{t=1}^I t \cdot PVF_t}{\sum_{t=1}^I PVF_t} \quad (1)$$

where:

- D = duration
- t = length of time (number of months, years, etc.) to the date of payment
- PVF<sub>t</sub> = present value of the payment (F) made at (t), or  $F_t / (1+i)^t$
- $\sum_{t=1}^I$  = summation from the first to the last payment (I).

This measure of duration is referred to as Macaulay's Duration, and is named after Frederick Macaulay, who first computed it in 1938 in his seminal study of the history of interest rates in the United States. Duration is a single number that is measured in units of time, e.g., months or years. For securities that make only one payment at maturity, duration is equal to maturity; for all other securities, it is shorter than term to maturity. Duration effectively converts a coupon security into its zero coupon (single payment or bullet) equivalent. For coupon bonds, for example, duration is equal to the term to maturity of an equivalent zero coupon bond that makes the same total payments and yields the same interest rate. The properties of duration have been de-

Figure 2

Assume interest rate increase of 200 basis points

Balance sheet					
Assets	Dollars*		Liabilities	Dollars*	
	Actual	Approx'd.		Actual	Approx'd.
Cash	100	100	CD (1 yr.)	589	588
BL (2½ yr.)	390	390	CD (5 yr.)	272	270
ML (30 yr.)	437	430	Net worth (K)	66	62
Total	927	920	Total	927	920

### Projected annual income statement for year

	Interest yield	Market value ÷ total assets	Interest ÷ total assets (percent)	
<b>Revenues</b>				
Cash	2	.11	0.2	
Loans	15	.89	13.4	13.6
<b>Expenses</b>				
Deposits	13	.93	12.1	12.1
Net income				1.5

### Summary accounts

K = \$66  
K/A = 7.1%  
NI = 1.5%

\*All accounts are valued at market (present value).

scribed elsewhere.<sup>2</sup> Most importantly for our purposes, at first approximation, duration relates changes in interest rates and percentage changes in bond prices linearly as follows:

$$\frac{\Delta S}{S} = -D \frac{\Delta i}{(1+i)} \approx -D \Delta i \quad (2)$$

where:

- S = price of a security
- i = yield to maturity
- Δ = change from previous value.

<sup>2</sup>G. O. Bierwag, George G. Kaufman, and Alden Toevs, "Duration: Its Development and Use in Bond Portfolio Management," *Financial Analysts Journal*, July/August 1983 and "Management Strategies for Savings and Loan Associations to Reduce Interest Rate Risk," *New Sources of Capital for the Savings and Loan Industry: Conference Proceedings* (Federal Home Loan Bank of San Francisco), December 1979. A more technical analysis appears in George G. Kaufman, G. O. Bierwag, and Alden Toevs, eds., *Innovations in Bond Portfolio Management: Duration Analysis and Immunization*, Greenwich, Conn.: JAI Press, 1983 and in Gabriel A. Hawawini, ed., *Bond Duration and Immunization*, New York: Garland Publishing, 1982.

**Figure 3**

**Assume interest rate decrease of 200 basis points**

Balance sheet					
Assets	Dollars*		Liabilities	Dollars*	
	Actual	Approx'd.		Actual	Approx'd.
Cash	100	100	CD (1 yr.)	612	612
BL (2½ yr.)	410	410	CD (5 yr.)	331	330
ML (30 yr.)	580	570	Net worth (K)	147	138
Total	1,090	1,080	Total	1,090	1,080

**Projected annual income statement**

	Interest yield	Market value $\frac{\Delta}{\text{total assets}}$	Interest $\frac{\Delta}{\text{total assets}}$ (percent)	
<b>Revenues</b>				
Cash	-2	.09	-0.2	
Loans	11	.91	10.0	9.8
<b>Expenses</b>				
Deposits	9	.87	7.8	7.8
Net income				2.0

**Summary accounts**

K = \$147  
 K/A = 13.5%  
 NI = 2.0%

\*All accounts are valued at market (present value).

Equation 2 is more accurate the smaller are the interest rate changes.

Now we can readily see why the value of each account changed when interest rates increased. All we need to do is compute the duration for each account by Equation 1 and multiply by the 200 basis point increase in interest rates. The duration of each account is shown in Figure 1. (For ease of following the analysis, the durations shown are rounded.) As was noted earlier, the duration of securities that generate periodic flows before maturity is less than their term to maturity. Thus, the initial duration of the 30-year monthly amortized mortgage yielding 13 percent is only seven years (7.14 years precisely). The durations of the single payment CDs are equal to their maturities. Because the bank will generate a constant stream of earnings as shown in the projected income statements unless interest rates change, the return on capital is projected to be constant. Thus, the duration of capital is the same as the duration of a fixed-

coupon perpetual bond (consol) and approximates 1/i. The initial return on capital is \$18 per \$100 or 18 percent, and its initial duration is approximately 5½ years.<sup>3</sup> The longer are the durations, the proportionately greater will be the price change predicted by Equation 2 for a given change in interest rates. Note also that: 1) duration, unlike maturity, has a linear relationship with price sensitivity and 2) the longer the duration of a security, the smaller is its interest sensitivity as defined at the beginning of this article.

The change in the market value of each account predicted by Equation 2 is shown in Figure 4. Changes in interest rates of 200 basis

**Figure 4**

**Using duration to measure risk exposure for individual accounts**

Approximate changes in the market value of balance sheet accounts for 200 basis point increase in interest rates (see equation 2):

BL:	-1.25 (+200) = - 2.5%, (-\$10)
ML:	-7.0 (+200) = -14.0%, (-\$70)
1 yr. CD:	-1.0 (+200) = - 2.0%, (-\$12)
5 yr. CD:	-5.0 (+200) = -10.0%, (-\$30)
K:	-5.5 (+200) = -11.0%, (-\$11)*
TA:	-4.0 (+200) = - 8.0%, (-\$80)
TL:	-2.65 (+200) = - 5.3%, (-\$53)*

\*Price effect (PE) only. But there is also an income effect (IE) = (TA<sub>PE</sub> - TL<sub>PE</sub>) = (80-53) = \$27

Thus, total effect = PE + IE = :

TL: -53 + (-27) = -\$80

K: -11 + (-27) = -\$38

points are large relative to the capabilities of Equation 2. As a result, the predictions are only rough approximations and will be less accurate with longer durations. In actuality, changes of 200 basis points are unlikely to occur all at one time. In addition, the predictions are distorted because rounded rather than precise values of duration are used. The approximate changes in

<sup>3</sup>If the bank's earnings are expected to increase, duration would be greater than the consol value, as can be seen from Equation 1. If earnings are expected to decrease, duration will be shorter. Although capital has no maturity date, its duration is not constant. It changes when interest rates change.

values will approach the actual changes in value as the assumed interest rate changes decrease in size and the precise values of duration are used. Nevertheless, for all the accounts except capital and total liabilities, the computed dollar changes are reasonably close to the actual changes between Figures 1 and 2. For example, the actual decline in the market value of the 5-year CD is \$28 (from \$300 to \$272) and the approximated decline is \$30 (from \$300 to \$270). To the extent that changes in prices reflect the degree of interest rate risk assumed, duration represents a good first-approximation measure of risk because it is proportional to the price change. For example, the price of the five-year CD will change five times as much for a given change in interest rates as the price of the one-year CD. This makes it five times as risky, which is reflected in a duration five times as great.

For capital and total liabilities, the computed declines are substantially smaller than the actual declines. This occurs because the approximated market value of the assets declines by \$80 when interest rates increase by 200 basis points, while the sum of deposits and capital declines by only \$53 (42 + 11), or \$27 less. The additional \$27 loss must be charged against capital. When this is done, the approximated value of capital declines by \$38, directly by \$11, as shown in Figure 4, and indirectly by \$27, from the additional loss in assets. (The actual value of capital declines by \$34.) The sharp decline reflects the combined effects of the 200 basis-point increase in the discount rate on capital from 18 to 20 percent and the decline in projected earnings on capital from 18 to 15 percent. Now also the approximated market value of total liabilities declines by \$80 (53 + 27), the same amount as the decline in assets, and the two sides of the balance sheet balance. Thus, as formulated, Equation 2 cannot be used directly for approximating changes in the value of capital.

With a small adjustment, however, durations can be used to measure the overall interest rate exposure of the institution. As is evident from analyzing the changes in the balance sheet and income statement above, the increase in interest rates did not affect every account equally. It is thus necessary to specify precisely what

account is most important to the institution. The selection of such a target account whose value is to be controlled and the assumption of a particular degree of risk exposure in that account is the function of the bank's senior management. In this article, we will focus on only two accounts: 1) the nominal value of capital and 2) the capital-to-asset ratio.<sup>4</sup> The first is most likely of primary concern to the shareholders of the bank and the second, to bank examiners. While the interest rate sensitivity or risk of individual accounts is related to the duration of the account, the interest rate risk of a target account is related to the difference, or gap, between the average duration of the assets of the institution and the average duration of the deposits. The duration gap measures for the two accounts discussed above are:

$$K: (D_A - wD_P) \quad (3)$$

$$K/A: (D_A - D_P)$$

where:

- $D_A$  = average duration of assets
  - $D_P$  = average duration of deposits
  - $w$  = a weight defined as  $P/P+K = P/A$
- The proofs for these relationships are derived in the Appendix.

The average durations for the measure of duration given in Equation 1 are obtained by weighting the durations for the individual securities by their relative market values. Substituting into Equation 3 the appropriate values of duration from Figure 1, yields the values of the duration gap for each account. These are shown in Figure 5. Note that the duration gaps differ for the two accounts, reinforcing the need for the

<sup>4</sup>It is also possible to use net income as a target account and to develop appropriate measures of duration gap. The duration gap measure for economic income is complex and has been derived by G. O. Bierwag. It is available from the author upon request (George Kaufman, Research Department, Federal Reserve Bank of Chicago, Box 834, Chicago IL, 60690). A duration gap measure for net income using current bank accounting practices has been derived in Alden Toevs, "Gap Management; Managing Interest Rate Risk in Banks and Thrifts," *Economic Review* (Federal Reserve Bank of San Francisco), Spring 1983.

**Figure 5**

**Using duration gap to measure interest rate risk exposure for target accounts**

**Duration gap formulas:**

$$K : (D_A - wD_P) = 4.0 - .9 (2.3) = 1.9 \text{ years}$$

$$K/A : (D_A - D_P) = 4.0 - 2.3 = 1.7 \text{ years}$$

$$\text{where } w = (P/A)$$

Approximate changes in market values for 200 basis point increase in interest rates (see equation 4):

$$K : -1.9 (+200) = \$-38$$

$$K/A : -1.7 (+200) (0.9) = -3.1\%$$

**Formulas for immunization (IRR = 0):**

$$K : D_A = wD_P$$

$$K/A : D_A = D_P$$

bank to identify a primary target account. The gap is 1.9 years for capital and 1.7 years for the capital-to-asset ratio.

The estimated impact of interest rate changes on the target accounts can be obtained by substituting the relevant duration gap into Equation 2, as follows:

**For capital:**

$$\begin{aligned} \frac{\Delta K}{A} &= -(D_A - wD_P) \Delta i \\ &= -(4 - .9 \times 2.3) \Delta i \\ &= -1.9 \Delta i \end{aligned}$$

or

$$\begin{aligned} \frac{\Delta K}{K} &= -(D_A - wD_P) \left( \frac{A}{K} \right) \Delta i \\ &= -(1.9) (10) \Delta i \\ &= -19 \Delta i \end{aligned} \quad (4)$$

**For capital-asset ratio:**

$$\begin{aligned} \Delta \left( \frac{K}{A} \right) &= -(D_A - D_P) \left( \frac{1 - K}{A} \right) \Delta i \\ &= -(4 - 2.3) (.9) \Delta i \\ &= -1.5 \Delta i \end{aligned}$$

Multiplying each duration gap by the 200-basis-point increase in interest rates yields the decrease in the value of the respective account as a per-

cent of total footings. Capital now decreases by 3.8 percent of total assets, or the full \$38, a combination of the \$11 price effect and the \$27 income effect (See Figure 4).

The above examples clearly demonstrate that the actual changes in the market value of the balance sheet accounts attributed to interest rate changes are proportional to the duration of the accounts. Accurate measures of duration yield close approximations of these actual changes.

The institution can change its degree of interest rate exposure to any extent it wishes by changing the composition of its balance sheet in such a way as to obtain the desired duration gap for its target account. The greater the duration gap, the greater is the institution's risk exposure for a particular target account; and conversely, the smaller the gap, the smaller its exposure. Moreover, the relationship is linear. For example, if in the above example the duration gap for capital were twice as large, say 3.8 years, then the value of capital would decline twice as much, or \$76 for a 200-basis-point increase in interest rates.

An institution can also eliminate its risk exposure to zero by setting its target account duration gap to zero. This can be seen by using a value of zero in Equation 4. The bank is then said to be "immunized," and unexpected interest rate changes will not change the market value of the target account. The decision on how much interest rate risk exposure to assume and the strategy of how to achieve this is referred to as interest rate risk management and is discussed in the next section.

Before introducing risk management, it is useful to emphasize a number of points:

1. If the institution does not specify a target account, it cannot measure its interest rate risk exposure accurately.
2. Interest rate exposure is directly related to the absolute size of the duration gap for the target account; the greater the gap, the greater the risk exposure.

3. Interest rate exposure can be removed or immunized in a target account by setting the appropriate duration gap to zero.
4. If the market value of total assets changes more or less with changes in the price level, maintaining a constant capital-to-asset ratio (immunizing) maintains a constant real value of capital.
5. Although the bank balance sheet used in our examples includes only securities traded on the cash market, duration analysis applies equally well to securities traded on the futures and options markets. The durations of these securities can be computed and included in the appropriate duration gap measure to measure the overall interest rate exposure of the bank.

### Managing interest rate risk

Like any private business firm, a depository institution attempts to maximize its profits. However, profit maximization presumes a desired level of risk exposure. Expected profit and risk are directly related. The greater the risk of loss assumed, the greater must be the expected profit required to compensate for the higher likelihood of loss, and, conversely, the smaller the risk exposure assumed, the smaller can the expected profit be. The desired risk-return trade-off for an institution is determined by its senior management and may be expected to differ from bank to bank. Interest rate risk is only one type of risk a bank assumes. It generally also assumes credit quality risk, liquidity risk, foreign exchange rate risk, and so on. Thus, managing interest rate risk is part of overall risk management.

To manage any risk accurately, a bank must predict the probability of possible outcomes of undertaking the risky activity. To manage interest rate risk, it is necessary to predict, at minimum, the direction of interest rate changes. The effect of an interest rate change will differ depending on whether the bank has a positive

gap (the duration of assets is greater than the duration of the appropriately weighted deposits) or a negative gap in the relevant target account. As may be seen in Equation 2, decline in interest rates will increase the nominal value of a bank's capital account if its capital duration gap is positive and decrease the value if its gap is negative. An increase in interest rates will have the opposite effect, decreasing the nominal value of the capital account if the duration gap is positive and increasing it if it is negative. The greater the gap, the greater will be the gain or loss. Thus, the bank must determine both the direction and the size of its gap on the basis of its predicted interest rates. Correct predictions will increase capital and incorrect predictions will decrease capital. A bank may pursue two interest rate risk strategies; a passive (immunization) strategy or an active strategy.

**Immunization.** For whatever reasons, a depository institution may wish to maintain a constant nominal value of its target account regardless of changes in interest rates and immunize its interest rate risk exposure. This is a complete hedging strategy. It should be noted that banks generate profits if they assume interest rate risk and manage it correctly, and that this income may be reduced or lost altogether when it decides to immunize. On the other hand, by immunizing the bank also decreases its chances of suffering losses if the risk is mismanaged. As discussed earlier, to immunize fully the bank needs to set the appropriate duration gap to zero.

Assume that the bank chooses the nominal value of capital as its target account and wishes to immunize its current market value. In our example in Figure 1, the initial value of capital is \$100. To immunize capital, the bank needs to restructure its balance sheet so that from Equation 3:

$$D_A - wD_P = 0 \quad (5)$$

Initially  $D_A = 4$  years,  $D_P = 2.3$  years and  $P/A = .9$ . This yields a duration gap of  $4 - .9(2.3) = 1.9$  years. The bank is not immunized. It can reduce the gap to zero either by shortening the duration of its assets by 1.9 years to 2.1 years or by length-

ening the duration of its deposits to 4.4 years so that  $.9(4.4) = 4$  years. It can do so either on the cash or the futures market. We will assume that the bank prefers to lengthen its deposits on the cash market. It can do so by reducing the dollar volume of its one-year CDs from \$600 to \$125 and increasing the volume of its five-year CDs from \$300 to \$775. As is shown in Figure 6, this increases the duration of the deposits from 2.3 to 4.4 years and satisfies Equation 5.

Now let interest rates increase 200 basis points as before. (For the sake of ease in tracing the mechanics, the examples are created using the approximate duration values rather than the precise duration values. Thus, the bank will not be perfectly immunized in actuality). Except for capital, which remains at \$100, the approximated market value of each account declines. Because the composition of assets was not changed, the decline in their market value is the same as in Figure 2. The composition of the deposits was changed, however, by reducing the proportion of shorter-term deposits. As a result, the decline in the value of total deposits is greater than before. Capital remains unchanged because the decline in the market value of the deposits is now exactly equal to the decline in the market value of the assets. Although capital remained unchanged, the values of the other two summary accounts did not. The capital-to-asset ratio increased to 10.8 percent, and net income increased to 2.0 percent of assets. Thus, immunization of the capital account does not imply immunization of other accounts. As shown in Figure 6, the gap for the capital-to-asset ratio is  $-0.44$  when the gap for capital account is 0.

Because, as can be seen from Equation 1, the change in interest rates changes the duration of all securities, Equation 5 is no longer satisfied after the change in rates. Thus, the bank is no longer immunized and must restructure its balance sheet so that it is immunized against the next interest rate change. Moreover, as may also be seen from Equation 1, the durations of securities decline, even if there is no interest rate change, just from the passage of time. To remain immunized, the bank must continually restructure its balance sheet to offset this duration "drift". For larger institutions that buy or sell Fed

**Figure 6**  
**Immunize K and set DGAP = 0 years**  
**when  $D_A = 4$  and  $D_P = 2.3$**

Strategy: Set  $D_A - (P/A)D_P = 0$   
 Currently:  $D_A = 4$ ,  $(P/A)D_P = 2.1$  years  
 $DGAP_K = 4 - 2.1 = 1.9$  years  
 Can satisfy by:  
 1. By shortening TA to  $D_A = 2.1$  years  
 2. Lengthening P to  $D_P = 4.4$  years [ $(P/A)D_P = .9(4.4) = 4.0$ ]  
 Can act on:  
 1. Spot market  
 2. Futures market  
 Assume lengthening P on cash market by changing mix:

Assets	Dollars*	D (yrs.)†	Liabilities	Dollars*	D (yrs.)†
Cash	100	0	CD (1 yr.)	125	1
BL (2½ yr.)	400	1.25	CD (5 yr.)	775	5
ML (30 yr.)	500	7.0	Net worth	100	5.5
Total	1,000	4.0	Total	1,000	2.65

**Deposit duration**  
 $D_P = \frac{125(1) + 775(5)}{900} = 4.44$  years

**If interest rates increase by 200 basis points:**

Assets	Dollars*		Liabilities	Dollars*	
	Actual	Approx'd.		Actual	Approx'd.
Cash	100	100	CD (1 yr.)	123	122
BL (2½ yr.)	390	390	CD (5 yr.)	704	698
ML (30 yr.)	437	430	Net worth	100	100
Total	927	920	Total	927	920

\*All accounts are valued at market (present value)

†Approximate.

**Projected annual income statement:**

	Interest yield	Market value ÷ total assets	Interest ÷ total assets (percent)
<b>Revenues</b>			
Cash	2	.11	0.2
Loans	15	.89	13.4
<b>Expenses</b>			
Deposits	13	.89	11.6
Net income			2.0

**Summary accounts**

K = \$100  
 K/A = 10.8%  
 NI = 2.0%

**Analysis (duration gap)**

$K : 4 - .9(4.44) = 0$  yrs.  
 $K/A : 4 - 4.44 = - .44$  yrs.

funds daily in the normal course of their business, this is not a problem. It is more of a problem for smaller institutions.

It is important to note that even though an immunized institution as a whole does not assume interest rate risk, the durations of the individual securities on the bank's balance sheets need not be matched, and the institution may still engage in interest rate intermediation in individual accounts. In our example, the bank has 30-year mortgages financed in part by one-year deposits. The reduction in overall risk exposure is achieved through diversification across individual securities with different durations. A portfolio of a given average duration can be structured from an almost infinite number of individual securities with different durations.

**Active management.** Many banks do not wish to eliminate interest rate risk altogether, but prefer to manage it. Because accepting risk exposure assumes that the bank will suffer losses if interest rates change in the wrong direction, the decision to accept such exposure presupposes that the bank is willing to predict interest rates and believes it can do so successfully. (If it is not, it is better off to immunize and to assume no risk for the same expected return.) Indeed, to determine the desired direction and magnitude of the duration gap, it is necessary, at minimum, for the bank to forecast the direction in which interest rates will change. If rates are predicted to increase, the gap should be negative, so that the average duration of the assets is shorter than that of the weighted deposits. This would make the bank behave as if it were a net liability, whose value declines as interest rates rise. The bank will benefit from an interest rate rise. On the other hand, if rates are predicted to decline, the institution would be better off if the gap were positive. Then the bank behaves like a net asset, whose value increases as interest rates decline.

Assume the bank predicts that interest rates will decline. It will restructure its portfolio to obtain a positive duration gap. The precise value of the gap it chooses depends on its risk-return preferences. The larger the gap, the higher the potential return but the higher also the risk of loss. The decision as to the precise risk-return matrix to assume and thus the value of the gap to

achieve is generally made by the bank's top management in consultation with the Asset and Liability Management Committee. Assume that the bank predicts that interest rates will decline in the next period and prefers to accept risk in the value of its capital consistent with a value of a positive duration gap of 1 year so that:

$$D_A - wD_P = 1 \quad (6)$$

It can achieve this value in our example either by shortening the duration of its assets from 4 to 3.1 years or by lengthening the duration of its deposits from 2.3 to 3.3 years. (We again use the approximate durations.) For every 100 basis points interest rates decline, the bank's capital value will rise by \$10 (100 basis points  $\times$  1 year gap = 1% of total assets).

In Figure 7, the bank lengthens the duration of its deposits on the cash market to 3.3 years by reducing the dollar amount of one-year CDs from \$500 to \$375 and increasing the dollar amount of five-year CDs from \$300 to \$525. Now, contrary to the bank's expectations, let interest rates increase by 200 basis points rather than decrease. The bank is worse off. Assets again decline to \$927 as before, but the market value of capital declines by \$17 to \$83. The bank has lost its bet on interest rates and has paid the price. At the same time, its capital-to-asset ratio declines to 9.0 percent and its net income remains basically unchanged. Each of these changes is easily predictable using duration analysis and assuming alternative interest rate scenarios. As was noted earlier, to win with an active policy, the bank must both predict interest rates and be right. As before, the interest rate increase changes the durations of the accounts differently and thus the value of the duration gap. To maintain a gap of 1 year, or any other target amount, the bank must restructure its balance sheet accordingly.

The changes in the values of the different summary accounts for a 200-basis-point increase in interest rates for alternative value of the duration gap in terms of capital are summarized in Figure 8. The changes may even be in different directions. For example, if the duration gap for capital is set at 0.5 years, the market value of capital will decline by about \$10 when interest

**Figure 7**

**Set DGAP for K = 1 year when D<sub>A</sub> = 4 and D<sub>P</sub> = 2.3**

Strategy: Set  $D_A - (P/A) D_P = 1$  year

Currently:  $D_A = 4, (P/A) D_P = 2.1$  years

$DGAP_K = 1.9$  years

Can satisfy by:

1. Shortening TA to  $D_A = 3.1$  years
2. Lengthening P to  $D_P = 3.3$  years [ $(P/A) D_P = 3.3$  and  $.9D_P = 3$ ]

Can act on:

1. Spot market
2. Futures market

Assume lengthening P on cash market by changing mix:

Assets	Dollars* D (yrs.)†		Liabilities	Dollars* D (yrs.)†	
Cash	100	0	CD (1 yr.)	375	1
BL (2½ yr.)	400	1.25	CD (5 yr.)	525	5
ML (30 yr.)	500	7.0	Net worth	100	5.5
Total	1,000	4.0	Total	1,000	3.5

**Deposit duration**

$$D_P = \frac{375 (1) + 525 (5)}{900} = 3.33 \text{ years}$$

**If interest rates increase by 200 basis points:**

Assets	Dollars*		Liabilities	Dollars*	
	Actual	Approx'd.		Actual	Approx'd.
Cash	100	100	CD (1 yr.)	368	367
BL (2½ yr.)	390	390	CD (5 yr.)	476	473
ML (30 yr.)	437	430	Net worth	83	80
Total	927	920	Total	927	920

**Projected annual income statement**

	Interest yield	Market value — total assets	Interest — total assets (percent)	
<b>Revenues</b>				
Cash	2	.11	0.2	
Loans	15	.89	13.4	13.6
<b>Expenses</b>				
Deposits	13	.91	11.8	11.8
Net income				1.8

**Summary accounts**

K = \$83  
 K/A = 9.0%  
 NI = 1.8%

**Analysis (duration gap)**

K : 4 - 3 = 1 yr.  
 K/A: 4 - 3.3 = 0.7 yr.

\*All accounts are valued at market (present value).

†Approximate.

**Figure 8**

**Changes in summary accounts for alternative duration gaps targeting capital account when interest rates increase by 200 basis points**

GAP (years)	Approximate changes in summary accounts*		
	K	(K/A)	NI
1.9	\$-38	-3.1%	-0.2%
1.0	-20	-1.2	0.0
0.5	-10	-0.2	+0.1
0	0	+0.8	+0.2

\*Using Equation 4.

rates increase by 200 basis points, the capital-to-asset ratio declines somewhat by 0.2 percentage points, and net income increases by .08 percentage points.

**Practical problems with applying duration gap analysis**

Although theoretically appealing, duration gap analysis has some practical problems that have limited its use to date. Duration gap analysis imposes strenuous data demands. It requires complete data on each account (security) or, at minimum, each homogeneous group of accounts on the bank's balance sheet, including not only information on contract (coupon) interest rate and maturity but also on when a variable rate account (security) can be repriced (its contract rate changed) before maturity and any constraints on the amount by which it can be repriced. In addition, data on prepayment and other call provisions; due-on-sale, early deposit withdrawal, and other put provisions; and any other options that are included and the conditions for when and how they may be exercised are required. This information requires full access to the bank's account origination files. The less information on individual accounts that is available, the less reliable will be the computed duration gaps.

Variable rate contracts and contracts that contain option provisions have effective maturities that are shorter than their nominal maturities. For example, if a 10-year variable rate bond can be repriced at \$100 at the beginning of every year, its price behavior resembles that of a one-year bond rather than a 10-year bond. Likewise, a 10-year bond with a call option permitting the

borrower to buy back (prepay) the bond at no more than a maximum price will behave like a shorter term bond when interest rates decrease so that the probabilities of a call are sufficiently high. The computation of durations for cash flows that involve either repricing or the exercise of option provisions requires forecasts of interest rates to determine when the cash flow pattern will be changed and by how much. The best forecasts to use for this purpose are the rate forecasts that are implicit in the term structure of interest rates at the time.<sup>5</sup>

A number of types of bank deposit accounts, such as demand deposits, savings, NOWs, SNOWs, and MMDAs, do not have specific maturity dates. Depositors may redeem these accounts at any time at par value. The accounts effectively have a put option exercisable by the holder on the bank at any time. What are the durations of such deposit accounts?

On the one hand, it may be argued that these are one-day accounts. If market rates of interest increase and the bank does not raise its deposit rates accordingly, either in cash or in services, the depositor may withdraw the funds. This is particularly likely in a world of increasing deregulation in which institutions across the street are able and likely to compete by offering market rates. The deposits may be effectively viewed as variable rate accounts that are repriced at par every day. Unlike our earlier examples, their market value will never decline below par value as interest rates rise. Their durations would be one day. This treatment makes it difficult for a bank to structure its balance sheet to produce zero or even small positive or negative duration gaps.

On the other hand, in the old world of regulation and deposit rate ceilings and to some extent even today, all deposits are not equally interest sensitive. If a bank's deposit rates lag increases in market rates, all deposits will not leave the bank immediately. "Core" deposits will remain for some time and flow out only slowly. It may be possible to assign accurate probabilities

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<sup>5</sup>For example, if the current interest rate on one-year fixed rate bonds is 10 percent and on two-year bonds is 11 percent, the implied rate on a one-year bond for delivery next year is approximately 12 percent.

to the timing of the net outflows, depending on the difference between the market and deposit rates. From this it is possible to compute the effective decline in the market value of the remaining deposit accounts by assuming them to be equivalent to certificates of deposits with maturity dates equal to the predicted outflow dates. Their durations would also be equivalent and thus would be longer than one day. If interest rates increase, it is then possible to value these deposits at less than their par value.

However, the correct duration to assign these deposits cannot be determined arbitrarily by the desirability of the assumptions. Rather, the actual price behavior of the accounts when interest rates change must be used. Otherwise the interest rate sensitivity of the bank is misgauged. The correct duration awaits additional research. (To the extent that interest rate deregulation has increased the availability of deposit accounts without specific maturity dates, it may have made it more difficult for banks to structure small positive or negative duration gaps and to decrease their interest rate exposure.)

Because the value of a security's duration is determined by the interest rate, changes in interest rates change its duration and may force a restructuring of the portfolio in order to maintain the desired duration gap. Moreover, even if interest rates did not change, periodic restructuring is necessary in a dynamic framework because the durations of coupon securities do not decline or age at the same rate as does time. They generally decline more slowly, although at times duration can increase as time passes. Thus, the durations of the two sides of the balance sheet are unlikely to change equally over time and continual updating or restructuring of the balance sheet is required. Restructuring, of course, is costly. But most depository institutions operate, at least in the Fed funds market, daily, so that restructuring at the margin should not be much of a burden.

In effect, every day is a new day for managing the gap. Although there may be long duration accounts on the balance sheet, the relevant time horizon for asset and liability management is only to the next restructuring date. Only in this

interval is the interest rate exposure of the institution uncertain. The disposition of cash flows beyond this interval is of no immediate concern for ongoing institutions. Through time, long duration accounts become shorter duration accounts and maturing accounts are, at maturity, either removed from the balance sheet or rolled over into the same or other accounts, whose durations are then included in the gap measure. At any moment, only an account's interest sensitivity matters.

We have made a number of simplifying assumptions in the analysis. One of these was to assume that the yield curve is flat and that when interest rates change, they all change by the same amount. This is highly unlikely. But the duration measure defined in Equation 1 is dependent on it. Different and more complex assumptions about the shape of the yield curve and changes in interest rates yield different and more complex measures of duration. If the actual process that governs interest rate changes, referred to as the stochastic process, were known, the correct duration formula could be used. But this process is not known with certainty. The theory, however, assumes that the correct duration measure is used. Moreover, securities of different default risk classes may be subject to different stochastic processes. Thus, the bank is likely to use an incorrect measure of duration and this introduces a source of error.<sup>6</sup>

In addition, the theory applies strictly only to securities that are free of the risk of default. Yet, many bank accounts, particularly on the asset side, have default risk. This introduces additional inaccuracies into the computation of the correct duration measure. Lastly, the analysis abstracts from transactions costs and taxes. Introduction of these complicates the analysis further.

### Advantages of duration gap analysis

Despite these disadvantages, duration gap analysis has substantial advantages over alterna-

<sup>6</sup>The nature of this risk is discussed in G. O. Bierwag, George G. Kaufman, and Alden Toevs, "Bond Portfolio Immunization and Stochastic Process Risk," *Journal of Bank Research*, Winter 1983.

tive techniques for measuring interest rate risk exposure accurately. The most widely used alternative measure technique involves classifying all asset and liability accounts by their terms to maturity or to first permissible repricing, whichever comes first.<sup>7</sup> The accounts are grouped in a number of maturity-period "buckets;" e.g., one day, one to three months, three to 12 months, one to five years and so on. Net balances, or maturity gaps in dollars, are computed for each bucket. The larger are the net balances in the shorter maturity buckets, the more interest sensitive and less price sensitive the institution.

Duration analysis considers the timing of coupon and other intermediate cash flows as well as the timing of the final payment at maturity. This is particularly important for mortgages and other amortized loans for which the intermediate flows are significantly larger than the final payment. Yet the maturity bucket approach classifies such accounts only by the date of the final payment or of the first permissible repricing.

For practical purposes, the number of maturity categories must be limited. What should be the maturity cutoffs for each bucket? Should the shortest-term bucket include accounts maturing or eligible for repricing in 1-30 days, 1-60 days, or 1-90 days? The same question applies to the other bucket categories. Changing the limits of the buckets can give a different picture of a bank's interest rate sensitivity. Figure 9, which groups the accounts in the balance sheets shown in Figure 1 and 6 in a number of alternative ways, illustrates this problem.

The more limited the number, the wider the category. But the wider the category, the less accurate is the informational content of each category. For example, a category of 6 to 12 months is frequently used, e.g., on the new Federal Reserve call report. This category would encompass 182-day securities as well as 364 day securities. If these were zero coupon single payment instruments so that the terms to their maturities were equal to their durations, Equation 2 indicates that the price sensitivity of the

<sup>7</sup>Alden L. Toevs, *op. cit.*; Barrett F. Binder and Thomas W. F. Lindquist, *Asset/Liability and Funds Management at U.S. Commercial Banks*, Rolling Meadows, IL.; Bank Administration Institute, 1982.

**Figure 9**  
**Alternative maturity gap measures**

Maturity bucket	Assets	Liabilities	Gap	Assets	Liabilities	Gap	Assets	Liabilities	Gap	Assets	Liabilities	Gap
<b>A. Balance sheet from figure 1</b>												
				<i>(dollars)</i>								
0-3 mos.	100	0	+100	} 100	600	-500	} 100	600	-500	} 500	600	-100
3-6 mos.	0	0	0									
6 mos.-1 yr.	0	600	-600									
1-2½ yrs.	400	0	+400	400	0	+400	400	300	+100			
2½-5 yrs.	0	300	-300	0	300	-300						
5-10 yrs.	0	0	0	0	0	0	} 500	100	+400	} 500	400	+100
>10 yrs.	500	100	+400	500	100	+400						
<b>Total</b>	<b>1,000</b>	<b>1,000</b>	<b>0</b>	<b>1,000</b>	<b>1,000</b>	<b>0</b>	<b>1,000</b>	<b>1,000</b>	<b>0</b>	<b>1,000</b>	<b>1,000</b>	<b>0</b>
<b>B. Balance sheet from figure 6</b>												
0-3 mos.	100	0	+100	} 100	125	- 25	} 100	125	- 25	} 500	125	+375
3-6 mos.	0	0	0									
6 mos.-1 yr.	0	125	-125									
1-2½ yrs.	400	0	+400	400	0	+400	400	775	-375			
2½-5 yrs.	0	775	-775	0	775	-775						
5-10 yrs.	0	0	0	0	0	0	} 500	100	+400	} 500	875	-375
>10 yrs.	500	100	+400	500	100	+400						
<b>Total</b>	<b>1,000</b>	<b>1,000</b>	<b>0</b>	<b>1,000</b>	<b>1,000</b>	<b>0</b>	<b>1,000</b>	<b>1,000</b>	<b>0</b>	<b>1,000</b>	<b>1,000</b>	<b>0</b>

364 day security to a given interest rate change is exactly twice that of the shorter security. Thus, if the 182 day security was the only security on the asset side of the balance sheet and the 364 day security the only security on the liability side, the maturity bucket would indicate no gap and no interest sensitivity. Yet the bank's liability side would in fact be twice as price sensitive as the asset side. In reality, there will be larger numbers of securities with different maturities in all the buckets so that the average maturity in each is unlikely to be at one extreme of the maturity range. Nevertheless, accuracy is sacrificed.

As noted, the maturity gap analysis yields a number of gap values equal to the number of maturity categories used. These individual gaps cannot be simply summed. The overall degree of risk exposure is thus difficult to summarize. It is not readily observable, for example, from any of the alternative maturity gap groupings in Figure 9B that the balance sheet is one that immunizes the dollar value of capital. The gap in any one bucket, even the shortest one, is unlikely to be representative of the overall interest rate sensitivity of the institution. The impact of the value of the gap in the shortest term bucket can be more than offset by the value of the gap in the next shortest bucket, so that the longest gaps can be in the same direction as the shortest gaps. More

importantly, measured this way, the risk exposure is difficult to manage. A different strategy is required for each bucket. To immunize, for example, cash flows must be matched in each bucket. This involves considerable management and transactions costs. It is reasonable to assume that some of the bucket gaps are internally offsetting and that the use of external transactions to achieve the same objective is inefficient. In contrast, duration analysis yields a single number and only a single gap to manage. Any internal cancelling is already accounted for.

Maturity bucket gaps must generally be managed with securities in the same maturity category; e.g., a gap in the 6 to 12 month bucket can be changed most easily by buying or selling other securities in this maturity range. This constrains management. The larger the number of buckets used to gain greater accuracy, the more constrained is management. On the other hand, duration gaps can be managed with a very wide range of maturities. For example, as was seen earlier, the duration of a 30-year amortized fixed rate mortgage when interest rates are 13 percent is nearly equivalent to that of a seven-year zero coupon bond. Moreover, even durations on individual securities on the two sides of the balance sheet need not be equal or different by the size of the gap as long as the average durations of all

securities are. The desired target gap value may be achieved by diversifying among individual securities of different but offsetting durations. Thus, bank management has an almost unlimited choice of maturities and can continue to provide a range of interest rate intermediation services to its customers within a given degree of net interest rate risk exposure to the institution. Even if the bank wishes to immunize itself, it may still engage in a wide range of interest rate intermediation for individual securities; it does not have to match cash flows in each maturity bucket. That is, a bank can simultaneously engage in macro immunization and micro interest rate intermediation and continue to accommodate its customers, who have a wide range of maturity preferences, with an equally wide range of products.

Lastly and perhaps most importantly, the strenuous data demands made by duration analysis are not any more severe than those that alternative systems, including maturity gap analysis, would impose if they were to be equally accurate. All measuring techniques must forecast interest rates to know when repricing will occur and options will be exercised. The relative simplicity claimed for some alternative systems cannot be obtained by sweeping such problems under the rug. Only a complete and thorough cost-benefit analysis can differentiate among the alternative techniques. Simplicity and reduced cost is likely to be achieved only at the cost of reduced accuracy. And, because once a computerized information system is in place it is costly to change, banks should plan their systems for asset and liability management models of the future. Duration-based models are in a relatively early stage of development and require further refinement. They appear, however, a most promising tool for accurate asset and liability management.

## Summary

Interest rate risk continues to be a problem of increasing importance to many depository institutions. In order to manage this risk correctly, it is first necessary to measure it accurately. This article has discussed how the re-

cently developed technique of duration analysis can be applied to this problem. Duration gap is both an accurate measure of an institution's interest rate risk exposure and, because it is a single number, a simple concept to manage. But its application is complex, and the data required is costly. However, most of these complexities and costs also apply to alternative measures of interest rate risk, if they are to be equally accurate, including maturity gap analysis. Banks and other depository institutions should consider the use of duration analysis to measure and manage their interest rate exposure reliably and to maximize their long-run profits.

## Appendix

### Approximate proofs of duration gap measures\*

Assume:

- 1) In market values:  $A = P + K$  and  $\Delta A = \Delta P + \Delta K$
- 2) All interest rates for same default risk class securities are the same and all changes in interest rates are equal.

#### 1. Target account K

Let  $K = 0$  (immunize)

Then from above assumptions:  $\Delta A = \Delta P$ .

Recall the basic equation:

$$\frac{\Delta A}{A} = -D_A \Delta i$$

or

$$\Delta A = -D_A \Delta i A$$

Thus as  $\Delta i_A = \Delta i_P$

$$D_A A = D_P P$$

or

$$D_A = (P/A) D_P$$

It follows that DGAP is:

$$(D_A - w D_P)$$

#### 2. Target account: K/A

Let  $K/A = 0$  (immunize)

Define  $K/A = c$

so  $K = cA$ .

Then from above assumptions:

$$A = P + cA$$

$$\Delta A = \Delta P + c \Delta A$$

$$A(1 - c) = P$$

And  $\Delta A(1 - c) = \Delta P$

Using the basic equation with  $\Delta i_A = \Delta i_P$ :

$$D_A A(1 - c) = D_P P$$

or

$$D_A = D_P$$

It follows that DGAP is:

$$(D_A - D_P)$$

\*More detailed proofs are found in F.M. Redington, "Review of the Principle of Like-Office Valuations," *Journal of the Institute of Actuaries*, 1952, pp. 286-340 (reprinted in Gabriel Hawawini, ed., *Bond Duration and Immunization*, New York: Garland Publishing Co., 1982, pp. 286-340); Myron A. Grove, "On 'Duration' and the Optimal Maturity Structure on the Balance Sheet," *The Bell Journal of Economics*, Autumn 1974, pp. 698-709; and the references cited in footnote 2.