A stable money demand: Looking for the right monetary aggregate

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Introduction and summary

The stability of a money demand relationship has been a major concern in monetary economics for the last 50 years. It is conventional to call the relationship between real money, a nominal interest rate, and a measure of economic activity a money demand relationship. A stable relationship between these variables helps answer important questions such as the following: What is the average growth rate of money that is consistent with price stability, given the average growth of the economy and a stable nominal interest rate? Knowledge about the response of money demand to changes in the nominal interest rate may also help quantify the welfare gains from a low average inflation rate.

In an essay in honor of Allan Meltzer, Lucas (1988) reassesses the evidence on the stability of the money demand estimated by Meltzer (1963) and justifies that stability not only on empirical grounds but also on theoretical ones. He shows that there is a theoretical equilibrium relationship between real money, a nominal interest rate as a measure of the opportunity cost of money, and gross domestic product (GDP) as a measure of transactions that is not exactly a money demand, but that is indeed stable. He estimates that equilibrium relationship using the monetary aggregate M1 as the measure of money with data up to 1985 and argues that there is a stable relationship between those variables with a unitary income elasticity and with a strong negative response of real balances to the nominal interest rate (see box 1 for definitions of the different monetary aggregates).

The relationship estimated by Lucas (1988) holds very well until the mid-1980s but not well at all after that. This could be because the demand for money is not a stable relationship after all, contrary to what the simple model would suggest. Another conclusion, which is our view, is that the measure of money is not a stable measure. In particular, we argue that technological innovation and changes in regulatory practices in the past two decades have made other monetary aggregates as liquid as M1, so that the measure of money should be adjusted accordingly. We show that once a more appropriate measure of money is taken into consideration, the stability of money demand is recovered.

Banking deregulation in the 1980s and 1990s and financial innovation in the 1990s associated with the development of electronic payments indeed suggest we need to reconsider the measure of transactions demand for money. Until the end of the 1970s, the transactions demand for money was well approximated by M1. Since then, however, a series of sweeping regulatory reforms and technological developments in the banking sector have significantly changed the way banks operate and the way people use banking services and conduct transactions. First, the Depository Institutions Deregulation and Monetary Control Act of 1980 abolished most of the interest rate ceilings that had been imposed on deposit accounts since the Banking Act of 1933 and authorized nationwide negotiable orders of withdrawal accounts (NOWs), which are interest-bearing checking accounts classified in M1. Furthermore, the Garn–St Germain Depository Institutions Act of 1982 authorized money market deposit accounts (MMDAs), interest-bearing savings accounts that can be used for transactions with some restrictions. MMDAs are classified in M2.
These two major banking reforms blurred the traditional distinction between the monetary aggregates M1 and M2 in their transactions and savings roles. Second, the rapid development of electronic payments technology and, in particular, the growing use of credit cards and the automated clearinghouse (ACH) as means of payment, reinforced the effect of the banking reforms in slowing down the growth of M1. Both credit cards and ACH transactions can be settled with MMDAs and, therefore, with M2 rather than M1. Third, the widespread adoption of retail sweep programs (discussed in detail later) by depository institutions since 1994, which reclassify checking account deposits as saving deposits overnight, reduced the balances that were classified in M1 by almost half.

These fundamental changes in the regulatory environment and the transactions technology justify the use of a different measure of money after 1980. The measure MZM (money zero maturity) includes balances that can be used for transactions immediately at zero cost and was initially proposed by Motley (1988) and Poole (1991) as a more appropriate measure of the transactions demand for money (see box 1). We show that changing the monetary aggregate measure from M1 to MZM from 1980 onward preserves the long-run relationship between real money, the opportunity cost of money, and economic activity up to a constant factor.

In the next section, we show evidence of the difficulty in explaining the behavior of M1 with the behavior of GDP and the nominal interest rate. Then, we discuss why MZM, rather than M1, is an appropriate measure of the transactions demand for money in the past two decades. Finally, we estimate a money demand equation derived from a simple transaction technology model, using M1 as the monetary aggregate before 1980 and MZM after 1980 and obtain evidence in support of the stability of money demand.

### An unstable demand for M1

Figures 1 and 2 reproduce figures 1 and 4 in Lucas (2000), extending the data through 2003.1 Figure 1 suggests that, over the course of the past century, movements in the ratio of M1 to nominal GDP have been inversely related to movements in the short-term nominal interest rate. Following Meltzer (1963), Lucas (1988) uses data up to 1985 to estimate a money demand equation, using M1 as the measure of money and a short-term nominal interest rate as the measure for the opportunity cost of money, and confirms Meltzer’s result that the income elasticity is about 1.0 and the interest elasticity is high.2 Lucas (1988) reports an interest rate semi-elasticity between 0.05 and 0.1, which for an interest rate of 4 percent corresponds to an interest elasticity between 0.2 and 0.4. Using data from 1900 through 1994, Lucas (2000) reports an interest elasticity of 0.5, consistent with a shopping time3 model for money demand. The money demand equation derived in Lucas (2000) is

$$\frac{M_t}{P_t} = \alpha Y_t i_t^{-\gamma},$$

where $M_t$ is the monetary aggregate measured by M1, $P_t$ is the price level, $Y_t$ denotes the aggregate output level, $i_t$ is the short-term nominal interest rate, and the interest elasticity is $\gamma = 0.5$, while $\alpha$ is a constant term. Real money responds to output with a unitary elasticity and negatively to the nominal interest rate with a relatively large elasticity, so that in response to a 1 percent increase in the nominal interest rate,
the real money demand declines by 0.5 percent. Output is a measure of transactions, and people demand more money when the volume of transactions is higher. The unitary income elasticity is consistent with real money growing at the same average rate as output. The negative response of the demand for money to the nominal interest rate makes sense because the short-term nominal interest rate is the foregone return from holding non-interest-bearing, but liquid, money balances.

Figure 2 plots the actual and estimated real balances using the money demand equation above with an interest elasticity $\gamma = 0.5$. Clearly, one would expect a larger reaction of real balances to the lower interest rates in the 1980s and 1990s. A lower elasticity of 0.32, instead of 0.5, would still not get close to being consistent with the actual low growth in M1. This is apparent from Figure 3, where we plot the logarithm of the ratio of M1 to nominal GDP against the logarithm of the nominal interest rate for the period 1900–2003. Figure 3 indicates that there could be a different money demand relationship for each of the three periods 1900–79, 1980–94, and 1995–2003. The solid line corresponds to the estimated elasticity of 0.32 for the entire 1900–2003 period. The interest elasticity for the three subperiods would be 0.26, 0.12, and −0.07, respectively, so that the response to the interest rate movements over time would be less and less pronounced. The constant term also changes across the three periods, corresponding to the increased inability to explain the low growth in M1 with movements in economic activity and the nominal interest rate.

Ball (2001) argues that the data after 1987 represent evidence against a stable money demand. He estimates a linear relationship between the logarithm of real money, the logarithm of output, and a nominal interest rate for subperiods of 1903–94. For the period 1903–87 the evidence is consistent with a stable relationship with a unitary income elasticity and a relatively high interest elasticity, as shown by Lucas (1988) and Stock and Watson (1993). However, the need to account for the low reaction of M1 to lower interest rates and higher output after 1980 lowers both the estimated interest elasticity and income elasticity. The relatively low income and interest elasticity in the postwar period (1947–94) are significantly different from the unitary income elasticity and relatively high interest elasticity in the prewar period (1903–45), leading Ball to argue against a stable long run money demand.4

![FIGURE 1](image1.png)

**FIGURE 1**
M1/nominal GDP and the nominal interest rate, 1900–2003

*Source: Authors’ calculations and Lucas (2000).*

![FIGURE 2](image2.png)

**FIGURE 2**
Actual and estimated real balances M1/P, 1900–2003 (interest elasticity of 0.5)

*Source: Authors’ calculations and Lucas (2000).*
Measuring money used for transactions

In this section, we argue that M1 was a good measure of money used for transactions before major developments in banking regulation and financial innovation starting in the early 1980s. Since then, a measure such as MZM has become more appropriate. Figure 4 (p. 54) shows the trend growth of all four monetary aggregates: M1, M2, M3, and MZM since 1959 (data for MZM are available since 1974). In particular, since 1980, M1 has grown at a low rate (5.1 percent) and flattened after 1994. In contrast, average MZM growth has been 9 percent since 1980. The rapid expansion in MZM is evident in the figure; its value surpassed that of M2 in 2001.

Before 1980, M1, consisting of currency, non-interest-bearing demand deposits, and a very small amount of interest-bearing checkable deposits (see figure 5 (p. 54) and discussion in the next section) was the primary transaction monetary aggregate. The main components of M2, other than M1, were savings deposits, mostly passbook savings accounts on which checks could not be written, and small time deposits. Neither could be directly used for transactions. The other component of M2, retail money market mutual funds (MMMFs), a nonbank financial instrument (some have restricted check-writing capacity) developed in the mid-1970s and remained very small, as shown in figure 5. Therefore, there was a clear distinction between M1 and the components of M2 other than M1 before 1980. The former could be used for transactions at zero cost and did not bear interest, while the latter were interest-bearing instruments that could not be directly used for transactions. Since then, this distinction has become

![Figure 3: M1/nominal GDP and nominal interest rate, 1900–2003](image-url)

Notes: Estimated 1900–2003: $\ln \left( \frac{M_1}{PY} \right) = -2.46 - 0.32 \ln i$. Estimated 1900–79: $\ln \left( \frac{M_1}{PY} \right) = -2.12 - 0.26 \ln i$. Estimated 1980–94: $\ln \left( \frac{M_1}{PY} \right) = -2.19 - 0.12 \ln i$. Estimated 1995–2003: $\ln \left( \frac{M_1}{PY} \right) = -1.85 + 0.07 \ln i$. 

Federal Reserve Bank of Chicago
less clear-cut. Three major developments in banking regulation and financial innovation are responsible for the change.

Financial innovation and regulatory reform since 1980

Banking deregulation

The banking deregulation that ensued in the late 1970s and early 1980s changed the banking industry landscape from a highly regulated one into a fairly competitive one. An unavoidable consequence of the deregulation was the blurring of the various components of M1 and M2 as transaction/saving instruments.

The reform started in the 1970s when many commercial banks and depository institutions were struggling to survive in the high inflation, high interest rate environment, with their hands tied by many regulations, in particular, Federal Reserve Regulation Q. This regulation prohibited interest payment on demand deposits and imposed interest-rate ceilings on time deposits and savings deposits. The first move toward deregulation was the authorization granted by several northeastern states to state-chartered mutual saving banks, and later other depository institutions, to offer NOWs, an interest-bearing transaction account. Other products or services designed to provide consumers with more efficient cash management tools developed at the same time. For example, commercial banks and thrifts were able to provide prearranged automatic transfer services (ATS) from consumers’ savings accounts to their checking accounts, customers could transfer their savings balances to checking remotely, and federally chartered credit unions were allowed to issue share drafts. These innovations were officially sanctioned by the Depository Institutions Deregulation and Monetary Control Act in 1980. More specifically, the act eliminated most of the interest rate ceilings on time deposits and savings accounts and authorized the use of checkable NOW accounts and other interest-bearing accounts (such as ATS and share draft accounts at credit unions) by individuals and non-profit organizations. The privilege was extended to all levels of government agencies in 1982. The only exception is demand deposits of corporations, on which the 1933 prohibition of interest payment remains in effect today. These regulatory changes allowed depository institutions to compete more effectively for funds; they also removed the impediments for depositors to earn the market rate of return on their transaction balances. The direct consequence of the act is the prevalent use of interest-bearing checking accounts.

A second major regulatory banking reform was the Garn–St Germain Depository Institutions Act of 1982. It authorized the creation of money market deposit accounts (MDAs) to compete with MMMFs. Classified as an M2 account, an MMDA
is an interest-bearing account that carries no reserve requirements. The account offers limited transaction capacity: no more than six withdrawals by check or pre-authorized transfer per month, but no limit on deposits or number of withdrawals from an ATM, by mail, or at a branch. This act led to a substantial increase in the use of checkable savings accounts for transactions.

The deregulatory measures of the early 1980s, allowing for interest payments on checking accounts and checking privileges on savings accounts, blurred the distinction between transaction and saving deposits, consequently blurring the distinction between M1 and M2.

Electronics payments

Following the banking deregulation in the 1980s, the rapid development of electronic payments in the 1990s also fostered the use of components of broader monetary aggregates for transaction purposes. Credit cards are particularly responsible for this. Credit cards are often used as a substitute for cash, check, and debit card transactions. Monthly balances on a credit card can be paid with an automated clearing house (ACH) transaction or a check written on a checking account or checkable savings account. The fact that there is a single payment at a certain date reduces the need to maintain high daily balances in checking accounts to meet the uncertain sequence of transaction and payment flows. This reduction is reinforced by the fact that it is possible to use checkable savings accounts to pay for credit card balances. The total number of credit and debit card transactions almost tripled in 1990s, from 10.8 billion in 1990 to 30 billion in 2000 (Humphrey, 2002).

The ACH is another important development in electronic payments. ACH is a nationwide mechanism that processes electronically originated batches of credit and debit transfers. ACH credit transfers include direct deposit payroll payments and payments to contractors and vendors. ACH debit transfers include consumer payments on insurance premiums, mortgage loans, and other kinds of bills. This form of electronic bill payment is a substitute for checks. A share of these transactions is from checkable savings accounts, classified in M2, instead of from checking accounts. The Federal Reserve Banks operate the nation’s largest ACH operation, which in 2000 processed more than 80 percent of commercial interbank ACH transactions.

In 1991, the Federal Reserve processed 1,119 million commercial (not including government) ACH transactions (valued at $5,549 billion), while in 2003 the number jumped to 5,588 million transactions ($13,952 billion), an annual increase of 14.3 percent in volume (8 percent in value).

Retail sweep programs

A third important development leading to the confounding roles of M1 and M2 for transactions and savings was the adoption of retail sweep programs that reclassify checking account deposits as savings deposits overnight. Since 1994, commercial banks have started using deposit-sweeping software to dynamically reclassify the balances in checking accounts above a certain level as MMDAs and to reclassify them back when the balances on the checking accounts are too low. By adopting the practice, depository institutions avoid reserve requirements on the reclassified portion of the checking account (the reserve requirement on demand deposits, ATS, NOW, and other checkable deposits can be as high as 10 percent, depending on the size of the institution). The software effectively creates a shadow MMDA for every checking account, based on the customer’s payment patterns, subject to the constraint that the number of “transfers” (reclassifications) from an MMDA to a checking account does not exceed six each month. The shadow account is included in M2, but not in M1.

More and more banks are adopting the retail sweep programs. As indicated by figure 6, the total amount of sweeps of transaction deposits into MMDAs has been rising steadily since 1994, from zero to an
amount nearly equal to transaction deposits in M1. According to the Federal Reserve Board’s estimates, as of December 2003, the sweeps of transaction deposits into MMDAs were approximately $575.5 billion, while total transaction deposits in published M1 were $621.3 billion. The widespread use of retail sweep programs substantially affected the growth of M1. The nominal value of M1 has been almost flat since 1994.

**MZM as a better measure of transaction balances since 1980**

As a result of the financial innovations and regulatory reforms since 1980, components of the “transactions” aggregate M1 bear interest, and components of the “savings” aggregate M2 are used for transactions. These changes call for a reconsideration of the measure of transactions demand for money and its opportunity cost. More specifically, if there is to be a stable, long-run relationship between real money, its opportunity cost, and transactions, a different measure of money and its opportunity cost may be necessary to sustain the relationship.

Motley (1988) and Poole (1991) argue that the present classification of monetary aggregates (M1, M2, M3) is inherently arbitrary, in particular in light of the banking industry developments discussed above. They believe that the important distinction should be whether the deposit has a specified term to maturity. For example, NOW accounts in M1 and MMDAs in M2 are nonterm deposits, but small and large denomination time deposits in M2 and M3 are term assets. Nonterm deposits can be readily converted into transaction balances, or in other words, are fully liquid. On the other hand, term deposits that have to be liquidated before maturity incur the cost of an early withdrawal penalty. In an environment free of government regulation, and within the limits of technology...
constraints, agents’ portfolio decisions depend on their liquidity preferences and the return on the assets. The term/nonterm distinction of monetary aggregates is aligned with private agents’ incentives.

Motley proposed classifying all non-term deposits, money that can be accessed without notice and at par, as a new monetary aggregate. Poole coined the name MZM (money zero maturity) for the measure. Specifically, MZM is defined as

\[ MZM = M_2 - \text{Small denomination time deposits} + \text{Institutional MMMFs} \]

Institutional MMMFs, currently classified in M3, are interest-bearing checkable accounts that allow holders to get around the zero-interest demand deposits restriction.

The demand for money

In appendix 1, we show that it is possible to derive from a simple stochastic general equilibrium monetary model the equilibrium relationship

\[ \frac{M}{P} = \alpha Y \left( i - i^m \right)^{-\nu} \]

which is a variant of equation 1 that accounts for the fact that money may earn interest. This is an exact equilibrium relationship of observable economic variables. As pointed out in Lucas (2000), this is reason to think that the empirical analog to that relationship, which will have to account for measurement error, is a stable relationship. The equilibrium relationship in equation 2 is not exactly a money demand function, computed from the decision by households on how much money to hold, given economic variables out of their control, namely the prices of goods and assets and endowments. It does, however, look like the money demand functions that are commonly estimated.

In this section, we estimate the empirical counterpart of the money demand equation above using ordinary least squares (OLS). First, like Lucas (1988, 2000), we use M1 as the measure of money and a short-term nominal interest rate as its opportunity cost. As mentioned before, the estimated interest elasticity is 0.32, lower than the 0.5 reported by Lucas (2000) for the period 1900–94. If we estimate the elasticity for three subperiods, 1900–79, 1980–94, and 1995–2003, the interest elasticities are lower, 0.26, 0.12, –0.07, respectively. It would also be apparent that the curves would be shifted down.

Next, we estimate equation 2 using M1 as the measure of money for the period 1900–79 and MZM for the period 1980–2003. Because components of M1 bore no interest before 1980 (mostly cash and demand deposits) and components of MZM are interest-bearing after 1980 (NOWs, MMDAs, MMMFs), we assume that \( i^m = 0 \) before 1980 and we set \( i^m \) to MZM’s own rate after 1980. MZM’s own rate is a weighted average of the returns on the different components of MZM. We allow different intercepts for the two periods, because it is not reasonable to impose the coincidence of the two series, M1 and MZM, in 1980, but we do impose a common interest elasticity. The estimated money demand equation is as follows,

\[
\begin{align*}
1900–79: \ln \left( \frac{\bar{M}}{PY} \right) &= -2.07 - 0.24 \ln i; \\
1980–2003: \ln \left( \frac{\bar{MZM}}{PY} \right) &= -1.8 - 0.24 \ln \left( i - i^m \right).
\end{align*}
\]

If we allowed for separate interest elasticity for the two periods in the regression, the two elasticities would be 0.26 and 0.2, respectively, for the first and second periods.\(^\text{10}\)
Figure 7 plots the logarithm of M1/nominal GDP for the period 1900–79 and that of MZM/nominal GDP for the subsequent period 1980–2003 against the logarithm of the opportunity cost of using these balances, along with the linear regression lines. The roughly common elasticity across the two periods suggests that the response of the money aggregate to changes in its opportunity cost, in percentage terms, has remained stable over the last century, as long as one uses the appropriate definition of monetary aggregate and its opportunity cost. The upward shift of the function (smaller intercept) reflects the fact that MZM and M1 include different liquid assets, even if all are zero maturity. In figure 8, we plot the actual and estimated real money demand using M1 for the period 1900–79 and MZM for the period following the deregulation and financial innovation.

Conclusion

While real M1 has increased very little in the last quarter century, nominal interest rates have come down considerably. If the interest elasticity were the one reported by Lucas (2000), we would expect a substantial increase in M1 that did not occur. This could indicate that the money demand relationship estimated by Meltzer (1963) and Lucas (1988), among many others, is not a stable equilibrium relationship. Instead, we argue that M1 is not the appropriate measure of money, following the regulatory reforms and innovation in electronic payments since the early 1980s. If we use an alternative, more appropriate measure of money, that is, MZM or money zero maturity, the long-run relationship between money and its opportunity cost is preserved. We estimate the interest elasticity to be 0.24, so that a 1 percent increase in the opportunity cost of holding money induces a 0.24 percent decline in real money balances.

Why do we care about estimating a stable money demand at the cost of an unstable measure of money? In addition to the theoretical interest of this issue, there is also a practical aspect to it. It is a worthy objective of a monetary authority to provide elastic11 liquidity at stable prices. A stable estimate of money demand, whatever the appropriate monetary aggregate might be, is an important tool in performing this task.
To be able to make comparisons, we use the same data as Lucas (2000) for relevant data analysis and figures. In particular, M1, real GDP, the price deflator, and the nominal interest rate are constructed, as in Lucas (2000), from different data sources for different periods. See the appendix for a detailed description of the data used in this article.

The income and interest elasticities measure the percentage increase in real money in response, respectively, to a 1 percent increase in real GDP and a 1 percent decline in the nominal interest rate. The semi-elasticity measures the percentage increase in real money induced by a decline in the interest rate of 100 basis points.

In a shopping time model, there is a transactions technology relating the volume of transactions to time and money used in performing those transactions.

The relatively low income elasticity is indistinguishable from a time trend in money demand.

The NOWs were first introduced in Massachusetts and New Hampshire in 1972, then Connecticut, Maine, Rhode Island, and Vermont in 1976, followed by New York in 1978. See Laporte (1979).

Business customers have several ways to minimize the loss of interest on demand deposits. One way is the sweep programs developed during the 1960s and 1970s that allow business demand deposits to be swept overnight into interest-bearing accounts such as repurchase agreements and money market mutual funds.

The term checking account is used to mean demand deposits and other checkable accounts, such as NOW accounts, classified in M1. Checkable savings accounts are accounts classified in M2 that have checking privileges.

The Federal Reserve Board makes monthly estimates available on the nationwide change in NOW accounts attributable to the implementation of sweeps during the month. These are not the current amounts being swept, and no data are available regarding the aggregate volume of deposits currently affected by sweep programs. Depositories do not report to the Federal Reserve the size of their sweep programs.

The MZM data and the data on the rate of return on MZM are provided by the Federal Reserve Bank of St. Louis.

Our results are consistent with those of Carlson et al. (2000), who find a stable cointegrating relationship between real MZM, an opportunity cost measure, and a measure of economic activity, using data for the period 1976-98. The income elasticity is not different from one.

Elastic currency is the wording used in the 1913 Federal Reserve Act that established the Federal Reserve System.
APPENDIX 1: MONETARY MODEL

Here, we consider a simple transaction technology monetary model and derive an equilibrium relationship between real money, the opportunity cost of money, and output that holds exactly. That stable relationship justifies on theoretical grounds the stability of the empirical money demand equation estimated in this article.

The economy consists of an infinitely lived representative household/firm and a government. Production uses labor according to the linear technology

\[ Y_t = A_t n_t \]

where \( Y_t \) is output and \( n_t \) is time used for production. \( A_t \) is a stochastic technological parameter realized in the beginning of period \( t \). The history of these shocks up to period \( t \) (or state at \( t \)) is denoted by \( A_t \). The initial realization \( A_0 \) is given.

Households have preferences over consumption \( c_t \) described by the utility function:

\[
E_0 \sum_{t=0}^{\infty} \beta^{\frac{1}{1-\sigma}} - \frac{1}{1-\sigma},
\]

where \( \beta \) is a discount factor.

Households conduct transactions according to the Cobb–Douglas transaction technology

\[ c_t = \xi(A_t s_t)\left(\frac{M_t}{P_t}\right)^{1-\nu}, \]

where \( M_t \) is money balances, \( P_t \) is the price of the good in units of money, and \( s_t \) is the time used for transactions. The technology parameter is the same for the two technologies, production of the good, and transactions.

The total amount of time used for transactions and for the production of the good is normalized to one.

\[ s_t + n_t = 1. \]

The government issues money \( M^s_t \) and makes transfers to the households \( T_t \).

In the beginning of period \( t \), households enter an assets market where they purchase money balances \( M_t \) that pay net interest \( i_t^m \) in the following period, as well as nominal bonds \( B_t \) that pay interest \( i_t^m \) and \( Z_{t+1} \) units of state-contingent nominal securities, with price \( z_{t+1} \), normalized by the probability of occurrence of state \( A_{t+1} \), in units of currency at \( t \) that pay one unit of money at the beginning of period \( t+1 \) in a particular state \( A^t \). Subsequently, they enter a goods market where they purchase consumption with \( M_t \), according to the transaction technology in equation 4. They also receive total income \( P_t A_t(1-\nu) \), as well as nominal transfers, net of taxes, \( T_t \). The period by period budget constraints are

5) \[ M_t + B_t + E_{t+1} z_{t+1} Z_{t+1} \leq (1 + i_{t+1}^m) M_{t+1} - P_{t+1} c_{t+1} + (1 + i_{t+1}^m) B_{t+1} + Z_t + P_{t+1} A_{t+1}(1 - s_{t+1}) + T_t. \]

A competitive equilibrium is a set of prices and quantities such that a) households choose \( \{c_t, s_t, B_t, M_t, Z_{t+1}\}^{\infty}_{t=0} \) to maximize utility in equation 3 subject to the restrictions in equations 4 and 5 together with a no-Ponzi games condition on the holdings of assets, given \( \{P_t, i_t^m, z_{t+1}\}^{\infty}_{t=0} \) and \( \{T_t\}^{\infty}_{t=0} \);

b) the government satisfies

\[ M_t = M^s_t, \]

9) \[ c_t = A_t(1 - s_t). \]

We could derive a money demand equation using the first order conditions of the households’ problem. That equation, however, would be a function of all the prices, including the prices on the state contingent nominal debt, as well as unobservable shocks, and, therefore, could not be directly estimated using simple econometric methods. Instead, the first order conditions can be used to derive the following relationship

10) \[ \frac{m_t}{c_t} = \alpha \left( i_t^m - i_{t+1}^m \right)^{-\nu}, t \geq 0, \]

where \( m_t \) denotes real money balances, \( m_t = M_t/P_t \), and

\[ \alpha = \left( \frac{1-\nu}{\nu} \right)^{\frac{1}{1-\nu}}. \]

As pointed out by Lucas (1988), this equation is not exactly a money demand, rather it is an equilibrium relationship between real money, consumption, and the opportunity cost of holding money that holds exactly in this stochastic environment. Given
that in this simple model consumption coincides with output, $c_t = Y_t$, equation 10 can be rewritten as

$$11) \quad \frac{M_t}{P_t} = \alpha Y_t \left( i_t - i^m_t \right)^{-\nu},$$

with interest elasticity equal to the Cobb–Douglas transactions technology parameter $\nu$. Note that the derived income elasticity is one.

The assumptions on the homogeneity of the transaction technology and technology progress in the two sectors, as well as assumptions on the utility function, imply that the long-run income elasticity is one. Alternative assumptions could imply a trend in money demand. Empirically, this could be captured by a time trend or by an income elasticity different from one, as in Ball (2001). Instead, we argue that the evidence is consistent with a stable long-run money demand with a unitary income elasticity and no time trend, if the monetary aggregate is appropriately defined to capture the technological and regulatory innovations since 1980.

\[1\] Lucas (2000) reports the interest elasticity to be $\nu = 0.5$. He justifies this result by arguing that equation 10 with $\nu = \frac{1}{2}$ is an approximation to the equilibrium relationship when the transaction technology is Baumol–Tobin. In fact, if the transaction technology was Baumol–Tobin, $s_t = \eta \left( \frac{c_t}{m_t} \right)$, the money demand equation 10 would be,

$$\frac{m_t}{c_t} = \omega \left( \frac{A}{c_t} \right) \left( i_t - i^m_t \right)^{-\frac{1}{2}},$$

where $\omega = \eta^{\frac{1}{2}}$. The approximation amounts to ignoring the term $\left( \frac{A}{c_t} \right)^{\frac{1}{2}}$. 

APPENDIX 2: DATA USED IN FIGURES AND REGRESSIONS

The following is a list of data used in the figures and regressions for this article. Unless explicitly specified, all monetary aggregates are in billion of dollars and are not seasonally adjusted annual data (we take the December value of each year as the entire year’s value).¹

**M1**

**M2**

**M3**

**MZM**

**Other checkable deposits** (quarterly frequency)
FRB data available through Haver Analytics (FMOTN in USECON).

**MMMFs** (quarterly frequency)
FRB data available through Haver Analytics (FMGMN in USECON).

**Institutional money market mutual funds** (quarterly frequency)
FRB data available through Haver Analytics (FMIOMN in USECON).

**Transaction deposits swept into MMDAs** (Cumulative)
FRB data available through Haver Analytics (FMSWEEP in USECON).

**Demand deposits**
FRB data available through Haver Analytics (FMDN in USECON).

**Price deflator**
1929–2003 (2000 = 100): BEA data available through Haver Analytics (DAGDP in USECON or USNA).²

**Real GDP**
1900–28 (millions of 1929 dollars), Kendrick (1961, Table A-III).
1929–2003 (in chained 2000 dollars), BEA data available through Haver Analytics (GDPHA in USECON or USNA).³

**Nominal interest rate**
1900–69: Friedman and Schwartz (1982, table 4.8, column 6), defined as short-term commercial paper rate.
1970–2003: three-month commercial paper, FRB data available through Haver Analytics, FFP3 in USECON.

**Opportunity cost**
1900–79: M1’s opportunity cost is defined as the nominal interest rate for this period.
Three-month T-bill rate: FRB data available through Haver Analytics (FTBS3 in USECON).

¹We follow Lucas (2000).
²These two series overlap in 1929 and using the ratio of the two series’ values in 1929, we construct a new price deflator that goes from 1900 to 2003, with 2000 = 1.0.
³From these two series, we construct a new real GDP in 2000 dollars using the new price deflator (2000 = 1.0).
REFERENCES


Poole, William, 1991, testimony before the U.S. Congress, Committee on Banking, Finance, and Urban Affairs, subcommittee on Domestic Monetary Policy.
