Robert Barsky and Theodore Bogusz

Introduction and summary

Economic commentators often assert that major asset price booms and busts are closely associated with variations in the terms of borrowing to fund risky asset purchases. One important narrative focuses on changes in borrowing costs arising from variation in the riskless interest rate, which may result from a variety of causes-notably central bank policy actions in the short run and changes in world saving and associated capital flows over a longer horizon. For example, Allen and Gale (2000) contend that so-called bubble episodes typically begin with "financial liberalization or a conscious decision by the central bank to increase lending." After a period of perhaps several years, these authors continue, the central bank's policy stance tightens, interest rates rise, and the bubble collapses.¹ Greenspan (2010) also notes a connection between interest rates and asset prices but stresses the role of global savings patterns and other determinants of long-term rates rather than the short-term policy rates that are most closely connected to the actions of central bankers.²

What does economic theory have to say about the extent to which exogenous changes in short-term and/or long-term riskless rates ought to affect asset prices, and by what channels? In this article, we examine the implications of three key theoretical models of asset booms and busts, focusing on a variety of channels through which interest rates might affect real asset prices.

After providing a bit of empirical motivation via a brief look at data from Japan's stock and land price boom and bust of 1985–91, we study implications of the central model of traditional asset pricing, in which price is simply expected discounted future dividends. Here the focus is on the way in which the riskless interest rate affects the fundamental value of assets. Leverage does not play a central role because of the celebrated Modigliani–Miller theorem, which says that the total value of titles to an asset's payoffs is independent of how they are divided into debt and equity claims. Using first the simple Gordon formula, and then Campbell and Shiller's log-linearized dynamic Gordon model, we derive quantitative implications for the effects of innovations in the short-term rate on an asset that could be thought of as land or the stock of an unlevered firm.

The key result of this perfect markets model is that the extent to which increases in the riskless interest rate lower fundamental asset values is an increasing function of the persistence of short-term interest rates and a decreasing function of the risk premium. This observation has important implications for debates over whether or not central banks are likely to cause

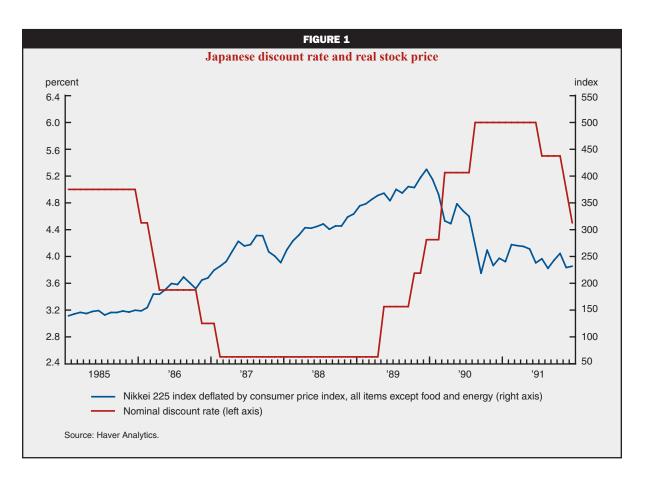
Robert Barsky is a senior economist and research advisor and Theodore Bogusz is a senior associate economist in the Economic Research Department of the Federal Reserve Bank of Chicago. © 2014 Federal Reserve Bank of Chicago

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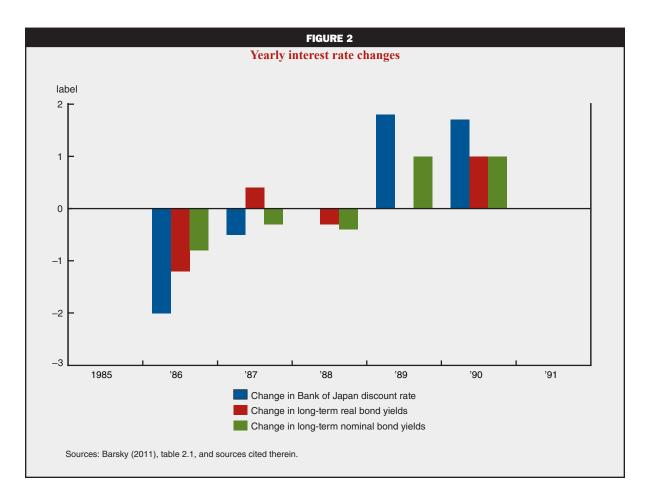
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large cycles in real asset prices by varying policy rates. In order to have such effects in this standard model, central bankers must be able to create highly persistent changes in real policy rates—put differently, they must exert major effects on real *long-term* interest rates. Monetary theory, however, suggests that the ability of central banks to effect permanent changes in real rates is limited (Shiller, 1980).³ These two observations in combination suggest that central banks also have limited ability to create booms and busts in real asset markets.

While traditional asset-pricing theory focuses on fundamentals—discounted future real cash flows there are alternative theories in which market imperfections play an important role. One such model, originating with Allen and Gale (2000) and developed more fully in Barlevy (2014), is based on the idea that "speculators" borrow (without sufficient collateral) from "banks" in order to buy risky assets. Banks are unable to differentiate between "entrepreneurs" that are safe to lend to and speculators that default if the payoff from the asset is disappointing. In this model, the price is pushed above its (social) fundamental value because of the default option. We introduce a borrowing limit (a "haircut" in finance terminology) and show that its magnitude impacts the size of the bubble. In the one-period model we sketch explicitly, the interest rate affects both the fundamental value and the size of the bubble, but the channel for the latter is essentially the same as that for the fundamental value.

In the Allen-Gale-Barlevy model-unlike the fundamental valuation model-leverage is absolutely essential; there is no Modigliani-Miller theorem. Further, the model's indispensable constraint on short sales makes it a model of "limits to arbitrage," in the sense of Shleifer and Vishny (1997). We consider another model in this class-the "natural buyers" model, based in our case (as in Miller, 1977; Geanakoplos, 2010; and Simsek, 2013) on heterogeneous beliefs, the natural buyers being those most optimistic about the dividend payout of the asset. In general, the natural buyers borrow in order to leverage their purchases of the asset. Increasing the effective demands of the natural buyers raises the equilibrium price, an effect that may be limited by high interest rates and possible credit constraints. This results in an asset price that is determined by a combination of beliefs, interest rates, and borrowing limits. In particular, we use a simple model from Barsky and Bogusz (2013) to illustrate the channels through which the interest rate can affect the asset price in this sort of model. The significant new interest



rate channel that this sort of model adds to the mix is what might be called an "affordability" effect. Higher borrowing costs may make it impossible for collateralconstrained natural buyers to fully roll over loans used to buy the asset, and the resulting drop in "cash in the market" necessitates a lower level of the asset price.⁴

A key question is whether the models incorporating limits to arbitrage might produce larger effects of temporary interest rate changes on asset prices than are seen in the perfect markets model. Though we by no means rule out that possibility, the simple examples that we construct do not have this property. In the imperfect markets models we present, the effects of interest rates on asset prices never exceed the effects on fundamental value.

Empirical motivation: Japanese stock prices, 1985–90

Figure 1 is a time-series plot of the raw monthly data on the Bank of Japan's nominal discount rate and the Nikkei 225, perhaps the best-known index of Japanese stock prices, divided by the core consumer price index (all items, except food and energy). The

plot shows that the discount rate halved between late 1985 and early 1987. As the discount rate fell from 5 to 2.5, the Nikkei 225 increased by 50 percent. From March 1987 until April 1989, the interest rate remained constant at 2.5 percent, its lowest level prior to the deep recession that followed the Nikkei's collapse. During this period, the Nikkei 225 increased a further 50 percent. From one perspective, this suggests the possibility that the low interest rate environment was fueling an asset boom. An alternative perspective might emphasize the fact that stock prices continued to rise rapidly without a further lowering of the discount rates. For a period of several months in 1989, the interest rate and asset price rose together, an indication that the relationship between interest rates and asset prices is complex and inconsistent with unidimensional causality in either direction. As the discount rate rose by a further 1.75 percentage points between late 1989 and the middle of 1990, stock prices declined by more than a third. These rate increases in 1989 and 1990 correspond rather closely with the collapse of the Nikkei. In particular, we see from figure 1 that the sharp rise in the discount rate from 4.25 percent to 5.25 percent

in February 1990 corresponds to a drop of 14 log points in the real Nikkei that same month. This, compounded by subsequent drops in the Nikkei—as well as in various indexes of land and housing prices—earned Yasushi Mieno (rightly or wrongly) the distinction of being called the "governor who pricked Japan's bubble economy."⁵

From mid-1991 through the middle of the subsequent year, the discount rate fell steadily as the stock price continued to decline. Thus, figure 1 illustrates the Allen–Gale notion of a so-called bubble that begins in an environment of falling policy rates and ends in a period characterized by sharp rate increases, but it also indicates that the relationship between interest rates and real asset prices is complex and requires a theoretical framework to facilitate meaningful discussions regarding causality. We turn now to the question of what theories might be relevant and what they might teach us.

How do interest rates affect fundamentals?

As noted in the introduction, the long-term rate features heavily in discussions of interest rate effects on asset prices. While only the discount rate is under the direct control of the central bank, figure 2 illustrates via a bar graph the association in annual data between changes in the basic discount rate, nominal and real long-term rates, and the Nikkei. In 1986, the Bank of Japan (BOJ) cut its discount rate by 200 basis points. In 1987, the discount rate was reduced by another 50 basis points. The discount rate was increased sharply in 1989 and 1990. Figure 2 shows that BOJ alterations of the nominal discount rate were associated with changes in both nominal and real long-term bond yields. We now turn to a consideration of several models that might shed light on the effect of short- and longterm interest rates on asset prices.

The most orthodox account of the impact of interest rate changes on asset prices focuses on the effect on the *fundamental* value of the asset—the expectation of the asset's stream of future cash flows discounted by an appropriate discount factor that we will call ρ . For now, we assume this is constant over time, though we will relax this later. More precisely, if P_t is the *fundamental* real asset price at time t and D_{t+i} is the real dividend or service flow received by the asset holder at time t + i, then

$$P_t = E_t \sum_{i=1}^{\infty} \frac{D_{t+i}}{(1+\rho)^i}$$

Because of the uncertain nature of the cash flows from real assets, they should be discounted not at the riskless interest rate r, but at a higher rate that includes

a risk premium that we will call θ .⁶ Thus, the full discount rate ρ is the sum of the riskless interest rate *r* (which is the primary focus of this article) and the risk premium

$$\rho = r + \theta$$

It is both convenient and revealing to represent the present value formula heuristically in a compact form known universally as the Gordon formula,

$$\frac{D_t}{P_t} = \rho - g$$
, or in terms of the asset price,

$$P_t = \frac{D_t}{\rho - g} = \frac{D_t}{r + \theta - g}$$
.⁷ The Gordon formula is de-

rived when ρ and g are constant over time, but what we are interested in, of course, is the effect of *changes* in r. Differentiating the Gordon formula will give the correct answer for the effect of unanticipated and permanent changes in r. If, on the other hand, r follows a stochastic process other than a random walk, a somewhat more complicated approximate formula is required (see below).

Let us begin with the effect on P_t of a permanent change in the riskless rate r. We have

$$\frac{d\log(\mathbf{P}_t)}{dr_t} = -\frac{1}{r+\theta-g}.$$

This derivative becomes large as g gets close to $r + \theta$. Thus, the presence of the risk premium θ puts a damper on the extent to which a change in r can affect the fundamental asset price. Since θ has historically been quite large (Mehra and Prescott, 1985), the dampening effect due to the presence of the risk premium is quantitatively important, and it will substantially reduce the effect of changes in the interest rate.

So far, we have focused on permanent changes in *r*. When *r* follows a stochastic process that renders interest rate changes less than permanent, it remains true that the presence of the risk premium reduces the effect on asset prices of changes in the riskless rate. In addition, however, the effect on the fundamental value of a shock to *r* is further reduced relative to the case when *r* is shocked permanently.⁸ A way to see this is to work with the log-linear approximation to the Gordon formula that holds when required rates of return and dividend growth rates are nonconstant over time but follow stationary stochastic processes (the Campbell–Shiller "dynamic Gordon growth model;" see Campbell, Lo, and MacKinlay, 1997, pp. 260–267). The formula reads

$$p_t \cong \frac{k}{1-\gamma} + E_t \left[\sum_{j=0}^{\infty} \gamma^i [(1-\gamma)d_{t+1+j} - r_{t+1+j}] \right],$$

where $\gamma = \frac{1}{1 + \exp(\mu_{d-p})}$ and μ_{d-p} is the population

mean of the log dividend–price ratio. Here, the lowercase p_t refers to the log of the price. Because our concern is with the role of the interest rate, it will

prove useful to define
$$p_{rt} \equiv \sum_{j=0}^{\infty} \gamma^{i} r_{t+1+j}$$
, the part of

this expression for the log-linearized asset price that depends on current and expected future short-term interest rates.

Campbell, Lo, and MacKinlay (1997) offer as an example the special case in which $E_t[r_{t+1}] = \overline{r} + x_t$ with $x_t = \phi x_{t-1} + \xi_t$. In this instructive example, we have

$$p_{rt} = \frac{\overline{r}}{1 - \gamma} + \frac{x_t}{1 - \gamma \phi}.$$

This provides a satisfying framework in which to discuss the role of the persistence of interest rate disturbances. We see immediately that the effect of an interest rate shock on the fundamental price is increasing in the persistence parameter ϕ and the discount factor γ (in fact they appear completely symmetrical). Note that ϕ measures the persistence of innovations in the interest rate. As ϕ goes to one, we effectively replicate the permanent shocks to *r* associated with the simple Gordon formula shown at the beginning of this section.⁹

The intuition for the importance of interest rate persistence is rather straightforward. The asset is presumed to be long lived, yielding cash flows for many years to come. A highly persistent increase in the interest rate raises the rate at which even cash flows that are expected to arrive far in the future are discounted, reducing significantly the present value of the sum total of expected future cash flows. A transitory increase in the interest rate, on the other hand, affects only the present value of cash flows expected to arrive in the near future. How about the role of the risk premium? Since the discount factor γ is inversely related to the mean dividend price ratio (which in turn is increasing in the average risk premium), this can be regarded as the reappearance in the dynamic context of the earlier point that a large risk premium puts a damper on the effect of an interest rate innovation on the asset price. The intuitive reason is that a higher baseline required

return shortens the duration of the asset and reduces the importance of interest rates in the more-distant future.

How persistent would interest rate shocks have needed to be to rationalize the rise in the Nikkei between 1985 and 1986 entirely in terms of the drop in the discount rate? Translating the persistence parameter ϕ into the half-life of the interest rate response to an exogenous shock of the appropriate magnitude and loosely calibrating to the Japanese financial data from this period suggest that the required degree of persistence corresponds to a half-life of about 13 years. If the change in the interest rate came from a monetary policy shock, this half-life is far too large to be plausible. If instead of a shock to the policy rate we were dealing with a drop in interest rates due to long-term capital flows, 13 years might be a quite reasonable half-life. Thus it matters a great deal where interest rate shocks come from. Contrary to first impressions, in the perfect markets present value model with rational expectations monetary policy does not seem to be a good candidate for explaining the large swings in the Nikkei during this period.

Limits-to-arbitrage models with a nontrivial role for leverage

Risk-shifting models

In the previous section, we examined the effect of interest rates on fundamentals. Our analysis shows that large fluctuations in asset prices due to exogenous interest rate movements cannot be explained entirely through fundamentals. However, a number of asset-pricing models deviate from the standard Gordon model. One such model is the monetary bubble model we dismissed in the introduction. In this section, we will examine two asset-pricing models that may yield more dramatic results. In both the risk-shifting and heterogeneity models, the price depends not only on fundamentals, but also on the ability of agents to borrow funds. It seems plausible that interest rates may have a larger effect in these models.

We sketch here a simple one-period version of a model proposed by Allen and Gale (2000) and analyzed in a much richer context by Barlevy (2014). This variety of model arises from the moral hazard that is induced when some agents are able to buy risky assets largely with borrowed money and default in one or more lowpayout states, shifting the risk to lenders. In these models, there is typically a kind of "bubble," in the sense that the price of the risky asset rises above its social valuation due to the subsidy implicitly received by the borrower as a result of the option to default. One question we will ask is whether a sufficiently high riskless interest rate can "pop the bubble." Suppose that there is a good state that occurs with probability q, in which the asset pays a high liquidating dividend of H at the end of the period. With probability 1-q, a bad state occurs in which the asset pays nothing at all. In the benchmark model, the asset is purchased by agents that we will call "speculators," entirely with funds borrowed from agents that we will call "banks." Speculators pay back the loan in the good state and default in the bad one. Banks cannot identify speculators because they are pooled with a third set of agents that we (following Barlevy, 2014) call "entrepreneurs." Though they would not lend to recognizable speculators, in the pooling equilibrium these risk-neutral banks earn an expected return just high enough to compensate for the "riskless rate" r yielded by an outside activity.

The one element we add to the existing models is a borrowing limit measured in terms of a margin or "haircut" h, so that speculators can borrow only B = (1 - h) P per unit of the risky asset priced at P. The (social) fundamental value F of the asset is qD / (1 + r). With free entry to the pool of speculators, the price of the asset will be this fundamental value plus the expected value of the subsidy (1 - q) B. Thus we have the equilibrium condition

$$P = \frac{qD}{(1+r)} + (1-q)(1-h)P$$

The solution for the asset price, therefore, is

$$P = \frac{1}{1+r} \left[\frac{qD}{1-(1-q)(1-h)} \right]$$

Not surprisingly, in the case in which the entire asset purchase can be funded by borrowing (that is, when

$$h = 0$$
), this collapses to $P = \frac{D}{(1+r)}$. The speculator

is willing to pay up to the full present value of the dividend in the good state, because in the bad state the purchase price is effectively refunded. This captures the intuition in the popular description of moral-hazardfilled financial transactions as "heads I win, tails you lose."

Note that in this zero-haircut case, the level of

the bubble is $(1-q)\frac{D}{1+r}$, that is, the price $\frac{D}{(1+r)}$ minus the fundamental $\frac{qD}{1+r}$. Although one might

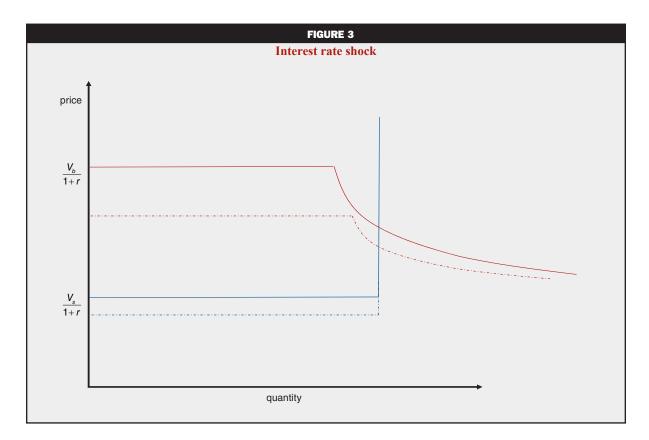
imagine that a sufficiently high interest rate would deter potential speculators and cause the price to revert to its fundamental level, this is not the case in the model described. These expressions show that a rise in the interest rate lowers the magnitude of the bubble component in exactly the same proportion that it lowers the fundamental value. However, there is another—somewhat unexpected—channel by which a rise in the interest rate can burst the bubble. Recall that the speculator is able to borrow only because in the context of a pooling equilibrium lenders cannot distinguish him from a productive entrepreneur. If the interest rate rises above the marginal efficiency of investment, the entrepreneurs will drop out of the loan market, revealing the identities of the speculators to the banks and leaving them unable to borrow.

Natural-buyer (heterogeneous beliefs) models

Another way to analyze the possible role of leverage in major asset price fluctuations is with a model in which agents have heterogeneous beliefs about fundamentals. Optimists face collateral constraints limiting their ability to borrow, and pessimists face short-sales constraints limiting their ability to sell the risky asset to the optimists. Here, we examine a simple and highly stylized one-period model of heterogeneous beliefs borrowed from Barsky and Bogusz (2013).

The model incorporates two types of agents, optimists and pessimists, and two goods, a final consumption good called "coconuts" and a coconut-yielding asset called "trees." Optimists and pessimists each receive an endowment of coconuts and trees. In equilibrium, optimists buy trees from the pessimists using coconuts, some of which are borrowed from the pessimists. At the end of the period, trees yield coconuts and agents consume. Optimists and pessimists differ in their beliefs about the number of coconuts that trees will yield at the end of the period, and pessimistswhich might be thought of as money market funds that cannot afford to "break the buck"-additionally have a particular concern with avoiding the downside risk associated with defaults on loans that they make. Aside from purchasing trees, agents can always store coconuts to earn a riskless return f. They can also take out loans at the interest rate r.

We define the following variables: S_i is agent *i*'s storage; b_i is the amount borrowed; q_i is the quantity of trees purchased (negative if the trees are sold); $E_i(Y)$ refers to agent *i*'s expected yield of trees at the end of the period; and T_i refers to agent *i*'s initial endowment of trees. The optimistic agents (*i* = 1), which might be thought of as hedge funds, are risk neutral and thus have expected utility $U_1 = s_1(1 + f) - b_1(1 + r) + E_1[Y](q_i + T_i)$. The "money market" agents are principally concerned with avoiding losses resulting from their lending and



will charge haircuts sufficient to render the loans riskless. Thus, in addition to the budget constraint and the nonnegative storage constraint faced by both kinds of agents, the borrower faces the collateral constraint

$$b_i \leq \frac{L_j(T_i + q_i)}{1 + r}$$
, where L_j is the worst state of the world

from the pessimists' point of view. Since even in the worst state of the world the lender is paid in full, loans are entirely riskless, and this ensures that the return on storage will be equal to the interest rate in equilibrium.

There are two channels through which movements in the interest rate affect asset prices in this model. Which channel the interest rate innovation operates through depends on what set of constraints are binding. The first way a rise in the interest rate affects the price is by reducing the fundamental value of both the optimists and the pessimists, since the perceived fundamental

can be written as $\frac{E_i(Y)}{1+r}$. This channel is relevant in

two different scenarios. The first is the case in which the short-sales constraint binds for the pessimists but the collateral constraint does not bind for the optimists. In this scenario, the price of the asset will be the optimists' fundamental value. The other scenario is when the short-sales constraint doesn't bind, but the collateral constraint does bind. In this scenario, the price of the asset will be the pessimists' fundamental value.

With the right constraints, however, interest rate movements can have rich effects in a heterogeneity model through an entirely different mechanism. When collateral constraints bind alongside short-sales constraints, a scenario we refer to as "cash in the market," the rise in the interest rate tightens the borrowing constraint shown above and reduces the *affordability* of the asset to the optimists. Since the optimists are able to borrow less than they were before the interest rate increase, the equilibrium asset price must fall. It follows that no matter what combinations of constraints are binding, a rise in the interest rate will reduce the asset price. Figure 3 illustrates the response equilibrium price to an exogenous interest rate shock.

Here, the red line corresponds to the optimists' demand curve and the blue line corresponds to the pessimists' supply curve. Note that the optimists can afford to pay their full valuation up to a certain quantity of the risky asset. This makes their demand curve horizontal. At a certain point, the collateral constraint begins to bind, and the optimists' demand curve is bent downward. The supply curve has a similar structure. As long as the pessimists are the marginal holders of the risky asset, the price will be their valuation and the supply curve is horizontal. However, once the

pessimists hit their short-selling constraint and no longer hold any of the risky asset, the trees can sell at a price above the pessimists' beliefs about the trees' fundamental value.

The dashed line corresponds to the supply and demand curves under a higher interest rate than before. Looking at the buyers' demand curve, we see there are two effects of the interest rate. First, it lowers

fundamentals, since the optimists' valuation is $\frac{E(Y)}{1+r}$.

However, there is an additional effect of increasing the interest rate, which is to lower the amount of lending the pessimists are willing to do.

Note that while leverage is a *sine qua non* for nontrivial results in Allen and Gale (2000) and Barlevy (2014), the natural-buyers model has content even in its absence. When the binding constraints are such that the equilibrium price is one of the agents' full valuation of the asset, there need not be any leverage at all. However, changes in the degree of leverage make the natural-buyers model a rich framework for studying large credit-induced movements in asset prices. Cash-in-the-market pricing is, of course, a result of leverage and the forces constraining it, and here the degree to which the optimist is able to lever himself has a crucial role to play.

In the cash-in-the-market region, we can also derive an expression for the effect (in logs) of a change in the interest rate on the asset price when the buyer is collateral constrained:

$$\frac{d\ln(p)}{dr} = -\frac{1}{(\frac{n(1+r)}{h}+1)(1+r)}$$

Here, $\frac{n}{b}$ is the ratio of risk capital to borrowing, which

in this model is the inverse of a measure of leverage. This says the effect of an interest rate change on price is increasing in leverage. Not surprisingly, the effect of interest rate movements on asset prices is larger when a greater fraction of the asset is paid for with borrowed funds.

Thus, the heterogeneity model is suggestive of two ways that exogenous interest rate movements might cause fluctuations in asset prices: one through fundamentals and another through the affordability channel. It is sometimes argued that the "fire sales" resulting from the sudden inability of natural buyers to roll over the loans needed to fund their positions can cause drops in asset prices that greatly exceed fundamentals, and Geanakoplos (2010) constructs a rather complex edifice with this property. In our oneperiod model, however, it is not hard to see that the effect of the interest rate on market price never exceeds its effect on the fundamental value as perceived by either the optimists or pessimists. For example, in the cash-in-the-market region, the effect on the market price approaches the fundamental effect only as the haircut approaches zero and the asset is funded entirely through borrowing.

Conclusion

We have discussed several models in which exogenous changes in the riskless interest rate, collateral constraints, or both have potentially major effects on real asset prices. We began with models in which credit constraints are absent and riskless interest rates impact asset prices through their effects on fundamental value. We found that in order to have large effects on fundamental real asset prices, large changes in the short-term interest rate have to be highly persistent. In addition, we showed that the presence of a substantial risk premium also puts a damper on the effects of interest rate changes on fundamentals.

We then moved to models with limits to arbitrage, in which the extent to which agents are able to purchase risky assets on margin is a key determinant of asset prices. The simple models we studied did not suggest that the asset price effects of interest rate changes are substantially larger in this context than in the classic model based on fundamentals. Geanakoplos (2010), in the elaborate fire-sales model mentioned earlier, is able to generate collapses in asset prices in excess of those justified by worsened fundamentals in response to certain sequences of bad news that necessitate deleveraging. It appears possible, but by no means certain, that some of the same considerations could rationalize large responses of asset prices to changes in interest rates. We are investigating this further in ongoing research.

NOTES

¹We use the term "bubble" as it appears in popular discourse without necessarily adopting the academic criterion that asset prices depart from fundamentals during these episodes.

²Similar arguments appear in a number of speeches by former Federal Reserve Board Chair Ben Bernanke, available on the website of the Board of Governors of the Federal Reserve System, www.federalreserve.gov.

³Shiller's paper is now 35 years old. Yet his "hypothesis 3" (that there is a policy effectiveness interval beyond which anticipated monetary policy cannot force real interest rates to depart from the natural rate) is fully consistent with current New Keynesian dynamic stochastic general equilibrium (DSGE) models. The length of this interval depends on the degree of price and wage rigidity and can be as short as a few months or as long as several years. See also Stanley Fischer's introduction to the volume in which Shiller's paper is found (Fischer, 1980).

⁴An additional alternative theory of asset pricing that warrants mention, but is not taken up further in this article, is the standard model of "rational bubbles." The simplest textbook version describes an asset with no fundamental value that trades at a positive price only because of the self-fulfilling rational expectation that the asset can be resold to someone else in the future. The only restriction on the asset price is that it must offer the market return by growing at the expected rate of interest. Thus, these models determine only the *growth rate* of the asset price, not its *level*, a point acknowledged but unfortunately underplayed in a prominent recent paper by Galí (2014). Galí argues—unconvincingly in our view that raising interest rates may well exacerbate rather than mitigate bubbles. Since a satisfactory resolution of this and other policy questions requires determining asset price *levels*, we do not discuss this model further.

⁵Mayumi Otsuma, 2012, "Mieno, governor who pricked Japan's bubble economy, dies," *Bloomberg*, April 18, available at www.bloomberg.com/news/2012-04-18/mieno-governor-who-pricked-japan-s-bubble-economy-dies-at-88.html.

⁶This premium, which compensates for *systematic* risk, is typically represented as a covariance between the asset's ex post return and the return on a broad index, such as the market portfolio or aggregate consumption.

Note that it is necessary that $\rho > g$, or the above present value will be infinite. There is an extensive technical literature about possible worlds in which this condition does not hold.

⁸The remainder of this paragraph, as well as the next one, draws heavily on Barsky's (2011) chapter on the Japanese bubble.

9To see this, note that

$$\frac{dp_{rl}}{dx_l} = \frac{1}{1 - \gamma} \approx \frac{1}{\exp(\mu_{d-p})} = \frac{1}{\rho - g} = -\frac{d\log(P_l)}{dr_l}$$

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