

Is there evidence of the new economy in U.S. GDP data?

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Introduction and summary

Economic theory suggests that temporary cyclical fluctuations in real gross domestic product (GDP) adversely affect the economic well-being of households. For example, when the economy experiences a cyclical downturn, companies lay off workers with resulting negative consequences for the workers and their families. Thus, it is not surprising that cyclical fluctuations in GDP receive a lot of attention from policymakers. Indeed, there is considerable empirical research that shows that cyclical fluctuations in GDP play an important role in the practical conduct of U.S. monetary policy.¹ In general, the U.S. Federal Reserve (Fed) tightens monetary policy (increases interest rates) when the cyclical component of GDP rises and loosens monetary policy (reduces rates) when the cyclical component of GDP falls.

Unfortunately, economists cannot observe the cyclical component of GDP. This is because observed GDP is made up of two *unobserved* components. The first, called the trend component, refers to the upward sloping part of GDP. For example, figure 1, panel A plots the trend component of GDP under the assumption that it is a constant linear trend (green line). The second, called the cyclical component, refers to the fluctuations around the trend component. Figure 1, panel B plots the cyclical component of GDP that is related to the constant linear trend plotted in panel A.

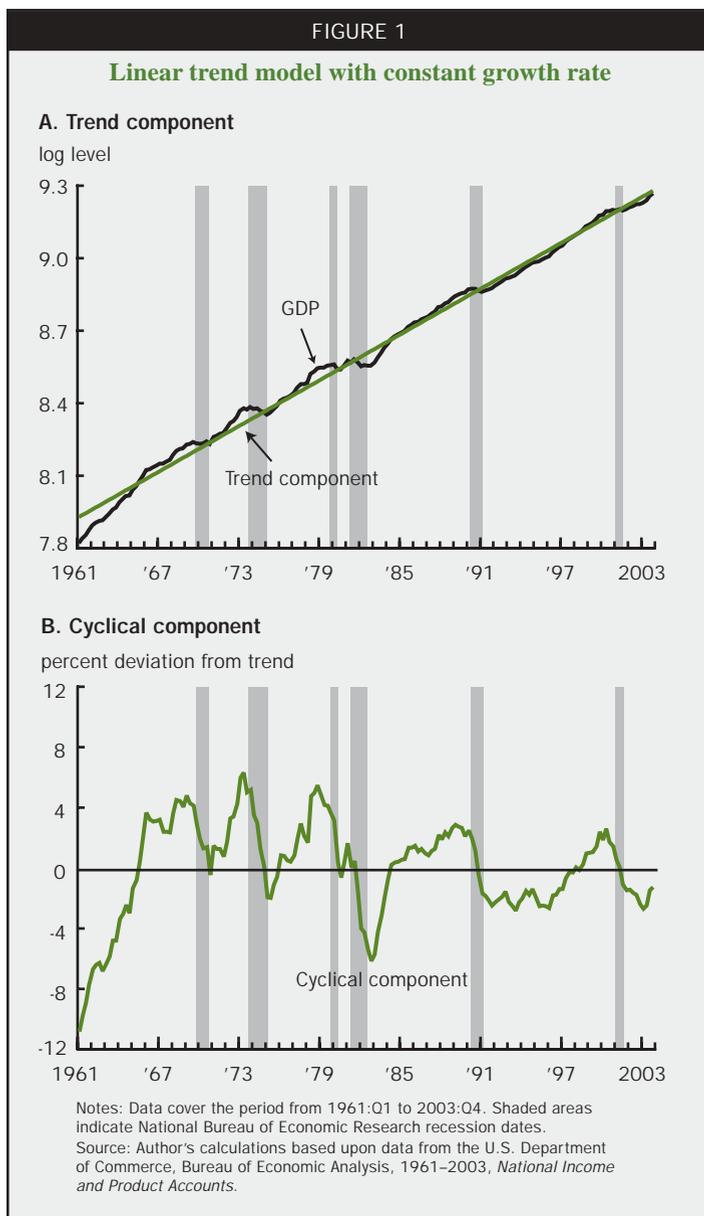
Economists typically identify the policy-important cyclical component by first making assumptions that allow them to isolate the trend component and then backing out the cyclical component. In general, the biggest challenge in isolating the trend component is estimating its slope. The slope of the trend component is determined by the trend growth rate of GDP (that is, the growth rate of output that would exist if there were no cyclical fluctuations in GDP). Higher trend growth rates imply a steeper trend component.

The debate over the true value of the trend growth rate received a lot of attention in the late 1990s from economic analysts and policymakers. Analysts argued, based on strong observed growth of labor productivity (GDP per worker), that the trend growth rate of GDP had increased significantly. If an increase in the trend growth rate had occurred, this type of structural change would have meant that economists could no longer rely on their longstanding rules of thumb about the relationship between observed GDP and the unobserved cyclical component in formulating policy. This led to speculation by the analysts that the U.S. was a *new economy* in the late 1990s, in which all the old rules about actual, trend, and cyclical fluctuations of GDP no longer held true.

In this article, I test whether there was in fact significant change in the trend growth rate of U.S. GDP over the new economy era. I do so by applying both long-established and newer techniques of extracting the trend component of U.S. GDP data and then testing to see if the implied trend growth rate of U.S. GDP (that is, its average slope) over the new economy era is significantly higher than the implied trend growth rate of U.S. GDP over the preceding productivity slowdown era.

Irrespective of the method used to extract the trend component, I find that the implied annual trend growth rate of U.S. GDP was about 3 percent over the productivity slowdown period, which is considerably higher than the typical 2.5 percent estimate based on productivity data, and about 3.25 percent over the new economy era. Although I find a positive difference between the new economy and productivity slowdown era estimates, it is not statistically significant. I conclude

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that, at least in terms of GDP data, the U.S. was the same *old economy* in the late 1990s.

A simple linear trend model of GDP

An economy is like a biological organism in that it grows exponentially over time. For example, in the simple case of an economy that is growing at the constant rate μ per time period, the size of this economy measured by GDP at time t is given by

$$1) \quad X_t = \Phi e^{\mu t},$$

where X_t denotes the level of GDP at time t and Φ is a constant. Economists generally do not work with the level of GDP, but instead prefer to work with the log of GDP (log GDP). The main reason for this transformation is that growth rate calculations using log GDP are linear, while similar calculations using the level of GDP are non-linear. For example, if we take the log of both sides of equation 1 and denote log GDP at time t by x_t , it follows that:

$$2) \quad x_t = A + \mu t,$$

where $A = \log(\Phi)$. In this simple case, log GDP is a linear function of a constant A and a time trend t with coefficient μ . If we were to plot this relationship with the value of log GDP on the vertical axis and time along the horizontal axis, the intercept of log GDP with the vertical axis would be A and the slope of log GDP as we move along the horizontal axis would be μ . An increase in A would shift up log GDP by a constant amount across all periods, so economists call changes in the constant a *level shift*. Raising the growth rate of GDP μ increases the slope of log GDP across all periods, so economists call changes in the growth rate a *slope change*. Models that economists actually use to explain the evolution of GDP over time essentially build on this simple model by allowing for some type of variation in the constant and slope.

Allowing for cyclical variation around the trend

The first significant departure from the simple linear trend model described by equation 2 is that log GDP can be additively decomposed into a trend component τ_t and a cyclical component c_t as follows:

$$3) \quad x_t = \tau_t + c_t.$$

The trend component captures the upward sloping part of GDP (which was explained in the simple model in equation 2 by a linear trend), while the cyclical component captures fluctuations around the trend component (this component was ignored in equation 2).

However, economists do not observe either the trend or cyclical component of GDP. Economists typically proceed along one of three paths in estimating

these unobserved components. First, they estimate the trend component directly and determine the cyclical component as the difference between observed log GDP and the estimated trend component. Second, they estimate the cyclical component directly and determine the trend component as the difference between observed log GDP and the estimated cyclical component. Or, finally, they jointly estimate the trend and cyclical components.

Estimating a constant growth rate model of GDP

Early attempts at estimating the trend component took the direct approach by assuming, as in the simple example above, that log GDP had a *linear* trend with constant slope μ :

$$4) \quad \tau_t = A + \mu t,$$

where t denotes the linear trend, A is a constant, and the t subscript denotes the date at which the trend is being measured. Just as in the simple model discussed above, the slope coefficient μ is the trend growth rate of GDP. The cyclical component is simply the difference between the linear trend and log GDP as follows:

$$5) \quad c_t = x_t - (A + \mu t).$$

All the elements of this decomposition are plotted in figure 1. Starting with figure 1, panel A, the black line is log GDP x_t , while the green line is the estimated linear trend, $\hat{\tau}_t = \hat{A} + \hat{\mu}t$, where \hat{A} and $\hat{\mu}$ are estimated using ordinary least squares.² According to my estimate of the linear trend model, the constant annual growth rate of GDP over the entire sample of quarterly GDP data from 1961:Q1 to 2003:Q4 was 3.16 percent. Figure 1, panel B plots the implied cyclical component c_t , which is multiplied by 100 so that it can be interpreted as a percentage deviation from the trend component. This figure shows that this relatively simple method of estimating the trend component produces a cyclical measure of GDP that has turning points that line up closely with the National Bureau of Economic Research's (NBER) peak and trough dates, captured by the gray bars.

Estimating a time-varying growth rate model of GDP

Virtually all the recent research aimed at estimating the trend growth rate of GDP has focused on whether it changed significantly over the so-called new economy era from 1995:Q4 to present. Much of this investigation was fueled by the spectacular increase in the observed trend growth rates of labor productivity (GDP per worker) in the mid-1990s. To understand

why this is important, we need to review the way that the trend growth rate of GDP is typically estimated. Economists have long recognized that the trend growth rate of GDP is the sum of the trend growth rate of labor productivity and the trend growth rate of employment. This led to the popular bottom-up approach in estimating trend GDP growth, whereby researchers estimate the trend growth rate of productivity and the trend growth rate of employment directly and simply add these components together to get the implied trend growth rate of GDP. Gordon (2003) and many others have used this approach and argued on the basis of their estimates that there was a significant variation in the trend growth rate of GDP over the new economy era.

In particular, Gordon found that the trend growth rate of labor productivity rose significantly from an annual growth rate of 1.5 percent estimated over the productivity slowdown era from 1973:Q1 to 1995:Q3 to 2.5 percent over the new economy era. Under the typical assumption that the trend growth rate of employment is 1 percent, which is based on a constant long-run labor force participation rate and trend growth of the labor force of 1 percent, Gordon concluded that the implied trend growth rate of GDP over the productivity slowdown era was 2.5 percent and that there was a significantly higher implied trend growth rate over the new economy era of 3.5 percent.

In contrast to these researchers, I take a more direct approach to testing whether the trend growth rate of GDP changed over the new economy era. I use the techniques used by Gordon (2003) and others to estimate the trend growth of productivity directly to estimate the trend growth of GDP directly. A possible advantage of this approach is that it does not require auxiliary assumptions about the trend growth of employment, since it uses the same method to estimate the trend growth rate of both productivity and employment.

Following Gordon's approach to estimating the trend growth rate of productivity, I allow the parameters that govern the slope of the trend component of GDP to vary over the sample. If this exercise shows there has been a statistically significant variation in the parameters that govern the slope of the trend before and after the new economy era, this would imply that we are indeed in a new economy. Alternatively, if I find that variation in the slope over these periods is not statistically significant, this would support the conclusion that the U.S. was the same old economy in the latter half of the 1990s.

A simple time-varying linear trend model of GDP

A useful starting point on this path is the time-varying (discrete jump) linear trend model that was

Table 1

Trend growth rate of GDP

Estimation method	Estimated trend growth rate			Estimated change in trend growth rate		
	Estimation period			Comparison period		
	1961:Q1 -2003:Q4	1961:Q1 -1972:Q4	1973:Q1 -1995:Q3	1961:Q1-1972:Q4	1973Q1-1995:Q3	1995:Q3-2003:Q4
Productivity studies (Gordon)						
	3.50	2.50	3.50	1.00*		1.00*
Linear trend models						
Linear trend (constant)	3.16					
Linear trend (time-varying/discrete jump)		2.94	3.39	0.94*		0.45*
Frequency domain						
Band-pass filter	4.06 ^a	2.98 ^a	3.26 ^a	1.08 ^{a*}		0.28 ^a
Unobserved component models						
Univariate models (Watson)						
Univariate unobserved component (constant)	3.30					
Univariate unobserved component (discrete-jump)		2.92	3.29	1.28*		0.37
Univariate unobserved component (unit-root)	4.20	3.18 ^a	3.12 ^a	0.54 ^{a*}		-0.06 ^a
Multivariate models (Gerlach & Smets)						
Multivariate unobserved component (constant)	3.32					
Multivariate unobserved component (discrete-jump)	4.12	3.00	3.14	1.12*		0.14
Multivariate unobserved component (unit-root)	3.91 ^a	3.14 ^a	3.02 ^a	0.77 ^{a*}		-0.12 ^a

^aDenotes average over period.

*Denotes statistically significant at 5 percent level.

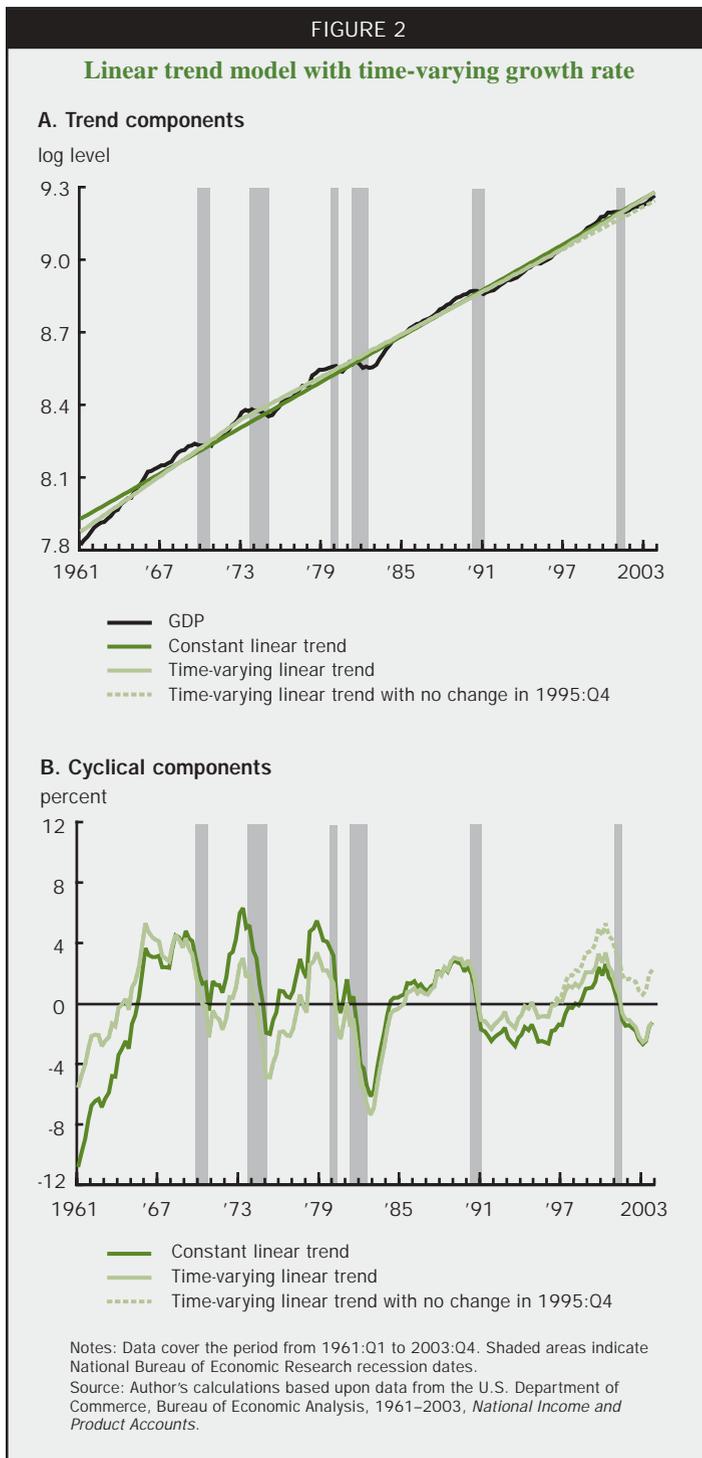
widely used in the 1970s. This model allows the constant and slope of the trend component to vary over discrete intervals:

$$6) \quad \tau_i = A_{i-j} + \mu_{i-j}t,$$

where A_{i-j} is the constant and μ_{i-j} is the trend growth rate over the time interval from i to j .³ Following the productivity trend growth literature, I allow for variation in trend component parameters over three periods: the productivity slowdown era from 1973:Q1 to 1995:Q3, the new economy era from 1995:Q4 to 2003:Q4, and the pre-slowdown era from 1961:Q1 to 1972:Q4.

My estimates of the discrete jump linear trend model, reported in table 1 on the previous page, suggest that the trend growth rate of GDP was 3.88 percent in the pre-productivity slowdown era, well above the productivity slowdown estimate of 2.94 and new economy era estimate of 3.39.⁴ More importantly, I find that the difference between the new economy and the productivity slowdown trend growth rate estimates is statistically significant, which suggests that the U.S. was a new economy post 1995:Q3.

Figure 2 reveals the impact on the cyclical component of allowing for a time-varying trend growth rate. Differences between trend components are inversely related to differences between cyclical components: As the trend component shifts up, the cyclical component decreases. Although the difference between the constant (dark green line) and discrete jump (light green line) trend components in figure 2 appears to be small, the percentage point difference between the constant (dark green line) and discrete jump (light green line) growth rate cyclical components is quite large. For example, the cycle in 1996:Q1 is -2.7 percent for the constant trend growth rate model and -0.9 percent for the discrete-jump trend growth rate model. A variation of this size would likely generate a different policy response from the Fed, which highlights the importance to policymakers of estimating the true trend growth rate.



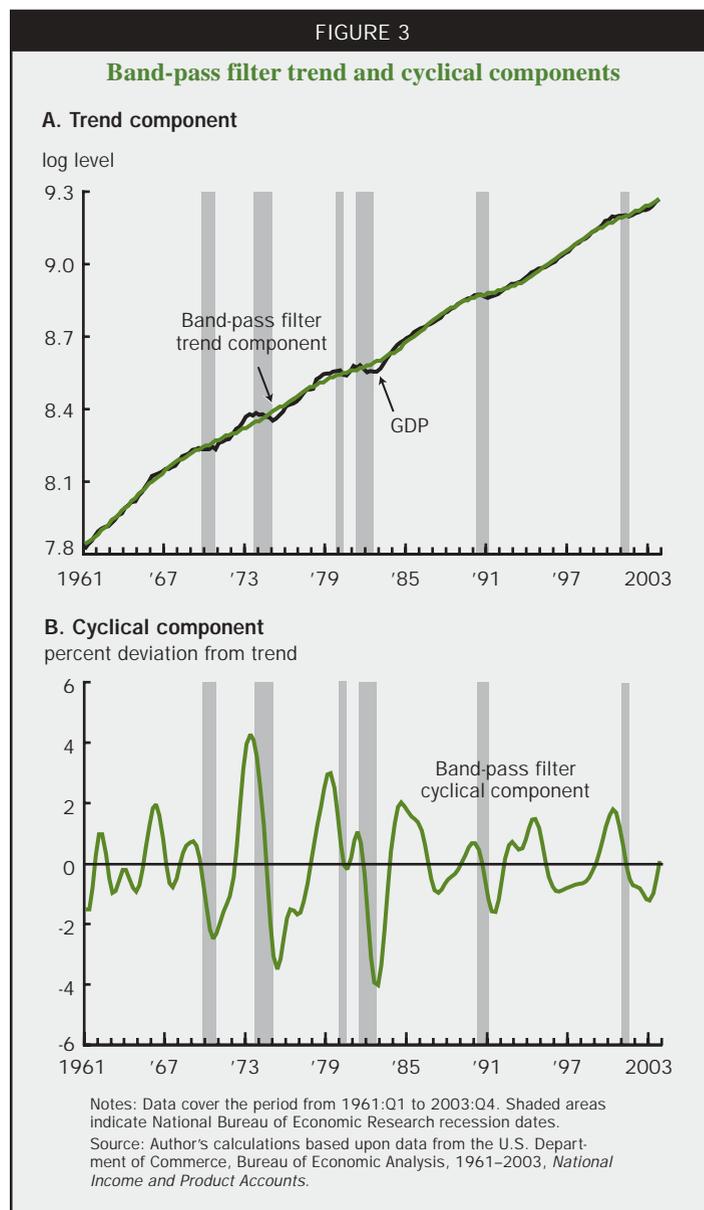
A more important experiment for the current exercise is a comparison of the cyclical component assuming no change in the trend growth rate (dashed green line, which shows what the discrete jump growth rate cyclical component would have been if

the trend growth rate were held constant at its 1995:Q3 level over the post 1995:Q3 new economy period) and the cyclical component when the trend growth rate is allowed to change (light green line). This figure appears to support the new economy theory, because it shows that the policy-important cyclical component that incorporates changes in the trend growth rate of GDP lies everywhere below the same cyclical component assuming no change. This suggests that the Fed would have responded to the change in growth rates over the new economy era by tightening monetary policy less aggressively than if it had maintained the growth rate of the productivity slowdown era.⁵

Does GDP have a linear trend?

Developments in the field of econometrics during the 1980s called into question the usefulness of the simple linear trend model for policy analysis. Armed with new and powerful statistical techniques, economists such as Nelson and Plosser (1982) explored the trend properties of economic time series and discovered that many U.S. time series, including GDP, had stochastic rather than deterministic trends as in the linear trend models.⁶ Stochastic trends are more general than the deterministic linear trend models described above. The primary difference is that they allow for significant variation in the level of the trend component. In other words, the constant term A in the linear model is a random variable in the stochastic trend model. This development meant that the widely used linear trend models were misspecified.

Economists reacted to this challenge by developing new approaches to modeling economic time series with stochastic trends, known as frequency domain and unobserved component techniques. These methods revealed that the simple linear trend models (including the discrete jump linear trend model estimated above) were poor representations of the data. In particular, they provided misleading results on the nature of the trend and cyclical components of GDP. In light of this finding, economists have largely relied on frequency domain and unobserved component techniques to isolate the trend and cycle components of economic time series.



Frequency domain estimates of the trend and cyclical components

Frequency domain techniques were made popular by the modern business cycle literature starting in the 1980s. According to this paradigm, fluctuations in the data at the so-called business cycle frequencies of between 18 months and eight years are considered cyclical movements, c_t , while long-run fluctuations occurring at frequencies of greater than eight years make up the trend component τ_t . The main advantage of this approach over unobserved component methods is that it can isolate the noisy short-run movements of economic time series that are a nuisance to

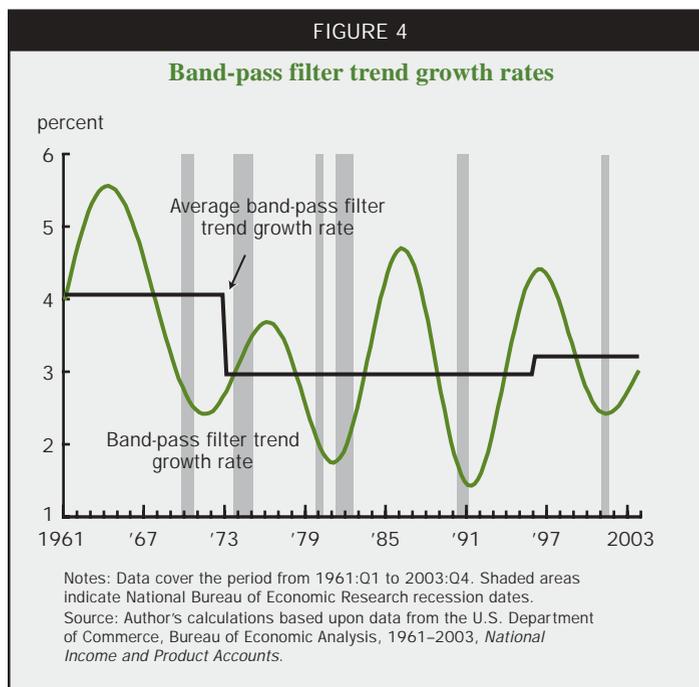
policymakers. Fluctuations in the data occurring at frequencies of less than 18 months are regarded as noise, η_t . Using this approach, log GDP is the sum of three components, trend, cycle, and noise:

$$x_t = \tau_t + c_t + \eta_t.$$

The most convenient way of extracting these three components from time series data is via an approximate band-pass filter (BPF). Approximate BPFs are essentially centered moving averages. The problem with these approximate filters is that the filtered data ends up being much shorter than the unfiltered time series, because the moving average requires a significant amount of data at the beginning and end of the sample, up to three years in the case of quarterly GDP data. My analysis of the trend growth rate of GDP relies on the approximate BPF method developed by Christiano and Fitzgerald (2003), which is designed to filter the data over the entire sample, thus preserving the sample size.

Figure 3 on page 17 plots the trend and cyclical component of GDP generated by a BPF. The most obvious difference from figure 2 is that the BPF cyclical component is considerably smoother than its linear trend counterparts. This reflects the fact that the BPF cycle does not include the highly irregular noise component. Another advantage of the frequency domain approach over the unobserved component method is that it can endogenously identify changes in the trend growth rate. Figure 3 shows that, in contrast to the discrete-jump linear trend model, the slope changes of the BPF trend are numerous and smooth. The extent of these growth rate changes is revealed in figure 4, which plots the implied annual growth rate of the BPF trend component (green line).

Given the variation in the implied trend growth rate, I test for significant change in the trend growth rate of GDP by testing if the average growth rate of the BPF trend component over the new economy era is significantly higher than the average growth rate of the BPF trend component over the preceding productivity slowdown era (black line). I find that the average BPF trend growth rates are 2.98 for the productivity slowdown period and 3.26 percent for the new economy era. In contrast to the discrete-jump linear trend model, the difference between these estimates is not statistically significant, which suggests that the U.S. was the same old economy in the late 1990s.



Unobserved component techniques

Another group of economists led by Watson (1986) took a completely different route to decomposing GDP into its trend and cyclical components by applying unobserved component (UC) techniques. In contrast to the frequency domain approach, UC methods require strong assumptions about the exact form of the trend and cyclical components. Watson's initial UC model of log GDP responded directly to the work of Nelson and Plosser (1982) by allowing log GDP to have a stochastic trend. In particular, Watson's model assumed that the trend component of log GDP was a random walk with drift.

The random walk with drift assumption meant that the trend component τ_t depended on its most recent past observation τ_{t-1} , a random component $\varepsilon_{\tau t}$, and a constant term, typically called drift μ :

$$7) \quad \tau_t = \mu + \tau_{t-1} + \varepsilon_{\tau t}.$$

In the absence of random fluctuations ($\varepsilon_{\tau t} = 0$), the trend component grows at a rate equal to the drift μ . However, the trend component does not always grow at the trend growth rate because positive random fluctuations lead to trend growth in excess of the drift, while negative random fluctuations cause the trend to grow by less than the drift. It is important to note that while fluctuations in the random component $\varepsilon_{\tau t}$ have a permanent effect on the level of the trend component, they do not have a permanent effect on the trend

growth rate. Therefore, the long-run or trend growth rate of GDP is measured by the drift term μ .

Watson's model assumes the cyclical component is a second order autoregression, which means that the current cyclical component c_t depends on its most recent past two observations c_{t-1} , c_{t-2} , and a random component ε_{ct} :

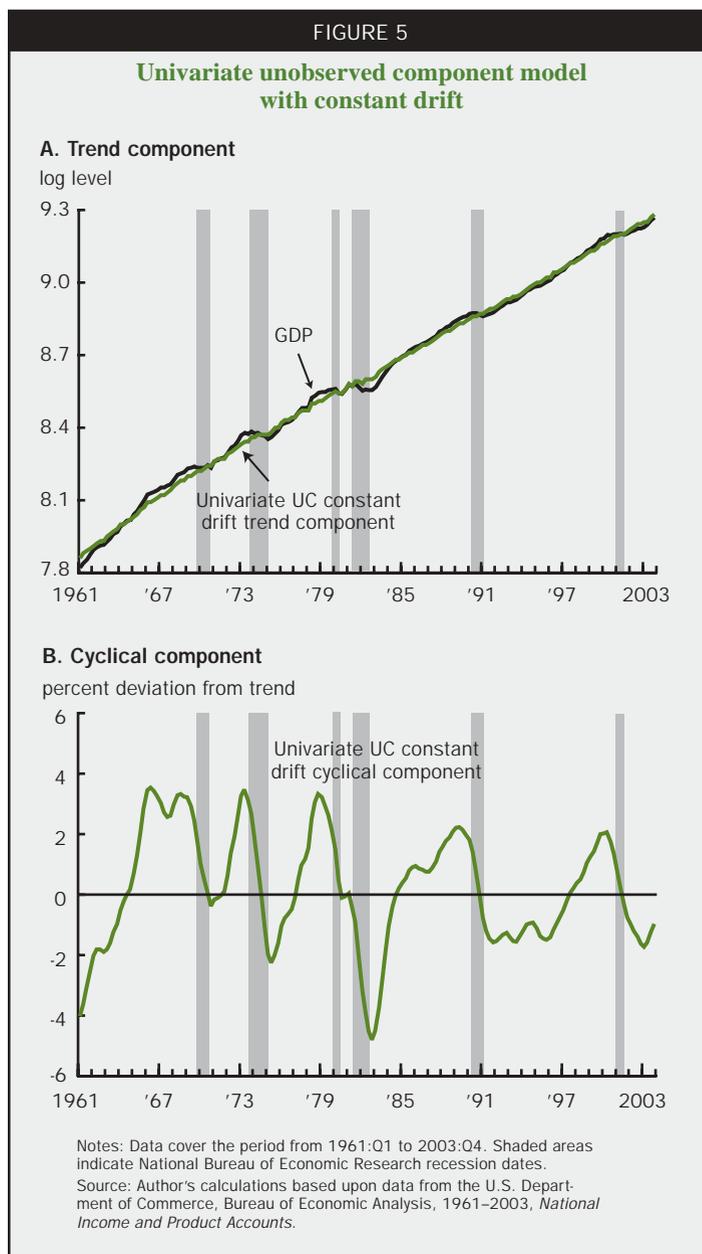
$$8) \quad c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + \varepsilon_{ct}.$$

The cyclical component is assumed to be a stationary process, which means that random shocks to the cycle ε_{ct} have no permanent effect on the level of the cycle or log GDP. This requires that $\phi_1 + \phi_2 < 1$. Finally, the noise component cannot be identified, so log GDP x_t is assumed to be the sum of the trend and cyclical components.

Watson's UC model of log GDP is often referred to as a univariate UC model because although there are many unobserved components, there is only one observed component. The unobserved components are the trend, cyclical, random trend, and random cycle components, while the observed component is log GDP. The unobserved components are identified by assuming that the random trend ε_{τ_t} and random cycle ε_{ct} components are uncorrelated.

Table 1 reports the trend growth rate estimates from Watson's univariate UC model with a constant drift.⁷ Despite additional data, the move to chain-weighted real GDP indexes, and recent changes in the measurement of business investment, my estimate of the trend growth rate of GDP is very close to that first reported by Watson. At 3.30 percent, it is slightly higher than the constant growth rate estimate from the linear trend model.

Figure 5 plots the univariate UC constant drift trend and cyclical components. Panel A reveals that the UC trend component is not as smooth as the linear trend component. This highlights level shifts of log GDP caused by fluctuations in the random trend component. Panel B shows that the univariate UC cyclical component also has turning points that closely match the NBER business cycle dates.



Unobserved component time-varying trend growth rate models

Watson's model assumes that variations in the growth rate of the trend are temporary, so it needs to be modified to test for permanent changes in the trend growth rate. I build on Watson's model by introducing a time-varying drift term μ_t that allows the growth rate of the trend to change permanently. I consider two cases, a discrete-jump model that allows for lumpy changes in the trend growth rate and a unit-root model that allows for smooth changes in the trend growth rate.

Discrete-jump trend growth rate

The first case assumes that changes in the trend growth rate of GDP take on discrete jumps. In particular, the trend growth rate is assumed to jump to a new level μ_{i-j} for a fixed period ($t = i$ to j). Under this assumption, the trend component is a random walk with drift:

$$9) \quad \tau_t = \mu_t + \tau_{t-1} + \varepsilon_{\tau t}$$

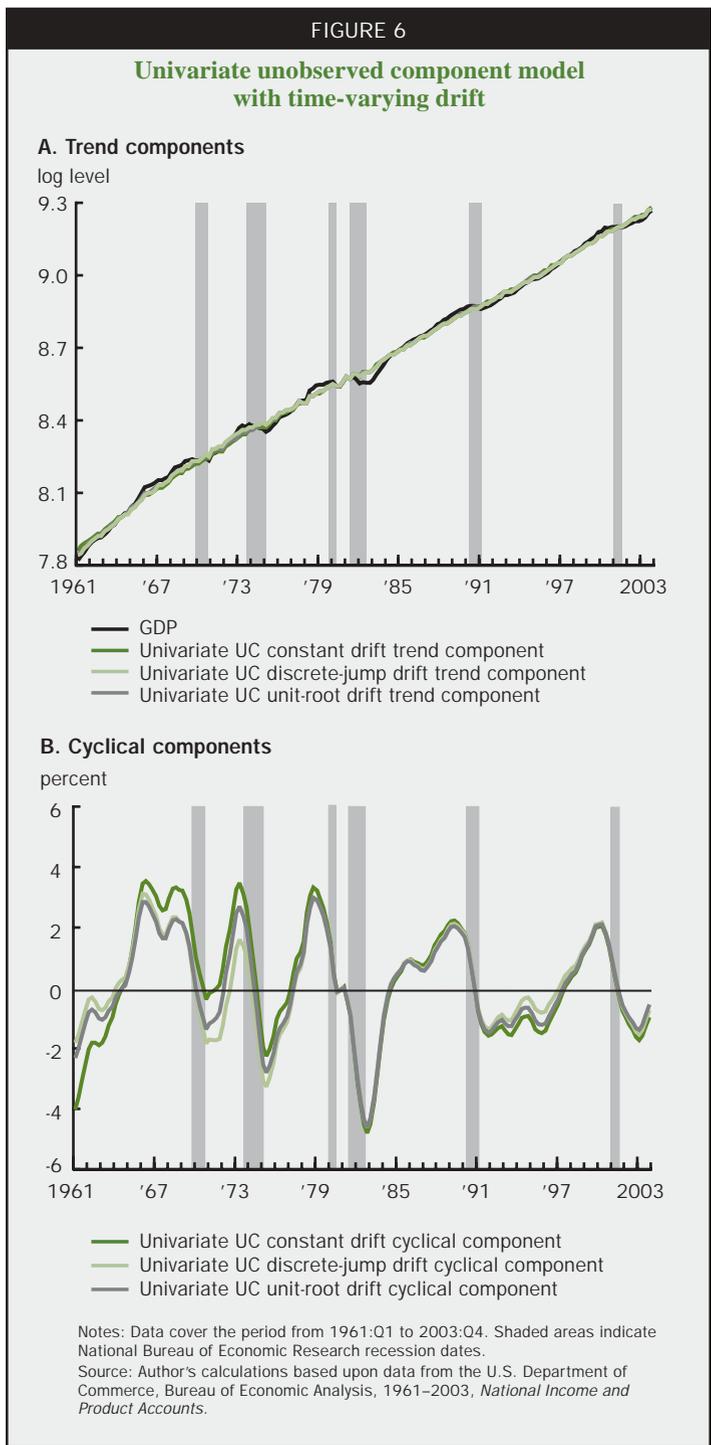
but now the drift μ_t is allowed to vary over fixed periods:

$$10) \quad \mu_t = \mu_{i-j} \text{ for } i \leq t \leq j.$$

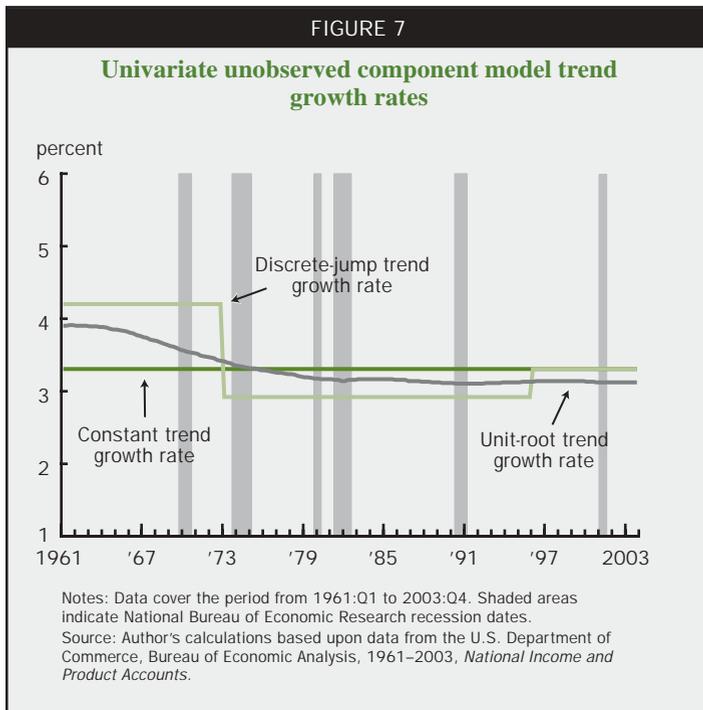
This is analogous to the time-varying discrete-jump linear trend model studied above. Just as in the linear trend case, I allow the drift term to vary over three periods that are widely viewed by empirical researchers to be periods in which the trend growth rate of productivity changed significantly: the productivity slowdown era; the new economy era; and the pre-productivity slowdown era. I test for significant change in the trend growth rate over the new economy era by testing whether the drift term over the new economy period is significantly higher than the drift term over the preceding productivity slowdown period.

Table 1 reports my estimates of trend growth rates for all the models I estimate across the three periods. In the case of the univariate discrete-jump model, the trend growth rate over the new economy era (3.29 percent) is higher than that for the productivity slowdown era (2.92 percent), but the difference between the two rates is not statistically different from zero. Therefore, I cannot reject the null hypothesis that the U.S. was the same old economy in the late 1990s. In contrast, my estimates suggest that the difference between the pre-slowdown and productivity slowdown trend growth rates is a statistically significant 1.28 percent.

Figure 6 plots the univariate UC trend and cyclical components under the discrete-jump assumption. The discrete-jump drift cyclical component (light green line) lies slightly above the constant-drift cyclical component (dark green line) over the early part of



the new economy period. This gap diminishes over the latter part of the 1990s, so that the two curves are virtually identical around 2000. Just as in the discrete-jump linear trend model, level differences of this size would likely generate a different policy response from the Fed, which again highlights the importance to policymakers of estimating the true trend component.



However, this comparison of the constant and discrete-jump drift cyclical components is uninformative when it comes to answering the question of whether changes in the trend growth rate had a bearing on monetary policy. As in the linear case, we need to compare the cyclical components under the assumption of no change in the trend growth rate in 1995:Q4 in the discrete-jump linear trend model (the old economy path, not plotted) and the plotted cycle, which allows for changes in the trend growth rate (the new economy path). Irrespective of where the estimation sample ends post 1995:Q3, I find that the path of the cyclical component, assuming no change in the growth rate from 1995:Q3 onwards, is close to the cycle that allows for changes in the trend growth rate in 1995:Q4. This finding suggests that if the Fed used this unobserved component model to estimate the trend and cycle component, but failed to factor in a change in the trend growth rate in 1995:Q4, its monetary policy response would have been indistinguishable from its response with a change in the trend growth rate.

Unit-root growth rate

The next time-varying model follows Harvey and Todd (1983) and Clark (1987) in assuming that the trend component is a random walk with a time varying drift:

$$\tau_t = \mu_t + \tau_{t-1} + \varepsilon_{\tau t}$$

but now I allow the drift μ_t to vary in a smooth way by allowing it to be also a random walk process:

$$11) \quad \mu_t = \mu_{t-1} + \varepsilon_{\mu t}$$

where the current trend growth rate μ_t depends on its most recent past observation μ_{t-1} , plus a random component $\varepsilon_{\mu t}$. Under this assumption, fluctuations in trend growth come from two sources: changes in the random trend component $\varepsilon_{\tau t}$, which permanently change the level of the trend component, and changes in the random drift shock $\varepsilon_{\mu t}$, which permanently change the slope of the trend (or long-run trend growth rate).

The unobserved trend, cyclical, and time-varying drift components are identified by assuming that

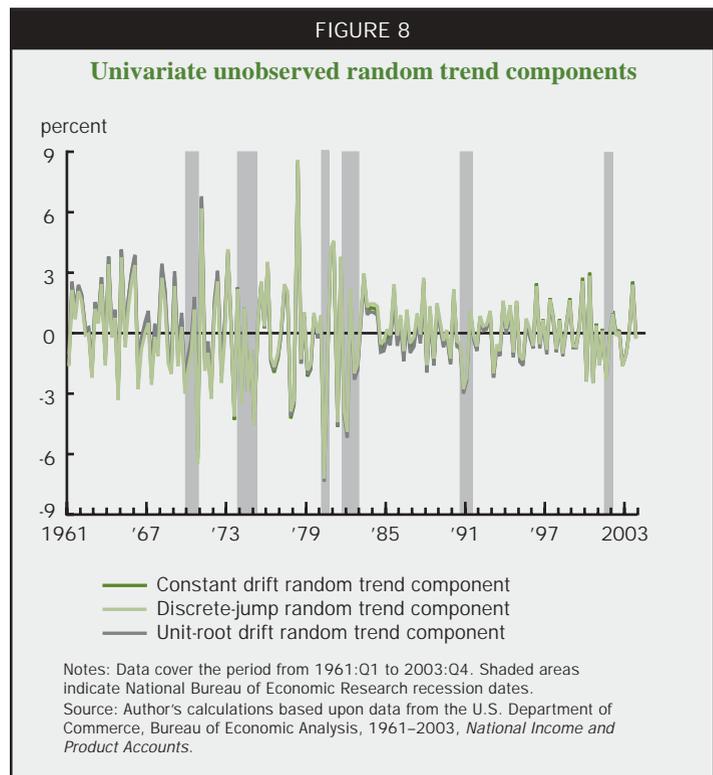
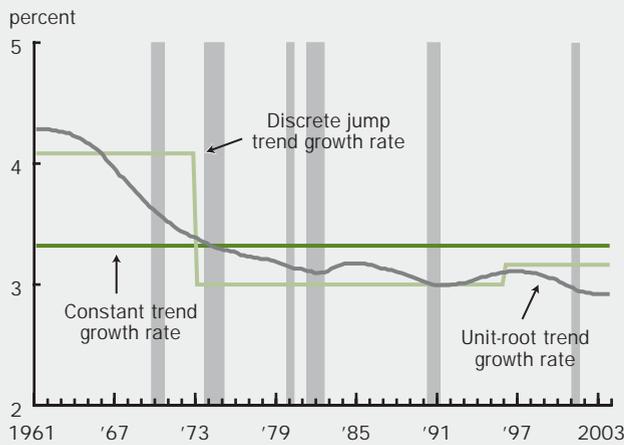


FIGURE 9

Multivariate unobserved component model trend growth rates



Notes: Data cover the period from 1961:Q1 to 2003:Q4. Shaded areas indicate National Bureau of Economic Research recession dates. Source: Author's calculations based upon data from the U.S. Department of Commerce, Bureau of Economic Analysis, 1961–2003, *National Income and Product Accounts*.

the random trend component ε_{tr} , random cycle component ε_{ct} , and random drift component ε_{ur} are uncorrelated. In this case, I test for significant change in the underlying trend growth rate by testing whether the average time-varying drift μ_t over the new economy era is significantly higher than the average time-varying drift over the preceding productivity slowdown era.

Figure 7 plots the unit-root trend growth rate for the Watson model (dark grey line) against its discrete-jump (light green line) and constant trend growth rate (dark green line) counterparts. This figure suggests that after a dramatic fall in the trend growth rate over the pre-productivity slowdown era, the trend growth rate held steady at about 3.12 percent over the latter part of the productivity slowdown era and into the new economy era. A comparison of the average growth rates over these periods indicates that there has not been a statistically significant variation in the average trend growth rate over the latter part of the estimation period (see point estimates in table 1). Hence, there is no evidence of the new economy in the GDP data.

The point estimates of the trend and cyclical component parameters of the unit-root model are slightly different to the model with a constant drift. This is echoed in figure 6 by the similarity of the unit-root (dark grey line) and the constant drift (dark green line) trend and cyclical components, especially over the productivity slowdown and new economy eras. The only noticeable difference occurs in the early part of the

sample, which reflects the relatively high trend growth rate estimates over the 1960s.

If the underlying trend growth rates did not change, what then explains the high observed GDP growth rates in the late 1990s? Figure 8 suggests that the high GDP growth rates of the late 1990s were the result of a level shift in the trend component, which was driven by a sequence of relatively large positive random fluctuations that had a permanent effect on the level of the trend, but no permanent effect on the trend growth rate.

Multivariate unobserved component models of GDP

One of the drawbacks of Watson's model is that despite carrying the label of a structural economic model, it is in fact atheoretical in that it embodies no behavioral economic relationships. Various authors have attempted to add behavioral content to Watson's model by employing multivariate UC models that link the un-

observed trend and cyclical components not only to observed log GDP, but also to observed price inflation π_t . This is typically done by adding a so-called Phillips curve to Watson's univariate model, which provides a link between changes in the level of price inflation and changes in the cyclical component of GDP.

For example, Gerlach and Smets (1999) (hereafter GS) add the following Phillips curve:

$$12) \Delta\pi_t = \alpha_1\Delta\pi_{t-1} + \alpha_2\Delta\pi_{t-2} + \alpha_3\Delta\pi_{t-3} + \gamma c_{t-1} + \varepsilon_{\pi t}$$

Their relationship allows for rich dynamics in the evolution of changes in the rate of inflation ($\Delta\pi_t = \pi_t - \pi_{t-1}$) through the autoregressive coefficients ($\alpha_1, \alpha_2, \alpha_3$). In general, γ is assumed to be positive, so that a widening gap between actual GDP and the trend, captured by the cyclical component c_t , leads to higher price inflation.

GS also modify the specification of the cyclical component of the model by incorporating information on the real federal funds rate (difference between the level of the Federal Reserve's target interest rate and the average level of price inflation):

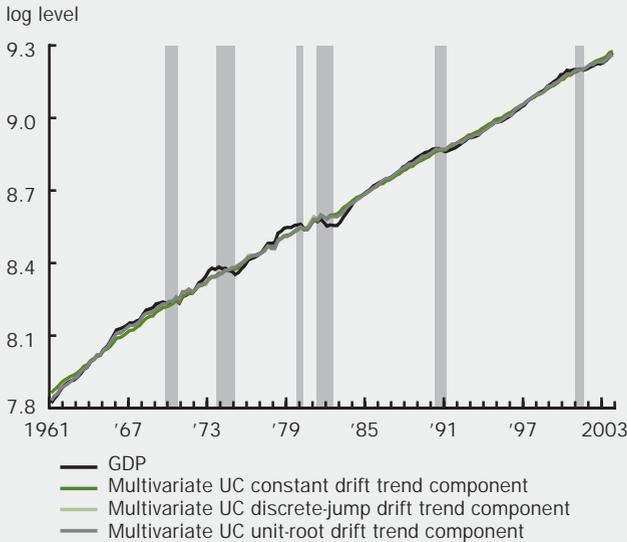
$$13) c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + \lambda(r_{t-1} - \bar{\pi}_{t-1}) + \varepsilon_{ct}$$

where r_t denotes the nominal U.S. federal funds rate and $\bar{\pi}_t$ denotes the average level of price inflation over the previous four quarters. GS argue that this

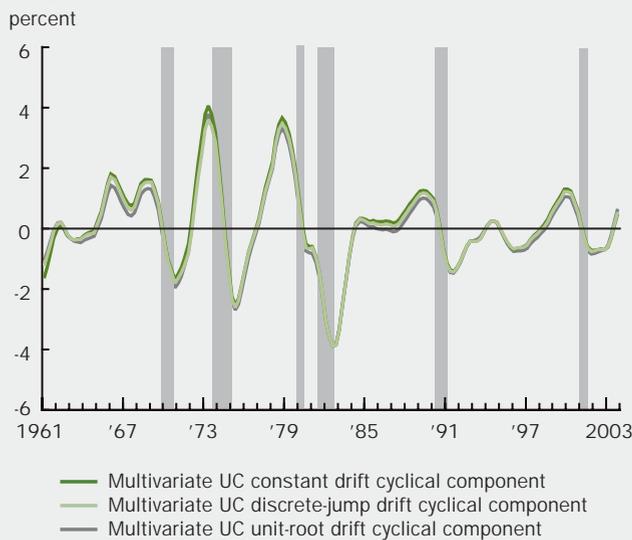
FIGURE 10

Multivariate unobserved component models

A. Trend components



B. Cyclical components



Notes: Data cover the period from 1961:Q1 to 2003:Q4. Shaded areas indicate National Bureau of Economic Research recession dates.
Source: Author's calculations based upon data from the U.S. Department of Commerce, Bureau of Economic Analysis, 1961–2003, *National Income and Product Accounts*.

modified cyclical equation is essentially a reduced-form aggregate demand function with λ assumed to be negative, so that a rising real interest rate decreases aggregate demand. The other equations in the GS model are the same as the constant and time-varying drift UC models described above.

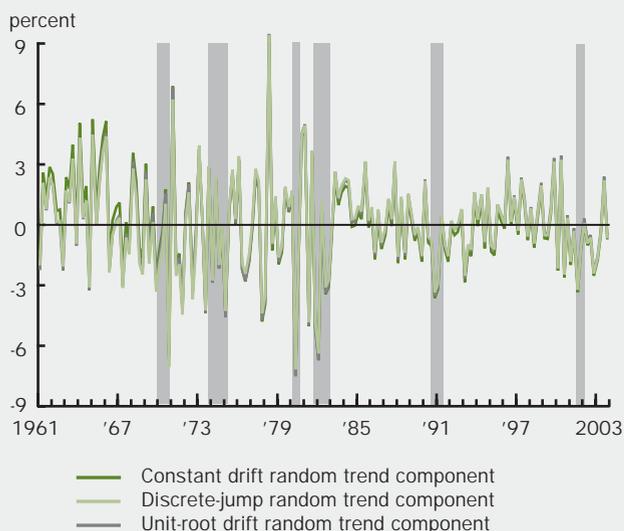
One of the key observations driving speculation that the U.S. had experienced a significant increase in its trend growth rate of GDP was that inflation was considered to be relatively low for such a rapidly expanding economy. To understand this argument, we must examine the relationship between inflation and trend growth embodied in the Phillips curve. An increase in the trend growth rate of GDP would, other things being equal, raise the level of the trend component and lower the level of the cyclical component, which would in turn imply a lower rate of increase in price inflation through the Phillips curve relationship. In light of this, GS's model is particularly well suited to exploring the existence of the new economy, since it allows inflation also to affect the measurement of the trend component.

Moving onto the trend growth rate estimates, table 1 shows that the estimated multivariate UC constant drift model has an underlying trend growth rate of 3.32 percent, which is marginally higher than the estimated constant trend growth rate from the univariate UC model. The multivariate UC discrete-jump model parameters, on the other hand, suggest that while there was significant change in the trend growth rate in the earlier part of the sample, there was not a statistically significant increase in the trend growth rate of GDP in the 1990s. A similar picture emerges from the multivariate UC unit-root drift estimates reported in figure 9 (dark gray line). Based on these results, the trend growth rate changed little from the end of the productivity slowdown era through to the new economy period. In fact, the unit-root drift estimates suggest that after rising slightly in the late 1990s, the trend growth rate actually fell below the levels recorded in the productivity slowdown period.

Parameters governing the cyclical components are virtually identical across all three multivariate UC models. This finding is echoed by the similarity of the multivariate UC cyclical components under the three trend growth rate assumptions plotted in figure 10. These cyclical components suggest that, other things being equal, if the Fed had relied on

FIGURE 11

Multivariate unobserved random trend components



Notes: Data cover the period from 1961:Q1 to 2003:Q4. Shaded areas indicate National Bureau of Economic Research recession dates.
Source: Author's calculations based upon data from the U.S. Department of Commerce, Bureau of Economic Analysis, 1961–2003, *National Income and Product Accounts*.

these multivariate UC models to estimate the cyclical component of GDP, its monetary policy response would have been invariant to its trend growth rate assumption over the late 1990s.

Overall, these multivariate UC models estimates provide no evidence in favor of the new economy, so what was the factor underlying the strong growth of

GDP? Turning to estimates of the multivariate UC random trend components in figure 11, we see even stronger evidence than in the univariate UC model case that there is a clustering of positive random fluctuations to the level of the trend component of GDP in the multivariate model in the late 1990s. Again, this suggests that the high GDP growth rates of the late 1990s were the result of a level shift in the trend component, which was driven by a sequence of relatively large, positive random fluctuations that had a permanent effect on the level of the trend, but no permanent effect on the trend growth rate.

Conclusion

New economy advocates argue that the high productivity growth rates of the second half of the 1990s ushered in a permanent increase in the trend growth rate of U.S. GDP. I test formally whether there was a statistically significant change in the trend growth rate of GDP over the late 1990s. Using a number of widely used approaches to estimate the trend component of GDP, I find that there was variation of the trend growth rate of GDP over the latter half of the 1990s, but it was not statistically different from zero. I conclude that, at least in terms of GDP data, the U.S. was the same *old economy* in the late 1990s.

NOTES

¹See Taylor (1993) for details.

²Details of data sources and dates used for estimation are reported in appendix A.

³I estimate the time-varying A and μ using standard linear spline and knot regression techniques, which restrict the estimated trend to be a continuous line; see Greene (1990) pp. 248–251 for details.

⁴At 2.94, my estimate of the productivity slowdown growth rate is slightly higher than the typical trend growth rate based on productivity growth rate estimates. As Seskin (1999) shows, this upward revision to the trend growth rate can be explained by the shift to current chain-weighted data, which also incorporates revisions to the measurement of business fixed investment that raised the average growth rate of GDP over the entire sample by roughly 0.3 percentage points.

⁵Orphanides and van Norden (2002) highlight problems in measuring the cyclical component of GDP in real time. The methods used in this article are subject to their critique of real-time estimates

of the cycle. However, my main objective here is not to estimate a real-time cycle, but to document whether the trend growth rate of GDP changes over the latter half of the 1990s using the best available data, so their criticism is not relevant for the trend growth rate results presented here.

⁶Trend properties of the data used in this article are reported in appendix A. In particular, table A1 reports augmented Dickey–Fuller tests for log GDP. According to these tests, log GDP has a stochastic trend.

⁷Details of the techniques used to estimate the UC models are reported in appendix B. I use the following conventions when reporting parameter estimates or plotting unobserved components: The cycle is expressed as percentage deviations from the trend, while the underlying growth rate of the trend is expressed as annualized percentage rates. Plots of the unobserved cycle and trend components refer to the two-sided estimates generated by the Kalman smoother. Appendix C reports estimates of the other parameters of the unobserved component models.

APPENDIX A: TREND PROPERTIES OF GDP AND INFLATION DATA

An important assumption in Watson’s (1986) and Gerlach and Smets’ (1999) models is that the log of GDP has a unit root. Gerlach and Smets’ model goes one step further in assuming that price inflation also has a unit root. This section reports the results of augmented Dickey–Fuller (ADF) tests for nonstationarity using quarterly U.S. real chain-weighted gross domestic product (GDP) and consumer price index (CPI) data from 1961:Q1 to 2003:Q4. Note, the quarterly CPI data are calculated as averages of the three months in the quarter.

The left-hand side of table A1, panel A reports ADF t-statistics for cases with a constant and time trend, using various lags of the change in the log of GDP ($\Delta x_t = x_t - x_{t-1}$). These test statistics do not allow me to reject the null of a unit root in the log of GDP at typical levels of significance.

A potential time-varying model of the trend growth rate of GDP is a unit root without drift. A time-varying trend growth rate with a unit root would require a unit root in the change in the log of GDP Δx_t . The right-hand side of table A1, panel A reports ADF t-statistics for the change in the log of GDP. I am able to reject the null of a unit root in the growth rate of GDP at conventional levels of significance when a constant is included in the regression. This implies that the trend growth rate of real GDP is a stationary process. However, Stock and Watson (1998) argue that if the variance of the innovation to the trend growth rate is small, the growth rate of GDP will have a nearly unit moving average (MA) root. It is well-known that unit-root tests have a high rejection rate in the presence of large MA roots, which means that the reported ADF statistics are consistent with a model that has a unit root in the trend growth rate with a small innovation variance.

TABLE A1		
Unit-root tests		
A: Real GDP		
Lags	$x_t = \log(\text{GDP}_t)$ Constant, trend	$\Delta x_t = x_t - x_{t-1}$ Constant
2	-2.77	-7.41
4	-2.38	-7.09
8	-2.02	-5.05
12	-2.10	-4.69
B: CPI inflation		
Lags	$\pi_t = \log(\text{CPI}_t) - \log(\text{CPI}_{t-1})$ Constant, trend	$\Delta \pi_t = \pi_t - \pi_{t-1}$ Constant
2	-2.51	-9.47
4	-2.73	-7.36
8	-2.70	-6.06
12	-1.90	-5.36
Source: Author's calculations based on GDP and CPI data from 1961:Q1 to 2003:Q4.		

Panel B of table A1 repeats these experiments for inflation π_t , measured as the change in the log of the CPI. The left-hand side suggests that at conventional levels of significance, I cannot reject the null of a unit root in inflation. The right-hand side suggests that I can reject the null of a unit root in the change in the level of inflation ($\Delta \pi_t = \pi_t - \pi_{t-1}$) at conventional levels of significance. This implies that $\Delta \pi_t$ is a stationary process.

APPENDIX B: ECONOMETRIC ISSUES

I estimate all models using maximum likelihood. In each case the likelihood function is evaluated using the Kalman filter on the model's state space representation. I simplify the estimation by transforming the models so that they are specified in first differences rather than levels of real GDP. For example, I estimate the univariate model with constant drift or drift with discrete jumps using the following structure:

$$\Delta x_t = \mu_{i,j} + \Delta c_t + \varepsilon_{\pi} \text{ for } i \leq t \leq j$$

$$c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + \varepsilon_{ct}$$

The advantages of this data transformation are twofold. First, the computation costs are lower because the state vector ξ_t is reduced to current and lagged values of the GDP gap:

$$\xi_t = [c_t \quad c_{t-1}]'$$

Second, these components are assumed to be stationary, so I can use the exact likelihood to estimate the model by specifying the initial values of the state vector ξ_0 and the initial covariance matrix of the associated estimation error P_0 in terms of the population moments of the state vector:

$$\xi_0 = 0 \text{ and } P_0 = \sigma_c^{*2} \begin{bmatrix} 1 & \phi_1/(1-\phi_2) \\ \phi_1/(1-\phi_2) & 1 \end{bmatrix},$$

where $\sigma_c^{*2} = \frac{\sigma_c^2}{1-\phi_1^2-\phi_2^2-2\phi_1\phi_2/(1-\phi_2)}$. This avoids the many problems associated with estimating models in levels,

such as the unobserved component estimates depending critically on initial values.

The set up of the unit-root models is more complicated. The state vector is expanded to include current and lagged values of the growth rate:

$$\xi_t = [c_t \quad c_{t-1} \quad \mu_t \quad \mu_{t-1}]'$$

and I must use the conditional likelihood to estimate the model's parameters. I follow Harvey (1993) by explicitly using Δx_0 as an estimator of μ_0 and noting that the variance of the associated estimation error is $E[(\Delta x_0 - \mu_0)^2] = E[(\Delta c_t + \varepsilon_{\pi})^2]$, which implies the following initial state vector and associated estimation error covariance matrix:

$$\xi_t = [0 \quad 0 \quad \Delta x_0 \quad \Delta x_{-1}]'$$

$$P_0 = \sigma_c^{*2} \begin{bmatrix} 1 & \frac{\phi_1}{(1-\phi_2)} & \left(1 - \frac{\phi_1}{(1-\phi_2)}\right) & \left(\frac{2\phi_1 - \phi_1^2 - \phi_2}{(1-\phi_2)}\right) \\ \frac{\phi_1}{(1-\phi_2)} & 1 & \left(\frac{\phi_1}{(1-\phi_2)} - 1\right) & \left(1 - \frac{\phi_1}{(1-\phi_2)}\right) \\ \left(1 - \frac{\phi_1}{(1-\phi_2)}\right) & \left(\frac{\phi_1}{(1-\phi_2)} - 1\right) & 2\left(1 - \frac{\phi_1}{(1-\phi_2)}\right) + \frac{\sigma_{\varepsilon}^2}{\sigma_c^{*2}} & \left(\left(\frac{2\phi_1 - \phi_1^2 - \phi_2}{(1-\phi_2)}\right) - 1\right) \\ \left(\frac{2\phi_1 - \phi_1^2 - \phi_2}{1-\phi_2}\right) & \left(1 - \frac{\phi_1}{(1-\phi_2)}\right) & \left(\left(\frac{2\phi_1 - \phi_1^2 - \phi_2}{(1-\phi_2)}\right) - 1\right) & 2\left(1 - \frac{\phi_1}{(1-\phi_2)}\right) + \frac{\sigma_{\varepsilon}^2}{\sigma_c^{*2}} \end{bmatrix}$$

Fortunately, the innovations to the inflation equation are stationary variables, so I can follow the same approach to initializing the multivariate models.

Table C1									
Univariate unobserved component model									
A: Constant drift									
Parameter estimates					Summary statistics				
μ	ϕ_1	ϕ_2	σ_τ	σ_c	SE	Q(16)	LLF		
3.30 (0.24)	1.68 (0.13)	-0.72 (0.14)	2.56 (0.24)	0.38 (0.12)	0.81	20.49	-207.33		
B: Discrete jump									
Parameter estimates					Summary statistics				
μ	ϕ_1	ϕ_2	ϕ_τ	ϕ_c	SE	Q(16)	LLF		
μ 6101:7204 $-\mu$ 7301:9504	μ 96Q1:0304 $-\mu$ 7301:9504				0.80	21.82	-204.60		
2.92 (0.28)	1.28 (0.55)	0.37 (0.90)	2.55 (0.25)	0.37 (0.12)					
C: Unit-root drift									
Parameter estimates					Summary statistics				
ϕ_1	ϕ_2	σ_τ	σ_μ	SE	Q(16)	LLF			
1.68 (0.13)	-0.74 (0.13)	2.59 (0.23)	0.08 (0.06)	0.81	20.35	-208.31			

Notes: Standard errors in parentheses. SE denotes equation standard error. Q(n) is the Box-Ljung test for randomness of the errors distributed χ^n . LLF denotes the log of the likelihood function.
Source: Author's calculations based on GDP data from 1961:Q1 to 2003:Q4.

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